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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report summarizes our theoretical and experimental findings. It is divided into 4 sections. Section 1 contains a general introduction to the problem of partially coherent sources. Section 2 summarizes the main known results on Schell model and quasi-homogeneous sources. Section 3 contains a description of the experimental set up and of the accuracy of the instrumentation. Section 4 is a preprint of a recently submitted paper unauthored jointly with Dr. E. Collett and detailing the main experimental results obtained under ARO support.		

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1. Introduction

A very common, almost traditional, method of analysis of many optical phenomena has been to consider the electromagnetic field as either strictly coherent or strictly incoherent. This procedure has led to mathematical idealizations of common experimental conditions that have often made it difficult to interpolate between the extreme limits and to predict even qualitatively how partially coherent radiation will behave.

The notion of partial coherence has been traditionally associated with specific stochastic properties of the source so that the possibility of producing variable coherence sources has not been entertained seriously until recent times.

In retrospect it is rather surprising that, starting from the well known Van Cittert-Zernike theorem, more active investigations have not been carried out in an attempt to produce sources with arbitrary complex degrees of coherence in the far field. It is well known that the Van Cittert-Zernike theorem relates the intensity distribution across a planar extended incoherent source and the complex degree of coherence of the radiation in the far field. Thus, at least in principle, if the primary source distribution is artificially modified by a suitable apodization of the intensity profile, the far field coherence can be altered in a predictable way as a function of the modified source intensity. Perhaps, in part, this possibility which has indeed been explored in the past with suitable arrangements of lenses and pinholes, was not taken too seriously because of the excessive loss of intensity caused by the filtering process.

In more recent times, the resurgence of interest in the laws of radiometry and in the equations of radiative transfer has led to the realization that the far field properties of radiation are intimately connected with both the intensity

distribution and the complex degree of coherence of the source. (Most relevant references are listed in the references section of the enclosed preprint). Clearly it would have been difficult to appreciate this fact if one insisted in dealing with completely incoherent source models.

In quite general terms, the radiant intensity of physical optics is directly proportional to the "diagonal" element $W(R\hat{s}, R\hat{s})$ of the cross spectral density function $W(\vec{r}_1, \vec{r}_2)$

$$J(\hat{s}) = \lim_{R \rightarrow \infty} R^2 W(R\hat{s}, R\hat{s}) \quad (1.1)$$

where $\hat{s}$ is the unit vector in the direction of observation. It is also well known that the cross spectral density function $W(\vec{r}_1, \vec{r}_2)$ obeys the Helmholtz equations

$$\nabla_2^{(i)} W(\vec{r}_1, \vec{r}_2) + k^2 W(\vec{r}_1, \vec{r}_2) = 0 \quad (1.2)$$

where the index i in the Laplacian indicates differentiation either with respect to the variable $\vec{r}_1$ or $\vec{r}_2$. Using standard mathematical techniques for solving the Helmholtz equation, the cross spectral density function in the far field can be related to its values at all pairs of points in the source plane. More specifically eq. (1.1) has been shown to take the form

$$J(\hat{s}) = (2\pi k)^2 \cos^2 \Theta \tilde{W}^{(0)}(k\hat{s}_\perp, -k\hat{s}_\perp) \quad (1.3)$$

where $\tilde{W}^{(0)}(\vec{f}_1, \vec{f}_2)$ is the spatial Fourier transform of the cross spectral density function in the source plane

$$\tilde{W}^{(0)}(\vec{f}_1, \vec{f}_2) = \frac{1}{(2\pi)^4} \iint W^{(0)}(\vec{r}_1, \vec{r}_2) e^{-i(\vec{f}_1 \cdot \vec{r}_1 + \vec{f}_2 \cdot \vec{r}_2)} d^2 r_1 d^2 r_2$$

The vector $\hat{s}_\perp$ is the projection of the unit vector $\hat{s}$ in the plane of the source and Θ is the angle between $\hat{s}$ and the normal to the source plane.

Equation (1.3) is an important result that shows clearly the close relation between the far field intensity distribution and the source properties (both intensity distribution and complex degree of coherence).

We owe it, however, to the inquisitive investigations of Wolf and coworkers to uncover a significant amount of physical consequences which are contained in germ in eq. (1.3), although not in a self evident form. The effect of these investigations has been to elucidate the physical implications of these results, but also to stimulate a number of experimental investigations in an area that could be properly labelled as beam optics.

We have been concerned with the experimental verification of the above theoretical results on partially coherent sources. In the course of our work we have not only succeeded in the stated goal, but we have also begun to explore possible areas of applications. These will be discussed separately in a forthcoming proposal.

Here we concern ourselves with two separate questions. We first review the main theoretical results which have been derived on the basis of eq. (1.3). Next we discuss the details of our experimental set up and the results of our tests.

2. Theoretical Background

A fairly general model of planar light sources is one which is characterized by a spectral degree of spatial coherence

$$\mu^{(o)}(\vec{r}_1, \vec{r}_2) = \frac{W^{(o)}(\vec{r}_1, \vec{r}_2)}{\sqrt{I^{(o)}(\vec{r}_1) I^{(o)}(\vec{r}_2)}}$$

which, in the source plane, is only a function of the difference $\vec{r}_1 - \vec{r}_2$ between the position vectors $\vec{r}_1$ and $\vec{r}_2$ (i.e. $\mu^{(o)}(\vec{r}_1, \vec{r}_2) \equiv g(\vec{r}_1 - \vec{r}_2)$). Sources of this type appear to have been considered first by Schell, and will be referred to

as Schell model sources. The Fourier transform of the cross spectral density function (Eq. (1.4)) is given by

$$\tilde{W}^{(0)}(k\hat{s}_1, -k\hat{s}_1) = \frac{1}{(2\pi)^4} \iint d^2r_1 d^2r_2 e^{-ik\hat{s}_1 \cdot (\vec{r}_1 - \vec{r}_2)} g(\vec{r}_1 - \vec{r}_2) \sqrt{I^{(0)}(\vec{r}_1) I^{(0)}(\vec{r}_2)} \quad (2.2)$$

Equation (2.2) makes it very obvious to what extent the complex degree of coherence and the intensity distribution in the source plane are indeed jointly responsible for the shape of the far field intensity distribution.

Because it is desirable to elucidate this point with a minimum amount of algebraic labor, Wolf and Collett and, independently, Baltes, Steinle and Antes have considered the special case of a source having a Gaussian intensity profile and a Gaussian degree of coherence. It is expected that, at least, qualitatively the results of this model calculation should provide a fair representation for a number of more complex situation, where the analytic handling of eq. (2.2) is no longer so straightforward. In this case, one assumes

$$I(\vec{r}) = A \exp(-r^2/2\sigma_I^2) \quad (2.3)$$

$$g(\vec{r}) = \exp(-r^2/2\sigma_g^2) \quad (2.4)$$

where σ_I is the size of the illumination area and σ_g the so-called coherence length at the source. The integral (2.2) reduces to the product of two similar integrals of the type

$$W_x = \int dx_1 dx_2 e^{-\frac{(x_1 - x_2)^2}{2\sigma_g^2}} e^{-ik s_{1x}(x_1 - x_2)} e^{-\frac{x_1^2}{4\sigma_I^2}} e^{-\frac{x_2^2}{4\sigma_I^2}} \quad (2.5)$$

The evaluation of Eq. (2.5) and of the similar one containing y_1, y_2 as variables of integration and s_{1y} as the projection of the unit vector $\hat{s}$ on the source plane is lengthy but quite simple. The resulting far field intensity distribution becomes

$$J(\hat{s}) = \frac{k^2 A \sigma_g^2 \sigma_I^2 \cos^2 \theta}{(1 + (\sigma_g/2\sigma_I)^2)} \exp\left(-\frac{k^2 s_1^2 \sigma_g^2}{2[1 + (\sigma_g/2\sigma_I)^2]}\right) \quad (2.6)$$

Thus, if we let

$$s_1^2 = \sin^2 \theta ; \quad \Delta = \frac{1}{k \sigma_g} \sqrt{1 + (\sigma_g/2\sigma_I)^2}, \quad J(0) = \left(\frac{\sigma_I}{\Delta}\right)^2 A \quad (2.7)$$

we arrive at the simple result

$$J(\theta) = J(0) \cos^2 \theta \exp(-\sin^2 \theta / 2\Delta^2) \quad (2.8)$$

The far field distribution is characterized by two experimentally relevant parameters, $J(0)$ and Δ . The latter in particular gives a measure of the angular spread of the beam in the far field. It is related to σ_g and σ_I in a way that indicates quite clearly the sensitive dependence between the measurable far field parameters and the source characteristics.

