AD-A077 548 AIR FORCE FLIGHT DYNAMICS LAB WRIGHT- TTERSON AFB OH F/G 1/3
RECOVERY SYSTEM PRELIMINARY DESIGN, A IMPLIFIED APPROACH TO DE--ETC(U)
SEP 79 M W HIGGINS
UNCLASSIFIED AFFDL-TR-79-3007 NL

1 OF 1
AD
A077548

END
DATE
FILMED
12-79
DDC

AFFDL-TR-79-3007

LEVEL II

AD A077548

RECOVERY SYSTEM PRELIMINARY DESIGN, A SIMPLIFIED APPROACH TO DETERMINING STAGING, TIMING AND ALTITUDE REQUIREMENTS FOR FAST INFLATING PARACHUTES

MICHAEL W. HIGGINS
RECOVERY CREW STATION BRANCH
VEHICLE EQUIPMENT DIVISION

September 1979

TECHNICAL REPORT AFFDL-TR-79-3007
Final Report for Period 13 February 1976 – 1 June 1976

Approved for public release; distribution unlimited.

AIR FORCE FLIGHT DYNAMICS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

NOTICE

When Government drawings, specifications, or other data are used for any purpose other than in connection with a definitely related Government procurement operation, the United States Government thereby incurs no responsibility nor any obligation whatsoever; and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, it is not to be regarded by implication or otherwise as in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

This report has been reviewed by the Information Office (IO) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

MICHAEL W. HIGGINS
Project Engineer/Scientist

R. HARLEY WALKER
Group Leader
Subsystems Group

FOR THE COMMANDER

Ambrose B. Nutt
Director
Vehicle Equipment Division
Air Force Flight Dynamics Laboratory

Copies of this report should not be returned unless return is required by security considerations, contractural obligations, or notice on a specific document.

AIR FORCE/56780/1 November 1979 — 160

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
AFFDL-TR-79-3007

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle)
RECOVERY SYSTEM PRELIMINARY DESIGN, A SIMPLIFIED APPROACH TO DETERMINING STAGING, TIMING AND ALTITUDE REQUIREMENTS FOR FAST INFLATING PARACHUTES.

5. TYPE OF REPORT & PERIOD COVERED
Final Report
13 Feb 1976-1 June 1976

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)
MICHAEL W. HIGGINS

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Air Force Flight Dynamics Laboratory
AFFDL/FER
Wright-Patterson Air Force Base, Ohio 45433

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
Program Element 62201F
Project 6065, Task 606505,
Work Unit 60650504

11. CONTROLLING OFFICE NAME AND ADDRESS
Air Force Flight Dynamics Laboratory
AFFDL/FER
Wright-Patterson Air Force Base, Ohio 45433

12. REPORT DATE
September 1979

13. NUMBER OF PAGES
66

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Parachutes	Remotely Piloted Vehicles
Reefing	Preliminary Recovery System Design
Recovery Systems	Reefing Analysis
Vehicle Recovery	Crew Escape Systems
Ram Air Parachutes	Deceleration

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Determining a recovery system design that will take a specified act of initial conditions and will operate within a given set of constraints to provide a required final condition is a complex task. To perform this task, current design practices make extensive use of both person hours and computer time in an analytical "cut and try" process. This report documents an analytical technique that takes a specified set of inputs (initial conditions,

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered) final conditions and operating constraints) and outputs a "reasonable" recovery system preliminary design for fast inflating parachutes. The output includes the number of operating stages, the drag area of each operating stage, the reefing cutter times, and resulting altitude losses. The technique assumes a vertical trajectory, step function increases in parachute drag area, and that the recovery system is a point mass.

The limitations of the analytical technique are discussed and recommendations are made with respect to reducing or removing the effects of these limitations. The analytical technique is applied to three different sets of conditions and constraints as example applications. Use of the analytical technique documented in this report will significantly reduce the assets required to arrive at a final recovery system design.

FOREWORD

The analytical technique documented in this Technical Report was derived by personnel from the Recovery and Crew Station Branch, Air Force Flight Dynamics Laboratory (AFFDL/FER) while working on projects in support of the Remotely Piloted Vehicle System Programs Office, (specifically Job Order Numbers 19645003 and 19640301 titled Kevlar Parachute Recovery System and Parafoil Recovery System, respectively). The analytical technique was then documented under Job Order Number 60650504 titled, Decelerator Technology Document.

The author is indebted to Ralph Speelman, Harley Walker, and Charles Babish of AFFDL/FER, Major Guy Clatterbuck and Major Randy Johnson of the Remotely Piloted Vehicle Office, and Capt. Tom Jonack of the 6511th Test Squadron for their asisstance, support, and use of the results of this efforts.

TABLE OF CONTENTS

LIST OF SYMBOLS

ALT	Altitude (= ALT_o - Z); ft
C_DS	Drag Area; ft^2
D	Drag ($\equiv q\, C_DS$); lbs
DAR	Drag Area Ratio for one parachute stage $\left(\equiv (C_DS)_{R_{min}} / (C_DS)_{R_N}\right)$
DF	Deceleration Force $\equiv (DLL/X_S)-W_T$; lbs
DLL	Design Limit Load; lbs
F	Force; lbs
g	Standard Acceleration of Gravity ($\equiv$ 32.2) ft/sec^2;
m	Mass ($\equiv W_T/g$); slugs
k	Constant associated with the amount of q decay obtained from each operating stage when the payload drag area is not negligible. See Appendix; $\left(K_n \equiv [1+\alpha_n [\frac{W_T X_S}{DLL+q_o K_1 K_2 \ldots K_{n-1} (C_DS)_{M/S} (X_S-1)} -1]]\right)$
M	Total number of parachute stages in the recovery system.
N	Total number of operating stages in this recovery system. $\left(\equiv \sum_{m=1}^{M} N_m\right)$
N_m	Total number of operating stages within the m^{th} parachute stage
q	Dynamic Pressure ($\equiv \frac{1}{2} \rho u^2$); lbs/ft^2
t	Time; sec
u	Vertical Velocity ($\equiv dz/dt$); ft/sec

LIST OF SYMBOLS (Continued)

u	Vertical Velocity ($\equiv dz/dt$); ft/sec
v	Horizontal velocity ($\equiv dx/dt$); ft/sec
W_T	Total System Weight (includes both vehicle and recovery system); lbs
X_S	Opening Shock Factor
x	Horizontal displacement; ft
$\dot{x}$	Horizontal velocity ($\equiv dx/dt$); ft/sec
$\ddot{x}$	Horizontal acceleration ($\equiv d^2x/dt^2$); ft/sec^2
z	Vertical displacement from initial altitude (positive down; ft
$\dot{z}$	Vertical velocity ($\equiv dz/dt$); ft/sec
$\ddot{z}$	Vertical acceleration ($\equiv d^2z/dt^2$); ft/sec^2

Subscripts

a	allowable
drogue	of the drogue chute
DR_n	at disreef of the n^{th} operating stage
F	at the final condition
m	the number of this parachute stage ($1 \leq m \leq M$)
M/S	of the man/seat combination
max	maximum
min	minimum
n	the number of this operating stage ($1 \leq n \leq N$)
N	at the last operating stage of the recovery system
o	at recovery system initiation
R_n	maximum allowable of the n^{th} operating stage

LIST OF SYMBOLS (Concluded)

System	of the system
T	at the terminal condition
T_n	at the terminal condition associated with the n^{th} operating stage

Operators

$\frac{d}{dt}$	Take the derivative of the following variable(s) with respect to time.
exp( )	The term within the brackets is to be used as a power of e ($\exp(x) \equiv e^x$).
ℓn ( )	Natural logarithm of the term within brackets
(t)	The preceding variable is a function of time.

Greek Symbols

α_n The ratio of dynamic pressure reduction utilized in the n^{th} operating stage to the dynamic pressure reduction available within that stage.

$$\alpha_1 \equiv \frac{q_o - q_{DR_1}}{q_o - q_{T_1}}$$

$$\alpha_n \equiv \frac{q_{DR_{n-1}} - q_{DR_n}}{q_{DR_{n-1}} - q_{T_n}}$$

If no subscriptis specified, α refers to the overall design ratio of actual to available dynamic pressure reduction to be obtained by each operating stage.

ρ	Air density; slugs/ft^3
θ	Nose down dive angle (from horizontal); deg
$\Delta_{\_\_n}$	The change in the intervening variable which occurs during the operation of the n^{th} stage.

SECTION 1
INTRODUCTION

A recovery system consists of three subsystems; a decelerator subsystem, a terminal descent subsystem, and an impact attenuation subsystem. For the purposes of this report, the term recovery system will be used to denote the decelerator and fast inflating terminal descent parachute subsystems. The impact attenuation subsystem will not be addressed in this report.

A recovery system designer is faced with two problems when he is asked to design a recovery system to a given set of initial (deployment) conditions, final (terminal descent) conditions, and operating constraints (maximum allowable design limit load, etc.). The first problem is to determine whether or not it is physically possible for any system to meet his specific requirements. If it is not physically possible, the designer must modify his set of conditions and constraints until he has a set that is both physically possible and capable of maximizing the size of his allowable recovery envelope. Once the recovery system designer has determined that he has a set of conditions and constraints that is within the realm of physical possibility, he then faces his second problem, that of determining the specific design details of this recovery system.

The current procedure for solving the recovery system designer's problems is a relatively complex process involving extensive use of computer time to "try out" and modify recovery system designs which the designer selects. Some of the variables considered during this process are the initial and final conditions, the total number of operating stages required to meet these conditions (see Figure 1 following), the total number of parachute stages required

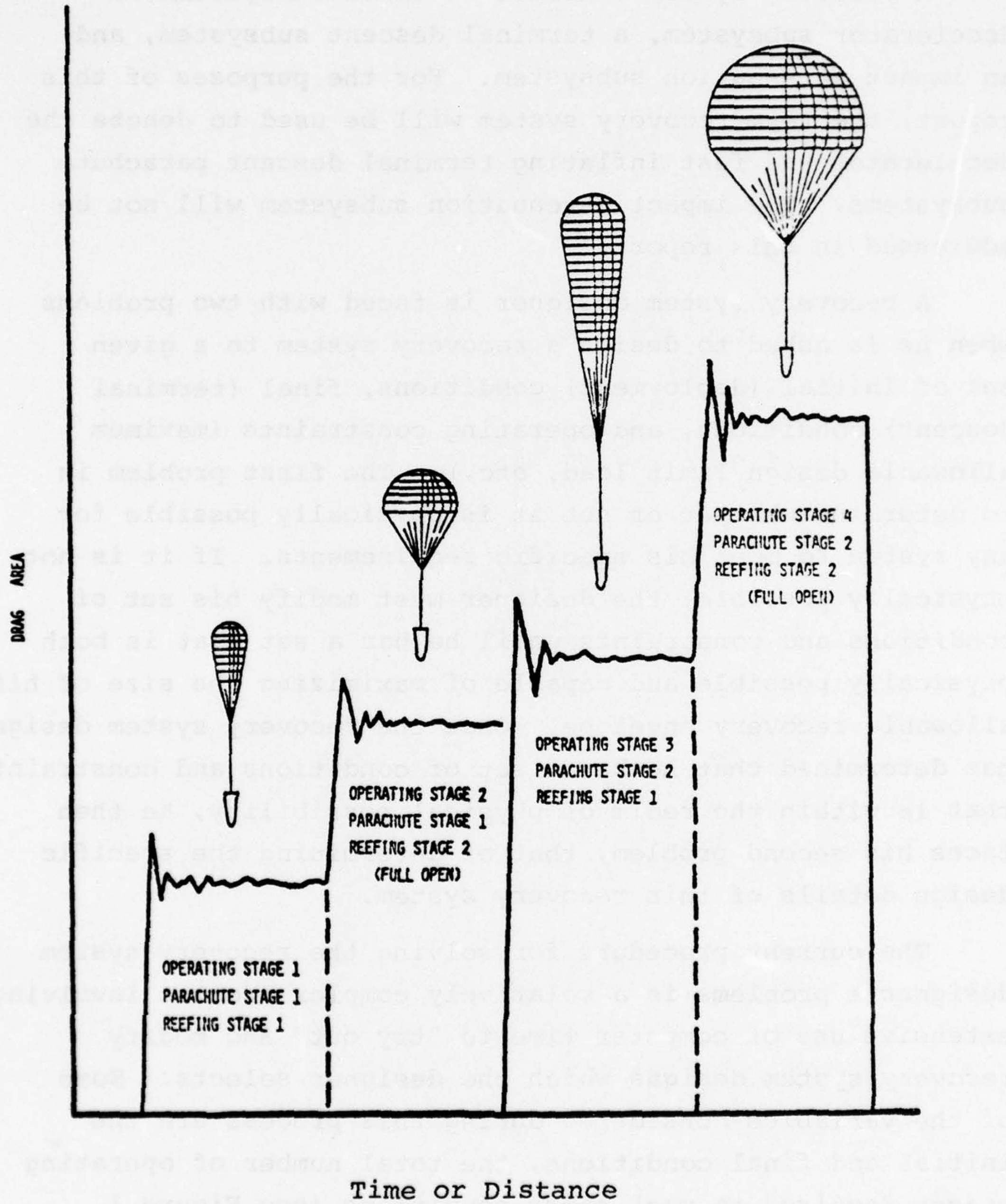


Figure 1. Example Multi-Parachute Stage Recovery System.

to provide the total number of operating stages and the drag area, cutter times, and altitude requirements of each operating stage. If the recovery system designer is faced with a wide range of conditions and a severe set of constraints the design process can consume a large number of person hours and computer time.

