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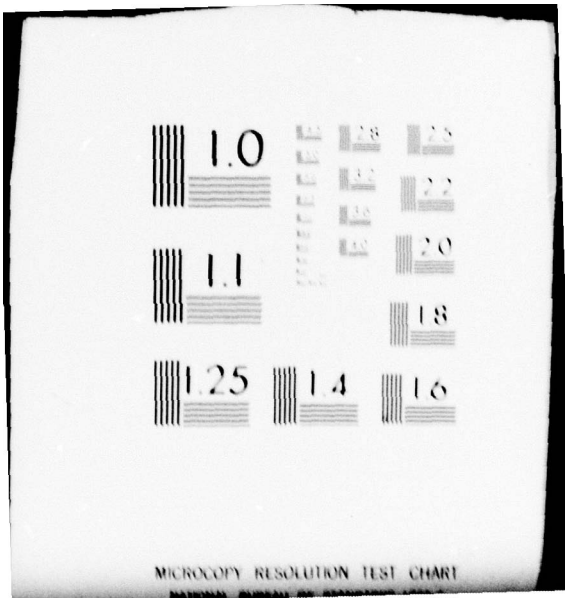
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AIR FORCE INSTITUTE OF TECHNOLOGY

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MARKOV SIMULATIONS OF ONE- AND TWO-DIMENSIONAL WEATHER DATA BASES

THESIS

Presented to the Faculty of the School of Engineering

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science

by

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Captain USAF

Graduate Operations Research

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Preface

This study had a major goal of extending Markov simulation methods, especially as applied to weather data, to two or more dimensions. These methods should be computationally simple, since they would be designed to produce a weather data field as only one of many possible inputs to a more complex simulation model used for another purpose, such as weapon system effectiveness. Examples performed using the models described in this paper show that most of these methods are realistic and all of them are quite easy to apply.

I would like to thank Dr. David Barr of the AFIT Mathematics Department for suggesting this topic to me and for his constant guidance during the research, along with allowing this to be an independent study as much as possible. Also, I would like to thank Capt Garry Jackson of AFFDL for his ideas, which led to the formulation of this problem, and for his constant interest in the outcome of the research, and I would like to thank Lt Col Jon Hobbs from the AFIT Systems Management Department for his helpful suggestions during the writing of this thesis. Finally, I would like to thank God, who constantly sustained me throughout this period.

Steven Schroeder

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Abstract

In many cases, weather is only one of many inputs to a simulation model, so a realistic but simple weather simulation method should be included in the model. This paper has three major areas of concern: (1) A fairly extensive review of applications of one-dimensional Markov and semi-Markov chains to weather data simulations, (2) A consideration of factors involved in and methods that are appropriate for extending Markov concepts to simulations of gridded data in two or more dimensions, and (3) Evaluation of the proposed methods in terms of realism and simplicity of application. A discussion of the general characteristics of real weather variables and observed weather data in the context of simulating weather as a stochastic process is also given.

The data base used for the example consisted of gridded weekly maps of temperature departures from normal in an area of the United States. For most analyses, the data was converted to five states, from state 1 (coldest) to state 5 (warmest). In the real data, it was rare to have an occurrence of unequal or nonconsecutive states in adjacent grid points. Such occurrences were called "unusual transitions," and one criterion for evaluating the realism of a weather simulation scheme was the frequency of generating these transitions.

Most of the proposed two-dimensional simulation methods produced data fields that were basically quite realistic, but they also produced "unusual transitions" more frequently than in the original data. A further suggested method, no more complex than the other methods, was found to greatly reduce the frequency of these "unusual transitions" in the simulated data fields. In this method, the probability distribution at any interior point of the grid depends on the states observed

at the grid points immediately west, north, and northeast of each point, and the interior of the grid is simulated by rows. All of the proposed simulation methods are very easy to perform on a computer.

MARKOV SIMULATIONS OF ONE- AND TWO-DIMENSIONAL WEATHER DATA BASES

I. Introduction

Overview of Weather Modeling

Weather is still an important factor in the effectiveness of almost all weapon systems. Fighter aircraft, heat-seeking missiles, helicopters, and cruise missiles are only a few of the systems whose performance can be significantly degraded by certain adverse weather conditions such as icing, low ceilings, or low visibility. Since these conditions occur very frequently in many areas of military interest, especially Europe, weather is a major concern of combat planners and needs to be modeled so it can be incorporated into planning procedures.

Categories of weather models. Weather models can be divided into three basic categories: deterministic models, stochastic models, and models that are a combination of both.

(1) Deterministic weather models use initial input conditions (based on weather observations) and physical laws of the atmosphere to produce explicit forecasts of conditions at a later time. The scales of motion and interactions involved require modeling at least the Northern or Southern Hemisphere for realistic results, if forecasts are made for a period of more than a few hours. Small-scale or temporary phenomena such as thunderstorms or fog cannot be adequately modeled, and accumulated errors cause accuracy to decrease substantially after about 24 hours. Because the physical laws are universal, the models may be applied almost anywhere (even on other planets with an atmosphere).

(2) Stochastic models are based on climatology, or a knowledge of past weather conditions. Climatology is usually a statistical description of the weather and is often expressed in the form of probabilities of occurrences of given weather conditions. A forecast made by a stochastic model will consist of the probabilities of various outcomes, based on current and recent weather conditions. Because this is a statistical rather than a meteorological approach, probabilities must be generated for each separate location. The data may be further stratified by season of year and time of day.

(3) Many operational computer-based models are combinations of the deterministic and stochastic types. An example is a "probability of precipitation" forecast. The expected conditions at a given point over a specified time interval are forecast deterministically. The uncertainty due to inaccuracy of the forecast is expressed in probabilistic form, with the probabilities determined from an empirical model based on the forecast model outputs.

Basic reasons for weather modeling. The major reasons for desiring to model the weather can be divided into two categories: short-range planning and long-range planning. Short-range planning as used here refers to the time period for which specific forecasts are available, generally no more than a few days into the future. Long-range planning is considered to concern time periods for which no forecast is available. Long-range planning can also include attempts to understand present and past climates (including the ice ages), since any past climate could possibly recur in the future.

For short-range forecasting, deterministic models are quite effective in most cases and are routinely produced at major forecasting

centers. Some small-scale phenomena that cannot be adequately modeled can be forecast stochastically using conditional probabilities (such as the probability of fog tomorrow morning, given the weather conditions observed today) obtained from a "conditional climatology."

Long-range weather planning is most effectively accomplished using a stochastic approach, since no exact knowledge of weather conditions can be obtained beyond a certain distance into the future. Simulation is a tool frequently used in long-range planning. Deterministic models have had some use in investigating past climates (such as in simulation of the last ice age), but the purpose has only been to obtain estimates of averages. In general, deterministic models are too complex and use too much computer time for use in most simulations of Air Force interest, while stochastic models (such as Markov chains) can be computationally simple and still provide sufficient realism.

Simulation and Weather Modeling

Much work has been done to simulate the performance of weapon systems under varying combat situations. Weather is often included as a factor in the simulation models, with varying degrees of realism. In general, the weather module of a simulation model should be as computationally simple as possible and should not introduce any more complexity than necessary. However, it should include enough of the characteristics of the actual weather (such as frequencies of occurrence, persistence, and variability, in space as well as time if needed) to be realistic in terms of the weapon systems and scenarios considered. Since a simulation is generally assumed to take place at some unspecified future time, for which the only thing known about the weather conditions is that they will be different from present and past

weather, an appropriate approach is to describe the available weather data base statistically in terms of probability distributions. Then, random procedures would be used to generate a statistically reasonable weather sequence, which would probably not be the same as any actually observed weather sequence. Only the parameters that generate the sequence would need to be stored in the computer, rather than the entire series of available data.

One problem in any simulation is to decide what weather data (if any) is needed. This will not be a major concern of this thesis, since meteorological reasoning and experimental data can be used on a case-by-case basis to determine what data needs to be included in each simulation. It is assumed here that data is already available or can be obtained, since for most land areas of the world, the weather has been continuously observed at specific locations for 20 to 100 years or more. Most of this data is archived in computer-readable format at the Air Force Environmental Technical Applications Center (ETAC), Scott Air Force Base, Illinois, and the National Climatic Center (NCC), Asheville, North Carolina. Most detailed data bases covering areas of the earth's surface or volumes of the earth's atmosphere on a small scale have been available only since the early 1970's. The major Air Force data base of this type is the three-dimensional nephanalysis data base (3D NEPH), archived at ETAC, which incorporates weather satellite observations and surface and upper-air weather data to provide a worldwide 15-layer analysis of atmospheric moisture and cloud cover (nephanalysis), with a resolution of approximately 25 nautical miles. On a large scale, daily Northern Hemisphere surface weather maps have been published back to 1899, and upper-air weather maps are available back to 1940 in the Daily Series, Synoptic Weather Maps (U.S. Department of Commerce, 1940 to

Present) and Historical Weather Maps (U.S. Department of Commerce, 1899 to 1939) series. This paragraph, of course, is not an exhaustive list of available data, but illustrates the fact that weather data is already available for almost any desired purpose.

The next problem in incorporating weather into a simulation model is to appropriately describe the data so that the data characteristics that are important for the simulation are included in the model. For example, to simulate a long, large-scale military exercise, or a conventional war lasting a year or more, it may be appropriate to assume average weather conditions or conditions that match the normal probability distributions of the weather phenomena. To simulate a shorter-duration event, such as a Warsaw Pact attack on western Europe, both changes and persistence from day to day should be included in the model. Different prevailing weather conditions during the period of this event (such as a severe winter cold spell, a mild and rainy winter period, a typical showery period in the summer, or a summer drought and heat wave) would lead to considerable differences in the effectiveness of weapon systems against various types of targets. In predicting the fuel costs for a building (or a base) during a heating season, the overall severity of the winter would be the important factor, rather than individual cold waves. However, to predict the performance of a solar heating system (possibly installed in the same building), day-to-day factors, such as cold or cloudy periods, are the most important factors.

Many procedures have been extensively developed and theoretically justified for providing a statistical description of the climate at a given location. A summary of the weather data that is recorded at a single station over a period of time can be called a "point climatology." For most stations of military interest, point climatologies are

already available from ETAC or NCC. However, published statistical summaries usually do not include data about persistence, or positive correlation between observations, which must be considered in most situations that extend over a fairly short period of time. When there is no persistence, a simulated data sequence can be generated by Bernoulli trials, or repeated random sampling from the unconditional probability distribution of weather states. If there is persistence, data sequences can be generated using Markov or semi-Markov processes. Efforts in this area are described more fully in the literature survey section.

The procedures for developing an "area climatology," or a statistical description of the characteristics of the weather conditions simultaneously occurring over an area or in a given volume of atmosphere, are not very well explored, especially for weather occurring on a small scale. The data that is available in map or gridded data form is concerned almost exclusively with averages or extremes over large areas, such as in the Climatic Atlas of the United States (U.S. Department of Commerce, 1968). In a simulation concerned with a small area over a short period of time, such as aerial combat, the significant meteorological factors are often not well defined, or are not based on routinely measured weather data. (For example, horizontal and vertical visibilities in the atmosphere are not always the same. Visibility observations from an airplane are usually approximate and are infrequently reported). Generally, the appropriate statistical distributions must be obtained on a case-by-case basis from the raw data that is used.

Motivation for This Project

It has been found that the cruise missile is quite vulnerable to ice accumulation, since it is designed to fly low, with little reserve power or fuel. In winter in eastern Europe and the USSR, icing conditions are frequent and may cover a large area for as long as several days.

Once this problem was identified, it was desired to find out the expected proportion of cruise missiles that could be lost under various realistic operating conditions. If the number lost could be large, icing protection may need to be built into the cruise missile even though this could reduce its payload and range.

ETAC was given responsibility to investigate this problem. Their approach was to select a given area of 3D NEPH data and count the number and duration of icing encounters (occurrences of specified combinations of temperature and liquid water content) along a simulated path as in figure 1 for each data time available. The probabilities of encounter of these icing situations could be used to estimate the probability of losing a cruise missile due to icing.

If further simulations are desired after this data is analyzed, there are many possible approaches to developing a simulation model, two of which are of interest here. In both cases, mutually exclusive weather conditions (or states) that correspond to different intensities of cruise missile icing could be defined.

The first approach is most suitable if the probability of loss of a single cruise missile due to icing is to be investigated. In this case, a one-dimensional approach that would simulate a sequence of weather conditions along a path could be used. This could be either

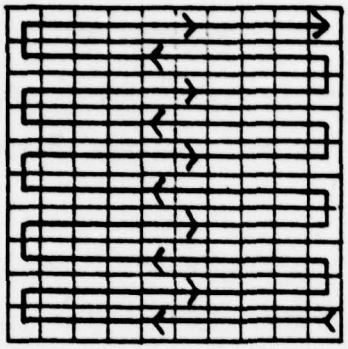

Figure 1. Simulated Path Used by ETAC to Analyze Icing Duration

a Markov or a semi-Markov model, and chapter IV describes some examples of applications of both types of models.

The second method is more appropriate if the paths of a large number of cruise missiles launched during a short time interval are to be simulated. In this case, a two-dimensional approach should be used, assuming that the cruise missiles are to fly at nearly a constant level above the ground. A two-dimensional data field would be generated and a large number of cruise missile paths through this data field would be simulated. The proportion of cruise missiles lost due to icing would be recorded, and this procedure would be repeated with different generated data bases until the desired statistics have been found.

Goals of This Thesis

In view of the preceding areas of concern, the first goal of this thesis is to summarize the existing literature in the area of simulation using one-dimensional Markov and semi-Markov processes, especially as

applied to weather data, for the benefit of users. The second goal is to investigate the possibility of extending Markov concepts to two dimensions, and to report on work that has already been done in this area. The third goal is to perform examples of the suggested simulation procedures to show the feasibility of the methods and to reveal any operational problems. Basically, the two-dimensional modeling procedures are concerned with simulating events occurring over an area at an instant of time. These concepts may be extended to n dimensions (including time as one of the possible dimensions), although complexity of the model would increase greatly.

