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A REMARK ON TWO-DIMENSIONAL FINITE AUTOMATA.(U)

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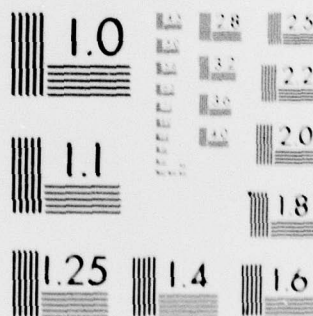
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A REMARK ON TWO-DIMENSIONAL
FINITE AUTOMATA

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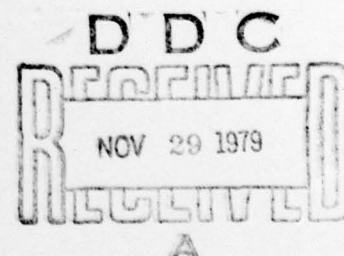
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ABSTRACT

Let $S2\text{-APMOTA}(m)$ be an area-preserving two-dimensional multipass on-line tessellation acceptor over square array input languages whose pass number is bounded by m . It is proved that an open problem "Is $L(2\text{-NA}) \subseteq L(S2\text{-AMPOTA}(1))$?" proposed in a previous paper by Inoue and the present author has a positive solution.

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In [1], we showed that the class of sets accepted by nondeterministic two-dimensional on-line tessellation acceptors properly contains that accepted by two-dimensional non-deterministic finite automata (i.e., $L(2-NA) \subsetneq L(2-OTA)$) and also that $L(2-DOTA)$ is incomparable with $L(2-NA)$ and $L(2-DA)$. This result (Theorem 4.1 in [1]) was the main theorem of [1]. Also we defined in [2] an area-preserving two-dimensional multipass on-line tessellation acceptor (i.e., 2-AMPOTA) and defined in [3] a 2-AMPOTA whose pass number is bounded by m (i.e., 2-AMPOTA(m)). Further, the 2-AMPOTA(1) over square array input languages was denoted by S2-APMOTA(1). In this notation, we proposed in [3] an open problem "Is $L(2-NA) \subsetneq L(S2-AMPOTA(1))$?".

In [1], the inputs were rectangular array languages. In this note, we consider square array languages. We prove that Theorem 4.1 in [1] is also valid for square array languages, and hence show that the open problem in [3] has a positive solution.

Let us consider a set $\Sigma = \{0,1,c,e,\emptyset\}$ of input symbols and also the input square array languages surrounded by the special boundary symbol #. In this note, we treat exclusively input languages such as shown in Fig. 1.

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#	#	#	#	.	.	#	#	#	#	#	#	.	.	#	#	#
#	a_{01}	a_{02}	.	.	.	a_{0n}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
#	c	c	c	c	c	c	c	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
.	c	c	c	c	c	c	c	c	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	.
.	c	c	c	c	c	c	c	c	c	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	.
#	c	c	c	c	c	c	c	c	c	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
#	a_{11}	e	a_{12}	e	a_{13}	e	.	.	.	e	a_{1n}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
#	$\emptyset$	a_{21}	e	a_{22}	e	a_{23}	e	.	.	.	e	a_{2n}	$\emptyset$	$\emptyset$	$\emptyset$	#
#	$\emptyset$	$\emptyset$	a_{31}	e	a_{32}	e	a_{33}				e	a_{3n}	$\emptyset$	$\emptyset$	$\emptyset$	#
#	$\emptyset$	$\emptyset$	$\emptyset$										$\emptyset$	$\emptyset$	$\emptyset$	#
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$											$\emptyset$	#
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	a_{l1}	e	a_{l2}	e					e	a_{ln}	#
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	c	c	c	#
.	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	c	c	.
.	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	c	.
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	#
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	#
#	#	#	#	.	.	#	#	#	#	#	#	.	.	#	#	#

Figure 1

In Fig. 1, a_{ij} is 0 or 1.

Now, we consider as a chunk a part of the form
as shown in Fig. 2:

#	#	#	#	.	.	#	#	#	#	#	#	#	.	.	#	#	#
#	a_{01}	a_{02}	.	.	.	a_{0n}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
#	c	c	c	c	c	c	c	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
.	c	c	c	c	c	c	c	c	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	.
.	c	c	c	c	c	c	c	c	c	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	.
#	c	c	c	c	c	c	c	c	c	c	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
#	a_{11}	e	a_{12}	e	a_{13}	e	.	.	.	e	a_{1n}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
#	$\emptyset$	a_{21}	e	a_{22}	e	a_{23}	e	.	.	.	e	a_{2n}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
#	$\emptyset$	$\emptyset$	a_{31}	e	a_{32}	e	a_{33}	.			e	a_{3n}	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	#
#	$\emptyset$	$\emptyset$	$\emptyset$												$\emptyset$	$\emptyset$	#
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$												$\emptyset$	#
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	a_{ℓ_1}	e	a_{ℓ_2}	e						e	a_{ℓ_n}	#
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	c	c	c	c	#
.	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	c	c	c	.
.	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	c	c	.
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	c	.
#	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	c	c	c	c	c	c	#
#	#	#	#	.	.	#	#	#	#	#	#	#	.	.	#	#	#

Figure 2

Let us denote the diagonal parts of the chunk by p_1 and p_2 respectively (Fig. 3):

