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MILITARY DECISION FROM THE VIEWPOINT OF GAME THEORY,(U)

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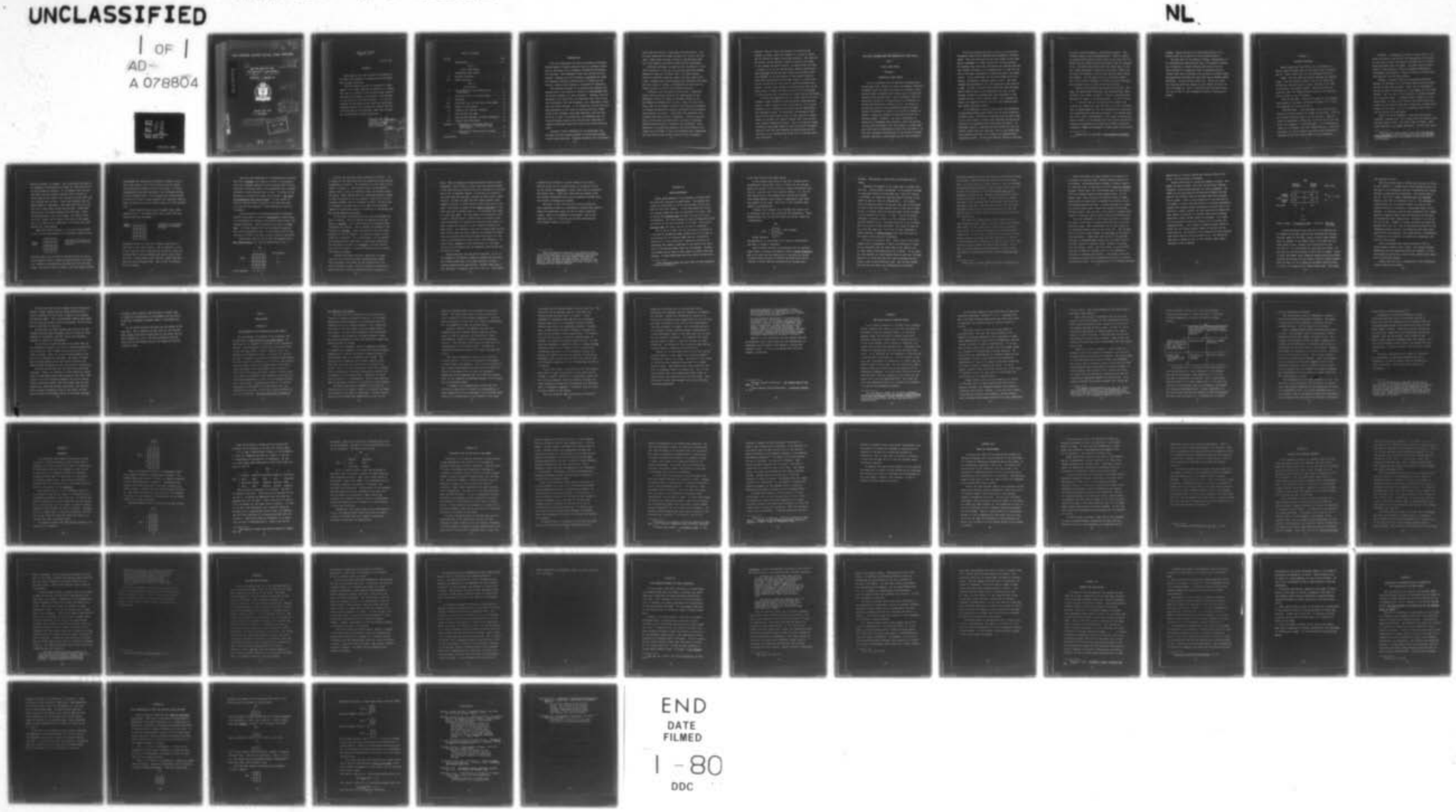

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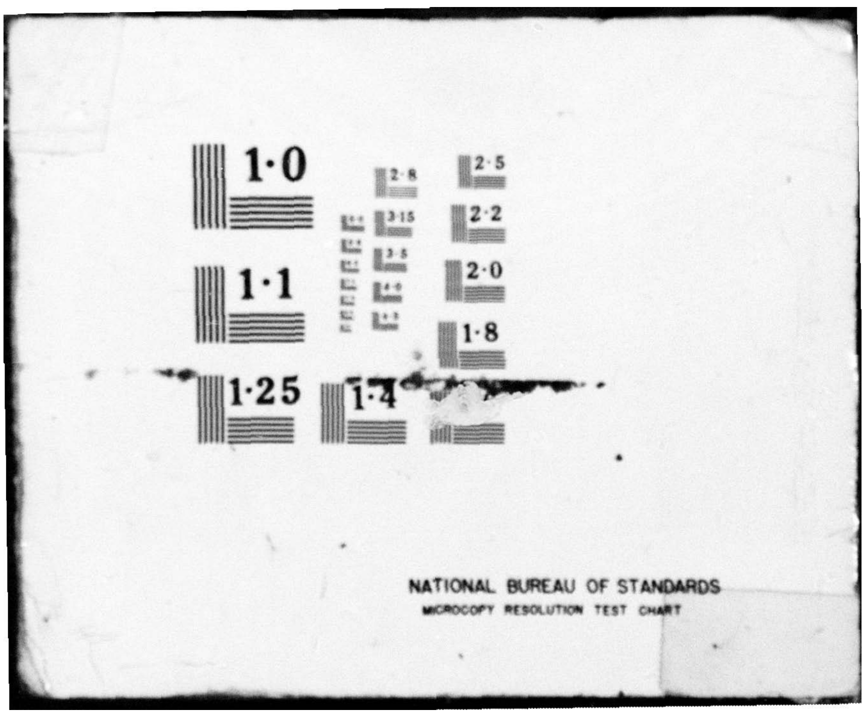
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THE UNITED STATES NAVAL WAR COLLEGE

MILITARY DECISION FROM THE VIEWPOINT OF GAME THEORY

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ADVANCED STUDY GROUP

May 1957

NAVAL WAR COLLEGE
Newport, R.I.

19 July 1957

FOREWORD

This paper is for use by staff and students of the Naval War College in connection with the Naval War College curriculum.

"Military Decision from the Viewpoint of Game Theory" is a study conducted by Captain R. P. Beebe, USN, while a student in the Advanced Study Group at the Naval War College, and presents a new approach to the problem of military decision. It is an expression of the author's attitudes and considered judgment and does not necessarily express the official opinions of the President, Naval War College, nor of the Chief of Naval Operations.

Charles H. Lyman
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INTRODUCTION

The vital importance of correct decisions in military affairs makes it highly desirable that all possible assistance be furnished to the Commander responsible for making the decision. The Commander's Staff was evolved for this purpose. Manuals are also furnished, suggesting the steps the Commander should go through prior to making his decision to insure that nothing is overlooked or neglected.

The detailed estimate of a situation may indicate the desirability of one course of action so overwhelmingly that the actual decision is a mere formality. However, when the estimate is not thus clear, the Commander must rely in the final analysis on his own judgement, past experience, training, and traits of mind. It is situations of this kind that give importance to the study of military history, formal study at War Colleges, peacetime exercises, and the like. All this is done in the hope that the Commander will, in times of stress, draw on all of his mental resources to formulate decisions which will prove to be sound after the event.

Because of their awareness if its importance, the military services became students of the decision making process many years ago. Economic interests and the academic world were far behind in realizing its significance. But, as their awareness has grown, so has their work on the subject. Today there is much of value to the military planner to be found in the work of other social disciplines.

Military decisions are made in what is known as a "conflict situation". That is, one side has goals and desires opposed to those of the other side. The same conflict exists in politics, business, games, and many other activities. Students have been studying such situations for years with the object of trying to determine what each side can expect to gain in view of the opposing interests.

Recently these speculations were given a tremendous boost toward practical usage by the publication of the monumental "Theory of Games and Economic Behavior", by von Neumann and Morgenstern in 1944. Here, for the first time, some of the simpler conflict situations were subjected to rigorous mathematical analysis and proof of the theorems which predict the result. Since that time numerous other scholars have built on this foundation. Practical application of the theory has been made in tactical situations, weapons system analysis, logistics, and economics. More important to the military planner, the implications of the theory can be brought to bear on situations more complicated and less precise than those to which it was originally applied. Many of these conclusions are tentative and limited in scope. Much work remains to be done in applying the Theory of Games to military decision. But, much as we dislike placing another technical straw on the burden of the present day Naval officer, it appears that the time is coming, if it is not already here, when he should have some knowledge of the way game theory can influence decision making. It is the purpose of this paper to give a simplified explanation of what game theory is, and how it can be applied in its pure form to various military situations. We will then discuss the implications that can be drawn from the theory and show how the line of reasoning it advocates may aid in arriving at sound military decisions.