Equation (2.8) also represents the basis for a remarkable theorem first stated by Wolf and Collett:

Two Schell-model sources, whose intensity distribution and degree of coherence and both Gaussian will generate fields with identical far zone intensity distributions if the rms widths σ_I and σ_g are such that each of the quantities

$$\begin{aligned} \Delta^2 &= (1/k \sigma_g)^2 + (1/2k \sigma_I)^2 \\ J(0) &= A (\sigma_I/\Delta)^2 \end{aligned} \quad (2.9)$$

are the same for both sources.

While this theorem is rigorously valid, as stated, only for Gaussian Schell model sources it seems reasonable to expect that at least qualitatively the main conclusions should hold under less restrictive conditions provided that the intensity profile and the degree of coherence in the source plane are smooth functions of their arguments and roughly shaped as Gaussians.

Another class of sources has been the subject of extensive investigations because of the numerous remarkable properties that have been uncovered. Suppose that as in the case of the Schell model sources the complex degree of coherence in the source plane is a function of only the difference between the spatial coordinates r_1 and r_2 and that, in addition, $g(r_1 - r_2)$ is appreciably different from zero over a domain whose linear dimensions are much smaller than the transverse dimensions of the illuminated area, but still much larger than the wavelength of light. Suppose, furthermore, that the spatial variations of the intensity profile are slow enough that $I(\vec{r})$ is practically constant over a coherence length (the range of $g(\vec{r}_1 - \vec{r}_2)$). A source that satisfies these criteria has been called quasi-homogeneous by Carter and Wolf.

Mathematically, one can characterize a quasi-homogeneous source by requiring the cross spectral density function to be of the form

$$W(\vec{r}_1, \vec{r}_2) \cong I\left(\frac{1}{2}(\vec{r}_1 + \vec{r}_2)\right) g(\vec{r}_1 - \vec{r}_2) \quad (2.10)$$

with $I(\vec{r})$ a slowly varying function across the illuminated area and $g(\vec{r})$ a rapidly varying function (over a spatial scale of many wavelengths). If Eq. (2.10) is satisfied, the Fourier transform of the cross spectral density function has been shown to take the form

$$\tilde{W}(\vec{f}_1, \vec{f}_2) = \frac{1}{(2\pi)^4} \iint d^2r d^2r' I(\vec{r}) g(\vec{r}') e^{-i(\vec{r} \cdot (\vec{f}_1 + \vec{f}_2) + \vec{r}' \cdot (\vec{f}_1 - \vec{f}_2)/2)} \quad (2.11)$$

It is clear that unlike the case of Schell model sources (cfr. eq. (2.2)), the present Fourier spectrum of the cross spectral density function factorizes into the product of two function,

$$\tilde{W}(\vec{f}_1, \vec{f}_2) = \tilde{I}(\vec{f}_1 + \vec{f}_2) \tilde{g}(\frac{1}{2}(\vec{f}_1 - \vec{f}_2)) \quad (2.12)$$

which are, respectively, the Fourier transforms of the intensity distribution and of the degree of coherence across the source. It is also clear that $\tilde{I}(\vec{f})$ is a rapidly varying function of the spatial frequency $\vec{f}$, while $\tilde{g}$ is instead slowly varying.

From equation (1.3) it follows that the far field radiant intensity is given by

$$J(\hat{s}) = (2\pi k)^2 \tilde{I}^{(0)}(0) \tilde{g}^{(0)}(ks_{\perp}) \cos^2 \theta \quad (2.13)$$

i.e. $J(\hat{s})$ is proportional to the Fourier transform of the complex degree of coherence in the source plane. The complex degree of coherence in the far zone on the other hand is given by

$$\mu^{(\infty)}(R_1 \hat{s}_1, R_2 \hat{s}_2) = \frac{\tilde{I}^{(0)}(k(\hat{s}_{1\perp} - \hat{s}_{2\perp}))}{\tilde{I}^{(0)}(0)} e^{ik(R_1 - R_2)} \quad (2.14)$$

i.e., it is proportional to the Fourier transform of the intensity distribution in the source plane.

Equations (2.13) and (2.14) express a remarkable reciprocity theorem and clarify the role of the source intensity distribution and degree of coherence in determining the far zone behavior of the field (both intensity and coherence). These equations also represent a generalization of the classic Van Cittert-Zernike theorem in the sense that the source is no longer restricted to be spatially

incoherent. Of course, quasi-homogeneity is required. There is a number of remarkable predictions that follow from the above reciprocity relations.

- a) The far field intensity distribution is independent of the size of the illumination area as well as of the actual intensity distribution in the source plane.
- b) The far field degree of coherence is independent of the coherence of the source.
- c) By appropriate selection of parameters it is possible to produce highly directional beams in spite of the global incoherence of the quasi-homogeneous source.

The properties a) and b) are a direct consequence of the reciprocity theorem (eqs (2.13) and (2.14)). Property c) is not self evident and it was demonstrated for the first time by Collett and Wolf in a communication that has stimulated a considerable amount of interest. These authors have been able to show that a quasi-homogeneous source with a Gaussian degree of coherence

$$g^{(0)}(\vec{r}_1 - \vec{r}_2) = \exp(-|\vec{r}_1 - \vec{r}_2|^2 / 2\sigma^2) \quad (2.15)$$

produces a far field radiant intensity of the form

$$J(\hat{s})/J_{(0)} \simeq \cos \theta \exp\left[-\frac{1}{2}(k\sigma)^2 \sin^2 \theta\right] \quad (2.16)$$

Clearly if $k\sigma \gg 1$ the angular beam spread can be made quite small, and in fact the beam itself can be made practically indistinguishable from a laser beam of properly chosen beam cross section. Thus complete spatial coherence of the source is not necessary to obtain a highly directional beam. Moreover, because of property (a), highly directional beams can be produced regardless of the detailed shape of the intensity distribution of the source.

In addition to the far field calculations leading to Eqs. (2.15) and (2.16), the behavior of the beam radius has also been considered in the literature for the case of a Gaussian Schell model source. After specializing

the calculation to the quasi-homogeneous limit, the full width at half-height of the beam as a function of distance from the plane of the source takes the form

$$d_{1/2}(z) = 2\sqrt{\ln 2} \left(2\sigma_I^2 + 2z^2/k^2\sigma_g^2 \right)^{1/2} \quad (2.17)$$

where σ_I , as usual, is proportional to the transverse dimension of the illumination region and σ_g is the coherence length of the source. The main goal of the present investigation was to identify a realizable quasi-homogeneous source and to test experimentally some of its properties. In the next section we discuss the details of our set up and give evidence of the highly directional propagation properties of quasi-homogeneous sources, of the nature of the far field intensity distribution and of the spatial variation of the beam size.

3. Description of the Experimental Activities

The experimental aspects of this work were divided into two main tasks. The first was to produce a source with the desired coherence properties, while the second was to design a system by which we could characterize the statistical properties of the source. In the following sections these two undertakings are discussed in more details than in the enclosed reprint. The discussion of the experimental results instead is left to the preprint of our joint paper with Dr. Collett.

a. Selection of the Source

In the preliminary phases of this project we decided to produce a quasi-homogeneous source by altering the spatial coherence properties of light from a laser using a rotating phase screen. The rotation has the purpose of allowing an ensemble average over the field configurations. We decided against the use of imaging or spatial filtering in order to minimize the complications associated with these methods.

