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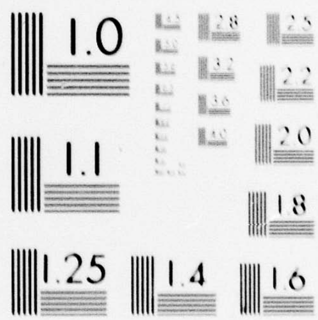

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aerospace engineering department

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CONTRACT NO. N00014-[illegible]

TECHNICAL REPORT NO. 3275-79-4

DECEMBER 15, 1979

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Contract No. N00014-76-C-0150
Task No. NR 064-534

Technical Report No. 3275-79-4

Dec[redacted]79

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David H. Allen* and Walter E. Haisler**

ABSTRACT

The authors have previously proposed a theoretical model for predicting structural response of elastic-plastic-creep materials subjected to variable temperature loadings. This model utilizes the combined isotropic-kinematic hardening rule in conjunction with the classical rate independent plasticity theory. The resulting constitutive law has been implemented into the finite strain finite element code AGGIE I. The authors have found it necessary to alter the formulation previously presented in order to obtain a concise numerical formulation. That modification is presented in this paper. In addition, a section is included which outlines supplementary information necessary for computer program input, followed by a computational procedure which is intended to aid programmers who wish to use this model. Finally, the results of several sample cases are presented.

INTRODUCTION

In our publication entitled "The Application of Thermal and Creep Effects to the Combined Isotropic-Kinematic Hardening Model for Inelastic

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Structural Analysis by the Finite Element Method"[1], we presented an infinitesimal strain constitutive law intended for use with elastic-plastic materials subjected to thermal loadings. The model accounted for time dependent behavior through an uncoupled creep term. The resulting law was utilized in a finite element formulation which uses a total Lagrangian formulation, and the resulting equations of motion were presented at that time.

The impetus for this new law resulted from a survey of the literature on temperature dependent constitutive laws. It was determined that most finite element programs available which account for thermal effects utilize either isotropic or kinematic hardening and thus may be inadequate for modelling the Bauschinger effect during cyclic loading. This statement has been previously verified for isothermal cases by Haisler et al.[5]

We found subsequent to our first publication that a similar model had been previously proposed by Yamada[16,17]. However, his model contains significant variations from that proposed by us. Specifically, he does not account for elastic modulus changes due to temperature, and his model is constructed within an Eulerian framework. More importantly, his determination of the hardening modulus departs significantly from that presented herein. This latter point will be elaborated upon later in this report.

During implementation of our model into the finite element code AGGIE I[6], we found it necessary to make a significant modification to the model in order to obtain computational results. This alteration as well as a short review of the model will be presented in the following section.

An added purpose of this publication is to present the supplementary information necessary to input the model to a computer program. Thus, we have included a section detailing these additions, as well as a computational procedure intended to clarify the model for those who wish to pursue the topic in greater depth.

In the final section the results of several sample problems are presented as a verification of the completed computer program.

REVIEW AND MODIFICATION OF THE THEORY

Recall that in the incremental theory of plasticity, the work-hardening rule is defined by a yield function in stress space, which for temperature dependent combined isotropic-kinematic hardening is

$$F(S_{ij} - \alpha_{ij}) = k^2 \left(\int d\bar{\varepsilon}^P, T \right), \tag{1}$$

where

S_{ij} = second Piola-Kirchhoff stress tensor,

α_{ij} = coordinates of the yield surface center in stress space,

$d\bar{\varepsilon}^P_{ij}$ = uniaxial plastic strain increment,

T = temperature,

and $\int d\bar{\varepsilon}^P$ represents the plastic strain history dependence in the yield function. Note that the explicit temperature dependence on the right hand side of equation (1) precludes kinematic hardening due to temperature changes. A schematic representation of equation (1) is shown in Figure 1.

Differentiating equation (1) gives the following consistency condition during plastic loading:

$$\frac{\partial F}{\partial S_{ij}} dS_{ij} - \frac{\partial F}{\partial S_{ij}} d\alpha_{ij} - 2k \frac{\partial k}{\partial \bar{\varepsilon}^P} d\bar{\varepsilon}^P - 2k\frac{\partial k}{\partial T} dT = 0, \tag{2}$$

where the term $\frac{\partial F}{\partial S_{ij}}$ represents $\frac{\partial F(S_{ij} - \alpha_{ij})}{\partial S_{ij}}$ evaluated at $S_{ij} - \alpha_{ij}$ which can be seen to be equivalent to $\frac{\partial F}{(\partial S_{ij} - \alpha_{ij})}$.

Since during neutral loading the plastic strain increment and $d\alpha_{ij}$ are zero, it is apparent that a statement governing loading is

$$\frac{\partial F}{\partial S_{ij}} dS_{ij} - 2k\frac{\partial k}{\partial T} dT > 0. \tag{3}$$

In addition to the statement of consistency during loading given by (2), the following associated flow rule is employed:

$$dE^P_{ij} = d\lambda \frac{\partial F}{\partial S_{ij}} . \tag{4}$$

The stress at any time is assumed to be given by

$$S_{ij} = D^t_{ijmn} (E_{mn} - E^P_{mn} - E^C_{mn} - E^T_{mn}), \tag{5}$$

where D^t_{ijmn} is the elastic constitutive tensor at time t. Incrementation of equation (5) was shown in our previous paper[1] to give

$$dS_{ij} = D^{t+\Delta t}_{ijmn} (dE_{mn} - dE^P_{mn} - dE^C_{mn} - dE^T_{mn})$$

$$+ dD_{ijmn}(E^t_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}), \tag{6}$$

where

$D^{t+\Delta t}_{ijmn}$ = elastic modulus tensor at time t+Δt,

dD_{ijmn} = change in the elastic constitutive tensor (due to a temperature change) during time step Δt,

E_{mn} = total strain tensor,

E^P_{mn} = plastic strain tensor,

E^C_{mn} = creep strain tensor,

E^T_{mn} = thermal strain tensor,

and all superscripts t indicate measurement at time t. The superscript $t+\Delta t$ on the elastic constitutive tensor is necessary due to the fact that the time step is finite rather than infinitesimal.

The final expression required to complete the constitutive law is the hardening rule. According to Ziegler's modification a tensorially correct statement is

$$d\alpha_{ij} = d\mu(S_{ij} - \alpha_{ij}), \tag{7}$$

where μ is a scalar to be determined by the consistency condition [equation (2)].

The purpose of the above equations is to obtain a mapping giving the total stress increment in terms of the total strain increment and known quantities. To accomplish this it is necessary to solve equations (2), (4), (6) and (7) to obtain the stress increment tensor. Unfortunately, these comprise nineteen equations in twenty unknowns: dS_{ij}, dE^P_{ij}, $d\alpha_{ij}$, $d\lambda$, and $d\mu$. One would normally consider this problem underspecified, but it will turn out that $d\lambda$ is an artificial parameter which need not be determined in order to obtain a solution. Incidentally, it is possible to obtain $d\lambda$ after the constitutive law is solved by determining the plastic strain increment.

The constitutive law is thus obtained by substituting equation

(4) into equations (2) and (6) and solving for the stress increment tensor. The resulting equation is

$$dS_{ij} = \left(D^{t+\Delta t}_{ijmn} - \frac{D^{t+\Delta t}_{ijvw} \frac{\partial F}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} D^{t+\Delta t}_{tumn}}{\frac{\partial F}{\partial S_{tu}} \frac{d\alpha_{tu}}{d\lambda} + 2k \frac{\partial k}{\partial \bar{\varepsilon}^{P}} \frac{d\bar{\varepsilon}^{P}}{d\lambda} + 2k \frac{\partial k}{\partial T} \frac{dT}{d\lambda} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) x$$

$$(dE_{mn} - dE^{C}_{mn} - dE^{T}_{mn})$$

$$- \left(\frac{D^{t+\Delta t}_{ijwv} \frac{\partial F}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} dD_{tumn}}{\frac{\partial F}{\partial S_{tu}} \frac{d\alpha_{tu}}{d\lambda} + 2k \frac{\partial k}{\partial \bar{\varepsilon}^{P}} \frac{d\bar{\varepsilon}^{P}}{d\lambda} + 2k \frac{\partial k}{\partial T} \frac{dT}{d\lambda} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) x$$

$$(E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn})$$

$$+ dD_{ijmn}(E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \quad . \tag{8}$$

The above constitutive law differs from that obtained by Yamada[16,17] in that the term containing dT is written as a separate portion of the stress increment in the latter's derivation. It will be shown, however, that the two are mathematically equivalent. It should also be noted that the above derivation differs slightly from that proposed in our previous paper. We have chosen this new form due to its clarity, although either formulation is mathematically correct. Unfortunately, in the form presented in equation (8) it is not possible to determine the stress increment due to the occurrence of several undetermined parameters on the right hand side. Yamada has accounted for these terms by introducing the plastic work rate. We have chosen a slightly different method for determining the effect of these unknowns.

As presented in a previous paper by us[8], we assume that there exists a scaler parameter c, called the hardening modulus, which when multiplied by the plastic strain increment and subtracted from the stress increment will be parallel to a tangent to the yield surface. Mathematically, this may be stated as

$$(dS_{ij} - c\,dE^{P}_{ij})\,\frac{\partial F}{\partial S_{ij}} = 0 \ . \tag{9}$$

It can be seen from an examination of the consistency condition [equation (2)], that in order for the above statement to be valid, the following must hold:

$$c\,dE^{P}_{ij}\,\frac{\partial F}{\partial S_{ij}} = \frac{\partial F}{\partial S_{ij}}\,d\alpha_{ij} + 2k\frac{\partial k}{\partial \bar{\varepsilon}^{P}}\,d\bar{\varepsilon}^{P} + 2k\frac{\partial k}{\partial T}\,dT \ . \tag{10}$$

If one employs the normality condition in equation (10) it is found that

$$c\frac{\partial F}{\partial S_{pq}}\,\frac{\partial F}{\partial S_{pq}} = \frac{\partial F}{\partial S_{tu}}\,\frac{d\alpha_{tu}}{d\lambda} + 2k\frac{\partial k}{\partial \bar{\varepsilon}^{P}}\,\frac{d\bar{\varepsilon}^{P}}{d\lambda} + 2k\frac{\partial k}{\partial T}\,\frac{dT}{d\lambda} \ , \tag{11}$$

which when substituted into the constitutive relation [equation (8)] results in

$$dS_{ij} = \left(D^{t+\Delta t}_{ijmn} - \frac{D^{t+\Delta t}_{ijmn}\,\frac{\partial F}{\partial S_{vw}}\,\frac{\partial F}{\partial S_{tu}}\,D^{t+\Delta t}_{tumn}}{c\frac{\partial F}{\partial S_{pq}}\,\frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs}\,\frac{\partial F}{\partial S_{pq}}\,\frac{\partial F}{\partial S_{rs}}} \right)(dE_{mn} - dE^{C}_{mn} - dE^{T}_{mn})$$

$$- \left(\frac{D^{t+\Delta t}_{ijvw}\,\frac{\partial F}{\partial S_{vw}}\,\frac{\partial F}{\partial S_{tu}}\,dD_{tumn}}{c\frac{\partial F}{\partial S_{pq}}\,\frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs}\,\frac{\partial F}{\partial S_{pq}}\,\frac{\partial F}{\partial S_{rs}}} \right)(E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn})$$

$$+ dD_{ijmn}(E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn}) \ . \tag{12}$$