It should be emphasized at the outset that we are not seeking a magic formula to solve difficult planning situations. Such a result is far in the future, if it can be deduced at all. Neither do we offer hope of simplifying the decision process. If anything, considering an estimate from the game theory point of view requires a higher degree of analysis and logical thought than does the present standard planning doctrine. But it is hoped an understanding of what game theory is and the type of reasoning behind it will aid the Commander in marshalling his own abilities to the maximum when faced with a difficult planning situation.

MILITARY DECISION FROM THE VIEWPOINT OF GAME THEORY

PART I

BASIC GAME THEORY

CHAPTER I

INTRODUCING GAME THEORY

It is no accident that Von Neumann and Morgenstern titled their book, "Theory of Games and Economic Behavior". Economists have long been interested in creating a mathematical model to predict the interactions of buyers and sellers in the market place. Early economists tried to extend the actions of a single person in the market to include the desires and results of all. It should be obvious that this so called "Robinson Crusoe" economy cannot be projected to predict the actions of numbers of people. Crusoe can produce the greatest good for the greatest number - himself. But when a multitude of people try to maximize their desires, their interaction as they plot and scheme to do this makes it certain that many will have to compromise on "less than the best". As this has tremendous implications to the overall national economy, interest in the problem is more than academic.

To use an analogy that may be closer to the military situation, every one is familiar with diagrams of football plays. They plot graphically how every member of the team has an assignment to clear the way for the ball carrier. If everyone does their job, the solid line representing the ball carrier proceeds up the diagram until it runs out the top of the page; presumably he crosses the goal line standing up. In an actual game the situation seems to be different. Occasional touchdown plays do occur. But most coaches are satisfied if they can grind out a few yards at a time. In fact, they know from experience that the play which looks so good on the blackboard will not, on the average, realize the maximum gain when subjected to the counter actions of the opposing team. Is it possible to deduce the expected return? It is to this question that game theory addresses itself.

It is important to note at this point the difference in philosophy between game theory and the more optomistic football coach. The theory recognizes from the start that it is a rare thing to achieve one's maximum desires in opposition to an opponent with conflicting interests. It addresses itself to deducing the best return one can reasonably expect over the long run. And it also shows that this return is the one to be sought in a nonrepetitive situation as well. It has very definite things to say when no plan of action promises a satisfactory result. The football coach is less straightforward. There is reason to believe that he must be well aware of the fact that his plays will not always produce the results they are designed to do. Yet what coach would admit it; would come out and say, "this play might be good for a yard but I doubt it?". Of course his excuse is that he must keep up morale among the players. In its place this is a very good reason. But if this line of thought is carried over into the planning process itself it may have serious results.

Interest in reasonable expectations is not new. Huygens, the great Dutch astronomer and mathematician, while writing on the theory of games of chance, realized he was skirting the edges of something bigger. In 1657 he wrote, ". . . I believe that in considering these things more closely the reader will soon see that it is not a question only of simple games but that the foundation is being laid for interesting and deep speculations".[1] Von Neumann made one of his first contributions to the theory by writing on poker in 1928. It is probably for this reason that he continued his researches in solving the economic problem by working on "Games of Strategy"; of which poker is the prime

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[1]Quoted by J.D. Williams in The Compleat Strategyst. p. vi.

example. Hence the use of the term "Game Theory" for a process that can be applied to the bloodiest battle.

Now just what is the Theory of Games? Essentially, it is a mathematical demonstration that, if opposing interests act rationally to achieve desired ends that can be set forth validly in a numerical scale of expected returns, returns that vary according to the success of various plans, the appropriate strategy for each side can be deduced mathematically. It says no more than this. As will be seen, the limiting words and phrases of this definition will return to plague us. But to illustrate what the pure theory is, let us turn to some examples in which it does give an answer.

CHAPTER II

MATCHED STRATEGIES

Before going into Game Theory, it is necessary to have in mind the definition of some terms used in discussing it. Other definitions will be introduced later on.

GAME. The word game has several meanings in ordinary usage. For our purpose a game is the set of rules that define what can and cannot be done, the size of the bets or penalties, and payoff methods. These rules must be complete, must not change during the play of the game and must be known to all contestants.

PLAY OF THE GAME. A play of the game is one complete run through of the game, including the paying off of bets and penalties at the end of play.

ZERO-SUM GAME. A Zero-sum game in one in which the gains of one side balance the losses on the other. That is, no outside influence takes a cut of the bets. Poker in which no percentage is taken out of the pot to pay for the beer is an example of a zero-sum game. If a percentage is taken out it becomes, (not very imaginatively), a non-zero-sum game. As can be readily appreciated the mathematical analysis of the two types of game is quite different.

STRATEGY. A strategy is a plan of action that is complete and ready to use before the commencement of the game. It takes into account the rules of the game and all intelligence available about the enemy. The great value of game theory to the military is that it analyzes situations of "incomplete intelligence". There is hardly a military plan that does not fit this description.

PERSON. A person is one of the opposing interests. Bridge, for instance, is a two person game; north and south are out to beat east and west. Poker can be considered a two person game by analyzing it as you against all the other players. War, in spite of the millions of persons involved is a two person game. A fighter versus a bomber is a two person game. In fact, most conflict situations can be resolved into two person games and a large part of game theory treats of this field. All of the examples used in Part I are two person, zero-sum games.

Now to our first example.[2] A man and his wife are planning a camping trip. For reasons that will appear later the man's name is Bill. His wife's name is Rhoda. Bill likes mountains, the higher the better. On the other hand,

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[2]The matrices used in Part I are taken from <u>The Compleat Strategyst</u>, by J.D. Williams; but the approach used in describing them has been changed to match the purpose of this paper.

Rhoda is allergic to heights. She is perfectly willing to go camping in the mountains but wants the camp set up at as low an altitude as possible. It so happens that the area into which they are going is covered with a network of Forest Service fire roads, four running north and south and four running east and west. After some argument Bill and Rhoda decide to compromise. They agree that Bill will select one of the roads running east and west. Rhoda will select one of the roads running north and south. Where the intersection is will be the camp. Of course it will not do for either one of them to lose face at this game; and it is a game by our definition. So they go into deep thought for a few minutes.

Take first Bill's problem. Let us list the altitude of all the road intersections in a properly oriented table.

Bill's Roads					
	1	7	2	5	1
	2	2	2	3	4
	3	5	3	4	4
	4	3	2	1	6

Altitude in thousands of feet of the 16 road intersections.

Now Bill wants to go as high as possible and he is led to the delights of Road 1, where the highest peak of all beckons. However, he immediately realizes that this is dream stuff. He does not dare to select a plan that might realize the maximum but would lead to disaster if Rhoda is skillful in her choice. He is led to look at all the roads, with particular attention to their low points. A little consideration shows him that Road 3 has a minimum altitude of three thousand feet. If he chooses this road the worst he can do is to get a three thousand foot campsite, while if Rhoda is careless he can do better, even up to five thousand feet.

As Bill feared Rhoda is just as smart about these things as he is. She knows better than to moon over the table of her road altitudes --

Rhoda's Roads

1	2	3	4
7	2	5	1
2	2	3	4
5	3	4	4
3	2	1	6

Altitude of road intersections in thousands of feet.

thinking how nice it would be to camp on road three or road four at the lowest points. She is impelled to look at her roads for their peaks and her inspection leads her to select Road 2 with a maximum altitude of three thousand feet. If she chooses this road the highest the camp can be is three thousand feet. And if Bill is careless it can be lower.

Now note that something of a coincidence has occurred. Bill has a strategy, Bill Road 3, or Bill 3, that guarantees him a minimum altitude of three thousand feet or more. Rhoda has a strategy, Rhoda 2 that guarantees her a maximum altitude of three thousand feet or less. If either of the opponents is careless the other can do better. But under no circumstances can they do worse. In fact either of them can announce their strategy to the other and there is no way for the opponent to take advantage of this intelligence.

This coincidence of guaranteed maximum and minimum payoffs being equal is called a saddlepoint in game theory. If a saddlepoint is present, both persons should follow the strategy indicated. Their strategies are said to be matched. If either departs from the indicated strategy he will suffer unnecessary loss. If both depart from it the situation is fluid -- but one of them is bound to get hurt.

Having gone this far let us turn the example into a game theory matrix; for that is exactly what it is.

