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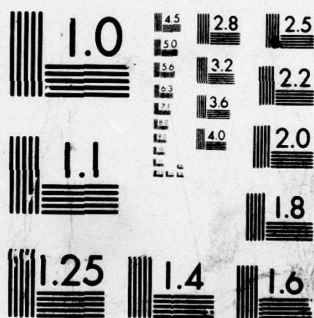

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The following questions are discussed for nonlinear systems with input Gaussian processes. How can the system be represented? How can the system be identified from the input and output processes? Does knowledge of the way the system responds to a certain Gaussian input determine the way it will respond to another Gaussian input or to a deterministic input? Does knowledge of the system and the statistics of the output determine the statistics of the input?

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NONLINEAR SYSTEMS WITH GAUSSIAN INPUTS: REPRESENTATION, IDENTIFICATION AND INVERSE PROBLEMS

Stamatis Cambanis*
Department of Statistics
University of North Carolina
Chapel Hill, North Carolina 27514

Summary

The following questions are discussed for nonlinear systems with input Gaussian processes. How can the system be represented? How can the system be identified from the input and output processes? Does knowledge of the way the system responds to a certain Gaussian input determine the way it will respond to another Gaussian input or to a deterministic input? Does knowledge of the system and the statistics of the output determine the statistics of the input?

Introduction

In studying nonlinear systems with random inputs one is faced with three broad classes of problems. In the first class the nonlinear system and the input are known, and the problem is to represent the output (system representation) and to analyze its statistical properties (system analysis). In the second class the input and output are known, or their joint statistics, and the problem is to identify the nonlinear system (system identification). And in the third class the nonlinear system and the output or its statistics are known and the problem is to identify the input or its statistics (inverse problem).

Here we concentrate on the special but significant case where the random input is Gaussian, and we summarize certain recent results on these classes of problems, which lead to several open questions. The list of references is by no means exhaustive and no mention is made of results for non-Gaussian inputs.

The nonlinear system is denoted by θ and the input and output processes by $X = \{X_t, t\epsilon T\}$ and $Y = \{Y_t, t\epsilon T\}$ respectively (where the parameter set T is, say, an interval on the real line and of course the input and output could have distinct parameter sets). Unless otherwise stated it will be assumed throughout that

(1) the Gaussian input X is mean square continuous, with mean zero (for simplicity) and continuous covariance function $R(t,s)$, and

(2) the nonlinear system θ is such that the output Y has finite second moments: $EY_t^2 < \infty$, $t \epsilon T$.

All integrals are over T, which is thus deleted.

System Representation and Analysis

It is shown in [7] that for each $t \epsilon T$,

$$Y_t = EY_t + \sum_{n=1}^{\infty} \int\ldots\int f_n(t;t_1,\ldots,t_n)X_{t_1}\ldots X_{t_n} dt_1\ldots dt_n,$$

where the series converges in quadratic mean, the integrals are multiple Wiener integrals and each kernel $f_n(t;\cdot)$ is a symmetric function on T^n belonging to a Hilbert space $\lambda_2(\otimes^n R)$ (for detailed definitions the reader is referred to [7]). The terms

$$Y_{n,t} = \int\ldots\int f_n(t;t_1,\ldots,t_n)X_{t_1}\ldots X_{t_n} dt_1\ldots dt_n$$

are homogeneous forms in X of degree n, and they are uncorrelated for distinct values of n. This representation generalizes the corresponding result of Wiener's for white noise X.

Thus the action of the system θ on the Gaussian input X is represented by the sequence of (deterministic) kernels $\{f_n\}$, which depends not only on the system θ but also on the input X through its covariance function R. This representation looks very much like a Volterra kernel expansion with the important difference that the multiple integrals are Wiener rather than Lebesgue integrals. Also the kernels $f_n(t;t_1,\ldots,t_n)$ do not necessarily vanish, as Volterra kernels do, when some t_i is larger than t; but they have this property when the system is nonanticipatory, i.e., when each Y_t depends only on the past $\{X_s, s\leq t, s\epsilon T\}$ of X.

Having represented the output of the nonlinear system, we would like to study its statistical properties. The most interesting question is to find or describe the distribution of Y or of Y_n, even for fixed t, i.e., its univariate distribution. Y_1 is of course a Gaussian process. The distribution of the quadratic form $Y_{2,t}$ is fully described through its characteristic function [6,11]. For $n \geq 3$ the distribution of $Y_{n,t}$ is not known but Arveson [1] has characterized all homogeneous forms of the same degree in white noise X which have the same distribution, and it would be of interest to do this for other Gaussian inputs and for systems with finite degree. Another remarkable result is that the tail of the (univariate) distribution of the finite degree form

$$Y_t^N = EY_t + \sum_{n=1}^{N} Y_{n,t}$$

determines the degree N [11].