The simplicity of the above formulation can be seen when one uses the normality condition in conjunction with equation (9) to obtain

$$c = \frac{\frac{\partial F}{\partial S_{ij}} dS_{ij}}{\frac{\partial F}{\partial S_{ij}} dE^{P}_{ij}} = \frac{2}{3} \frac{d\sigma_x}{d\bar{\varepsilon}^{P}} \quad . \tag{13}$$

Thus, for isothermal loading it can be seen that the hardening modulus is simply two thirds the instantaneous slope of the uniaxial stress-plastic strain diagram. However, for non-isothermal loadings this is not the case. To see this note that since the uniaxial stress is a function of both the plastic strain history and temperature, an increment of uniaxial stress is given by

$$d\sigma = \frac{\partial \sigma}{\partial \bar{\varepsilon}^{P}} d\bar{\varepsilon}^{P} + \frac{\partial \sigma}{\partial T} dT \quad . \tag{14}$$

Combining equations (13) and (14) gives

$$c = \frac{2}{3} \left(H' + \frac{\partial \sigma}{\partial T} \frac{dT}{d\bar{\varepsilon}^{P}}\right) \quad , \tag{15}$$

where H' is the instantaneous slope of the stress-plastic strain diagram. Substituting the above equation into the constitutive relation [equation (12)] , then rearranging and employing the normality condition, gives

$$dS_{ij} = C^{t}_{ijmn}(dE_{mn} - dE^{C}_{mn} - dE^{T}_{mn}) + dP_{ij} \quad , \tag{16}$$

where

$$C^{t}_{ijmn} = D^{t+\Delta t}_{ijmn} - \frac{D^{t+\Delta t}_{ijvw} \frac{\partial F}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} D^{t+\Delta t}_{tumn}}{\frac{2}{3} H' \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \quad , \tag{17}$$

and

$$dP_{ij} = -\left(\frac{D^{t+\Delta t}_{ijvw} \frac{\partial F}{\partial S_{vw}} \frac{\partial F}{\partial S_{tu}} dD_{tumn}}{\frac{2}{3} H' \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) \times$$

$$(E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn})$$

$$+ dD_{ijmn}(E^{t}_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn})$$

$$+\left(\frac{\sqrt{\frac{2}{3} \frac{\partial F}{\partial S_{tu}} \frac{\partial F}{\partial S_{tu}}}\; D^{t+\Delta t}_{ijmn} \frac{\partial F}{\partial S_{mn}} \frac{\partial \sigma}{\partial T}}{\frac{2}{3} H' \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{pq}} + D^{t+\Delta t}_{pqrs} \frac{\partial F}{\partial S_{pq}} \frac{\partial F}{\partial S_{rs}}} \right) dT \quad , \qquad (18)$$

and the terms H' and $\frac{\partial \sigma}{\partial T}$ may be determined from uniaxial stress-strain data at time t as described later. Thus, it can be seen that **c** is superscripted at time t since the hardening parameters are determined there. Due to its length the derivation of the above constitutive equation is not covered in detail here, but may be obtained from the authors on request.

The translation of the yield surface in stress space may now be obtained by substituting equation (7) into equation (2) and solving for $d\mu$. The resulting relation is

$$d\mu = \frac{\frac{\partial F}{\partial S_{ij}} dS_{ij} - 2k\frac{\partial k}{\partial T} dT - 2k\frac{\partial k}{\partial \bar{\varepsilon}^{P}} d\bar{\varepsilon}^{P}}{(S_{mn} - \alpha_{mn}) \frac{\partial F}{\partial S_{mn}}} \quad . \qquad (19)$$

Equation (19) thus guarantees that the state of stress will remain consistent with the yield surface during loading even under non-isothermal conditions.

To summarize, then, the constitutive law is obtained by solving, in the following order, equations (18), (17), (16), (19) and (7).

An application of the constitutive law to the principle of virtual work within a total Lagrangian framework yields the following matrix algebra equations:

$$[M] \left\{ \ddot{u}_\alpha^{t+\Delta t} \right\} + ([K_L]^t + [K_{NL}]^t) \left\{ \Delta u_\alpha \right\} = \left\{ R^{t+\Delta t} \right\} - \left\{ F^t \right\} , \qquad (20)$$

where the quantities in the above equation are as defined in reference 1. Solving the above equations and employing equilibrium iteration gives the displacement increment for a given load step.

APPLICATION OF THE THEORY TO THE COMPUTER CODE AGGIE I

Although the above model is mathematically correct, certain additions must be made in order to obtain a computational form. In this section are presented those additional requirements which were necessary to input the model to the code AGGIE I. It should be emphasized, however, that these additions are not essential to the model itself, and, as such, variations can be made without affecting the validity of the model.

A. The Yield Function

A function describing the onset of permanent deformation must be utilized to describe equation (1). The yield function utilized in AGGIE I is the Von Mises yield criterion for isotropic materials, which is given by

$$F(\sigma_i) = \frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] = k^2 \quad , \tag{21}$$

where σ_1,σ_2 and σ_3 are principal stresses and k is the current uniaxial yield surface size.

B. Creep Strain Calculation

In this theory the creep strain is considered to be an initial strain which can be uncoupled from the instantaneous plastic strain. There are currently three methods of calculating the creep strain in the program AGGIE I. One is the use of empirical equations from microphenomenological materials science literature. The second consists of interpolating a set of creep strain versus time curves at varying stress levels. The final method makes use of viscoelasticity theory. Because of the complexity of these calculations, further discussion is omitted from this paper. The reader is referred to references (13) and (14) for a more detailed discussion.

C. Determination of Thermal Strain

For the form of the constitutive law given by equation (16) it can be shown that the thermal strain for isotropic materials can be approximated by

$$E_{ij}^{T} = \delta_{ij} \int_{T_R}^{T} \alpha_T dT \quad , \tag{22}$$

where α_T is a temperature dependent material properly, T_R is the reference temperature and δ_{ij} is the Kroneker delta. In metals literature it is customary to define

$$\alpha = \frac{1}{(T - T_R)} \int_{T_R}^{T} \alpha_T dT \quad , \tag{23}$$

so that

$$E_{ij}^{T} = \delta_{ij}\alpha(T - T_R) \quad . \tag{24}$$

It can be seen that whenever α_T is independent of temperature α_T and α are identical. Since both α_T and α are reported in various materials handbooks, the option to use either is specified in the code AGGIE I. When α_T is employed the thermal strain is calculated by integrating equation (22), and when α is chosen the thermal strain is determined by solving equation (24).

D. Material Input Data

In order to facilitate easy input, the materials data are input as piecewise linear curves. The necessary data are the uniaxial stress-strain curves for various temperatures, a coefficient of thermal expansion versus temperature curve, and a Poisson's ratio versus temperature curve. These data are shown schematically in Figure 2.

E. Calculation of Equivalent Uniaxial Yield Surface Size and Gradients.

We have found it convenient to construct a set of k vs. $\bar{\epsilon}^P$ curves as shown in Figure 3 from the input stress-strain data shown in Figure 2. This is accomplished by a transformation on both the uniaxial stress and strain. The ith abscissa may be obtained from

$$\bar{\epsilon}_i^P = \epsilon_{xi} - \sigma_{xi}/E_j \quad , \tag{25}$$

where E_j is the uniaxial elastic modulus of a stress-strain curve at T_j. The ith ordinate is determined using

$$k_i = \bar{\sigma}_{xj} + \beta \cdot (\sigma_{xi} - \bar{\sigma}_{xj}) \quad , \tag{26}$$

where $\bar{\sigma}_{xj}$ is the initial yield stress at temperature T_j and β is the ratio of isotropic to kinematic hardening. β may be obtained from a uniaxial cyclic loading test. Utilizing equations (25) and (26) will allow one to construct the data shown in Figure 3.

In order to obtain the current value of k it is first necessary to determine the uniaxial plastic strain, which is given by

$$\bar{\epsilon}^P = \sqrt{\frac{2}{3} E_{ij}^P E_{ij}^P} \quad . \tag{27}$$

Thus, for the current temperature and uniaxial plastic strain level it is possible to obtain k, as shown in Figure 4. A linear interpolation gives

$$k = k_H - (k_H - k_L)\frac{(T_H-T)}{(T_H-T_L)}\quad , \tag{28}$$

where T is the current temperature. In addition, it is possible to obtain the yield surface temperature gradient, which is

$$\frac{\partial k}{\partial T} = \frac{k_H - K_L}{T_H - T_L}\quad . \tag{29}$$

It should be noted that $\frac{\partial \sigma}{\partial T}$ may be obtained in a similar fashion from a stress vs uniaxial plastic strain curve. Also, one may estimate $\partial k/\partial \bar{\varepsilon}^P$ by

$$\frac{\partial k}{\partial \bar{\varepsilon}^P} = \left(\frac{\partial k}{\partial \bar{\varepsilon}^P}\right)_H - \left[\left(\frac{\partial k}{\partial \bar{\varepsilon}^P}\right)_H - \left(\frac{\partial k}{\partial \bar{\varepsilon}^P}\right)_L\right]\frac{(T_H-T)}{(T_H-T_L)}\quad , \tag{30}$$

and H' may be determined similarly from a stress vs uniaxial plastic strain curve.

F. Elastic Strain Increment

The elastic strain increment is obtained by solving equation (6):

$$dE^E_{mn} = DI^{t+\Delta t}_{ijmn}\,[dS_{ij} - dD_{ijmn}\,(E^t_{mn} - E^{Pt}_{mn} - E^{Ct}_{mn} - E^{Tt}_{mn})]\quad , \tag{31}$$

where $DI^{t+\Delta t}_{ijmn}$ represents the elastic compliance tensor at time t+Δt.

COMPUTATIONAL PROCEDURE

Using the above modifications and extensions of the theoretical model it is possible to establish a computational procedure which when programmed will completely define the constitutive law.