The purpose of this report is to describe an analytical technique that determines what constitutes a set of conditions and constraints that is both physically possible and likely to result in a reasonable preliminary design for a recovery system.

Some of the major limiting assumptions associated with the analytical technique described in this report are:

1) The vehicle trajectory is assumed to be vertical.
2) The parachute is assumed to undergo step function increases in drag area.
3) The recovery system is a point mass.

The effect of these assumptions is to limit the application of the analytical technique presented in this report to vehicles whose trajectory is primarily vertical and to that portion of the recovery system which uses fast inflating parachutes (drogue and possibly ram-air parachutes). These and other assumptions and their effects are discussed further in Section V.

Use of the preliminary design technique described in this report will result in the identification of a "reasonable" recovery system preliminary design. The recovery system designer can then use this design as an input to his computer trajectory programs for further refinement. By beginning with the preliminary design resulting from the technique described in this report, it should be possible for the designer to substantially reduce the assets required to arrive at a recovery system final design.

The approach followed in this report is to determine a general expression for the drag area of each operating stage, the maximum allowable number of operating stages in each parachute stage, and the minimum number of parachute stages required (these expressions are referred to as the staging requirements of the recovery system). The equations of motion are then solved to determine a general expression for the time and altitude requirements of each stage. The expressions are then applied to the design of three example recovery systems and the assumptions, recommendations, and the conclusions are discussed.

SECTION II
DERIVATION OF STAGING REQUIREMENTS

The objective of this section is to derive expressions for:

1) The drag area and appropriate dynamic pressures for each operating stage and for the final operating stage.
2) The total number of operating stages required.
3) The maximum number of operating stages in each parachute stage.
4) The total number of parachute stages.

Where feasible, these expressions will be in explicit terms of the initial and final conditions and the operating constraints.

During the derivation of the above expressions, we will make a number of assumptions. These assumptions and their effects will be discussed in Section V.

A. DRAG AREAS ANL TERMINAL AND DISREEF DYNAMIC PRESSURES

The purpose of reefing a parachute is to allow a vehicle to be decelerated from an initial condition to a final condition without exceeding a given force constraint. This constraint can be mathematically expressed for the first operating stage as:

$$F_{max} = DLL = q_o (C_D S)_{R_1} X_{S_1} \qquad (1)$$

where:

F_{max}	$\equiv$	Maximum allowable force (lbs)
DLL	$\equiv$	Design limit load (lbs)
q_o	$\equiv$	Initial dynamic pressure (lbs/ft^2)
$(C_D S)_{R_1}$	$\equiv$	Maximum allowable drag area of the first operating stage (ft^2)
X_{S_1}	$\equiv$	Opening shock factor of the first operating stage

Equation (1) can be used to identify the maximum allowable drag area for the first operating stage.

$$(C_D S)_{R_1} = \frac{DLL}{q_o X_{S_1}} \qquad (2)$$

Given the total system weight (W_T) including the weights of the vehicle and the recovery system, and given the drag area value of the first reefed stage, we can identify the first stage terminal dynamic pressure (q_{T_1}).

For vertical descent under standard gravitational conditions:

$$D_1 = W_T$$

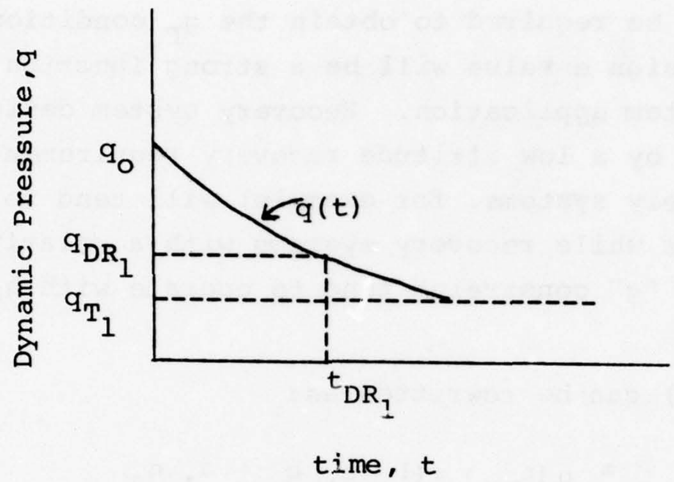
and

$$D_1 = (C_D S)_{R_1} q_{T_1} = W_T.$$

Therefore,

$$q_{T_1} = \frac{W_T}{(C_D S)_{R_1}} \qquad (3)$$

The reefed parachute will require time to decay the dynamic pressure from the initial condition q_o to the point where we can safely proceed (disreef DR) to the next stage. To impose some constraints on the time required to obtain a specified amount of dynamic pressure decay, we have defined quantity α as shown below.



$$\alpha_1 \equiv \frac{q_o - q_{DR_1}}{q_o - q_{T_1}}$$

More generally,

$$\alpha_n \equiv \frac{q_{DR_{n-1}} - q_{DR_n}}{q_{DR_{n-1}} - q_{T_1}} \qquad (4)$$

where

n = number of this operating stage = 1, 2, ...N

N = total number of operating stages in the recovery system.

As can be seen above, α is defined as the ratio of the actual dynamic pressure reduction obtained to the dynamic pressure reduction available. In general, α can vary between 0 and 1. As α approaches 1, we are saying we want to get all of the available q decay and are willing to live with the time and altitude loss that will be required. As α approaches 0, we are saying that we must have a fast acting system and are willing to live with the increased number of operating stages that will be required to obtain the q_F condition. Selection of the design α value will be a strong function of the recovery system application. Recovery system designs which are driven by a low altitude recovery requirement (personnel recovery systems, for example) will tend to operate with low α values while recovery systems with a relatively low deceleration "g" constraint tend to operate with high α values.

Equation (4) can be rewritten as:

$$q_{DR_1} \equiv q\,(t_{DR_1}) = (1 - \alpha_1)\, q_o + \alpha_1\, q_{T_1} \qquad (5)$$

where q_{DR_1} and t_{DR_1} are respectively defined as the dynamic pressure and the time at which first stage disreef occurs.

Combining Equations (1), (3), and (5) yields:

$$\begin{aligned} q_{DR_1} &= (1 - \alpha_1)\, q_o + \alpha_1 q_o \frac{W_T\, X_{S_1}}{DLL} \\ &= q_o \left[1 + \alpha_1 \left[\frac{W_T\, X_{S_1}}{DLL} - 1\right]\right] \end{aligned} \qquad (6)$$

Now that we know q_{DR_1}, we can calculate the maximum allowable drag area for the second operating stage as:

$$(C_D S)_{R_2} = \frac{DLL}{q_{DR_1} X_{S_2}}$$

$$= \frac{1}{q_o} \frac{DLL}{\left[1 + \alpha_1 \left[\frac{W_T X_{S_1}}{DLL} - 1\right]\right] X_{S_2}} \qquad (7)$$

The second stage terminal dynamic pressure is therefore:

$$q_{T_2} = \frac{W_T}{(C_D S)_{R_2}}$$

$$= q_o \frac{W_T X_{S_2}}{DLL} \left[1 + \alpha_1 \left[\frac{W_T X_{S_1}}{DLL} - 1\right]\right] \qquad (8)$$

Following the same logic as Equations (5) and (6) and realizing the q_{DR_1} is the q_o for this stage, the disreef dynamic pressure for the second operating stage can be written as:

$$q_{DR_2} = (1 - \alpha_2) q_{DR_1} + \alpha_2 q_{T_2}$$

$$= (1 - \alpha_2) q_o \left[1 + \alpha_1 \left[\frac{W_T X_{S_1}}{DLL} - 1\right]\right]$$

$$+ \alpha_2 q_o \frac{W_T X_{S_2}}{DLL} \left[1 + \alpha_1 \left[\frac{W_T X_{S_1}}{DLL} - 1\right]\right] \qquad (9)$$

$$= q_o \left[1 + \alpha_1 \left[\frac{W_T X_{S_1}}{DLL} - 1\right]\right] \left[1 + \alpha_2 \left[\frac{W_T X_{S_2}}{DLL} - 1\right]\right]$$

Knowing the disreef conditions for the second stage, we can therefore calculate the maximum allowable drag area for the third operating stage to be:

$$(C_D S)_{R_3} = \frac{DLL}{q_{DR_2} X_{S_3}} \tag{10}$$

$$= \frac{1}{q_o} \frac{DLL}{\left[1 + \alpha_1 \left[\frac{W_T X_{S_1}}{DLL} - 1\right]\right] \cdot \left[1 + \alpha_2 \left[\frac{W_T X_{S_2}}{DLL} - 1\right]\right] X_{S_3}}$$

The third stage terminal dynamic pressure is therefore:

$$q_{T_3} = \frac{W_T}{(C_D S)_{R_3}} \tag{11}$$

$$= q_o \frac{W_T X_{S_3}}{DLL} \left[1 + \alpha_1 \left[\frac{W_T X_{S_1}}{DLL} - 1\right]\right] \cdot \left[1 + \alpha_2 \left[\frac{W_T X_{S_2}}{DLL} - 1\right]\right]$$

And the third stage disreef dynamic pressure is:

$$q_{DR_3} = (1 - \alpha_3)\, q_{DR_2} + \alpha_3\, q_{T_3}$$

$$= q_o \left[1 + \alpha_1 \left[\frac{W_T X_{S_1}}{DLL} - 1\right]\right] \cdot \left[1 + \alpha_2 \left[\frac{W_T X_{S_2}}{DLL} - 1\right]\right] \cdot \left[1 + \alpha_3 \left[\frac{W_T X_{S_3}}{DLL} - 1\right]\right] \tag{12}$$

That's enough steps to establish a pattern. It's now time to simplify the equations. If we assume that the α values, the reefed shock factors, X_S, and the weights, W_T, and the design limit loads, DLL, are the same for each operating stage, Equations (10), (11), and (12) become:

$$(C_D S)_{R_n} = \frac{DLL}{q_o X_S \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{n-1}} \qquad (13)$$

$$q_{T_n} = q_o \frac{W_T X_S}{DLL} \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{n-1} \qquad (14)$$

$$q_{DR_n} = q_o \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{n} \qquad (15)$$

When the last operating stage (N) has been reached, the equations become:

$$(C_D S)_{R_N} = \frac{DLL}{q_o X_S \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{N-1}} \qquad (16)$$

$$q_{T_N} = q_s \frac{W_T X_S}{DLL} \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{N-1} \qquad (17)$$

$$q_{DR_N} = q_F = q_o \left[1 + \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{N} \qquad (18)$$

where q_F is the final dynamic pressure to be reached by this recovery system.

B. NUMBER OF OPERATING STAGES

The objective of this subsection is to derive an expression for the total number of operating stages required to go from q_o to q_F without exceeding DLL.

We will begin by solving Equation (18) for N. Equation (18) can be rewritten as:

$$\frac{q_F}{q_o} = \left[1 + \alpha\left[\frac{W_T X_S}{DLL} - 1\right]\right]^N \tag{19}$$

Solving for N yields:

$$N = \frac{\ln\left[\frac{q_F}{q_o}\right]}{\ln\left[1+\alpha\left[\frac{W_T X_S}{DLL} - 1\right]\right]} \tag{20}$$

Equation (16) can also be used to obtain an expression for N

$$N = \frac{\ln\left[\frac{DLL}{q_o (C_D S)_{R_N} X_S}\right]}{\ln\left[1 + \alpha\left[\frac{W_T X_S}{DLL} - 1\right]\right]} + 1 \tag{21}$$

C. NUMBER OF PARACHUTE STAGES

Equations (20) and (21) identify the required number of operating stages for total system operation. If there were no lower limits on the reefed drag areas that could be achieved on a parachute, we could perform the entire recovery function with one parachute. Unfortunately, there is a lower limit on how much we can reduce the drag area of a given parachute.

