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6 THE DISCOUNT RATE FOR DEFENSE DECISIONMAKING:  
SOME NEW CONSIDERATIONS

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the same, both ways of calculating present discounted value yield identical results. *Relative* price changes in inputs and outputs are definitely important. If the analyst has some notion of how relative prices are expected to move, it is fairly easy to incorporate this in the present value calculations.

#### THE DISCOUNT RATE WITHOUT ACCOUNTING FOR PROJECT RISKINESS

For the moment let us ignore the problems of risk. There are basically two views on how the discount rate ought to be selected. The first view is that the discount rate ought to reflect the (social) opportunity cost of capital, which is also known as the intertemporal marginal rate of transformation (MRT). According to this view, only by discounting future costs and benefits at the rate that could be earned by the best alternative private project can society be guaranteed that a public undertaking does not displace a private undertaking that yields more. Central to this viewpoint is that the (social) opportunity cost of capital can in fact be measured. There have been a number of attempts to measure the opportunity cost of capital; the most widely acknowledged of these are separate studies by Harberger,<sup>1</sup> Stockfish,<sup>2</sup> and Haveman.<sup>3</sup> The basic methodology is to make an assumption about where the marginal dollar of resources will come from--that is, by borrowing or taxing--and to estimate the incidence of the additional taxes or borrowing on various capital-using sectors. The estimates of incidence provide the weights by which the pre-tax rate of return on capital in each sector

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<sup>1</sup>Arnold Harberger, "On Measuring the Social Opportunity Cost of Public Funds," *The Discount Rate in Public Investment Evaluation*, Report 17, Conference Proceedings from the Committee on the Economics of Water Resources Development of the Western Agricultural Economics Research Council, Denver, Colorado, December 17-18, 1968.

<sup>2</sup>Jacob Stockfish, "Measuring the Opportunity Cost of Government Investment," Research Paper P-490, Institute for Defense Analyses, March 1969.

<sup>3</sup>Robert H. Haveman, "The Opportunity Cost of Displaced Private Spending and the Social Discount Rate," *Water Resources Research*, Vol. 5, October 1969, pp. 947-957.

is multiplied.<sup>1</sup> Separate studies by Harberger, Stockfish, and Haveman give different results because different assumptions are made by each author. In Harberger, the social opportunity cost of capital is a weighted average of the after-personal-income-tax rate of return to savers and the pre-corporate-income-tax cost of capital. Stockfish calculates the pre-tax rate of return in several corporate sectors and takes a weighted average of that with the rate of return in the non-corporate sector. Haveman assumes that additional government revenue will be financed completely through the personal income tax, on which the relevant rate of return is a weighted average of various consumer borrowing rates.

The second major view on the discount rate is that one should use society's rate of time preference, which is also called the intertemporal marginal rate of substitution (MRS).<sup>2</sup> In an ideal world with no taxes, externalities, or market imperfections to drive a wedge between society's MRT and MRS, the opportunity cost rate and time preference rate would be the same. In the real world, taxes, differential costs of information, and monopolies act to create a difference between society's opportunity cost rate and rate of time preference. Some adherents to the time preference view suggest that society's MRS can be inferred from households' decisions regarding savings and consumption or borrowing and lending. Other economists suggest that the MRS is different for different classes of projects and that for a particular class it is whatever society wants it to be. Still other economists suggest that it can be

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<sup>1</sup>The existence of different sectoral rates of return implies either differences in risk among sectors, difference in taxes, or imperfections in the capital market. In the three studies cited above, differences in risk have not been purged from the sectoral rate-of-return data. Consequently, the estimates of social opportunity cost of capital include risk premia.

The primary difference in taxes occurs between the corporate and noncorporate sector. Many economists assume 100 percent shifting of the corporate income tax, so if the corporate income tax is 50 percent, then the rate of return in the corporate sector is twice that of the non-corporate sector.

<sup>2</sup>Heuristically speaking, the intertemporal MRT is the most efficient rate at which society is able to transform resources today into resources tomorrow, whereas the intertemporal MRS is the rate at which society is willing to forego resources today for resources tomorrow leaving utility unchanged.

inferred for a class of projects from past voter-consumer referenda by whether such projects were accepted or rejected.