		RED 1	RED 2	RED 3	RED 4	Row minimum
BLUE	1	7	2	5	1	1
	2	2	2	3	4	2
	3	5	3	4	4	3*
	4	3	2	1	6	1
Column maximum		7	3*	5	6	

This is the way game theory problems are written. Let us agree for the time being that the payoff for each strategy interaction is correct. The derivation of this value is discussed in chapter 5. Bill becomes Blue, the maximizing player; that is, Blue is seeking to make the greatest possible gains. Rhoda becomes Red, the minimizing player; seeking to keep the payoff as low as possible. From now on we will consider problems from the viewpoint of Blue. As far as Red is concerned the result will be the same if he considers himself the minimizing player or constructs a new matrix with Red in the maximizing position.

To the right of the matrix we put the row minimums for Blue to inspect. Blue is seeking the maximum of the minimums, or maximin. It is indicated with an asterisk after inspecting the figures. Red is seeking the minimum of the maximums, or minimax. Below the matrix we put the column maximums for Red to inspect. The smallest figure is indicated by an asterisk. It is the minimax. If the maximin and minimax are the same value, a saddlepoint exists and the strategies are said to be matched. We have already seen what will happen if either contestant does not follow the matched strategy.

Consider this matrix now as a game that is played over and over with both persons simultaneously naming their strategy. The figure at the intersection to be paid in dollars to Blue. For it is another convention of game theory that positive numbers represent payments to

Blue. Then in playing this game Red would always pay Blue something. And it would not take long even playing randomly for fun for Red to realize that his strategy 2 is the only one to use to minimize losses. Or for Blue to realize in this case that his strategy 3 brings the best return, three dollars. To make the game fair Red should demand a side payment from Blue of three dollars before every play of the game. This then, is known as the value of the game, a concept that will be useful later on. As described this would be a very dull game. But Game Theory says nothing on such a point. Neither does it require that a strategy be sensible providing it is complete and abides by the rules. It must, however be rational, by the original definition of game theory. For instance a military strategy might provide that you run like the devil whenever you sighted the enemy. This is perfectly valid from the game theory point of view however reprehensible it might seem to your superiors. In other words, judgement of the suitability of a strategy still has to be supplied by the Commander, game theory will not do it for him.

Having come this far what have we gained? As far as the example is concerned it might be argued that any sensible person could come to the same conclusion without knowing anything about game theory. Without going into a full analysis, it appears that this is not so. Blue should certainly not use Blue 2; and Red should not use Red 1. This is taken care of in game theory, as a matter of fact, by the concept of dominance, a factor that will be discussed more fully later. After this there does not appear to be any further elimination that would be apparent without using the concept of minimax and maximin.

The point is that the solution of this problem has been rigorously proven as a mathematical theorem that applies under any conditions, under any set of rules, using any values. This is Von Neumann's achievement. And is the departure point for "interesting and deep speculations" that Huygens lacked in his day.[3]

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[3] Lest the reader be deceived by the apparent simplicity of the examples it is well to remember rigorous proof that the rules of game theory problem working give the right answer, and only the right answer, required the genius of the foremost mathematician of the country. The late Dr. John Von Neumann himself said that he had to use mathematics "far beyond calculus".

CHAPTER III

MIXED STRATEGIES

Before going further, it is necessary to become familiar with more concepts of game theory. Von Neumann describes two variations of the two person game such as was used for the first example. In one Blue makes his choice of strategy before Red does and announces it to Red. This is called the <u>minorant game</u> of Blue; that is, he is at a disadvantage. The other subgame is when Red makes his choice in advance of Blue and announces it to Blue. This is the <u>majorant game</u> for Blue; that is, he had the advantage.

Now it can be shown[4] that the doctrine of arriving at a decision by estimating enemy capabilities is the same as the solution of the minorant game that is, the maximin. Similarly it can be shown that a decision arrived at by estimating the enemy's intentions is the same as the solution of the majorant game, that is, the minimax. Further, the maximin may be equal to but cannot be greater than the minimax. To make deductions from these facts we will have

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[4]The reasoning behind the statements in this paragraph is given in Appendix A.

to go still further into game theory.

It has already been noted in the first example that a "saddlepoint" was present; that is, there was one strategy which each opponent should follow or suffer unnecessary loss. This is a special case of the general theory. In a four by four matrix such as the example even filling the squares with random numbers would produce a saddlepoint only about ten percent of the time. What then is the situation when there is no saddlepoint?

Consider as an example the following two person, two by two game; that is, each person has two strategies. Again, please accept that the payoffs shown are correct under the circumstances.

		RED 1	RED 2	Row minimums
BLUE	1	3	6	3
	2	5	4	4*
Column maximums		5*	6	

The maximin is 4, the minimax is 5; hence no saddlepoint. How then to choose a strategy?

Here we enter the most controversial part of applying game theory to military situations, that of <u>mixed strategies</u>. And it is here that the difficulty of applying the theory to obtain exact answers to military problems that are not capable of being expressed mathematically becomes most evident. Nevertheless, some useful conclusions may be drawn.

Consider the example to be a game that is played over and over. Look at Blue's situation. If he chooses strategy 1 he may gain three or he may gain six. But if he sticks to strategy 1 it would not take Red long to realize it and limit his gain to three by playing Red 1. If he plays Blue 2 he may get five or he may get four, but if he sticks to this it will not take long for Red to limit him to four. He can, of course, play the minorant game; choose strategy 2 and be assured of a return of four. Can he do better? Certainly he should be able to as the average of all the possible payoffs is $4\frac{1}{2}$. He might gain this if he is lucky. But can he gain it with certainty? The answer is that he can, over the long run. To do this he must determine a grand strategy, or <u>mixed strategy</u> that tells him when to use either of his two pure strategies.

The way it is done is this. Recall that in the first example with a saddlepoint it made no difference if either of the opponents told the other what he was going to do. In other words, intelligence was not a factor. In our present case however, an intelligence service could pay off handsomely if it could tell Red just which strategy Blue was going to use at any particular time. How can Blue insure that Red will not gain this information by any means, including deducing a pattern of operation from previous plays of the game? There is only one sure means of Blue doing this. He should choose a chance device so that the strategy is selected wholly by chance in the correct proportion. If he does this properly he is assured <u>in the long run</u> of averaging a payoff of $4\frac{1}{2}$. Neither contestant knows which strategy Blue will use next. In fact, Blue has restored the game to the condition that intelligence collection by Red cannot hurt him. Although Blue cannot tell Red which Strategy he will use on any particular play, he can tell Red his grand, or mixed strategy and Red cannot profit by this information.

The rules for computing the proper odds for use of each strategy by both players are quite simple for games of this size.[5] In the example Blue should favor strategy 2 three times out of four; Red should play both strategies randomly but in even odds. If Red does not counter Blue's mixed strategy of playing his pure strategies at odds of 1:3 by playing Red's strategies at odds of 1:1 it will cost Red more than $4\frac{1}{2}$ points per game on the average. So if Red is careless Blue may do better than a payoff of $4\frac{1}{2}$, but under no circumstances can he do worse <u>over the long run</u>.

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[5]The rules for simple games are given in Appendix B.

Recall that Blue can assure himself of a payoff of 4 by playing the minorant game, thereby assuring himself of the maximin. By playing mixed strategy he assures himself of a payoff of $4\frac{1}{2}$, a clear gain of $\frac{1}{2}$ point <u>with no increase in risk</u>. Recall also that the minorant game is the equivalent of a decision based on enemy capabilities. We can deduce, then, that a doctrine of decision based on enemy capabilities is essentially conservative and does not gain the maximum possible success. A mixed strategy will always do better by some factor between the maximin and minimax. Provided, again, that the play of the game is repeated often enough to bring the laws of chance into play statistically.

This last statement is the stumbling block, of course. For many, if not most, military situations are of the non-repetitive type. The circumstances of the next action do not reproduce exactly those of the prior one. And for game theory to give an exact answer the game <u>must</u> be the same on each repetition. Admittedly this is one of the greatest barriers to extending game theory into the broad field of military decision in general terms. However, we should note two things. First, that the concept of mixed strategies can be used as a general guide to the Commander's thoughts in reaching his decision. And second, there are situations in which mixed strategies will work on a purely mathematical basis. As a working tool it is finding increased application in tactical studies and weapons system evaluation. Let us consider an example.

Suppose that Blue is flying his bombers in pairs. One bomber carries the bomb, the other carries radar jamming equipment, anti-missile missiles, or other equipment. The bomber in the lead position obtains more defense from the guns of the follower than the follower does from the guns of the leader. Blue is only interested in the survival of the bomber; as usual Tail End Charlie is strictly expendable. Suppose that weapons system evaluation has shown that in an attack by a single fighter the lead plane has an eighty percent chance of survival if attacked, while the following plane has a sixty percent chance if attacked. If a plane is not attacked its survival chances are, of course, one hundred percent. Just offhand a commander might decide to settle for the eighty percent chance of survival in the lead position. An estimate of the situation in the standard form would indicate that he should do so. But before he does this it might be well to run through a game theory analysis of the situation.