Each parachute design has a minimum allowable drag area ratio (DAR_{min_a}) associated with it. An attempt to reef a parachute below its DAR_{min_a} value is likely to result in failure of the parachute to inflate.

Each operating stage has a drag area ratio associated with it. If one parachute stage is used, the minimum drag area ratio for that parachute stage is:

$$DAR_{min} = \frac{(C_D S)_{R_{min}}}{(C_D S)_{R_N}} = \frac{(C_D S)_{R_1}}{(C_D S)_{R_N}} \tag{22}$$

The requirement that the minimum drag area ratio for a parachute stage must always be greater than a specified DAR_{min_a} value results in the following inequality:

$$(DAR)_{min_a} \leq (DAR)_{min} = \frac{(C_D S)_{R_1}}{(C_D S)_{R_N}} \tag{23}$$

Inserting Equations (2) and (16) into Equation (23) will result in

$$(DAR)_{min_a} \leq \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1\right]\right]^{N-1} \tag{24}$$

The procedure to determine whether or not we will need more than one parachute stage will be as follows: either Equation (20)or (21) will be solved to yield the required value for N. This value and the other design parameters will be inserted into the right hand side of Equation (24). If the resulting numerical value is indeed greater than the DAR_{min_a}

value for the selected specified parachute design, one parachute stage will be sufficient. If not, more than one parachute stage will be required.

If more than one parachute stage is required, Equation (22) becomes:

$$DAR_{min_m} = \frac{(C_D S)_{R_{min_m}}}{(C_D S)_{R_{max_m}}} \tag{25}$$

$$m = 1, 2 \ldots M$$

Where the subscript m refers to each parachute stage, the value M is the total number of parachute stages, $(C_D S)_{R_{min_m}}$ refers to the minimum drag area in the m^{th} parachute stage and $(C_D S)_{R_{max_m}}$ refers to the maximum drag area in the m^{th} parachute stage. Combining Equations (16), (23), and (25) yields:

$$(DAR)_{min_{a_m}} \leq (DAR)_{min_m} = \frac{(C_D S)_{R_{min_m}}}{(C_D S)_{R_{max_m}}}$$

$$= \frac{\left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1\right]\right]^{N_{max_m} - 1}}{\left[1 + \alpha \left[\frac{W_T X_S}{DLL}\right]\right]^{N_{min_m} - 1}} \tag{26}$$

Where N_{max_m} and N_{min_m} respectively, refer to the operating stage number associated with the maximum and minimum drag areas on the m^{th} parachute stage.

Taking the logarithms of both sides of Equation (26) yields:

$$\ln (DAR)_{min_{a_m}} \leq (N_{max_m} - 1) \ln \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right] - (N_{min_m} - 1) \ln \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right] \tag{27}$$

or

$$\ln(DAR)_{min_{a_m}} \leq (N_{max_m} - N_{min_m}) \ln \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right] \tag{28}$$

or

$$\ln(DAR)_{min_{a_m}} \leq (N_m - 1) \ln \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right] \tag{29}$$

where N_m is the number of operating stages within the m^{th} parachute stage ($N_m \equiv N_{max_m} - N_{min_m} + 1$).

Taking the absolute value of both sides of equation (29) and rearranging will ultimately yield:

$$N_m \leq \frac{\left| \ln (DAR) min_{a_m} \right|}{\left| \ln \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right] \right|} + 1 \tag{30}$$

Equation (30) identifies the maximum allowable number of operating stages that can be put into the m^{th} parachute stage without exceeding the $(DAR)_{min_{a_m}}$ constraint to convert from

N_m to the conventional terminology "number of reefing stages", it should be remembered that a parachute with two operating stages (reefed open and full open) is said to have one stage of reefing. A parachute with three operating stages (first stage reefed open, second stage reefed open and full open) is said to have two stages of reefing. In general, a parachute with N_m operating stages is said to have $N_m - 1$ stages of reefing.

To solve for the total number of parachute stages, we must remember that:

$$N = \sum_{m=1}^{M} N_m \leq \sum_{m=1}^{M} \left[\frac{\ell n (DAR)_{min_{a_m}}}{\ell n \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]} + 1 \right] \tag{31}$$

Assuming that $(DAR)_{min_{a_m}}$ has the same value for each parachute yields:

$$N \leq M \left[\frac{\ell n (DAR)_{min_a}}{\ell n \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]} + 1 \right] \tag{32}$$

Solving for M yields:

$$M \geq \frac{N}{\dfrac{\ell n (DAR)_{min_a}}{\ell n \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]} + 1} \tag{33}$$

Using equations (20) and (21) can yield:

$$M \geq \frac{\ell n \left(\frac{q_F}{q_o} \right)}{\ell n (DAR)_{min_a} + \ell n \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]} \tag{33a}$$

or

$$M \geq \frac{\ln\left[\dfrac{DLL}{q_o \, (C_D S)_{R_N} \, X_S}\right] + \ln\left[1 + \alpha\left[\dfrac{W_T X_S}{DLL} - 1\right]\right]}{\ln(DAR)_{min_a} + \ln\left[1 + \alpha\left[\dfrac{W_T X_S}{DLL} - 1\right]\right]} \tag{33b}$$

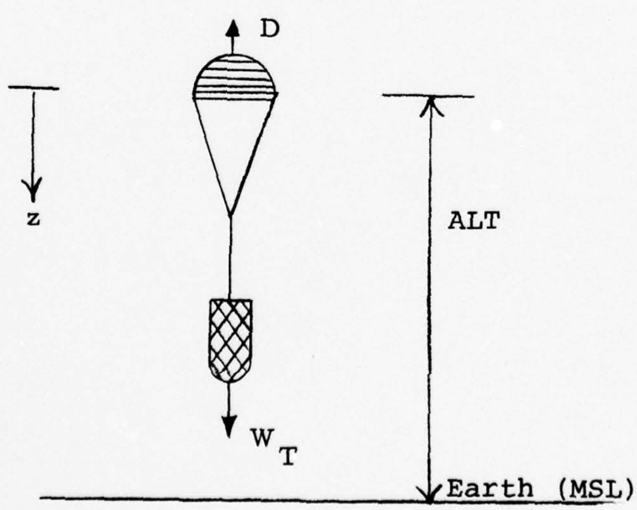
SECTION III

DERIVATION OF TIME AND ALTITUDE REQUIREMENTS

In this section we will derive an expression for the equations of motion for a vehicle decelerating by means of a parachute having a constant drag area. The vehicle will be assumed to be in subsonic vertical descent under standard gravitational conditions. We will solve the equations of motion to obtain an expression for the vehicle's response to a step function increase in drag area. Using this expression, we will develop expressions for the time and altitude requirements of each operating stage. We will then determine expressions for the time and altitude requirements for operation of the recovery system.

A. DERIVATION OF THE EQUATIONS OF MOTION

The forces acting on a vertically descending vehicle with a negligible vehicle drag area and a parachute of constant drag area are shown in the sketch below.



Looking at the sketch, we can write:

$$\Sigma F_z = m\ddot{z} = W_T - D \tag{34}$$

Equation (34) can be rewritten as:

$$m\ddot{z} = W_T - \frac{1}{2}\rho \dot{z}^2 (C_D S) \tag{35}$$

Defining $u \equiv \dot{z}$ and $\dot{u} \equiv \ddot{z}$, inserting into Equation (35) and rearranging yields,

$$\dot{u} = \frac{W_T}{m} - \frac{\rho(C_D S)}{2\,mg}\, g\, u^2 \tag{36}$$

Realizing that:

$$u_T^{\,2} = \frac{2\,mg}{\rho\,(C_D S)} = \frac{2\,W_T}{\rho\,(C_D S)} \tag{37}$$

Results in:

$$\frac{du}{dt} = \dot{u} = g\,(1 - \frac{u^2}{u_T^{\,2}}) \tag{38}$$

Equation (38) is an expression of the differential equation of motion. Note that by solving this expresssion for u, we will have an expression for the vertical velocity as a function of time, gravity, and the terminal velocity u_T.

B. SOLUTION OF THE EQUATIONS OF MOTION

The objective of this subsection is to solve Equation (38) for u(t) and z(t) and thus obtain an expression for both the velocity decay and altitude loss of the vehicle as a function of time.

We begin by rewriting Equation (38) as:

$$\int_{u_o}^{u(t)} \frac{du}{(u_T^2 - u^2)} = \int_{t_o}^{t} \frac{g\,dt}{u_T^2} \qquad (39)$$

Carrying out this integration, Equation (39) becomes:

$$-\frac{1}{2u_T}\,\ell n\left[\frac{u - u_T}{u + u_T}\right]\Bigg|_{u_o}^{u(t)} = \frac{gt}{u_T^2}\Bigg|_{t_o}^{t} \qquad (40)$$

Evaluating Equation (40) at its boundary conditions and regrouping yields:

$$\frac{2g}{u_T}(t - t_o) = \ell n\left[\frac{(u_o - u_T)}{(u_o + u_T)} \cdot \frac{(u(t) + u_T)}{(u(t) - u_T)}\right] \qquad (41)$$

In order to solve for u(t), we must take both sides of Equation (41) as powers of e to get:

$$\exp\left(\frac{2g(t - t_o)}{u_T}\right) = \frac{(u_o - u_T)}{(u_o + u_T)} \cdot \frac{(u(t) + u_T)}{(u(t) - u_T)} \qquad (42)$$

Regrouping Equation (42) will eventually result in:

$$u(t) = u_T \left[\frac{\left[\frac{u_o + u_T}{u_o - u_T} \cdot \exp\left(\frac{2g(t - t_o)}{u_T}\right)\right] + 1}{\left[\frac{u_o + u_T}{u_o - u_T} \cdot \exp\left(\frac{2g(t - t_o)}{u_T}\right)\right] - 1}\right] \qquad (43)$$

Equation (43) may also be integrated to give an expression for the change in altitude of the vehicle as a function of time, z (5). This may be done by integrating both sides of Equation (43).

For $z_o = 0$;

$$z(t) = \int_{t_o}^{t} u(t)\ dt$$

$$= u_T \int_{t_o}^{t} \frac{\frac{u_o + u_T}{u_o - u_T} \cdot \exp\left(\frac{2\,gt}{u_T}\right)}{\left[\frac{u_o + u_T}{u_o - u_T} \cdot \exp\left(\frac{2\,gt}{u_T}\right)\right] - 1}\ dt \qquad (44)$$

$$+ \int_{t_o}^{t} \frac{1}{\left[\frac{u_o + u_T}{u_o - u_T} \cdot \exp\left(\frac{2\,gt}{u_T}\right)\right] - 1}\ dt$$

Integrating Equation (44) and assuming $t_o = 0$ will eventually result in:

$$z(t) = \frac{u_T^2}{g}\ \ell n \left[\frac{u_o}{u_T} \frac{\left[\exp\left(\frac{2gt}{u_T}\right) - 1\right]}{2} + \frac{\left[\exp\left(\frac{2\,gt}{u_T}\right) + 1\right]}{2} \right] - u_T t \qquad (45)$$

Equations (43) and (45) represent our desired expressions. We have met the objective of this subsection.

C. INDIVIDUAL OPERATING STAGE REQUIREMENTS

The objective of this subsection is to obtain equations for the time and altitude requirements of each operating stage. We will express these equations in terms of the parameters (α, W_T, X_S, DLL) used in the staging equations of Section II.