In our initial effort we attempted to construct the required phase screen from a ground glass plate. Several plates were ground with numerous grit sizes varying from 60 to 220. In all of our plates we discovered two distinct size groupings of inhomogeneities. One group was characterized by typical transverse dimensions $\leq 1 \mu\text{m}$ which caused large angle scattering of the incident laser light. The second group consisted of large inhomogeneities ($>100 \mu\text{m}$) which caused small angle scattering. These two groups were always present. Attempts to create a plate with a more uniform distribution of sizes resulted in the creation of more small inhomogeneities. We found it impossible to make a disk with inhomogeneities that were closely grouped in the 10-100 μm range without at the same time producing numerous small inhomogeneities as well. The far field distribution which resulted from one of these ground surfaces had an envelope that was almost identical to that produced by the laser at the same distance. The intensity profile was also extremely noisy as shown in Fig. 1. The interpretation of this result is fairly obvious. The large inhomogeneities are responsible for the small angle scattering profile and for the large intensity fluctuations which occur as they cross the illuminated region. This we considered to be a sufficient reason for abandoning this approach and to attempt different methods. Our most successful phase screens were produced by spraying glass blanks with a fine mist of clear acrylic finish. Observation under the microscope revealed a uniform distribution of sizes of inhomogeneities everywhere on the surface. The size of the inhomogeneities, in addition, could be varied by changing the thickness of the coating. We have been able to prepare phase screens whose appearance varied from that of a frosted glass to that of a shower glass. With these phase screens we have observed far field intensity distributions which were both highly directional and independent of the illumination spot size $w_{1/2}(0)$ (See figures 3a and 3b of enclosed preprint).

The above phase screens are mounted on a shaft supported by ball bearings and rotated by a servo mechanism to obtain vibrationless rotation over a reason-

able range of angular frequencies (5-1000 rpm.). They are illuminated from behind at normal incidence with a 10mW He-Ne laser which is spatially filtered and expanded. The expansion factor of the beam is variable so that spot sizes ranging from 1 to 10 mm on the phase screen can be obtained. The collimation of the expanded beam is rather critical and must be checked at various distances from the illuminated area.

b. Measurement System

The analysis of the statistical properties of a partially coherent source should require measurements of the degree of spatial coherence and of the intensity distribution at various distances from the source plane. In our case it would be enough to obtain the complex degree of spatial coherence $g(r_1-r_2)$ at the source and the far field intensity distribution. A verification of the reciprocity theorem would then confirm the quasi-homogeneous nature of the source. At this time a direct measurement of $g(r_1-r_2)$ in the source plane is very difficult because of the small range of the coherence length ($\sigma_g < 100\mu\text{m}$). Future investigations will be directed to solving this problem.

For the moment we have decided to focus on the measurement of the far field distribution, and to establish the properties of the source from this information. In designing the far field measuring system we have decided to allow the field to propagate freely, in contrast with some other recent investigations (See reference 6 of enclosed preprint) where the presence of intervening optical components makes the analysis of the data less unambiguous.

A schematic diagram of the experimental set up is shown in Fig. 1 of the enclosed preprint. This figure shows the mirrors and translation stages which are mounted on a 4'x8' optical table. The table is supported by vibration isolation mounts to attenuate mechanical disturbances which would limit severely the detection of narrow beams in the far field. The system of mirrors used in our measurements has allowed us to obtain a free propagation path from the source to the detector of up to 13 meters. For illustration sake, the equivalent optical

path is shown in Fig. 2. The mirrors are 50 mm diameter, 12.5 mm thick Pyrex blanks polished to one tenth wave flatness and coated with an evaporated layer of silver and a thorium fluoride overcoat. The silver coating was found necessary in order not to attenuate the beam excessively owing to the many reflections.

Using the accepted far field criterion $Z \gg k\sigma_I\sigma_g$, (See reference 11 of the enclosed preprint), we have estimated that the far field properties of sources with coherence length as large as 150 μm could be studied with the present set up. Of course if needed the range can be easily extended by the addition of more mirrors.

The scanning of the intensity distribution is performed with the joint linear translation of the detector in the far field and the angular rotation of the source. The translation of the detector (an EMI photomultiplier) is produced with the help of a stepper motor driven linear translation stage capable of 6.25 μm steps. This linear translation corresponds to an angular step size of 0.5 μrad at a source to detector distance of 12.5 m. Note, however, that because of the limited amount of light that is available at these distances, a 50 μm aperture has been used routinely in our measurements. Under these conditions the angular resolution of the linear scanning system has been typically 5 μrad at the same distance.

With reference to Fig. 2, we see that the maximum angular beam which can yield any information is defined by the angle subtended by the aperture of the last mirror and the center of the source. (typically 6 mrad). This limitation on the maximum angular scan sets a practical lower limit on the coherence length, σ_g , of the sources to 25-50 μm . This limitation is not acceptable for some of our sources, which are characterized by considerably smaller coherence lengths, because it prevents the scanning of the full intensity distribution.

Large angular scans of the far field intensity distribution have been accomplished by rotating the source itself about an axis passing through the source plane and perpendicular to the direction of propagation. This method does not suffer from the limitation of the linear translation system because rotations can be performed through any angle. It does, however, suffer from resolution problems because the stepper motor that drives the rotation stage is only capable of 0.1 mrad per step. This rotation system can be used to scan broad distributions (angular divergence > 10 mrad) by itself, or narrower distributions jointly with the translation stage.

The phase screen is placed so that the axis of the rotation stage is in the plane of the phase screen, is perpendicular to the axis of propagation of the laser light and passes through the center of the illuminated area. Naturally the rotation of a large piece of hardware like the laser is likely to lead to mechanical instabilities. For this reason we have designed a system of mirrors to reflect the light from the stationary laser in such a way as to illuminate the phase screen at normal incidence regardless of its angular orientation. A schematic drawing of the system is shown in Fig. 3.

The axis of the laser beam that illuminates the phase screen defines the scattering angle θ . This was measured in an absolute way by using a calibration procedure based on the measurement of the diffraction orders of a known grating. The calibration procedure is described in Fig. 4 and the accompanying caption.

Figure Captions

1. The far field intensity distribution obtained using a rotating ground glass as a phase screen (noisy trace) is shown together with the laser profile at the same distance (about 5m in this experiment). The near coincidence between the envelope of the scattered intensity and of the laser intensity profile is due to the large inhomogeneities on the surface of the phase screen. This feature was typical of all the ground glasses produced in this work. We have also observed a much broader and weaker intensity background resulting from the small size inhomogeneities. This cannot be seen in this limited scan.
2. The equivalent optical path of the light from the source. θ is the scattering angle and α , defined by the aperture of the last mirror is the maximum angular range of the linear translation stage.
3. Schematic drawing of the multiple reflection system that provides laser illumination of the phase screen at normal incidence regardless of the angular position of the rotating stage. The shaded mirror is stationary while the components contained within the heavy dashed line are rotated about the axis as shown.
4. A typical plot of a calibration run in which the phase screen is replaced by a coarse transmission grating of known line density. The horizontal axis corresponds to the angular rotation of the rotation stage. The angular position of the diffraction maxima act as calibration markers.