			RED				
			attack follower	attack leader	blue odds		
			1	2			
BLUE	bomber follow	1	60	100	20	1	or 1:2
					or	or	
	bomber lead	2	100	80	40	2	
Red odds			20	40			
			or				
			1	2			
			or				
			1:2				

Value of game $\frac{1 \times 60 + 2 \times 100}{3} = 86\ 2/3\ \%$ (8% more than 80%)

A mixed strategy in the form of a 2:1 preference for the bomber in the lead position has increased the average survival chance to 86 2/3%, a gain of about eight percent over the chances if the bomber stays in the lead and the enemy finds it out. Against this strategy Red's best strategy is a 2:1 preference for attacking the leader. If Red does not do this the gain to Blue will be higher. Clearly then, the bomber force commander should position his bomb carriers by rolling a die or using a table of random numbers. With the die, if 1 or 2 comes up the bomber should follow; if 3,4,5, or 6 comes up the bomber should lead. This gives the required 1:2 odds.

Now you may well have a feeling that leaving the choice of his tactics to a chance device shows irresponsibility on the part of the commander; that he is abdicating his responsibility to make military decisions. But clearly this is not so. All of the cogent reasoning that the commander wishes to put into the decision is in it. The weighing of probabilities and the relative weight to be given each course of action is necessary to using game theory on the problem. It requires a more precise evaluation and a higher degree of logical thought than does deriving the decision by considering enemy capabilities. Deception is as old as the art of war. Changes in tactics to throw the enemy off balance often pay off. But, if successive decisions of the commander reveal a pattern, the enemy may well take advantage of this intelligence. The random device is deliberately chosen by the commander to prevent this happening, and only for this reason.

Other situations where these principles could be applied should readily come to mind. The contest between submarines and anti-submarine forces for example. Or the stationing of carriers in a fast carrier attack force under air or sub-surface attack.

Having come this far, a summing up of the conclusions already drawn may be useful.

We have seen that the old argument between basing estimates on enemy capabilities and enemy intentions has a direct counterpart in the theory of games in the minorant and majorant games. This correspondence between game theory and our planning process is most interesting. We will have more to say about it later on.

We have seen the rather obvious fact that if two contestants have matching strategies, that is, there is a saddlepoint in the matrix, both must follow the indicated strategy or suffer the consequences.

In the field of mixed strategy we have seen that, in the repetitive situation, the use of a mixed strategy will, in the long run, give a bigger payoff than the use of a single strategy. Rules for correct proportionment of the use of each strategy are available. The Theory of Games does have application now in the fields of tactics and weapon system evaluation and this use should be understood.

But what we have done thus far must have raised many questions. Where, for instance, do these numerical expressions of anticipated gains from strategies come from? What happens if the two opposing commanders do not use similar scales of military worth? What about games that are only played once, like most military situations? How can the idea of mixed strategies be used in that case? We have seen how useful intelligence can be in the mixed strategy situation, can we derive from the theory a useful scale to judge the effectiveness of intelligence and the effort to put into learning the enemys plans or concealing your own?

All of these questions and more will be tackled in the next part. But in doing so we will have to, for the most part, depart from strict application of the theory that we have followed up to now. And draw inferences from the basic work of Von Neumann that go far beyond what he had in mind in 1944.

PART II

IMPLICATIONS

CHAPTER IV

THE ESTIMATE OF THE SITUATION AND GAME THEORY

Let us consider the Estimate of the Situation form as laid down in Joint Action of the Armed Forces. The first part of it comprises a statement of the Commander's Mission. The second part contains the intelligence data on own and enemy forces, terrain, weather, and so on. Together with standard doctrine, the work up to now may be said to compare to the Rules of the Game in game theory. The next item in the Estimate is to "note all the possible courses of action within the capabilities of the enemy which can effect the accomplishment of your mission." After this, "note all practicable courses of action open to you which if successful will accomplish your mission." This would correspond to a list of the strategies available to both sides. In game theory form, it is done by arranging the strategies on a blank matrix form, own strategies on the left or Blue side, enemy strategies at the top or Red side. No values would yet be placed in the squares of the matrix.

The next step in the Estimate of the Situation is "Analysis of Opposing Courses of Action." In the Naval Operational Planning Manual it is recommended that the Commander write the opposing courses in two columns and test each of his own courses of action against each of the enemy's. In each cross comparison he is admonished to visualize the interaction, estimate probable losses, and conclude whether the enemy can effectively oppose the proposed course of action.

Even if the Commander uses the suggested procedure, it appears that comparing the courses of action and enemy capabilities in matrix form gives a clearer picture of the situation than the recommended two columns. The point by point comparison will usually require a good deal more text than can conveniently be placed in the matrix. But, as a final summary and visual aid, displaying the interaction of several strategies in matrix form is superior to the column method. Certainly the extra work to make it out is negligible. And for injecting the game theory point of view it is essential, as will appear.

The Manual enjoins the Commander to consider all capabilities of the enemy in order to eliminate the danger of deception by one that is overlooked. It then issues a warning that listing enemy capabilities is not for the purpose of deciding which one the enemy will actually employ. "To base a plan solely on what we think the enemy is going to do is extremely dangerous." From the game theory point of view, this is a clear recommendation for the Commander to play the minorant game.

Later on, however, the Manual notes that the Commander may well consider special knowledge of the enemy in order to determine his intentions. "Such knowledge may reward the Commander with outstanding success." From the game theory point of view this is a recommendation of the majorant game; provided the Commander actually has deduced the course his enemy will follow.

That this contradiction exists is well known in military circles. It is not the purpose of this paper to take sides in the argument. But it can be pointed out that a game theory approach can well set more realistic limits to the gains to be won than the "extremely dangerous" if the Commander does not follow the minorant game, to the "possible outstanding success" of an accurate use of the majorant game.

It appears to be unqualifiedly correct that the doctrine of arriving at a decision by consideration of enemy capabilities is the equivalent of the minorant game. However the correspondence of an estimate of the enemy intentions and the majorant game is not as clear cut. The definition of the majorant game is, "Red makes his decision first and announces it to Blue." If the Commander is estimating what Red will do it becomes a majorant game only if he estimates correctly and Red actually does follow the capability selected. Furthermore, if Red himself follows a doctrine of basing his action on an estimate of Blue capabilities, his decision will be from his point of view the minorant game, or minimax, in a matrix from Blue's point of view. In this case the difference in payoff to Blue in a mixed strategy situation played once in regard to the two methods of estimating enemy action is the difference between the maximin for capabilities and the minimax for intentions. Ordinarily these limits would be much less extreme than the "extremely dangerous" or the "outstanding success" of the Manual.

On the other hand, if Red does not "act rationally", which from the game theory point of view means using a minimax or maximin strategy or a combination between the two, and the Commander does not correctly estimate the enemy intentions he may well find himself in a situation that is "extremely dangerous." Provided, that is, there is such a result in the matrix.

Thus, by using the game theory point of view the

Commander is provided with a more accurate aid to exercising his judgement than the remarks in the Manual. For instance, suppose the difference in results to be obtained by estimating enemy capabilities on the one hand, and enemy intentions on the other if the enemy makes a "rational decision", is small. But a strategy selected on the basis of enemy intentions shows a possibility of a very unfavorable result if the enemy does not, in fact, use the intention estimated. The Commander then would probably deem it unwise to base his estimate on enemy intentions. The reverse of this situation might make an estimate based on intentions more attractive.

Opposed to the discussion above is the fact that an estimate based on enemy capabilities protects against "irrational" acts by the enemy. Any failure by the enemy to follow his best capability may result in a larger return to Blue. This feature does seem attractive, and will be discussed further in Chapter VII. But, if the enemy deduces that our estimates are based solely on capabilities, his course of action becomes clear. That this point is more than academic can be seen from the following quotations:

> By General Westphall of the Whermacht "Their /the Allies/ desire to undergo as little risk as possible prevented them from seizing their chances of bringing an early decision.[6]"

> By Field Marshall Kesselring: "I believe this development /of new tactics at Anzio/ was due to a cardinal error of our German Propaganda, which could not do enough to taunt the enemy for their lack of initiative, thereby goading them into a gradual change of operational principles. The method of cautious and calculated advance according to plan with limited objectives gave place to an inspirational strategy which was perfected through the months remaining till the end of the war.[7]"

Implicit in the discussions of this chapter has been an assumption that a value scale for the interactions of strategies can be placed in the matrix to provide the guidance needed. It is to this problem that the next chapter is addressed.