A strong case for ignoring current market decisions by individuals has been made in separate articles by Marglin<sup>1</sup> and Feldstein.<sup>2</sup> In essence these authors believe that individuals are irrationally myopic, that future generations are underrepresented in current capital markets, and that society, acting collectively, may (should) desire a distribution of income among generations different from that which it generates through individual behavior. Accordingly, it would not be inconsistent to borrow at say 15 percent to increase current consumption while voting increased taxes for a project yielding 8 percent, because by calling upon the government's power to tax, the individual can essentially guarantee that the other individuals in society will be compelled to contribute to the project as well.

If the argument is accepted, then the proper policy is to lower the market rate(s) of interest for all investments using monetary and fiscal instruments. At the lower rate of interest, the rate of return required by investors would also be lower, leading presumably to the acceptance of some projects that were previously rejected. If monetary and fiscal policies can be used to reduce the interest rate (or interest rates), then a separate social discount rate is unnecessary; but if the use of monetary and fiscal policies is inhibited, then a "second-best" policy may be to use a social discount rate lower perhaps than either the MRT or MRS in the evaluation of public projects. Such a move, as Hirshleifer points out, would be extreme.<sup>3</sup>

Despite the appearances to the contrary, the two views are not devoid of overlap. Harberger's calculation of the social opportunity cost of capital includes the after-personal-income-tax rate of return

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<sup>1</sup>Stephen A. Marglin, "The Social Rate of Discount and the Optimal Rate of Investment," *Quarterly Journal of Economics*, Vol. 77, February 1963, pp. 95-112.

<sup>2</sup>Martin S. Feldstein, "The Social Time Preference Discount Rate in Cost-Benefit Analysis," *Economic Journal*, Vol. 74, June 1974, pp. 360-379.

<sup>3</sup>See Jack Hirshleifer, "Social Time Preference," Discussion Paper 18, Department of Economics, University of California at Los Angeles, April 1972.

on savings, which is presumably related to the rate of time preference.<sup>1</sup> Using a two-period model, several authors<sup>2</sup> have shown that with no externalities in either government or private investment in the first period, the appropriate discount rate is a weighted average of the intertemporal MRT and intertemporal MRS, the weight attached to the MRT being the proportion of the marginal dollar of government investment that is drawn away from private investment. On this last point, some economists have argued that government investment in fact produces positive spillovers on private investment. They argue essentially that although a dollar of government investment may displace some private capital formation, the effect of the flow of services from the government project may be to increase downstream private capital formation. This concept is particularly appealing if government investment is different in character from private investment.<sup>3</sup> As an example, one might expect some positive effects on private investment from government investment in the economic infrastructure. Under the assumption that the output from an additional dollar of government investment increases private investment by exactly the same amount that private investment is decreased as a result of government financing, then the appropriate discount rate according to the hybrid view is the intertemporal MRS. However, under the assumption that an additional dollar of government investment displaces one dollar of private investment with no positive

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<sup>1</sup>Harberger's calculated social opportunity cost of capital is in effect a weighted average of the rate of return on capital and the after-tax interest rate on savings. Because individuals save for reasons not confined to the desire to optimize consumption streams, I am in doubt as to whether the after-tax interest rate on savings is the social rate of time preference. It seems more likely that the after-tax interest rate on consumer borrowing is closer to the social MRS. Stockfish, on the other hand, calculates a weighted average of the rate of return on capital in the corporate and noncorporate sectors. His implicit weight of zero on an MRS component explains why his estimate is higher than Harberger's.

<sup>2</sup>See David F. Bradford, "Constraints on Government Investment Opportunities and the Choice of the Discount Rate," *American Economic Review*, Vol. 65, December 1975, pp. 887-899; and Peter Diamond, "The Opportunity Cost of Public Investment: Comment," *Quarterly Journal of Economics*, Vol. 82, November 1968, pp. 682-688.

<sup>3</sup>Indeed, legal restrictions on the kinds of investments the government may undertake tend to reinforce the dissimilarities between government and private investments.

external effect in the other direction, then the government should use a discount rate equal to the intertemporal MRT. This is perhaps best represented by the case in which a government project is a perfect substitute for a private project.

