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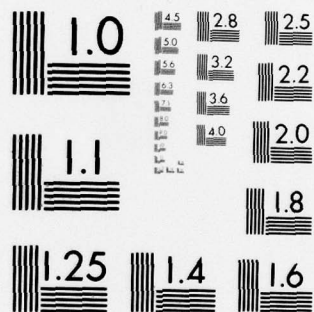
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## RESEARCH REPORT

RANK WEIGHTING IN MULTIATTRIBUTE  
UTILITY DECISION MAKING: AVOIDING  
THE PITFALLS OF EQUAL WEIGHTS

WILLIAM G. STILLWELL  
WARD EDWARDS

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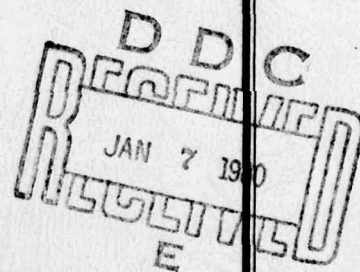
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RANK WEIGHTING IN MULTIATTRIBUTE  
UTILITY DECISION MAKING: AVOIDING  
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William G. Stillwell  
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Ward Edwards

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# SUMMARY

The notion of "dominance" in multiattribute utility decision contexts leads to a change in the considered alternative set. The implications of this set change are discussed in relation to the conditions of Wainer's (Wainer, 1976) "equal weights theorem" and the resulting sensitivity to weighting of importance dimensions demonstrated. Data from three multiattribute decision making studies are examined using four rank weighting techniques as well as equal weights. Rank weighting of importance dimensions demonstrate marked improvement of approximation as reflected in both Pearson and rank order correlations for measures of overall utility across alternatives within the nondominated subset. Implications for multiattribute utility application are discussed.

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## 1. Introduction

A number of recent articles have compared equal weighting to various statistical weighting schemes for linear prediction and decision models (Dawes and Corrigan, 1974; Newman, 1977; Newman, Seaver, and Edwards, 1976). Dawes and Corrigan's (1974) seminal article provided evidence that simplified weighting procedures are strikingly robust. Both equal and random weighting procedures provided excellent approximations to a defined optimal set of weights in such diverse settings as graduate admissions, psychiatric diagnosis, and an abstract decision task.

Expanding this notion, Einhorn and Hogarth (1975) derived the minimum correlation between standard linear regression and unit weighting composites as a measure of the degree of similarity of those composites and in conclusion stated "The minimum is high for most applied situations." They contend that unit weighting is a viable alternative to standard regression methods "because unit weights: (1) are not estimated from the data and therefore do not consume degrees of freedom; (2) are estimated without error; and (3) cannot reverse the true relative weights of the variables." In addition, of course, equal weights require no elicitation.

Wainer (1976) took this argument one step further. In defense of his "Equal weights theorem" Wainer showed that under "very general circumstances" equal weights can replace the least squares weights for the sample from which the regression weights were derived with little or no loss in accuracy. He also showed that equal weights will hold up better than weights derived from multiple regression when new samples are examined.



Although Wainer made some errors in the estimation of loss of explained variance (Laughlin, 1978) given his conditions, the conclusions reached were both tenable and important. In fact, in many real prediction situations equal weights provide remarkably good approximations to differential weights, again measured by correlations between predictions. Wainer's conditions are the following. (a) All predictor variables (attributes) should be oriented properly; that is, for all of them, either more should be better than less or less should be better than more. (b) Predictor variables (attributes) should be positively intercorrelated.

The former condition is easy to ensure; an appropriate rescaling will always solve the problem. Even peaked (nonmonotonic) functions become monotonic when the nonmonotonic measure is rescaled into utility or some other monotonic measure of desirability.

Condition B is the heart of the matter. Considered as a requirement for prediction -- which, after all, is what correlations are intended to do -- it is not a problem. GRE scores will correlate positively with GPAs over a population of applicants to most graduate departments. The same comment can be made about many, perhaps most, of the simple descriptive measures that are typically used in examples by proponents of equal weights.

But description, by itself, is not usually the point of any real-world application of these ideas. Description is typically intended to be useful. A description can, it seems to us, be useful only if it serves as a basis for one or more decision. And indeed the examples used by the proponents of equal weights are decision examples: admission to graduate school, assignment to a psychiatric classification (with differential implications for treatment), and so on.



Once we conclude that decision, rather than mere description, is our goal, a set of considerations quite different from those relevant to description alone become relevant -- even crucial. This paper examines some of those considerations and their consequences; we are looking into others. Some of the ideas of the riskless part of multiattribute utility measurement (MAUM) are very similar to those that enter into the equal-weights discussion and will be discussed briefly in this paper as they apply. (For expositions of various approaches to and applications of MAUM, see Keeney and Raiffa, (1976); Edwards, (1971; 1973; 1978).

Newman, in work done earlier but published in 1977, showed that in at least some multiattribute decision contexts equal weighting of attributes could lead to substantial changes in the resulting evaluations. Following up Newman's earlier work, Newman, Seaver and Edwards (1976), in a simulation study, found that equal weighting led to poor selection ordering in most situations that have substantial negative correlations among pairs of dimensions. Keren and Newman (1978) demonstrated additionally that the presence of suppressor variables (defined by Conger, 1974) among dimensions could lead to marked inferiority of equal weighting to regression weighting, even for pure prediction situations.

The key factor that makes decision situations different from prediction situations is the idea of irrelevance, or domination. If a single option is to be chosen in a multiattributed contest, then only options on the Pareto frontier are admissible candidates. Dr. David Seaver, while a graduate student in this laboratory, first saw the point. Unpublished work he performed in 1973 (Note 1) demonstrated that while correlations between aggregate measures of overall utility were insensitive to model

changes (additive, multiplicative, and quasi-additive) for either an entire set of 1000 options or the nondominated subset of 100 options, within the nondominated subset these correlations were extremely sensitive to dimension weights. Specifically, equal weights were found to produce measures of overall utility which markedly differed from those produced by the defined set of optimal weights.