1. Time Requirements

We can write Equation (41) as:

$$t_{DR_1} - t_o = \frac{u_{T_1}}{2g} \ln\left[\frac{(u_o - u_{T_1})}{(u_o + u_{T_1})} \cdot \frac{(u_{DR_1} + u_{T_1})}{(u_{DR_1} - u_{T_1})}\right] \qquad (46)$$

Remembering Equation (6), and solving for u_{DR_1} yields:

$$u_{DR_1} = u_o \sqrt{1 + \alpha_1\left[\frac{W_T X_{S_1}}{DLL} - 1\right]} \qquad (47)$$

Expressing u_o and u_{T_1} as:

$$u_o = \sqrt{\frac{2q_o}{\rho}} = \sqrt{\frac{2DLL}{\rho X_{S_1} (C_D S)_{R_1}}} \qquad (48)$$

$$u_{T_1} = \sqrt{\frac{2 W_T}{\rho (C_D S)_{R_1}}} \qquad (49)$$

and inserting Equations (47), (48), and (49) into Equation (46) results in:

$$t_{DR_1} - t_o = \frac{1}{2g}\sqrt{\frac{2 W_T}{\rho (C_D S)_{R_1}}} \ln\left[\frac{\sqrt{\frac{DLL}{W_T X_{S_1}}} - 1}{\sqrt{\frac{DLL}{W_T X_{S_1}}} + 1} \cdot \frac{\sqrt{(1-\alpha)\frac{DLL}{W_T X_{S_1}} + \alpha_1} + 1}{\sqrt{(1-\alpha)\frac{DLL}{W_T X_{S_1}} + \alpha_1} - 1}\right] \qquad (50)$$

Equation (50) can be written in a more general form as:

$$\Delta t_n \equiv t_{DR_n} - t_{DR_{n-1}}$$

$$= \frac{1}{2g}\sqrt{\frac{2\,W_T}{\rho(C_D S)_{R_n}}}\;\ell n\left[\frac{\sqrt{\frac{DLL}{W_T X_S}} - 1}{\sqrt{\frac{DLL}{W_T X_S}} + 1}\cdot \qquad (51)\right.$$

$$\left.\cdot\frac{\sqrt{(1-\alpha)\frac{DLL}{W_T X_S} + \alpha} + 1}{\sqrt{(1-\alpha)\frac{DLL}{W_T X_S} + \alpha} - 1}\right]$$

Inserting Equation (13) into Equation (51) results in:

$$\Delta t_n = \frac{u_o}{2g}\sqrt{\frac{W_T X_S}{DLL}}\left[1 + \alpha\left[\frac{W_T X_S}{DLL} - 1\right]\right]^{\frac{n-1}{2}} \cdot \qquad (52)$$

$$\cdot\;\ell n\left[\frac{\left(\sqrt{\frac{DLL}{W_T X_S}} - 1\right)}{\left(\sqrt{\frac{DLL}{W_T X_S}} + 1\right)}\cdot\frac{\left(\sqrt{(1-\alpha)\frac{DLL}{W_T X_S} + \alpha} + 1\right)}{\left(\sqrt{(1-\alpha)\frac{DLL}{W_T X_S} + \alpha} - 1\right)}\right]$$

Equation (52) represents our desired solution for the time requirements of each operating stage.

2. Altitude Requirements

If we insert Equation (46) into Equation (45) we can arrive at:

$$z_{DR_1} - z_o = \frac{u_{T_1}^2}{2g} \cdot \ln \left[\frac{u_o^2 - u_{T_1}^2}{u_{DR_1}^2 - u_{T_1}^2} \right] \quad (53)$$

Inserting Equations (47), (48), and (49) into Equation (53) and regrouping will eventually yield:

$$z_{DR_1} - z_o = \frac{W_T}{g\rho (C_D S)_{R_1}} \ln \left[\frac{1}{1 - \alpha_1} \right] \quad (54)$$

Expressing the equation in a more general form will result in:

$$\Delta z_n \equiv z_{DR_n} - z_{DR_{n-1}}$$

$$= \frac{W_T}{g\rho (C_D S)_{R_n}} \cdot \ln \left[\frac{1}{1 - \alpha} \right] \quad (55)$$

Inserting Equation (13) into Equation (55) yields:

$$\Delta z_n = \frac{u_o^2}{2g} \quad \frac{W_T X_S}{DLL} \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{n-1} \ln \left[\frac{1}{1 - \alpha} \right] \quad (56)$$

Equation (56) represents our desired solution for the altitude requirements of each operating stage. We have met the objective of this subsection.

D. RECOVERY SYSTEM REQUIREMENTS

The objective of this subsection is to obtain equations for the time and altitude requirements for that portion of the recovery system operation where our fast inflating parachute assumption is valid.

1. Time Requirements

We begin by assuming that each operating stage has the same design values (W_T, X_S, DLL, α, etc.). We then examine Equation (51) and realize that the total operating time may be expressed as:

$$t_N = \Delta t_1 + \Delta t_2 + \ldots\ldots + \Delta t$$

$$= \left[\frac{1}{\sqrt{(C_D S)_{R_1}}} + \frac{1}{\sqrt{(C_D S)_{R_2}}} + \ldots + \frac{1}{\sqrt{(C_D S)_{R_N}}} \right]$$

$$\cdot \frac{1}{2g} \sqrt{\frac{2 W_T}{\rho}} \cdot \ln [A] \qquad (57)$$

Where:

$$A = \left[\frac{\sqrt{\frac{DLL}{W_T X_S}} - 1}{\sqrt{\frac{DLL}{W_T X_S}} + 1} \right] \cdot \left[\frac{\sqrt{(1-\alpha) \frac{DLL}{W_T X_S} + \alpha + 1}}{\sqrt{(1-\alpha) \frac{DLL}{W_T X_S} + \alpha - 1}} \right]$$

Equation (57) can also be written as:

$$t_N = \left[\sum_{n=1}^{N} (C_D S)_{R_n}^{-\frac{1}{2}} \right] \cdot \frac{1}{2g} \sqrt{\frac{2 W_T}{\rho}} \cdot \ln [A] \qquad (58)$$

Another form of Equation (58) can be derived from Equation (52) as:

$$t_N = \left[\sum_{n=1}^{N} \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{\frac{n-1}{2}} \right] \cdot \frac{u_o}{2g} \sqrt{\frac{W_T X_S}{DLL}} \cdot \ln [A]$$

(59)

Equations (58) and (59) represent our desired expressions for time.

2. Altitude Requirements

The altitude expressions can be derived from Equation (55) as:

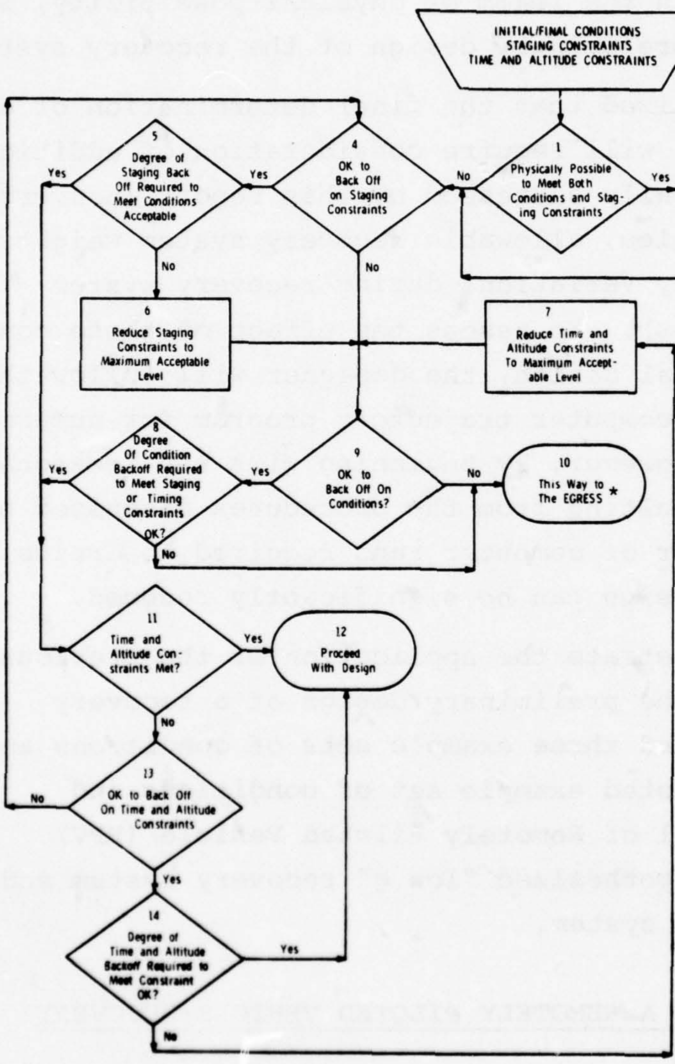
$$z_N = \left[\sum_{n=1}^{N} (C_D S)_{R_n} \right]^{-1} \cdot \frac{W_T}{g\rho} \cdot \ln \frac{1}{1-\alpha} \qquad (60)$$

Equation (60) may also be derived from Equation (56) as:

$$z_N = \left[\sum_{n=1}^{N} \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]^{n-1} \right] \frac{q_o}{\rho g} \; \frac{W_T X_S}{DLL} \cdot \ln \frac{1}{1-\alpha} \qquad (61)$$

SECTION IV
EXAMPLE APPLICATIONS

The objective of this section is to demonstrate how the equations derived in the previous sections may be used to produce a "reasonable" preliminary design of a fast inflating parachute recovery system. The general thought process used in applying the previously derived equations is shown in the following flow chart.



*Design not possible using this technique

As can be seen from the preceding flow chart, the recovery system initial design technique described in this report considers such items as initial and final conditions, staging constraints, and time and altitude constraints. The flow chart depicts the general thought process we will go through to determine a realistic, physically possible set of conditions and constraints for our recovery system design. Once we have established that the conditions and constraints, staging, timing, and altitude are within the realm of physical possibility, we can proceed with the preliminary design of the recovery system.

It should be realized that the final determination of a recovery system design will require consideration of additional variables not specifically addressed by this report (nonvertical initial trajectory angles, allowable recovery system weights and volumes and density variations during recovery system operation, for examples). To assess the effect of these considerations on the final design, the designer will follow the current practice of a computer trajectory program for numerous "fine tuning" runs. However, by beginning with the "reasonable" preliminary design resulting from the procedures discussed in this report, the number of computer runs required to arrive at an acceptable final design can be significantly reduced.

In order to demonstrate the application of the previously derived equations to the preliminary design of a recovery system, we have selected three example sets of conditions and constraints. The selected example set of conditions and constraints are typical of Remotely Piloted Vehicle (RPV) recovery systems, a hypothesized "low g" recovery system and a crew escape recovery system.

A. EXAMPLE DESIGN OF A REMOTELY PILOTED VEHICLE RECOVERY SYSTEM

The objective of this subsection is to use the equations derived in Sections II and III to obtain a "reasonable" preliminary recovery system for a set of conditions and constraints which are representative of the Remotely Piloted Vehicle (RPV) environment. RPV recovery systems generally are required to be capable of recovering a wide range of vehicle weights at a wide range of initial conditions. RPV recovery systems will usually require a large (80-100 ft diameter), slow inflating terminal descent parachute, will not be strongly driven by time and altitude requirements, and will make use of existing parachute designs.

Because of the above considerations, the example we have selected requires that we design a recovery system capable of recovering two classes of vehicles, selects the q_F condition on the basis of the maximum allowable dynamic pressure at which we can deploy the main parachute, places more importance on the staging rather than the time and altitude constraints, and asks that for vehicle deceleration we use an existing $(C_D S)_{R_N}$ = 100 ft^2 parachute (15.3 ft diameter ribbon) if possible.

The example RPV recovery system problem is defined as follows.

	Class I	Class II
Initial Conditions		
q_o (psf)	748	371
W_T (lbs)	4500	6000
ALT_o (ft)	10000	10000
Staging Constraints		
M	≤ 1.0	≤ 1.0
N	≤ 3.0	≤ 3.0
$(DAR)_{min_a}$	.22	.22
DLL (lbs)	16800	16800
Time and Altitude Constraints		
$t_{DR_{N-1}}$* (sec)	≤ 12.0	≤ 12.0
z_N (ft)	≤ 7000	≤ 7000
Final Conditions		
q_F (psf)	≤ 50	≤ 50
ALT_F (ft)	≥ 3000	≥ 3000

*This constraint is based on the longest reefing cutter time delay readily available in the designer's inventory.