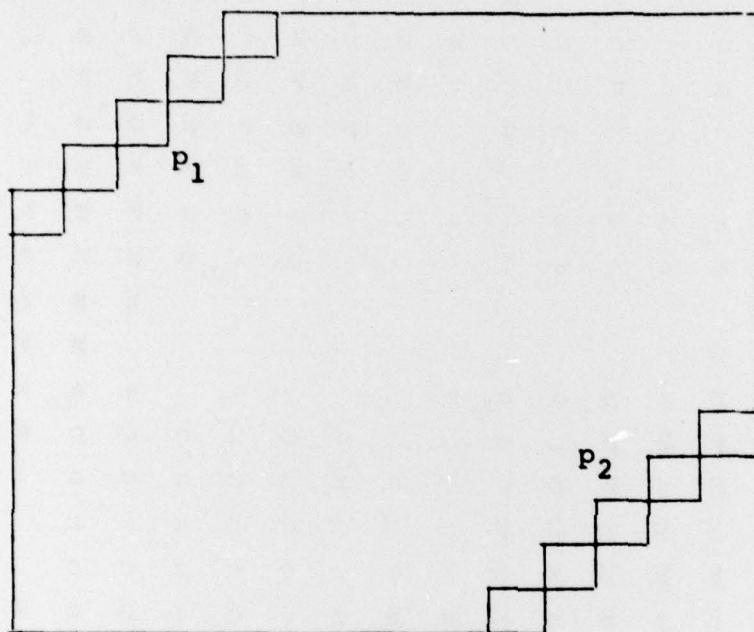
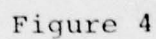


Figure 3

#	r_1	#
$\vdots$	$\vdots$	$\vdots$
#	r_2	#

This chunk plays the same role as the chunk in [1], and the diagonals p_1 and p_2 correspond to the rows r_1 and r_2 , respectively.

A chunk as in Fig. 4 is called an (ℓ, n) -chunk.



Let M be a 2-NA and x, y be any different (ℓ, n) -chunks. Then M -equivalence of x and y is defined in a similar way as in [1]. Note that M always enters or exits a chunk at the diagonals p_1 or p_2 .

Now, let us consider a set T of pictures such as shown in Fig. 1 in which there exists some i ($1 \leq i \leq l$) such that $a_{i1}a_{i2}\dots a_{in}$ is the same as the head $a_{01}a_{02}\dots a_{0n}$ of the top row.

Theorem 1. (1) $T \in L(2\text{-DOTA})$
 (2) $T \notin L(2\text{-NA})$.

Proof:

(1) is shown without difficulty.

(2) is provable as follows:

By the same considerations as in [1], there are at most $s = (2^{2(n+2)k+1}, 2^{(n+2)k})$ M -equivalence classes of $(2^n, n)$ -chunks, where k is the number of states of M . We denote those classes by $C_1, C_2, \dots, C_s$.

There are 2^n different strings over $\{0,1\}$ of length n . Let us denote those strings by $R_1, R_2, \dots, R_{2^n}$. Here, we distinguish chunks depending on the appearances of these rows. For example, $[R_1]$ means a chunk in which only rows corresponding to R_1 appear (see Fig. 5).

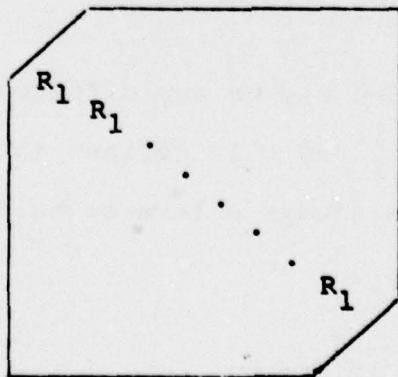


Figure 5

$[R_1, R_2]$ means a chunk in which only rows corresponding to R_1 and R_2 appear (see Fig. 6).

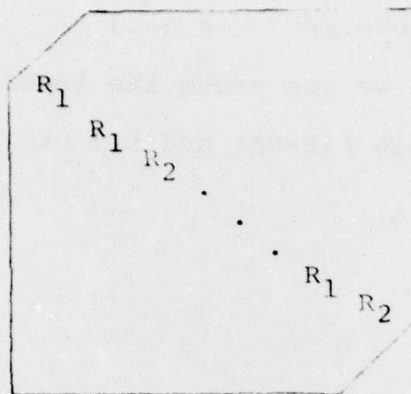


Figure 6

According to this characterization we know that there are v different languages for $(2^n, n)$ -chunks, where

$$v = {}_{2^n}C_1 + {}_{2^n}C_2 + \dots + {}_{2^n}C_{2^n} = 2^{2^n}.$$

Since $v > s$ for large n , we know the following fact:

There exists some R_j such that R_j appears in some chunk C_ρ but does not appear in some chunk C_σ ($\rho \neq \sigma$) where C_ρ is M -equivalent to C_σ . Thus, we can prove that $T \in L(2-NA)$ implies a contradiction by making use of the definition of M -equivalence of chunks. Therefore, we get (2). //

Theorem 2. $L(2-NA) \subsetneq L(S2-AMPOTA(1))$.

Proof:

$L(S2-AMPOTA(1))$ is the same as $L(2-OTA)$ over the square array.

Thus, it is provable in the same way as in [1] that an $S2-AMPOTA(1)$ can simulate a 2-NA. Therefore, from Theorem 1 this theorem follows.//

From Theorem 1, we can prove the incomparability of $L(S2-DAMPOTA(1))$ with $L(2-NA)$ and $L(2-DA)$ by the same method as in [1].

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- [1] K. Inoue and A. Nakamura: Some properties of two-dimensional on-line tessellation acceptors, Information Sciences 13, 1977, 95-121.
- [2] K. Inoue and A. Nakamura: Two-dimensional multipass on-line tessellation acceptors, to appear in Information and Control; also TR-628, Computer Science Center, University of Maryland, College Park, MD, January, 1978.
- [3] K. Inoue, I. Takanami and A. Nakamura: Pass-bounded two-dimensional multipass on-line tessellation acceptors, Trans. IECE 61-D, 10, 1978, 735-742 (in Japanese).

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