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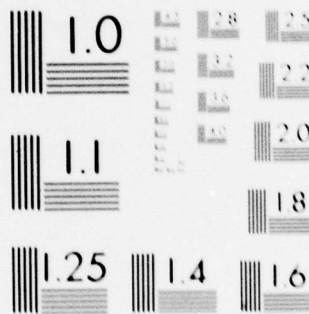
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# A CLASS OF COMMUTATIVE MARKOV MATRICES

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ABSTRACT

~~We examine~~ a class of Markov matrices which  
arise in a simple model of a defense system. *HAS BEEN EXAMINED*  
The model illustrates a Markov chain which is  
not time-homogeneous but is still amenable to  
analytic treatment. The matrices are shown to  
be commutative and the class is shown to be  
closed under matrix multiplication. The  
matrices are also shown to be diagonalizable  
and the eigenvectors have a simple form, name-  
ly composed of elements of Pascal's triangle.  
A description of the defense system model is  
given and the implications of the mathematical  
results for this model are discussed.



## INTRODUCTION

A defense system model, described later, leads to the formulation of a class of matrices with parameters  $n$  and  $p$ , and with elements  $A_{ij}$ , where

$$A_{ij} = \begin{cases} 0 & \text{if } i < j \\ \binom{i}{j} p^{i-j} q^j & \text{if } i \geq j \end{cases}$$

for given  $p$  and  $q$  with  $0 \leq p \leq 1$  and  $1 = p+q$ . When  $i \geq j$ ,  $A_{ij}$  represents the probability of  $j$  failures in  $i$  Bernoulli trials, given  $q$ , the probability of a failure in a single trial. For simplicity, this class of matrices will be referred to as type  $\alpha$ .

For type  $\alpha$  matrices, we establish a proof of multiplicative commutativity by the diagonalization process via eigenvalues and eigenvectors. In doing so, we discover the intriguing result that the complete set of linearly independent eigenvectors forms a lower triangular matrix whose rows are those of Pascal's triangle. The eigenvectors are independent of  $p$  and  $q$ . Also, we show that the class of matrices of type  $\alpha$  is closed under matrix multiplication.

After the derivation of these mathematical results, the rest of the paper is devoted to the explanation of the role of type  $\alpha$  matrices in the model, and to the interpretation of the mathematical results for this model.

#### MATHEMATICAL RESULTS

##### Theorem 1.

For any positive integer  $n$ , let  $A$  be an  $(n+1)$  by  $(n+1)$  matrix of type  $\alpha$ , with  $p \geq 0$ ,  $q \geq 0$  and  $p+q = 1$ .

Then  $A$  is diagonalizable with  $A = SDS^{-1}$ , where

$D = \text{Diag} \{1, q, \dots, q^n\}$  and, independent of  $p$  and  $q$ ,  $S$  is simply a lower triangular matrix whose rows coincide with those of Pascal's triangle (if presented in the form of a right triangle).

##### Proof.

Since  $1, q, q^2, \dots, q^n$ , with,  $0 \leq q \leq 1$ , are the  $n+1$  distinct eigenvalues of matrix  $A$ ,  $A$  can be diagonalized to  $D = \text{Diag} \{1, q, \dots, q^n\}$

Mathematical induction on  $n$  will be used to prove  $AS=SD$ .

From here on, appropriate subscripts will be used to indicate sizes of matrices under investigation;

therefore:

$$A_{n+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ p & q & 0 & 0 & \dots & 0 \\ p^2 & 2pq & q^2 & 0 & \dots & 0 \\ | & & & & & | \\ | & & & & & | \\ | & & & & & | \\ \binom{n}{0} p^n & \binom{n}{1} p^{n-1} q & \binom{n}{2} p^{n-2} q^2 & \dots & \dots & q^n \end{bmatrix}$$

and

$$S_{n+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & 0 & \dots & 0 \\ 1 & 2 & 1 & 0 & \dots & 0 \\ | & & & & & | \\ | & & & & & | \\ | & & & & & | \\ \binom{n}{0} & \binom{n}{1} & \binom{n}{2} & \dots & \dots & \binom{n}{n} \end{bmatrix}$$

When  $n=1$ ,  $A_2 = \begin{bmatrix} 1 & 0 \\ p & q \end{bmatrix}$  and  $S_2 = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ .

It can be easily verified that  $A_2 S_2 = S_2 D_2$ .

Assuming  $A_n S_n = S_n D_n$ , we want to establish  $A_{n+1} S_{n+1} = S_{n+1} D_{n+1}$ . Matrix block multiplication yields:

$$A_{n+1} S_{n+1} = \left[ \begin{array}{c|c} A_n S_n & \begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \end{array} \\ \hline [p^n, \binom{n}{1} p^{n-1} q, \dots, (S_n)] + q^n [ \binom{n}{0}, \binom{n}{1}, \dots, \binom{n}{n-1} ] & q^n \end{array} \right]$$

and

$$S_{n+1} D_{n+1} = \left[ \begin{array}{c|c} S_n D_n & \begin{array}{c} 0 \\ 0 \\ \vdots \\ 0 \end{array} \\ \hline \binom{n}{0}, \binom{n}{1} q, \dots, \binom{n}{n-1} q^{n-1} & q^n \end{array} \right]$$

Since  $A_n S_n = S_n D_n$ , all that remains to show, after comparison of  $A_{n+1} S_{n+1}$  with  $S_{n+1} D_{n+1}$ , is that:

$$[(\binom{n}{0})p^n, (\binom{n}{1})p^{n-1}q, \dots, (\binom{n}{n-1})pq^{n-1}] [S_n] + q^n [(\binom{n}{0}), (\binom{n}{1}), \dots,$$

$$(\binom{n}{n-1})] = [(\binom{n}{0}), (\binom{n}{1})q, \dots, (\binom{n}{n-1})q^{n-1}] \quad (1)$$

To establish the equality, we first expand a typical, say  $(r+1)$ st, term of the R.H.S. of equation 1:

$$q^r \binom{n}{r}, \text{ where } r = 0, 1, 2, \dots, n-1,$$

$$q^r \binom{n}{r} = q^r \binom{n}{r} (p+q)^{n-r} \text{ since } p+q = 1. \text{ Expanding } (p+q)^{n-r}:$$

$$= q^r \binom{n}{r} [(\binom{n-r}{0})p^{n-r}q^0 + (\binom{n-r}{1})p^{n-r-1}q^1 + \dots + (\binom{n-r}{n-r-1})p^1q^{n-r-1} + (\binom{n-r}{n-r})p^0q^{n-r}]$$

$$= q^r [(\binom{n}{r})(\binom{n-r}{0})p^{n-r}q^0 + (\binom{n}{r})(\binom{n-r}{1})p^{n-r-1}q^1 + \dots + (\binom{n}{r})(\binom{n-r}{n-r})p^0q^{n-r}]$$

$$= q^r [(\binom{n}{r})(\binom{r}{r})p^{n-r}q^0 + (\binom{n}{r+1})(\binom{r+1}{r})p^{n-r-1}q^1 + \dots + (\binom{n}{n})(\binom{n}{r})p^0q^{n-r}],$$

since  $\binom{n}{r} \binom{n-r}{k} = \binom{n}{r+k} \binom{r+k}{r}$ , for  $k < n-r$

Thus,  $q^r \binom{n}{r}$  has the following expansion:

$$q^r \binom{n}{r} = \binom{n}{r} \binom{r}{r} p^{n-r} q^r + \binom{n}{r+1} \binom{r+1}{r} p^{n-r-1} q^{r+1} + \dots + \binom{n}{n} \binom{n}{r} p^0 q^n .$$

The formation of the L.H.S. of equation 1 indicates that its  $(r+1)$ st term is the sum of  $q^n \binom{n}{r}$  and a dot product. While  $q^n \binom{n}{r}$  is simply the  $(r+1)$ st element in  $q^n [ \binom{n}{0}, \binom{n}{1}, \dots, \binom{n}{n-1} ]$ , the dot product is that of the first  $n$  elements of the  $(n+1)$ st row in  $A_{n+1}$  and the  $(r+1)$ st column in  $S_n$ , this column being equal to  $[0, \dots, 0, \binom{r}{r}, \binom{r+1}{r}, \dots, \binom{n-1}{r}]$ . After expansion, the dot product is equal to  $\binom{n}{r} \binom{r}{r} p^{n-r} q^r + \binom{n}{r+1} \binom{r+1}{r} p^{n-r-1} q^{r+1} + \dots + \binom{n}{n-1} \binom{n-1}{r} p q^{n-1}$ . This sum plus  $q^n \binom{n}{r}$  is indeed identical to the earlier expansion for the  $(r+1)$ st term of the R.H.S. of equation 1. The equality  $A_{n+1} S_{n+1} = S_{n+1} D_{n+1}$  is now established, and this completes the proof.