Table 1 gives the results of this analysis. Of particular interest are the correlations of weighting procedure 1 (the defined optimal weight

set) with weighting procedure 2 (differential and correctly rank ordered but non-optimal) and weighting procedure 1 with weighting procedure 3 (equal weights). The explicit implication of this analysis is that weights do matter when nondominated options are being considered, but are not crucial as long as they are correctly rank ordered and to some extent differential. Implicit in the analysis is that negative correlations exist between dimensions within the nondominated set of alternatives. Although his paper did not report this analysis, Seaver looked at the correlations between dimensions within the non-dominated set of alternatives and found that indeed many were negative. In fact 9 out of a possible 10 of these correlations for the 5 dimensions were negative. And it is these negative correlations between dimensions that are responsible for the poor performance of equal weights.

Following up on the insight of Seaver and the implications of his study, the importance of which is only now beginning to be understood,

TABLE 1

Correlations between overall utilities for  
Different models and weighting schemes.

|                       |                | all<br>alternatives | admissible<br>alternatives |
|-----------------------|----------------|---------------------|----------------------------|
| # alternatives        |                | 1000                | 100                        |
| # dimensions          |                | 5                   | 5                          |
| Composi-<br>tion Rule | F <sub>1</sub> | .980                | .995                       |
|                       | F <sub>2</sub> | .690                | .878                       |
|                       | F <sub>3</sub> | .825                | .867                       |
|                       | F <sub>4</sub> | .660                | .504                       |
| # dimensions          |                | 5                   | 5                          |
| Weighting procedures  | 1 x 2          | .970                | .959                       |
|                       | 1 x 3          | .752                | .514                       |
|                       | 1 x 4          | .335                | -.157                      |
|                       | 1 x 5          | .164                | -.274                      |
|                       | 2 x 3          | .870                | .692                       |
|                       | 2 x 4          | .511                | .053                       |
|                       | 2 x 5          | .324                | -.109                      |
|                       | 3 x 4          | .869                | .757                       |
|                       | 3 x 5          | .743                | .630                       |
|                       | 4 x 5          | .968                | .971                       |

- 1: (81, 49, 25, 9, 1) [optimal]  
 2: (9, 7, 5, 3, 1)  
 3: (1, 1, 1, 1, 1)  
 4: (1, 3, 5, 7, 9)  
 5: (1, 9, 25, 49, 81)

models

$$F1(x) = \sum_{i=1}^5 x_i + \sum_{\substack{i,j \\ i \neq j}} \sum_j x_i x_j$$

$$F2(x) = \sum x_i + \prod x_i$$

$$F3(x) = (X_1 \cdot X_2 + X_3 \cdot X_4) \cdot X_5$$

$$F4(x) = (X_1 + X_2) \cdot X_3 + X_4 \cdot X_5$$

NOTE from "Correlation Analyses of Approximations and Sensitivity for Multi-attribute Utility."  
 By D.A. Seaver, unpublished manuscript, Mathematical Psychology Program, University of Michigan, 1973.



McClelland (1978) demonstrated the ubiquity of significant negative correlations among dimensions in multiattribute utility applications. In addition he examined the problems that accrue from the use of equal weighting of dimensions in those applications.

The argument for negative correlations among dimensions relevant to decision is extremely straightforward. Good sense and formal argument alike would require that, in any decision situation, the chosen alternative should not be clearly inferior to some other alternative. Option B is clearly inferior to, or, in more technical language, is dominated by option A if A is at least as good as B on every relevant dimension, and definitely better on at least one. In such a situation, no reasonable person would bother to consider option B; he would simply eliminate it from the option set. The Pareto frontier mentioned above is simply the highly reduced set of options that remain for consideration after dominated options are eliminated.

But elimination of options inevitably changes the correlations among all pairs of dimensions. Dimensions that may have been positively correlated in the original option set almost inevitably become negatively correlated in the reduced one. This means, of course, that within the reduced option set condition B of Wainer's Equal Weights theorem is almost inevitably violated.

Consider the following simple example. You have decided to buy a new car. Your options are:



TABLE 2  
Alternative Set

|       |    |    |    |    |
|-------|----|----|----|----|
| Car 1 | 10 | 22 | 24 | 42 |
| Car 2 | 37 | 77 | 99 | 96 |
| Car 3 | 89 | 85 | 28 | 63 |
| Car 4 | 9  | 10 | 7  | 51 |
| Car 5 | 2  | 1  | 52 | 7  |
| Car 6 | 36 | 54 | 32 | 29 |
| Car 7 | 2  | 81 | 29 | 0  |
| Car 8 | 5  | 91 | 0  | 0  |

Note: Entries for this table randomly generated

Table 3 shows the correlations between dimensions for the eight cars.

TABLE 3  
Attribute correlations: complete choice set

|                | A <sub>1</sub> | A <sub>2</sub> | A <sub>3</sub> | A <sub>4</sub> |
|----------------|----------------|----------------|----------------|----------------|
| A <sub>1</sub> | 1.0            |                |                |                |
| A <sub>2</sub> | .42            | 1.0            |                |                |
| A <sub>3</sub> | .20            | .06            | 1.0            |                |
| A <sub>4</sub> | .57            | .07            | .57            | 1.0            |

All correlations are slightly to moderately positive. But several cars are dominated. For instance, car 1 has values (10, 22, 24, 42) while car 2 has values (37, 77, 99, 96). No rational person would choose car 1 when he/she could choose car 2, no matter what the weighting of the attributes.