Below is the computational procedure for non-isothermal combined isotropic-kinematic hardening used by AGGIE I. The procedure is used in an equilibrium iteration loop and is therefore performed several times for each load step. The first time through the procedure during each load step the stresses are calculated for the previous load step in A. through O. The materials data are then updated to the current step in P. through X. These data are then used in another routine to establish the total strain increment. On each succeeding iteration the stresses are calculated for the current step in A. through O. and steps P. through X. are not performed. The program variables used in this outline are defined in a table following the procedure.

A. If this is the first load step and the first iteration on that load step (KSTEP. EQ. 1. AND. ICOUNT. NE. 3) calculate the initial temperature change.

B. Obtain change in elastic constitutive tensor due to temperature change during current step.

1. Interpolate input materials data to obtain elastic modulus, Poisson's ratio, and coefficient of thermal expansion for temperature at start of current step.

2. Construct elastic constitutive tensor for temperature at start of current step.

3. Interpolate input materials data to obtain elastic modulus and Poisson's ratio for temperature at end of current step.

4. Construct elastic constitutive tensor for temperature at end of current step.

5. Subtract elastic constitutive tensor at start of step from elastic constitutive tensor at end of step to obtain change in elastic constitutive tensor during current step.

C. Calculate the total strain increment for current step.

D. Calculate thermal strain increment for current step.

1. If IALPHA = 1 the coefficient of thermal expansion is a function of temperature [equation (22)].

2. If IALPHA = 0 the coefficient of thermal expansion is not a function of temperature [equation (24)].

E. Obtain initial estimate of elastic strain increment.

1. Subtract creep strain increment from total strain increment.

2. Subtract thermal strain increment from total strain increment.

3. Initialize elastic strain increment to elastic plus plastic strain increment.

F. Calculate a trial stress increment assuming the total strain increment to be purely elastic [equation (6)].

G. Calculate trial stress state.

H. Check whether trial stress state causes yielding [equation (1)].

If $F>k^2$ and IPEL = 1 go to I.

If $F>k^2$ and IPEL = 2 go to K.

If $F = k^2$ and IPEL = 1 set IPEL = 2 and go to N.

If $F = k^2$ and IPEL = 2 go to K.

If $F<k^2$ go to N.

I. The material was behaving elastically at the beginning of this time step, but will yield during this strain increment. Determine the part of the strain increment taken elastically. Set IPEL = 2 and IELAS = 2.

J. Update stress state to yield surface.

K. The material is now behaving plastically.

1. Determine the number of subincrements, M.

2. Determine the strain subincrement.

3. Determine the temperature subincrement.

4. Interpolate materials data to obtain elastic modulus and Poisson's ratio at end of subincrement.

5. Construct elastic constitutive tensor at end of subincrement.

6. Subtract elastic constitutive tensor at start of subincrement from elastic constitutive tensor at end of subincrement to obtain change in elastic constitutive tensor during subincrement.

L. Calculate the stress tensor minus the yield surface center.

M. Obtain the plastic stress increment by looping through this section M times.

1. Update elastic constitutive tensor to end of subincrement.

2. Calculate $\partial F/\partial S_{ij}$.

3. Calculate H', $\partial\sigma/\partial T$, $\partial k/\partial\bar{\varepsilon}^P$ and $\partial k/\partial T$ for current value of $\bar{\varepsilon}^P$.

4. Calculate effective modulus tensor C_{ijmn} [equation (17)].

5. Calculate dP_{ij} for current subincrement [equation (18)].

6. Calculate stress increment for current subincrement [equation (16)].

7. Calculate the elastic strain subincrement [equation (31)].

8. Subtract the elastic strain subincrement from the total strain subincrement to obtain the plastic strain subincrement.

9. Update total elastic strain increment for current step.

10. Update equivalent plastic strain [equation (27)].

11. Determine the size of the subsequent yield surface [equation (28)].

12. Calculate the new position of the yield surface center [equations (19) and (7)].

13. Update temperature to end of subincrement.

14. Calculate stress tensor minus yield surface center.

N. Update stresses, strains and elastic strains to end of step.

O. Check for equilibrium iteration. If equilibrium iterating, the procedure is complete. If not, continue.

P. Calculate the temperature change and temperature increment for this step.

Q. Obtain the change in elastic constitutive tensor due to temperature change during current step.

1. Interpolate input materials data to obtain elastic modulus, Poisson's ratio, and coefficient of thermal expansion for temperature at start of current step.

2. Construct elastic constitutive tensor for temperature at start of current step.

3. Interpolate input materials data to obtain elastic modulus and Poisson's ratio for temperature at end of current step.

4. Construct elastic constitutive tensor for temperature at end of current step.

5. Subtract elastic constitutive tensor at start of step from elastic constitutive tensor at end of step to obtain change in elastic constitutive tensor during current step.

6. Update elastic constitutive tensor to end of current step.

R. Calculate thermal strain increment for current step.

1. If IALPHA = 1 the coefficient of thermal expansion is a function of temperature [equation (22)].

2. If IALPHA = 0 the coefficient of thermal expansion is not a function of temperature [equation (24)].

S. Calculate creep strain increment for current step.

T. Check for yielding:

If IPEL = 1 go to V.

If IPEL = 2 go to U.

U. Calculate effective constitutive tensor for plastic action.

1. Find $\partial F/\partial S_{ij}$.

2. Determine the stress increment for the current step.

3. Find H', $\partial\sigma/\partial T$, $\partial k/\partial\bar{\varepsilon}^P$, and $\partial k/\partial T$.

4. Determine effective constitutive tensor [equation (17)].

V. Calculate new yield surface size. If IPEL = 2 go to X.

W. Set effective constitutive tensor equal to elastic constitutive tensor for elastic action.

X. Find dP_{ij} for current step [equation (18)].

PROGRAM VARIABLES

KSTEP - load step number

ICOUNT - equilibrium iteration flag

EQ.3 - equilibrium iteration

NE.3 - not equilibrium iteration

IELAS - first yield control flag

EQ.1 - material has not yielded

EQ.2 - material has yielded

IALPHA - thermal strain type flag

EQ.1 - constant alpha

EQ.2 - variable alpha

IPEL - elastic-plastic control flag

EQ.1 - material is elastic on previous step

EQ.2 - material is plastic on previous step

APPLICATION OF THE MODEL TO SPECIAL CASES

In order to provide a better understanding of the model several sample cases are considered in this section. The purpose of these cases is both to verify the accuracy of the constitutive law and to provide several economical test cases which may be used to debug the model when it is input to a finite element code. The calculations are simplified by modelling the response of uniaxial bars under small strain conditions and subjected to quasi-static loadings. The stress tensor S_{ij} is therefore replaced with the uniaxial stress σ, and the strain tensor E_{ij} is replaced with the uniaxial strain ε. Also, the elastic constitutive tensor D_{ijmn} is replaced with the elastic modulus E, and the yield surface translation tensor α_{ij} becomes α. For simplicity, there is assumed to be no creep.

The stress increment is first predicted by assuming zero plastic strain and solving equation (6):

$$d\sigma = E^{t+\Delta t}(d\varepsilon - d\varepsilon^{T}) + (E^{t+\Delta t} - E^{t})(\varepsilon^{t} - \varepsilon^{Pt} - \varepsilon^{Tt}) \quad . \qquad (32)$$

If the loading condition is satisfied [equation (1) or (3)], then the stress increment is updated by using equations (16) through (18) which reduce to

$$d\sigma = \left(E^{t+\Delta t} - \frac{E^{t+\Delta t} \frac{\partial F}{\partial \sigma} \frac{\partial F}{\partial \sigma} E^{t+\Delta t}}{H' \frac{\partial F}{\partial \sigma} \frac{\partial F}{\partial \sigma} E^{t+\Delta t}} \right)(d\varepsilon - d\varepsilon^{T})$$

$$-\left(\frac{E^{t+\Delta t}\frac{\partial F}{\partial \sigma}\frac{\partial F}{\partial \sigma}\, dE}{H'\frac{\partial F}{\partial \sigma}\frac{\partial F}{\partial \sigma} + E^{t+\Delta t}\frac{\partial F}{\partial \sigma}\frac{\partial F}{\partial \sigma}}\right)(\varepsilon^{t} - \varepsilon^{Pt} - \varepsilon^{Tt})$$

$$+ dE(\varepsilon^{t} - \varepsilon^{Pt} - \varepsilon^{Tt}) + \left(\frac{\frac{\partial F}{\partial \sigma} E^{t+\Delta t}\frac{\partial F}{\partial \sigma}\frac{\partial \sigma}{\partial T}}{H'\frac{\partial F}{\partial \sigma}\frac{\partial F}{\partial \sigma} + E^{t+\Delta t}\frac{\partial F}{\partial \sigma}\frac{\partial F}{\partial \sigma}}\right) dT . \tag{33}$$

The yield surface translation scalar [equation (19)] reduces to

$$d\mu = \frac{2(\sigma-\alpha)d\sigma - 2k\frac{\partial k}{\partial T}\, dT - 2k\frac{\partial k}{\partial \bar{\varepsilon}^{P}}\, d\bar{\varepsilon}^{P}}{(\sigma-\alpha)2(\sigma-\alpha)} , \tag{34}$$

and the yield surface translation increment tensor is simplified to

$$d\alpha = d\mu(\sigma-\alpha) \tag{35}$$

By replacing σ_1 in equation (21) with σ, the uniaxial stress, it can be seen that the yield surface gradient is $\frac{\partial F}{\partial \sigma} = 2\,(\sigma - \alpha)$. Finally, the elastic strain increment is determined from equation (31) to be

$$d\varepsilon^{E} = \frac{d\sigma - dE(\varepsilon^{t} - \varepsilon^{Pt} - \varepsilon^{Tt})}{E^{t+\Delta t}} . \tag{36}$$

A. Case Number 1 - Elastic Unloading

The first case involves an axial bar which is loaded isothermally to some state of stress outside the initial yield surface and then slowly cooled, as shown in Figure 5. For simplicity, hardening is assumed to be isotropic.