#### NUMERICAL ESTIMATION AND RECOMMENDATIONS

Having dealt with the theory of the discount rate a bit, let us turn to some numerical recommendations by economists. Some of the nominal discount rates in Table 1 were calculated from data; the origins of the DoD and OMB recommended rates are less clear. In all applicable cases, we calculated a real discount rate by subtracting a geometrically computed average of the inflation rate during the six years prior to the year of the estimate.

The range of the recommended real discount rates results of course from the different assumptions made by each author. Haveman's 6-percent recommendation might be favored by those who adhere to time preference theory. Estimates between 7.5 percent and 10 percent probably reflect weightings of the opportunity cost and time preference rates. A real rate between 8 and 10 percent seems to be justified on the basis of Table 1.

#### RISK CONSIDERED

Even in the absence of all uncertainty about future benefits and future costs, it is still necessary to discount. Some authors have recommended that to account for inherent project uncertainty, a project-specific risk premium be attached to a risk-free discount rate, while others have argued that uncertainty is not necessarily an exponential (or even monotonic) function of time, and therefore cannot be correctly handled through the discounting procedure.

The question of whether risk premia ought to be included arises because when data from the private sector are used to estimate an appropriate discount rate for government projects, the greater the inherent private riskiness, the greater on average the observed rate of return. Should the government also have to earn this higher rate of return when it undertakes a more risky project? The primary technical issue is then whether private risk is a social risk as well. The

Table 1  
RECOMMENDATIONS ON THE DISCOUNT RATE

Author	Year <sup>a</sup>	Recommended Nominal Rate (%)	Adjusted for Expected Inflation <sup>b</sup> (%)
Krutilla and Eckstein	1958	6.0	4.58
Hirshleifer, DeHaven, and Milliman	1960	10.0	8.39
Bain, Caves, and Margolis	1966	6.0	4.65
Haveman	1966	7.3	5.95
DoD Directive <sup>c</sup>	1956	--	10.00
Stockfisch <sup>d</sup>	1949-1965	12.0	10.67
Harberger	1968	10.68	8.33
Baumol	1968	10.0	7.65
OMB Directive <sup>e</sup>	1972	--	10.00
Dorfman <sup>f</sup>	1975	(f)	7.50

<sup>a</sup>This column refers to the year (or years) to which the recommended nominal rate applies and not necessarily to the year of publication of the recommended nominal rate.

<sup>b</sup>The adjustment for (expected) inflation was made by calculating a geometric average of the rates of inflation in the six years prior to the year of the estimate and subtracting it from the nominal rate. This geometric average rate of inflation  $\theta^t$  was calculated from the equation

$$1 + \theta^t = \prod_{j=t}^{j=t-5} (1 + \theta_j)^{1/6}$$

where  $\theta_j$  is the rate of inflation in year  $j$  and  $t$  is the year of the estimate.

<sup>c</sup>DoD Instruction 7041.3, December 19, 1966.

<sup>d</sup>If the anticipated rate of inflation were calculated using the entire 1949-1965 period, the adjusted recommended rate would be 10.4 percent.

<sup>e</sup>OMB Circular No. A-94, March 27, 1972.

<sup>f</sup>Dorfman's estimate is based almost completely on theoretical considerations. The estimate shown in the table, however, relies on parameters derived from U.S. experience in the 1960s.

argument that a private risk is *not* a social risk is that when the risks associated with individual projects are pooled and averaged over the entire population, the social risk approaches zero in the limit. Therefore, for purposes of calculating the social opportunity cost, one should use the rate of return less the risk premium.

The counterargument runs as follows: the pooling of risks is not sufficient to reduce the social risk to zero. A necessary condition for a zero social risk is a zero average covariance among the rates of return; this, of course, can occur if the rate of return on each project is an independent random variable, or if there is a significant negative covariance among some projects. The existence of business cycles is some indication that individual rates of return are in fact positively correlated. In other words, if the rate of return to a particular project is correlated with national income--not an unreasonable assumption for most projects--the social risk cannot be zero; after all, monetary and fiscal policy are not perfect instruments of national income management.