Similarly car 3 dominates car 4, and 2 dominates 5 and 6. Cars 2, 3, 7, and 8 are the remaining options upon which a decision analysis would have some bearing. Table 4 shows the intercorrelations for this reduced choice set. Attribute  $A_2$  has a high negative correlation with both  $A_3$  and  $A_4$  in violation of Wainer's condition B.

TABLE 4

Attribute correlations: reduced choice set

|       | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|-------|-------|-------|-------|-------|
| $A_1$ | 1.0   |       |       |       |
| $A_2$ | -.10  | 1.0   |       |       |
| $A_3$ | .20   | -.89  | 1.0   |       |
| $A_4$ | .67   | -.62  | .84   | 1.0   |

What does this violation mean? In practice it means that if equal weights were used in multiattribute utility measurement of either additive or multiplicative types, the two negatively correlated dimensions would, to some extent, cancel each other. The magnitude of the cancellation effect would depend on the size of the correlation and on the weights of the dimensions. In the car example, assume an individual's true weights to be the following:

| $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|-------|-------|-------|-------|
| .4    | .4    | .1    | .1    |

A linear multiattribute utility analysis of the alternative cars would yield these results:

|       | <u>Total Utility</u> | <u>Rank</u> |
|-------|----------------------|-------------|
| Car 2 | 65.10                | 2           |
| Car 3 | 78.70                | 1           |
| Car 7 | 36.10                | 4           |
| Car 8 | 38.40                | 3           |

The same analysis using equal weights produces the following conflicting results:

|       | <u>Total Utility</u> | <u>Rank</u> |
|-------|----------------------|-------------|
| Car 2 | 77.25                | 1           |
| Car 3 | 66.25                | 2           |
| Car 7 | 28.00                | 3           |
| Car 8 | 24.00                | 4           |

The correlation between these two sets of utilities is .90. But equal weights would lead to selection of car 2 while the decision maker's true weights would lead to the selection of car 3. The potential loss in utility calculated from the decision maker's own weights is a striking 17.3%. This paper presents a set of simplified weighting procedures that avoid the problems of equal weights and at the same time simplify the assessment task.

## II. Rank Weights

Weighting procedures are usually compared on two criteria, ease of elicitation (for both subject and analyst) and numerical quality. The extremes of these dimensions are represented by equal and ratio weights. Equal weights present no elicitation problem, but the numbers



give no information for discrimination between non-dominated alternatives. Weights that have appropriate ratio properties are the most difficult to assess by any procedure, but provide the maximum discrimination power. These ratio properties, essential to multiattribute utility measurement, are implicit in the elicitation procedures reviewed and invented in the Keeney-Raiffa book (1976), and are explicitly the basis of Edwards' procedure (Edwards, 1973; 1977; Edwards, Guttentag and Snapper, 1975). Newman (1977) and Newman, Seaver and Edwards (1976) discuss equal versus ratio weights thoroughly but leave unanswered the question: How simple can elicitation be and still give adequate representation for a good decision?

Obviously equal weights are the simplest to assess, and in a number of prediction contexts they have been shown to be of sufficient quality to justify their use (Newman, Seaver, and Edwards, 1976; Dawes and Corrigan, 1974). The concern of this paper is those situations in which some level of differential weighting is necessary, yet the difficulty of eliciting ratio weights demands some intermediate procedure. Rank weighting provides a simplified assessment procedure and a method of assigning weights that does not have the limitations of equal weighting.

This paper examines four rank weighting procedures. All four fall between equal and ratio weights both in assessment ease and in number quality. Each derives information from the rank ordering of the attributes in the attribute set. This information determines the weight given the attribute in the subsequent MAUM. We have named the weighting techniques rank sum, rank reciprocal, decision rule rank and rank exponent.

Rank sum weights are the standard rank weighting technique considered in the literature.  $N$  attributes are ranked and each attribute is weighted  $(N-R_i+1)$  where  $R_i$  is the rank position of the attribute. Each weight is then normalized by  $\sum_{i=1}^N (N-R_i+1)$  so that the weights sum to 1.0. Table 5 shows rank sum weights as well as rank reciprocal and rank exponent weights for 5 dimensions. Equal weights would, of course, assign a weight of .2 to each dimension. The problem of tied ranks among dimensions will be discussed later.

TABLE 5

## Rank weights for 5 dimensions

| Dimension<br>Rank | Rank Sum<br>Weight | Rank Reciprocal<br>Weight | Rank Exponent<br>Weight ( $Z=1.5$ ) |
|-------------------|--------------------|---------------------------|-------------------------------------|
| 1                 | .333               | .438                      | .396                                |
| 2                 | .267               | .219                      | .284                                |
| 3                 | .200               | .146                      | .184                                |
| 4                 | .067               | .088                      | .035                                |

For rank reciprocal weights, the reciprocal of an attribute's rank is taken as that attribute's weight. That is, the weights before normalization are 1, 1/2, 1/3, 1/4, and 1/5. Each weight is then normalized, as in other rank weight procedures. Decision rule rank weights require one more piece of information from the decision maker. Once he has rank ordered the attributes, he must judge the weight of the most important dimension. Then equal, rank sum, or rank reciprocal weights are used; the method of choice is that which most closely approximates the weight elicited for the first dimension.