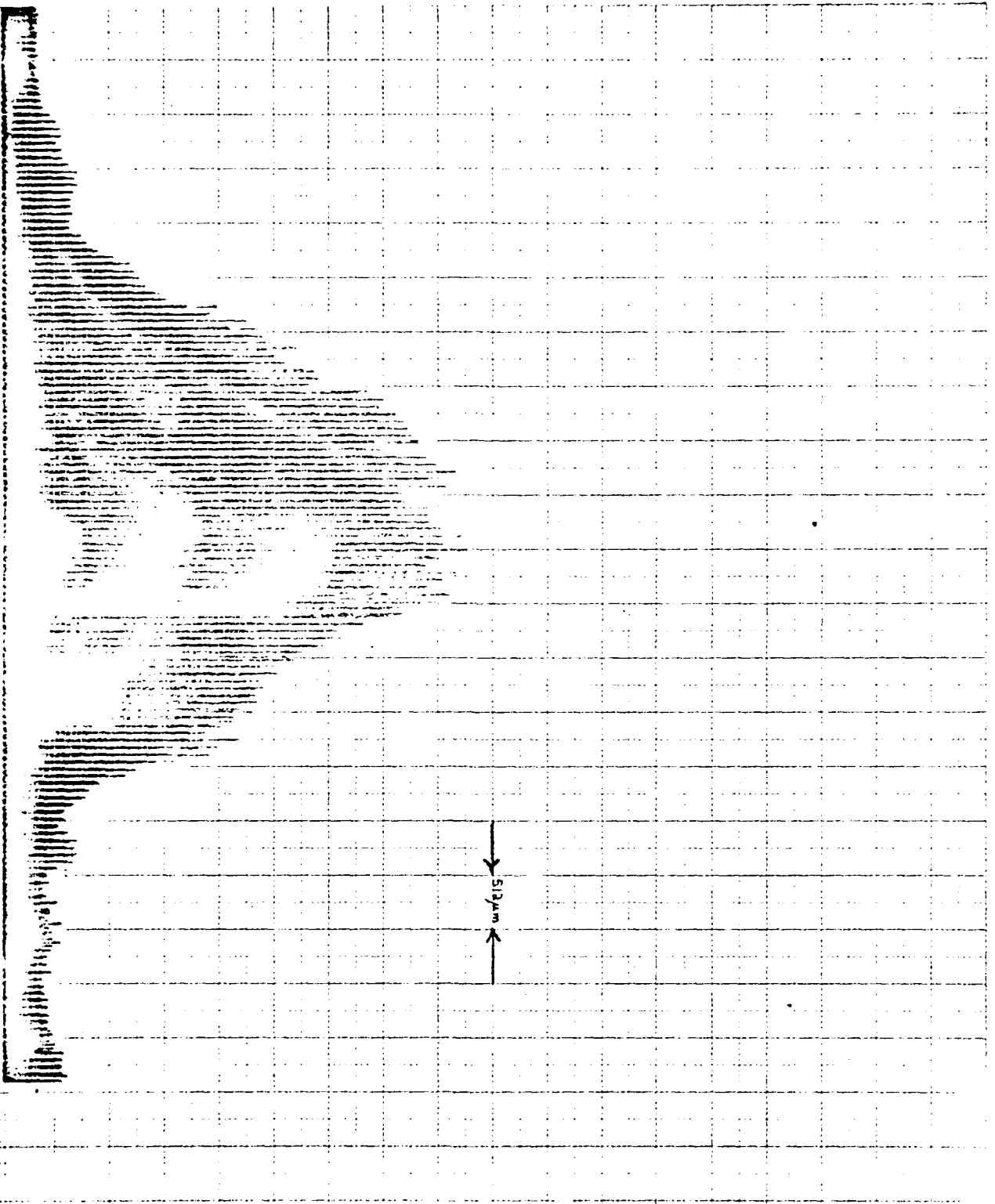


Fig. 1

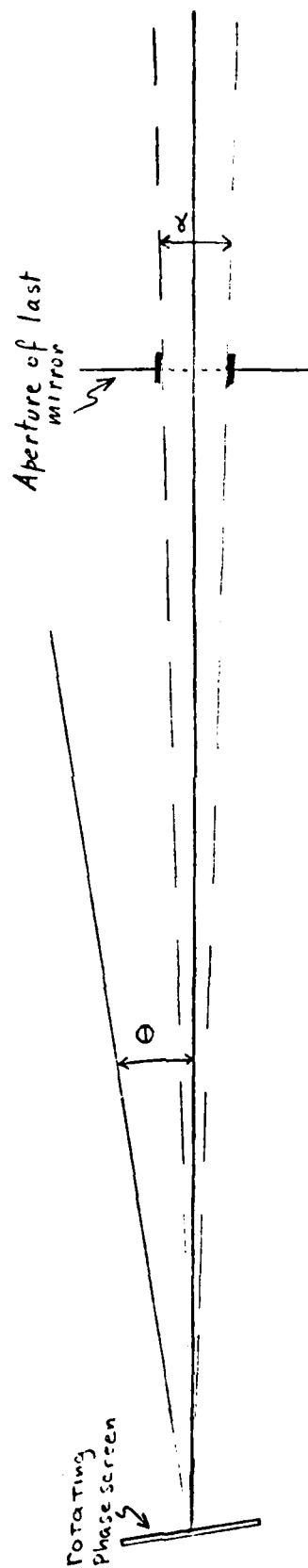


Fig. 2

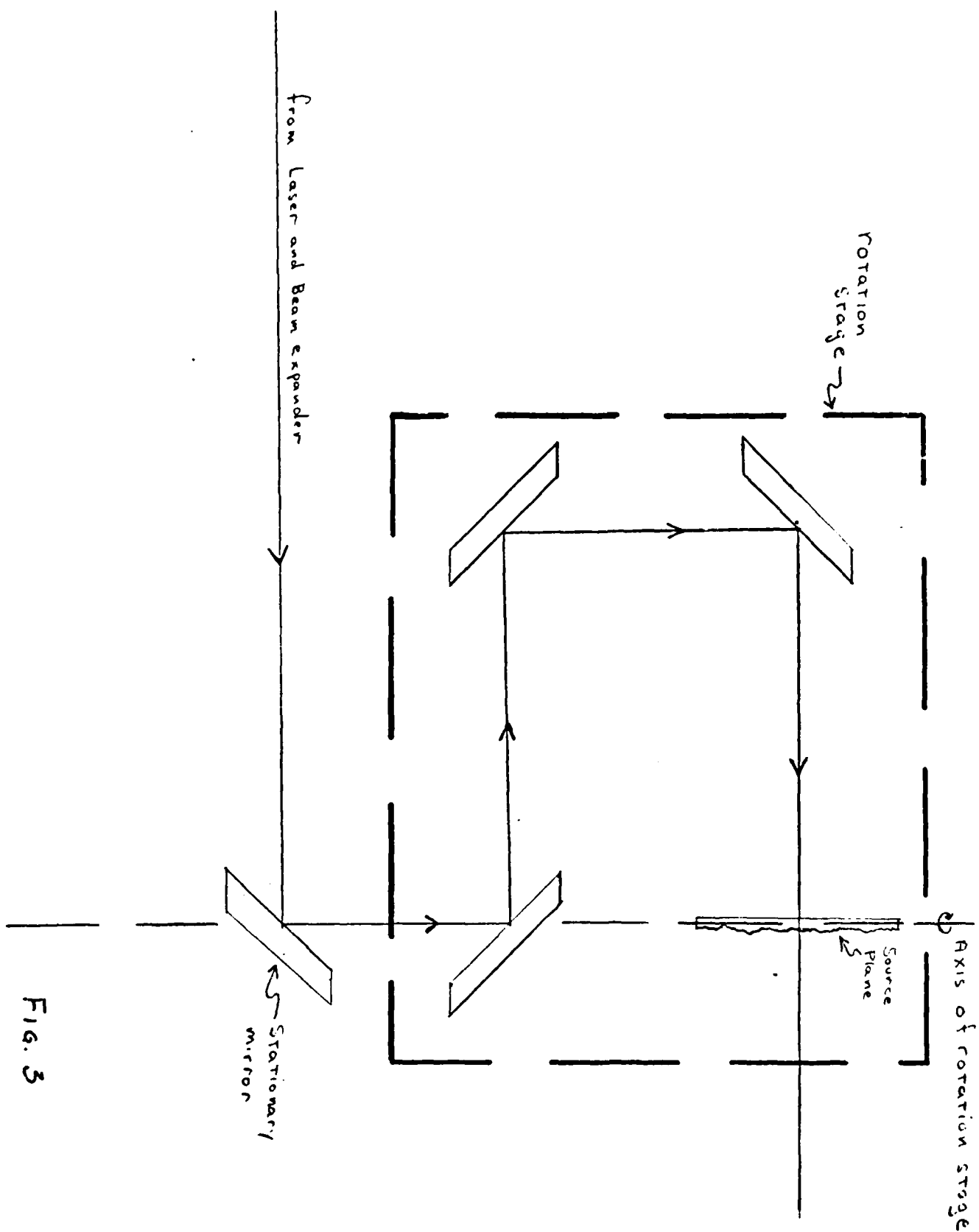


FIG. 3

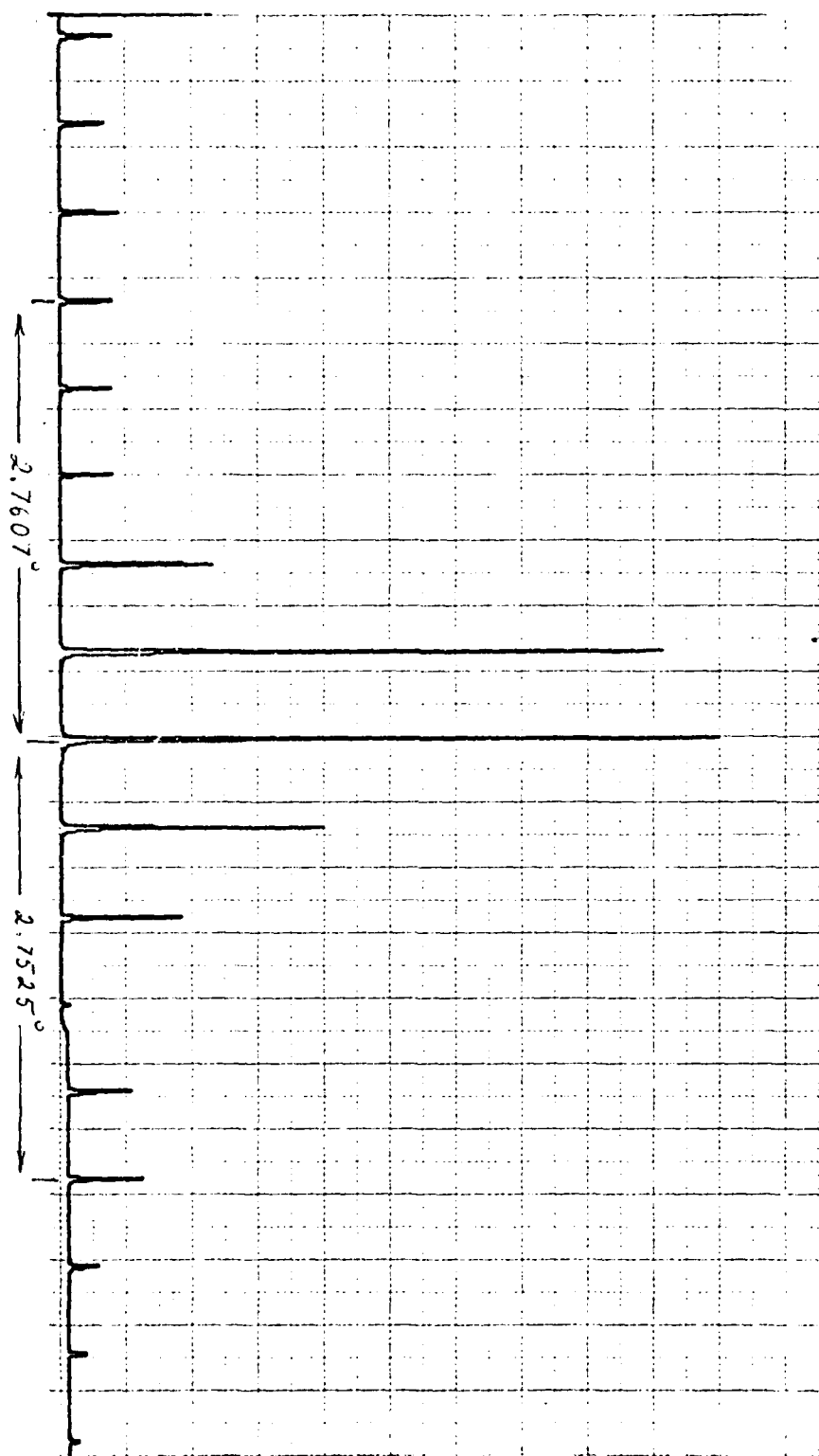


FIG. 4

GENERATION OF HIGHLY DIRECTIONAL BEAMS FROM A
GLOBALLY INCOHERENT SOURCE*

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ABSTRACT

We have carried out a series of observations of the far field intensity distribution produced by a class of partially coherent planar sources. Our results confirm that, under the Collett-Wolf quasi-homogeneity conditions, highly directional beams can be produced even from a source which is globally incoherent. The far-zone pattern is shown to be largely independent of the intensity profile in the source plane and that the beam radius varies as predicted by Foley and Zubairy in the paraxial approximation. We show experimentally how the coherence length in the source plane can be measured directly from the far zone intensity distribution.

1. INTRODUCTION

The properties of the electromagnetic radiation produced by partially coherent planar sources have been the subject of active investigations in recent times.¹⁻⁴ Among the theoretical models that have been proposed an especially appealing one, commonly known as the Schell-model⁵ has been explored in some details^{1b} in an effort to clarify the relation between the state of coherence of the source and its far-zone intensity distribution.

The theoretical predictions that have emerged, with regard, for example, to the highly directional character of certain radiation patterns, have stimulated experimental investigations into the practical realization of such sources.⁶ Our paper contains a description of additional experimental studies of this problem.

Our main goal is to analyze the field radiated by a special kind of Schell-model source (the quasi-homogeneous source)^{1d} under the simplest experimental conditions, so as to provide an unambiguous test of their predicted properties. In Section II we summarize the relevant features of the Schell model sources and of their quasi-homogeneous limit. In Section III we give details of our experiments and discuss the main results.

2. SHELL-MODEL SOURCES AND QUASI-HOMOGENEOUS SOURCES

A Schell-model source⁵ is defined by the requirement that the cross-spectral density function⁷ at frequency ω will be of the form

$$W_{\omega}(\vec{r}_1, \vec{r}_2) = \sqrt{I(\vec{r}_1)} \sqrt{I(\vec{r}_2)} g(\vec{r}_1 - \vec{r}_2) \quad (2-1)$$