Furthermore, economists opposing the use of a riskless discount rate correctly observe that the private investor can diversify his portfolio at negligible marginal cost by participating in stock markets. The individual can reduce his private risk to the average covariance among projects, which is an irreducible social risk. Therefore, the pooling argument is valid only if the government can provide more efficient diversification than can private markets for risk-bearing.<sup>1</sup>

Although there are a number of techniques for handling uncertainty in present discounted value calculations, I recommend incorporating the major uncertainties explicitly. This can be done by identifying alternative states of the world in which materially different benefits and

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<sup>1</sup>Bailey and Jensen argue that the government is even less able to distribute risks than are private markets. In the case where both private risk markets and the government are imperfect distributors of risk--the most likely case according to Bailey and Jensen--then the risk premium for public projects must be the same as that demanded in the private sector for bearing that risk. See Martin J. Bailey and Michael C. Jensen, "Risk and the Discount Rate for Public Investment," in *Studies in the Theory of Capital Markets*, Michael C. Jensen (ed.), Praeger, New York, 1972.

costs may be realized, and calculating the *present certainty-equivalent value*. First described by Hirshleifer in his classic 1966 article<sup>1</sup> on the state-preference approach to decisions under uncertainty, this concept is described below in the context of defense decisionmaking.

#### THE STATE-PREFERENCE APPROACH

The state-preference approach is ideally suited for an analysis of the appropriate discount rate under uncertainty. Under the state-preference approach, the outcome of a given investment--for example, the benefits of a particular public project--in any subsequent period depends on the state of the world in that period. The state of the world in some future period is of course uncertain, but under the state-preference approach, it is assumed that all possible future states can be enumerated. Further, those future states are assumed to be mutually exclusive. These two assumptions are not altogether unreasonable if the relevant states of the world are vastly different from one another, as for example, war versus peace, or prosperity versus depression.

A private investor considering a potential investment will rationally want to contemplate the return he will obtain in each of the relevant future states. The income resulting from this investment will generally be different in each of these future states, and therefore we may picture the risk-averse investor as willing to exchange his claims on future income in some states for claims on future income in other states. The establishment of markets for various contingent claims on future income will enable the private investor to make such trades and achieve his desired diversification. A perfectly competitive market in contingent claims on future income produces the usual Pareto efficiency.<sup>2</sup> The independent trading decisions by many individuals establish a set of prices (to be paid now) for one dollar of income in each of many future states of the world. These prices naturally reflect the market's

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<sup>1</sup>See Jack Hirshleifer, "Investment Decisions Under Uncertainty: Applications of the State Preference Approach," *Quarterly Journal of Economics*, Vol. 80, May 1966, pp. 252-277.

<sup>2</sup>In a Pareto efficient situation, no individual can be made better off without making someone else worse off.

collective wisdom about the probabilities of each of the relevant states, the relative desirability of income in each of the states, and time-preference.

Using the prices established for contingent claims, it is possible to specify a decision rule for government investments that generalizes the familiar present value criterion from the riskless to the risky case. The rule is that the government ought to do all projects whose present certainty equivalent value (PCEV) is greater than zero. The PCEV is given by

$$V_0 = -p_0 C_0 + \sum_{j=1}^T \sum_{i=1}^{n_j} p_{ij} S_{ij} \quad (1)$$

where  $V_0$  is the PCEV,  $p_0$  is the price of current claims generally taken to be one, and  $C_0$  is the current cost of the project.  $S_{ij}$  is the net dollar benefits occurring in the state  $i$  and time period  $j$ , and  $p_{ij}$  is the state-time price to be paid now to obtain one dollar in state  $i$  and time period  $j$ .

A simple example may help illustrate this rule. Suppose there is one future period in which one of two possible states must occur, e.g., war or peace. Suppose further that all costs are incurred in the present and all benefits are realized in the future time period. The PCEV is then given by

$$V_0 = -C_0 + p_{1a} S_{1a} + p_{1b} S_{1b} \quad (2)$$

where  $C_0$  is the cost of the project,  $S_{1a}$  and  $S_{1b}$  are the benefits occurring in state  $a$  and state  $b$ ,  $p_{1a}$  and  $p_{1b}$  are the prices for future contingent claims. The price of current claims is taken as the numeraire.