Because the focus of this investigation is statistical rather than meteorological, it will not be primarily concerned with meteorological processes. Markov chains provide a statistical description of, rather than a physical explanation for, weather phenomena. Of course, when a large data base is available, the statistical description should agree with physical explanations.

The purpose of this study will be to develop procedures suitable for simulation rather than forecasting, although the procedures could be applied to a "conditional climatology" approach to short-range forecasting.

Overview

Chapter II describes some of the characteristics of actual weather variables and observed weather data.

Chapter III briefly summarizes some of the theory of appropriate Markov and semi-Markov processes and defines terms and notation to be used in this report.

Chapter IV describes some of the applications of Markov processes

to weather data series, including first-order and higher-order Markov chains, semi-Markov processes, and statistical tests of Markov order.

Chapter V discusses several additional topics that become especially important when attempting to model a two-dimensional data field by a Markov process, including reversibility and inverse probabilities, conditional probabilities, and correlation.

Chapter VI describes a regression-type scheme for simulating a two-dimensional data base as a preliminary to extending Markov concepts to two or more dimensions. The methods Galbraith and Walley (1976) and Pickard (1977) used to simulate a two-dimensional data field by Markov procedures are described, and an additional suggested procedure is shown.

Chapter VII gives examples of simulations produced by the Markov models described in chapter VI. These examples show that all of the suggested methods can generate combinations of weather states that are unusual in the real data. A suggested refinement of Pickard's method reduces the frequency of these unusual combinations. Three of the four suggested methods do produce data fields that are reasonably realistic, and all of the simulation methods are easy to perform on a computer.

Chapter VIII summarizes the results and identifies some areas of possible further research.

II. Weather Data Characteristics and Analysis

Introduction

Weather is naturally occurring and is difficult or impossible to reproduce under laboratory conditions, so it must be observed to be described, explained, and understood. Weather observations seek to describe various conditions that can be extracted from the environment, are measurable, and appear significant. It is important to know what the major characteristics of the underlying real variables are, and how the observed data differs from the real variables. The extensive general discussion of weather variables in this chapter is prepared for the benefit of users who may not have extensive experience in working with weather data. Also, the discussion of data analysis is given for those who may not be familiar with some of the techniques used.

As described before, the basic purposes of a weather model are to aid in short-range and long-range planning. Current weather observations are a key to forecasting the future weather, and a long series of observations is needed to provide an adequate climatology in the form of a statistical description of the climate of a location. Climatological data is used for various planning studies throughout the Air Force, including simulation models.

This chapter will show that available weather data is in, or can be transferred into, a form suitable for modeling as a stochastic process. However, the meteorological processes involved and the limitations on the data should be considered to avoid obtaining useless results.

It is possible to construct weather variables that will not have all of the properties named in this chapter, but these variables can be converted into variables that are suitable for stochastic modeling.

For example, the cumulative precipitation measured at a station since its establishment could be divided into two states: (1) under 1000 inches, and (2) 1000 inches or over. Once 1000 inches has been observed, state 2 is entered and is never left, so state 2 is an "absorbing state" and state 1 is a "transient state," a state that is never reentered once left. Weather data modeled as in this thesis cannot have absorbing or transient states. If annual (or monthly or daily, etc.) precipitation is recorded instead, this problem is eliminated and the data has the desired properties. In this thesis, it will be assumed that all variables will either be a measurement at a specific time or a cumulation over a specified length of time, and such data can be suitable for stochastic modeling.

Characteristics of Real Weather Variables

Continuous. Since weather occurs continuously in time and space, at least above the molecular level, real weather data is fundamentally continuous. Therefore, most real weather variables are either continuous (such as temperature) or are integrals of continuous variables over discrete time intervals (such as daily precipitation).

Deterministic. Since weather is governed by physical laws, it is deterministic, meaning that same conditions will lead to the same result. The fundamental laws include three equations of motion (one in each space dimension) and equations for the conservation of mass and energy. Sometimes an equation for the conservation of moisture is added. These are nonlinear partial differential equations and are true at any point in the atmosphere at any given time. In principle, if all of the conditions at a given time are known, the weather conditions at any following time can be perfectly forecasted. Actually, these

equations, as operationally used, are incomplete since some factors are left out.

Dependent. Weather data is never truly independent, regardless of the separation of time and distance. A small change at one location will have an eventual effect (although it may be slight) on the weather at every other location.

Multivariate. The large number of factors and interactions that produce weather mean that weather is a very complex process. It is multivariate, with vector and tensor (or matrix) notation required to describe the weather at a point in space and time.

Nonlinear. Weather is nonlinear, as shown by the form of the governing physical equations, which cannot be analytically solved because of their nonlinear form. Also, effects may not appear to be proportional to the cause, and interactions between weather elements can amplify or damp out the changes that would occur from each weather element separately.

Scales of motion. Weather occurs on many scales of motion simultaneously. These are generally divided into three categories by meteorologists: macroscale (horizontal extent approximately 1000 kilometers to planetary scale), mesoscale (about 10 to 1000 kilometers), and microscale (under 10 kilometers). These are not precise divisions between categories. Examples of generally microscale phenomena are turbulence, tornados, and thunderstorms. Squall lines, lake effect snowstorms, urban pollution, and tropical storms are generally considered mesoscale phenomena. Macroscale phenomena include air masses, frontal systems, and planetary waves (jet streams). Natural or man-made geographic features such as cities, rivers, lakes, mountain ranges,

and oceans have effects on the weather on generally the same scale as the size of the geographic feature.

Scales of velocity. The physical laws of the atmosphere contain solutions for all three types of air motions that occur in the atmosphere. Only the synoptic (or meteorological) motions, which describe the movement of air masses, are significant in the weather. Meteorological motions include velocities from zero, in calm air, to those observed in tornados and jet streams. Gravity waves, the second type of motions, arise on discontinuities between air masses and are sometimes visible in the atmosphere as a set of parallel rows of clouds. Sound waves are the third type of motions and are also obtained as solutions of the basic equations. Neither gravity nor sound waves are of meteorological significance although they do occur in the atmosphere. They can cause entirely spurious results in numerical weather forecasts, so they are removed from the set of permissible solutions to the governing physical equations by various mathematical filtering methods.

Scales of time. For a given phenomenon, the time scale may vary independently of the scale of distance or velocity. For example, a city will have some effect on the weather as it develops over decades or centuries, although its horizontal extent is only a few miles, while a low pressure area will last only a few days or weeks and may cover thousands of miles.

In descending order, some of these time scales are as follows. The causes given are quite speculative for scales longer than a year.

Thousands of years or longer: ice ages and interglacial periods. These may be caused by regular changes in the earth's orbit, such as the tilt of the earth's axis, or by changes in volcanism.

Centuries: long climatic cycles such as the "Little Ice Age" from about 1600 to 1850. Causes could include the longer cycles of heat storage in the deep ocean layers.

Years to decades: the quasi-biennial oscillation (an approximately 26-month cycle noticed mainly in the tropical stratosphere), drought and wet cycles, or warm and cool periods. A cycle averaging approximately 22 years in length is easily visible in Ohio temperature data and in the data from many other locations. These may be caused by stable or nearly stable oscillations in the earth-atmosphere-ocean system.

Annual or seasonal: the annual temperature cycle and seasonal monsoons. Abnormal weather may also occur on a seasonal scale. Examples are the recent extremely cold winters in the central and eastern United States, and the hot summers of 1975 and 1976 in western Europe. These were caused by persistent upper air patterns on the scale of a few months.

Days to weeks: air mass movements and tropical and extratropical storm systems. These phenomena are of major concern to forecasters and can be forecasted fairly well using the physical equations described previously.

Hours to one day: daily solar heating cycles, thunderstorms, squall lines, urban air pollution dispersion, and icing occurrences.

Seconds to minutes: Gusts, turbulence, and diffusion of pollution from a single source. These are caused or affected by such factors as the viscosity of the atmosphere and the characteristics of flow over rough surfaces.

Periodic components. Many of the motions occurring on various time scales are periodic. All of these periodicities are irregular in

some way because of the number of interactions that occur simultaneously.

Inhomogeneous. A result of the periodicities and differing scales of motion is that the probabilities of occurrence of given weather phenomena are not constant with time or from one location to another.

Not in Steady state. Weather has been occurring under approximately the same conditions (such as topography and ocean levels) throughout recent human history, but the presence of long climatic cycles means that probability distributions for weather events do not stay constant even when averaged over long periods. It appears from geological investigations that there is no such thing as a "normal" climate for the earth.

Statistically distributed. Every weather variable has a probability distribution or density function of some form, generally not exactly matching any theoretical distribution function. The meteorological theory is not well enough developed to predict the exact statistical form of the distribution. One major task in almost any weather investigation is to find an appropriate distribution function for a given set of weather data.

Characteristics of Observed Weather Data

Transformation of real variables. In reducing actual weather to observed data, the real weather variables are transformed in many ways, both intentionally and unintentionally. This section describes some characteristics that are significant in considering measured weather data. All of the properties of real weather variables still hold for observed weather data, except for the continuous and deterministic properties, which are modified.

Differing methods of observation. Some variables, such as temperature, wind speed and direction, or cloud cover, are observed directly by an instrument or observer.

Other variables must be calculated because they are not routinely measured, cannot be directly measured, or reflect applications of measured variables to special purposes. Several examples can be given:

(a) Relative humidity is calculated from the air temperature and wet bulb or dew point temperature observations.

(b) Vertical air motion cannot be satisfactorily measured using existing instruments, so equations have been developed to calculate vertical velocities from other observed variables.

(c) The geostrophic wind does not really exist but is useful for certain forecasting purposes. It describes the wind that would occur in the absence of friction as the Coriolis force acts on air masses of differing densities.

(d) The number of heating or cooling degree days has been found to be quite accurately proportional to the amount of heating or cooling required, when comparing one time period or location with another. If a day averages 65 degrees Fahrenheit or colder, the number of degrees by which the daily average temperature is lower than 65F is equal to the number of heating degree days. Similarly, if the average daily temperature is 65F or warmer, the number of degrees it is above 65F is equal to the number of cooling degree days. For example, a day averaging 50F has 15 heating degree days and a day averaging 70F has 5 cooling degree days. The heating or cooling degree days can be accumulated separately over a period such as a month or season to give a measure of the energy needed to heat or cool during the period. Of

course, a heating degree day does not cancel out a cooling degree day.

Discreteness and delays. Weather variables are observed in a discrete manner (1) in scales of measurement, due to the limitations on the precision or resolution in measurement and reporting, (2) in frequency of measurement, due to discrete observing intervals, smoothing caused by response lag times of instruments, or the need to accumulate data over a period of time, (3) by observing at discrete locations, and (4) by recording occurrence or nonoccurrence of a phenomenon. For some statistical purposes, the data can be considered continuous, but it is actually discrete, and all statistical techniques used in this thesis will be designed for handling discrete data. Note that discreteness in time means that there are delays in data availability.

Errors. Errors and discontinuities are introduced into weather data in many ways.

(1) Errors in measurement usually do not cause a systematic bias. They may often be reduced by averaging.

(2) Errors in calibration will not generally be reduced by averaging because of a systematic bias that may be present in the data. Averaging may help to detect this problem. For example, the duration of sunlight is registered by an instrument that responds to a certain level of illumination, so it is highly sensitive to calibration. From 1968 to 1978, Columbus, Ohio averaged 9.0 percent less of the possible amount of sunshine than Dayton, Ohio, while their daytime cloudiness averaged only 1.4 percent of the possible amount more than in Dayton. From 1957 to 1967, Columbus had 3.1 percent less sunshine and .8 percent more daytime cloudiness than Dayton. This data was compiled from Local Climatological Data issues from 1957 to 1978 for Columbus and

Dayton, Ohio (U.S. Department of Commerce, 1957 to 1978). The large difference in sunshine between Dayton and Columbus from 1968 to 1978 appears to be due more to differences in calibration than to actual physical differences.

(3) Elements of weather that are not instrumentally measured but are recorded by a human observer (such as type of precipitation or other weather, amount and type of cloud cover, and snow depth) introduce human biases and differences due to levels of training or experience.

(4) Changes in equipment can cause discontinuities similar to calibration errors. For example, a hygrothermometer, an electronic device to measure atmospheric temperature and moisture, will not respond in the same way as a mercury-in-glass thermometer. Most Weather Bureau (now National Weather Service) and Air Weather Service stations replaced mercury-in-glass thermometers with hygrothermometers in the late 1950's.

(5) Changes in station location cause a significant discontinuity. Until the 1930's, almost all professionally observed weather data was obtained in cities. Since the 1930's, the official source of weather data for most cities has been the airport, usually in a somewhat rural location. Even when a weather station stays at the same location in a city, the growth of the city changes the weather gradually.

(6) Administrative policies and changes in observing procedures or programs may introduce greater discontinuities than normally recognized. An example in the Air Force has been the elimination of the ROS (Remote Observing Site) as the source of weather observations at most bases. At some bases, the observer must walk all the way around

the operations building if he wants to see the whole sky when making a weather observation.

(7) Changes in public awareness have caused the number of reported tornado occurrences to increase greatly throughout this century. As weather effects on aircraft became more fully known, procedures for reporting significant weather were refined because of the need for a more complete data base.

Not complete. Since weather cannot be observed completely, only a limited number of variables are observed, and there are errors in the available data, observed weather data is not complete. There are several consequences of this: (1) Observed weather data is not completely deterministic, but can be considered to be a mixture of deterministic and random components. (2) A missing observation cannot be reconstructed exactly. (3) It is not possible to perfectly forecast the weather, regardless of the number of available weather observations. (4) The correlation between weather observations in space or time is not complete, although complete statistical independence usually cannot be assumed either.