Rank exponent weights require the same information from the subject as decision rule rank weights. The weight given the most important dimension ( $W$ ) is entered into the following formula:

$$W = \frac{(N)^Z}{\sum_{i=1}^N (N - R_i + 1)^Z} \quad (1)$$

which is then solved for  $Z$  via an iterative process.  $N$  is the number of dimensions in the ranking and  $R_i$  is the rank of the  $i^{\text{th}}$  dimension. Once  $Z$  is known, weights for the rest of the dimensions are determined. For instance the weight for the dimension ranked third would be:

$$W_3 = \frac{(N - 3 + 1)^Z}{\sum_{i=1}^N (N - R_i + 1)^Z}$$

Table 5 shows rank exponent weights calculated for 5 dimensions using an arbitrary  $Z$  value of 1.5.

The first step in any of these methods is to delete from the list of dimensions any dimension to which the respondent wishes to assign zero weight. Ties are permitted in all four methods. For instance, the ranks in a five-dimensional problem might be 5, 4, 2.5, 2.5, and 1.

Each of these methods is easy to apply to hierarchical multi-attribute utility structures (Value Trees). Each branch of the tree is weighted separately. We have seen no published discussion of the rather trivial arithmetic by which such separately obtained branch weights can be aggregated into final weights, combined with single-dimension utilities, and thereafter be appropriately disaggregated



to produce useful profiles showing how each option stands on each major value within the tree. Such a discussion, with a real world example, appears in Edwards (Note 2).

Rank exponent weights exhibit several interesting characteristics.  $Z=0$  defines the equal weights case and  $Z = 1$  defines rank sum weights. As  $Z$  increases, the set of normalized weights gets steeper and steeper. One great advantage of the procedure, considered as an approximation, is that it always gives exactly the same weight to the most important dimension that the decision maker does. This ensures that the procedure will yield a highly satisfactory approximation to the ultimate utilities from a multiattribute utility analysis.

### III. Three Examples

In this section we reanalyze the data from two decision problems to which MAUM has been applied. In addition we briefly review a third reanalysis, performed by Newman in this laboratory. Each raises an interesting issues about the use of rank weighting procedures. For detailed reports of the three problems see Edwards (Note 2), where MAUM was used to evaluate the desegregation plans for the Los Angeles Unified School District, Otway and Edwards (1977), a study of siting decisions for nuclear waste disposal, and Newman (1976) where automobile selection was at issue.

#### III. A. Desegregation

In July, 1977, the Los Angeles Unified School District asked Edwards to design a method of analysis of various desegregation plans submitted to the Los Angeles School Board as a near-final step in the prolonged Crawford desegregation case.

Seven plans were chosen for evaluation. Unfortunately, the plan finally adopted by the board could not be among them, since it was prepared too late for evaluation.

Prolonged discussions with senior District officers, School Board members, plaintiffs and intervenors in the Crawford case, and others led to a very complex Value Tree. The tree had four levels and a total of 144 twigs. (A twig is the lowest level of a downward-branching tree; measures of performance, i.e. single-dimension utilities, must be collected for each twig). The techniques Edwards used to elicit value dimension, importance weights, and location measures (single dimension utilities), were those of SMART (Edwards 1973; 1977), suitably modified to meet the size and political requirements of the problem and the lack of decision-theoretical expertise of the numerous respondents. The most important politically necessary compromise was the use of so complex a tree. 144 twigs is far too many! But none could be eliminated without offending one or another of the very numerous stakeholders.

Eight individuals and one group judged weights, using the standard SMART ratio judgment technique. Of the eight individuals, five were members of the Board; their weights were averaged to preserve individual anonymity. The other three were desegregation experts.

The hierarchical structure of the Value Tree made such judgments possible. Judgments were made separately for each branch; the smallest branch had only two values and the largest had 14. Several techniques were used to cut down excessive judgmental labor in the lower branches of the tree.

We reanalyze only the averaged weights of the five Board members here. Each of the four rank weighting techniques, as well as equal weights, were used to calculate the overall value of each plan. In the case of rank reciprocal and rank sum weights, the number of dimensions determined the weight given to each dimension in the analysis. For decision rule rank and rank exponent weights, the weight given the most important dimension for averaged ratio weights was used to determine the complete set of individual dimension weights.

The entire analysis was conducted for each set of weights. Raw data for plan values is, of course, that used in the original analysis. The plans were examined for admissibility and none found to be dominated. Values of overall benefit were calculated for each weighting technique for both the 1978-79 and 1981-82 fiscal years. Overall benefit shows strong correspondence between results for each of the rank weighting techniques as well as equal weights. To look at the exact strength of this relationship we calculated Pearson correlation coefficients, correlating aggregate benefits of each of the seven plans as evaluated by each approximation and with actual Board weights. No correlation not involving equal weights was less than .98; no correlation involving equal weights was higher than .92.

For decision making purposes a second criterion is change in the rank ordering of alternatives by the various techniques. To examine this criterion plans were rank ordered by each weighting technique and a Kendall tau coefficient calculated. A Kendall tau coefficient represents a difference between two proportions: the proportion of pairs



of measurements having the same relative order in a pair of rankings minus the proportion of pairs showing different relative order in two rankings (Hays, 1973). Thus it gives us a measure of the proportion of changes in the rank ordering in a pair of rank ordered sets. Table 6 shows the results of this analysis for the first and second levels of the MAUM structure.

To avoid over-interpreting Table 6 it is important to know that all weighting procedures led to the same plan being identified as best. Still we hope to be able to generalize to other decision problems where a  $k$  out of  $N$  selection rather than a 1 out of  $N$  is to be made. For the most general case the rank ordering of all alternatives is important. If 7 graduate applicants are to be chosen out of 8 who meet the minimum standards of selection, reversal of the rank ordering of the 7th and 8th applicants would lead to a suboptimal decision while reversal of the 1st and 2nd ranked applicants would make no difference.