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[6]General Siegfried Westphall. The German Army in the West, p. 167

[7]Field Marshal Albert Kesselring. A Soldier's Record, p. 238

CHAPTER V

THE VALUE SCALE OF MILITARY WORTH

In our opening examples of how game theory operates several matrices were used in a form that presented the strategies' worth in numerical terms. In the first example, about the altitudes of various road intersections, there was no doubt what the values should be. In the third example, about bombers in formation, the values were derived from separate work. Presumably, the figures on vulnerability were the best available from tests. It is *important* for the Commander to keep his eye on whether this is actually so. Are the figures on the same type of aircraft and armament he is using? Are the weather conditions the same? Are the tactics the same? Plainly there is a field here for the exercise of the Commander's judgment. He or his staff should not be too impressed with the data available to him unless it has passed every test for applicability.[8] Only after the value has passed this test is it suitable for inclusion in a matrix for determining strategy.

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[8]Colonel Haywood, USAF, in his thesis, <u>Military Doctrine of Decision and the Von Neumann Theory of Games</u>, p. 55, gives an example a bomber formation analysis on a mathematical basis which was found <u>not</u> applicable to actual service.

In the second example of the first part we used four arbitrary strategies, values 3, 4, 5, 6. Assuming this was a military situation where would one find these values to represent the military worth of various courses of action?

The plain truth is, there is no such scale of military worth. The most that can be gotten out of manuals on military planning is the instruction to rank enemy capabilities in their order of probability, and own courses of action in order of desirability. It is the lack of such a scale that prevents the use of game theory as a formula for calculating the proper decision. In view of all the imponderables in military situations the development of such a scale would be most difficult, particularly in view of the range of decision situations with which military Commanders are faced. It would appear, however, that useful work can be done on the lower, or tactical, end of the scale and extended further as experience with the subject grows.

If game theory points up the necessity for a scale of military worth, it also points up the fact that in making a decision the Commander actually does express a preference for various outcomes. If he has three courses of action, A, B, and C, and prefers A, he has decided that he values A more highly than B or C or any combination of A, B, and C. Such relationships are the first step in deducing a value scale.[9]

That a value scale is desirable may be one of the reasons for the development of such organizations as the Weapon System Evaluation Group, though its charter does not express the idea in such a form. In fact, it can be seen that a value scale is really required for logical usage of current doctrines of decision quite aside from any questions of game theory analysis. Game theory only points up the fact that such a development has been needed right along.

However, it is not necessary to have a scale of numerical values for opposing strategies to use the aid of game theory in arriving at a decision. As we have said, in considering his own courses of action the Commander must end up with a preference scale of some kind for them. This preference may be expressed in words or phrases, or might simply be the position of the courses on a list. In either event the preference ordering can be used in checking the comparison against the matrix form. As an example consider this student estimate for a Naval War

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[9] Von Neumann and Morgenstern, op. cit., pp. 16-20, have considerable discussion of the development of a numerical scale of utility from preferences expressed. They point out that the problem has been solved in other cases where it did not appear possible to do so.

College exercise in which air, sea, and amphibious forces were available to control an area. It has been suitably paraphrased to remove its security classification.

		RED	
		Deny Blue entry into the area by offensive air, sub, & mine warfare 1	Destroy Blue forces in ground action after Blue landing 2
B L U	Destroy Red forces which threaten control of the area by air, sea, and amphibious assault 1	Stands up well	Stands up reasonably well
E	Destroy Red forces which control area by air attack. 2	Does not give positive control	Will not control

The phrases used in the matrix boxes are extracted from the discussion of each comparison in the estimate. Blue selected strategy 1, and properly so. It is the only way to accomplish his mission; and it can succeed against either of the enemy capabilities. It is also the solution to the minorant game of Von Neumann, the maximin is "stands up reasonably well." Further inspection of the matrix will show that the maximin is also the minimax and Red's best strategy is 2. A saddlepoint is present and the strategies are matched.

But recall that we are not attempting to deduce a formula for solving the decision situation. The matrix is only an aid to the Commander in ensuring the best possible decision. Let us inspect it a bit more.

Blue 1 clashes directly with the sea and air forces in Red 1. The favorable result is predicated on Blue feeling his forces are capable of doing the job. The somewhat less favorable result for Blue 1 against Red 2 is based on the difficulty of countering Red ground reinforcement by air attack prior to the landing. Suppose Red thought his strategy 1 could hold off Blue; would he not use this instead of Red 2? Yet the Blue estimate shows Red 2 as Red's best strategy. Since there appears to be no reason why Red cannot implement both of his strategies should not there be a Red 3 that does this? It would appear that the estimate is somewhat incomplete and consideration should be given to a third course for Red, combining Red 1 and 2.

The significance of the last paragraph is this. The idea that a third capability for Red should be incorporated came from an inspection of the matrix and not study of the Estimate's text. All of the information was in the Estimate. But it was easier to spot this discrepancy while "playing" with the matrix and varying the relative values given to the interactions.[10]

An analysis of five other strategic war game estimates shows that three of them chose courses of action that did not correspond to the strategy indicated by applying game theory to the matrix with even the most casual scale of values. There may be sound reasons why this was done. Again we are not advocating solution by formula. But, at the minimum, such a result on the matrix should make the Commander pause and consider; the text of his estimate should reflect the reasons for such a preference. It does not appear this was done in these cases.

To sum up, construction of a matrix to represent the interaction of opposing strategies may lead a Commander to a clearer insight into the problem with which he is faced. Its use for such a purpose is recommended.

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[10] It is interesting to note that in the play of this war game Red actually used Red 1 instead of the proposed Red 3. Bue he did this because his own estimate gave Blue a greater capability for landing at widely separated points than Blue thought he possessed. Hence Red held his troops back for use when the landing point became clear. From the Blue point of view this was an "irrational" act by Red.

CHAPTER VI

DOMINANCE

In considering enemy capabilities and own courses of action, a detailed analysis of each coupled with a comparison of their interaction can lead to a great deal of unnecessary work when the Commander is sure he will not use some of the suggested courses of action and becomes convinced that the enemy will not use some of his possible capabilities. Game theory provides an accurate method of eliminating such strategies from the matrix by the concept called dominance.

From the Blue point of view, dominance exists if the elements of any row are equal to or superior to the corresponding elements of any other row. If this is the case the dominated row can be eliminated from the matrix. Such a strategy can only offer equal or inferior results compared to the strategy row that is dominant. Similarly Blue can estimate that, if Red acts rationally, Red will not use any column in which the values are equal to or greater than those of another column.

For example consider the first matrix presented, the selection of a campsite.

Rhoda

Bill		1	2	3	4
	1	7	2	5	1
Bill	2	2	2	3	4
	3	5	3	4	4
	4	3	2	1	6

Bill 3 is dominant over Bill 2, consequently there is no reason for Bill to consider Bill 2 further. In Rhoda's case every element of Rhoda 1 is equal or superior to the corresponding box in Rhoda 2. Consequently (as Rhoda is the minimizing player) she would certainly not select column 1, whatever else her decision might be based on. The reason for writing, "Bill is led to the delights of road 1, but he immediately realizes that this is dream stuff", is, in fact, dominance.

Having eliminated Rhoda 1 and Bill 2 the game reduces to:

Rhoda

Bill		2	3	4
	1	2	5	1
Bill	3	3	4	4
	4	2	1	6

None of the rows or columns are now dominant and attention can be focussed on solving this smaller game.

Let us take a hypothetical estimate of the situation in which the Commander has made up a matrix to aid him in visualizing the interaction of strategies. He uses as a value scale words descriptive of the result from his point of view.[11]

		Red				
		1	2	3	4	5
	1	Failure	Excellent	Excellent	Superior	Excellent
Blue	2	Good	Fair	Fair	Fair	Excellent
	3	Excellent	Defeat	Superior	Superior	Fair
	4	Good	Fair	Failure	Failure	Superior

None of Blue's strategies are dominant. But Red 5 is dominant over Red 2. Hence Red can be expected not to use Red 5. It should be emphasized that Blue does not <u>know</u> Red will not use Red 5. But if Red did so it would be an irrational act from game theory viewpoint. In view of the fact that strategists are constantly being admonished not to underestimate their enemy, the Commander may well feel justified in concluding that Red will not use Red 5. When this column is eliminated it is found that now Blue 2 dominates Blue 4. Blue 4 can then be

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[11]The matrix is taken from Colonel Haywood's thesis, <u>op</u>. <u>cit</u>.

eliminated. When this is done Red 4 dominates Red 3 and can be eliminated. And Red 3 in turn dominates Red 2 and can be eliminated. The matrix then becomes

		Red	
		1	2
	1	Failure	Excellent
Blue	2	Good	Fair
	3	Excellent	Defeat

It is, of course, much too easy and arbitrary to cut down a list of courses of action or capabilities by such a mechanical method. Many other things should be considered; such as the effect of an irrational act by the enemy, possible combinations of strategy, the precision of intelligence. Nevertheless, if the Commander uses the matrix as an aid, dominance or the lack of it can be a guide to his decision. Again, as in the selection of a course of action, lack of correspondence between the matrix and the estimate is a clear warning to pause and reconsider.

