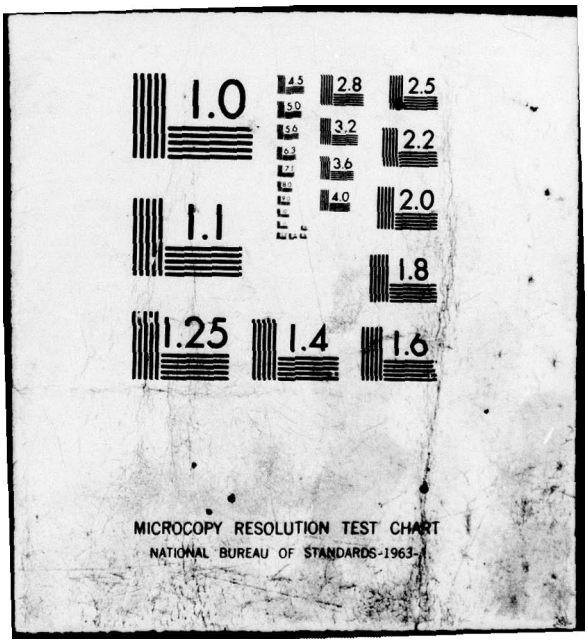
AD-A077 450 NAVAL RESEARCH LAB WASHINGTON DC F/G 9/5
THE INSTABILITY BOUNDARY AND GROWTH RATE CONTOURS OF THE GYROTR--ETC(U)
NOV 79 J Y CHOE , S AHN
UNCLASSIFIED NRL-MR-4106 NL

1 OF 1
AD
A077450

END
DATE
FILMED
1-80
DDC


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0
5.6
6.3
7.1
8.0
MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

NRL Memorandum Report 4106

The Instability Boundary and Growth Rate Contours of the Gyrotron Amplifier with Spreads in Energy and Momenta

JOON Y. CHOE AND SAEYOUNG AHN

Microwave Tube Staff
Electronics Technology Division

November 14, 1979

NAVAL RESEARCH LABORATORY
Washington, D.C.

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: NRL Memorandum Report 4106
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle): THE INSTABILITY BOUNDARY AND GROWTH RATE CONTOURS OF THE GYROTRON AMPLIFIER WITH SPREADS IN ENERGY AND MOMENTA
5. TYPE OF REPORT & PERIOD COVERED: Interim report on a continuing NRL problem.
6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s): Joon Y. Choe and Saeyoung Ahn
8. CONTRACT OR GRANT NUMBER(s): Memorandum rept.

9. PERFORMING ORGANIZATION NAME AND ADDRESS: Naval Research Laboratory, Washington, DC 20375
10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NRL Problem R18-35

11. CONTROLLING OFFICE NAME AND ADDRESS
12. REPORT DATE: November 14, 1979
13. NUMBER OF PAGES: 12

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
15. SECURITY CLASS. (of this report): UNCLASSIFIED
15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES: NRL-MR-4106

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Gyrotron
Instability boundary
Dispersion relation
Stabilization
Spreads in Energy and Momenta
Growth rate contours
Synchronization

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The combined effects of the spreads in energy and axial momentum of annular beam electrons in a gyrotron amplifier are spectrally investigated. A conveniently normalized dispersion relation is found for TE modes, and analyzed to obtain the instability boundary and growth rate contours. The spread effects prove to reduce the growth rate near the beam-waveguide synchronous mode, and enhance it for the mode out of synchronization. In particular the growth rate at the synchronous mode tends to peak when the electron distribution is thermally isotropic.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

251 950

THE INSTABILITY BOUNDARY AND GROWTH RATE CONTOURS OF THE GYROTRON AMPLIFIER WITH SPREADS IN ENERGY AND MOMENTA

The mechanism of the electron cyclotron maser instability[1-3] has been used successfully in the gyrotron oscillator[4] which provides a powerful source of millimeter and submillimeter microwave generation. Many reviews on the developments of theories and experiments of the gyrotron already exist in the literature.[5-6]

Recently the bandwidth and linear growth rate of a gyrotron amplifier has been investigated in theories[7-9] and experiments.[10,11] The measured linear gain of a gyrotron amplifier lies in the predicted range of the growth rate. Its poor efficiency[10] however was attributed to the velocity spread effect, i.e., the spreads in momentum and energy. It is our objective to investigate the effects of the spreads on the linear growth rates over a wide range of the mode spectra.

