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COMPACTNESS EFFECTS ON DRIFT OF ARCTIC PACK ICE

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April 1976-March 1977

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ABSTRACT

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I. Introduction

In the Arctic Ocean and its surrounding seas, an enormous number of ice floes of various sizes and shapes are floating on the surface. Most of these ice floes, under the influence of winds, ocean currents and tidal waves, are in constant but chaotic motion and are continuously being deformed. Any such floe, individually considered over a discrete period of time, represents a microscopic description of ice movement in the Polar Oceans. On the other hand, for many practical reasons, we are interested in the average motion of ice over a large area such as over 1000 kilometer radius or larger, and for an extended period of time, say several days, months or years. The description of these large scale movements of pack ice should be given by the average of local ice movement over a large area and an extended period of time. This is a macroscopic description of pack ice movement. In order to correlate the microscopic and macroscopic description of pack ice so that we may predict the pack ice deformation and movement, the development of a mathematical model is necessary. There are several mathematical models in literature (Fel'zenbaum 1958, Campbell, 1965, Doronin 1970, Coon et al, 1974 and Pai-Li 1977). In most of these models (Fel'zenbaum 1965, Campbell 1965, Coon et al 1974), the compactness N of pack ice, the amount of ice per unit area, has been disregarded, while others (Doronin 1970, Drogaitsev 1956, Nikiforov 1957, Rothrock 1970) considered the compactness only approximately. Since the compactness is important in the study of pack ice movement, particularly in the marginal region, it is useful to derive the exact equation of compactness of pack ice and to show the main effect of compactness on the ice drift. We are going to derive the equation of compactness of pack ice based on the two-phase flow model of Pai-Li (1977).

2. Pai-Li model of Arctic Pack Ice

The main concept of Pai-Li model is the two-phase flow approach in which the Arctic pack ice is considered as a mixture of solids (ice floes) and a fluid (sea water). Let the depth of ice be $H(x,y,t)$ which is in general a function of x and y, the two spatial coordinates on the surface of the ocean, and the time t. The maximum thickness of ice, H_o, is of the order of ten meters. We choose a typical length of the ocean L to represent the dimension of the surface of ocean which we are interested in such as $x \sim L$ and $y \sim L$. The representative length L may be of the order of 1000 km. Hence L is much larger than H_o. We may divide the ice packed ocean into two layers. In the upper layer, i.e., $z \leq H_o$, where z is the coordinate perpendicular to the surface of the ocean measured from the surface of the ice positive downward, we have a mixture of ice floes and sea water. Since $L \gg H_o$, the ice floes may be considered as small solid particles in a fluid (sea water). In this mathematical model, the concept of fluidization of solid particles in a fluid (Pai, 1971) is used. Hence these ice floes may be considered as a pseudo-fluid. For $z \leq H_o$, we have a mixture of two fluids: one is the sea water and the other is the pseudo-fluid of ice floes. We may extend the theory of two-phase flows of a mixture of small particles and a fluid to study the fundamental equations of dynamics of pack ice (Pai-Li, 1976).

Because of the fact that $L \gg H_o$, we may reduce the three-dimensional ice dynamics to an effective two-dimensional problem by integrating the three-dimensional equations of ice dynamics with respect to z, the vertical coordinate, with the limits z = O to z = H(x, y, t). The resultant equations depend only on two spatial coordinates x and y and time t with the thickness of ice H(x, y, t) as an unknown parameter to be determined. All the mathematical models (Fel'zenbaum 1958, Campbell 1965, Doronin 1970, Coon et al 1974) are based essentially on this effective two-dimensional approach, even though no one (except Pai-Li 1976) has previously presented the fundamental equations of the effective two-dimensional ice flow explicitly in this manner.

For $z > H$, the two-phase flow equation will be reduced to those of ocean water when the compactness is zero.

3. Derivation of the Equation of Compactness of Pack Ice (Pai-Li 1976).

The equation of continuity of the ice of the three-dimensional flow in the two-phase flow theory of ice dynamics is as follows:

$$\frac{\partial \bar{\rho}_i}{\partial t} + \frac{\partial \bar{\rho}_i u_i}{\partial x} + \frac{\partial \bar{\rho}_i v_i}{\partial y} + \frac{\partial \bar{\rho}_i w_i}{\partial z} = \sigma_i \qquad (1)$$

where $\bar{\rho}_i = \rho_i Z_i$ is the partial density of the pack ice, ρ_i is the species density of the pack ice and, Z_i, is the volume fraction of the ice in the mixture.