The rank order correlations in Table 6 demonstrate the clear superiority of all rank weighting techniques to equal weights. For the 1978-79 data a loss in tau for overall utility of 23.8% (.929-.691) results from the use of equal weights rather than the next poorest simplification. The resultant loss for 1981-82 is 31.1% (.929-.618). Thus, a much larger proportion of rank reversals occur with equal weights than with any of the rank weighting techniques.

TABLE 6

Kendall tau coefficients between plan values based on ratio weights  
and plan values produced by approximations.

1978-79

|                    | <u>Equal</u> | <u>Rank Sum</u> | <u>Rank Reciprocal</u> | <u>Decision<br/>Rule Rank</u> | <u>Rank<br/>Exponent</u> |
|--------------------|--------------|-----------------|------------------------|-------------------------------|--------------------------|
| Overall<br>Utility | .691         | .982            | .929                   | 1.000                         | 1.000                    |
| Branch A           | 1.000        | .982            | .964                   | .982                          | 1.000                    |
| Branch B           | .890         | .944            | .815                   | 1.000                         | 1.000                    |
| Branch C           | .929         | .982            | .929                   | 1.000                         | .982                     |
| Branch D           | .643         | .982            | .909                   | 1.000                         | .982                     |
| Branch E           | .857         | .857            | .929                   | .857                          | .909                     |
| Branch F           | .905         | 1.000           | 1.000                  | 1.000                         | 1.000                    |

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|                    |       |      |       |       |       |
|--------------------|-------|------|-------|-------|-------|
| Overall<br>Utility | .618  | .929 | .929  | 1.000 | .982  |
| Branch A           | 1.000 | .929 | .929  | 1.000 | 1.000 |
| Branch B           | .982  | .982 | .963  | .963  | .982  |
| Branch C           | .764  | .982 | .889  | 1.000 | .963  |
| Branch D           | .714  | .982 | .982  | .982  | .982  |
| Branch E           | .837  | .889 | .909  | .889  | .909  |
| Branch F           | .804  | .972 | 1.000 | .972  | 1.000 |

An additional test of weighting approximation is to compare sets of weights directly. The primary difference among the various approximations is the peakedness of the weight set. Equal weights, of course, have no peak. Rank sum weights are less peaked than rank reciprocal weights. Decision rule rank and rank exponent weights are determined by the judged weight of the most important dimension. Only the rank exponent procedure (among those considered here) can produce weights more peaked than rank reciprocal.

In order to examine the relative peakness of the approximation techniques relative to the judged ratio weights, regression analyses were performed using the ratio weights as predictors of weights from each approximation technique. Slopes of the regression lines indicate the peakedness of the weights of the approximation technique relative to ratio weights. A weight set that perfectly approximates the ratio weights would result in a slope of 1.0. Table 7 shows the results of this analysis. This analysis was conducted for the highest level of

the tree as well as each second level set of weights. Equal weights have no variance and therefore the slope would equal 0.0.

Intercepts for the analyses were all virtually zero and thus are not included in this table. For the three comparisons that are possible rank exponent weights always produce slopes that rank first or second in terms of absolute distance from 1.0. This is not surprising. Rank exponent weights guarantee that the weight to the most important dimension will be correct and thus the regression analysis will have an upper end point that is anchored to the correct ratio weight. It is interesting to note that in each comparison between rank reciprocal and rank sum



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TABLE 7

Slopes based on regression of approximate weights on ratio weights.

|                  | Rank Sum | Rank Reciprocal | Rank Exponent |
|------------------|----------|-----------------|---------------|
| <u>1st Level</u> | .641     | .955*           | 1.101         |
| <u>2nd Level</u> |          |                 |               |
| A Branch         | 1.551    | 2.279           | 1.005         |
| B Branch         | 1.078*   | 1.815           | 1.159         |
| C Branch         | .861*    | 1.214           | .996          |
| D Branch         | 1.136*   | 1.244           | .944          |
| E Branch         | 1.002*   | 1.367           | 1.165         |
| F Branch         | .806*    | 1.376           | 1.208         |

\*Weighting procedure selected by the decision rule rank procedure.

(For branch A the decision rule rank procedure led to the selection of equal weights.)

weights, the decision rule procedure would have chosen the weighting technique whose slope was closest to 1.0. In addition, for the case in which the decision rule procedure indicated equal weights, both rank sum and rank reciprocal weights were much too peaked (indicated by slopes much higher than 1.0).

### III. B. Nuclear waste disposal site selection

Otway and Edwards (1977) presents Multi-Attribute Utility Measurement (MAUM) as a method of evaluating proposed nuclear waste disposal sites. The paper demonstrates the usefulness of MAUM in combining technical information with the corresponding social attitudes. Although aware of the importance of these attitudes, technical experts typically have been unable either to measure them or to aggregate them with technical data. In the past, either or both of these problems have led the experts to make recommendations that overweight the importance of the engineering aspects of the problem. Of special interest is the recent development of methodologies for the measurement of social attitude toward technologies (Otway and Fishbein, 1976), hopefully solving the measurement problem. MAUM, of course, provides a means of aggregation.

The experts whose information was used for the analysis were ten high ranking specialists from eight countries with advanced nuclear energy programs. They were assembled for an international meeting concerned with nuclear waste disposal. The experts were very much concerned with the problems of disposal and the risks involved and were thus highly co-operative.