It is appropriate at this point to recall that due to the nonlinear nature of the problem, the solution is obtained through an iterative procedure. As described in references 7 and 15, that method assumes a strain increment and iterates to solution. It has been shown to converge for small load increments. Since it is not the intention of this paper to demonstrate the equilibrium iteration procedure the method used here will be to assume a reasonable strain increment based on engineering judgement and use that prediction to obtain the stress and strain increments for the load step. For the conditions shown in Figure 5, a reasonable assumption for the strain increment is

$$\Delta\varepsilon = -\frac{\sigma^t}{E^t} + \frac{\sigma^t}{E^{t+\Delta t}} + \alpha\Delta T \quad . \tag{37}$$

where ΔT is understood to be a negative quantity. From equation (32) the stress increment is predicted to be

$$d\sigma = E^{t+\Delta t}\left(-\frac{\sigma^t}{E^t} + \frac{\sigma^t}{E^{t+\Delta t}}\right) + (E^{t+\Delta t} - E^t)(3\varepsilon_y - 1.5\ \varepsilon_y) = 0. \tag{38}$$

Since the predicted stress increment is zero and the yield surface size will increase due to the temperature decrease, unloading is predicted. Thus, there is no plastic straining during the load step and the stress increment is precisely that given by equation (38). The elastic strain increment is determined by equation (31) to be

$$d\varepsilon^E = 0 - \frac{(E^{t+\Delta t} - E^t)\ (1.5\ \varepsilon_y)}{E^{t+\Delta t}} = -\frac{\sigma^t}{E^t} + \frac{\sigma^t}{E^{t+\Delta t}} \quad . \tag{39}$$

The total strain increment is thus

$$d\varepsilon = d\varepsilon^E + d\varepsilon^T = -\frac{\sigma^t}{E^t} + \frac{\sigma^t}{E^{t+\Delta t}} + \alpha\Delta T \quad . \tag{40}$$

It can be seen that the resulting strain increment is exactly that predicted by equation (37). Therefore, a convergent equilibrium iteration procedure will result in a strain and stress increment in accordance with engineering intuition. Since hardening is isotropic,

$d\mu = 0$, and the resulting yield surface translation tensor is ϕ. The above case is intended to illustrate the predictive capability of the model for elastic unloading.

B. Case Number 2 - Temperature Dependent Elastic Modulus

Material properties and load history for the second sample problem are shown in Figure 7. Hardening is assumed to be kinematic. The object of this test case is to determine whether the proper stress increment is predicted when the elastic modulus is temperature dependent. As an initial guess the strain increment is estimated to be.

$$d\varepsilon = 1.2\,\varepsilon_y + \alpha\Delta T \; . \tag{41}$$

The yield surface translation at the start of the step is found to be $\alpha = .5\,\sigma_y$. The loading condition [equation (1)] will be satisfied because the yield surface size is decreasing with temperature. Therefore, one must calculate the slope of the uniaxial stress-plastic strain curve:

$$H' = \frac{2\sigma_y - \frac{2}{3}\sigma_y}{6\,\varepsilon_y} = \frac{2\sigma_y}{9\varepsilon_y} \; , \tag{42}$$

where H' is obtained from the stress-strain curve for the temperature at the end of the step and at the uniaxial plastic strain level at the start of the step. This is required to obtain a consistent stress increment due to temperature effects and can be seen to be consistent with equation (15).

From equation (33), the resulting stress increment is

$$d\sigma = \left(\frac{2}{3}\frac{\sigma_y}{\varepsilon_y} - \frac{\frac{2}{3}\frac{\sigma_y}{\varepsilon_y} \cdot 2\sigma_y \cdot 2\sigma_y \cdot \frac{2}{3}\frac{\sigma_y}{\varepsilon_y}}{\frac{2\sigma_y}{9\varepsilon_y} \cdot 2\sigma_y \cdot 2\sigma_y + \frac{2}{3}\frac{\sigma_y}{\varepsilon_y} \cdot 2\sigma_y \cdot 2\sigma_y} \right) 3\varepsilon_y$$

$$- \left(\frac{\frac{2}{3} \frac{\sigma_y}{\varepsilon_y} \cdot 2\sigma_y \cdot 2\sigma_y \cdot - \frac{\sigma_y}{3\varepsilon_y}}{\frac{2\sigma_y}{9\varepsilon_y} \cdot 2\sigma_y \cdot 2\sigma_y + \frac{2\sigma_y}{3\varepsilon_y} \cdot 2\sigma_y \cdot 2\sigma_y} \right) (3\varepsilon_y - 1.5\ \varepsilon_y)$$

$$+ \frac{-\sigma_y}{3\varepsilon_y} (3\varepsilon_y - 1.5\ \varepsilon_y) + \left(\frac{2\sigma_y \cdot \frac{2\sigma_y}{3\varepsilon_y} \cdot 2\sigma_y \cdot \frac{-\sigma_y}{2\Delta T}}{\frac{2\sigma_y}{9\varepsilon_y} \cdot 2\sigma_y \cdot 2\sigma_y + \frac{2\sigma_y}{3\varepsilon_y} \cdot 2\sigma_y \cdot 2\sigma_y} \right) \Delta T = 0, \tag{43}$$

where it should be noted that $\partial\sigma/\partial T$ is calculated using the equivalent plastic strain and temperature at the start of the step. Thus, as can be seen from Figure 7, the model predicts the correct stress increment when the elastic modulus is temperature dependent. Since hardening is kinematic equation (34) gives

$$d\mu = \frac{2(1.5\ \sigma_y - .5\ \sigma_y) \cdot 0 - 2 \cdot \sigma_y \cdot \frac{\sigma_y}{3\Delta T}\ \Delta T}{(1.5\ \sigma_y - .5\ \sigma_y)\ 2\ (1.5\ \sigma_y - .5\ \sigma_y)} = \frac{1}{3}\ , \tag{44}$$

and the yield surface translation increment is given by [equation (35)]:

$$d\alpha = \frac{1}{3} (1.5\ \sigma_y - .5\ \sigma_y) = \frac{\sigma_y}{3}\ . \tag{45}$$

A graphical representation of the yield surface is shown in Figure 8.

C. Case Number 3 - Verification of H'

In the third sample case the bar is loaded isothermally to some state of stress outside the initial yield surface and then slowly heated and mechanically loaded, as shown in Figure 9. Hardening is assumed to be isotropic. The elastic modulus is independent of temperature so that this case checks the validity of H'. From Figure 9 one can estimate the total strain increment to be

$$d\varepsilon = 4\ \varepsilon_y + \alpha\Delta T\ . \tag{46}$$

Since equation (32) predicts a positive stress increment and the yield surface size decreases due to the temperature increase, equation

(1) predicts loading. Therefore, it is determined that

$$H' = \frac{\sigma_y}{7\epsilon_y} \tag{47}$$

Since the elastic modulus does not change the second and third terms disappear in equation (33) and the stress increment is found to be

$$d\sigma = \left(\frac{\sigma_y}{\epsilon_y} - \frac{\frac{\sigma_y}{\epsilon_y} \cdot 3\sigma_y \cdot 3\sigma_y \cdot \frac{\sigma_y}{\epsilon_y}}{\frac{1}{7}\frac{\sigma_y}{\epsilon_y} \cdot 3\sigma_y \cdot 3\sigma_y \cdot + \frac{\sigma_y}{\epsilon_y} \cdot 3\sigma_y \cdot 3\sigma_y} \right) 4\epsilon_y$$

$$+ \left(\frac{3\sigma_y \cdot \frac{\sigma_y}{\epsilon_y} \cdot 3\sigma_y \ \frac{-\sigma_y}{7\Delta T}}{\frac{1}{7}\frac{\sigma_y}{\epsilon_y} \cdot 3\sigma_y \cdot 3\sigma_y + \frac{\sigma_y}{\epsilon_y} \cdot 3\sigma_y \cdot 3\sigma_y} \right) \Delta T = .25\ \sigma_y \quad . \tag{48}$$

The predicted stress increment can be seen to be in accordance with the value given in Figure 9. Therefore, H' is verified for this loading case. Since the hardening is isotropic the yield surface translation tensor will be ϕ, as can be seen from Figure 10.

D. Case Number 4 - Combined Thermal and Mechanical Loading

The final sample problem illustrates a combined loading case for an axial bar. The loading conditions and material properties are shown in Figure 11. Hardening is assumed to be combined isotropic-kinematic with $\beta = .5$. From the figure it is seen that the strain increment can be estimated to be

$$d\varepsilon = 4.5\ \epsilon_y + \alpha\Delta T. \tag{49}$$

The yield surface translation at the start of the step is determined to be $\alpha = .25\ \sigma_y$. An evaluation of equation (32) will once again predict loading when used in equation (1). Therefore, H' is found to be

$$H' = \frac{2\sigma_y}{9\varepsilon_y} . \tag{50}$$

The stress increment, from equation (33), is

$$d\sigma = \left(\frac{2}{3}\frac{\sigma_y}{\varepsilon_y} - \frac{\frac{2\sigma_y}{3\varepsilon_y} . 2.5\ \sigma_y . 2.5\ \sigma_y . \frac{2}{3}\frac{\sigma_y}{\varepsilon_y}}{\frac{2}{9}\frac{\sigma_y}{\varepsilon_y} . 2.5\ \sigma_y . 2.5\ \sigma_y + \frac{2}{3}\frac{\sigma_y}{\varepsilon_y} . 2.5\ \sigma_y . 2.5\ \sigma_y}\right) 4.5\ \varepsilon_y$$

$$- \left(\frac{\frac{2}{3}\frac{\sigma_y}{\varepsilon_y} . 2.5\ \sigma_y . 2.5\ \sigma_y . \frac{\sigma_y}{3\varepsilon_y}}{\frac{2}{9}\frac{\sigma_y}{\varepsilon_y} . 2.5\ \sigma_y . 2.5\ \sigma_y + \frac{2}{3}\frac{\sigma_y}{\varepsilon_y} . 2.5\ \sigma_y . 2.5\ \sigma_y}\right)(3\varepsilon_y - 1.5\ \varepsilon_y)$$

$$+ \frac{-\sigma_y}{3\varepsilon_y}\ (3\varepsilon_y - 1.5\ \varepsilon_y)$$

$$+ \left(\frac{2.5\ \sigma_y . \frac{2}{3}\frac{\sigma_y}{\varepsilon_y} . 2.5\ \sigma_y . \frac{-\sigma_y}{2\Delta T}}{\frac{2}{9}\frac{\sigma_y}{\varepsilon_y} . 2.5\ \sigma_y . 2.5\ \sigma_y + \frac{2}{3}\frac{\sigma_y}{\varepsilon_y} . 2.5\ \sigma_y . 2.5\ \sigma_y}\right)\Delta T = .25\ \sigma_y . \tag{51}$$

Note that $\partial\sigma/\partial T$ should be obtained from the uniaxial stress plastic strain diagram rather than the uniaxial stress-strain diagram. This can be seen by noting that equation (14) implies that $d\sigma/dT$ is evaluated at constant plastic strain. An inspection of Figure 11 shows that equation (51) predicts the correct stress increment. The yield surface translation scalar is [equation (34)]

$$d\mu = \left[2\ (1.5\ \sigma_y - .25\ \sigma_y)\ .25\ \sigma_y - 2\ .\ 1.25\ \sigma_y . \frac{-9}{24}\frac{\sigma_y}{\Delta T} . \Delta T\right.$$

$$\left. - 2\ .\ 1.25\ \sigma_y . \frac{\sigma_y}{9\varepsilon_y} . 3.375\ \varepsilon_y\right] \Big/ \left[(1.5\ \sigma_y - .25\ \sigma_y)^2 . 2\right] = \frac{7}{30} . \tag{52}$$

giving a yield surface translation of

$$d\alpha = \frac{7}{30}(1.5\,\sigma_y - .25\,\sigma_y) = \frac{7}{24}\sigma_y . \qquad (53)$$

A careful examination of the given data will show that the yield surface translation tensor guarantees that the stress tensor remains consistent with the yield surface.