It is possible to express the price of a *certain* future claim as  $p_1$  by observing that such a claim can be purchased for  $p_{1a} + p_{1b}$ . Alternatively, if  $S_{1a} = S_{1b}$ , there is no uncertainty and again the contingent price is  $p_{1a} + p_{1b}$ . This leads directly to the definition of a riskless discount factor,  $\rho_1 = 1 +$  the discount rate, in a world with uncertainty, namely

$$\frac{1}{\rho_1} = p_1 = p_{1a} + p_{1b} \quad (3)$$

In order to apply the state preference approach to the choice of the discount rate, we must be able to relate the prices in Eq. (1) or Eq. (2) to society's resource constraints and "tastes." Suppose again there is one future period and two possible states, a and b, for this period. Society's opportunities to trade current consumption for future consumption on the margin *in state a* can be represented by the curve labeled  $T^a$  in Fig. 1. The curve  $T^a$  has been deliberately drawn to reflect the condition that increasingly greater amounts of current consumption must be sacrificed in order to gain equal increments of future consumption. The rate at which society can exchange current consumption for future consumption on the margin is just the slope of  $T^a$  at any point.

Similarly, *in state b*, society can also trade current consumption for future consumption, though the curve representing the rate at which this can be done may be different. I have labeled that curve  $T^b$  in Fig. 2.

Let us assume the government acts to maximize society's utility in each state by choosing the point along  $T^a$  (or  $T^b$ ) at which any feasible movement would leave society worse off. This optimal point occurs where the rate at which society *can* trade current consumption for future consumption is just equal to the rate at which society *is willing* on the margin to make that exchange.<sup>1</sup> The willingness *in state a* to exchange current consumption for future consumption is represented by the curve  $\tilde{u}^a$  in Fig. 1. The slope at the tangency of  $T^a$  and  $\tilde{u}^a$  unambiguously defines the appropriate discount factor *for state a*. By a similar process, the appropriate discount factor *for state b* is just the slope at the tangency of  $T^b$  and  $\tilde{u}^b$  in Fig. 2.

These slopes,  $\rho^a$  and  $\rho^b$ , however, need not be equal. This is the essence of the state-preference approach. Different tastes or resource constraints arising in different states of the world result in different discount rates for each state. If we know for example that state a were

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<sup>1</sup>In technical language, the rate at which society can make the trade is called the intertemporal marginal rate of transformation (MRT); the rate at which society is willing to make such trades is called the intertemporal marginal rate of substitution (MRS).