Theorem 2. Multiplicative commutativity holds for square matrices of type  $\alpha$ .

Proof. Let A and B be two  $(n+1) \times (n+1)$  matrices of type  $\alpha$ . Then, by Theorem 1, A and B can be diagonalized with  $A = SDS^{-1}$  and  $B = SD'S^{-1}$  where D and D' are diagonal matrices and S is the lower triangular matrix described in Theorem 1. The commutativity of A and B now is evident, since:

$$AB = SDS^{-1}SD'S^{-1} = SDD'S^{-1} = SD'DS^{-1} = BA .$$

We comment that the matrices S, with  $i, j$ th element equal to  $\binom{i}{j}$ , are seen in combinatoric settings, e.g., reference 1, page 45. Reference 1 also proves that the  $i, j$ th element of  $S^{-1}$  is given by  $(-1)^{i+j} \binom{i}{j}$ , so that  $S^{-1}$  is obtained directly from S by inserting "-" at appropriate locations. Hence, the spectral decomposition of  $A_{n+1}$  can be written down almost by inspection, i.e.,

$$A_{n+1} = \underline{u}_1 \underline{v}_1 + q \underline{u}_2 \underline{v}_2 + \dots + q^n \underline{u}_{n+1} \underline{v}_{n+1}$$

where the column vector  $\underline{u}_k$  is the  $k$ th column of  $S_{n+1}$

and the row vector  $\underline{v}_k$  is the  $k$ th row of  $S_{n+1}^{-1}$ .

A direct verification of Theorem 2 is instructive.

Suppose that the  $i, j$ th element of A is  $\binom{i}{j} q_1^j p_1^{i-j}$

and the  $j, k$ th element of B is  $\binom{j}{k} q_2^k p_2^{j-k}$ . Then the

$i, k$ th element of AB is given by

$$\begin{aligned} & \sum_{j=0}^n \binom{i}{j} \binom{j}{k} q_1^j q_2^k p_1^{i-j} p_2^{j-k} \\ &= \binom{i}{k} (q_1 q_2)^k \sum_j \binom{i-k}{j-k} (q_1 p_2)^{j-k} p_1^{i-j} \\ &= \binom{i}{k} (q_1 q_2)^k (1 - q_1 q_2)^{i-k} \end{aligned}$$

where we have used the identity  $\binom{i}{j} \binom{j}{k} = \binom{i}{k} \binom{i-k}{j-k}$

and recognized the binomial expansion of  $(p_1 + q_1 p_2)^{i-k}$ .

Clearly, the steps remain valid with  $q_1$  and  $q_2$  inter-

changed, and the commutativity of A and B follows.

Further examination of the proof shows that we have also shown that the class of matrices of type  $\alpha$  is closed under matrix multiplication. More precisely, Theorem 3. If A and B are matrices of type  $\alpha$  of the same order and with parameters  $q_1, q_2$ , respectively, then AB is of type  $\alpha$  with parameter  $q_1 q_2$ .

Theorem 3 will be useful for providing computational simplification in our model.

#### MODEL DESCRIPTION

The foregoing class of matrices arise in the context of a defense system model in which a cluster of  $n$  missiles attempts to penetrate several lines of defense. The number of missiles surviving the first line of defense is Binomially distributed with parameters  $n, q_1$ , i.e., there are  $i$  survivors with probability  $\binom{n}{i} q_1^i p_1^{n-i}$ ,  $i=0,1,\dots,n$ , and  $q_1 = 1-p_1$ . Suppose that  $n_1$  missiles survive the first line of defense. We assume that the number of missiles surviving the second line of defense is Binomially

distributed with parameters  $n_1, q_2,$  and etc.<sup>a</sup>

In short, we assume that the lines of defense are independent and produce attrition in accordance with a Binomial distribution - each line of defense having its own "success probability"  $p_k$  and associated

$$q_k = 1 - p_k .$$

It is convenient to regard the missiles' passage through a particular line of defense as a transition in a Markov chain (reference 2). For example, given  $n = 2$  and  $p_1 = 0.6,$  the matrix below is the transition matrix for penetration of the first line of defense.