Incidentally, the reader might find it profitable to look at the reduced matrix above and ponder the implications to be drawn from it in regard to basing his strategy on intentions or capabilities.

CHAPTER VII

IRRATIONAL ACTS ON THE PART OF THE ENEMY

Returning to our original definition of Game Theory, it was described as the selection of a strategy by opponents that make "rational" decisions. From the point of view of von Neumann and Morgenstern, working on economics, a rational decision was one in which a player sought to make the greatest feasible gain over the long run. Anyone that engages in business without such an ambition is driven by forces outside the realm of economics. Game theory completely falls apart without such an assumption. At least one of the opponents must make rational decisions. In the corresponding military situation, the Commander of Blue undoubtedly considers his estimate of the situation as the height of rationality. So this condition is satisfied; at least until the battle starts! What then is the effect of irrational decisions on the part of the enemy?

We have noted previously the minorant game has the advantage that if the enemy does not play his best strategy the gain to Blue will be even further increased. A decision based on estimate of enemy capabilities is the same course of action as the minorant game, except Blue does not announce his decision to Red. It would appear that the theory and reality part company here. For this reason. In game theory a strategy is a complete plan of action, prepared before the commencement of play, and taking into account all intelligence of the enemy and the physical environment, as well as the rules of the game. Nothing unexpected in the way of weather, terrain features, material failures and the like can come up. For by definition these have been anticipated in the formulation of the strategy. Furthermore, Red has anticipated all possible strategies of Blue and has only to pick one to counter the one selected by Blue.

While such a situation might exist in real life, it is more probable that some one of the factors mentioned would be imperfectly known or improperly estimated. We can say then, as a practical matter, in the use of a course of action chosen on an estimate of enemy capabilities an irrational act on the part of the enemy will probably result in benefit to Blue but it is not as certain as pure theory would have it. It follows that the better and more detailed the estimate, the greater chance that mistakes on the part of the enemy will benefit the Commander.

It might be well to interject at this point the idea that, in real life, the situation represented by the matrix of strategies is in reality three matrices; the matrix as estimated by Blue, the matrix as estimated by Red, and the matrix as it actually works out in play.[12] The closer a Commander's estimate comes to matching the situation in reality the better he is prepared to play the game. This leads to the same conclusion as the previous paragraph; the better and more detailed the estimate, the greater chance that mistakes on the part of the enemy will benefit the Commander.

In real life, when playing the majorant game, Red does not actually announce his decision to Blue. (Though it would be well to remember that a leader called Hitler did it with outstanding success -- for awhile.) Blue estimates Red's intentions. If he is right Blue gains over the result to be obtained by estimating enemy capabilities. But he must be right. At Pearl Harbor Blue was wrong, with devastating effect. Military historians are now generally agreed that the attack on Pearl Harbor was an "irrational" act on the part of the Japanese. In Normandy General Bradley was faced with a decision in the Avranches Gap situation.[13] He chose to station his

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[12]Footnote 10 of Chapter V notes the effect in a war game of the opposing commanders using different matrices.

[13]General Omar Bradley. A Soldier's Story. p. 369

reserves to support the Gap although he believed the enemy's best strategy was to withdraw. In an analysis of this battle from the game theory point of view Colonel Heywood discusses the strategies available to the American and German Commanders.[14] Von Kluge could either attack the Gap or withdraw. He decided to withdraw. Game theory analysis supports this conclusion. But Hitler ordered him to attack the Gap at all costs. The attack failed, his army was encircled and Von Kluge committed suicide. Bradley was well advised not to base his estimate on enemy intentions. Furthermore when the enemy committed his "irrational" act the payoff to Bradley was greatly increased, as the analysis shows it would be. Failure in this battle actually cost the Germans France.

Although Pearl Harbor is still a subject of much controversy, from the game theory point of view it does not appear that basing American dispositions on enemy intentions promised any particular advantage to offset the enemy's gain by doing the unexpected. Hence the decision could not be recommended. In General Bradley's case the

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[14] Colonel O. G. Haywood. "Military Decision and Game Theory." *Journal of the Operations Research Society of America*, Volume 2, Number 4, September, 1954.

increase in payoff if the enemy acted "irrationally" was very great if he based his estimate on capabilities as compared to the gain if he based his estimate on intentions; plus the fact that he could be in serious trouble with the Gap cut if he did not estimate enemy intentions correctly.

It appears that game theory analysis of the opposing strategies can be useful in such situations by indicating more clearly the limits of advantage and disadvantage if the enemy makes a mistake, acts stupidly, or uses a markedly different matrix than Blue.

CHAPTER VIII

VALUE OF INTELLIGENCE

In theory the limits set by game theory analysis of opposing strategies give precise measure to the advantages to be gained by obtaining information about your enemy, or his succeeding in gaining it about you. In practice the uncertainties of the situation, the "fog of War," difficulties of communication and the like, will make these measurements less susceptible of calculation. Nevertheless, a game theory point of view can again furnish the Commander with guidance which may be useful.

Recall that the minorant game is the equivalent of the enemy, Red, gaining complete information about Blue; while the majorant game is the equivalent of Blue gaining complete information about Red. If both play "rational" strategies, which we as Blue certainly expect to and which the enemy will most probably do, the difference in gain in the two situations is the difference between the maximin and the minimax. Even in a matrix made up of descriptive phrases it can be expected that the result will not vary more than the expected difference in return between one phrase and the next as to the actual return of the minimax or maximin.

It follows then that if the difference between the minimax and maximin is small the gain due to intelligence cannot be large. In this case the expenditure of large forces and resources in gaining this intelligence would not be justified. The converse is equally true.

These theoretical limits on the value of intelligence may not be applicable in actual practice, but they could furnish guidelines in doubtful cases. They might also be useful in confronting overzealous subordinates with the necessity of justifying a larger intelligence effort.

It will be seen that the philosophy of the game theory approach recommends choosing a strategy on the basis that the enemy will find it out. In the matched strategy situation there is, in theory, no way for the enemy to take advantage of knowledge of what strategy Blue will employ. In the mixed strategy situation, when repetitive, the use of a mixed strategy on an odds basis using a chance device to determine the exact pure strategy to use at any one time restores the game to the same situation; that is, the enemy cannot profit from knowing the mixed strategy. He can, however profit from knowing the pure strategy that is proposed to be used.

It can be used as a guide, then, that if the commander is using a mixed strategy, preparatory actions which are essential to all of the strategies to be employed can be undertaken without undue security precautions. This is often the case in large undertakings. For instance, there was no possibility of concealing from the Germans the build-up of American forces in England during the first years of World War II. However, the use of these forces, first in Africa and then in Normandy was guarded with maximum security.

It should be noted that the gain from intelligence of the enemy is limited to the difference between the maximin and the minimax only if the enemy follows a rational strategy. If he does not, that is, if he is stupid or makes a mistake, a Commander finding it out must be prepared to depart from the strategy indicated by game theory analysis in order to obtain maximum advantage of the intelligence. Von Neumann says, "All this may be summed up by saying that while our good strategies are perfect from a defensive point of view, they will (in general) not get the maximum out of the opponents (possible) mistakes."[13]

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[13]von Neumann and Morgenstern, op. cit., p. 164.

CHAPTER IX

SUPERIOR AND INFERIOR STRENGTH

In his remarks on poker, von Neumann points out some facts that may be useful in planning. He shows that a player who never bluffs will lose in the long run. Bluffing should be done for two reasons; one, the gain if it is successful; and second, the gain that is obtained if it is not successful. For, by bluffing and being called, a player gains by introducing uncertainty into his opponents mind. The bluffer is then in a position to collect when he has a really strong hand. Whereas if he never bluffed a high bet would cause the opponents to drop and his gain over the long run will be decreased.