Additional Constraints/Assumptions

$(C_D S)_N = 100 \text{ ft}^2$ (if possible)

$X_S = 1.5$

$$\rho = \frac{\rho_o + \rho_F}{2} = \frac{.001756 + .002177}{2} = .001967 \text{ slugs/ft}^3$$

In the case of a vertical trajectory, the requirement to cover both the Class I and Class II envelopes means that we have to design a recovery system capable of recovering a 6,000 lb vehicle at a q_o of 748 psf. To determine if it is physically possible to meet our initial and final conditions while living within our staging constraints, we use Equation (20).

$$N = \frac{\ell n \left[\frac{q_F}{q_o} \right]}{\ell n \left[1 + \alpha \left[\frac{W_T X_S}{DLL} - 1 \right] \right]}$$

Inserting the proper values into the right hand side of the above equation yields the minimum number of operating stages required to meet the initial and final conditions.

$$N = \frac{\ell n \left[\frac{50}{748} \right]}{\ell n \left[1 + 1 \left(\frac{6000(1.5)}{16800} - 1 \right) \right]} = 4.33$$

Since we know from our staging constraints that $N \leq 3$, we conclude that it is physically impossible to recover both vehicle classes using the same recovery system design without some change in either conditions or constraints. At this point, we will assume that we are willing to back off on the requirement that the same recovery system be capable of recovering both classes of vehicles. The scope of this example is now limited to designing a recovery system for the Class I vehicle.

We begin to design a Class I recovery system by again assessing whether or not it is physically possible to meet both the initial and final conditions and the staging constraints.

$$N = \frac{\ln\left[\frac{50}{748}\right]}{\ln\left[\frac{4500(1.5)}{16800}\right]} = 2.97$$

In other words, it is physically possible to design a system to meet initial and final dynamic pressures as well as the staging constraints. We now ask if that same system will be capable of meeting our time and altitude constraints as well.

In order to calculate our time and altitude requirements, we will need a numerical value for α. We obtain this value by solving Equation (19) for α and rounding N to the next highest integer of 3.

$$\alpha = \frac{\left(\frac{q_F}{q_o}\right)^{\frac{1}{N}} - 1}{\frac{W_T X_S}{DLL} - 1} = \frac{\left(\frac{50}{748}\right)^{1/3} - 1}{\frac{(4500)(1.5)}{16800} - 1} = .99$$

Solving Equation (59) for the time required before discreeting the 2nd stage yields:

$$t_n = t_{DR_2} = \left[\sum_{n=1}^{2}\left[1 + \alpha\left[\frac{W_T X_S}{DLL} - 1\right]\right]^{\frac{n-1}{2}}\right] \cdot \frac{u_o}{2g}\sqrt{\frac{W_T X_S}{DLL}}\ \ln[A]$$

$$= \left[1 + \sqrt{1 + .99\left[\frac{4500\ (1.5)}{16800} - 1\right]}\right]\frac{1}{2(32.2)}\sqrt{\frac{2(748)}{.001967}}\ \cdot$$

$$\cdot\sqrt{\frac{4500(1.5)}{16800}}\ \cdot \ln\left[\frac{\sqrt{\frac{16800}{4500\ (1.5)}} - 1}{\sqrt{\frac{16800}{4500\ (1.5)}} + 1} \cdot \frac{\sqrt{(1 - .99)\ \frac{16800}{4500\ (1.5)}} + .99 + 1}{\sqrt{(1 - .99)\ \frac{16800}{4500(1.5)}} + .99 - 1}\right]$$

$= 57.74$ sec.

Since the time constraint ($t_{DR_2} \leq 12.0$) is based on the maximum available reefing cutter time, we can't back off this value. If we assume that we are not willing to back off on our staging constraints, then our only remaining option is to back off on the initial condition, q_o.

We'll now determine, by trial and error, how much we'll have to back off on q_o. We begin by solving Equation (19) for q_o.

$$q_o = \frac{q_F}{\left[1 + \alpha\left[\frac{W_T X_S}{DLL} - 1\right]\right]^N}$$

Since we know α must be less than .99, we pick α = .8 and solve for q_o.

$$q_o = \frac{50}{\left[1 + .8\left[\frac{4500(1.5)}{16800} - 1\right]\right]^3}$$

$$= 352.68 \text{ psf}$$

That's a little bit steep for a q_o penalty; are there any other options? We remember that the above equation assumes equal α values for every operating stage. Yet our time constraint is based only on the second stage reefing cutter constraint. If we assume that $\alpha_N = \alpha_3 = .95$ and remember how we arrived at Equations 12 and 18, then the above equation becomes:

$$q_o = \frac{q_F}{\left[1 + \alpha\left[\frac{W_T X_S}{DLL} - 1\right]\right]^2 \cdot \left[1 + \alpha_N\left[\frac{W_T X_S}{DLL} - 1\right]\right]}$$

$$= \frac{50}{\left[1 + .8\,\frac{4500(1.5)}{16800} - 1\right]^2 \cdot \left[1 + .95\left[\frac{4500(1.5)}{16800} - 1\right]\right]}$$

$$= 425.99 \text{ psf}$$

According to Equation (59) this will require:

$$t_{DR_2} = \left[1 + \sqrt{1 + .8\left[\frac{4500(1.5)}{16800} - 1\right]}\right] \cdot \frac{1}{64.4} \sqrt{\frac{2(425.99)}{.001967}} \cdot$$

$$\cdot \sqrt{\frac{4500(1.5)}{16800}} \cdot \ell n\,[3.44]$$

$$= 13.80 \text{ sec.}$$

We're closer, but not quite there yet. Let's try reducing α to .75:

$$q_o = \frac{50}{\left[1 + .75 \;\; \frac{4500(1.5)}{16800} - 1\right]^2 \cdot \left[1 + .95\left[\frac{W_T X_S}{DLL} - 1\right]\right]}$$

$$= 381.03$$

and

$$t_{DR_2} = 11.13 \text{ sec}$$

We've got our design point bracketed. Selecting α = .77 results in:

$$q_o = 398.12 \text{ (psf)}$$

$$t_{DR_2} = 12.11 \text{ (sec)},$$

which is close enough, checking on our altitude constraint via Equations (61) and (56) results in:

$$z_N = z_3 = z_{DR_2} + \Delta z_3$$

$$= \left[1 + \left[1 + .77\left[\frac{4500(1.5)}{16800} - 1\right]\right]\right] \frac{1}{32.2} \cdot \frac{(398.12)}{.001967}$$

$$\cdot \frac{4500(1.5)}{16800} \quad \ell n\left[\frac{1}{1 - .77}\right]$$

$$+ \frac{404799}{64.4} (.40) \; (.54)^2 \quad \ell n\left[\frac{1}{1 - .95}\right]$$

$$= 5714 + 2201$$

$$= 7915 \text{ feet;}$$

which exceeds our altitude constraint by 915 feet.

If this performance is unacceptable, we can further reduce α until we meet the altitude constraint. For this example we will assume that the performance is acceptable and we will proceed to determine our preliminary reefing system design.

We can determine the drag area of each stage from Equations (2), (7), and (10) as:

$$(C_DS)_{R_1} = \frac{16800}{398.12(1.5)} = 28.13 \text{ ft}^2$$

$$(C_DS)_{R_2} = \frac{16800}{(398.12)(1.5)\left[1 + .77\left[\frac{4500(1.5)}{16800} - 1\right]\right]} = 52.16 \text{ ft}^2$$

$$(C_DS)_N = \frac{16800}{(398.12)(1.5)\left[1 + .77\left[\frac{4500(1.5)}{16800} - 1\right]\right]^2} = 96.70 \text{ ft}^2$$

We can now determine what the effects of using a $(C_DS)_N = 100 \text{ ft}^2$ parachute are on our recovery system performance. Solving Equation (16) for q_o and inserting $(C_DS)_N = 100 \text{ ft}^2$ indicates that if we use the existing parachute our q_o will be:

$$q_o = \frac{16800}{100(1.5)\left[1 + .77\left[\frac{4500(1.5)}{16800} - 1\right]\right]^2} = 384.98 \text{ psf}$$

The larger (100 ft^2) drogue chute and the reduced q_o will reduce the α_N value required.

We will need to know the α_N value to determine the time and altitude requirements of the last operating stage. We can determine what the required α_N value is by solving Equations (12) and (18) for the last α value in the equation, i.e.,

$$\alpha_N = \alpha_3 = \frac{\left[\dfrac{q_F}{q_o\left[1+\alpha\left[\dfrac{W_T X_S}{DLL} - 1\right]\right]^2}\right] - 1}{\dfrac{W_T X_S}{DLL} - 1} = .925.$$

We can now proceed with the detailed design of our recovery system using the $(C_D S)_N = 100\ ft^2$ parachute. We do this by using Equations (13), (51), and (55) to calculate the following table.

Operating Stage No.	α_n	$(C_D S)_{R_n}$ Eqn. (13)	Δt_n Eqn. (51)	t_n $\Sigma\Delta t_n$	Δz_n Eqn. (55)	z_n $\Sigma\Delta z_n$
1	.77	29.09	6.87	6.87	3589	3589
2	.77	53.94	5.04	11.91	1936	5525
3	.925	100.00	7.10	19.01	1840	7365

The last remaining check is to see that we have met our $(DAR)_{min_a}$ constraint. Since we only have one parachute stage, this is expressed as:

$$\frac{(C_D S)_{R_{min}}}{(C_D S)_N} = \frac{29.09}{100} \quad .291 \geq (DAR)_{min_a} = .22$$

We have met this last remaining constraint, the design is valid.

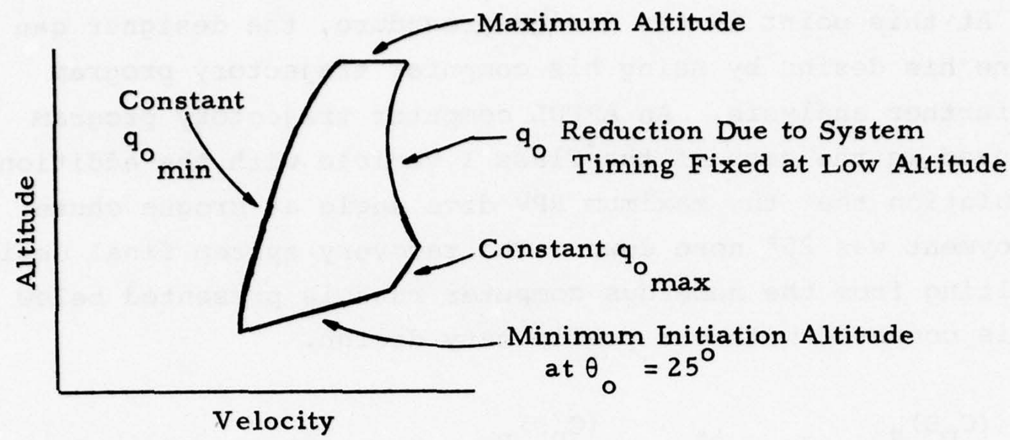
In summary, we have designed a system for the Class I vehicle which is capable of being deployed at q_o = 385 psf, uses the existing 100 ft^2 drogue chute, provides the required final dynamic pressure condition of 50 psf and meets all of the constraints but the altitude constraint (which it exceeds by 365 feet). If the failure to meet the altitue constraint is of concern, we can reduce α_1, α_2, and q_o slightly and repeat the above process. For purposes of this discussion, the altitude loss is assumed acceptable.

At this point in the design procedure, the designer can refine his design by using his computer trajectory program for further analysis. An AFFDL computer trajectory program was used in the case of the Class I vehicle with the additional stipulation that the maximum RPV dive angle at drogue chute deployment was 25° nose down. The recovery system final design resulting from the numerous computer runs is presented below and is compared with our preliminary design.

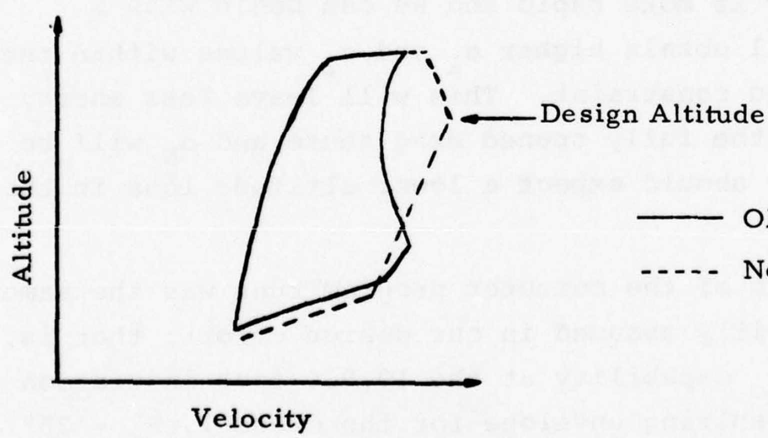
q_o	$(C_DS)_{R_1}$	α_1	Δt_1	$(C_DS)_{R_2}$	α_2	Δt_2	α_N	z_N
PRELIMINARY DESIGN (technique from this report), θ=90°								
385	29.1	.77	6.9	53.9	.77	5.0	.93	7365
FINAL DESIGN (AFFDL computer program), θ=25°								
430	26.9	.91	5.6	59.1	.91	6.4	.73	4040

Looking at the preceding table, the effects of the trajectory angel variation becomes evident. In the θ=25° case, the q decay is more rapid and we can begin with a higher q_o and will obtain higher α_1 and α_2 values within the same t_{DR_2} = 12 sec constraint. This will leave less energy to be removed by the fully opened drag chute and α_N will be lowered. We also should expect a lower altitude loss in the θ=25° case.