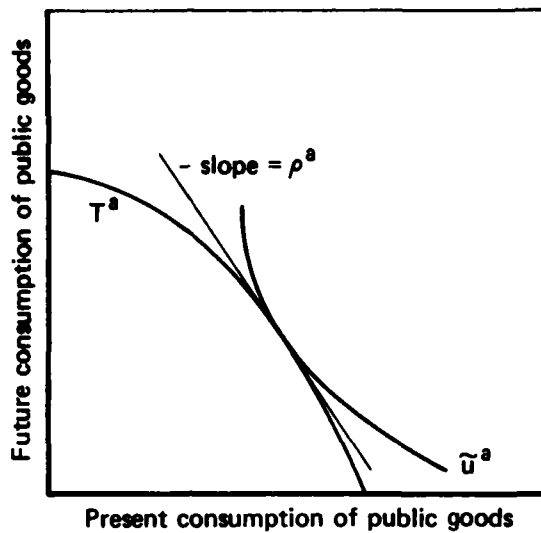


Fig. 1 — The discount factor in state a

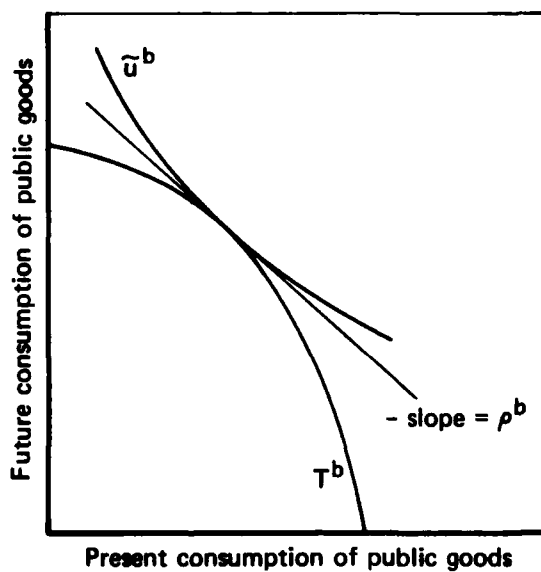


Fig. 2 — The discount factor in state b

certain to occur, then the appropriate state preference price would simply be the reciprocal of  $\rho^a$ . Of course the future is not known with certainty, but we can still calculate the appropriate state-preference prices for contingent claims if we adjust each discount factor by the probability that state will occur. If we let  $\pi^a$  be the probability that state a will occur and  $\pi^b = 1 - \pi^a$  be the probability that state b will occur,<sup>1</sup> then

$$P_{1a} = \frac{\pi^a}{\rho^a} \quad (4a)$$

and

$$P_{1b} = \frac{1 - \pi^a}{\rho^b} . \quad (4b)$$

#### A COMPARISON OF VARIOUS DISCOUNTING PROCEDURES

We can now use the state-preference theory to compare various discounting procedures. To do this let me draw upon a 1975 study at Rand concerning the automation of the Navy's FF-1052 class escorts.<sup>2</sup> The issue is sufficiently simple: an investment in certain equipment for the FF-1052 will produce dollar savings by allowing for a reduction in shipboard manning. These savings are roughly proportional to the number of ships of the FF-1052 class that are so automated. Let us consider two possible future states of the world, peace and war. If peace, which is also the current state of the world, continues through the next period, a certain level of savings will occur. If war occurs, the level of savings realized will undoubtedly be smaller because some portion of the ships will be lost in combat. However, a dollar's worth of savings may be valued differently in war than in peace. In particular, I will argue that in war a dollar's worth of savings will be valued higher because resources are scarcer. Even though the total dollar savings are less,

<sup>1</sup>I have assumed that the probability of state a is objectively known and that society behaves like a von Neumann-Morgenstern maximizer.

<sup>2</sup>See Robert Shishko, *The Economics of Naval Ship Automation: An Analysis of Proposed Automation of the DE-1052, R-1790-ARPA*, The Rand Corporation, November 1975.

the value attached to each dollar is higher so in essence these are partially offsetting effects.

If the probability of war is small, some analysts are inclined to ignore that possible state of the world and proceed to discount only the savings that occur in peacetime. Other analysts may calculate the *expected savings* and then discount. In fact, I have identified five separate discounting procedures that collectively exhibit varying degrees of sophistication.

Let state a be peace and state b war, and suppose the probability of war  $1 - \pi^a$  is small. We may calculate the "present value" of the proposed automation of the FF-1052 by one of five following equations:

$$V_0 = -C_0 + \frac{S_{1a}}{\rho_1} \quad (5a)$$

$$V_0 = -C_0 + \frac{S_{1a}}{\rho^a} \quad (5b)$$

$$V_0 = -C_0 + \frac{\pi^a S_{1a} + (1 - \pi^a) S_{1b}}{\rho_1} \quad (5c)$$

$$V_0 = -C_0 + \frac{\pi^a S_{1a} + (1 - \pi^a) S_{1b}}{\rho^a} \quad (5d)$$

$$V_0 = -C_0 + \left(\frac{\pi^a}{\rho^a}\right) S_{1a} + \left(\frac{1 - \pi^a}{\rho^b}\right) S_{1b} \quad (5e)$$

where  $C_0$  is the investment cost of the proposed automation;  $S_{1a}$  is the realized savings if future state a occurs;  $S_{1b}$  is the realized savings if future state b occurs;  $\rho_1$  has been previously defined in Eq. (3) as the riskless discount factor;  $\rho^a$  and  $\rho^b$  have been previously defined.

In Eqs. (5a) and (5b), the *most likely* benefits are discounted respectively by the riskless rate, and the (riskless) rate applicable to the *most likely* state. Equation (5c) is actually the procedure recommended by Arrow.<sup>1</sup> The expected savings are discounted by the

<sup>1</sup>See K. J. Arrow and Robert C. Lind, "Uncertainty and the Evaluation of Public Investment Decisions," *American Economic Review*, Vol. 60, June 1970, pp. 364-378.

riskless rate. In Eq. (5d), which is similar to Eq. (5c), the expected savings are discounted by the (riskless) rate applicable to the *most likely* state. Equation (5e) is the procedure recommended by Hirshleifer and is actually the PCEV. Here the savings in each state are valued by the prices for contingent claims in that state.

