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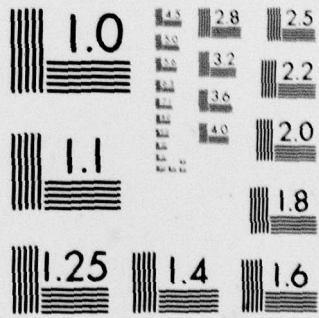
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SMOOTHING AND APPROXIMATION OF MULTIVARIATE FUNCTIONS

FINAL REPORT

LEVEL II

E. W. CHENEY

OCTOBER 31, 1979

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SMOOTHING AND APPROXIMATION OF MULTIVARIATE FUNCTIONS

Grant number DAAG29-77-G-0215, titled "Smoothing and Approximation of Multivariate Functions," was made to The University of Texas at Austin on 10 September 1977 and expired 9 September 1979. Principal investigator was E. W. Cheney, Professor of Mathematics.

The research results from this grant are reported in 8 technical papers which have been or will be published in the open scientific literature. They have also been issued in Preprint form at the Center for Numerical Analysis, University of Texas. At the end of this report, technical summaries of these papers will be presented.

The grant supported the principal investigator during four months in the summers of 1978 and 1979. In addition, the grant supported one Ph.D. student, John Respass, at a rate of half-time, during two academic years. Mr. Respass' dissertation will be completed in 1980. The grant provided a small amount of supporting service through the Center for Numerical Analysis. Several mathematicians from abroad who worked with the principal investigator were enabled to visit the University of Texas with funds from this grant. Finally, some travel funds from the grant enabled the principal investigator to accept an invitation to lecture at the Third Biannual Dundee Conference on Numerical Analysis (Dundee, Scotland, June 1979).

→ The goal of the research project was to proceed as far as possible in the study of a certain class of practical approximation problems. Namely, the problem is to reduce the complexity of multivariate functions by representing them precisely or approximately by combinations of univariate functions. A prototype problem is that of finding a best approximation to a function of two variables, $f(x,y)$, by a sum $g(x) + h(y)$. The prototype problem is thoroughly understood in the case that the functions involved are continuous and an approximation in the uniform sense is needed.

The general problem of approximating $f(x,y)$ by

$$(1) \quad \sum_{i=0}^n a_i(x) y^i + \sum_{i=0}^m b_i(y) x^i$$

with undetermined continuous coefficient functions $a_i(x)$ and $b_i(y)$ is very poorly understood, and our efforts have been directed towards it.

A typical practical computing problem which leads to this type of approximation is the integral equation

$$(2) \quad u(x) = \int_0^1 f(x,y)u(y) dy$$

in which $u(x)$ is an unknown function and the kernel function $f(x,y)$ is given. The theory shows that $u(x)$ can be determined easily if $f(x,y)$ is first approximated accurately in the form (1). The theory encompasses, of course, more general equations than (2) and more general forms than (1).

Research Reports by E. W. Cheney Supported by the Grant

1. "On Simultaneous Chebyshev Approximation" (with G. M. Phillips, J. H. McCabe), C A Report 131, February 1978. 8 pages. To appear in Journal of Approximation Theory 26 (1979).

This paper deals with the problem of minimizing an expression

$$\max_x \int |f(x,y) - g(x)| dy$$

when f is given and g ranges over a set of continuous functions. Several characterizations of the optimal function g are given. These generalize theorems previously proved in the literature. This paper also contains a bibliography of 30 references to works on "simultaneous approximation."

2. "On the Approximation of a Bivariate Function by Sums of Univariate Functions" (with W. A. Light), CNA Report 140, August 1978. 27 pages. To appear, Journal of Approximation Theory.

This paper offers a completely new proof of convergence of the Diliberto-Straus Algorithm for uniform approximation of a continuous bivariate function by sums of continuous univariate functions. Also, a special analysis of the discrete case (which is of importance in scaling problems of matrices) is given. In particular, the functions which have unique best approximations are completely characterized.

3. "On the Algorithm of Diliberto and Straus for Approximating Bivariate Functions by Univariate Ones" (with M. V. Golitschek). Numerical Functional Analysis and Optimization 1 (1979), 341-363. Also, CNA Report 141, August 1978.

This paper gives results on the efficiency of the Diliberto-Straus Algorithm for obtaining best approximations of the form

$$f(x,y) \approx g(x) + h(y)$$

using the Tchebycheff norm. For example, three steps produce an approximation within 50% of the ultimate final accuracy. Examples can be constructed, however, showing arbitrarily slow linear convergence. In the discrete case, best approximations possessing certain extra, favorable properties are shown to exist.

4. "The Approximation of Bivariate Functions by Sums of Univariate Ones Using the L_1 -Metric" (with W. A. Light, T. McCabe, G. M. Phillips). CNA Report 147, December 1978, 16 pages. Submitted for publication.

This paper establishes the existence of best approximations

$$f(x,y) \approx g(x) + h(y)$$

in an L_1 -setting. Thus the norm employed is the L_1 -norm, and the functions g and h can be arbitrary integrable functions. It is also proved, by an example, that the algorithm of Diliberto-Straus does not produce best approximations in the L_1 -norm.

5. "Multivariate Approximation with Tensor-product Spaces" (with W. A. Light). CNA Report 153, November 1979. 25 pages. Being readied for publication.

This report establishes the existence of best approximations to a function $f(x,y)$ by functions of the form

$$\phi(x,y) = \sum_{i=1}^n a_i(t) g_i(s) + \sum_{i=1}^m b_i(s) h_i(t)$$

when (1) the functions $g_i(s)$ and $h_i(t)$ are given as essentially bounded measurable functions; (2) the functions $a_i(t)$ and $b_i(s)$ are allowed to range over all integrable functions; and (3) the criterion for the approximation is the L_1 -norm.

Thus, we want to make this measure of the error small:

$$\iint |f(x,y) - \phi(x,y)| dx dy$$

6. "Approximation Theory in Tensor-product Spaces" (with John Respass).

This is on-going work connected with Mr. Respass' Ph.D. research. A representative result states that each continuous function of two variables, s and t , possesses a best approximation of the form $y(s)h(t) + \sum_{i=1}^n x_i(t)g_i(s)$ if h, g_1, \dots, g_n are continuous, h has no zeros, $\{g_1, \dots, g_n\}$ is a Tchebycheff set, and the functions y, x_1, \dots, x_n are free to range over all possible continuous functions.

7. "Approximating Multivariate Functions by Combinations of Univariate Ones," Proceedings of the 1979 Army Numerical Analysis and Computers Conference, El Paso, Texas, February, 1979, pp. 433-440.

This is an expository report, explaining several practical situations in which a multivariate function must be approximated by combinations of univariate ones. Progress on the existence of best approximations and on the algorithmic aspects of this branch of numerical analysis is outlined.

8. "Best Approximation in Tensor Product Spaces," CNA Report 151, August 1979, 8 pages. To appear, Proceedings of Dundee Conference on Numerical Analysis.

This is an expository account of the problem of representing multivariate functions by combinations of univariate ones. Applications are cited in the fields of numerical treatment of partial differential equations, numerical solution of integral equations, and preconditioning of matrices.

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