For two-phase flow of a mixture of ice and water, we have two different definitions of density for each species in the mixture. Let $V = V_i + V_w$ be the elementary volume of the mixture where V_i is the volume occupied by the ice and V_w is the volume occupied by the sea water. Let $M = M_i + M_w$ be the total mass of the mixture of ice and water in V where M_i is the mass of ice in V and M_w is the mass of water in V. The species density of ice is

$$\rho_i = \frac{M_i}{V_i} = \rho_i\,(T_i,\ p,\ S_i,\ \varepsilon\,) \qquad (2)$$

where ρ_i is in general a function of ice temperature T_i, ice pressure p_i, ice salinity S_i, and ice porosity ε.

The partial density of ice $\bar{\rho}_i$ in the mixture which is the value of density used in the two-phase flow theory (Pai, 1971), i.e., the Eq. (1) is

$$\bar{\rho}_i = \frac{M_i}{V} = \frac{M_i}{V_i} \cdot \frac{V_i}{V} = \rho_i Z_i \qquad (3)$$

where

$$Z_i = \frac{V_i}{V} = \text{volume fraction of ice in the mixture.} \qquad (4)$$

Similarly, we have species density of sea water and partial density of sea water in the mixture.

The source term of ice, σ_i, depends on the mass transfer by thermal and mechanical processes.

Now we integrate Eq. (1) with respect to z from z = O to z = H(x,y,t) and have the following equation

$$\int_0^H \frac{\partial Z_i \rho_i}{\partial t} dz + \int_0^H \frac{\partial Z_i \rho_i u_i}{\partial x} dz + \int_0^H \frac{\partial Z_i \rho_i v_i}{\partial y} dz + z_i \rho_i w_i \Big|_0^H = \int_0^H \sigma_i dz. \qquad (5)$$

Since the ice remains on the top layer of the ocean, we may assume that at z = O and z = H(x,y,t), w_i = O, or if we consider freezing and melting of ice, the rate of increase or decrease at the surface z = O and z = H may be assumed approximately equal. As a result, the fourth term on the left-hand side of Eq. (5) vanishes. We have also the following relations

$$\frac{\partial}{\partial t}\int_0^H Z_i \rho_i dz = Z_i \rho_i \frac{\partial H}{\partial t} + \int_0^H \frac{\partial Z_i \rho_i}{\partial t} dz \qquad (6)$$

and similar relations for the second and the third terms on the left-hand side of Eq. (5). Furthermore, since H(x,y,t) is much smaller with respect to the horizontal dimension L, we may assume that the variation of ρ_i, Z_i, u_i, and v_i with respect to z are negligibly small and we may replace their value by average values of these variables between the limits z = O and z = H at any given point x and y and time t. Hence we have the following relations

$$\int_0^H \rho_i Z_i dz = HN \rho_i \qquad (7a)$$

$$\int_0^H \rho_i Z_i u_i dz = HN \rho_i u_i \qquad (7b)$$

$$\int_0^H \rho_i Z_i v_i dz \qquad (7c)$$

$$\int_0^H \sigma_i dz = H\sigma_i = E_i \tag{7d}$$

where N is the compactness of the pack ice which is the average value of volume fraction of the pack ice between the limits z = O to z = H at any point on the surface of the ocean (x, y) and at a given time t. All the average variables u_i, v_i, ρ_i, N and σ_i (or E_i) are in general functions of x, y and t only.

Substituting Eqs. (6) and (7) in (5), we have the equation of continuity of pack-ice in polar ocean as follows:

$$\frac{\partial N\rho_i H}{\partial t} + \frac{\partial N\rho_i u_i H}{\partial x} = \frac{\partial N\rho_i v_i H}{\partial y} = N\rho_i \frac{D_i H}{Dt} + E_i \tag{8}$$

where

$$\frac{D_i}{Dt} = \frac{\partial}{\partial t} + \frac{\partial}{u_i \partial x} + v_i \frac{\partial}{\partial y} \tag{9}$$

Equation (8) may be written as an equation for compactness N as follows

$$\rho_i H \frac{D_i N}{Dt} = -N\rho_i H\left(\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y}\right) - NH\frac{D_i \rho_i}{Dt} + E_i \tag{10}$$

or

$$\rho_i \frac{D_i N}{Dt} = -N\rho_i\left(\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y}\right) - N\frac{D_i \rho_i}{Dt} + \sigma_i \tag{11}$$

It is interesting to notice from Eq. (11) that the compactness equation is independent of the thickness of the ice H(x, y, t). Such a result is consistent with our assumption that L $\gg$ H. This result is essential in that the pack ice may be considered as a two-dimensional continuum.