From von Neumann's point of view intelligence in a game consists mainly of deductions from the opponents previous plays. Random bluffing will prevent the deduction of a pattern in this manner. A bluff can be one component of a mixed strategy. If a player never bluffs he is in effect limited, at the maximum, to the payoff of the minorant game. But in a game in which strategies are not matched, such as poker, the payoff of a mixed strategy will be greater than this by some fraction between the maximin and the minimax. This value is also the value of the game which was previously referred to. In effect the value of the game is the breakeven point at which two equally strong and skillful opponents would arrive in the long run. So if one of such opponents plays the minorant game he will on the average gain less than the value of the game and will eventually be the loser. It can be seen, then, that playing the minorant game, that is playing capabilities, will eventually lose against an opponent of equal strength but superior strategic sense.

The correspondence between the poker situation, which has been proven to be mathematically correct, and the military situation, depends of course on the degree of correlation between this game of strategy and the type of situation with which the Commander is faced. As a game of strategy poker does have some relation to war, though it is not contended that in the usual situation this correlation will be very close. Nevertheless, it can be seen that the closer a military situation comes to resembling poker the more surely will the mathematical predictions of von Neumann operate in the long run.

This being so we can say, as a guide, that a Commander who bases his estimates on capabilities in a situation which resembles that of mixed strategies will probably lose in the long run if an equal strength opponent plays a mixed strategy. The only way to avoid this is to be stronger than the opponent. By and large being stronger than the enemy at the point of contact and playing capabilities has been the course followed by the United States in its military history. That it has produced a good record is unquestioned.

It would appear possible, however, that in the future we might not be able to assure this superiority at all times and in all places. Essentially it has depended on a long and careful buildup of our forces and those of our allies. That this will always be possible is being questioned by many. It is not the purpose here to take sides in this argument. But from the game theory point of view it can be pointed out that an equal strength force can only break even in the long run by playing mixed strategy; and an inferior force can only save itself by playing intentions if such a result is possible at all. Commanders who have always based their estimates on enemy capabilities will be under a distinct mental handicap in these situations. During the first months of the Korean War the United States forces were in a distinctly inferior position. Col. Haywood describes how at least one of the commanders met this situation.

> The fame attained by General Michaelis in the first year of the Korean struggle rested to a large extent on his ability to estimate the pattern of thought of opposing North Korean generals. Time and again he left his front

> manned by a skeleton force because he estimated that the Reds would attack from the flank or rear; and he was right. His actions were not based on unconsidered rashness. Rather he recognized that his troops were so outnumbered that he had to deploy strength only to areas where he expected attack. He could not afford the luxury of a conservative decision.[14]

If there is anything to the idea that we may not be as able to assure military superiority in the future as we have in the past it might be well to give more attention to such situations than is presently found in military manuals. Game theory can make distinct contributions to the discussion.

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[14] Military Decision and Game Theory, p. 39.

CHAPTER X

THE NON-ZERO-SUM GAME

It will be recalled that all of the examples given in Part I were zero-sum-games; that is, the gains of one side were balanced by the losses of the other. This same criteria applies to the more general discussions in Part II up to this point. This assumption appears to be valid or nearly so, for most military situations. However, it is quite possible to have conflict situations in which the zero-sum assumption does not apply. Examples can be found in several fields. If the Commander was told, "win if you can but under no circumstances lose," his scale of values would have to differ from those of an opponent who did not have this limitation. Determining the gain or loss from military action will also include the relative worth given to various components of the force. The United States with its traditional regard for human life would look on the outcome of a battle in a different light than an opponent to whom losses of troops were of slight concern. We have already experienced this in Korea. Forester's novel, The Gun, gives an extreme example of the relative valuing of materiel over the humans involved. With a satisfactory result to the participants, incidentally, though many of us would be inclined to question it. What, then, is the effect if the players do not use similar scales of worth?

Non-zero-sum games lead into mathematical complexities that are still being worked upon. In general, the solution in simple cases is to consider that there is a third player, called Nature, that supplies the difference in payoff required to balance the books. This is not the same as a Three Person game. For in this type, which we shall not consider at all, it is interesting to note the theory leads to quite positive indication that the only proper course in a game with more than two players is to form coalitions against one player. It then becomes a two person game again as long as the mutual payments to keep the partners satisfied hold the coalition together. (NATO?). However, when Nature is the third player forming a coalition with her is not possible.

That this concept of a third player, Nature, is usable can be seen from the situation in which two opposing Commanders each decide not to engage, but to decline battle. Presumably both get satisfaction from this course of action, a payoff. Where does this come from? The concept of Nature provides an answer. In effect the two opponents have formed a coalition.

It can be seen that a Commander who plays capabilities does not have to worry about the opponents scale of values. Any deviation from the correct strategy for the opponent as determined by the Commander can only increase the payoff to Blue; provided the estimate is correct and complete. Any deviation from the correct strategy by Red on account of Red using a different scale of values, will from the Blue point of view be the same as an irrational act.

In the mixed strategy situation the foregoing still applies, though the payoff may not be as great as it could be made by knowing the opposing commanders mind. The practical solution of such a problem is not yet available. But as long as we play a mixed strategy based on our own scale of value we are assured at least of the expectancy which would result if Red did, in fact, use the same scale. It is only in playing intentions that the enemy's scale of values enters into the solution of the game. Again, there is no formal method of solving this problem. But it should be obvious that a Commander playing intentions must, if he is to be successful, be first of all thoroughly grounded in the manners, morals, and past history of his opponent. Certainly, estimating what an enemy will do using, even subconsciously, ones own scale of values will inevitably lead to disaster. It would appear that a good deal of present day talk on the possible nature of future war falls into this error.

CHAPTER XI

SOME FURTHER THOUGHTS ON MIXED STRATEGIES

We have noted in the first discussion of mixed strategies (Chapter III) that the result is assured only after sufficient repetition to bring statistical probability into play. Also, successive actions in military affairs will, in general, not exactly reproduce the circumstances of the first play of the game. It would appear that this is a stumbling block to using the concept of mixed strategies.

However, it is interesting to note that von Neumann and Morgenstern, in deducing their theories on mixed strategy, solved the problem specifically for games played only once.[15] They show that the course of action in the mixed strategy situation played once should be selected by an odds measured random choice in the same manner as was indicated for repeated plays of the game. In fact a mathematical purist can well object that the theory is proven for the single play only. It has not been extended to include games played _n_ times. Williams, in The Compleat

[15] _Op. cit._ pp. 44-45 & 146-148 for discussion of this point.

Strategyst, devotes considerable attention to this point.[16] He cites as part of his discussion the following situation:

> Consider a non-repeatable game which is terribly important to you, and in which your opponent has excellent human intelligence of all kinds. Also assume that it will be murderous if your opponent *knows* which strategy you will adopt. Your only hope is to select a strategy by a chance device which the enemy's intelligence cannot master--he may be lucky of course but you have to accept some risk. Game theory simply tells you the characteristics your chance device should have.
>
> You may also adopt the viewpoint that you will play many one-shot games between the cradle and the grave, not all of them being lethal games, and that the use of mixed strategies will improve your batting average over this set of games.

There are several reasons why a military Commander faced with a mixed strategy picture in his matrix, in a single play situation, would be under pressure to follow the conservative course--play the minorant game on an estimate of enemy capabilities. In the first place, present doctrine favors this course; the training to which he has been subjected leads to this conclusion except in clearly exceptional circumstances. The usual pattern of superiors is to give the subordinate enough forces to accomplish the objective and then hold him strictly accountable for the results. Defeat can have a shattering

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[16]*Op. cit.*, pp. 206-207.

effect on a leader's career. Conservatism is thus reinforced. In addition the American tradition frowns on sacrificing men and materiel to set up a situation of success in the future. We even have trouble justifying such expenditures in training. All of these things reinforce the tendency to conservatism, of proceeding by massing superior forces and grinding out a victory. It has been successful in the past.

It should be evident by now that such a doctrine is not sufficient for situations of equal or inferior strength. And there is grave doubt that it accomplishes the result desired at minimum cost and time even when the force is superior. How can we change our doctrine to take advantage of the gains of mixed strategy?

Col. Haywood makes several sound suggestions in his thesis on this matter.[17] In the first place, it is not meant to press the superiority of mixed strategy unduly. The "fog of war" will have some effect in randomizing our (and the enemy's) strategies. He suggests, in lieu of a fully determined mixed strategy, that superiors in control of a number of subordinate units direct in a random fashion

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[17]Op. cit., pp. 79 ff.

that these subordinates base their actions on capabilities for a certain period and intentions for another. The directive should not be stereotyped, but made by a random device. The more weight that is given to intentions, the bolder the strategy. This would lead to defeat in some areas and better than average success in others. A subordinate must be judged only on the execution of his strategy, not the results. In the long run this course should show a more favorable result than following a doctrine of capabilities. At the very least it would keep the enemy from being able to count on our course of action. Hence it would force *him* to a more conservative strategy. It is precisely this method which von Neumann recommends to tame the overaggressive poker player.