CONCLUSION

It has been the purpose of this report to establish the procedure necessary to implement the temperature dependent elastic-plastic constitutive law previously proposed by these authors into a finite element code for structural analysis. In so doing, we have presented some modifications to the previously reported law, as well as additional information necessary for computer applicability. We have also included a computational procedure, and we have shown that our form of the constitutive law produces correct results for several uniaxial sample cases. In every sample case we have obtained identical results in the computer code AGGIE I, thus verifying not only the constitutive law but the finite element discretization and equilibrium iteration procedure as well. A future paper will compare the results of experiments to analysis for specimens subjected to cyclic mechanical and thermal loading.

ACKNOWLEDGEMENT

This research was supported by the office of Naval Research under Contract N00014-76-C-0150.

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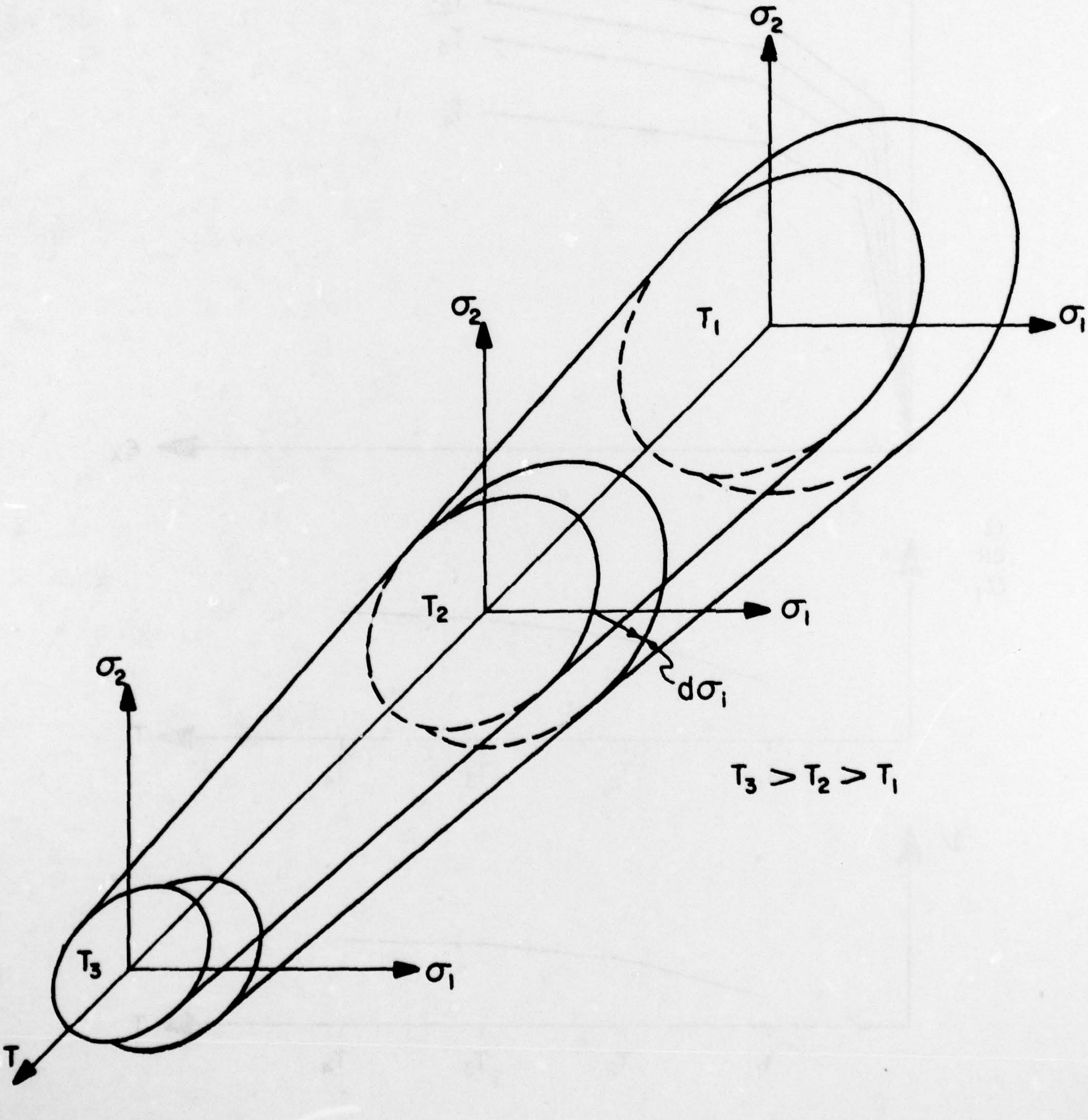

