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UNIFIED TREATMENT OF SOME INEQUALITIES
AMONG RATIOS OF MEANS

by

Emad El-Neweihi* and Frank Proschan**

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ABSTRACT

Using majorization and Schur-functions, Marshall, Olkin, and Proschan obtained a result concerning monotonicity of the ratio of means. This note shows that a slight extension of their result provides a unified method for obtaining and extending inequalities between means due to Chan, Goldberg, and Gonek, as well as deriving additional inequalities of the same type.

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Technical Information Officer

1. Introduction. Chan, Goldberg, and Gonek [1] show that:

$$(1) \quad \left[\frac{x^p + y^p}{(1-x)^p + (1-y)^p} \right]^{1/p} < \left[\frac{x^q + y^q}{(1-x)^q + (1-y)^q} \right]^{1/q},$$

where $0 \leq x < y$, $x + y < 1$, and $p < q$; and

$$(2) \quad \left[\frac{\sum_{i=1}^n x_i^{-p}}{\sum_{i=1}^n (1-x_i)^{-p}} \right]^{-1/p} \leq \left[\frac{\sum_{i=1}^n x_i^{-q}}{\sum_{i=1}^n (1-x_i)^{-q}} \right]^{-1/q}$$

where $0 \leq x_i \leq 1/2$ and $p > 0$. Strict inequality holds in (2) unless

$$x_1 = x_2 = \dots = x_n.$$

Earlier, Marshall, Olkin, and Proschan [2] showed:

$$(3) \quad \left[\frac{\sum_{i=1}^n \lambda_i a_i^r}{\sum_{i=1}^n \lambda_i b_i^r} \right]^{1/r} \text{ is increasing in } r,$$

where $a_1 \geq a_2 \geq \dots \geq a_n > 0$, $b_1 \geq b_2 \geq \dots \geq b_n > 0$, $\frac{b_1}{a_1} \leq \frac{b_2}{a_2} \leq \dots \leq \frac{b_n}{a_n}$, and $\lambda_i > 0$, $i = 1, \dots, n$, $\sum_{i=1}^n \lambda_i = 1$.

Result (3) was obtained using majorization and Schur-functions (for definitions see [2]).

The main purposes of this note are to show that using (3), (a) inequalities (1) and (2) can be proved in a *unified* way, (b) (1) and (2) can be *extended*, and (c) *additional* inequalities of a similar type can be obtained.

2. Main Results. Before we state and prove the main results, we present several remarks:

Remark 2.1. It is easy to verify that (3) holds even if certain of the a_i 's are equal to zero.

Remark 2.2. Careful inspection of the proof of (3) shows that in certain cases the ratio in (3) is *strictly* increasing in r .

We may now prove:

Theorem 2.3. Let $0 \leq x < y$, $x + y < 1$, $0 < \lambda < 1$, and $p < q$. Then

$$(4) \quad \left[\frac{\lambda x^p + (1 - \lambda)y^p}{\lambda(1 - x)^p + (1 - \lambda)(1 - y)^p} \right]^{1/p} < \left[\frac{\lambda x^q + (1 - \lambda)y^q}{\lambda(1 - x)^q + (1 - \lambda)(1 - y)^q} \right]^{1/q}.$$

Proof. Clearly $(1 - x)x < (1 - y)y$. Let $a_1 \equiv y$, $a_2 \equiv x$, $b_1 \equiv 1 - x$, and $b_2 \equiv 1 - y$. Inequality (4) follows from (3) by Remark 2.2. ||

Setting $\lambda = \frac{1}{2}$ in (4) we get (1) as a special case.

The same technique yields an extension of Inequality (2):

Theorem 2.4. Let $0 \leq x_i \leq \frac{1}{2}$, $i = 1, \dots, n$, $p > 0$, $\lambda_i \geq 0$, $i = 1, \dots, n$, and $\sum_{i=1}^n \lambda_i = 1$. Then

$$(5) \quad \left[\frac{\sum_{i=1}^n \lambda_i x_i^{-p}}{\sum_{i=1}^n \lambda_i (1 - x_i)^{-p}} \right]^{-1/p} < \left[\frac{\sum_{i=1}^n \lambda_i x_i^p}{\sum_{i=1}^n \lambda_i (1 - x_i)^p} \right]^{1/p}$$

unless $x_1 = x_2 = \dots = x_n$.

Proof. Let $x_{[1]} \geq x_{[2]} \geq \dots \geq x_{[n]}$ denote the decreasing rearrangement of $x_1, \dots, x_n$ from now on. Let $a_i \equiv x_{[i]}$, $b_i \equiv (1 - x_{[i]})^{-1}$, $i = 1, \dots, n$. Since $-p < p$ and $(1 - x_{[i]})^{-1} x_{[j]} \leq (1 - x_{[j]})^{-1} x_{[i]}$ for $i < j$, we have by (3):

$$(6) \quad \left[\frac{\sum_{i=1}^n \lambda_i x_i^{-p}}{\sum_{i=1}^n \lambda_i (1 - x_i)^p} \right]^{-1/p} < \left[\frac{\sum_{i=1}^n \lambda_i x_i^p}{\sum_{i=1}^n \lambda_i (1 - x_i)^{-p}} \right]^{1/p}$$

unless $x_1 = x_2 = \dots = x_n$ (see Remark 2.2). The desired result follows from (6). ||

Note that (2) is a special case of (5) by setting $\lambda_i = \frac{1}{n}$, $i = 1, \dots, n$.

Finally, Theorem 2.5 below yields an inequality similar to (1) and (2).

This illustrates that majorization and Schur-functions can be used to generate through (3) a host of inequalities similar to (1) and (2).

Theorem 2.5. Let $x_i \geq 0$, $\lambda_i > 0$, $i = 1, \dots, n$, $\sum_{i=1}^n \lambda_i = 1$, and $p < q$.

Then:

$$(7) \quad \left[\frac{\sum_{i=1}^n \lambda_i x_i^p}{\sum_{i=1}^n \lambda_i (1 + x_i)^p} \right]^{1/p} \leq \left[\frac{\sum_{i=1}^n \lambda_i x_i^q}{\sum_{i=1}^n \lambda_i (1 - x_i)^q} \right]^{1/q}.$$

Strict inequality holds in (7) unless $x_1 = x_2 = \dots = x_n$.

Proof. Let $a_i \equiv x_{[i]}$ and $b_i \equiv 1 + x_{[i]}$, $i = 1, \dots, n$. Since $\frac{1+x}{x}$ is decreasing, we apply (3) to get the desired result. By Remark 2.2, strict inequality holds in (7) unless $x_1 = x_2 = \dots = x_n$. ||

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1. F. Chan, D. Goldberg, and S. Gonek, On extensions of an inequality among means. Proc. Amer. Math. Soc. 42 (1974), 202-207.
2. A. W. Marshall, I. Olkin, and F. Proschan, Monotonicity of ratios of means and other applications of majorization. Inequalities, ed. by O. Shisha. Academic Press, New York (1967), 177-190.

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