Usable with caution. In spite of the difficulties mentioned, there is actually a vast quantity of accessible weather data of reasonably good quality. Generally, the errors are not large enough to significantly affect results if the user is aware of the data deficiencies. The existing data is usually suitable for most projects, since it is often possible to calculate new variables from the available data, and reconstruct the value of the variable into the past to produce a long data series. For example, heating and cooling degree days are calculated from daily temperatures, so that the heating and cooling days

can be calculated for many locations back to the 1800's even though the concept of heating or cooling degree days was not introduced until about 1915.

Properties of Observed Weather Data That Will Be Useful for Simulation

Random components. For simulation purposes, weather can be considered to have random components, because some factors are unmeasured. The limitations in, and uncertainties of, existing weather observations allow the data to be suitable for modeling as a stochastic process.

In many cases, it is appropriate to remove some or all of the nonrandom components from the raw data, as will be discussed in the next section. Data analysis would be performed on the residuals. All of the properties described in this section will be at least as valid for the residuals as for the unmodified data.

A finite range. Even though there is almost no theoretical limit to the range of many variables, such as the amount of precipitation that can fall, they are observed to occur in relatively limited ranges of values.

Discrete states. It is not possible to measure the weather with infinite precision. In general, the number of different values of a measurable meteorological variable is fairly small, although there may be a few hundred different values. Often, it is useful to combine these values into a smaller number of mutually exclusive ranges of values, which are called discrete states.

Recurrent. Recurrence means that any state can be obtained at any, or almost any, future time. There are no "absorbing states" or "transient states" (defined at the beginning of this section). Even if a certain value or range of values of a variable is not expected to

occur at certain times and is likely at the same location at other times (such as a temperature below freezing in the central United States, in July compared with January), it is often possible to remove nonrandom components from the data so that the states determined from the remaining components can be considered completely, or nearly completely, recurrent.

An exception to this property is a variable which is physically limited to one value at certain fixed times (such as sunshine intensity when the sun is below the horizon), or is so unlikely to deviate from a specified value at certain times that it may be considered to be fixed (such as the number of days with frost in Alabama in July). Possible procedures for handling such cases are discussed in the next section.

Aperiodic. There is some periodic component in almost any weather data, such as a daily or annual cycle, but there is a significant variable component superimposed on this, which may be considered a random (nonperiodic) component .

Homogeneous and in Steady State. Over a limited time or distance, probability distributions can be considered constant, In some cases (as in the preceding two paragraphs), removal of cyclic components may lead to constant probability distributions. Another consequence of the assumption of a steady state is that it is assumed that there is no significant unilateral trend in the data from one cycle to another, at least for the period of interest. If the removal of nonrandom components does not give reasonable results, a limited kind of steady state can be achieved by a method discussed in the next section.

Preliminary Data Analysis

Should deterministic components be removed from the data? If observed data were completely deterministic, it would not be necessary to model the data by any kind of random process. The exact values of any desired weather variable at any future time in any location would be specified by appropriate equations. However, observed weather data is considered to be a mixture of deterministic (predictable) and random components. In many cases, it is useful to remove the deterministic components from the data and perform statistical analyses on the remaining (random) components.

The first task is to determine if removing the deterministic components from the data will produce reasonable results. Occasionally, it may be best to remove some of the deterministic components but not attempt to convert the residuals to a constant probability distribution.

(1) If there is a procedure that will leave residuals which can be considered to be in a steady state (having a constant probability distribution), it will be generally easier to model the residuals rather than the unmodified data by a stochastic model. An example of this is temperature data. In many cases, the temperature can be standardized to the number of standard deviations from the average for the time of year (or day), and these transformed data values can be considered to have a constant (usually normal) probability distribution.

(2) If no procedure will produce an exact or nearly exact probability distribution of the residuals, a suitable transformation may still produce results that will be sufficiently accurate to be useful. For example, assume that monthly precipitation data is to be analyzed in an area with moderate or heavy precipitation, with very

few long dry periods. If the data is standardized by conversion to the percentage of average precipitation for the time of year, the probability distribution may vary somewhat, but the variations may be small enough to be ignored in a simulation.

An example of this could be monthly precipitation for Columbus, Ohio, at Ohio State University. Normals are given in U.S. Department of Commerce, Weather Bureau (1964) and percentile values for monthly precipitation are given by Miller and Weaver (1969). Based on this data, a monthly precipitation at least 50 percent above normal occurs from about 12 percent of the time in February to 20 percent of the time in March. Precipitation that is half or less of normal occurs from 8 percent of the time in April to 28 percent of the time in October. If substantial accuracy in predicting extreme events is not needed, these differences might be overlooked. An average distribution, generating monthly precipitation values more than 50 percent above normal about 17 percent of the time and values more than 50 percent below normal about 18 percent of the time, could be used.

(3) If there are relatively large or sudden variations in probability distributions throughout the year (or day, etc.), there may be no advantage produced by removing nonrandom components from the data. For example, in a fairly dry area with a definite rainy season, such as Arizona, the average precipitation for different months may vary (possibly) from .2 to 2 inches. For the month averaging .2 inches, probably in many years there is no measurable precipitation at all (0 percent of normal), while 2 inches of precipitation (1000 percent of normal) will probably occur occasionally. For the month averaging 2 inches, either 20 inches or no precipitation during the month would

probably never be recorded, so the probability distributions of percentage of normal would vary greatly throughout the year. In a case such as this, expressing the data in terms of percentiles may give more realistic results, but special procedures (as described later in this chapter) should be used if a variable is restricted to a single value at certain times. For example, snowfall may not occur at a location at all during some months of the year, so a percentile value would be meaningless during these months.

(4) In some cases, there are physical limits on the data values, as above, either on one end or both ends of the permissible range. For example, precipitation and wind speed are restricted to nonnegative values, and the proportion of the sky covered with clouds must be in the range from 0 to 1 (0 to 10 tenths). Sunshine intensity is zero when the sun is below the horizon. If there is a physical limit, and especially if one or the other of the limiting values occurs occasionally or frequently (such as months with no precipitation, when analyzing monthly precipitation data), it is usually best to work with the original data without removing the nonrandom components.

Techniques to use if nonrandom components can be removed. If it is reasonable to remove the deterministic components from a data series, analysis and simulation is a four-step process: (1) Separate the deterministic components from the data series and develop an equation or procedure to specify the values of the deterministic components. (2) Develop a statistical description of the residuals, or random components. (3) Simulate a series of random components using the statistical description. (4) Superimpose a series of deterministic values onto the series of random values to produce a final simulated

data series.

No complete discussion of data transformation techniques can be given here, but some possible techniques can be named. A very basic procedure is to standardize data to the number of standard deviations from the average value for the time of year (or day, etc.), or to partially standardize by using the percentage of the average value. Spectral analysis may be used to develop a smoothed curve of "normal" values throughout the cycle. In some cases where the data is restricted to nonnegative values, it may be useful to work with the logarithms of the data values rather than the data itself before standardization (with a small value added to each data point before taking the logarithm if there are any zero values, as suggested by Srikanthan and McMahon (1968). Another data transformation technique is to convert the data values into percentiles, unless a lot of the data values are the same. (For example, if 50 percent of the data values are zero, there is no distinction between the 10th and 40th percentiles). Basically, meteorological reasoning must be used to determine an appropriate transformation.

The preceding techniques are intended to be used with variables that have a large number of possible values, such as monthly precipitation measured in units of .01 inch. If it is desired to reduce the number of possible values to a small number of states, the basic data transformation should be performed first. If the data is already in the form of a small number of different states, probably no data transformation should be performed.

Techniques to be used if nonrandom components should not be removed. When realistic results will not be achieved by using a transformation such as those suggested above, or if the precision gained

is not necessary, or if some deterministic components are removed but the residuals do not have a constant probability distribution, it may be reasonable to simply ignore the lack of a steady probability distribution. For example, Yost and Aronson (1977) assumed that the average cloud cover in Albuquerque is the same throughout the year, even though the data shows that the average cloud cover varies from less than 2 to over 5 tenths at different times of the year, with an especially rapid rise in average cloudiness in early July.

If the differences in probabilities should not be ignored, the most common technique is to use different probability distributions for different times of the year (or day, etc.). These subintervals should be chosen so that the probabilities within each subinterval can be considered constant. For example, Haan, Allen, and Street (1975) used separate precipitation probability distributions for each month of the year in their simulation. More data would be required, of course, to give the same confidence for each probability distribution as when considering the data as a whole.

Another possible technique is used by Feyerherm and Bark (1965). They hypothesized that the probabilities involved in their data vary smoothly throughout the year, so they used spectral analysis to develop a smoothed probability cycle throughout the year for each independent probability. If there is a reasonable basis for making this hypothesis, this method could be considered a "variance reduction technique," to lead to both an accurate reproduction of the underlying trend and high confidence in the validity of the statistically-derived cycle.

In some cases, as mentioned previously, a variable is restricted to a single value, or is almost completely limited to a single value,

at some fixed times (such as sunshine intensity when the sun is below the horizon). It is best to simply specify the value of the variable at these times. If the times involved are somewhat variable, such as the season of no snowfall in most mid-latitude continental locations, random procedures may be appropriate to determine the starting and ending dates or times.

III. Theory of Markov and Semi-Markov Processes

Characteristics of Stochastic Processes

A stochastic process is used to statistically describe the operation of a system that behaves as follows:

(1) The system condition is observed at specified values of an independent variable. This variable is usually time (meaning that the system is observed at various times), but one objective of this thesis is to observe systems where the independent variable consists of points in space, and time is held constant at each observation of the system. Only discrete, equally-separated values of the independent variable will be considered here, and their values will usually be denoted by sequences of the nonnegative integers.

In higher-dimensional cases, a vector independent variable will be used. If a weather condition is measured over an area, the independent variable will be expressed as a two-element vector, as in a Cartesian coordinate system. This can be extended to higher dimensions, and time can be one of the dimensions.

(2) For each specified value of the independent variable (a given time, or a given point in space), the system is observed to be in exactly one of a number of states. The observed state can be called an outcome. In this thesis, the number of different states is generally considered to be finite.

(3) The state of the system is a random variable, meaning that the probabilities may be described by a probability vector, which has one element for each state and whose elements are the probabilities associated with each state. Since the processes defining the probabilities are not limited to any particular form, any vector whose components

are nonnegative and sum to 1 can be called a probability vector (Miller, 1977, page 7-3).

One-Dimensional Markov Processes

A general Markov process is a stochastic process in which the probability distribution at any observation depends only on a specified set of other actual observations. If these actual observations are known, knowledge of additional data values will not give any more information about the probability distribution. In particular, it is not necessary to know all of the other data values in a series to specify the probability distribution at an observation.

Most discussions of Markov processes are confined to one-dimensional processes, where the independent variable is a scalar, such as time. Assume that a system is observed at equidistant time intervals and that the system is observed to be in exactly one of N different states at each time. Then, in a first-order Markov process, the probability distribution of the state at any observation depends only on the value at the immediately preceding observation. More generally, a Markov chain of order r assumes that the probability distribution at each observation depends only on the last r available observations.

In a first-order Markov chain, where i and j represent states ($i = 1, \ldots, N$; $j = 1, \ldots, N$), let p_{ij} be the transition probability, or probability that a system which is in state i at one observation will be in state j at the next observation. The set of transition probabilities for all i and j combinations can be expressed as an N x N transition probability matrix (TPM), which will be normally denoted by P. P is a stochastic matrix, a matrix whose rows are stochastic vectors.

If $i \neq j$, a real transition occurs, meaning that the system enters a new state. If $i = j$, a virtual transition occurs, meaning that the system remains in the same state. This distinction will only become significant when discussing semi-Markov processes.

For the following discussion, the properties of observed weather data that were discussed in the last section of chapter II will be assumed to hold. Also, each state will be considered to be recurrent (observable at any or almost any time or location) and aperiodic (not occurring at absolutely fixed intervals), and the stochastic process will be considered to be in a steady state (the probability vector will be considered constant over the values of the independent variable). When each state is recurrent and aperiodic, each state is called ergodic. A more extensive discussion of each of these terms is given in standard textbooks about stochastic processes, such as Feller (1968, Chapter XV), and these conditions will define a certain type of Markov process.

Let $s(t)$ be the state observed at time t. Let $V(t) = (v_{t1}\ v_{t2} \cdots v_{tN})$ be the vector of unconditional probabilities for each state at time t. Then, the basic definition of a first-order Markov chain requires that

$$P\left[s(t) = j \mid s(t-1) = i,\ s(t-2) = h,\ s(t-3) = g, \ldots\right]$$
$$= P\left[s(t) = j \mid s(t-1) = i\right] = p_{ij}.$$

Feller (1968, chapter XV) shows, using slightly different notation, that

$$V(1) = V(0)P \quad \text{and} \quad V(t)P = V(0)P^{t},$$

where $V(0)$ is the initial unconditional probability distribution. As t approaches infinity, $V(t)$ approaches a limiting vector called $V = (v_1 v_2 \cdots v_N)$ if the Markov chain is ergodic, which is true for

most weather variables. This limiting vector is the steady-state distribution and does not depend on V(0). Since weather has been occurring long enough that it can be considered to be in a steady state (at least during the available period of record), this limiting distribution should be equal to the observed unconditional probability distribution. In general, the value of V is obtained from a transition probability matrix by solving V = VP, using the fact that the sum of all of the elements of V is 1 to obtain a unique vector.

As described earlier in this section, higher-order dependence can be treated using Markov concepts. The most frequently-used notation is called "expanded state representation." For a chain of order r, the state at an observation is denoted by an r-element vector consisting of the r most recent observations, in sequence. For two consecutive observations, the last r-1 elements of the state vector for the first observation must be identical to the first r-1 elements of the state vector for the second observation. For example, assume there are three states denoted by 0, 1, and 2. A second-order vector state of (2,1) denotes a present observation of state 1, with state 2 at the preceding observation. At the next observation, the only possible vector states are (1,0), (1,1), and (1,2).