FIGURE 1. Yield Surface as a Function of Stress and Temperature

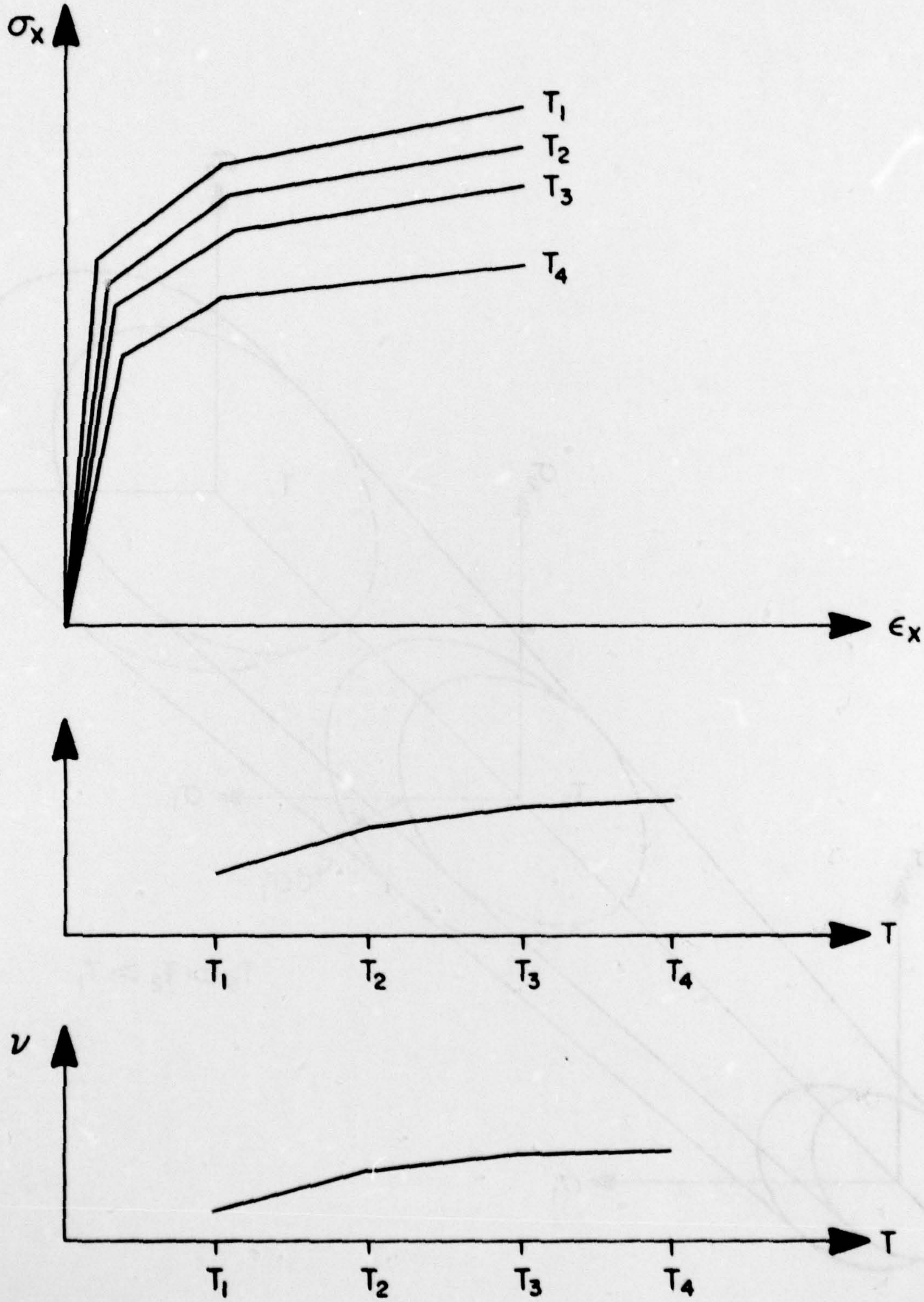

FIGURE 2. Materials Input Data

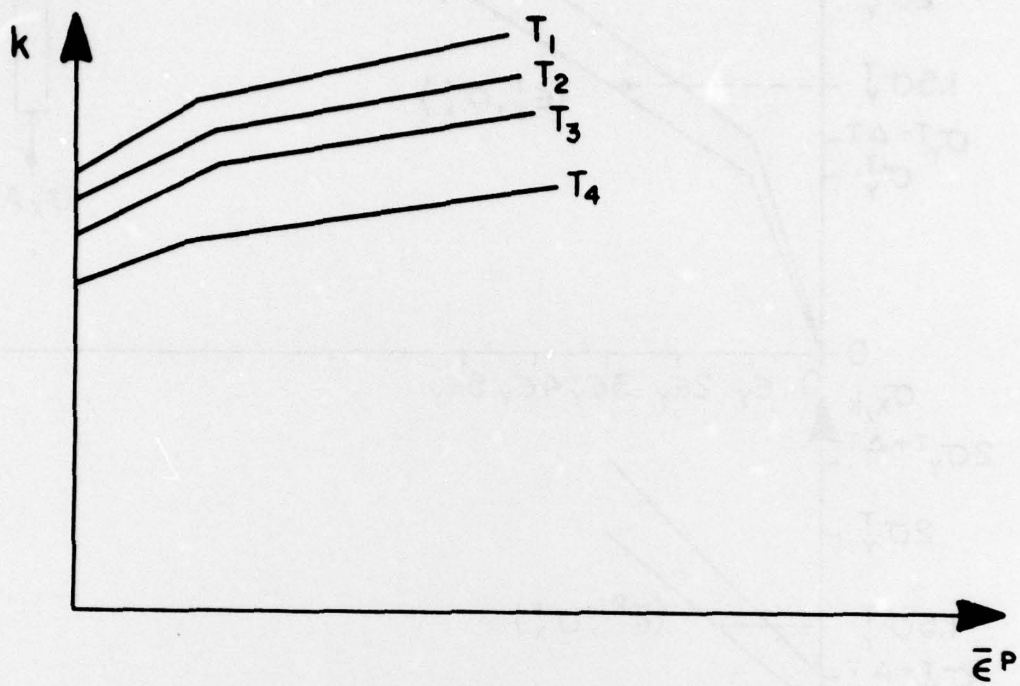

FIGURE 3. Construction of Yield Surface Size Vs. Uniaxial Plastic Strain Curves

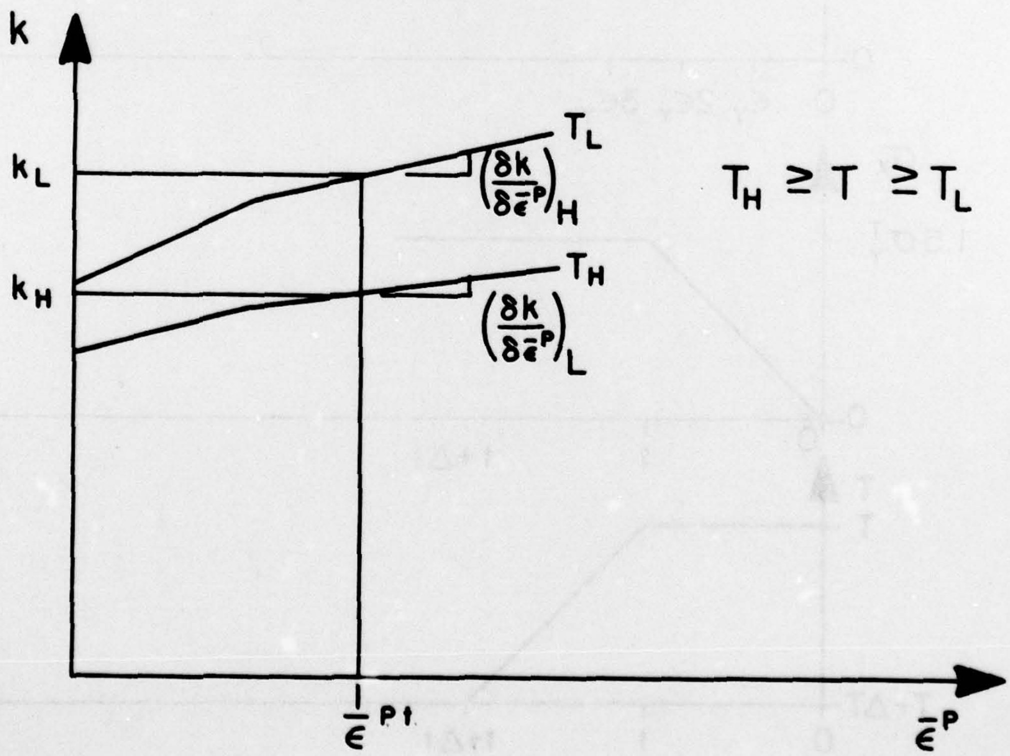

FIGURE 4. Interpolation of Yield Surface Size Vs. Uniaxial Plastic Strain Curves

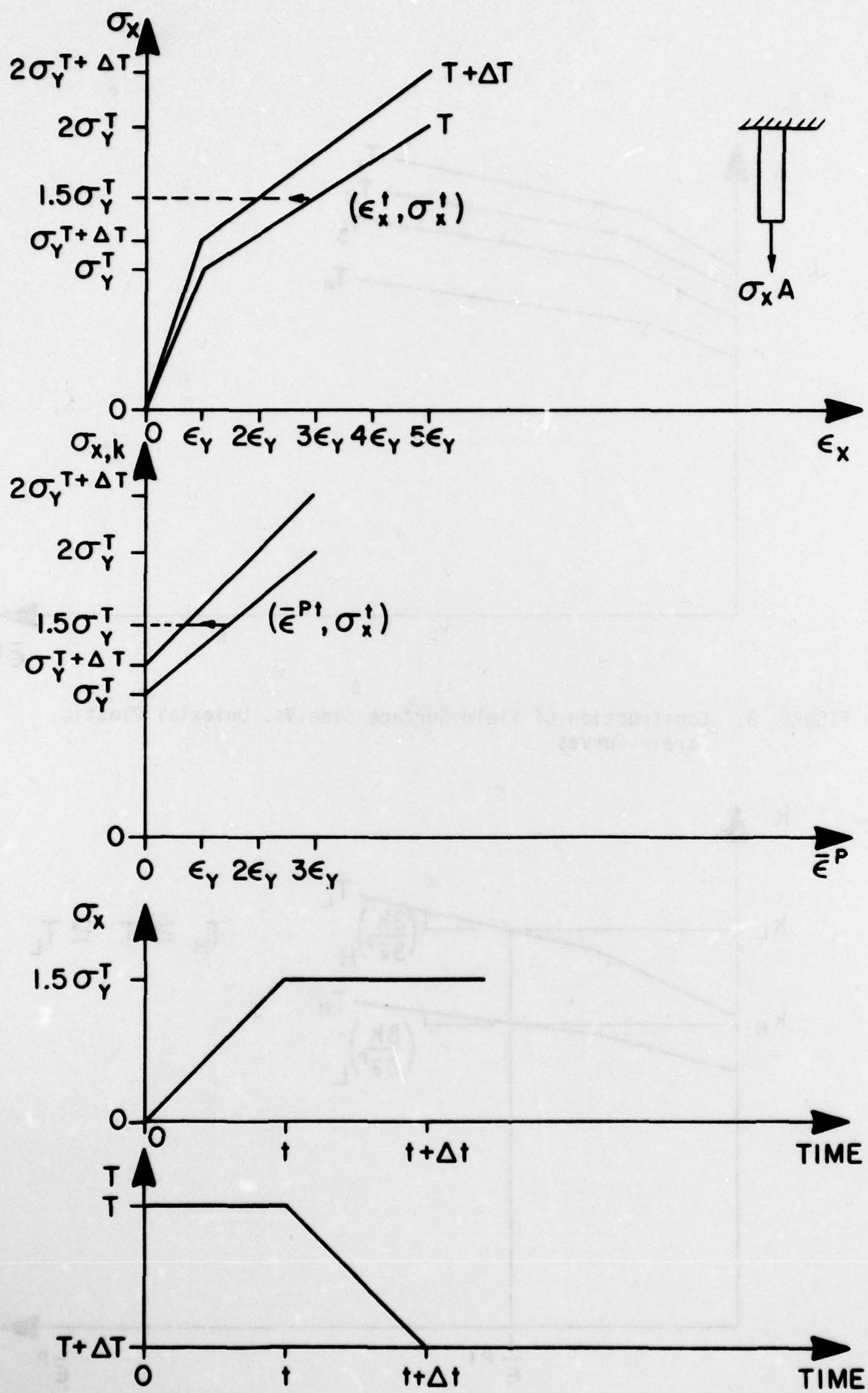