The objective of the computer program runs was the same as we have implicitly assumed in our design effort; that is, to maximize the q_o capability at the 10,000 foot initiation altitude. The resulting envelope for the Class I, θ_o = 25° case is shown below.



If we wish to increase our velocity (q_o) capability for the higher altitudes, we can do so by determining our operating times and drag areas at the densities associated with the higher altitudes. After determining the required operating times at the higher altitudes, we would then determine the effect that the longer operating times would have on low altitude performance. A rough sketch of the anticipated change in the envelope due to this procedure is shown as follows.



Use of this design procedure and realization of the physical consequences associated with the numerous assumptions will give the designer both a "reasonable" preliminary design and a rough indication of how to modify his design to meet additional conditions and constraints which are not directly addressed by this report. It is believed that, once the design procedure discussed in this report is fully understood, the process of arriving at an initial recovery system design may require less than one person day of time. Based on AFFDL experience, it is estimated that use of this technique can reduce the assets (person hours and computer time) required to establish a recovery system final design by 25 to 50 percent.

B. EXAMPLE DESIGN OF A "LOW G" RECOVERY SYSTEM

The objective of this subsection is to establish the preliminary design for the recovery system that was originally requested in the previous RPV recovery system example (i.e., one capable of recovering both Class I and Class II vehicles). This time we will address the problem without imposing any constraints on the number of operating stages, time or altitude. We want to determine what is required to provide a system having the following capability:

Initial Conditions

$$q_o = 748 \text{ psf}$$

$$W_T = 6{,}000 \text{ lbs}$$

$$ALT_o = 20{,}000 \text{ ft*}$$

Staging Constraints

$$(DAR)_{min_a} = .22$$

$$DLL = 16{,}800 \text{ lbs}$$

*We need some initial altitude to determine ρ. This initial altitude will be increased or decreased as required by z_N.

Final Conditions

$$q_F \leq \frac{W_T}{(C_D S)} = \frac{6000}{\frac{4500}{50}} = 67 \text{ psf}$$

$$ALT_F = 3,000 \text{ ft}$$

Additional Assumptions

$$X_S = 1.5$$

$$\rho = \frac{\rho_o + \rho_F}{2} = \frac{.001267 + .002177}{2} = .001722 \text{ slugs/ft}^3$$

Our first step will be to determine the minimum number of operating stages (N) required to meet our q_o and q_F conditions. Using Equation (20) and inserting $\alpha = 1.0$ yields:

$$N = \frac{\ln\left[\frac{q_F}{q_o}\right]}{\ln\left[\frac{W_T X_S}{DLL}\right]}$$

$$= \frac{\ln\left[\frac{67}{748}\right]}{\ln\left[\frac{9000}{16800}\right]} = \frac{-2.41}{-.62} = 3.87.$$

We'll need a minimum of four operating stages. Solving Equation (19) for α and inserting N = 4 indicates that this will allow a system α value of:

$$\alpha = \frac{\left(\frac{q_F}{q_o}\right)^{1/4} - 1}{\frac{W_T X_S}{DLL} - 1} = .98$$

Which according to Equations (59) and (61) will require:

$$t_N = t_F = \sqrt{\frac{2(748)}{.001722}}\ (.73)\ \frac{1}{64.4}\ \ell n\,[3603] \left\{ (.55)^{3/2} + .55 + \sqrt{.55} + 1 \right\}$$

$$= 101.97 \text{ sec}$$

and

$$z_N = z_F = \frac{6000(1.5)}{16800} \cdot \frac{748}{(.001722)(32.2)} \cdot \ell n \left[\frac{1}{1-.98}\right]$$

$$\cdot \left\{ (.55)^3 + (.55)^2 + (.55) + 1 \right\} = 56,653 \text{ ft}$$

Repeating the above procedures for an increasing number of operating stages and using Equations (30) and (33a) yields:

N	α	t_F	z_F	N_m	M
4	.98	101.97	56653	$\leq$ 2.5	$\geq$ 1.1
5	.82	50.44	29585	$\leq$ 3.2	$\geq$ 1.2
6	.71	41.52	24676	$\leq$ 3.8	$\geq$ 1.3
7	.63	37.54	22385	$\leq$ 4.4	$\geq$ 1.3
8	.56	34.64	20767	$\leq$ 5.0	$\geq$ 1.3
.	.	.	.	.	.
.	.	.	.	.	.
10	.46	31.48	18964	$\leq$ 6.3	$\geq$ 1.4

One thing that the above table points out is that our ALTO value may have to be greater than 20,000 feet. For the purpose of the calculations associated with this preliminary design, however, our design density will be maintained at .001722 slugs/ft^3.

To obtain some perspective on how the above table compares with an "ideal" case, we will allow N to approach infinity. In this case, we are effectively maintaining a constant deceleration force equal to:

$$\text{Deceleration Force} = DF = \left[\frac{DLL}{X_S} - W_T\right]$$

Since the change in kinetic energy is equal to a constant value, and since we are using the largest allowable force (DF), it then follows that the altitude loss is minimum.

$$\Delta \text{ Kinetic Energy} = \frac{1}{2} m(u_o^2 - u_F^2) = DF \cdot \text{dist} = \text{work done}$$

$$\Delta KE = \left[\frac{DLL}{X_S} - W_T\right] \cdot z_{min}$$

We also know that

$$\Delta \text{ momentum} = m(u_o - u_F) = DF \cdot \text{time}$$

The same logic used to explain the rationale for z_{min} applies to t_{min}

$$\Delta \text{ momentum} = \left[\frac{DLL}{X_S} - W_T\right] \cdot t_{min}$$

We can solve for z_{min} and t_{min} as:

$$z_{min} = \frac{u_o^2 - u_F^2}{2g\left[\frac{DLL}{W_T X_S} - 1\right]} = \frac{q_o - q_F}{\rho g\left[\frac{DLL}{W_T X_S} - 1\right]}$$

$$t_{min} = \frac{u_o - u_F}{g\left[\frac{DLL}{W_T X_S} - 1\right]} = \frac{\sqrt{\frac{2}{\rho}}\left[\sqrt{q_o} - \sqrt{q_F}\right]}{g\left[\frac{DLL}{W_T X_S} - 1\right]}$$

Inserting the proper values into the above equations gives us our absolute minimum time and altitude requirements (when N approaches infinity) and gives us a basis for evaluating the relative performance of a finite number of operating stages. The previous table therefore becomes:

N	α	t_F	z_F	N_m	M
4	.98	102.0	56653	$\leq$ 2.5	$\geq$ 1.1
5	.82	50.4	29585	$\leq$ 3.2	$\geq$ 1.2
6	.71	41.5	24676	$\leq$ 3.8	$\geq$ 1.3
7	.63	37.5	22385	$\leq$ 4.4	$\geq$ 1.3
8	.56	34.6	20767	$\leq$ 5.0	$\geq$ 1.3
.	.	.	.	.	.
.	.	.	.	.	.
10	.46	31.5	18964	$\leq$ 6.3	$\geq$ 1.4
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
∞	0	23.4	14171	$\leq$ ∞	$\geq$ 1.6

Several things become evident upon examining the above table. The major conclusion is that the higher we go in the total number of operating stages, the lower the payoff in terms of t_F and z_F reductions. The designer must look at the above table and determine at what point the further improvements in performance (lowered t_F and z_F) do not warrant the complexity associated with the addition of another operating stage. For this example we have selected N = 6. This was done for the following reasons:

1. It is obvious from the M values that we will need at least two parachute stages, therefore, we select M = 2.
2. The maximum number of reefed stages that anyone has currently put on a drogue parachute is two. We are not willing to push the state-of-the-art past this point, therefore, we select $N_m = 3$.
3. We are willing to live with the t_F and z_F values associated with six operating stages.

We now proceed with establishing the design details associated with our recovery system. Use of Equations (13), (51), and (55) results in the following table.

Stage Number	$(C_DS)_{R_n}$	Δt_n	t_n	Δz_n	z_n
1	14.97	10.77	10.77	8948	8948
2	22.34	8.82	19.59	5996	14944
3	33.32	7.22	26.81	4020	18964
4	49.70	5.91	32.72	2695	21659
5	74.15	4.84	37.56	1806	23465
6	110.61	3.96	41.52	1211	24676

If we are willing to live with this kind of time and altitude requirements and if we have the hardware required to reliably produce the specified time delays then the next step would be to establish staging q values and then finalize the design either by further hand calculations or by use of recovery systems designer's computer programs.

C. EXAMPLE DESIGN OF A CREW ESCAPE RECOVERY SYSTEM

The objective of this subsection is to obtain a "reasonable" recovery system preliminary design for a set of conditions and constraints which are representative of recovery systems used by personnel ejecting from high-speed aircraft. These recovery systems are usually "g" constrained (due to human acceleration tolerance limits), have to operate across a wide range of q_o conditions and are strongly driven by their minimum time and altitude constraints. These systems also have a high reliability requirement, a relatively large payload (man/ejection seat) drag area and a low terminal descent rate requirement. The example selected reflects the above requirements/constraints.

The fact that man/seat drag area is not negligible results in a requirement to rederive the staging, timing and altitude equations developed in Sections II and III. This has been done and the results are presented in the Appendix. Note that the form of the equations makes it impossible to derive an explicit expression for N, the total number of operating stages required. To determine this value, the calculations continue until the required final dynamic pressure is achieved (i.e., until $q_{DE_N} \leq q_F$.

The requirement for a low terminal descent rate (compatible with human tolerance limits) requires that a relatively large (28 foot diameter) parachute be used for the terminal descent phase. It is this author's opinion that the inflation time for such a parachute does not meet the fast inflating assumption used in deriving the equations. To assume otherwise would introduce significant departures between the analytical predictions and the real world. The q_F condition for this recovery system example is therefore based on a rough estimate of the maximum allowable dynamic pressure at which we can deploy the terminal descent parachute and therefore this design is only concerned with the man/seat deceleration parachute, the drogue parachute.

Initial Conditions

ALT_o = 2000 feet

q_o = 1200 psf

W_T = 400 lbs

Staging Constraints

$M \leq 1.0$

$N \leq 2.0$

DAR_{min_a} = .30

$\#g's = 30 \Longrightarrow DLL = 30 \cdot 400 = 12{,}000$ lbs

Time and Altitude Constraints

$t_F \leq 2.0$ sec

$z_f \geq 1000$ feet

Final Conditions

$$q_F = 250 \text{ psf}$$

$$ALT_F = 1000 \text{ feet}$$

Additional Assumptions

$$X_{S_{drogue}} = 1.5$$

$$\rho = \frac{\rho_o + \rho_F}{2} = \frac{.002242 + .002310}{2} = .002276 \text{ slugs/ft}^3$$

$$(C_D S)\ M/S = 7.0 \text{ ft}^2$$

To determine whether or not it is physically possible to design a system to meet the above conditions and staging constraints, we examine the equation

$$q_{DR_N} = q_o K_1 K_2 \ldots K_n$$

where K_n values are mathematically defined in the Appendix. We now attempt to determine if

$$\frac{q_{DR_N}}{q_o} = \frac{q_F}{q_o} = \frac{250}{1200} \geq K_1 \cdot K_2$$

with $\alpha = 1.0$

$$K_1 = \frac{W_T X_S}{DLL + q_o (C_D S)_{M/S} (X_S - 1)}$$

$$= \frac{400(1.5)}{12000 + (1200)(7.0)(.5)} = .0370$$

$$K_2 = \frac{W_T X_S}{DLL + q_o K_1 (C_D S)_{M/S} (X_S - 1)}$$

$$= \frac{(400)(1.5)}{12000 + (1200)(.0370)(7)(.5)} = .0494$$

$$K_1K_2 = (.0370)(.0496) = .0018$$

$$\frac{250}{1200} = .21 \geq .0018$$

It is physically possible to meet the staging constraints of this system. We must now determine whether or not it is possible to meet the time and altitude constraints.