Using this notation for the states, an appropriate notation for the probability of a transition from state (i,j) to state (j,k) is p_{ijk}. For a chain of order r, the probability of transition from state (i,j, . ., k) to state (j,k, . . ,l) is $p_{ij\ldots kl}$, where each state vector has r elements and the probability has r+1 subscripts. The transition probability matrix is $N^r \times N^r$ in size, with zero probabilities for impossible transitions. For example, 3-state first- and

second-order transition probability matrices are shown in figure 2, with probabilities that may be nonzero indicated by *. Most mathematical manipulation is the same as for first-order matrices. For example, V = VP will give the limiting value of the unconditional probability distribution of each expanded state.

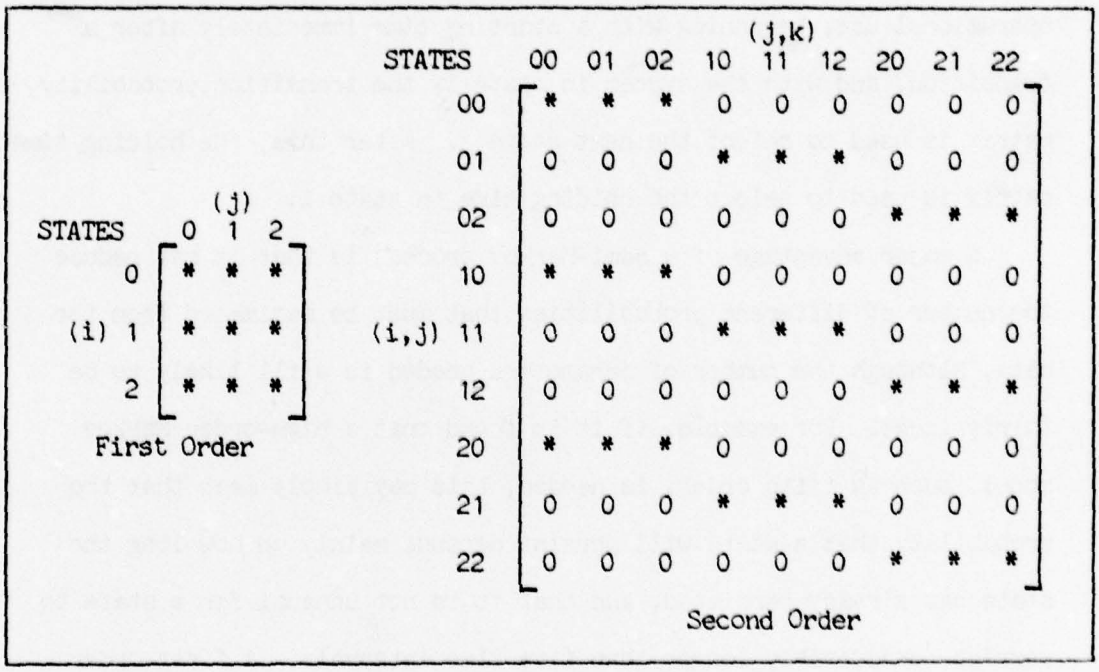

Figure 2. Transition Probability Matrices for First- and Second-Order 3-State Markov Chain

Semi-Markov Processes

Basic theory and practice. When discrete observation times are considered, a semi-Markov process differs from a Markov process basically only in allowing the times between transitions to be random variables of some arbitrary distribution. Merrill (1974) gives an extensive discussion of the theory and use of semi-Markov processes. For a transition from state i to state j, there is a transition probability p_{ij} and also a holding time vector H_{ij}. Each element of the holding time vector, denoted by $h_{ij}(t)$, with t = 1, 2, . . . , is the

probability that the system will remain in state i for t time units, given that the system will next enter state j. Therefore, the vector H_{ij} is a stochastic vector. In most operational cases, t is not allowed to exceed a certain value T. Then, the complete holding time matrix H is a three-dimensional matrix with dimensions N x N x T. For basic operational use, beginning with a starting time immediately after a transition, and with the system in state i, the transition probability matrix is used to select the next state j. After this, the holding time matrix is used to select the holding time in state i.

A major advantage of a semi-Markov process is that it may reduce the number of different probabilities that must be estimated from the data, although the number of parameters needed is still likely to be fairly large. For example, if it is found that a high-order Markov model, such as fifth order, is needed, this may simply mean that the probability that a state will persist depends mainly on how long the state has already persisted, and that it is not unusual for a state to persist considerably longer than five time intervals. A first-order semi-Markov model allowing holding times up to T = 24 hours may describe the data at least as well. If there are five states, a fifth-order Markov model will require 5^6 = 15625 different probabilities to be specified. Since each probability vector, with N = 5 elements, must add up to 1, there are N-1 = 4 independent probabilities in each vector, and the total number of independent probabilities is 5^5 x 4 = 12500. A first-order semi-Markov model, with T = 24, requires 5^2 = 25 probabilities for matrix P and 5^2 x 24 = 600 probabilities for the holding time matrix H, or a total of 625 probabilities. The number of independent probabilities involved in this case is 480, which will be explained

under alternative (2) below.

There are several alternatives or options in the operational use of a semi-Markov process.

(1) Allowing virtual transitions is optional. If virtual transitions are allowed, there is more than one way to obtain a given holding time in a state. The distinction between a three-hour holding time, compared to a holding time of one hour in a state, followed by a holding time of two hours in the same state, is meaningless. To obtain holding time distributions from a set of actual data, it is best to consider only real transitions, except when the holding time exceeds T, as discussed in the next paragraph.

(2) The decision to limit the maximum holding time to a particular value, T, to control the size of the holding time matrix, should be made from the data. There should be an insignificant probability of a holding time greater than T, if the state is not allowed to "renew" itself after it has persisted for T time intervals. To approximate the distribution of holding times greater than T, one possible approach is to allow virtual transitions with a probability p_{ii} which is equal to the probability of state i persisting longer than T time units. This approximates the probability of a holding time of T + t time units, given that the state has persisted T time units, by the probability of a holding time of t units.

In this case, $h_{ii}(t) = 0$ if $t \neq T$ and $h_{ii}(T) = 1$. To calculate the number of independent probabilities, each vector in P has N-1 = 4 independent probabilities (considering the five-state example used so far in this chapter), and each vector H_{ij} has T-1 = 23 independent probabilities. However, since H_{ii} is exactly specified, it has no

independent probabilities. Therefore, the number of independent probabilities is 5 x 4 (for P) + 5 x 4 x 23 (for H, excluding the H_{ii} vectors) = 20 + 460 = 480.

(3) A large amount of computer storage space could be saved by approximating each H_{ij} distribution by a formal distribution, so only the parameters of the distribution would need to be stored. For example, assume that H_{ij} can be approximated by a normal distribution. Then, only the mean and standard deviation would need to be stored for each (i,j) combination. For the five-state semi-Markov example given above, the total number of parameters to store would be 25 (for P) + 5^2 x 2 (for H) = 25 + 50 = 75, and the number of independent parameters is 20 (for P) + 50 (for H, since none of the parameters are independent) = 70. Appropriate statistical tests would need to be performed to justify the choice of a distribution.

Equivalence of discrete semi-Markov and Markov processes. As shown by Merrill (1974), a discrete semi-Markov process can be expressed by an expanded state discrete Markov process, so the theory of Markov processes can be shown to apply to discrete semi-Markov processes.

If the system condition at each observation is denoted by a two-element vector (i,t) containing the state i and the time t that the state has persisted, all possible combinations of states and persistence times can be specified uniquely. As in the expanded state representation of a higher-order Markov process, only certain transitions are possible. For example, if state (i,t) is observed at one time, at the next hour either the same state has persisted one more hour, denoted by state (i, t+1); or a new state j is observed, and it has persisted up to one hour, denoted by (j,1). With this notation, it is still

possible to limit the maximum time to a value T if desired. Then, state (i,T) is followed by either (i,1) or (j,1), and there is a total of NT possible states.

With this notation, a transition probability matrix can be prepared as for a higher-order Markov process, and its dimensions will be NT x NT. However, for each state, there are only N possible transitions (one transition from (i,t) to (i, t+1) and N-1 transitions from (i,t) to (j,1)), so the number of probabilities that must be found is N^2T. This expanded-state representation incorporates both the P and H matrices into one matrix, so the number of independent probabilities is the same using either representation. In the five-state semi-Markov example, each state probability vector has N-1 = 4 independent elements, and there are NT = 120 states, so the number of independent probabilities is 480.

A semi-Markov concept is more appropriate than an expanded-state Markov concept, as a way of thinking about the problem considered in Merrill's thesis, but a semi-Markov process appears to be fundamentally a one-dimensional process. Also, since the argument given in this section shows that a discrete semi-Markov process can be equivalently expressed as a Markov process, semi-Markov processes will not be a further major concern of this thesis.

Estimation and Statistical Testing

Introduction. In justifying the fact that applying a Markov-type process to a specific set of data is a useful way of describing some of the statistical characteristics of the data, there are two issues that need to be considered. The problem of estimation discusses how to obtain estimates of transition probabilities and other parameters

from a set of observed data. Statistical testing concerns the legitimacy of the results, and helps answer two questions: Does a certain hypothesized Markov chain reproduce the desired statistical characteristics of the data well? Is one Markov process better than another one in describing these statistical characteristics?

Maximum likelihood estimation of parameters. So far in this thesis, it has been assumed that estimates of transition probabilities and unconditional probabilities are obtainable from a set of observed data. Actually, it is true that the most intuitively appealing estimators are the maximum likelihood estimators. An unconditional probability of a state is best estimated by the number of occurrences of that state divided by the total number of observations. A transition probability from state i to state j is best estimated by the total number of transitions from state i to state j divided by the total number of transitions from state i. Anderson and Goodman (1957) give more details about maximum likelihood estimation of parameters under varying circumstances.

For example, assume that of a total of n observations and N states, there are n_i observations of state i. The true probabilities are given by vector $V = (v_1\ v_2\ .\ .\ v_N)$, where

$$v_N = 1 - \sum_{i=1}^{N-1} v_i \ .$$

Therefore, the likelihood function is

$$L = v_1^{n_1}\ v_2^{n_2} \ . \ . \ . \ v_N^{n_N} = \left\{ \prod_{i=1}^{N-1} v_i^{n_i} \right\} \left\{ 1 - \sum_{j=1}^{N-1} v_j \right\}^{n_N}$$

which is to be maximized. Taking the derivative of ln L with respect to each variable and setting each derivative to zero, for each value

of i from i = 1 to N - 1, gives

$$\frac{d(\ln L)}{d\, v_i} = \frac{n_i}{v_i} - \frac{n_N}{1 - \sum_{j=1}^{N-1} v_j} = 0 \ .$$

Solving these N-1 equations for each v_i, the general solution, which is the maximum likelihood estimator for v_i (denoted $\hat{v}_i$), is

$$\hat{v}_i = n_i / \sum_{j=1}^{N} n_j = n_i/n \ .$$

So, transition probability $\hat{p}_{ij}$, where n_{ij} is the observed number of transitions from state i to state j, is

$$\hat{p}_{ij} = n_{ij} / \sum_{j=1}^{N} n_{ij} \ .$$

For a higher order, let $n_{ij..kl}$ denote the number of observations of the sequence (i,j, . ., k,l), with $p_{ij..kl}$ being the corresponding transition probability. For order r, each term has r+1 subscripts. Then,

$$\hat{p}_{ij\ldots kl} = n_{ij\ldots kl} / \sum_{l=1}^{N} n_{ij\ldots kl} \ .$$

Expected frequencies of runs. Research which demonstrated the fact of persistence in weather data series (usually sequences of wet and dry days) led up to the use of Markov processes as a way to model weather data. This research was usually based on a study of the frequencies of wet and dry sequences of various lengths.

If there is no persistence, the number of sequences of length k will form a geometric series. Also, in two-state Markov processes, the number of sequences will have a geometric form, even though there is persistence from one observation to the next. This section derives the expected number of sequences of given lengths. These run length frequencies will be used in hypothesis testing of Markov order.

The terminology in this section will assume a data sequence of wet (W) and dry (D) days, although any other two-state sequence could be used. The total length of the series is N days. The probability of a wet day is p, and the probability of a dry day is 1-p = q. The expected number of sequences of length k is ap^k (wet) and $b(1-p)^k = bq^k$ (dry), since it is assumed that the probability of a wet (or dry) day is unaffected by the character of the preceding day, and where a and b are constants to be found from the data as described later.

Then, the total number of wet sequences is

$$W = ap + ap^2 + ap^3 + \ldots = ap(1 + p + p^2 + \ldots)$$
$$= ap/(1 - p) = ap/q ,$$

and the total number of dry sequences is

$$D = bq + bq^2 + bq^3 + \ldots = bq/(1 - q) = bq/p .$$

For a long data series, assume that the number of wet and dry sequences is equal, or ap/q = bq/p, although the numbers could differ by one regardless of the length of the series.

The total number of days in all wet sequences, or the total number of wet days, is

$$N_W = ap + 2ap^2 + 3ap^3 + \ldots$$
$$= (ap + ap^2 + ap^3 + \ldots) + (ap^2 + ap^3 + \ldots) + (ap^3 + \ldots) + \ldots$$
$$= ap(1 + p + p^2 + \ldots) + ap^2(1 + p + \ldots) + ap^3(1 + \ldots) + \ldots$$
$$= (ap + ap^2 + ap^3 + \ldots)(1 + p + p^2 + \ldots)$$
$$= ap(1 + p + p^2 + \ldots)^2 = ap/(1-p)^2 = ap/q^2 = pN$$

Similarly, the total number of dry days is $N_D = bq/p^2 = qN$. Of course, the total number of days is $N = ap/q^2 + bq/p^2 = pN + qN = (p + 1 - p)N = N$.

Solving for a and b, $a = q^2pN/p = q^2N$, $b = p^2qN/q = p^2N$.