We consider a gyrotron amplifier which consists of the metallic cylindrical waveguide with radius R, and the annular tenuous electron beam under the uniform axial magnetic field B_o inside the waveguide. We choose the equilibrium distribution function of beam electrons f_o, given in terms of the total relativistic mass factor γ, the axial linear momentum p_z, and the azimuthal canonical angular momemtun P_θ, to be a rectangular type function in γ, p_z and P_θ with the mean values given by $\hat{\gamma}$, $\hat{p}_z$ and $\hat{P}_\theta$, and their spreads by

Note: Manuscript submitted September 11, 1979.

$\hat{\gamma}\varepsilon_\gamma$, $\hat{p}_z\varepsilon_z$ and $\hat{P}_\theta\varepsilon_\theta$(Fig. 1). The linear dispersion relation for a $TE_{\ell ns}$ mode is then obtained for the eigenfrequency ω at the azimuthal mode number ℓ, the radial mode number n, and the magnetic harmonic number s, as a function of the axial wave number k.

In order to be managable we assume that the spreads are not large such that only their effects on the resonance terms are kept,[12] and further assume that the growth rate is small, i.e., $|\omega_i| << \omega_c/\hat{\gamma}$, where $\omega_c \equiv |e| B_o/mc$. Then the dispersion equation is reduced to the equation*

$$\Delta + a + \eta b F(\Delta, \varepsilon_\gamma, \varepsilon_z) = 0, \qquad (1)$$

where $\Delta \equiv \omega - \Omega_s$ represents the eigenfrequency shift from the Doppler-shifted cyclotron frequency of beam electrons Ω_s, defined by $\Omega_s \equiv kc\beta_z + s\omega_c/\hat{\gamma}$; $a \equiv (\Omega_s^2 - \omega_{\ell n}^2)/2\Omega_s$ refers to the degree of the synchronization between the beam mode Ω_s and the waveguide mode $\omega_{\ell n}$ defined by $\omega_{\ell n}^2 = c^2k^2 + \omega_{co}^2$ with ω_{co} as the cutoff frequency; $b \equiv (\Omega_s^2 - c^2k^2)/2\Omega_s$ measures the available energy; and η is the beam-waveguide coupling coefficient.[13] Here $c\beta_z$ is the average axial velocity of electrons. The small growth rate assumption ($|\Delta| << \omega_c/\hat{\gamma}$) favors the instability term (proportional to Δ^{-2}) over the stabilizing terms (proportional to Δ^{-1} or Δ^{o}) in the dispersion equation. Consequently equation (1) slightly overestimates the actual growth rate. The information on the influence of the spreads are contained in function F, defined by

*Equation (1) was first obtained by Uhm and Davidson (i.e., see Eq. 29 of the paper; Phys. Fluids 22, September 1979) for the TE_{on} mode. Here we generalized the case to the azimuthally asymmetric $TE_{\ell n}$ mode. The analysis is carried out in this report such as to keep the combined effect of both spreads maintained for the arbitrary axial wave number k.

$$F \equiv \frac{1}{4\varepsilon_1\varepsilon_2} \ln\left[\frac{\Delta^2-(\varepsilon_1-\varepsilon_2)^2}{\Delta^2-(\varepsilon_1+\varepsilon_2)^2}\right] \tag{2}$$

where $\varepsilon_1 \equiv |\Omega_s|\varepsilon_\gamma$ and $\varepsilon_2 \equiv |kc\beta_z|\ \varepsilon_z$. The function F is interchangeable with respect to ε_1 and ε_2, and an even function of ε_1 and ε_2; $F(\Delta, \varepsilon_\gamma = \varepsilon_z = 0) = \Delta^{-2}$ recovers the dispersion relation without spread.[13] Since the effects of ε_γ and ε_z are weighted by $|\Omega_s|$ and $|kc\beta_z|$, we find that the influence of the energy spread is dominant over that of the axial momentum for small $|k|$, while for large $|k|$ the reverse is true. In usual gyrotron devices, the wavenumber k is commonly chosen such that ε_1 and ε_2 are in the same order of magnitude.