where $\vec{r}_1$ and $\vec{r}_2$ are two arbitrary position vectors in the source plane, $I(\vec{r})$ is the source intensity at the position $\vec{r}$ and $g(\vec{r})$ is the complex degree of spatial coherence of the light in the source plane. The far zone radiant intensity

$J_\omega(\hat{s})$ in the direction $\hat{s}$ forming an angle θ with the perpendicular to the source plane is given by⁸

$$J_\omega(\hat{s}) = (2\pi k)^2 \cos^2 \theta \tilde{W}_\omega(k s_\perp, -k s_\perp) \quad (2-2)$$

where $\tilde{W}_\omega(f_1, f_1)$ is the four dimensional Fourier transform of the cross-power spectral density function (2-1) and $s_\perp$ is the projection of the unit vector $\hat{s}$ in the source plane.

It is clear from eqs. (2-1) and (2-2) that the far zone angular distribution of the light is intimately related to both the intensity and the degree of coherence of the source. If, in particular, $I(\vec{r})$ and $g(\vec{r})$ are both Gaussian functions of the form

$$I(\vec{r}) = A e^{-r^2/2\sigma_I^2} \quad (2-3)$$

$$g(\vec{r}_1, \vec{r}_2) = \exp \left(- |\vec{r}_1 - \vec{r}_2|^2 / 2\sigma_g^2 \right), \quad (2-4)$$

then the far field radiant intensity becomes^{1b}

$$J_\omega(\hat{s}) = J_\omega(0) \cos^2 \theta \exp(-\sin^2 \theta / 2\Delta^2) \quad (2-5)$$

where

$$J_\omega(0) = (\sigma_I / \Delta)^2 A \quad (2-6a)$$

$$\Delta = \frac{1}{k\sigma_g} \sqrt{1 + \left(\frac{\sigma_g}{2\sigma_I} \right)^2} \quad (2-6b)$$

Equation (2-5) also describes the far field radiant intensity produced by a laser with a Gaussian output distribution of the form

$$I_L(\vec{r}) = A_L e^{-r^2/2\delta_L^2} \quad (2-7)$$

provided δ_L is adjusted in such a way that the identity

$$\frac{1}{(2\delta_L)^2} = \frac{1}{\sigma_g^2} + \frac{1}{(2\sigma_I)^2} \quad (2-8)$$

is satisfied. If the source is characterized by a coherence length σ_g which is much larger than the wavelength of light and much smaller than the linear dimension of the source, i.e. if

$$k\sigma_g \gg 1, \sigma_g/\sigma_I \ll 1 \quad (2-9)$$

then, the angular divergence predicted by eq. (2-6) can be made quite small. In the limit specified by eq. (2-9) the Gaussian Schell model source becomes a quasi-homogeneous source.^{1d} The equivalent laser width σ_L in this limit, should be chosen equal to $\sigma_g/\sqrt{2}$. It is worth emphasizing that while the laser output is spatially coherent, at least under ideal conditions, the quasi-homogeneous source is, instead, globally incoherent.