Summarizing, the expected number of sequences is

Wet: (length k) $W_k = p^kq^2N$, (total) $W = pqN$

Dry: (length k) $D_k = p^2q^kN$, (total) $D = pqN = W$.

If there is persistence, the procedure is only slightly modified. Let x be the probability of a wet day following a wet day, and let y be the probability of a dry day following a dry day. Then the expected number of sequences of length k is $W_k = ax^k$ (wet) and $D_k = by^k$ (dry). The total number of sequences is $W = ax/(1-x)$ (wet) and $D = by/(1-y)$ (dry), and it is assumed that $W = D$. The total number of wet days is $N_W = ax/(1-x) = pN$ and the total number of dry days is $N_D = by/(1-y) = qN$, with $N_W + N_D = N$. Solving for a and b,

$$a = pN(1-x)^2/x \ , \qquad b = qN(1-y)^2/y \ .$$

Therefore, the number of sequences is

Wet: (length k) $W_k = pN(1-x)^2x^{k-1}$, (total) $W = pN(1-x)$

Dry: (length k) $D_k = qN(1-y)^2y^{k-1}$, (total) $D = qN(1-y) = W$.

If there are more than two different states in a sequence, probably the easiest method to calculate the number of sequences of each kind is to combine all states except one into one group and use this procedure with the two groups. The expected number of sequences of the group containing only one state would give the expected number of sequences of that state. Repeating this for each other state in turn would give the expected number of sequences for each state. For higher order dependence, the treatment of probabilities of short sequences becomes more complex and will not be considered here.

Statistical testing: the runs test. According to Mendenhall and Scheaffer (1973, chapter 15.6), the expected number of runs in a long

two-state series of length N, with probabilities p and 1-p = q, and where n_p is the number of observed occurrences of the state with probability p, n_q is the number of observations of the other state, and $n_p + n_q = N$, is

$$E(R) = \frac{2n_p n_q}{n_p + n_q} + 1$$

which may be modified to

$$E(R) = 2Npq + 1 .$$

Also, the variance is

$$V(R) = \frac{2n_p n_q(2n_p n_q - n_p - n_q)}{(n_p + n_q)^2(n_p + n_q - 1)} = \frac{2pq(2pq - 1/N)}{1/N - 1/N^2}$$

and $S(R) = \sqrt{V(R)}$.

Then, if a long series (usually considered to be $N \geq 30$) has R observed runs, the test statistic

$$Z = \frac{R - E(R)}{S(R)}$$

has an asymptotic normal distribution with zero mean and unit variance. If the calculated absolute value of Z is smaller than the critical value - Z_α for a selected α level (note that a one-tailed test is used), the hypothesis of a random (nonpersistent) series can be rejected. However, this test does not tell what the level of dependence is.

Statistical testing of observed and expected frequencies of run lengths: a chi-squared test. A large sample which is categorized into frequencies of occurrence of runs of given states can be tested for goodness of fit to a hypothesized distribution by a chi-squared test, as described by Elderton (1902) with slightly different notation.

Assume there are k frequency groups. When the expected number of observations in a group is less than 5 or 10, it is best to combine groups (and reduce the value of k accordingly) so that there are expected

to be at least 5 or 10 observations in each group. The observed frequency of the rth group is m_r and the theoretical frequency is M_r. Then, the chi-squared test statistic is

$$\chi^2 = \sum_{r=1}^{k} \frac{(m_r - M_r)^2}{M_r}$$

with k-1 degrees of freedom and the usual rejection regions.

This test can be used to demonstrate the fact of persistence in a data series by comparing the observed frequencies with the frequencies expected under the assumption of no persistence. This test can also be used to test the goodness of fit of a Markov chain or other model if the expected frequency can be calculated. However, this test does not test all of the independent probabilities in Markov chains with more than two states or of second or higher order, since it only tests sequences of observations of the same state.

A chi-squared test of Markov order. Several authors, including Anderson and Goodman (1957), Lee (1973), and Lowrie and Guthrie (1968) describe a general test that a Markov chain is of order r-1, rather than order r or higher. The notation of Lee appears to be most convenient for the purpose of this paper, so it will be used with only slight modifications to better match the notation used elsewhere in this thesis.

Let $n_{ij...k1}$ be defined as it was defined earlier in this section. Let

$$n_{ij...k-} = \sum_{i-1}^{N} n_{ij....k1}$$

where there are N states. Also, all probabilities are maximum likelihood estimators (although superscripts indicating that these values are

estimates will be omitted), so

$$p_{ij\ldots kl} = n_{ij\ldots kl}/n_{ij\ldots k-}, \text{ and}$$

$$p_{j\ldots kl} = \sum_{i=1}^{N} n_{ij\ldots kl} \Big/ \sum_{i=1}^{N} n_{ij\ldots k-} .$$

Then, the chi-squared test statistic is

H_o: chain is of order r-1

H_a: chain is order r but not order r-1

$$\chi^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} \cdots \sum_{k=1}^{N} \sum_{l=1}^{N} \left[n_{ij..k-} \left(\frac{(p_{ij\ldots kl} - p_{j\ldots kl})^2}{p_{j\ldots kl}} \right) \right]$$

with $N^{r-1}(N-1)^2$ degrees of freedom.

IV. Literature Survey - Statistical Analysis of Time Series Weather Data

Historical Overview

Attempts to apply Markov concepts to the analysis of weather data bases are mostly recent, but the number of published applications of Markov, semi-Markov, and modified Markov processes is already quite large. Therefore, this will not be an exhaustive survey of the subject, but will mainly focus on summarizing methods used and significant results obtained, with a goal of obtaining insights that will help in modeling a two-dimensional data base.

The fact that weather is persistent, or that there is a positive correlation between observations, was demonstrated statistically as early as 1916 (Newnham). Newnham and other authors found fewer short sequences and more long sequences than would be expected if the data is randomly distributed.

By the early 1950's, various authors attempted to fit various kinds of logarithmic series to the frequencies of the lengths of wet and dry spells. Also at this time, Brooks and Carruthers (1953) suggested the possibility of an underlying Markov chain, although their model was a modification of a basic Markov process.

Possibly the first published account of an application of a Markov chain to a weather data sequence considered sequences of wet and dry days during the winter season at Tel Aviv (Gabriel and Neumann, 1962). This study considered Markov chains up to third order. Within a few years, many published articles reported on the examination of other data and reexamination of the earlier studies under the Markov model. Many of these articles used modified Markov or non-Markov models to improve

the fit. Most studies considered only two-state Markov chains, usually sequences of wet and dry days. One of the first published weather models considering more then two states is by Lee (1973).

Extending Markov concepts to two or more dimensions was a nearly unexplored subject until recently, with possibly the first published articles being written by Galbraith and Walley (1976) and Pickard (1977). Pickard's article described a homogeneous generation scheme in two dimensions that leads to a stationary Markov field, a grid (or lattice) for which the probability distribution of states at each point depends only on the states that are observed on a specified set of neighboring points. The concept of a Markov field had been previously developed, but simulating such data fields had been done mainly by regression-type methods.

Statistical Analysis of Persistence

First, a few concepts that led up to the first published Markov applications should be summarized. As stated before, if there is no persistence, the number of wet (or dry) spells of various lengths would conform to a geometric series of the form $y = ax^n$, where y = number of spells (runs) of length n, x = unconditional probability of a wet (or dry) day, and a is a constant determined from the data.

Jorgensen's (1949) treatment of persistence is possibly the most sophisticated early approach, and his methods were designed to be suitable for operational use. He assumed that a geometric progression would determine the expected frequencies of lengths of winter wet and dry spells in San Francisco, in the absence of persistence. By using charts, he showed a comparison of the probability of N more wet (or dry) days, given that it has been wet (or dry) for M consecutive days.

with the probabilities given by the geometric series. These charts showed relatively little variation of the probabilities for different values of M (since M was at least 1), which helped to justify the Markov approaches used by later investigators.

After several similar studies demonstrated the fact of persistence in many weather sequences, several authors attempted to fit various kinds of series to the frequency counts of lengths of wet and dry spells. For example, Williams (1952) and Cooke (1953) fit logarithmic series of the form $y = (ax^n)/n$ to rainfall data for Harpenden, England, and Moncton, New Brunswick. Also, Williams discussed the logical difference between a wet and a dry day. A dry day requires at least 24 to 48 hours of no precipitation, since (for example) if precipitation stops immediately after the first observation on one day and resumes just before the observation at the end of the next day, the nearly 48-hour dry period would result in no dry observational days. A wet day could result from only a few seconds or minutes, or up to 24 hours, of precipitation during the observational day. Because of this logical difference, and the fact that a precipitation data series consists of alternating dry and wet periods, which could be controlled by different processes, these authors tested the wet and dry sequence frequency counts separately.

Longley (1953) used a logarithmic series of the form $\ln y = ax + b$, and tested statistically for deviations of the logarithms of frequencies of both dry and wet spells with various lengths from a straight line. He also used the frequency counts to estimate the probability of a wet day folowing a dry day, and a dry day following a wet day. While he compared these (maximum likelihood estimate) probabilities with probabilities derived from the slopes of the least-squares lines for dry and

wet spells, and found some substantial differences, he was very close to using a Markov model since the frequency-count procedure derived all of the probabilities needed for a first-order Markov chain.

In all of these cases, the statistical treatment of persistence resulted in more realistic descriptions of the distribution of the lengths of dry and wet spells. However, these models did not result in a simple-to-use scheme for simulation which would generate simulated data series with the characteristics that were found to be a satisfactory explanation for the observed data.

Two-State Markov Chains

Since the statistical methods for Markov chains were quite well developed before any published applications to weather data were made, some of the earliest articles did not confine their attention to first-order Markov chains.

Probably the earliest published paper is that of Gabriel and Neumann (1962). This paper considered Markov chains up to third order, based on winter (December to February) precipitation at Tel Aviv from 1923-24 to 1949-50. Meteorological reasoning using comparisons of daily precipitation probabilities for each month was used to justify treating these months together. The published data was given in a form suitable for immediately calculating the transition probabilities. Also, chi-squared tests demonstrated that the model fit the data reasonably well, even for first order. The advantage of a Markov model was demonstrated, since only a small number of parameters were required, from two for a first-order model to eight for a third-order model. All of the maximum likelihood estimates of transition probabilities were easily calculated from the data.

Due to the success of this attempt to fit weather data as a Markov process, investigators such as Caskey (1963) and Weiss (1964) considered two-state Markov chains of first order. Articles such as those written by Sakamoto (1970), Lowrey and Guthrie (1968), Feyerherm and Bark (1967), Denny, Kisiel, and Yakowitz (1974), and Medhi (1976) considered other orders (up to sixth order in Denny, Kisiel, and Yakowitz's paper), with the use of chi-squared tests to determine the most appropriate order for the Markov chain obtained. Sundararaj and Ramachandra (1975) considered only first-order two-state Markov chains, but applied separate chi-squared tests to the frequencies of lengths of dry and wet spells to show their individual contributions to the goodness of fit of the total Markov transition process. Feyerherm and Bark (1965) not only considered Markov chains of order 0 (unconditional probabilities), 1, and 2, but also allowed the transition probabilities to vary smoothly throughout the year. Their method was to use a Fourier series with harmonics of 1 to 12 periods per year to describe a smoothed annual curve for each independent probability involved. Since such smoothed transition probabilities could not be statistically tested for goodness of fit, chi-squared tests were performed on transition matrices based on several 40-day periods during the year.

Several authors, including Green (1964, 1965, and 1970) and Wiser (1965) considered modified Markov models, which are beyond the scope of this paper. Green's (1964) model was derived from a continuous-time model. These models could be considered equivalent to Markov chains of very high order (theoretically infinite order in some cases) or semi-Markov models.

Markov Chains with More Than Two States

Not much work has been reported describing Markov chains with more than two states. This is probably because of several reasons: (1) More independent parameters must be estimated. (2) The statistical tests may be less conclusive. (3) Few additional theoretical insights are expected to be obtained. (4) Some of the theoretical results, such as exact formulas for expected values and variances, are not as easy to express as when there are only two states. (5) Probably many studies have been performed for operational use and simulation, for which exact statistical goodness of fit is less important, and for which results and methods have not been widely reported.

One study which is in the operational category is by Yost and Aronson (1977), which presents an 11 x 11 transition matrix for average daily cloud cover (from 0 to 10 tenths) at Albuquerque. This model assumes homogeneity although the unconditional probability distribution for cloud cover varies throughout the year. The reasonableness of the resulting model was only discussed subjectively, but this was good enough for the operational use of the model in this case.

Another interesting operational approach designed for simulation was used by Haan, Allen and Street (1976) to simulate sequences of daily rainfall. The daily precipitation (measured to the nearest .01 inch) was separated into the following seven classes: none or trace, .01-.02", .03-.06", .07-.14", .15-.30", .31-.62", and at least .63". Transition matrices using these seven classes were computed for each month of the year. These transition matrices could be used to simulate a precipitation sequence by classes of precipitation amounts. However, simulated precipitation amounts to the nearest .01 inch were desired,

so uniform probability distributions were used within the middle five classes, and a shifted exponential distribution in the highest precipitation class. After the precipitation class for a day was determined, another random number was drawn to produce a precipitation value to the nearest .01 inch. The use of uniform probability distributions in the middle five classes resulted in a slight overestimation of the average annual precipitation in the simulations, and methods were described for dealing with this if the differences were considered significant.

One of the most complete treatments of the subject of fitting Markov chains to weather data sequences is that of Lee (1973). He considered semi-Markov processes, as well as Markov processes of up to third order, and the chi-squared statistical tests of order, for data series of seven different weather variables. The Markov processes considered had either two or three states. Satisfactory fits were obtained in most, but not all, cases. Where the model did not fit the data satisfactorily, the limited amount of data prevented higher order models from being tested.