The difficulties of implementing such a doctrine can be seen. Nevertheless, the gains to be found in it are real; and justify Commanders devoting considerable thought to the problem. If this chapter stimulates such thought it will have served its purpose.

CHAPTER XII

SUMMARY AND CONCLUSIONS

As stated in the Introduction, the purpose of this paper is to give a simplified look at what game theory is in the hope that military leaders will become interested enough to pursue the subject further. Game theory gives a different point of view on the subject of military decision. The fact it is different makes it desirable that Commanders become familiar with it on the chance that it may aid their thinking by injecting a new and fresh insight. This is the most valuable contribution that game theory can make at the present time. For, except in a limited range of problems, it is not available for actual solutions of military planning dilemmas. ". . . the theory of games is spectacular, if that is the word, only in an intellectual sense, and only in that sense can it be appreciated."[18] Yet who would deny that planning for the world situation in which we find ourselves today places a greater emphasis on rational thinking than on piling up weapons? To such a planner game theory has much to say.

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[18] McDonald, John. <u>Strategy in Poker, Business and War</u>. p. 18.

Although this paper is preliminary, there are certain conclusions which appear to be acceptable at the present time.

1. The use of the matrix form for representing the interaction of strategies is superior to that recommended in the Naval Planning Manual. It should be used now as the Commanders summary and visual aid in place of the Manual recommendation.

2. Familiarity with the concepts of maximin, minimax, matched strategy, mixed strategy, majorant and minorant games will enable the Commander to use his matrix as a test and check of his estimate. It is not recommended at the estimate. Nevertheless, it is believed that Col. Haywood's maxim holds good; "If the Commander is not prepared to make a matrix of the opposing strategies for the situation, he is not prepared to make a decision."[19]

3. If mathematical expressions of the worth of a strategy are available for inclusion in a matrix the theory will give the proper strategy to follow or indicate the range of choice. Due to the lack of such a scale of values to cover all the facets of the military problem this solution cannot be used. However, by using a qualitative scale satisfactory to himself the Commander can gain from the matrix relations

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[19] Military Decision and Game Theory. p. 46.

indications of the proper strategy subject to the range of uncertainty contained in his scale. These relations can be useful as a check against the body of the estimate. If they do not correspond he has a clear warning to stop and reconsider.

4. The matrix can provide, subject to the range of uncertainty of the scale of values, a measure of the worth of intelligence effort and the difference in payoff between basing an estimate on enemy capabilities and enemy intentions.

5. Game theory clearly points up the essential conservatism of our present doctrine of basing estimates on enemy capabilities. Unlike such a doctrine, it indicates a course of action if our own forces are equal to or inferior to those of the enemy.

6. Since the scale of values is the crux of the matter, game theory indicates in certain circumstances the importance of "knowing your enemy." It thus reinforces this military maxim.

APPENDIX A

CORRELATION OF ESTIMATES BASED ON INTENTIONS AND CAPABILITIES WITH GAME THEORY

The correlation of the majorant and minorant games with the military doctrine of basing decisions on enemy intentions or enemy capabilities was first pointed out, in unclassified form, by Col. Haywood in his Air War College thesis, Military Doctrine of Decision and the von Neumann Theory of Games.[20]

The line of reasoning for capabilities is as follows: It is based on the assumption that there is at least one course of action which promises success. This does not appear unreasonable. The Commander must apply the criteria of suitability, feasibility, and acceptability to his courses of action. If none of them pass these tests--which simply mean, in effect, that he can lick the enemy at acceptable cost--he is enjoined to return to his superior and present his conclusions. The superior may change the mission; or he may direct that the subordinate carry out the mission as given, being willing to pay the price in view of other considerations. In this case it becomes

[20]Op. cit., p. 20 ff.

suitable, feasible, and acceptable, "by direction." With one course of action available against any enemy capability it is obvious that it must be the maximin; i.e. the solution to the minorant game. If there is more than one action which will be suitable, etc. the one which has the higher maximum of the row minimums would be the choice; again the solution of the minorant game. Thus a decision based on enemy capabilities will counter the worst he can do to you.

In the case of the estimate based on enemy intentions corresponding to the majorant game, this is true only if the Commander does actually estimate the enemy intention correctly. In this case it fills the definition of majorant game just as well as if the enemy sent a message saying what he was going to do as the definition calls for.

APPENDIX B

THE COMPUTATIONS OF ODDS FOR USE WITH MIXED STRATEGY

The following is taken from the Compleat Strategyst, by Williams. This book, by the Head of the Mathematics Department of the Rand Corporation, is a condensation of game theory stressing the mathematical approach simplified to the point that no higher mathematics are involved. It is the most understandable source for the average person. Anyone wishing to become reasonably conversant with basic theory would do well to study this book and work all of the sample problems it contains.

Step 1. Look for a saddlepoint. If there is no saddlepoint the best grand strategy is a mixed strategy. If there is a saddlepoint, computing the odds will only lead you to a wrong conclusion.

Step 2. If there is no saddlepoint, compute the proper odds as follows. (Using the same matrix that illustrated the idea of mixed strategies.) Take Red's odds first.

Red

1	2
3	6
5	4

Subtract the numbers in the second row from those in the first and put the answers in two new boxes.

Red

1	2
-2	2

Then the oddment (a manufactured term for a single component of an expression of odds; thus, odds 4:5, 4 and 5 individually are oddments) for Red 1 is in the Red 2 box; thus

Red

1	
	2

and the oddment for Red 2 is in the Red 1 box, thus

Red

	2
-2	

One of these numbers will be negative, always. Disregard the minus sign. The odds then are Red 1: Red 2 = 2:2 or 1:1 Red should mix his strategies using a chance device with even odds; like flipping a coin.

Blue odds are computed similarly with everything turned 90 degrees.

Blue			
	1	3	6
	2	5	4

Subtract the second, or right hand column, from the first.

Blue	1	-3
	2	1

Then the oddment of Blue 1 is:

Blue	1	
		1

And the oddment of Blue 2 is:

Blue		-3
	2	

Blue's odds are Blue 1:Blue 2 = 1:3. It would be easiest to take this from an electronically calculated table of random numbers. (Under no circumstances manufacture your own - your method of thought will appear in it somewhere.) The method of doing this is in the book, it is too long to reproduce here.

Proof that the odds calculated give the right answer can be seen by calculating the <u>value of the game</u> for all four possible strategies, or more simply, one for Blue and one for Red. Thus:

Red plays at odds of 1:1. His average against Blue 1 is:

$$\frac{1 \times 3 + 1 \times 6}{1+1} = 4\ 1/2$$

Blue plays at odds of 1:3. His average against Red 1 is:

$$\frac{1 \times 3 + 3 \times 5}{1+3} = 4\ 1/2$$

And similarly for the other two strategies.

BIBLIOGRAPHY

Bradley, General Omar N. A Soldier's Story. New York: Henry Holt and Company, 1951.

Haywood, Colonel O. G., Jr. "Military Doctrine of Decision and the von Neumann Theory of Games." Project Rand, RM-528. The Rand Corporation, Santa Monica, California, 2 February 1951.

This paper was originally written as Colonel Haywood's thesis at the Air War College. It was published by the Rand Corporation as a working paper for their investigations of the theory for the Air Force. An abridged version appeared in the Air University Quarterly Review, 4:17, Summer, 1950.

_______. "Military Decision and Game Theory." Journal of the Operational Research Society of America. Volume 2, Number 4, November, 1954.

Levinson, Horace C. The Science of Chance. New York: Rinehart and Company, 1952.

Because of the close kinship of the theory of games and chance, this book is valuable as a manual on probability and the latest advances in statistical methods.

von Neumann, John and O. Morgenstern. Theory of Games and Economic Behavior. Princeton, N.J.: Princeton University Press, 1947.

McDonald, John. Strategy in Poker, Business, and War. New York: W.W. Norton and Company, 1950.

McKinsey, J.C.C. "Introduction to the Theory of Games." Project Rand, R-228, The Rand Corporation, 1 July 1952.

Extremely technical. Suitable only as a handbook for the mathematician.

Shubrik, Martin. Readings in Game Theory and Political Behavior. Garden City: Doubleday and Company, 1954.

Not of much interest to the military reader. But interesting in showing the way various social sciences are trying to apply game theory to their problems. It stretches implications even further than does this paper!

Williams, J.D. The Compleat Strategyst. New York: McGraw-Hill, 1954.

An excellent introduction to the study and solving of problems mathematically.