FIGURE 5. Materials Data and Load History for Case Number 1

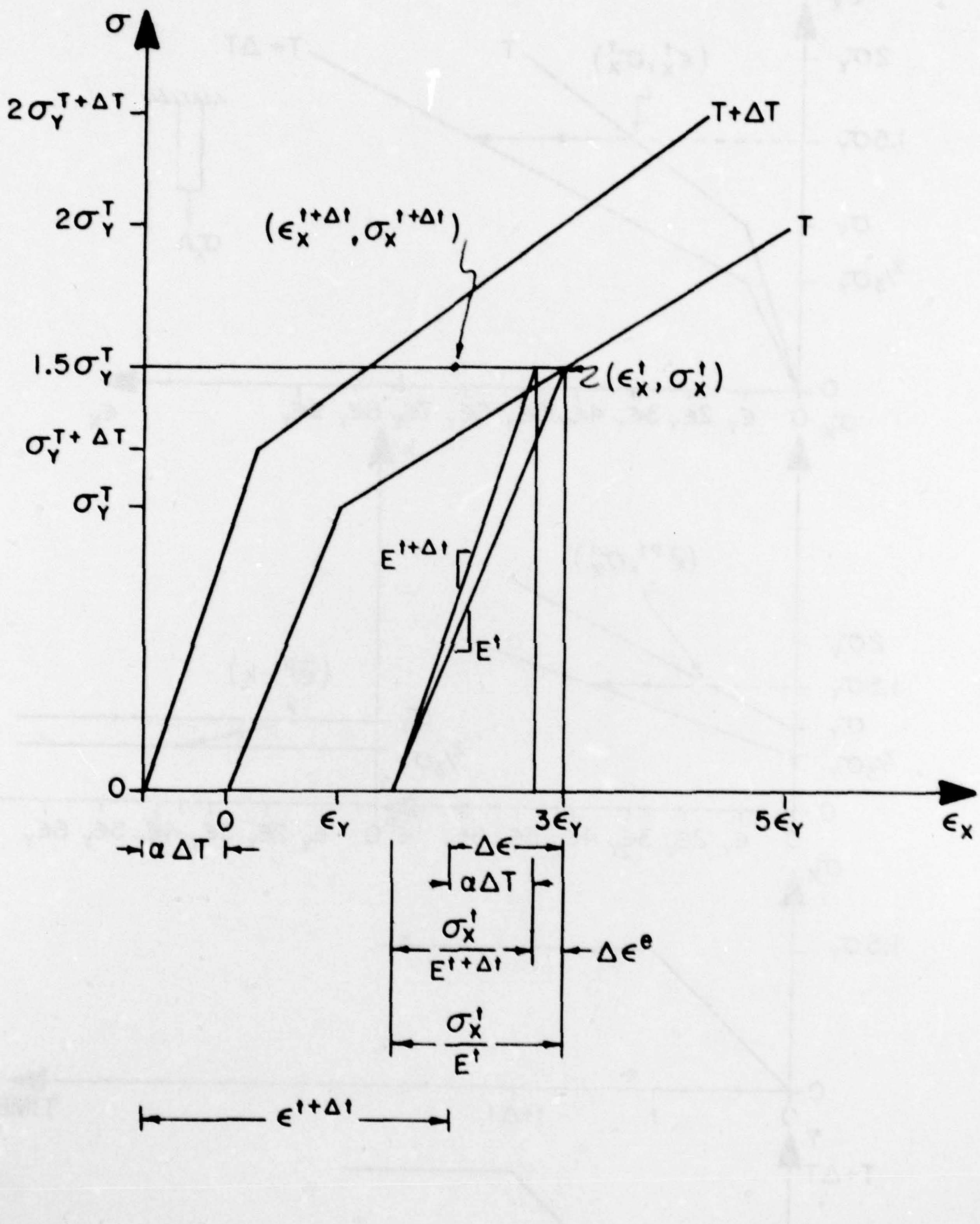

FIGURE 6. Determination of Total Strain Increment for Case Number 1

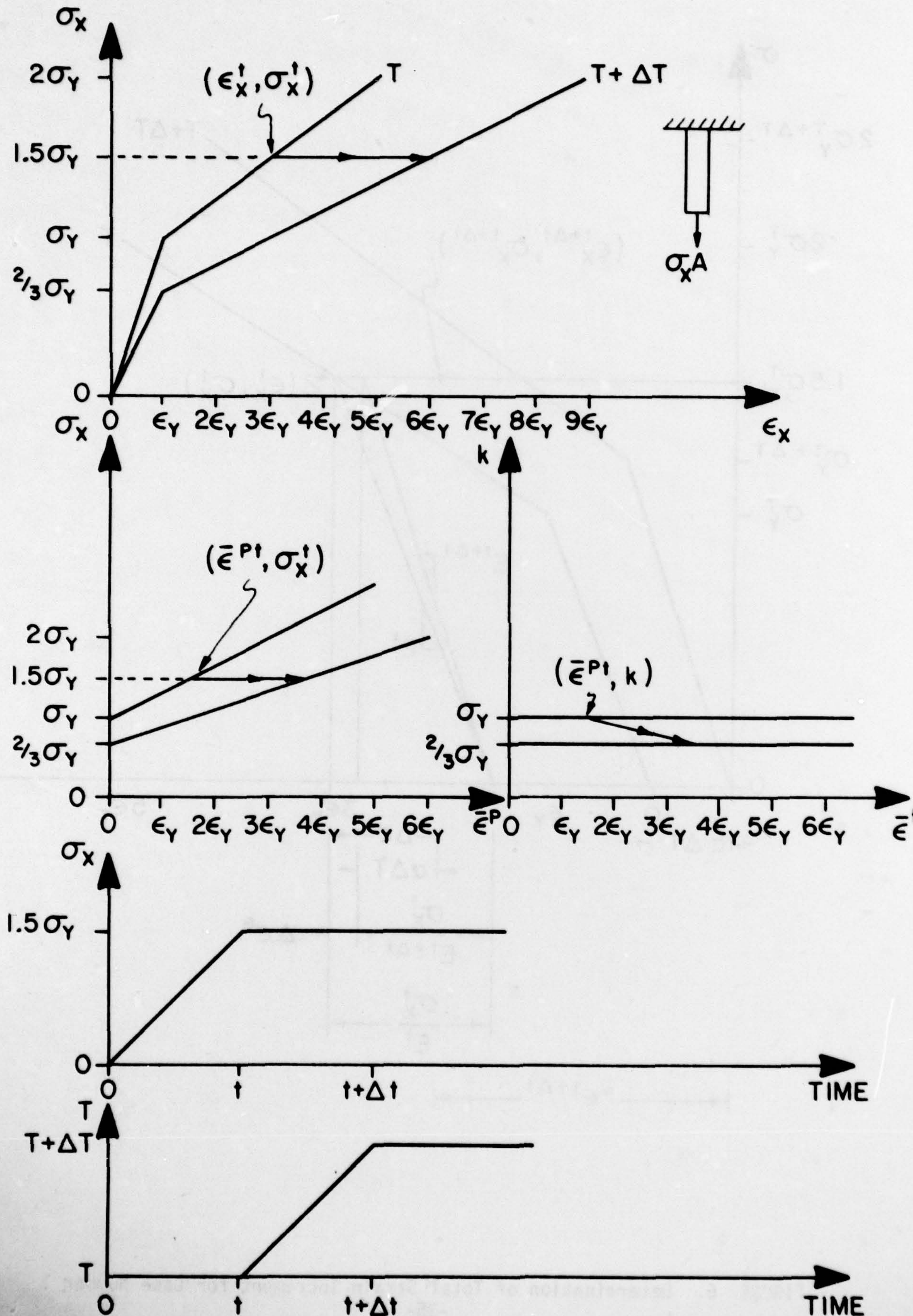

FIGURE 7. Materials Data and Load History for Case Number 2

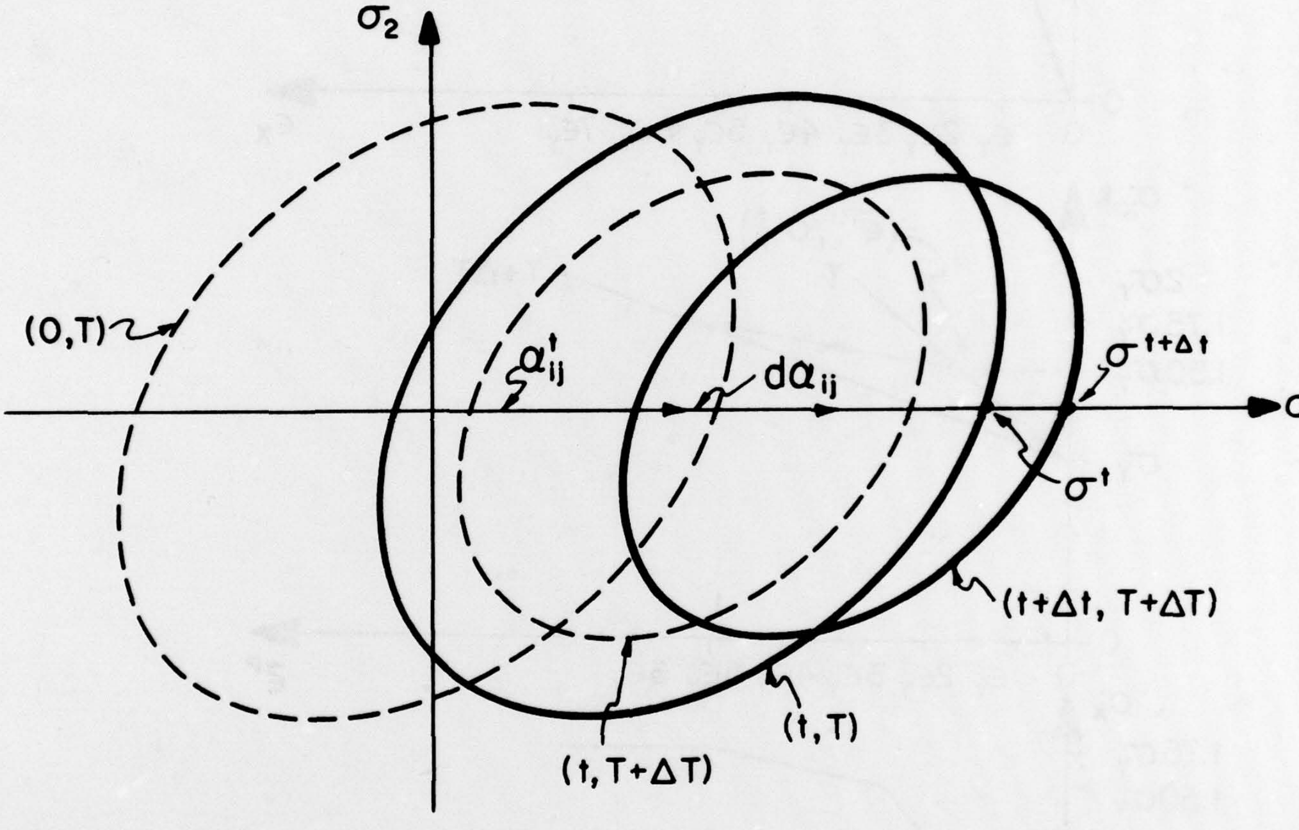

FIGURE 8. Graphical Representation of Yield Surface Translation for Case Number 2