Lameiro and Bryson (1978) considered a vector state with three components to simulate a weather data series for evaluating the performance of a solar heating system. The three components were temperature of stored water, insolation intensity, and ambient (outside air) temperature, each with several possible values. The simulation used a first-order process to generate values of the three variables for each hour and produced fairly accurate results, even though it is known that significant dependence of the values of these variables will extend over a much longer period than one hour. Of course, even a second-order matrix probably would have been too complex to use in this simulation.

Air Weather Service Technical Report 77-273 (Miller, 1977, chapter 7) summarizes various methods of Markov simulations of weather data, with a focus on multi-state and vector state phenomena. Two methods of dealing with large numbers of states are described, both based on linearization of the expanded-state Markov process.

Semi-Markov Applications

Since it is well known that discrete semi-Markov processes can be converted into expanded-state discrete Markov processes, and since it appears that semi-Markov processes are fundamentally one-dimensional, only a limited bibliography will be provided for the benefit of users. A fairly extensive general treatment of semi-Markov processes in both continuous and discrete time, with examples, was given by Nunn and Desiderio (1977). Few applications of semi-Markov processes to weather data have been published.

Lee (1973) included semi-Markov models in his treatment of seven weather variables. He described a testing procedure to determine if a Markov or semi-Markov model fits the data best.

Merrill (1974) developed a 10-state semi-Markov model for ceiling and visibility categories at Bitburg Air Base, Germany. This model was based on hourly observations, included holding times of up to 100 hours, and had a different set of transition matrices for each of four seasons. While this model would contain over 10,000 parameters for each season (10 x 10 x 100 holding times, 10 x 10 transition probabilities, and 10 unconditional probabilities, with some values zero because only real transitions were considered), few cases of holding times over 30 hours occurred. He suggested that the number of parameters could be reduced either by reducing the maximum holding times to between 25 and

40 hours, depending on the season, or by replacing the holding time probabilities by formal distributions. Other possibilities would be (1) to consolidate the holding time categories into a smaller number of nonuniform holding time intervals, with random procedures similar to those used by Haan, Allen, and Street (1976) to select any integer holding time length within each interval, or (2) to allow virtual transitions, with a probability equal to the probability of exceeding the maximum holding time in the matrix. The holding time in case of a virtual transition would be set to the maximum holding time, so that a state could (in rare cases) persist a very long time, just as it does in nature.

Alternative Statistical Tests of Markov Order

Determining the order of a Markov chain is actually a multiple decision process, and is a subjective decision if chi-squared tests are used and there are more than two feasible candidates for the order of the Markov chain. This is because, as the order of the Markov chain used to fit a data series is increased, the quality of the fit increases, until in the extreme case of a chain of order n - 1 based on a series of n observations, the series is exactly specified with no randomness. This quality of fit is expressed as a significance level, and the lowest-order model that exceeds an arbitrarily determined level of significance is accepted as the "true" Markov order of the data. If a different significance level is chosen, a different model may be chosen. For example, Gabriel and Neumann (1962) chose a first-order model to fit their data based on a significance level of .05. If the chosen level had been .10, a second-order model would have been required to fit the data.

A second type of approach is the likelihood ratio approach, first considered by Bartlett (1951). Basically, this method determines the likelihood of each alternative considered and uses an empirical procedure based on a ratio of the likelihoods to choose the proper Markov order. Two methods using this approach involve the Akaike Information Criterion (Tong 1975, Gates and Tong 1976, and Chin 1977) or the Schwarz Bayesian Criterion (Schwarz 1978). Each method is based on a slightly different assumption about the amount of "information" to discriminate between orders (or competing models) considered in the test. Both methods attempt to balance between underfitting, which has a large variance, and overfitting, which requires a large number of parameters (Gates and Tong 1976).

Both models contain a term which will be denoted by ${}_kH_m$, using the notation of Chin (1977). This term equals $(-2)\ln\lambda_{k,m}$, where $\lambda_{k,m}$ is the ratio of the maximum likelihood under the assumption of order k to the maximum likelihood assuming order m, with k smaller than m. The use of the logarithm of the likelihood is convenient because of the size of the numbers typically involved, and because, under certain conditions which are fulfilled by these Markov chain models, $(-2)\ln\lambda_{k,m}$ converges in distribution to a chi-squared variable with a number of degrees of freedom equal to the number of independent parameters in the model (Mendenhall and Scheaffer, section 10.8, 1973).

For a sequence of n data points, the likelihood is

$$L = P(x_1)\ P(x_2|x_1)\ P(x_3|x_1,x_2) \ldots P(X_n|x_1,x_2, \ldots x_{n-1}) .$$

If a Markov chain of order r is assumed, the dependence extends to no more than the past r observations, and the likelihood becomes

$$L_r = P(x_1)\ P(x_2|x_1)\ \ldots\ P(x_{r+1}|x_1, \ldots x_r)\ P(x_{r+2}|x_2, \ldots x_{r+1})$$
$$\ldots\ P(x_n|x_{n-r+1}, \ldots x_{n-1})$$
$$= P(x_1) \prod_{j=2}^{r} P(x_j|x_1, \ldots x_{j-1}) \prod_{i=r+1}^{n} P(x_i|x_{i-r}, \ldots x_{i-1}) ,$$

where L_r indicates likelihood under the assumption of order r.

More specifically, for order 0 (unconditional probabilities),

$$L_0 = \prod_{i=2}^{n} P(x_i) ,$$

for order 1,

$$L_1 = P(x_1) \prod_{i=2}^{n} P(x_i|x_{i-1}) .$$

and for "order -1" (equal probabilities, with S states),

$$L_{-1} = S^{-n} .$$

Maximum likelihood estimates based on the observations are used for the true probabilities, which are unknown. Where $n_{ij\ldots kl}$ is the number of observed sequences of transitions from i to j to . . to k to l, the maximum likelihood estimate of a transition probability is

$$\hat{p}_{ij\ldots kl} = P(x_l|x_i, x_j, \ldots x_k) = n_{ij\ldots kl} \Big/ \sum_{l=1}^{S} n_{ij\ldots kl} .$$

It is easiest to calculate the likelihood ratios for successive orders. In general, $\lambda_{r-1,r} = L_{r-1}/L_r$, and $\lambda_{k,m} = \lambda_{k,k+1}\lambda_{k+1,k+2} \ldots \lambda_{m-1,m}$. A convenient formula to calculate each λ value, according to Gates and Tong (1976), is

$$_{r-1}H_r = (-2)\ln\lambda_{r-1,r}$$
$$= 2 \sum_{i=1}^{S} \sum_{j=1}^{S} \ldots \sum_{k=1}^{S} \sum_{l=1}^{S} n_{ij\ldots kl} \left(\ln \frac{n_{ij\ldots kl}}{\sum_{l=1}^{S} n_{ij\ldots kl}} - \ln \frac{n_{j\ldots kl}}{\sum_{l=1}^{S} n_{j\ldots kl}} \right)$$

This quantity is distributed as a chi-squared variable with $S^{r+1} - S^r$ degrees of freedom and may be tested for significance if desired. For ${}_kH_m$, the number of degrees of freedom is $S^{m+1} - S^m - S^{k+1} + S^k$. As an example from Gates and Tong (1976), a 2-state chain of n = 120 points has 42 observations of state 0 and 78 of state 1. For first-order transitions reading from left to right, there are 25 observations in vector state (0,0), 17 in (0,1), 16 in (1,0), and 61 in (1,1), a total of 119 sequences of two observations.

The terms of ${}_0H_1$ are as follows:

$$i = 0, \quad j = 0: \quad 2n_{00} \left(\ln \frac{n_{00}}{n_{00} + n_{01}} - \ln \frac{n_0}{120}\right)$$

$$= 2\,(25)\left(\ln \frac{25}{25 + 17} - \ln \frac{42}{120}\right) = 2\,(13.2750827)$$

$$i = 0, \quad j = 1: \quad 2n_{01} \left(\ln \frac{n_{01}}{n_{00} + n_{01}} - \ln \frac{n_1}{120}\right)$$

$$= 2\,(17)\left(\ln \frac{17}{25 + 17} - \ln \frac{78}{120}\right) = 2\,(-8.0524471)$$

$$i = 1, \quad j = 0: \quad 2\,(16)\left(\ln \frac{16}{16 + 61} - \ln \frac{42}{120}\right) = 2(-8.3423132)$$

$$i = 1, \quad j = 1: \quad 2\,(61)\left(\ln \frac{61}{16 + 61} - \ln \frac{78}{120}\right) = 2(12.0689329),$$

so ${}_0H_1 = 17.89976$. This has been found to be very sensitive to rounding. Also, attempting to "correct" the first-order frequency counts so they total 120 instead of 119 will significantly change the likelihood ratio. The number of degrees of freedom is $2^2 - 2^1 = 3$.

As a check, use likelihood ratios:

$$\ln L_0 = n_0 \ln \left(\frac{n_0}{120}\right) + n_1 \ln \left(\frac{n_1}{120}\right) = 42 \ln \frac{42}{120} + 78 \ln \frac{78}{120}$$

$$= -77.69359666 .$$

$$\ln L_1 = \ln P(x_1) + \ln P(x_2|x_1) + \ldots + \ln P(x_{120}|x_{119})$$

$$= \ln\left(\frac{n_0}{120}\right) + n_{00}\ln\left(\frac{n_{00}}{n_{00}+n_{01}}\right) + n_{01}\ln\left(\frac{n_{01}}{n_{00}+n_{01}}\right)$$

$$+ n_{10}\ln\left(\frac{n_{10}}{n_{10}+n_{11}}\right) + n_{11}\ln\left(\frac{n_{11}}{n_{10}+n_{11}}\right)$$

$$= \ln\frac{42}{120} + 25\ln\frac{25}{42} + 17\ln\frac{17}{42} + 16\ln\frac{16}{77} + 61\ln\frac{61}{77}$$

$$= -68.74371583$$

$$(-2)\ln(L_0/L_1) = (-2)(\ln L_0 - \ln L_1) = -2(-8.94988083)$$

$$= 17.89976\ .$$

The first term in the L_1 equation should be explained further. Basically, in the series of 120 observations, considering sequences of two, there are 119 pairs. For 25 (= n_{00}) + 16 (= n_{10}) pairs = 41 pairs, the last number of the pair is 0. For 17 (= n_{01}) + 61 (= n_{11}) pairs = 78 pairs, the last number is 1. Since there are 42 observations of state 0 and 78 observations of state 1, and the first number in the series is not the last number of any of the pairs, it must always be 0. The first number of the sequence is 0 and there are 41 cases where the last number of a pair is 0. If the first observation were state 1, it would not be possible to have the observed number of pairs. This situation is automatically accounted for by using the $_{r-1}H_r$ formula.

Based on Akaike's (1974) development, the Akaike Information Criterion (AIC) is defined as

(-2) ln (maximum likelihood) + (2) (number of parameters)

$$= {}_kH_m - 2\,(S^{m+1} - S^m - S^{k+1} + S^k)\ ,$$

where m is the maximum order considered, k is any integer from 0 to m, and S is the number of states. If k = m, AIC = 0. Akaike's method is called Minimum AIC Estimate, or MAICE, and consists of choosing the

minimum value of AIC for k = 0 to m. This means that the data can be best modeled by a chain of order k. If the AIC values are all positive, except for AIC = 0 when k = m, the test is inconclusive, since all it can indicate is that the Markov order is m or larger.

As defined by Schwarz (1978), the Schwarz Bayesian Criterion (SBC) is

$$SBC = {}_kH_m - (S^{m+1} - S^m - S^{k+1} + S^k)(\ln n) ,$$

where n is the number of observations. Similarly to MAICE, the method is to find the value of k that minimizes SBC. Under Schwarz's assumptions, this test has a smaller probability of choosing the wrong order of k than MAICE. It definitely leans toward lower orders when there are at least eight observations.

Gates and Tong (1976) used the MAICE method to examine rainfall and sunshine data for Manchester and Liverpool and found that chains of low order (up to 2) fit the data best. They reexamined Gabriel and Neumann's (1962) data and concluded that a second-order model was better than a first-order model, which was different from the original conclusion. Chin (1977) examined January-February and July-August precipitation data for over 100 stations in the continental United States from 1949 to 1973. The appropriate Markov order was plotted on maps. Winter precipitation was modeled in most areas by chains of order 2, while summer precipitation was usually satisfactorily modeled by first-order chains. Areas which deviated from these orders were in well-defined geographical areas, and some tentative hypotheses for the physical reasons for these deviations were given.

The SBC method is so new that few applications have been published, although it would be easy to compare with MAICE. Katz (1979) disputed

Gate and Tong's (1976) conclusion that a second-order Markov chain fit the Tel Aviv data better than a first-order chain, and stated that the MAICE method has a .135 probability of estimating that the order is 2 when it should be 1.

V. Additional Topics

Characteristics of Two-Dimensional Real Weather Variables and Observed Weather Data

The statistical characteristics of real weather variables and observed weather data in two dimensions are similar to the characteristics of weather data, in general, as described in chapter II. A review of the characteristics will be given here to point out some issues that are important when attempting to model a two-dimensional data field. It is assumed here that time is a constant, so that the two dimensions are area dimensions, such as north-south and east-west.

Real two-dimensional weather data is, of course, still continuous, deterministic, dependent, multivariate, nonlinear, nonhomogeneous, occurring on different scales of motion, and statistically distributed. Observed weather data is still discrete, inaccurate, and incomplete. The issue of dependence, or correlation, is important enough to be discussed several ways, such as in terms of inverse probabilities, correlation, conditional probabilities, and directionality.

It will be assumed here that the data has been standardized as much as appropriate and feasible. For example, a grid of temperature data over a large area such as the eastern United States should be standardized so that the data values have the same mean and standard deviation. The data values could then be considered random components or residuals. After they are statistically described and a data field of residuals is simulated, the mean and standard deviation at each point would be used to transform the data back into the format of a temperature field.