TABLE 8

## Description of Six Hypothetical Nuclear Waste Disposal Sites

Value Dimension, Range and  
Scaling

|                                                                                                       | Site 1    | Site 2    | Site 3    | Site 4    | Site 5    | Site 6    |
|-------------------------------------------------------------------------------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|
| D1. Public attitude. 0 =<br>extremely negative;<br>100 = extremely positive                           | 40        | 20        | 10        | 40        | 60        | 70        |
| D2. Remoteness from popu-<br>lation center, km.<br>(0 km = 0; 160 km<br>= 100)                        | 40        | 12        | 12        | 120       | 40        | 120       |
| D3. Geospheric path length,<br>km (0 km = 0;<br>160 km = 100)                                         | 40        | 12        | 12        | 4         | 4         | 40        |
| D4. Proximity to natural<br>resources, km. (0 km =<br>0; 160 km = 100)                                | 50        | 150       | 50        | 50        | 15        | 15        |
| D5. Geologic disturbance<br>probability per year<br>(1 = 0; $10^{-6}$ = 100;<br>linear in exponent)   | $10^{-4}$ | $10^{-5}$ | $10^{-4}$ | $10^{-6}$ | $10^{-5}$ | $10^{-6}$ |
| D6. Relative migration rate<br>of critical nuclide<br>(1 = 0; $10^{-5}$ = 100;<br>linear in exponent) | $10^{-3}$ | $10^{-3}$ | $10^{-2}$ | $10^{-1}$ | $10^{-2}$ | $10^{-1}$ |
| D7. Transportation distance<br>km. (1600 km = 0;<br>0 km = 100)                                       | 1500      | 500       | 500       | 1500      | 150       | 150       |

Note: From "Application of a simple multiattribute rating technique to evaluation of nuclear waste disposal sites: A demonstration." by H.J. Otway and W. Edwards, International Institute for Applied Systems Analysis Research Memorandum, 1977.

For the purposes of the analysis six hypothetical sites were invented. One of the experts at the conference volunteered to perform this task. He did so by thinking of real sites that, in the pasts, had been suggested for the purpose. The relevant physical parameters were approximate figures. Public attitudes toward the hypothetical waste disposal sites were assigned on a random basis. Table 8 provides a numerical description of sites.

As in the previous problem the SMART procedure was used in each stage of the analysis. The experts described above provided the inputs. In addition, the authors provided suggestions for dimensions, which the experts accepted. These suggestions dealt primarily with public attitude. Scaling of the location measures was also provided by the authors, but in conjunction with the technical experts.

The final decision structure was not hierarchical. Each proposed waste disposal site was evaluated on seven dimensions. They were: public attitude toward the site remoteness from population center, distance from the nearest point used by the public (called geospheric path length by these nuclear specialists), proximity to natural resources, geologic disturbance probability, relative migration rate of critical nuclide, and the transportation distance between the site and the nuclear plant it serves. All other dimensions were assumed equal for all sites (for a discussion of nuclear waste disposal see Burkholder, 1976).

Importance weights of the value dimensions were judged twice. Thus, test-retest reliability of these judgments could be calculated. The authors perform these calculations and report a mean correlation of .93 with a standard deviation of .11. The lowest is .65. The second set of weights was used in the original analysis and will be used for the analyses in this paper. Correlations were also calculated between weights judged by all possible pairs of respondents for the second set of judgments. These were reported to range from .97 to -.27 with a mean of .39 and standard deviation of .35. Normalized averaged weights were then calculated across respondents and these means used in the utility analysis.

In order to examine the strength of Wainer's assertion that high negative correlations between dimensions occur rarely (Wainer, 1976), correlations were calculated for the values in this analysis. In seeming conflict with Wainer, some extremely high negative correlations occur. Over half (11 out of 21) interdimensional correlations are negative with several less than -.6 (see table 4, Otway and Edwards, 1977). Obviously some serious questions can be raised about the applicability of Wainer's result, at least for MAUM decision making contexts.

As originally scaled, the alternatives to be evaluated did not cover the full range of value on some attributes (see table 8). This is most striking for the third dimension, geospheric path length, where the range of the alternatives on that attribute covers only 22.5% of the range assigned to it. When a set of alternatives cover only a portion of the total range of an attribute the importance of that attribute is diminished proportionately (for linear utilities) to the part of the range not covered. That decrease is not represented in the original importance weight.



This problem can only be fully solved by judgmental methods; that is, reassessment of weights with attention to the range of values on alternatives. While a reassessment was not possible for this analysis, a transformation procedure was undertaken on the original weights in order to place all of the scaling value on the importance weighting. This placement is, of course, required for the use of rank weights. The transformation was as follows:

$$u'_{ij} = 100(u_{ij} - M_i)/R_i$$

$$W' = W_i R_i / S$$

where  $S = \sum_i W_i R_i$ .

$R_i$  is the range of  $u_{ij}$  in dimension  $i$  over the set of entities to be evaluated, and  $M_i$  is the minimum value of  $u_{ij}$  over those entities in dimension  $i$ . So  $u'_{ij}$  will have a minimum value of 0 and a maximum of 100 on each dimension, over the set of entities to be evaluated.

Table 9 shows the transformed weights as well as the rank weight from each of the three rank weighting procedures. Equal weights would

give a weight of .143 to each of the seven dimensions. The decision rule weight selection procedure would lead to the use of rank sum weights.

This constitutes all the information necessary for the analysis. Each alternative was evaluated using each set of rank weights as well as equal weights and the original ratio weights elicited from the experts. As in the LASD analysis, Pearson correlations were calculated on values of overall utility between weighting techniques across potential sites. Overall utility calculated from ratio weighting correlated .86, .99, .72, and .97 with overall utility calculated from equal, rank sum,

TABLE 9

Weights elicited from experts and those from rank weighting procedures for Otway and Edwards (1977).

| Value dimension                   | Transformed<br>Ratio<br>Weights | Rank Sum*<br>Weights | Rank Reciprocal<br>Weights | Rank Exponent<br>Weights $z = 1.62$ |
|-----------------------------------|---------------------------------|----------------------|----------------------------|-------------------------------------|
| Public Attitude                   | .246                            | .214                 | .193                       | .223                                |
| Distance from<br>City             | .087                            | .071                 | .064                       | .062                                |
| Geospheric Path<br>Length         | .088                            | .107                 | .077                       | .100                                |
| Proximity to<br>Natural Resources | .267                            | .250                 | .386                       | .267                                |
| Earthquake Probability            | .103                            | .143                 | .096                       | .139                                |
| Migration of<br>Critical Nuclide  | .153                            | .179                 | .129                       | .181                                |
| Transportation<br>Distance        | .055                            | .036                 | .055                       | .028                                |

\*Decision Rule Rank Weights would be the same as Rank Sum Weights

rank reciprocal and rank exponent weights respectively. For this analysis both rank sum and rank exponent are markedly superior to equal weights. It is especially interesting to note that the decision rule procedure led to the selection of rank sum weights, clearly the optimal choice from among the approximation procedures.