		<u>Out</u>		
		0	1	2
<u>In</u>	0	1	0	0
	1	.6	.4	0
	2	.36	.48	.16

<sup>a</sup>It is a standard result in probability theory that if the the random variable  $X,$  given  $N,$  is distributed as Binomial with parameters  $N, p,$  and if  $N$  is distributed as Binomial with parameters  $N^*, p^*,$  then the unconditional distribution of  $X$  is Binomial with parameters  $N^*, pp^*.$  The proof is virtually identical with that of theorem 3. Hence we have a probabilistic proof that the number of survivors of the second line of defense is distributed as Binomial with parameters  $n, q_1 q_2,$  and an induction argument gives the general result for several lines of defense.

Thus, if 2 missiles enter this line of defense, the probability that 1 missile emerges is 0.48. We see that this matrix is just  $A_3$  defined in the preceding section (with  $p = 0.6$ ). In general the number of incoming missiles might be a discrete random variable having probability distribution  $T$ , whereupon the number of missiles surviving the line of defense is found by pre-multiplying the transition matrix by the row vector  $T$ . In our 3x3 example, if the probabilities of having 0,1,2 incoming missiles were 0,.5,.5, respectively, then the computation would be

$$(0 \ .5 \ .5) \begin{bmatrix} 1 & 0 & 0 \\ .6 & .4 & 0 \\ .36 & .48 & .16 \end{bmatrix} = (.48 \ .44 \ .08)$$

so that the probability of having 0,1,2 missiles penetrate the first line of defense would be .48, .44, .08, respectively. This resultant vector serves as the input to the second line of defense, and by successive such vector-matrix multiplications the probability of having 0,1, or 2 missiles penetrate all the defenses can be calculated. In particular, it follows that if

there are  $L$  levels of defense and the transition matrix for each level is the matrix  $A_3(k)$ ,  $k=1,2,\dots,L$ , then the distribution of the number of penetrators is given by

$$(0 \quad .5 \quad .5) \prod_{k=1}^L A_3(k)$$

More generally, if there may be as many as  $n-1$  incoming missiles then the vector  $T$  will need to have  $n$  elements and the transition matrices will have to be of size  $n \times n$ . The reader should verify that the transition matrices for this general case are precisely the matrices  $A_n$  defined in the previous section, with  $p = p_k$  as appropriate. Thus, we have a Markov chain in discrete time which is not time-homogeneous, i.e., the transition matrices are not identical, and for which - as we shall see - closed form results are available.

#### DISCUSSION AND CONCLUSIONS

A non-trivial question is "Does the ordering of the lines of defense make any difference?". Mathematically, this is the same as asking if the ordering of the  $A_n(k)$  makes any difference in evaluating the

product  $\prod A_n(k)$ . It is well known that matrix multiplication is non-commutative in general, and since the  $A_n(k)$  matrices will not usually be identical (unless the  $p_k$ 's are identical) it is not obvious mathematically that these matrices will commute. However, the results in the previous section show that the  $A_n(k)$  matrices do commute and the ordering of the lines of defense makes no difference in this model.

The naive approach to evaluating the product  $\prod A_n(k)$  is to do it by brute force, performing the indicated matrix multiplications one at a time. A more sophisticated approach is to use theorem 1 and recognize that the product can be written as  $SDS^{-1}$  where  $D$  is the diagonal matrix whose  $i$ th diagonal entry is  $\prod q_k^i$ , requiring only two matrix multiplications. A still more sophisticated approach involves theorem 3, seeing that the product can be written down immediately using no matrix multiplications at all. This illustrates the practical utility of increased mathematical insight in analysis of our model.

Finally, we observe that our results for this model find application in a more general setting. For example, instead of missiles let the survivors be ships in a task force, and instead of levels of defense let the attrition occur because of a sequence of attacks on the task force. Intuitively, one feels that an optimal defense should be insensitive to the sequencing of attacks, even in elaborate models, so that there is some possibility of the kind of computational simplifications that we have seen in our simple model.

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