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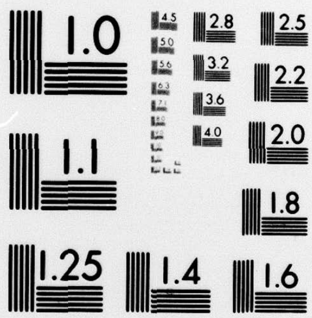

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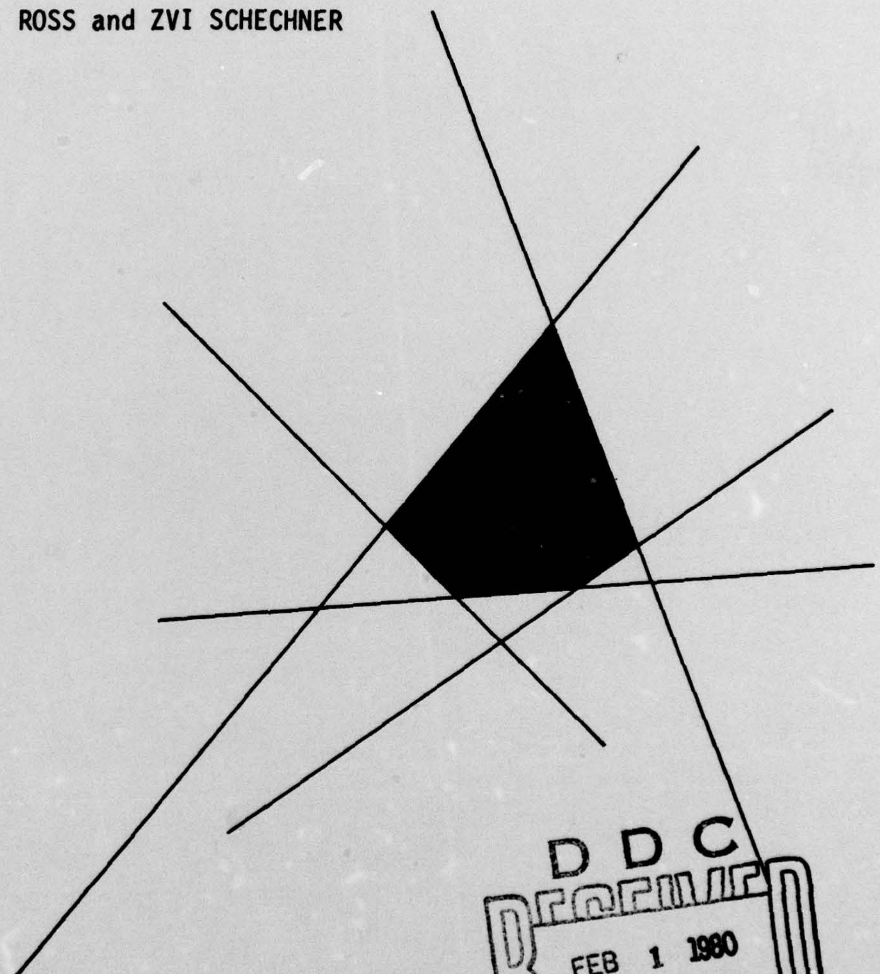
A NOTE ON FIRST PASSAGE TIMES IN BIRTH AND DEATH AND NONNEGATIVE DIFFUSION PROCESSES

by

CYRUS DERMAN, SHELDON M. ROSS and ZVI SCHECHNER



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ABSTRACT

Consider a birth and death process starting in state 0 . ~~Keilson~~ [1] ~~has shown by~~ analytical arguments* that the time of first passage into state n has an increasing failure rate (IFR) distribution. We present a probabilistic proof for this. In addition, our proof shows that for a nonegative diffusion process, the first passage time from state 0 to any state x is IFR.