Kendall tau coefficients were calculated as in the previous analysis. Tau was .733, .867, .333 and .733 between overall utilities calculated from ratio weights and those calculated from equal, rank sum, rank reciprocal and rank exponent weights respectively. Slopes were calculated between ratio weights and each approximation weight set. Slopes were .874, 1.27, and .990 between ratio weights and rank sum, rank reciprocal and rank exponent weights respectively.

It seems apparent from a comparison of the correlations and slopes that while rank sum weights prove superior for the alternatives in this problem, it is quite likely that rank exponent weight would prove superior generally. That is, for this analysis rank sum weighting resulted in higher correlations between measures of overall utility but rank exponent weights have slopes much closer to 1.0 and therefore for a different set of alternatives would prove the superior procedure.

### III. C. Automobile Selection.

A third brief example comes from work performed by J.R. Newman in his reanalysis of a study of optimal automobile design. The Automobile Club of Southern California developed a target program in which it sought to evaluate the contribution of various factors or attributes to an automobile's overall value to a cross section of the driving public. The club's members and other experts provided the list of attributes.



Eleven attributes were identified consisting of fuel economy, interior size, passing/acceleration ability, interior noise, exterior size, crashworthiness, luggage capacity, handling, ride quality, ease of entry and exit and maneuverability.. Expert staff of the Auto Club itself provided location measures on each dimension. 31 non-dominated 1978 automobiles were used as alternatives for the analysis.

Interdimensional correlations were calculated. Correlations between attributes cover almost the full range from -1 to +1. Almost half (27 of 55) of the correlations are negative with 6 lower than -.6. As previously stated, these high negative correlations imply severe problems when equal weights are used for the analysis. This implied problem is illustrated by the comparison of the results from the various weighting schemes. Again Pearson correlations were calculated between weighting techniques, across alternatives. The correlations of ratio weighting with equal, rank sum, rank reciprocal, and rank exponent are .726, .800, .925, and .820 respectively. Equal weights clearly provide the worst approximation to the ratio weights. Rank sum and rank exponent are similar and rank reciprocal is by far the best approximation to ratio weights. As in the nuclear disposal site selection reanalysis the decision rule selection procedure results in a choice of the weighting technique that has the highest correlation with ratio weights.

Kendall tau coefficients between overall utilities were calculated along with slopes between weight sets. Tau was .588, .632, .788, and .600 between the results from ratio weights and those from equal, rank sum, rank reciprocal and rank exponent weights respectively. Slopes between ratio and rank sum, rank reciprocal and rank exponent were .685, 1.397 and 1.231.

The information provided by these two indices seems to be in conflict. While the slopes suggest that for the more general situation rank exponent weights would provide the best approximation, the correlations between measures of overall utility point to rank reciprocal (for this set of alternatives at least). Still, all of the rank weight procedures result in higher correlation than do equal weights, lending evidence for the use of some rank weighting procedure.

#### Discussion

The use of rank weighting techniques as approximations to ratio weights in this study provided remarkably good results. In each of the three reanalyses at least one of the rank weighting approximations resulted in a Pearson correlation coefficient between ratio and rank weights for aggregate overall utilities above .9. Also, in each of the re-analyses equal weighting of attributes led to a sizable loss in correlation for measures of overall value. Thus, rank weighting of attributes can be said to provide good approximations to the "true" ratio weights. Within the conditions of this study and for the purposes of multiattribute decision making, rank weights seem to be sufficient improvements over equal weights to warrant the extra effort involved in their elicitation.

Our ability to validate an approximation technique is obviously bounded by the quality of the judgment or value that is being approximated. Because this study depends so heavily on the ratio weights elicited from the subjects, some rationale should be developed for their use as a criterion. The use of ratio weights as a criterion rests on at least two assumptions. First, we are assuming that an individual knows his/her

own mind; that is, that he/she is able to determine the rank ordering of dimension importance, the relative strengths of the dimension weighting and a true zero point. Secondly, we are assuming that an individual can express this information in quantitative, orderly fashion.

Support for both of these assumptions comes from three studies. Each study examines weighting in the context of the multiple cue learning paradigm (Hammond and Summers, 1972). Cook and Stewart (1975), in a task where subjects were asked to evaluate graduate student profiles, found that: 1) various methods for elicitation of subjective weights (the weights that the subjects say they are using) result in roughly equivalent judgments, i.e., consensual validation, and 2) these subjective weights correspond fairly closely to objective weights derived from the best fit linear model (i.e., linear regression). Both of these conclusions suggest that subjects can express their decision policies in an orderly quantitative fashion.