From the Appendix, we know

$$(C_DS)_{R_1} = \frac{DLL}{q_oX_S} - \frac{(C_DS)_{M/S}}{X_S}$$

$$= \frac{12,000}{1200\ (1.5)} - \frac{7}{1.5}$$

$$= 2.00\ ft^2$$

and assuming $\alpha = 1.0$

$$(C_DS)_{R_2} = \frac{DLL}{q_oK_1X_s} - \frac{(C_DS)_{M/S}}{X_S}$$

$$= \frac{12000}{(1200)(.0370)(1.5)} - \frac{7}{1.5}$$

$$= 175\ ft^2$$

but our DAR_{min_a} constraint means that the maximum allowable full open drag area value is really

$$(C_DS)_{R_N} = \frac{(C_DS)_{R_1}}{DAR} = \frac{2}{.3} = 6.67\ ft^2$$

Using this value in the DLL equations means that

$$DLL = q_{DR_1}\left((C_DS)_{M/S} + X_S(C_DS)_{R_N}\right)$$

or

$$q_{DR_1} = \frac{DLL}{(C_DS)_{M/S} + X_S(C_DS)_{R_N}} = \frac{12000}{7 + 1.5(6.67)}$$

$$= 706 \text{ psf}$$

the q_{T_1} value is

$$q_{T_1} = \frac{W_T}{(C_DS)_{M/S} + (C_DS)_{R_1}} = \frac{400}{7 + 2}$$

$$= 44.4 \text{ psf}$$

The α value is therefore

$$\alpha_1 = \frac{q_o - q_{DR_1}}{q_o - q_{T_1}} = \frac{1200 - 706}{1200 - 44.4} = .4275$$

The q_{T_2} value is

$$q_{T_2} = \frac{W_T}{(C_DS)_{M/S} + (C_DS)_{FD}} = \frac{400}{7 + 6.67} = 29.3 \text{ psf}$$

The α_2 value is

$$\alpha_2 = \frac{q_{DR_1} - q_F}{q_{DR_1} - q_{T_2}} = \frac{706 - 250}{706 - 29.3} = .6739$$

At this point we should note that it won't do us any good to try to select identical values for α_1 and α_2 because our α values are based on our q_o, q_F, and DAR_{min_a} constraints (we can't go to a larger full open drogue chute). We will therefore calculate what the performance of the resulting drogue chute portion of the system will be.

The K_1 and K_2 values are:

$$K_1 = 1 + .4275 \left[\frac{400\ (1.5)}{12000 + 1200(7)(.5)} - 1 \right] = .5883$$

$$K_2 = 1 + .6739 \left[\frac{400\ (1.5)}{12000 + 1200(7)(.5)} - 1 \right] = .3540$$

We now use the Equations from the Appendix to solve for the times required for drogue chute operation.

$$\Delta t_1 = \sqrt{\frac{1200}{2(.002276)}} \quad \frac{1}{32.2} \quad \sqrt{\frac{400(1.5)}{12000 + 1200(7)(.5)}}$$

$$\ell n \left[\frac{\left(\sqrt{\frac{12000 + 1200(7).5}{400(1.5)}} - 1\right)}{\left(\sqrt{\frac{12000 + 1200(7).5}{400(1.5)}} + 1\right)} \frac{\left(\sqrt{\frac{12000(.5883) + 1200(.5883)7(.5)}{400(1.5)}} + 1\right)}{\left(\sqrt{\frac{12000(.5883) + 1200(.5883)7(.5)}{400(1.5)}} - 1\right)} \right]$$

$$= 3.0687\ \ell n[1.1309]$$

$$= .38 \text{ sec}$$

$$\Delta t_2 = \sqrt{\frac{1200}{2(.002276)}} \quad \frac{1}{32.2} \quad \sqrt{\frac{400(1.5)(.5883)}{12000 + 1200(.5883)7(.5)}}$$

$$\ell n \left[\frac{\left(\sqrt{\frac{12000 + 1200(.5883)7(.5)}{400(1.5)}} - 1\right)}{\left(\sqrt{\frac{12000 + 1200(.5883)7(.5)}{400(1.5)}} + 1\right)} \cdot \frac{\left(\sqrt{\frac{(.3540)(12000 + 1200(.5883)7(.5))}{400(1.5)}} + 1\right)}{\left(\sqrt{\frac{(.3540)(12000 + 1200(.5883)7(.5))}{400(1.5)}} - 1\right)} \right]$$

$$= 2.4904\ \ell n[1.2889]$$

$$= .75 \text{ sec}$$

$$\Delta t_F = \Delta t_1 = \Delta t_2 = .38 + .75 = 1.13 \text{ sec}$$

We now use the Equations in the Appendix to solve for the altitude required for drogue chute operation.

$$\Delta z_1 = \frac{1200}{.002276(32.2)} \; \frac{400(1.5)}{12000 + 1200(7).5}$$

$$\ln \left[\frac{\dfrac{12000 + 1200(7).5}{400(1.5)} - 1}{\dfrac{.5883(12000 + 1200(7).5)}{400(1.5)} - 1} \right]$$

$$= 606.44 \ \ln[1.7468]$$

$$= 338 \text{ ft}$$

$$\Delta z_2 = \frac{1200}{.002276(32.2)} \; \frac{400(1.5)(.5883)}{12000 + 1200(.5883)7(.5)}$$

$$\ln \left[\frac{\dfrac{12000 + 1200\ (.5883)7(.5)}{400(1.5)} - 1}{(.3540) \left[\dfrac{12000 + 1200(.5883)7(.5)}{400(1.5)} \right] - 1} \right]$$

$$= 399.40 \ \ln[3.0670]$$

$$= 448 \text{ ft}$$

$$\Delta z_F = \Delta z_1 + \Delta z_2 = 338 + 448 = 786 \text{ feet}$$

In summary, we have established a preliminary design for the drogue chute portion of the recovery system which requires 1.1 seconds and 786 feet of altitude to provide the required dynamic pressure decay from 1200 to 250 psf. The system meets or exceeds all performance constraints levied on it. The next steps in the design process would be to set the system timing at the maximum initiation altitude and then determine the effect of this revised timing on the low altitude performance. These steps could be performed either by further hand calculations or by insertion into the designer's computer trajectory program.

The objectives of this subsection have been met.

SECTION V
DISCUSSION

The objective of this section is to briefly discuss (1) some of the assumptions that were made in the derivation of the staging, timing and altitude equations, (2) the effect of these assumptions on the resulting design procedures, and (3) some areas offering potentially high payoffs for future work.

The discussion is not intended to be a mathematically rigorous examination of all of the assumptions made throughout this report, and no attempt is made to extract every possible recommendation. The purpose of this discussion is to acquaint the reader with some of the larger assumptions we have made, their effect, and how to remove or reduce this effect. It is hoped that by this process, the reader will acquire additional insights into the current limitations of the design technique and will be able to extend or modify it to suit his specific requirements or goals.

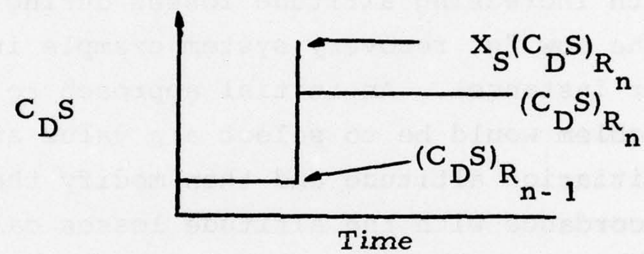
A. ASSUMPTIONS

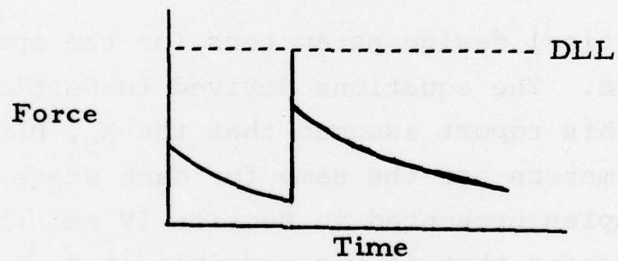
Some of the known assumptions made throughout this report and a brief discussion of their effects are listed below.

1. Vertical trajectory. The design technique presented in this report assumes a worst-case condition where the parachute is deployed from a vehicle descending vertically. In this case, the velocity decay versus time is minimized and the time and altitude required are maximized. If the designer's recovery system does not have to recover the vehicle from a vertical trajectory, use of this reports' design technique will result in a conservative estimate of the recovery system's performance capability. If a less conservative

estimate of the recovery systems performance capability is desired, the design resulting from this technique may be inserted into the designer's computer trajectory program for further refinement.

2. Step function increases in drag area. The technique presented in this report assumes that the time required to proceed from the drag area of one operating stage to the drag area of the next operating stage (filling time) is essentially zero. A better physical picture of what we are assuming can be obtained from the figures below.





As a result of the preceding model, this design technique is most accurate when dealing with fast opening parachutes (drogues and possibly ram-air hi-glide parachutes). When the design technique is applied to a slower opening parachute, it will result in a conservative (pessimistic) estimate of recovery system performance. The designer will have to evaluate when the zero filling time assumption introduces an unacceptable degree of error, the point in his system design at which this occurs and the conditions (time, altitude, dynamic pressures, etc.) at this point. These conditions will then be used to establish the required final conditions for that portion of the recovery system design where use of the design technique is acceptable.

3. Negligible variation in air density during system operation. The design technique presented in this report assumes that the air density, ρ, is constant during system operation. While this assumption may be acceptable for those systems experiencing relatively small altitude losses, it becomes increasingly suspect with increasing altitude losses during system operation (the low "g" recovery system example in Section IV, for instance). An initial approach to resolve this problem would be to select a ρ value at the maximum initiation altitude and then modify the ρ value in accordance with the altitude losses calculated for each operating stage.

4. Identical design parameters for the operation of each stage. The equations derived in Section II and III of this report assumed that the X_S, DLL, W_T and α parameters are the same for each stage. The design examples presented in Section IV and the Appendix indicated that it was possible to depart from these assumptions when desired but that the equations became a bit more complex. In using the design technique the designer will have to assess when it is to his benefit to depart from the assumption of identical design parameters.

5. Subsonic recovery system. The design technique presented in this report does not automatically check to make sure that the subsonic condition is

not violated and indeed, we have violated this assumption in some of our examples (the low "g" recovery system example). The reason for stating that the use of this design technique is limited to the subsonic regime is because the applicability of the technique has not yet been evaluated for the supersonic regime.

B. RECOMMENDATIONS

There are several areas where additional effort offers the potential for significantly increasing the capability of the design technique presented in this report. These areas are listed below and are generally keyed to the previous subsection on assumptions.

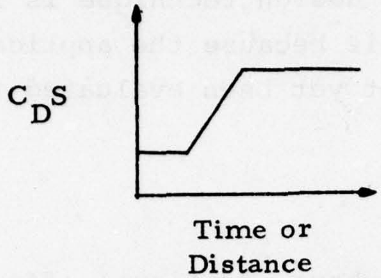
1. Solve for a more general form of the equations of motion. It can be shown that a more general form of Equation (35) can be written as:

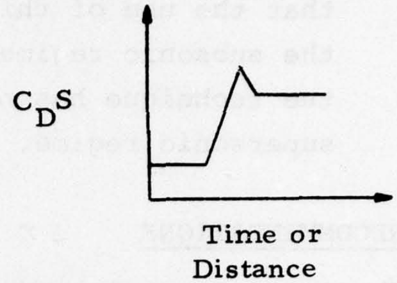
$$\frac{du}{u} - \frac{g}{u}\,dt = \frac{dv}{v}$$

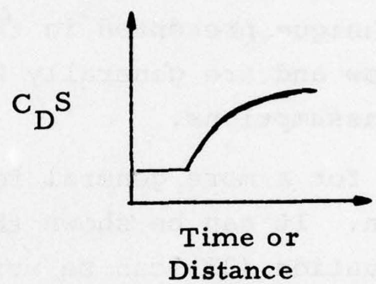
   where u and v are the vertical and horizontal velocities respectively. It is the author's personal belief that a solution can be obtained for the above expression. Such an expression would remove our dependence on the assumption of a purely vertical trajectory.

2. Consider other mathematical models for increases in drag area. The current technique assumes a step function model for increases in drag area. This assumption limits the application of the technique to fast opening parachutes. If the staging, timing and altitude equations can be rederived using other mathematical models for increases in drag (some

examples of which are illustrated below), it may be possible to extend the application of this design technique to slower opening parachutes.