4. Comparison of our Equation of Compactness of Pack Ice With Similar Equations in the Literature

It is interesting to compare our equation of compactness of pack ice N, (10) or (11) with those equations of compactness in the literature.

Rothrock (1970) gave the following equation of compactness [Eq. (4) of Rochrock, 1970]:

$$\rho_i N \frac{D_i N}{Dt} = -N\rho_i H\left(\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y}\right) - N\rho_i H\psi + \phi_2 \tag{12}$$

The differences between (10) and (12) are tri-fold:

(i) On the left-hand sides of these equations, H in (10) replaces N of Rothrock's (12). It is evident that N in (12) of Rothrock must be a printing error because with N, the dimensions of the term are not the same as those of all the other terms in (12).

(ii) NH $D_i \rho_i/Dt$ in Eq. (10) replaces N $\rho_i H \psi$ of Eq. (12) of Rothrock who had little information about his function ψ. We have no difficulty in explaining our term NH$D_i \rho_i/Dt$.

(iii) There are some basic differences between Eqs. (10) and (12) in concept. In general, the velocities of ice (u_i and v_i) are different from those of water (u_w and v_w) in the top layer $O \leq z \leq H$. The distinction between these velocities should be noted and in Eq. (10), the use of operator D_i/Dt should be kept in mind. In Rothrock's analysis (1970), such a distinction had not been mentioned. Rothrock separated the source term into two parts and in (12), only one part of the source term, i.e., the rate of formation of ice by freezing open water is included. In Rothrock's analysis, if one makes the distinction between freezing at the surface $z = O$ and freezing at $O < z < H$, the variation with z of all variables should be considered and one cannot use the effective two-dimensional approach. For effective two-dimensional approach, the source term should be the average total source involving the mixture of ice and water at the point (x,y) and time t. Hence our $E_i = H \sigma_i$ is different from Rothrock's ϕ_2.

Various Russian authors used simplified equations for compactness N. We are going to compare their simplified equations with our exact equation of compactness (8), (10), or (11).

Drogaitsev (1956) considered the conservation of the mass of ice per unit area and obtained the following equation of compactness for two dimensional ice dynamics:

$$\frac{\partial N \rho_i H}{\partial t} + \frac{\partial N \rho_i H u_i}{\partial x} + \frac{\partial N \rho_i H v_i}{\partial y} = 0 \qquad (13)$$

Comparing Eq. (13) with our exact equation (8), we see that Drogaitsev neglected the source term E_i. Furthermore, he missed the important term due to the variation of the thickness of ice H. This is the main difference between the effective two-dimensional treatment of ice dynamics and the ordinary two-dimensional approach obtained by simply dropping the variation with z-coordinate. For instance, if we drop the z-variation in the three-dimensional equations of motion, we will never get the important terms due to air and water stresses on the surface of the ice. Thus such a simple equation of continuity (13) is not consistent with the correct effective two-dimensional equations of motion (15) and (16). Therefore (13) is correct only if we neglect the source term and if we assume that the thickness of ice H is a constant.

Nikiforov (1957) and Doronin (1970) used the following equation for compactness:

$$\frac{\partial N}{\partial t} = -\left(\frac{\partial N u_i}{\partial x} + \frac{\partial N v_i}{\partial y}\right) \qquad (14)$$

If we compare Eq. (14) with (11), we see that both Nikiforov and Doronin neglected the source term σ_i and the variation of the species density of ice. Hence (14) is valid only if we assume that there is no source term and that the variation of species density of ice ρ_i is negligible.