The growth rate, viz. the imaginary part of Δ, depends on several experimental parameters: V (beam voltage), I (beam current), $\beta_\perp/\beta_z$ (ratio of the velocity in perpendicular to axial direction), r_o (beam center location), s, ℓ n, R and k in addition to ε_γ and ε_z. By introducing a normalization equation

$$Q \equiv (\eta b)^{1/3}\ \tilde{Q}, \tag{3}$$

the dispersion equation (1) becomes

$$\tilde{\Delta}+\tilde{a}+F\ (\tilde{\Delta},\ \tilde{\varepsilon}_1,\ \tilde{\varepsilon}_2) = 0 \tag{4}$$

The advantage of normalization (3) becomes apparent when we note that the reduced dispersion equation (4) now becomes a function of mere dimensionless numbers $\tilde{\Delta}$, $\tilde{a}$, $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ and that the experimental parameters are absorbed in normalizing factor $(\eta b)^{1/3}$. The scaling factor $(\eta b)^{1/3}$, $\tilde{a}$, $\tilde{\varepsilon}_1/\varepsilon_\gamma$, and

$\tilde{\varepsilon}_2/\varepsilon_z$ as functions of the wave number k are illustrated in Fig. 2. It shows a TE_{011} mode with V = 71.5 kV, I = 9.5A, $\beta_\perp/\beta_z = 1.5$, and r_o and R optimized to give highest growth rates.[10] Note that $\tilde{a} \gtrless 0$ for $\Omega_s \gtrless \omega_{\ell n}$ and the upper limit of $\tilde{a}$ is determined by the wall radius R. We also note that the scaling factor $(\eta b)^{1/3}$ is in the order of 10^{-2} and remains relatively unchanged over k except for the stable regions at the edges in Fig. 2. The weighting factor for ε_γ is much larger ($\sim$40) than that for ε_z ($\sim$3). Therefore, in order to have equivalent effect, only a fraction of the momentum spread ($\varepsilon_\gamma \leq 0.1\ \varepsilon_z$) is sufficient for the energy spread.

Examination of the dispersion equation (4) yields that there exists an upper limit of the growth rate, given by $|\tilde{\Delta}_i| \lesssim \min\{3^{\frac{1}{2}}/2,\ \pi/4\tilde{\varepsilon}_1\tilde{\varepsilon}_2\}$. The absolute maximum growth rate $\tilde{\Delta}_i = 3^{\frac{1}{2}}/2$ occurs for the zero temperatured ($\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2 = 0$) electron beam at the beam-waveguide synchronous ($\Omega_s = \omega_{\ell n}$) mode ($\tilde{a} = 0$). This shows that the rate of energy extraction from beam electrons is highest when the beam and the waveguide modes are synchronous. The possible maximum gain reduces as the spread ($\tilde{\varepsilon}_1$ or $\tilde{\varepsilon}_2$) increases. Equation (4) also shows that the instability region is bounded in a $\tilde{a}$-space by $\tilde{a}_+ > \tilde{a} > \tilde{a}_-$. The instability boundaries $\tilde{a}_+$ and $\tilde{a}_-$ are given in Fig. 3. The lower bound $\tilde{a}_-$ is a decreasing function of $\tilde{\varepsilon}_2$(or $\tilde{\varepsilon}_1$), while the upper bound $\tilde{a}_+$ is a decreasing function of $|\tilde{\varepsilon}_1 - \tilde{\varepsilon}_2|$ with its maximum at $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2$. Since the upper limit of $\tilde{a}$ is experimentally imposed by wall radius R (see Fig. 2) unless the spreads are very large, in which case it is determined by $\tilde{a}_+$ it is found that for small $\tilde{\varepsilon}$ the unstable range in k is determined by $\tilde{a}_-$. Therefore we conclude that the unstable bandwidth in the wave number k becomes broader as the spreads increase.