When the nonlinear system is time-invariant, in the sense that $f_n(t;t_1,\ldots,t_n) = f_n(t_1-t,\ldots,t_n-t)$, and T is the real line, then

$$Y_{n,t} = \int\ldots\int f_n(t_1-t,\ldots,t_n-t)X_{t_1}\ldots X_{t_n} dt_1\ldots dt_n$$

and if X is stationary, Y is strictly stationary.

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Wiener asked the following question: What is the class of all stationary processes that are time-invariant transformations of white noise X? It turns out that all such processes are strongly mixing [11], and that every strictly stationary ergodic process can be approximated in law by such processes (this is a profound result of Wiener's which was clarified by Nisio--see [11]). Analogs of these results for more general stationary Gaussian inputs X than white noise are shown in [7].

System Identification

The sequence of kernels $\{f_n\}$ which represents the action of the system θ on the Gaussian input X can be determined from the joint statistics (e.g. moments) of the input and output processes X and Y [8]. However, as was pointed out, these kernels describe the action of the system only on the Gaussian input X (the kernels depend on the covariance function R of X), and the following question arises: Does knowledge of the way the system responds to one Gaussian input determine its response to another Gaussian input or to a deterministic input?

It is reasonable to expect that knowing how the system responds to a Gaussian input X, i.e., knowing the kernels $\{f_n\}$, would at most reveal how the system responds to another Gaussian input X' which is equivalent to X, and to a deterministic input which is a sample function of X (or X'); and would provide no information on how the system may respond to a Gaussian input X' which is singular to X. This question is considered in [8]. It is shown that the kernels $\{f_n\}$ in the representation of the system θ acting on the Gaussian input X determine the kernels $\{f_n'\}$ in the representation of the system acting on an equivalent Gaussian input X', provided either one of the following conditions is satisfied: (i) the nonlinear system is of finite order, (ii) the Radon-Nikodym derivative of the two Gaussian processes is a bounded random variable. However the conjecture is that this result should be true for general equivalent Gaussian inputs and for general (infinite degree) nonlinear systems.

It is also shown that for some sequence $N_m \to \infty$, with probability one

$$Y_t = EY_t + \lim_{m\to\infty} \sum_{n=1}^{N_m} L\int\cdots\int h_n^{N_m}(t;t_1,\ldots,t_n)X_{t_1}\cdots X_{t_n}\,dt_1\cdots dt_n$$

where L indicates Lebesgue integral and the kernels $\{h_n^N\}$ are continuous in $(t_1,\ldots,t_n)$ and can be found from the kernels $\{f_n\}$. Thus knowledge of the way the system acts on a Gaussian input X determines how the system will act on almost all the sample functions of X. This deterministic input-output representation depends strongly on the input covariance R, since R determines the kernels $\{h_n^N\}$ and (up to a zero probability set) the deterministic functions for which the representation is valid. It is remarkable though that, under the extremely weak assumption on the system that its output has finite second moment, one can obtain for a small class of deterministic inputs a representation identical in form to the representation

$$y(t) = \lim_{m\to\infty}\left\{k_0^{N_m} + \sum_{n=1}^{N_m} L\int\cdots\int k_n^{N_m}(t;t_1,\ldots,t_n)\,x(t_1)\cdots x(t_n)\,dt_1\cdots dt_n\right\}$$

obtained by Fréchet [3] for all x(t) in C[a,b] or in $L_2[a,b]$ under the assumption that the system is continuous, in the sense that for each fixed t, y(t) is a continuous functional on C[a,b] or $L_2[a,b]$.

A classical property of "smooth" linear systems, namely time-invariant linear systems with transfer function, is that they can be identified from their response to exponential inputs: knowledge of the way such a linear system responds to exponential inputs of arbitrary frequency determines its transfer function and hence how the system will respond to any input with finite energy. One should likewise determine classes of "smooth" nonlinear systems which can be identified from their response to appropriate Gaussian inputs. The conjecture here is that if a nonlinear system θ when acting on a deterministic input in, say, $L_2[a,b]$ has a Volterra input-output relationship

$$y(t) = k_0(t) + \sum_{n=1}^{\infty} L\int\cdots\int k_n(t;t_1,\ldots,t_n)\,x(t_1)\cdots x(t_n)\,dt_1\cdots dt_n\ ,$$

then knowledge of the way the system responds to a Gaussian input with strictly positive definite covariance function determines its Volterra kernels $\{k_n\}$ and hence how the system will respond to any (admissible) deterministic or random input. This is shown in [8] under some technical regularity conditions on the system θ, which are satisfied when θ has finite degree; its proof under no additional assumptions eludes us at present.

Inverse Problems

Unlike system analysis and identification problems, where the nonlinear systems studied are allowed to have memory and/or anticipation, the study of inverse problems has so far been confined to memoryless time-invariant nonlinear systems θ:

$$Y_t = f(X_t)\ , \qquad t \in T\ ,$$

where f(x) is a function defined on the real line. In this case the multiple Wiener integrals in the representation of Y_t become Hermite polynomials in X_t.