5. Effect of Compactness of Ice on Ice Drift

In studying the effect of compactness of ice on ice drift, we should solve the equation of compactness together with the equations of motion of ice and the equations of motion of air above the ice layer and those of water under the ice as shown in references by Fel'zenbaum (1958) and Campbell (1965). The effective two-dimensional equation of motion of ice according to Pai-Li model (1976) including the compactness of ice are as follows:

$$\frac{D_i u_i}{Dt} - f v_i = -\frac{1}{\rho_i}\frac{\partial p}{\partial x} + \frac{1}{N\rho_i}\left(\frac{\partial \tau_{ixx}}{\partial x} + \frac{\partial \tau_{ixy}}{\partial y}\right) - \frac{\tau_{ixzo}}{\rho_i NH} + \frac{\tau_{ixzH}}{\rho_i NH} + \frac{K(N)}{N\rho_i}(u_i - u_w) \qquad (15)$$

$$\frac{D_i v_i}{Dt} + f u_i = -\frac{1}{\rho_i}\frac{\partial p}{\partial y} + \frac{1}{N\rho_i}\left(\frac{\partial \tau_{ixy}}{\partial x} + \frac{\partial \tau_{iyy}}{\partial y}\right) - \frac{\tau_{ixzo}}{\rho_i NH} + \frac{\tau_{iyzH}}{\rho_i NH} + \frac{K(N)}{N\rho_i}(v_i - v_w) \qquad (16)$$

where $f = 2\Omega \sin\phi$ = coriolis force factor, ϕ is the latitude, and Ω is the angular velocity of the Earth. The terms τ_{ixzo} and τ_{iyzo} are respectively the xz and yz wind stresses on the top of the surface of ice floes, z = O and the terms τ_{ixzH} and τ_{iyzH} are respectively the xz and yz water stresses on the lower surface of the ice floes z = H.

There are two types of interaction forces in (15) and (16): (i) One is the friction force due to the difference in velocities between ice and water i.e., slip velocity $(u_i - u_w)$ and the other is the internal stresses of ice floes due to their random motion, τ_{ixx}, τ_{ixy}, and τ_{iyy}. We shall discuss the solution of Eqs. (10), (15) and (16) in another paper. Here we shall derive a formula for wind ice drift which shows the effect of compactness for a simplified case.

Similar to the solution of Fel'zenbaum (1958), we neglect the inertial term as well as the interaction force terms. Then the basic equations of motion are reduced to a set of linear differential equations for which the principle of superposition is applicable. In other words, as shown by Fel'zenbaum, we may divide the ice drift into two parts: One is the pure wind drift and the other is the gradient drift. As we intend to compare our results with the U. S. Naval Fleet Weather Facility ice drift model, and their model is for wind ice drift only, we shall consider only the pure wind drift caused directly by the driving effect of wind on ice in this section. In practice, the effect of permanent currents is added vectorially to the wind drift to determine the total drift of pack ice.

Similar to Fel'zenbaum's simplified solution for the modulus of the pure wind drift component u_{ia} for the deep water part of the Arctic Ocean, we have the following formula:

$$u_{ia} = \frac{\rho_a}{\rho_w} \sqrt{\frac{A_a}{A_w}} \frac{V_G}{\sqrt{1 + 2m + 2m^2}} \tag{17}$$

where ρ_a = density of air $\simeq 0.0013$ gr/cc; ρ_w = density of sea water $\simeq 1.028$ gr/cc; A_a = turbu= lent exchange coefficient of air; A_w = turbulent exchange coefficient of sea water, and V_G is the geostropic wind. Eq. (17) may be derived from Eq. (58) of Fel'senbaum's paper (1958). By comparing our equations of motion of ice with Fel'zenbaum's equations, we conclude the Eq. (17) may be derived from our equations (15) and (16) under similar approximation provided that we replace ρ_i by $N\rho_i$. In other words the parameter m for our theory should be defined as follows:

$$m = \frac{N\rho_i}{\rho_w} ah \tag{18}$$

where h is the thickness of ice and a

$$a = \sqrt{\frac{f}{2A_w}} \tag{19}$$

It is understood that in Fel'zenbaum's analysis, N = 1 is implicitly assumed.

Even though for actual prediction of ice drift we have to solve the fundamental equations of ice dynamics, we may estimate some first order effects of compactness by considering some typical values of the following quantities:

According to Fel'zenbaum (1958), we may assume

$$\frac{A_a}{A_w} \simeq 150 \tag{20}$$

and

$$\frac{\rho_i ah}{\rho_w} \simeq 1 \tag{21}$$

Substituting Eqs. (18), (20) and (21), into (17), we have

$$u_{ia} = 0.0154 \frac{V_G}{\sqrt{1 + 2N + 2N^2}} = G(N).V_G \tag{22}$$

when G(N) is the pure wind drift factor defined in Eq. (22). Equation (22) shows some essential effect of compactness on pure wind drift. The main effect of compactness is that as compactness N decreases, the pure wind drift increases.