The practical realization of quasi-homogeneous sources has been the subject of recent experimental investigations.⁶ The main result reported by DeSantis et al. in Ref. (6) is that, by means of suitable optical elements and amplitude filters, a globally incoherent source can be made to produce a far field intensity distribution that approximates well that of a laser beam of properly chosen spot size.

We too have been concerned with the practical realization of a quasi-homogeneous source, and with the study of the coherence and propagation properties of the radiated field. We have been especially interested in carrying out measurements of the field properties using the least number of optical elements between the source and the detector, so as to minimize the uncertainties introduced by added components, and to provide an unambiguous interpretation of the results.

Under quasi homogeneous conditions (2-9) and according to eqs. (2-5) and (2-6), the normalized far zone intensity distribution $J_\omega(\hat{s})/J_\omega(o)$ should be

practically independent of the linear dimensions c_I of the source. Furthermore, the full width at half height of the beam ($d_{1/2}(z)$) at a distance z from the source plane is predicted to behave according to the relation

$$d_{1/2}(z) = 2\sqrt{\lambda n 2} \left\{ 2\sigma_I^2 + \frac{2z^2}{k^2 c_g^2} \right\}^{1/2} \quad (2-10)$$

in the paraxial approximation.⁹

It is clear from Eqs. (2-1) and (2-2) that if the source intensity distributions $I(\vec{r})$ is practically constant over several coherence lengths in the source plane, the normalized far zone angular distribution becomes directly proportional to the Fourier transform of the degree of coherence $g(\vec{r})$, and is independent of $I(\vec{r})$. A test of this reciprocity relation based on the direct measurement of $g(\vec{r})$ and $J_\omega(\hat{s})$ will be reported in a future publication.

It is interesting to observe that the value of the correlation length measured from far field data is in excellent agreement with that obtained from the measurement of the mean square radius as a function of distance. This lends strong indirect support to the validity of the reciprocity relation.^{1d}

3. DESCRIPTION OF THE EXPERIMENT AND RESULTS

A convenient source for our purposes has been obtained by illuminating a suitable phase screen with a Gaussian laser beam. The phase screen has been created by spraying a finishing mist (Krylon #1306) on a clear glass blank (thickness 3.2 mm, diameter 130 mm). The spray has been found to produce a uniform rough coating of controllable thickness (the thickness is easily adjusted by varying the amount of deposition). Original attempts to construct rough surfaces by grinding glass blanks with various kinds of grinding compounds have been much less successful because of the difficulties with producing a fairly uniform distribution of inhomogeneities of reasonable size.¹⁰

The glass samples have been mounted on the rotating shaft of a synchronous motor which could turn the glass with negligible vibrations. The motor has been mounted on a rotating platform driven by a stepper motor capable of providing angular steps of 0.005° . By a suitable combination of carefully positioned mirrors on the rotating platform we have managed to produce highly accurate rotations of the source with the illuminating laser beam always perpendicular to the back surface of the phase screen but without the need for mounting the laser itself on the rotating stage. In addition, the receiving photomultiplier could be kept fixed, thus eliminating the difficulties associated with angular scans at large distances from the source.

A 10 mW He-Ne laser, used as the primary source, was expanded and accurately collimated with a beam expanding telescope. This allowed us to vary the diameter of the illumination area of the source from 0.8 to approximately 8 mm (intensity full width at half-height).

The beam emerging from the phase screen was allowed to propagate through different path lengths with the help of an array of mirrors (2" diameter) which have been coated for maximum reflectivity at $6328\overset{\circ}{\text{Å}}$. As a detector we have used a cooled photomultiplier mounted on a linear translation stage for additional fine scanning. A schematic diagram of our set-up is shown in Fig. 1.

As a necessary preliminary step to subsequent measurements we have carried out an absolute angular calibration of the rotation stage by replacing the rough rotating surface with a rather coarse transmission grating and by measuring the position of a sufficient number of diffraction orders in the far field.

In our first series of measurements we have carried out an angular scan of the laser beam profile without the rough surface. Both the output beam from the laser as well as the expanded and recollimated beams have been observed to be Gaussian to a high degree of approximation at various distances of up to

13 m from the laser source. Subsequently, different rough surfaces have been positioned on the rotating stage and the respective angular intensity distributions measured at a distance of 12.5 m. The results obtained with two typical surfaces (henceforth labelled S_1 and S_2 for simplicity) are shown in Fig. 2 and Figs. 3a,b. In either case, changing the size of the illuminating beam did not affect the far field pattern in a measurable way as shown in Fig. 2. The measured beam divergences ($d_{1/2}/2L$, where L is the distance between the source and the detector) are 27.0 mrad for S_1 and 2.78 mrad for S_2 . (The factor of 10 between the two values of the beam divergence is completely accidental and of no direct significance). It is clear that in either case the intensity distributions across the source had large enough linear dimensions to satisfy the quasi-homogeneity criterion. As an additional qualitative test, a slice of the illumination area was blocked off, again without affecting the far field pattern in an obvious way. From the observed far field intensity profile we have estimated the size of the coherence areas in the respective source planes for S_1 and S_2 . The results are listed in Table I under column $\sigma_g^{(a)}$.

We have also measured the beam diameter $d_{1/2}(z)$ as a function of the distance between the detector and the source. The results are displayed in Fig. 4 and Fig. 5. There is good agreement with the predicted linear behavior of $d_{1/2}(z)$ (eq. (10)) at large distances from the source and with the expected departure from linearity in the Fresnel zone. The values of the coherence length entered in Table I are divided into two columns. The results labelled $\sigma_g^{(a)}$ correspond to the coherence length obtained assuming that a distance of 12.5 m is sufficiently large to be in the far field and using eq. (6) in the quasi-homogeneous limit. The second column labelled $\sigma_g^{(b)}$ represents the result of a two-point fit of the data to eq. (10). The fit was performed using the measured values of $d_{1/2}$ in the source plane ($z = 0$) and at $z = 1.25$ m. When Leader's

criterion¹¹ ($z \gg k \sigma_g \sigma_I$) is satisfied it is expected that $\sigma_g^{(a)}$ and $\sigma_g^{(b)}$ will agree. This is in fact borne out by our measurements.

In conclusion we have shown that the illuminated phase screens used in our experiments behave as quasi-homogeneous sources when illuminated by collimated laser light. The radiated fields are highly unidirectional in spite of the global incoherent character of the fields in the source plane, and the observed far field patterns are essentially independent of the intensity distribution in the source plane. We have also verified that the beam size is well described by eq. (10) in both the Fresnel and far field regions, as predicted in Ref (9).

A direct experimental test of the reciprocity theorem will be discussed in a subsequent publication.

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TABLE I

Summary of the relevant data

Source	Distance to Detector (m)	Angular Divergence (mrad)	$\sigma_g(a)$ (μm)	$\sigma_g(b)$ (μm)
S_1	12.5	27.0	8.8	8.8
S_2	12.5	2.78	85.4	87.2

FIGURE CAPTIONS

Fig. 1 Schematic diagram of our experimental set up. The mount containing the rotating rough surface is such that the stationary laser beam impinges on the back surface of the glass always in the perpendicular direction as if the laser was rigidly mounted on the rotation stage (as shown in the figure only for illustration purposes).