Equations (5c) and (5e) give the same answer if and only if the probabilities assigned to states are proportional to the prices of the contingent claims. From Eqs. (3) and (4) recall that

$$\frac{1}{\rho_1} = p_{1a} + p_{1b} = \frac{\pi^a}{\rho^a} + \frac{1 - \pi^a}{\rho^b} \quad (3')$$

If  $\rho^a = \rho^b$  by some coincidence, then the above condition holds and the equality of Eq. (5c) and Eq. (5e) can be seen directly.

A numerical example will help illustrate how the choice of the discounting procedure can affect the decision to accept or reject the proposed project. Suppose the following values hold:  $\pi^a = 0.9$ ,  $C_0 = 1.1$ ,  $S_{1a} = 1.50$ ,  $S_{1b} = 0.50$ ,  $\rho^a = 1.30$  and  $\rho^b = 1.05$ . In state b, war, I have assumed that only one-third of the FF-1052s will survive, so  $S_{1b}$  is only one-third of  $S_{1a}$ . From the information above,  $\rho_1 = 1.27$ . Table 2 presents the "present values" calculated from Eqs. (5a) through (5e).

Table 2  
PRESENT VALUE OF HYPOTHETICAL SHIPBOARD  
AUTOMATION PROJECT USING FIVE DIFFERENT  
DISCOUNT PROCEDURES

<u>Equation</u>	<u><math>V_0</math></u>
5a .....	+ .081
5b .....	+ .054
5c .....	+ .003
5d .....	- .023
5e .....	- .014

If Eqs. (5a) and (5b) are used to evaluate the proposed automation, the project is accepted. If Eqs. (5d) and (5e) are used, the project is rejected; the project roughly breaks even if Eq. (5c) is used to evaluate costs and benefits. The selection of the discount procedure is probably more important for the acceptance or rejection of a given project than the choice of the discount rate per se. What one counts as part of the benefits in future periods and how those benefits are incorporated into the present value calculation are nonnegligible considerations.

I need not have picked for my working example two such radically different states of the world as war and peace to illustrate this point for military investments. Consider for a moment, several "peaceful" states of the world,  $a_1, a_2, \dots, a_n$ , in which the opponent has achieved various degrees of success in countermeasures. For example, in the case of the automation of the FF-1052, the opponent may have developed a way to make our FF-1052s so vulnerable that we voluntarily decide to retire them. The downstream benefits of automation will then not be realized. If the probabilities of achieving various degrees of success in countermeasures were known, we could treat this problem explicitly using the state-preference approach. To illustrate in a simple model how the ability of the opponent to develop countermeasures that reduce the benefits of a military project can be dealt with analytically, let there be two future states of the world,  $a$  and  $a'$ . In future state  $a$ , no countermeasure is deployed, but in future state  $a'$ , a countermeasure is deployed. Suppose the benefits in all future periods  $j = 1, 2, \dots, T$  are  $S$  if state  $a$  occurs and zero if state  $a'$  occurs. The current cost of the project is  $C_0$ . The PCEV is given by:

$$V_0 = -C_0 + S \sum_{j=1}^T \left( \frac{\pi_j^a}{\rho_j^a} \right) \quad (6)$$

where  $\pi_j^a$  is the probability that state  $a$  will occur in future period  $j$  and  $\rho_j^a$  is the appropriate discount factor for state  $a$  in period  $j$ . ( $\pi_j^a / \rho_j^a$  is the price of contingent claims in state  $a$  and period  $j$ .) The assumption of zero benefits in state  $a'$  is very convenient because no assumption about  $\rho_j^{a'}$  is needed.