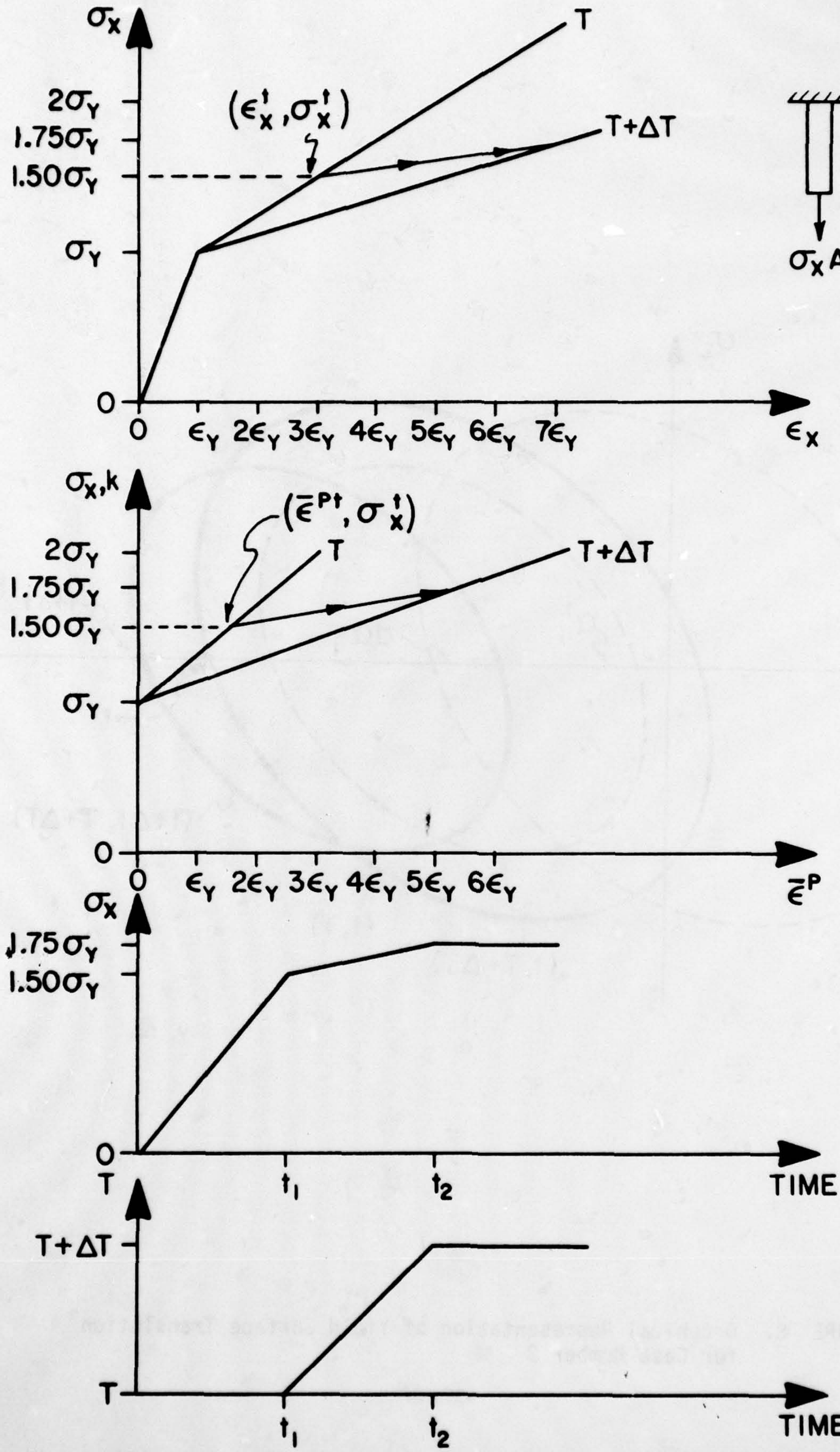

FIGURE 9. Materials Data and Load History for Case Number 3

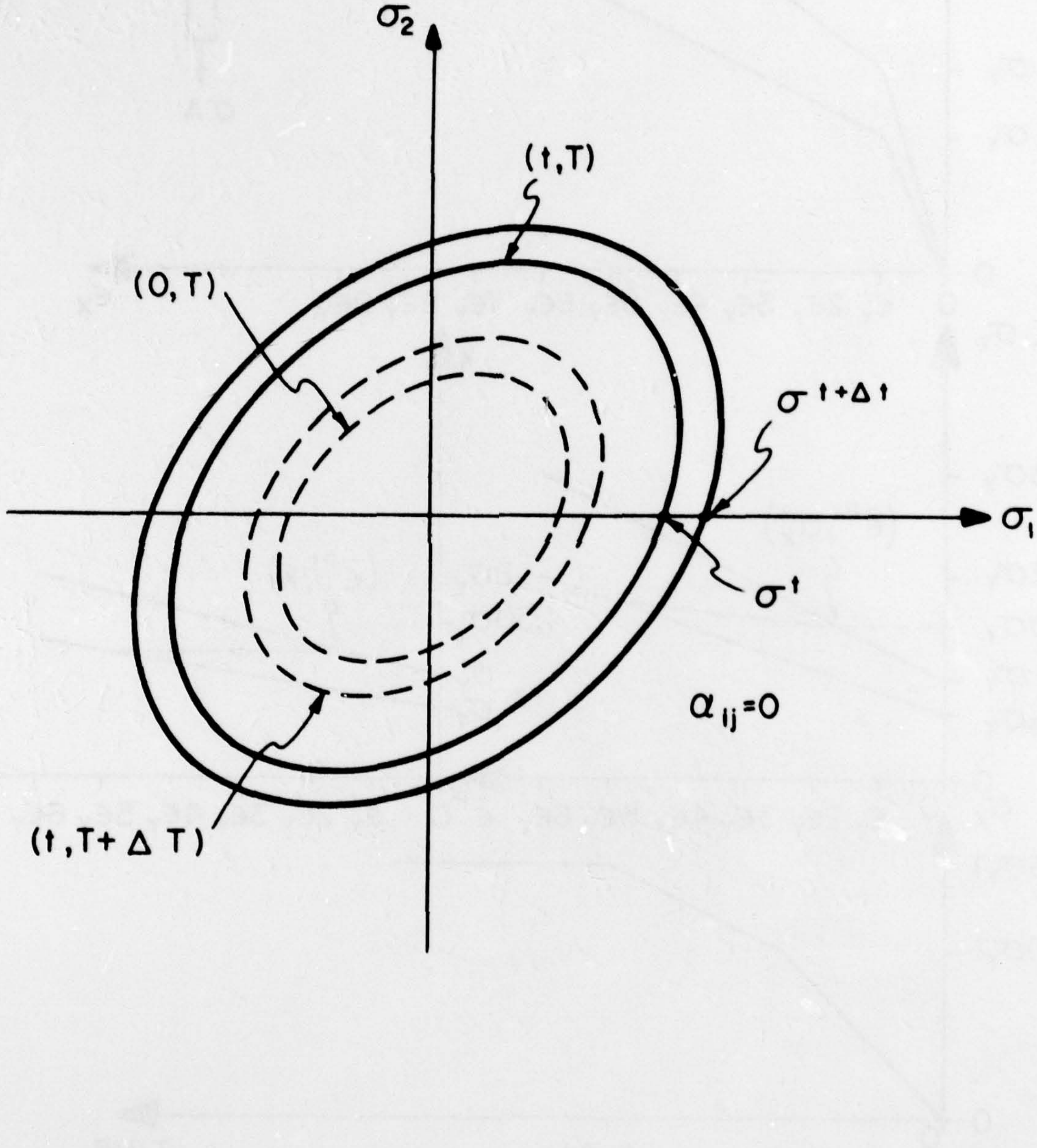

FIGURE 10. Graphical Representation of Yield Surface Expansion for Case Number 3

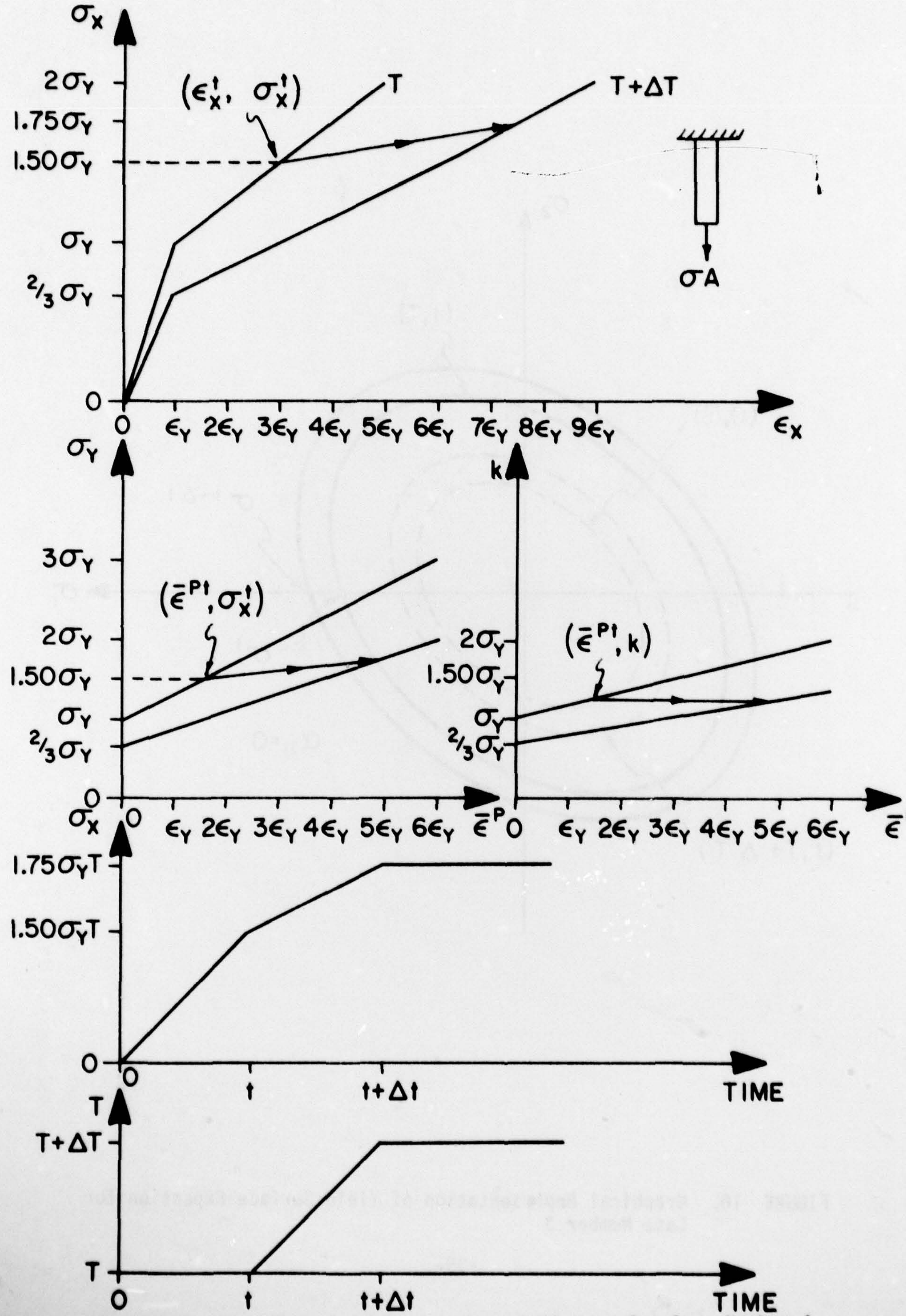

FIGURE 11. Materials Data and Load History for Case Number 4

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4. TITLE (and Subtitle): Thermoplastic Analysis Using The Finite Element Code AGGIE I	5. TYPE OF REPORT & PERIOD COVERED: Interim
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s): David H. Allen and Walter E. Haisler	8. CONTRACT OR GRANT NUMBER(s): N00014-76-C-0150
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Department of Aerospace Engineering, Texas A&M University, College Station, Texas 77843	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: NR 064-534
11. CONTROLLING OFFICE NAME AND ADDRESS: Office of Naval Research, Department of the Navy, Arlington, Virginia 2217	12. REPORT DATE: December 15, 1979
	13. NUMBER OF PAGES: 40
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report): Unclassified
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

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18. SUPPLEMENTARY NOTES

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Finite Element
Thermoplasticity
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Workhardening
Isotropic-Kinematic

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The authors have previously proposed a theoretical model for predicting structural response of elastic-plastic-creep materials subjected to variable temperature loadings. This model utilizes the combined isotropic-kinematic hardening rule in conjunction with the classical rate independent plasticity theory. The resulting constitutive law has been implemented into the finite strain finite element code AGGIE I. The authors have found it necessary to alter the formulation previously presented in order to obtain a concise

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numerical formulation. That modification is presented in this paper. In addition, a section is included which outlines supplementary information necessary for computer program input, followed by a computational procedure which is intended to aid programmers who wish to use this model. Finally, the results of several sample cases are presented.

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