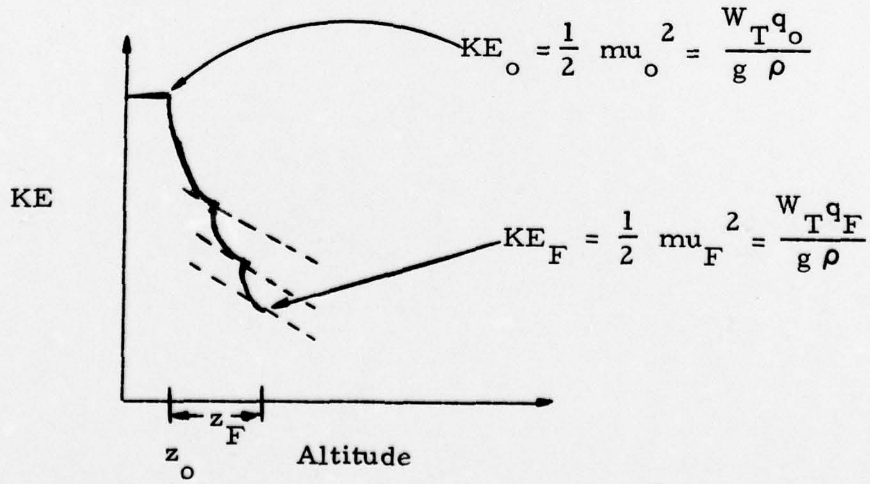






3. Consider using a mathematical model to express the variation of air density with altitude. In Section III the air density (ρ) was assumed to be constant throughout the altitude variation experienced during the operation of the recovery system. This assumption introduces increasing amounts of error with those systems experiencing large altitude losses during recovery systems operations. Insertion of a mathematical model of $\rho(z)$ into equations similar to those derived in Section III would reduce the effect of this error source.
4. Consider other criteria for staging. The current design technique assumes that we will proceed to the next operating stage when an α percentage of the available q decay is obtained. The equations derived

in Section II and III also assume that we want equal α values for each operation stage. If our recovery system design must be strongly oriented towards either a time requirement or an altitude loss requirement, then it may be possible that a design technique oriented towards optimizing either of these parameters would have different α values for each stage of operation. One approach towards determining what values would minimize the time or distance requirements of a recovery system having a fixed number of operating stages would be to plot either the momentum versus time (for the time constrained system) or the kinetic energy versus distance (for the altitude constrained system). Once the plot was drawn we could attempt to determine the maximum slope for staging and still meet our constraints on the number of operating stages. Once the value of that slope was determined, it should be possible to determine the α value associated with that slope for each operating stage. The following plot of kinetic energy versus distance may be of some assistance in following the logic for the vertical trajectory case where we are attempting to minimize the altitude losses.



If it turns out to be possible to carry out the above steps, we should be able to mathematically identify the α values for each stage that will minimize the altitude or time losses associated with recovery systems operations.

5. Evaluate the applicability of the design technique discussed in this report to the supersonic regime.

SECTION VI
CONCLUSIONS

1. Use of the design technique described in this report will result in "reasonable" recovery system preliminary designs for fast inflating parachutes.

2. This design technique appears to offer potential for modifications that may result in preliminary designs which come closer to final designs.

3. Until such time as this design technique can be expanded to include consideration of the effects of the variables which it currently ignores, the recovery systems designer will have to rely on his computer trajectories to refine his recovery system preliminary design into a final design. Nevertheless, use of this analytical technique will significantly reduce the assets required to arrive at a final recovery system design.

Obviously, use of the equations and procedures presented in this report must not be a matter of blind faith. The recovery systems designer must have a good working knowledge of the numerous assumptions made, their effects, and when the assumptions invalidate the application of the design technique to his specific problem. The design technique does appear to have resulted in a powerful tool for use by the designer. Presentation of this tool and demonstration of its use has been the reason for the preparation of this report.

APPENDIX

STAGING EQUATIONS FOR PAYLOADS WITH NONNEGLIGIBLE DRAG AREAS

Rewriting Equations (1) through (5) for payloads with nonnegligible drag areas for the first operating stage yields:

$$DLL = q_o \left((C_D S)_{M/S} + X_S \quad (C_D S)_{R_1} \right)$$

$$C_D S_{R_1} = \frac{\frac{DLL}{q_o} - (C_D S)_{M/S}}{X_S}$$

$$q_{T_1} = \frac{W_T}{(C_D S)_{R_1} + (C_D S)_{M/S}}$$

$$= \frac{W_T}{\frac{DLL}{q_o X_S} - \frac{(C_D S)_{M/S}}{X_S} + (C_D S)_{M/S}}$$

$$= q_o \frac{W_T X_S}{DLL + q_o (C_D S)_{M/S} (X_S - 1)}$$

$$\alpha_1 = \frac{q_o - q_{DR_1}}{q_o - q_{T_1}} \Rightarrow q_{DR_1} = q_o (1 - \alpha_1) + q_{T_1} \alpha_1$$

$$q_{DR_1} = q_o (1 - \alpha_1) + q_o \alpha_1 \frac{W_T X_S}{DLL + q_o (C_D S)_{M/S} (X_S - 1)}$$

$$= q_o \left[1 + \alpha \left[\frac{W_T X_S}{DLL + q_o (C_D S)_{M/S} (X_S - 1)} - 1 \right] \right]$$

$$= q_o \; K_1$$

And for the second operating stage:

$$DLL = q_{DR_1}\left((C_DS)_{M/S} + X_S(C_DS)_{R_2}\right)$$

$$(C_DS)_{R_2} = \frac{\dfrac{DLL}{q_{DR_1}} - (C_DS)_{M/S}}{X_S} = \frac{DLL}{q_oK_1X_S} - \frac{(C_DS)_{M/S}}{X_S}$$

$$q_{T_2} = \frac{W_T}{(C_DS)_{M/S} + (C_DS)_{R_2}}$$

$$= \frac{W_T}{\dfrac{DLL}{q_oK_1X_S} - \dfrac{(C_DS)_{M/S}}{X_S} + (C_DS)_{M/S}}$$

$$= q_o \frac{W_TK_1X_S}{DLL + q_oK_1(C_DS)_{M/S}(X_S - 1)}$$

$$\alpha_2 = \frac{q_{DR_1} - q_{DR_2}}{q_{DR_1} - q_{T_2}}$$

$$q_{DR_2} = q_{DR_1}(1 - \alpha_2) + q_{T_2}\alpha_2$$

$$= q_oK_1(1 - \alpha_2) + q_o\alpha_2 \frac{W_TK_1X_S}{DLL + q_oK_1(C_DS)_{M/S}(X_S - 1)}$$

$$= q_oK_1\left[1 + \alpha_2\left[\frac{W_TX_S}{DLL + q_oK_1(C_DS)_{M/S}(X_S - 1)} - 1\right]\right]$$

$$= q_oK_1K_2$$

And for the general operating stage:

$$(C_DS)_{R_n} = \frac{DLL}{q_{DR_{n-1}}\, X_S} - \frac{(C_DS)_{M/S}}{X_S}$$

$$= \frac{1}{X_S}\left[\frac{DLL}{q_{DR_{n-1}}} - (C_DS)_{M/S}\right]$$

$$= \frac{1}{X_S}\left[\frac{DLL}{q_o K_1 K_2 \ldots K_{n-1}} - (C_DS)_{M/S}\right]$$

$$K_n = \left[1 + \alpha_n\left[\frac{W_T X_S}{DLL + q_o K_1 K_2 \ldots K_{n-1}(C_DS)_{M/S}(X_S - 1)} - 1\right]\right]$$

Note: $K_o \equiv 1$

$$q_{T_n} = \frac{W_T}{(C_DS)_{M/S} + (C_DS)_{R_n}}$$

$$= q_o \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1}\ (C_DS)_{M/S}\ (X_S - 1)}$$

$$q_{DR_n} = q_o K_1 K_2 \ldots K_n$$

Note: Solving for n or N is impossible.

Equation (46) can be expressed as:

$$\Delta t_n = \frac{u_{T_n}}{2g}\ \ell n\left[\frac{(u_{DR_{n-1}} - u_{T_n})}{(u_{DR_{n-1}} - u_{T_n})} \cdot \frac{(u_{DR_n} + u_{T_n})}{(u_{DR_n} - u_{T_n})}\right]$$

$$= \sqrt{\frac{2q_{T_n}}{\rho}}\ \frac{1}{2\ g}\ \ell n\left[\frac{\left(\sqrt{q_{DR_{n-1}}} - \sqrt{q_{T_n}}\right)}{\left(\sqrt{q_{DR_{n-1}}} + \sqrt{q_{T_n}}\right)} \cdot \frac{\left(\sqrt{q_{DR_n}} + \sqrt{q_{T_n}}\right)}{\left(\sqrt{q_{DR_n}} - \sqrt{q_{T_n}}\right)}\right]$$

$$= \frac{1}{\sqrt{2\rho}\, g} \sqrt{q_o \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}}$$

$$\ln \left[\frac{\left(\sqrt{q_o K_1 K_2 \ldots K_{n-1}} - \sqrt{q_o \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right)}{\left(\sqrt{q_o K_1 K_2 \ldots K_{n-1}} + \sqrt{q_o \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right)} \right.$$

$$\left. \frac{\left(\sqrt{q_o K_1 K_2 \ldots K_n} + \sqrt{q_o \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right)}{\left(\sqrt{q_o K_1 K_2 \ldots K_n} - \sqrt{q_o \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right)} \right]$$

$$\Delta t_n = \sqrt{\frac{q_o}{2\rho}}\ \frac{1}{g} \sqrt{\frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}}$$

$$\ln \left[\frac{\left(1 - \sqrt{\frac{W_T X_S}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right)}{\left(1 + \sqrt{\frac{W_T X_S}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right)} \right.$$

$$\left. \frac{\left(\sqrt{K_n} + \sqrt{\frac{W_T X_S}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right)}{\left(\sqrt{K_n} - \sqrt{\frac{W_T X_S}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right)} \right]$$

$$\Delta t_n = \sqrt{\frac{q_o}{2\rho}} \; \frac{1}{g} \sqrt{\frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}}$$

$$\ln \left[\frac{\left(\sqrt{\frac{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}{W_T X_S}} - 1 \right)}{\left(\sqrt{\frac{DLL + q_o K_1 K_2 \ldots K_{n} 1 (C_D S)_{M/S} (X_S - 1)}{W_T X_S}} + 1 \right)} \right.$$

$$\left. \cdot \; \frac{\left(\sqrt{\frac{DLL\ K_n + q_o K_1 K_2 \ldots K_n (C_D S)_{M/S} (X_S - 1)}{W_T X_S}} + 1 \right)}{\left(\sqrt{\frac{DLL\ K_n + q_o K_1 K_2 \ldots K_n (C_D S)_{M/S} (X_S - 1)}{W_T X_S}} - 1 \right)} \right]$$

Equation (53) can be expressed as:

$$\Delta Z_n = \frac{u_{T_n}^{\;2}}{2g} \; \ln \left[\frac{u_{DR_{n-1}}^{\;2} - u_{T_n}^{\;2}}{u_{DR_n}^{\;2} - u_{T_n}^{\;2}} \right]$$

$$= \frac{2q_{T_n}}{\rho 2g} \; \ln \left[\frac{q_{DR_{n-1}} - q_{T_n}}{q_{DR_n} - q_{T_n}} \right]$$

$$= \frac{q_o}{\rho_g} \; \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}$$

$$\ln \left[\frac{q_o K_1 K_2 \ldots K_{n-1} - q_o \dfrac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}}{q_o K_1 K_2 \ldots K_n - q_o \dfrac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right]$$

$$= \frac{q_o}{\rho g} \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}$$

$$\ell n \left[\frac{1 - \dfrac{W_T X_S}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}}{K_n - \dfrac{W_T X_S}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}} \right]$$

$$\Delta Z_n = \frac{q_o}{\rho g} \frac{W_T X_S K_1 K_2 \ldots K_{n-1}}{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}$$

$$\ell n \left[\frac{\dfrac{DLL + q_o K_1 K_2 \ldots K_{n-1} (C_D S)_{M/S} (X_S - 1)}{W_T X_S} - 1}{\dfrac{DLL\ K_n + q_o K_1 K_2 \ldots K_n (C_D S)_{M/S} (X_S - 1)}{W_T X_S} - 1} \right]$$

☆U.S.Government Printing Office: 1979 — 657-084/244