Two other studies which lend support to the notion that subjects can accurately describe their decision policies are those of Schmitt (1978) and John and Edwards (Note 3). In each of these studies subjects were trained in a weighting model using the multiple cue learning paradigm (Hammond and Summers, 1972) and then asked to describe their subjective judgment as to the weighting of dimensions. Each study used a number of elicitation techniques for the report of these weights. In both of these studies the method of eliciting the subjective weight mattered little. More important, though, for the purposes of this discussion is the fact that both studies found a very strong relationship between predictions derived from the statistical model (regression model)



and those derived from subjects' subjective policy statements. Schmitt sums up the findings of both studies when he says "The results also suggest that weights in policy-capturing studies (e.g., Borman and Dunnette, 1974) would not need to be statistical weights. The rational judgments of cue weights are equally as good as the least-squares weights derived from regressing subjects' judgments on the cues in the sense of producing predicted values that correlate highly with optimal predictions of the criterion of interest."

An additional point supporting the use of rank weighting approximations involves the problems suggested for the use of equal weighting. The correlations between attributes found in Otway and Edwards (1977) and Newman (1978) raise serious doubts as to the stated generality of conditions for the "equal weights theorem" detailed in Wainer (1976) and expanded in Wainer (1978). Correlations were not calculated between attributes for the LASD analysis. This was due to the complexity imposed by the hierarchical MAUM structure. Values at any level of the hierarchy were dependent on both weights and values from lower levels. In each of the other two analyses, however, several dimensions correlated negatively in excess of  $-.6$ . As previously stated, the frequency of the occurrence of the high negative correlations in the two studies where examination was possible suggest that Wainer's conclusions are not justified in view of the severe and common violations of his condition.

In addition to the rank ordering of attributes, an additional bit of information is required for two of the rank weighting techniques. For rank exponent and decision rule rank weights, the proportion of the total weight given to the most important dimension must be elicited from

the decision maker. For the purposes of these re-analyses it was assumed that the weight given to the first dimension by ratio weights was the correct weight for that dimension. Clearly, whether subjects can provide estimates of this quantity is an empirical question.

Work in progress will seek to test this question. Using an external criterion of utility weighting, this work will test rank weighting procedures against both the established "true" weight and the weights derived from subjects' judgments of those "true" weights. If in fact subjects can provide estimates of the most important dimension's weight which are both reliable and congruent with the established "true" weights, the results of the current study indicate that a simple decision rule strategy for selection of weight approximation procedure is remarkably effective for decision purposes. In each of the two analyses where one level of attributes was involved, the simple decision rule procedure for selection between equal, rank sum, rank reciprocal and ratio weighting led to selection of the technique which correlated highest with the results of ratio weighting. For the LASD analysis, the use of the decision rule strategy on each level of the hierarchy led to a remarkably good approximation of the analysis using the entire set of elicited ratio weights. The quality of this approximation (Pearson  $r = .996$ ) is so good as to indicate that the effort involved in the elicitation of over 150 ratio weights was unnecessary and the decision rule rank weighting procedure a more than adequate approximation. In addition, rank exponent weights seem to consistently provide estimates of the ratio weights which have near the same level of peakedness. Looking at the regression analyses between the set of ratio weights and rank exponent weights for each

problem we found that slopes were generally near the correct 1.0. This finding suggests that rank exponent weights will provide good approximations to ratio weights in most instances.

The other possibility is that judges will not be able to reliably provide the weight to the most important dimension. In this case we still find that the rank sum procedure provides an approximation that is better than equal weights. In sum, we have a situation where rank weighting has been shown to be superior to equal weights throughout the reanalyses conducted in this paper. This is true whether or not the more stringent assumptions necessary for the use of the decision rule and the rank exponent weighting procedures are justified. Certainly, the moderate increase in effort necessary for the elicitation of rank ordering of dimensions is justified.



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Washington, DC 20330

Air Force Office of Scientific Research  
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Attention: Mr. John Atkinson  
Griffis AFB  
Rome, NY 13440

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600 Independence Avenue  
Washington, DC 20546

Other Institutions

Department of Psychology  
The Johns Hopkins University  
Attention: Dr. Alphonse Chapanis  
Charles and 34th Streets  
Baltimore, MD 21218

Institute for Defense Analyses  
Attention: Dr. Jesse Orlansky  
400 Army Navy Drive  
Arlington, VA 22202

Director, Social Science Research Institute  
University of Southern California  
Attention: Dr. Ward Edwards  
Los Angeles, CA 90007

Perceptronic, Incorporated  
Attention: Dr. Amos Freedy  
6271 Variel Avenue  
Woodland Hills, CA 91364

Stanford University  
Attention: Dr. R. A. Howard  
Stanford, CA 94305

Director, Applied Psychology Unit  
Medical Research Council  
Attention: Dr. A. D. Baddeley  
15 Chaucer Road  
Cambridge, CB 2EF  
England

Department of Psychology  
Brunel University  
Attention: Dr. Lawrence D. Phillips  
Uxbridge, Middlesex UB8 3PH  
England

Decision Analysis Group  
Stanford Research Institute  
Attention: Dr. Miley W. Merkhofer  
Menlo Park, CA 94025

Decision Research  
1201 Oak Street  
Eugene, OR 97401

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University of Washington  
Attention: Dr. Lee Roy Beach  
Seattle, WA 98195

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University of Virginia  
Charlottesville, VA 22901

Professor Raymond Tanter  
Political Science Department  
The University of Michigan  
Ann Arbor, MI 48109

Professor Howard Raiffa  
Morgan 302  
Harvard Business School  
Harvard University  
Cambridge, MA 02163

Department of Psychology  
University of Oklahoma  
Attention: Dr. Charles Gertys  
455 West Lindsey  
Dale Hall Tower  
Norman, OK 73069

Institute of Behavioral Science #3  
University of Colorado  
Attention: Dr. Kenneth Hammond  
Room 201  
Boulder, Colorado 80309

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of overall utility across alternatives within the nondominated subset.  
Implications for multiattribute utility application are discussed.

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