Inverse Probabilities and Reversibility

When using a Markov chain for simulation where the independent variable is distance along a line rather than time, there is no fundamental direction and it is desirable to see if the transition probability matrix is the same regardless of direction. Also, in some cases, it is useful to reverse time and reconstruct a Markov chain backwards in time. For example, we may want to achieve a certain result (or state) ten years in the future and we have some control over policies in the near future. Repeated simulation runs starting ten years in the future with the desired state and working back to the present may give a range of policies that might possibly lead to the goal. Then, simulation using each of the possible policies forward into the future would show how likely each of the policies is to reach the goal.

The probabilities for a Markov chain in a reversed direction are called inverse probabilities. Of course, these probabilities must be between 0 and 1 and the inverse transition probability matrix must be a stochastic matrix. The inverse TPM will be denoted by $P^{(-1)}$ (in contrast to P^{-1}, the inverse of the TPM), with elements $p_{ij}^{(-1)}$, the probability that an observation in state j is preceded by an observation in state i. If matrix $P^{(-1)}$ is used to construct a sequence backward in time (or distance), it will construct a chain with the same statistical properties as when using P forward in time. Also, using $P^{(-1)}$ to construct a forward sequence is equivalent to using P to construct a sequence backwards in time or distance.

It is easy to demonstrate that a Markov chain is not automatically reversible, or that the probabilities are not the same in both directions, by giving a counterexample. Let the TPM be

$$P = \begin{bmatrix} .8 & .1 & .1 \\ .4 & .5 & .1 \\ 0 & .1 & .9 \end{bmatrix},$$

with states 1, 2, and 3. If the system is in equilibrium, the unconditional probability vector V can be shown, by solving VP = V, to be $V = [\ .\overline{3} \quad .1\overline{6} \quad .5\ ]$, where a bar over a digit or set of digits indicates infinite repetition, such as $.\overline{3} = .33333\ldots$ This transition matrix will be used frequently as an example.

In this example, the probability of a transition to state 3, given that state 1 is the present state, is .1. However, if we assumed that P could also be used to construct the sequence backwards, the probability of state 3 preceded by state 1 would be p_{31}, which is zero.

It is also possible to show that the inverse of the TPM, or p^{-1}, is not in general the inverse TPM. Using the same example, the inverse is

$$P^{-1} = \begin{bmatrix} 1.375 & -.25 & -.125 \\ -1.125 & 2.25 & -.125 \\ .125 & -.25 & 1.125 \end{bmatrix}$$

which is not a stochastic matrix even though its rows still add up to 1.

To lead to a development of the inverse TPM, it is useful to consider the expected unconditional probability that a randomly selected observation will be in state i followed by state j, assuming a steady state situation. The unconditional probabilities can be expressed in the form of a matrix, which will be called the expected proportion matrix (EPM). It is denoted by E, with elements E_{ij}, the unconditional probability that a randomly selected observation is in state i and is followed by state j.

The EPM is obtained from the TPM by multiplying each row by the unconditional probability for that row, or $E_{ij} = p_{ij}v_i$. For example, using P as above, the third row is $[\,0 \quad .1 \quad .9\,]$ and the unconditional probability of state 3 is .5. Therefore, the unconditional probability that a sequence of two consecutive observations will be (3,2) is (.1)(.5) = .05. The entire EPM is

$$E = \begin{bmatrix} .8(.\overline{3}) & .1(.\overline{3}) & .1(.\overline{3}) \\ .4(.1\overline{6}) & .5(.1\overline{6}) & .1(.1\overline{6}) \\ 0(.5) & .1(.5) & .9(.5) \end{bmatrix} = \begin{bmatrix} .2\overline{6} & .0\overline{3} & .0\overline{3} \\ .0\overline{6} & .08\overline{3} & .01\overline{6} \\ 0 & .05 & .45 \end{bmatrix}$$

The sum of all elements of an expected proportion matrix is 1.

When a sequence of observations is considered in reverse order, the EPM based on the original sequence is transposed to give the unconditional probabilities of pairs of states in the reversed sequence. For example, the number of occurrences of (2,1) in forward order (= E_{21}) is the same as the number of pairs of (1,2) in reverse order. The probability obtained from this frequency count is equal to the probability that a randomly selected observation is in state 1 and is preceded by an observation of state 2. Since the unconditional probability that a single observation is in state 1 is constant regardless of direction, the conditional probability that an observation is in state i, given that it is preceded by state j, is $p_{ij}^{(-1)} = E_{ji}/p_i = p_{ji}p_j/p_i = P(x_i|x_j)$. This result is confirmed by Feller (1968, chapter XV, section 12). In this case,

$$E^T = \begin{bmatrix} .2\overline{6} & .0\overline{6} & 0 \\ .0\overline{3} & .08\overline{3} & .05 \\ .0\overline{3} & .01\overline{6} & .45 \end{bmatrix}$$

so,

$$P^{(-1)} = \begin{bmatrix} .2\overline{6}/.\overline{3} & .0\overline{6}/.\overline{3} & 0/.\overline{3} \\ .0\overline{3}/.1\overline{6} & .08\overline{3}/.1\overline{6} & .05/.1\overline{6} \\ .0\overline{3}/.5 & .01\overline{6}/.5 & .45/.5 \end{bmatrix} = \begin{bmatrix} .8 & .2 & 0 \\ .2 & .5 & .3 \\ .0\overline{6} & .0\overline{3} & .9 \end{bmatrix}$$

It can be easily checked that $VP = VP^{(-1)} = V$ in this case. The difference between P and $P^{(-1)}$ is fairly substantial here, but in most real data, the differences should be smaller. Note that if $P = P^{(-1)}$, then $p_{ij}p_i = p_{ji}p_j$.

If it is desired to develop a nondirectional matrix from the data, an intuitively appealing procedure is simply to count the total number of forward and backward transitions of each kind by considering the data sequence both forward and backwards. The TPM obtained from this procedure is equivalent to the average of P and $P^{(-1)}$. This matrix, which will be called R, will be shown to be reversible.

For a 3 x 3 matrix, the average of P and $P^{(-1)}$ is

$$R = \begin{bmatrix} p_{11} & \frac{1}{2}\left(p_{12} + \frac{p_{21}p_2}{p_1}\right) & \frac{1}{2}\left(p_{13} + \frac{p_{31}p_3}{p_1}\right) \\ \frac{1}{2}\left(p_{21} + \frac{p_{21}p_1}{p_2}\right) & p_{22} & \frac{1}{2}\left(p_{23} + \frac{p_{32}p_3}{p_2}\right) \\ \frac{1}{2}\left(p_{31} + \frac{p_{13}p_1}{p_3}\right) & \frac{1}{2}\left(p_{32} + \frac{p_{23}p_2}{p_3}\right) & p_{33} \end{bmatrix}$$

or in general, $R_{ij} = \frac{1}{2}\left(p_{ij} + \frac{p_{ji}p_j}{p_i}\right)$.

To verify that R is reversible, it will be shown that $R_{ij}p_i = R_{ji}p_j$:

$$R_{ij}p_i = \frac{1}{2}\left(p_{ij} + p_{ji}p_j/p_i\right)p_i = \frac{1}{2}\left(p_{ij}p_i + p_{ji}p_j\right)$$

$$R_{ji}p_j = \frac{1}{2}\left(p_{ji} + p_{ij}p_i/p_j\right)p_j = \frac{1}{2}\left(p_{ji}p_j + p_{ij}p_i\right) = R_{ij}p_i$$

The fact that VR = V can be easily shown from the associative property of matrix multiplication.

For the example used so far in this chapter,

$$R = \begin{bmatrix} .8 & .15 & .05 \\ .3 & .5 & .2 \\ .0\overline{3} & .0\overline{6} & .9 \end{bmatrix}$$

In a real problem, once P and $P^{(-1)}$ are found, a statistical test such as a chi-squared test of Anderson and Goodman (1957) could be used to show whether $P^{(-1)}$ is significantly different from P. If there is no significant difference, and if it is desired to make the simulated data sequence reversible, using R as the transition probability matrix would generate a reversible simulated sequence.

Conditional Probabilities and Invariance of Generation

When simulating an area data base on a Cartesian-coordinate type of grid, there are many ways to generate a data field. It is desirable for the generation process to produce a statistically homogeneous field. This could be verified in terms of conditional probabilities. If the field is homogeneous, then the probability distribution at a grid point (i+h+a, j+k+b), given the state observed at point (i+a, j+b), is the same regardless of the values of a and b, or the conditional probability distribution is invariant under translation. Also, the conditional probability at point (i,j) given the state at point (i+h, j+k) should be the same regardless of the variation on the basic generation scheme used (such as generation by rows instead of by columns). This property could be called invariance of generation.

In the case of a basic first-order transition from one point to

the next, the TPM and the conditional probability matrix (CPM) are the same, since $p_{ij} = P(x_j|x_i)$. The CPM for the transition in reverse is $P^{(-1)}$ because $p_{ij}^{(-1)} = P(x_i|x_j)$.

The conditional probabilities should first be investigated for first-order schemes of generating a gridded data base, as shown in figure 3. The conditional probability distribution at point j, given the probability distribution at point i, will be compared for all four cases.

Under first-order generation schemes, the paths from one point to another can be divided into three types, as shown in figure 4, and the conditional probability structures resulting can be applied to the cases depicted in figure 3. Assume that the transition matrix along an arrow is P, and that the transition matrix in reversed direction is $P^{(-1)}$. For example, a type 2 structure would be generated by the transition matrix obtained from $P^{(-1)}P$.

For a type 1 situation, the total transition matrix is $PP = P^2$, so $P(x_b|x_a) = p_{ab}^{(2)}$. Since the transition matrix for type 2 is given by $P^{(-1)}P$, matrix multiplication can be used to show that $P(x_b|x_a) = \sum_{c=1}^{N} p_{ac}^{(-1)} p_{cb}$. For the P matrix used as an example so far in this chapter, the conditional probability matrices are

TYPE 1

$$P^2 = \begin{bmatrix} .68 & .14 & .18 \\ .52 & .30 & .18 \\ .04 & .14 & .82 \end{bmatrix}$$

TYPE 2

$$P^{(-1)}P = \begin{bmatrix} .72 & .28 & .10 \\ .36 & .30 & .34 \\ .0\overline{6} & .11\overline{3} & .82 \end{bmatrix}$$

Intuitively, it appears that, for $P(x_b|x_a)$, type 3 and type 2 are the same, since the unconditional probability distribution at point c is the same in both cases. This can be shown to be true by finding the conditional probability in a type 3 situation:

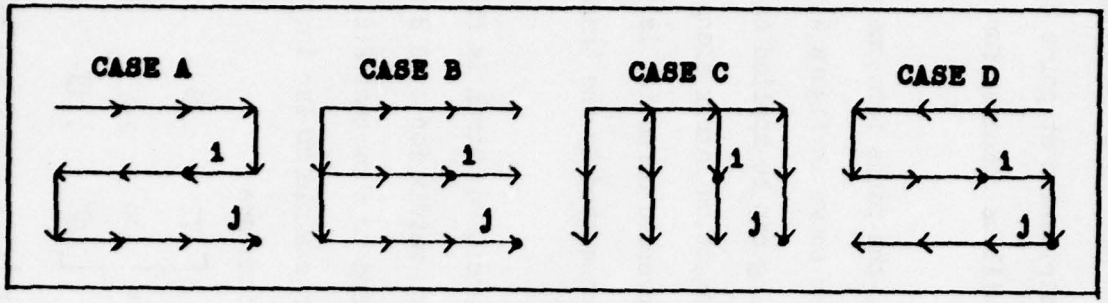

Figure 3 - Possible First-Order Schemes for Generating a Two-Dimensional Gridded Data Base

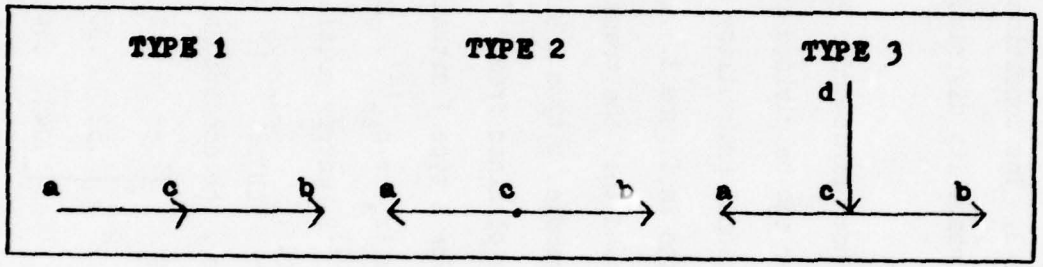

Figure 4 - Basic First-Order Transition Structures

$$P(x_b|x_a) = \frac{P(x_b \cap x_a)}{P(x_a)} = \frac{\sum_{d=1}^{N} P_d \sum_{c=1}^{N} P_{dc}P_{ca}P_{cb}}{P_a}$$

$$= \frac{\sum_{c=1}^{N} P_{ca}P_{cb} \sum_{d=1}^{N} P_d P_{dc}}{P_a} = \frac{\sum_{c=1}^{N} P_{ca}P_{cb}P_c}{P_a} \quad \text{(note 1)}$$

$$= \frac{\sum_{c=1}^{N} P_{ac}^{(-1)} P_a P_{cb} P_c}{P_c P_a} \quad \text{(note 2)} = \sum_{c=1}^{N} P_{ac}^{(-1)} P_{cb}$$

Note 1: This step uses V = VP. If each transition probability is p_{dc}, a term-by-term expansion of each side of V = VP gives $p_c = \sum_{c=1}^{N} p_d p_{dc}$.

Note 2: This step substitutes $p_{ac}^{(-1)}\, p_a/p_c$ for p_{ca}.

This shows that a type 3 structure has the same conditional probabilities as type 2. Therefore, there are only two basic first-order structures, shown as type 1 and type 2. These two structures may be generalized as in figure 5, where r backward and s forward steps occur between points i and j.