Finally we proceed to find the eigenvalue $\tilde{\Delta}$ by numerically solving eq. (4) for given $\tilde{a}$, $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. The results are shown in Fig. 4. The growth rates contours are drawn in $\tilde{\varepsilon}_1$-$\tilde{\varepsilon}_2$ space for four representative values of $\tilde{a}$ (1,0,-1,-2). The instability region in spread space becomes broader as $\tilde{a}$ decreases due to $\tilde{a}_+$ curve (see Fig. 3), but as $\tilde{a}$ further decreases (e.g., $\tilde{a} = -2$), a new stable region appears near $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2 = 0$ due to $\tilde{a}_-$ curve. The maximum growth rate in spread space occurs at $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2 = 0$ in general, but as $\tilde{a}$ decreases, the point, marked by a filled square, moves along $\tilde{\varepsilon}_1 = 0$ (or $\tilde{\varepsilon}_2 = 0$) line (e.g., $\tilde{a} = -2$). This phenomenon is readily explained when we note that at out of synchronization mode ($\tilde{a} \neq 0$) the spreads provide a fraction of electrons that are synchronous, while at the synchronous mode ($\tilde{a} = 0$) the spreads reduce the number of synchronous electrons. Gyrotron amplifiers are usually operated near $\tilde{a} = 0$. With parameters given in Fig. 2, the minimum spreads needed to stabilize the instability are found to be 3.4% and 49.2% for the energy and the momentum at the synchronous case. Near the beam-waveguide synchronous mode (i.e., $\Omega_s \sim \omega_{\ell n}$) the instability is stabilized with the spreads for low $\tilde{\varepsilon}$, while the growth rate is maximum at $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2$ for high $\tilde{\varepsilon}$. The reverse is true for an out-of synchronization mode (i.e., $|\Omega_s - \omega_{\ell n}| >> 0$). The opposite behavior of the growth rate on the spreads at high $\tilde{\varepsilon}$ may be significant if the spreads increase during the interaction stage.

We note that at the synchronous mode the growth rate becomes highest along the $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2$ line in Figs. 4 (the case $\tilde{a} = 0$). At the synchronous condition where the conducting wall radius R is adjusted to have the beam mode Ω_s curve graze the waveguide $\omega_{\ell n}$ curve, the line $\tilde{\varepsilon}_1 = \tilde{\varepsilon}_2$ corresponds that $\beta_\perp^2 \varepsilon_\perp = 2\beta_z^2 \varepsilon_z$, where $\varepsilon_\perp$ refers to the spread in the transverse momentum. This means that

the thermal temperature of the beam electrons is isotropic. In other words, at the synchronous wave number the amplifier becomes most efficient when the beam distribution is isotropic.

It is worthwhile to emphasize that the reduced dispersion equation (4) coupled with the normalization equation (3) describes the entire spectrum of mode s, ℓ, n, and k. Once the mode is identified, eq. (3) determines the numerical values of $\tilde{a}$, $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ to find the growth rate $\tilde{\Delta}_i$ via eq. (4), which in turn is transformed to the actual Δ_i through eq. (3). Therefore, the mode spectral analysis is reduced to determination of the scaling factor eq. (3).

The authors would like to acknowledge the benefit of the useful discussions with H. Uhm.

References

1. R. Q. Twiss, Aus. J. Phys. 11, 564 (1958).

2. J. Schneider, Phys. Rev. Letts. 2, 504 (1959).

3. A. V. Gaponov, Izv. VUZ. Radiofizika 2, 450 (1959) and Addendum, ibid, p. 837 (1959).

4. P. Sprangle and A. T. Drobot, IEEE Trans. MTT-25, 528 (1977); K. R. Chu, NRL Memo Rep. 3672 (1977); M. Read and R. Lucey, JAYCOR Rep. 2061, NRL Contract No. N00173-77-C-0215 (1978).

5. A. V. Gaponov, M.I. Petelin and V. K. Yulpatov, Radio Phys. Quant. Elect. 10, 9, 794 (1967).

6. J. L. Hirshfield and V. L. Granatstein, IEEE Trans. MTT-25, 522 (1977); See also V. A. Flyagin, A. V. Gaponov, M. I. Petelin and V. K. Yulpatov, ibid, No. 6, p. 514 (1977)

7. K. R. Chu, Phys. Fluids 21, 2354 (1978).

8. H. Uhm, R. C. Davidson and K. R. Chu, Phys. Fluids 21, 1866 (1978).

9. J. Y. Choe and S. Ahn, NRL Memo Rep. 4041 (1979).

10. V. L. Granatstein, L. Seftor, L. Barnett, M. Read, K. R. Chu and P. Sprangle, IEEE Int'l. Conf. on Plasma Sci., June 4-6 (1979).