If we make the assumptions that there is a constant probability in each period that no countermeasure will be deployed and that  $\rho_j^a = \rho^j$ --that is,  $\rho_j^a$  is just a constant  $\rho$  compounded  $j$  times--then Eq. (6) can be rewritten as:

$$V_0 = -C_0 + S \sum_{j=1}^T \left(\frac{\pi}{\rho}\right)^j . \quad (6')$$

Is it possible to treat the formulation in Eq. (6') as if the benefits  $S$  are certain, but discounted at  $\rho$  plus a risk premium? In other words, is there a  $\delta$  such that

$$\left(\frac{\pi}{\rho}\right)^j = \left(\frac{1}{\rho + \delta}\right)^j ? \quad (7)$$

Equation (7) is easily solved for  $\delta$ , and one obtains  $\delta = \delta(\rho) = \left(\frac{1-\pi}{\pi}\right)\rho$ . The "appropriate risk premium" is proportional to  $\rho$ . For example, if  $\pi = 0.9$ , then  $\delta = 0.11\rho$ ; if  $\rho = 0.09$ , then  $\rho + \delta = 0.10$ .

It is easy to demonstrate that if the probability of countermeasure deployment in each period varies from period to period, then the risk premia are not constant over time. If the probability of countermeasure deployment increases in each period, then the appropriate risk premia also increase. Consider the more complex example in which the probability that no countermeasure will be deployed in period  $k$  is given by  $e^{-\beta k}$ ,  $k = 0, 1, 2, \dots$ . This functional form suggests that at the outset the probability of no countermeasure deployment is high but diminishes rapidly as the system matures. For this reason this form is perhaps more useful in the evaluation of military investments. To calculate the risk premium in period  $j$ ,  $\delta_j$ , we must solve

$$\frac{\prod_{k=1}^{j} e^{-\beta k}}{\rho^j} = \left(\frac{1}{\rho + \delta_j}\right)^j . \quad (8)$$

A closed-form solution is given by:

$$\delta_j = (-1 + e^{\beta(j+1)/2})\rho \quad (9)$$

which means, for example, that in the fifth period, with  $\beta = .05$ ,  $\delta_j = .16\rho$ ; if  $\rho = .09$  as in the previous illustration,  $\rho + \delta_j = .105$ . In general, attaching a period-specific risk premium to the discount rate in each period may be formally equivalent to accounting for alternative states of the world, but it should be clear that no universal set of premia will work for all projects; each project's own risk pattern must be considered.

#### GAME THEORETIC CONSIDERATIONS

The *ex post* rate of return on a military investment is partially under the control of the opponent because the opponent can devote his own resources to countermeasures. The greater the amount of resources devoted to countermeasures presumably the greater the opponent's chance of success, but on average the probability of the opponent achieving a countermeasure is *not* independent of the number of different projects undertaken. *The existence of many projects dilutes the opponent's ability to commit his limited resources to countering any one of them.* If an opponent has a limited budget for countermeasures and is maximizing his utility subject to that constraint, the introduction of a new project diverts his funds away from countering the original set of projects; the probability of achieving a countermeasure on each of these projects in general will be lower. Success in countering one project--that is, an outcome in which our rate of return on the investment is low--is likely to be negatively correlated with success in countering the rest of the portfolio. In other words, for a portfolio of military projects, the covariance between the rate of return on a new project and the rate of return on an existing portfolio of projects is likely to be negative.<sup>1</sup> Compare this to a new civilian project. The covariance between the rate of return on a new civilian project and the rate of return on the existing portfolio could be positive, negative or zero, but the

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<sup>1</sup>This is one of the reasons the United States relies on the "triad" for deterrence instead of on a single system. A high probability of developing a countermeasure for one element of the triad is associated with high opponent expenditures on countermeasures. Fewer resources are then available to the opponent to counter the other elements of the triad, resulting in a diminished probability of countering those elements.

larger the original portfolio--that is, the more it resembles national income--the more likely the covariance is to be positive. *Thus, the "pooling" argument often advanced for civilian projects must a fortiori be stronger for military projects.*

#### SUMMARY

The choice of the discount rate is just a part of the larger issue. In evaluating public investments, particularly military projects, states of the world in which tastes, production possibilities, or benefits differ from those of the current state or most likely future state, are too often ignored. It should be possible to improve our assessments of military projects if the effect of possible countermeasures is directly incorporated into the present value calculations.

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