*were shown earlier

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0. INTRODUCTION

Consider a birth and death process starting in state 0. Keilson [1] has shown by analytical arguments that the time of first passage into state n has an increasing failure rate (IFR) distribution. We present a probabilistic proof for this. In addition, our proof shows that for a nonnegative diffusion process, the first passage time from state 0 to any state x is IFR.

1. THE RESULT

Let $\{X_j(t) , t \geq 0\}$, $j = 1,2$ be independent and identically distributed birth and death processes. Let

$$T_j = \min \{t : X_j(t) = n\} .$$

We start by showing that, given $T_j > t$, the conditional state at time t is stochastically increasing in the initial state.

Lemma:

For $i_2 > i_1$

$$P\{X_2(t) \geq k \mid X_2(0) = i_2 , T_2 > t\}$$
$$\geq P\{X_1(t) \geq k \mid X_1(0) = i_1 , T_1 > t\} .$$

Proof:

Assume throughout the proof that all probability statements are conditional on the events $X_2(0) = i_2$, $T_2 > t$, $X_1(0) = i_1$, $T_1 > t$; and consider sample paths for the two processes conditional on these events. Define

$$T = \inf \{s : X_2(s) = X_1(s)\}$$

and let it be ∞ if $X_2(s) > X_1(s)$ for all s ; and define the process $\{X_3(s) , s \geq 0\}$ by

$$X_3(s) = \begin{cases} X_2(s) & \text{for } s < T \\ X_1(s) & \text{for } s \geq T . \end{cases}$$

Now it is easy to see from the strong Markov property that $\{X_3(s) , s \geq 0\}$ has the same distribution as $\{X_2(s) , s \geq 0\}$. As $X_3(s) \geq X_1(s)$ for all s , the result follows.||

We now show that if $X_1(0) = 0$ and $T_1 > t$, then $X_1(t)$ increases stochastically in t .

Theorem 1:

$P\{X_1(t) \geq k \mid X_1(0) = 0 , T_1 > t\}$ is increasing in t .

Proof:

For $s < t$, we have by conditioning on $X_1(t - s)$ and using the Markov property that

$$P\{X_1(t) \geq k \mid X_1(0) = 0 , T_1 > t\}$$

$$= \sum_i P\{X_1(s) \geq k \mid X_1(0) = i , T_1 > s\}P\{X_1(t-s) = i \mid X_1(0) = 0 , T_1 > t\}$$

$$\geq P\{X_1(s) \geq k \mid X_1(0) = 0 , T_1 > s\}$$

where the last inequality follows from Lemma 1.||

Recalling that a continuous random variable X having distribution F and density f is said to be IFR if its failure rate function $\lambda(t)$ defined by

$$\lambda(t) = f(t)/(1 - F(t))$$

is increasing in t. If X is the life of an item, then $\lambda(t)$ can be interpreted as the instantaneous probability density that a t year old item will fail.

Corollary 1:

If $X_1(0) = 0$ then the first passage time to state n is IFR.

Proof:

It is easy to see that

$$\lambda(t) = \theta_{n-1}P\{X_1(t) = n - 1 \mid X_1(0) = 0 , T_1 > t\}$$

where θ_{n-1} is the birth rate at state $n - 1$. The result now follows from Theorem 1.||

Remark:

Our method of proof also shows that the first passage time from state 0 to any state x is IFR for any nonnegative diffusion process (that is, for any nonnegative Markov process on the real line whose sample path is continuous with probability 1). The key thing is that as in the case of birth and death processes, if we have 2 independent and identically distributed diffusion processes which start at time 0 in different states, then either the initially larger one remains larger or they meet at some point (since their sample paths are continuous) and if they meet their probabilistic structure is identical after they meet (by the strong Markov property). The above remains true even if we are conditioning on the event that the first passage time has not occurred by time t .

REFERENCE

[1] Keilson, J., "Markov Chain Models - Rarity and Exponentiality," ORC 74-32, Operations Research Center, University of California, Berkeley, California (October 1974).