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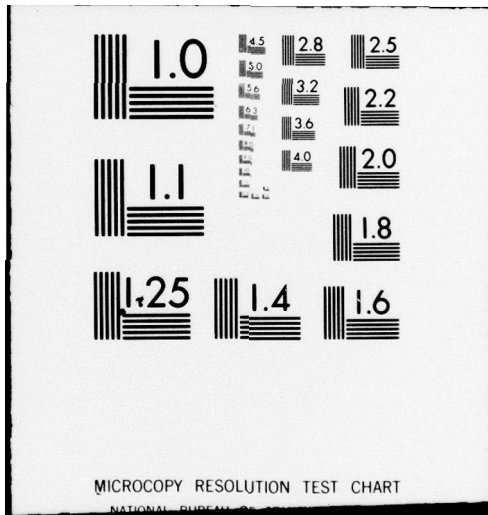
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## POWER SERIES FOR THE CALCULATION OF BESSEL FUNCTIONS FOR LARGE ARGUMENTS

Provides  $J_\nu(x)$  and  $Y_\nu(x)$ , accurate to at least 14 places.

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## INTRODUCTION

Goldstein and Thaler (ref 1) have employed the phase-amplitude method to develop mathematical expressions for the calculation of Bessel functions of real order and argument for large values of the argument. The power series reported in reference 1 allow  $J_\nu(x)$  and  $Y_\nu(x)$  to be calculated within an accuracy of six to eight figures. Presented herein are higher order terms of the power series that allow  $J_\nu(x)$  and  $Y_\nu(x)$  to be calculated within an accuracy of 14 to 16 figures.

## THE POWER SERIES

It can be shown (reference 1) that the Bessel functions  $J_\nu(x)$  and  $Y_\nu(x)$  can be written in terms of an amplitude and a phase function:

$$J_\nu(x) = \left[ \frac{B_\nu(t)}{t} \right] \sqrt{\frac{2t}{\pi}} \cos \Phi_\nu(t)$$

and

$$Y_\nu(x) = \left[ \frac{B_\nu(t)}{t} \right] \sqrt{\frac{2t}{\pi}} \sin \Phi_\nu(t) .$$

The amplitude function is expanded into a power series about  $t = 0$ :

$$[B_\nu(t)/t] = 1 + a_2 t^2 + a_4 t^4 + a_6 t^6 + a_8 t^8 + a_{10} t^{10} + a_{12} t^{12} + a_{14} t^{14} + \dots ,$$

where

$$a_2 = \frac{1}{4} \alpha$$

$$a_4 = \frac{5}{32} \alpha^2 - \frac{3}{8} \alpha$$

$$a_6 = \frac{15}{128} \alpha^3 - \frac{37}{32} \alpha^2 + \frac{15}{8} \alpha$$

$$a_8 = \frac{195}{2048} \alpha^4 - \frac{611}{256} \alpha^3 + \frac{1821}{128} \alpha^2 - \frac{315}{16} \alpha$$

$$a_{10} = \frac{663}{8192} \alpha^5 - \frac{4199}{1024} \alpha^4 + \frac{29811}{512} \alpha^3 - \frac{2223}{8} \alpha^2 + \frac{2835}{8} \alpha$$

$$a_{12} = \frac{4641}{65536} \alpha^6 - \frac{103649}{16384} \alpha^5 + \frac{713065}{4096} \alpha^4 - \frac{1944363}{1024} \alpha^3 + \frac{513495}{64} \alpha^2 - \frac{155925}{16} \alpha$$

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1. Bessel Functions for Large Arguments, by M Goldstein and RM Thaler; *Mathematical Tables and Other Aids to Computation*, vol 13, no 65, 1958, p 18-26

$$a_{14} = \frac{16\,575}{262\,144} \alpha^7 - \frac{595\,595}{65\,536} \alpha^6 + \frac{438\,325}{1024} \alpha^5 - \frac{35\,994\,327}{4096} \alpha^4 + \frac{84\,480\,939}{1024} \alpha^3 \\ - \frac{41\,365\,215}{128} \alpha^2 + \frac{6\,081\,075}{16} \alpha$$

The phase function is similarly expanded into a power series about  $t = 0$ :

$$t \left[ \Phi_\nu + \left( \nu + \frac{1}{2} \right) \frac{t}{2} \right] = 1 + b_2 t^2 + b_4 t^4 + b_6 t^6 + b_8 t^8 + b_{10} t^{10} + b_{12} t^{12} \\ + b_{14} t^{14} + \dots,$$

where

$$b_2 = \frac{1}{2} \alpha$$

$$b_4 = \frac{1}{24} \alpha^2 - \frac{\alpha}{4}$$

$$b_6 = \frac{1}{80} \alpha^3 - \frac{7}{20} \alpha^2 + \frac{3}{4} \alpha$$

$$b_8 = \frac{10}{1792} \alpha^4 - \frac{95}{224} \alpha^3 + \frac{807}{224} \alpha^2 - \frac{315}{56} \alpha$$

$$b_{10} = \frac{7}{2304} \alpha^5 - \frac{35}{72} \alpha^4 + \frac{1975}{192} \alpha^3 - 58 \alpha^2 + \frac{315}{4} \alpha$$

$$b_{12} = \frac{21}{11\,264} \alpha^6 - \frac{3045}{5632} \alpha^5 + \frac{64\,319}{2816} \alpha^4 - \frac{427\,071}{1408} \alpha^3 + \frac{247\,365}{176} \alpha^2 - \frac{155\,925}{88} \alpha$$

$$b_{14} = \frac{37\,751}{30\,457\,856} \alpha^7 - \frac{3927}{6656} \alpha^6 + \frac{145\,013}{3328} \alpha^5 - \frac{1\,851\,351}{1664} \alpha^4 + \frac{19\,472\,013}{1664} \alpha^3 \\ - \frac{10\,091\,115}{208} \alpha^2 + \frac{6\,081\,075}{104} \alpha$$

and

$$\alpha \equiv \left( \nu^2 - \frac{1}{4} \right).$$

Attention is also called to a typographical error in reference 1 for  $b_8$ . The form shown herein is the correct one.

## CONCLUSION

The power series developed by Goldstein and Thaler are extended to allow the Bessel functions  $J_\nu(x)$  and  $Y_\nu(x)$  to be calculated accurately to at least 14 figures.

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