11. H. R. Jory, S. J. Evans, S. J. Hegji, J. F. Shively, R. S. Symons and N. J. Taylor, IEEE Int'l. Conf. on Plasma Sci., June 4-6 (1979).

12. We note that the growth rate is dominantly governed by the beam resonance term (Δ in eq. (1)), and as can be seen in the expression for Δ ($\equiv \omega - \Omega_s$), it is independent of P_θ. Therefore, the effect of the spread in the canonical angular momentum is much smaller than those in the energy or axial momentum.

13. J. Y. Choe and S. Ahn, NRL Memo Rep. 4035 (1979).

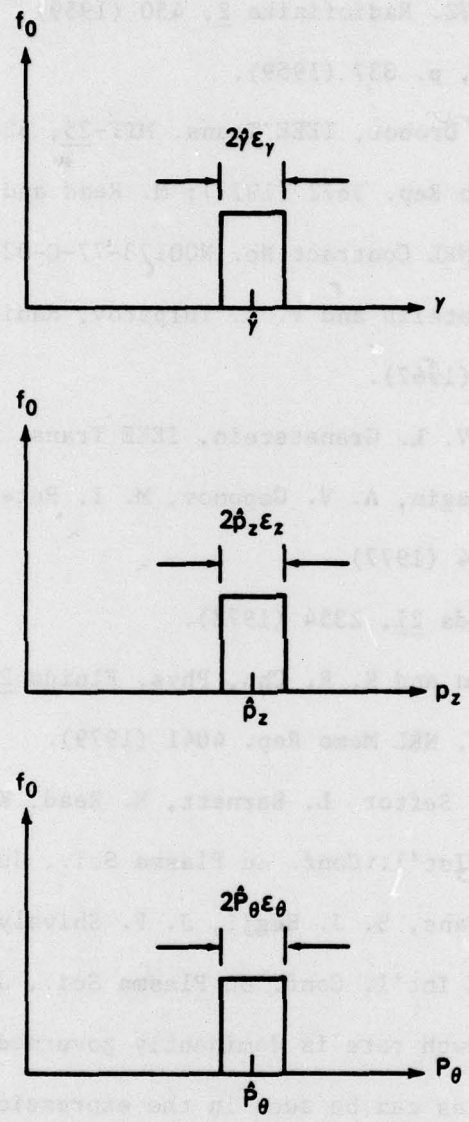


Fig. 1 - Equilibrium distribution function of beam electrons

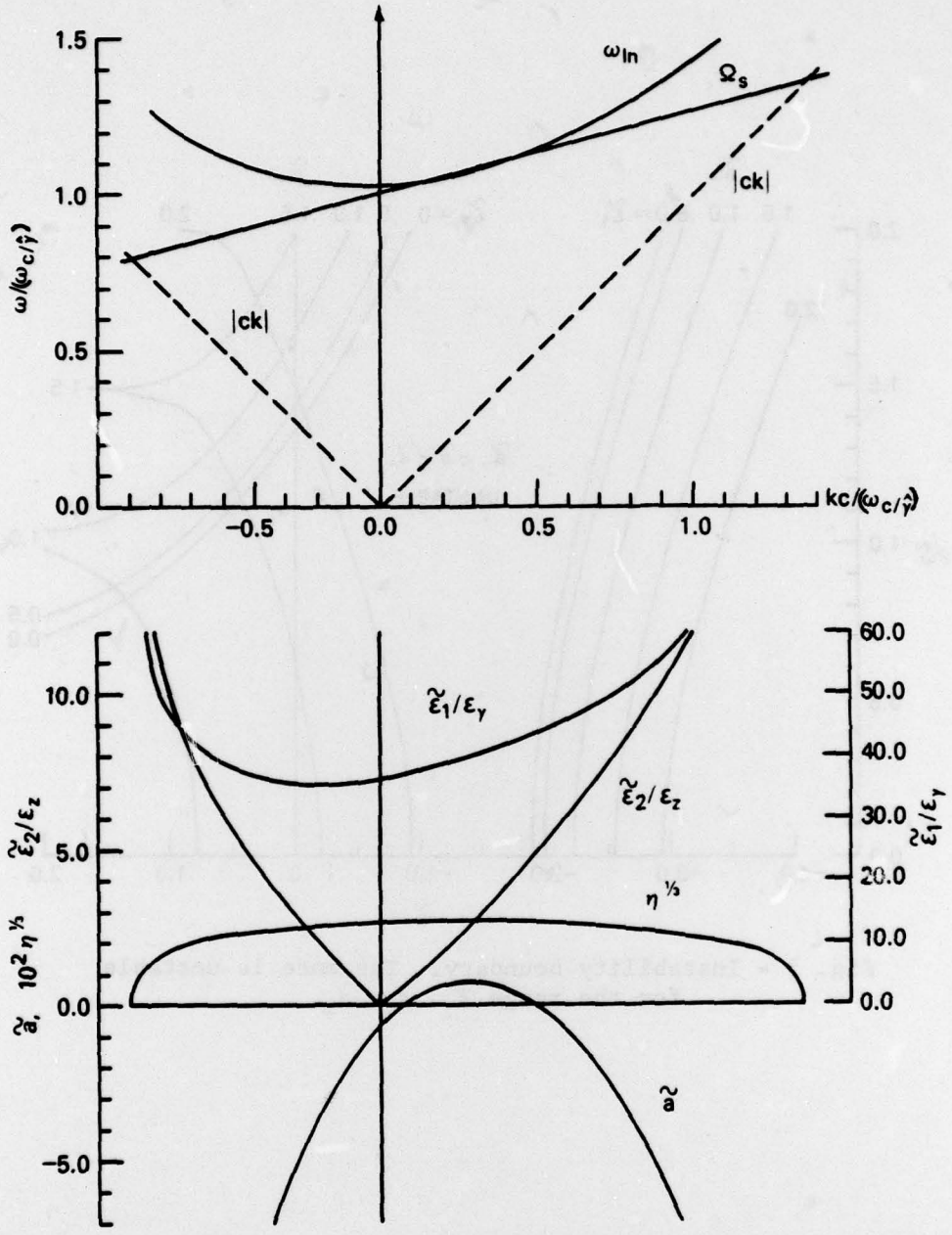


Fig. 2 - Doppler shifted beam mode (Ω_s) and the waveguide mode ($\omega_{\ell n}$) vs. the axial wavenumber k, and the corresponding normalization factor $(\eta b)^{1/3}$, $\tilde{\varepsilon}_1/\varepsilon_\gamma$, $\tilde{\varepsilon}_2/\varepsilon_z$ and the normalized $\tilde{a}$ vs. k. For actual parameters see text.