Fig. 2 Far field angular intensity distribution from the rough surface S_1 illuminated by a laser beam of diameter 7 mm. The detector was positioned at 12.5 m from the plane source. The solid circles represent the intensity distribution from the same surface illuminated by a laser beam of diameter 3.5 mm.

Fig. 3a,b Far field angular intensity distributions from the rough surfaces S_1 (a) and S_2 (b) illuminated by a laser beam of diameter 7 mm. The detector was positioned at 12.5 m from the source. The vertical scale is in arbitrary units (A.U.).

Fig. 4 Behavior of the beam diameter $d_{1/2}(z)$ (full intensity width at half-height) as a function of z for the rough surface S_1 . The solid line corresponds to eq. (10) (a rescaled version of eq. (3.7) in Ref. (8)).

Fig. 5 Behavior of the beam diameter $d_{1/2}(z)$ (full intensity width at half-height) as a function of z for the rough surface S_2 .

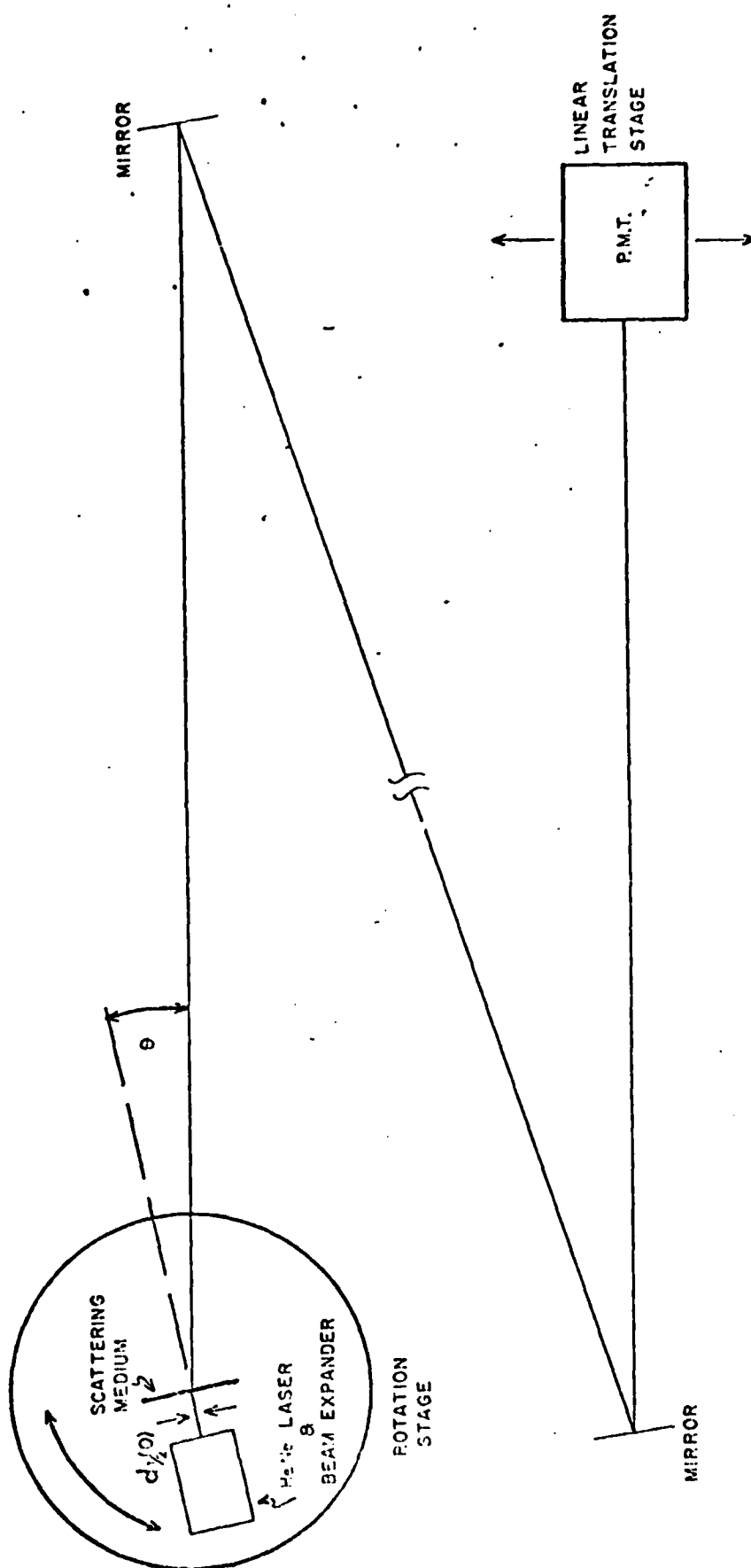


Figure 1

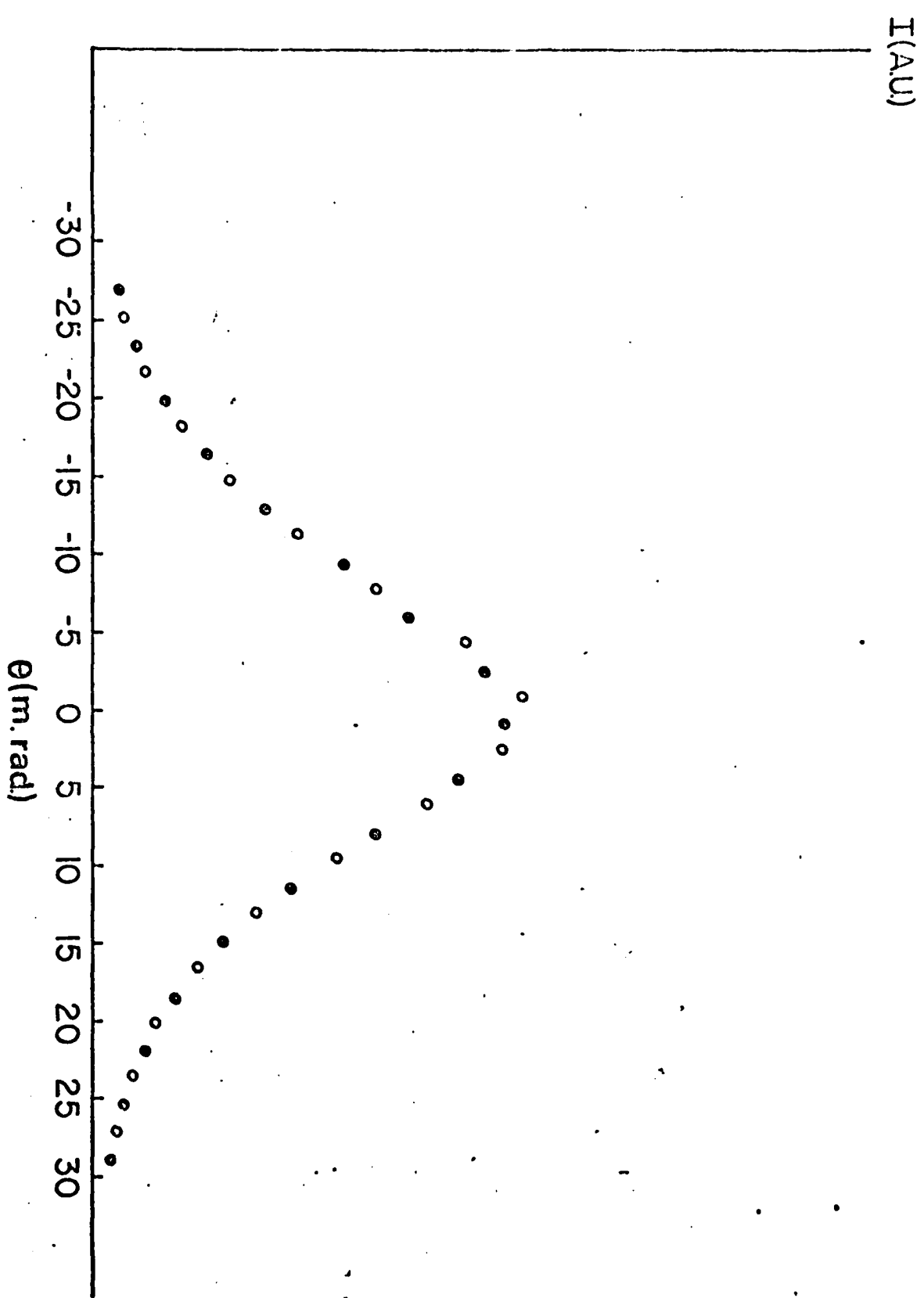
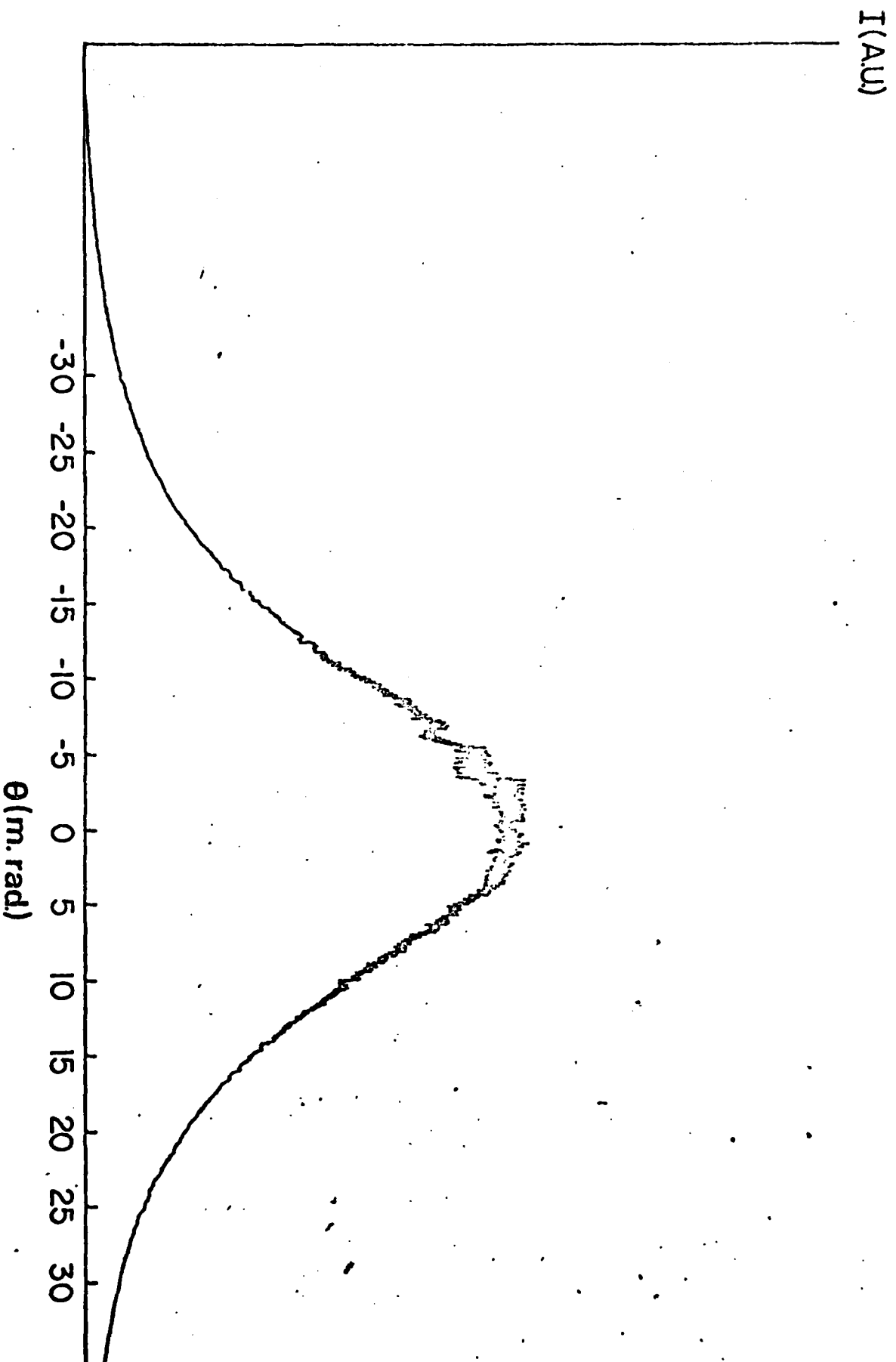
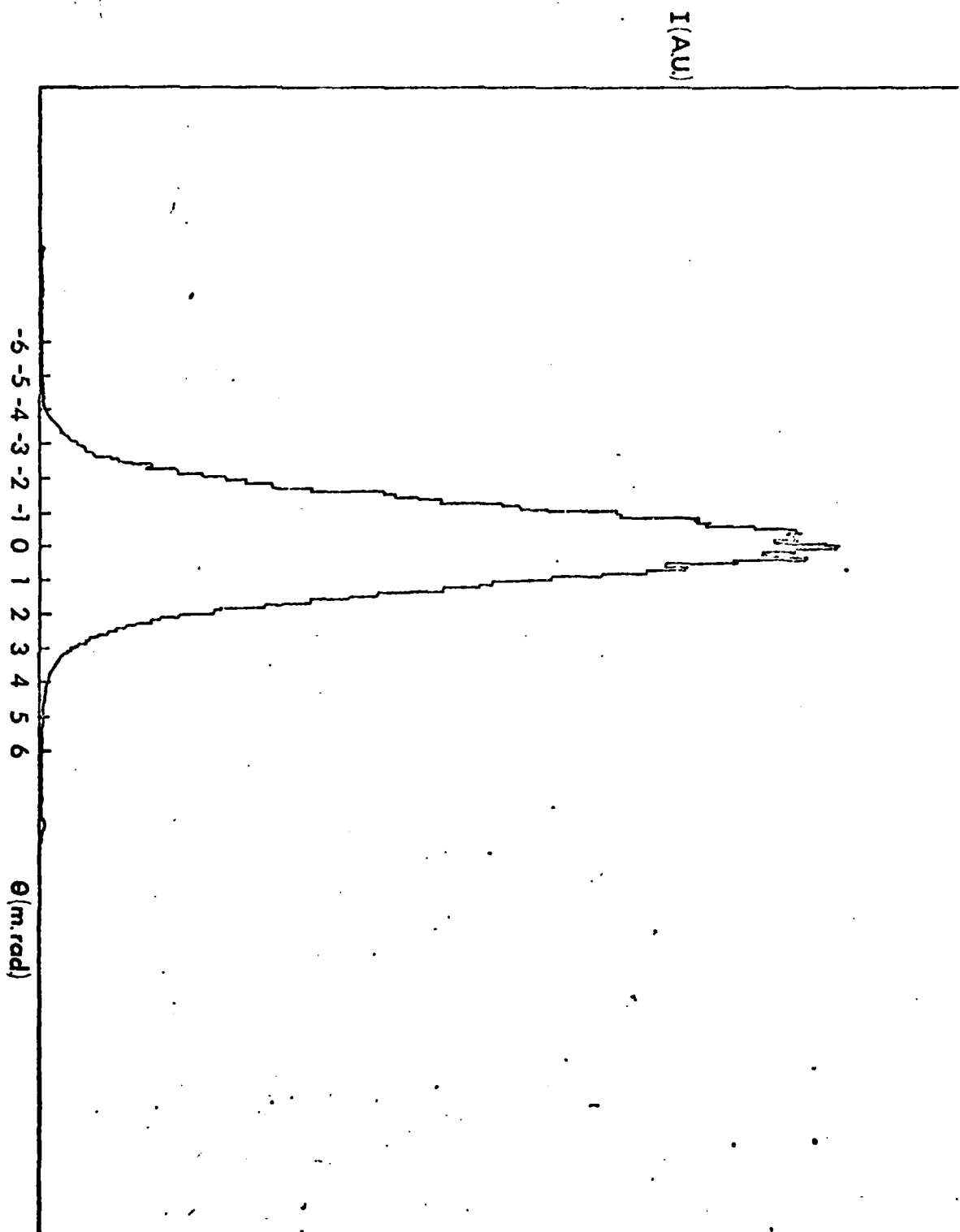


Fig. 2.4





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