Grünbaum [4,5] showed that, when the Gaussian input X is stationary and has zero mean, its covariance function R(t) can be identified from the joint moments of all orders of the output process Y, for certain classes of even nonlinearities f(x), including interval-windows $f(x) = 1_{(-a,a)}(x)$. In [2] it is shown that when the mean of the input is zero, the covariance function R(t) can be constructively identified for several classes of nonlinearities from only the mean and correlation functions of the output Y. Included here are hard- and soft-limiters, quantizers, even and odd nonlinearities, as well as quite general nonlinearities. Also all interval-windows $f(x) = 1_{(a,b)}(x)$ are found for which arbitrary covariances can be identified.

The constructive identification of the mean function $s(t)$ of the Gaussian input X from the output mean and correlation functions is considered in [9] when the input covariance function $R(\tau)$ is known. This is an equivalent formulation of the following interesting problem: a (nonrandom and unknown) signal $s(t)$ in additive Gaussian noise N with mean zero and known covariance function $R(\tau)$, $X_t = (t) + N_t$, is passed through a memoryless nonlinearity $f(x)$, and the problem is to identify the signal $s(t)$ from the mean and correlation functions of the output process Y. Note that in the absence of noise the signal $s(t)$ cannot in general be identified from $f[s(t)]$. It is shown in [9] that arbitrary signals

(i) can be identified when the nonlinearity is monotonic, such as a hard- or soft-limiter or an infinite-interval-window;

(ii) can be identified up to a global sign when the nonlinearity is symmetric around some point x_0, bounded below or above, and monotonic on $[x_0,\infty)$, such as a full wave even νth-law device or a finite-interval-window.

The problem of identifying both the mean function $s(t)$ and the covariance function $R(\tau)$ of the input X remains open at present.

(i) implies in particular that a signal can be identified from the first two moment functions of the hardlimited version of the signal plus noise: $Y_t = \operatorname{sgn}[s(t) + N_t]$. The question thus arises whether the signal $s(t)$ can actually be estimated from the binary output Y_t. Note that in the absence of additive noise the signal $s(t)$ cannot be identified or estimated from the binary output $\operatorname{sgn}[s(t)]$. It is shown in [10] that by deliberately adding noise $\{N_k\}$ to the periodic samples of the signal $\{s(\frac{k}{W})\}$ prior to hardlimiting, the signal $s(t)$ can be estimated consistently from the binary sequence $\{\operatorname{sgn}[s(\frac{k}{W}) + N_k]\}$ as the sampling rate W tends to infinity. The estimate $\hat{s}_W(t)$ is in fact shown to converge to the signal $s(t)$ with probability one and also to be asymptotically normal. The estimator consists of a time-varying linear system followed by a memoryless time-invariant nonlinearity, which can be made linear by a proper choice of the noise distribution! These results hold for all bounded and uniformly continuous signals and, in addition to the hardlimiter, for further monotonic and nonmonotonic nonlinearities.

References

1. W.B. Arveson, Nonlinear Spectral Theory with Applications to Nonlinear Noise, University of California, monograph in manuscript form, 1979.

2. S. Cambanis and E. Masry, On the reconstruction of the covariance of stationary Gaussian processes observed through zero-memory nonlinearities. IEEE Trans. Information Theory IT-24 (1978) 485-494. Part II, IEEE Trans. Information Theory IT-26 (1980), to appear.

3. M. Fréchet, Sur les fonctionnelles continues, Ann. Ec. Norm. 27 (1910), 193-216.

4. F.A. Grünbaum, Identifying a Gaussian signal after a zero-memory filter, Adv. Math. 14 (1974), 173-182.

5. F.A. Grünbaum, Looking at a Gaussian process through a window, J. Multivariate Anal. 4 (1974), 401-408.

6. T. Hida, Topics on nonlinear filtering theory, P.R. Krishnaiah, ed., Multivariate Analysis - IV (1977), 239-245, North-Holland.

7. S.T. Huang and S. Cambanis, Stochastic and multiple Wiener integrals for Gaussian processes, Ann. Probability 6 (1978), 585-614.

8. S.T. Huang and S. Cambanis, On the representation of nonlinear systems with Gaussian inputs, Stochastics 2 (1979), 173-189.

9. E. Masry and S. Cambanis, Signal identification after noisy nonlinear transformations, IEEE Trans. Information Theory IT-26 (1980), to appear.

10. E. Masry and S. Cambanis, Consistent estimation of continuous-time signals from nonlinear transformations of noisy samples, (1979), manuscript.

11. H.P. McKean, Wiener's theory of nonlinear noise, in Stochastic Differential Equations, SIAM-AMS 6 (1973), 191-209.