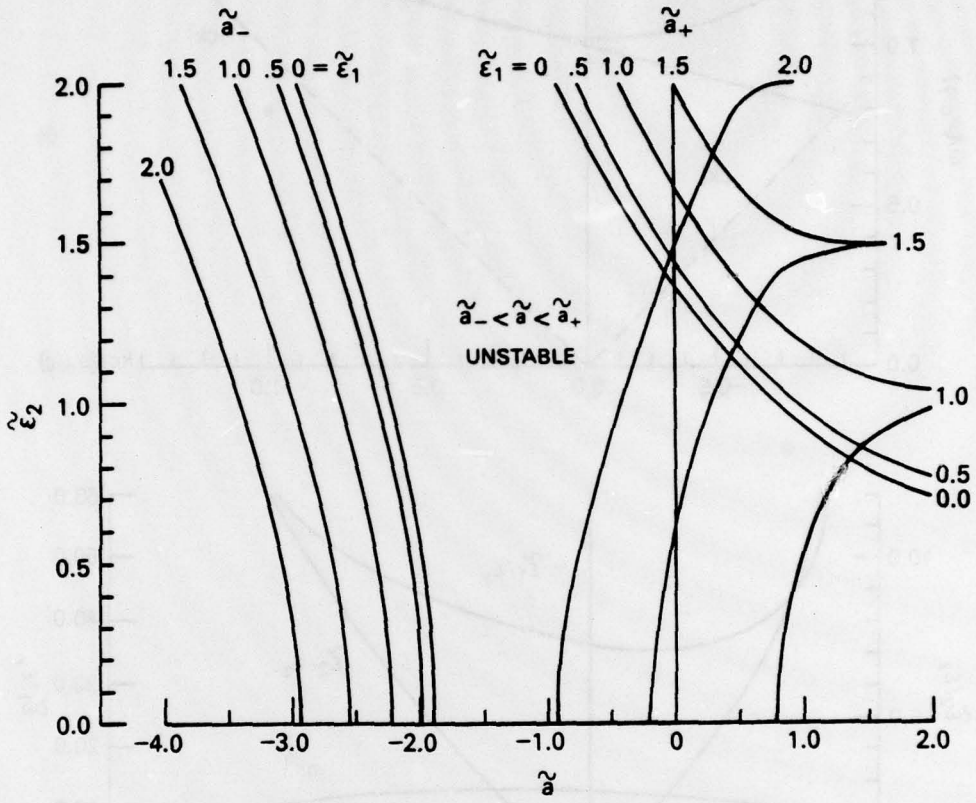


Fig. 3 - Instability boundary. The mode is unstable for the range $\tilde{a}_- < \tilde{a} < \tilde{a}_+$.

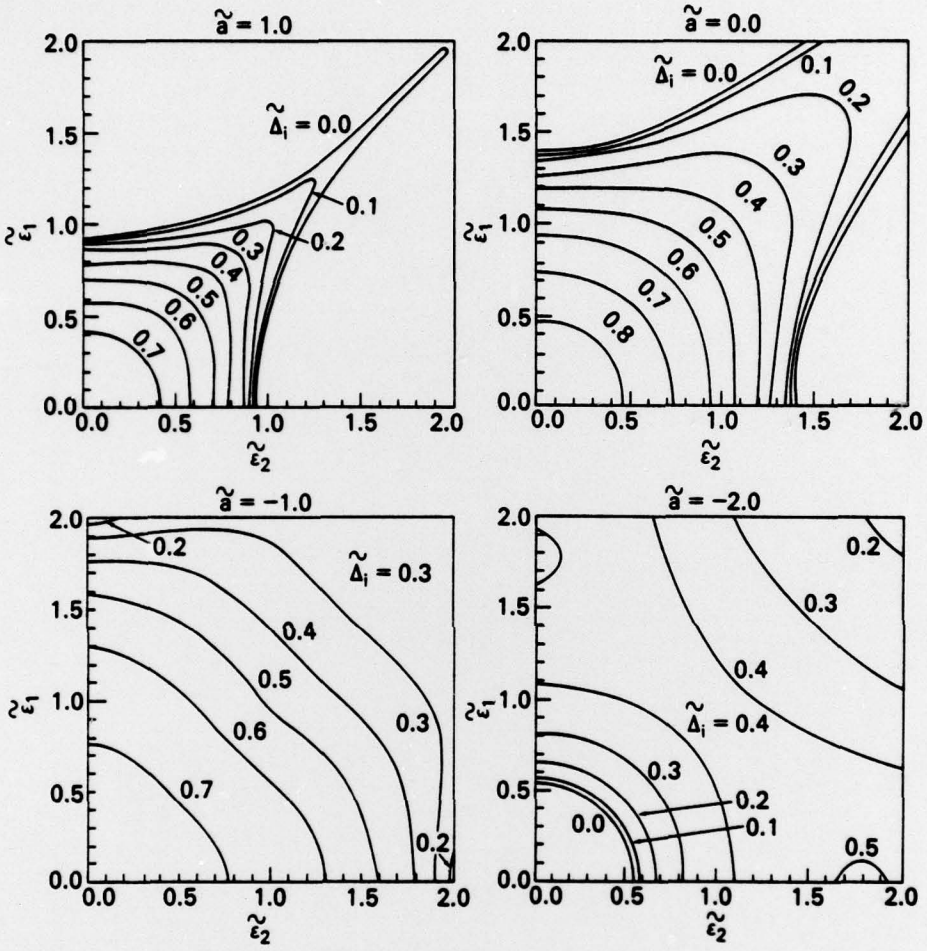


Fig. 4 - Growth rate contours for $\tilde{a}$ = 1, 0, -1, -2