It is interesting to compare Eq. (22) with the empirical formulas of pure wind drift as used currently by the U.S. Naval Fleet Weather Facility (FWF) sea ice division as follows:

The empirical formulas used by FWF are

$$u_{ia} = 0.02 \; V_s \cong 0.02 \; V_G \quad \text{for} < 2 \text{ knots} \tag{23a}$$

$$u_{ia} = 0.025 + 0.00772 \; V_G \quad \text{for} \; V_G \geq 2 \text{ knots} \tag{23b}$$

where V_s is the surface wind velocity.

Equation (23a) is essentially Zubov's formula (1945). The "exact" coefficient based on Zubov's and Nansen's data (Zubov 1945) is

$$\vec{u}_{ia} = 0.0155 \; \vec{V}_s = 0.0077 \; \vec{V}_G \tag{24}$$

But Zubov (1945) in his book rounded the figures and wrote

$$\vec{u}_{ia} = 0.02 \; \vec{V}_s = 0.01 \; \vec{V}_G \tag{24a}$$

where Zubov assumed $2\vec{V}_s = \vec{V}_G$. We should compare our formula (22) with the "exact" formula of Zubov (24). FWF assumed that for low speed, $\vec{V}_G < 2$ knots, $\vec{V}_s = \vec{V}_G$. Hence they have

$$\vec{u}_{ia} = 0.0155 \; \vec{V}_s \cong 0.0155 \; \vec{V}_G \cong 0.02 \; \vec{V}_G \tag{24b}$$

Equation (23b) is based on the empirical formula by Skiles (1968). The slope of the line $\vec{u}_{ia}$ vs $\vec{V}_G$ of (23b) is equal to Zubov's value. If we assume that the slip velocity $(\vec{u}_{ia} - \vec{u}_w)$ in Eqs. (15) and (16) as a constant, we may obtain the constant term in (23b) by choosing proper friction coefficient. We shall discuss this point in another paper.

From (22) we have the following values:

N =	0.1	0.3	0.5	0.7	0.8	1
G(N) =	0.0139	0.0115	0.0097	0.0084	0.0077	0.0069

Our formula (22) is identical to Zubov's formula (24) if we take $N \cong 0.8$.

Pure wind drift would be deflected from the isobaric drift to the right at an angle β given by

Fel'zenbaum [(59) of Fen'zenbaum 1958]

$$\tan \beta = \frac{1}{1 + \frac{1}{m}} = \frac{1}{1 + \frac{\rho_w}{\rho_i N h a}} \qquad (25)$$

Again, in our theory, we may replace ρ_i by $N\rho_i$ in Fel'zenbaum's formula. By the approximation (21), (25) becomes

$$\tan \beta = \frac{1}{1 + \frac{1}{N}} \qquad (26)$$

Equation (26) gives the following values

N = 0.1	0.3	0.5	0.7	0.8	1
β = 5°12'	13°	18°26'	22°22'	24°18'	26°34'

Both (22) and (26) show qualitatively the effect of compactness N on the pure wind drift and the deflection angle of pure wind drift from isobasic drift. However, because the approximation of (21), we do not expect that these formulas give exact quantitative results from field observation. For using these formula to check the field observations, we should take the actual variation of ice thickness into consideration.

6. Conclusions and Recommendations

(1) We have given the accurate equation of compactness of pack ice in Arctic Ocean based on Pai-Li model (1976) of two-phase flow theory and compare it with the simplified equations of compactness of pack ice in literature.

(2) A simple formula for pure wind ice drift with compactness effect is derived. This formula agrees well with empirical formula of Zubov and those used by U.S. Naval Fleet Weather Facility for ice drift prediction. Our formulas show qualitatively the effect of compactness on pure wind ice drift and the deflection of pure wind ice drift from isobaric direction. As the compactness decreases, the pure wind ice drift increases for a given geostropic wind and the deflection angle decreases.

(3) We shall use our fundamental equations of ice dynamics based on Pai-Li model of two-phase flow theory for the calculation of ice drift in Arctic Seas, particularly in the marginal sea area where the effect of compactness of ice would be important.

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