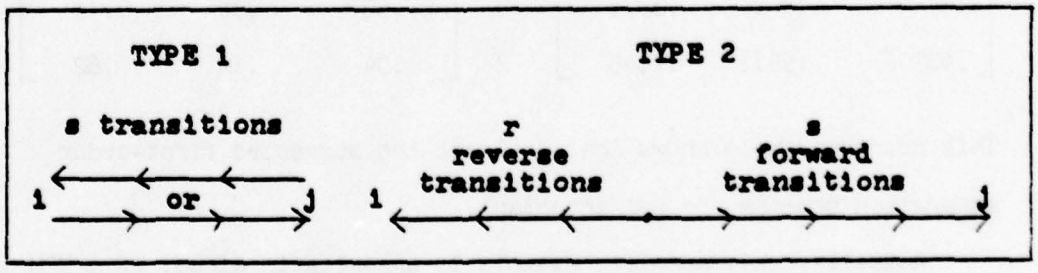

Figure 5. Final Generalized First-Order Transition Structures

In type 1, the overall TPM from i to j is p^s, so $P(x_j|x_i) = p_{ij}^{(s)}$.

In type 2, the overall TPM from i to j is given by $P^{(-r)}P^{s}$, so

$$P(x_j|x_i) = \sum_{k=1}^{N} p_{ik}^{(-r)} p_{kj}^{(s)}.$$

The four sample cases given in figure 3 will now be compared in terms of conditional probabilities, with the P matrix given earlier in the chapter used as an example. The overall transition matrices are as follows:

Case A: (6 forward steps) P^6

Case B: (2 reverse and 4 forward steps) $P^{(-2)}P^4$

Case C: (1 reverse and 3 forward steps) $P^{(-1)}P^3$

Case D: (2 forward steps) P^2 .

The numerical values of the matrices are

Case A

$$\begin{bmatrix} .465088 & .165984 & .368928 \\ .460992 & .170080 & .368928 \\ .202944 & .165984 & .631072 \end{bmatrix}$$

Case B

$$\begin{bmatrix} .511168 & .169056 & .319776 \\ .362688 & .170080 & .467232 \\ .204992 & .163936 & .631072 \end{bmatrix}$$

Case C

$$\begin{bmatrix} .5872 & .1688 & .2440 \\ .4144 & .1880 & .3976 \\ .13706 & .15813 & .7048 \end{bmatrix}$$

Case D

$$\begin{bmatrix} .68 & .14 & .18 \\ .52 & .30 & .18 \\ .04 & .14 & .82 \end{bmatrix}$$

This counterexample shows that at least the suggested first-order generation schemes are not invariant.

Actually, this specific example is unrealistic since, if a TPM is not reversible, it would not be used to generate data in all directions. However, if some reversible matrix R is used, and is assumed to be constant in all directions, the overall transition probability matrix from point i to point j would range from R^6 in cases A and B to R^2 in

case D, and the conditional probabilities would not be the same.

This section has shown that at least some first-order generation schemes on a two-dimensional grid are not invariant. Since these were expressed in terms of processes that could occur on a line or in a time series, this demonstrates that present Markov chain applications are not invariant when the TPM is not reversible. This is not a problem in most applications since the proper matrix would be used in each direction of simulation, but could be a problem in two-dimensional simulation since using even reversible matrices does not guarantee invariance.

Correlation Issues

Along with the issue of conditional probabilities when generating a two-dimensional data field, it is useful to consider the correlation structure of the data and see if this structure is preserved by various simulation processes.

A Markov field, as defined by Besag (1974), assumes that the correlation between two points depends only on the distance between them and not on the direction. More specifically, assume that the points on a gridded two-dimensional data field (or lattice) are denoted by coordinates (i,j) relative to some point (0,0), which can be moved around on the grid, if desired. This is a stationary Markov field if (1) the probability distribution at point (i,j) is conditional only on a specified set of adjacent points such as (i-1,j), (i,j-1), (i+1,j), and (i,j+1) and (2) if the probability distribution is the same for any interior point on the lattice.

One way of describing the "order" of a Markov field is given in figure 6, although terminology could differ. The probability distribution at point 0, for a Markov field of order n, depends only on

4	4	3	4	4
4	2	1	2	4
3	1	0	1	3
4	2	1	2	4
4	4	3	4	4

Figure 6. Order of Dependence in a Gridded Two-Dimensional Markov Field:

The probability distribution at point 0, in a Markov field of order n, depends only on the data values at points numbered n or less.

the data values at points numbered n or less. Generally, only first-order Markov fields have been considered in actual investigations because of the rapid increase in complexity with higher orders.

Another way to describe the correlation structure is to define a "line transect" (Whittle 1954) as a line that crosses the data field. If the data field is a Markov field, the process on the line is a one-dimensional Markov process.

Using actual data, the correlation structure of a two-dimensional data field is often expressed by a correlogram in the form of a table, whose entries are the correlations between the data value at a relative point (0,0) and the values at other points with coordinates of (i,j) on this relative coordinate system. One example, based on orange trees in an orchard (Patankar 1954 and Whittle 1954) showed a correlation structure of values gradually decreasing in all directions away from the point of interest, from around .54 to .55 for adjacent points, to about .2 to .3 for points 8 to 12 units apart. The correlation of a point with itself (1.0) was a "spike" in the center of a rounded "mound." The authors remarked that this appear to be a quite common situation in analysis of gridded data bases. The fact that the correlation for

points one unit or more apart is quite a bit lower than 1.0 shows that there is a substantial amount of individual variability, while the gradual decline with distance shows that there is still a considerable amount of structure in the data field. If the correlation decreases fairly uniformly in all directions (such as $r_{(a,b)}$ being approximately equal to $r_{(b,a)}$, where $r_{(a,b)}$ indicates the average correlation between points with coordinates of (i,j) and (i+a, j+b), for all possible (i,j) points on the grid), there is no significant trend in the data. As stated before, any known trend should be removed from the data before this analysis is performed.

If it is assumed that the states are interval scaled (such as 0,1,2,. .), it is possible to use the standard formula for correlation to derive the correlation structure of a one-dimensional first-order process, or a process in two dimensions considered in terms of one-dimensional paths. The process is assumed to be in steady state, or VP = V. The correlation derived is the "expected correlation," with the unconditional probabilities distributed according to V and the transition probabilities distributed according to P. The standard correlation coefficient formula is given by

$$r = \frac{n\sum xy - \sum x \sum y}{[n\sum x^2 - (\sum x)^2][n\sum y^2 - (\sum y)^2]}$$

In this formula, x can be considered to be the state at point i. The expected number of observations in state x_i in n observations is np_i.

Also, y is the state at point j, or x_j. Point j does not need to be adjacent to point i. Out of n trials, the expected number of cases with state x_i followed by state x_j at point j is np_ip_{ij}.

The values of the terms in the correlation coefficient formula can be calculated as follows. There are N states and n observations.

$$E(\Sigma x) = \sum_{i=1}^{N} np_i x_i$$

$$E(\Sigma y) = \sum_{i=1}^{N} np_i \sum_{j=1}^{N} p_{ij} x_j$$

$$E(\Sigma x^2) = \sum_{i=1}^{N} np_i x_i^2$$

$$E(\Sigma y^2) = \sum_{i=1}^{N} np_i \sum_{j=1}^{N} p_{ij} x_j^2$$

$$E(\Sigma xy) = \sum_{i=1}^{N} np_i x_i \sum_{j=1}^{N} p_{ij} x_j$$

Next, it will be shown that $E(\Sigma x) = E(\Sigma y)$ and $E(\Sigma x^2) = E(\Sigma y^2)$. This is intuitively reasonable because the same unconditional distribution is maintained before and after the transition. The last step of each proof used the fact that $VP = V$.

Show that $E(\Sigma x) = E(\Sigma y)$:

$$E(\Sigma y)/n = \sum_{i=1}^{N} p_i \sum_{j=1}^{N} p_{ij} x_j = (p_1 p_{11} x_1 + p_1 p_{12} x_2 + \cdots + p_1 p_{1N} x_N)$$

$$+ (p_2 p_{21} x_1 + p_2 p_{22} x_2 + \cdots + p_2 p_{2N} x_N)$$

$$+ (p_N p_{N1} x_1 + p_N p_{N2} x_2 + \cdots + p_N p_{NN} x_N)$$

$$= (p_1 p_{11} + p_2 p_{21} + \cdots + p_N p_{N1}) x_1$$

$$+ (p_1 p_{12} + p_2 p_{22} + \cdots + p_N p_{N2}) x_2 + \cdots + (p_1 p_{1N} + p_2 p_{2N} + \cdots + p_N p_{NN}) x_N$$

$$= p_1 x_1 + p_2 x_2 + \cdots + p_N x_N = \sum_{i=1}^{N} p_i x_i = E(\Sigma x)/n$$

Show that $E(\Sigma x^2) = E(\Sigma y^2)$:

$$E(\Sigma y^2)/n = \sum_{i=1}^{N} p_i \sum_{j=1}^{N} p_{ij} x_j^2 = (p_1 p_{11} x_1^2 + p_1 p_{12} x_2^2 + \cdots + p_1 p_{1N} x_N^2)$$

$$+ (p_2 p_{21} x_1^2 + p_2 p_{22} x_2^2 \cdots + p_2 p_{2N} x_N^2) + \cdots + (p_N p_{N1} x_1^2 + p_N p_{N2} x_2^2 + \cdots + p_N p_{NN} x_N^2)$$

$$= (p_1 p_{11} + p_2 p_{21} + \cdots + p_N p_{N1}) x_1^2 + \cdots + (p_1 p_{1N} + p_2 p_{2N} + \cdots + p_N p_{NN}) x_N^2$$

$$= p_1 x_1^2 + \cdots + p_N x_N^2 = \sum_{i=1}^{N} p_i x_i^2 = E(\Sigma x^2)/n$$

Using these factors, the expected correlation coefficient between points i and j becomes

$$E(r_{i,j}) = \frac{\sum_{i=1}^{N} p_i x_i \sum_{j=1}^{N} p_{ij} x_j - (\sum_{i=1}^{N} p_i x_i)^2}{\sum_{i=1}^{N} p_i x_i^2 - (\sum_{i=1}^{N} p_i x_i)^2}$$

Note that all factors of n (the number of observations) canceled out. Also, both terms in the denominator were identical.

The correlation can be generalized to cases where there is more than one transition between point i and point j by substituting the correct transition probabilities for p_{ij}. The only term that changes in the formula is the left term in the numerator. For example, in case A in the last section, the correlation coefficient could be calculated by substituting $p_{ij}^{(6)}$ for p_{ij}. Using P as the TPM for cases A through D, these and other correlations are calculated as follows. Where $X = [\,1\ 2\ 3\,]$, some of the constants needed are $(\Sigma p_i x_i)^2 = 4.69\overline{4}$ and $\Sigma p_i x_i^2 = 5.5$, and the denominator is $5.5 - 4.69\overline{4} = .80\overline{5}$. Let $r_{(s)}$ indicate the correlation between points s forward transitions apart; let $r_{(-r)}$ indicate the correlation between points r reverse transition apart; and let $r_{(-r,s)}$ indicate the correlation between points r reverse

transitions and s forward transition apart. (This notation applies only to points on a line, and this notation is used only in this section). Then, some typical expected correlations can be calculated as follows. The cases (A to D) are shown in figure 3.

Case	Expected Correlation Coefficient
One transition, forward	$E(r_{(1)}) = .81379$
One transition, reverse	$E(r_{(-1)}) = .81379$
Two transitions, forward (case D)	$E(R_{(2)}) = .65655$
Two transitions, reverse	$E(r_{(-2)}) = .65655$
One step reverse, one forward	$E(r_{(-1,1)}) = .68966$
Case A	$E(r_{(6)}) = .27104$
Case B	$E(r_{(-2,4)}) = .30918$
Case C	$E(r_{(-1,3)}) = .45992$

It is interesting to compare $E(r_{(2)})$ with $E(r_{(1)})$. It would be expected that $E(r_{(n)}) = E(r_{(1)})^n$, but $E(r_{(1)})^2 = .66226$, slightly higher than the real correlation. Also, case A should have a correlation of $E(r_{(1)})^6$ .29046. This is also slightly higher than the real correlation. This non-exponential trend of correlation is confirmed by Lloyd (1974), who uses an eigenvalue approach for computing correlation. He states that the correlation is strictly exponential for a two-state Markov chain, but that the trend is more complicated when there are more than two states.

Under invariance of generation, the correlation between i and j in cases A, B, C, and D should be the same. However, they range from .2710 to .6566, using P as the example of a transition matrix. The "true" correlation is probably approximately equal to .6566, the correlation that results from taking a "shortest path" from i to j, as in

case D.

It is not difficult to prove that as n approaches infinity, $E(r_{(n)})$ approaches zero. As $n \to \infty$, $p_{ij}^{(n)} \to p_j$, so

$$\sum_{i=1}^{S} p_i x_i \sum_{j=1}^{S} p_{ij} x_j = \left(\sum_{i=1}^{S} p_i x_i\right)\left(\sum_{j=1}^{S} p_j x_j\right) = \left(\sum_{i=1}^{S} p_i x_i\right)^2 ,$$

and the numerator of $E(r_{(\infty)})$ would be zero.

The expected correlation for an equal number of steps forward or backward is equal, as expected. However, the expected correlation for one step backwards and one step forward (as in a type 2 structure, mentioned earlier in this chapter) is not the same as the expected correlation for two steps forward or backward. This shows why the correlation is not the same for case A as for case B. If a reversible TPM is used, the correlation would be the same for a constant number of steps regardless of the combination of steps.

Directionality and Inhomogeneity

So far in this chapter, it has been assumed in many cases that the transition matrix is reversible and is also totally nondirectional. In many cases, deterministic components of the data can be removed and the data can be processed so that it is homogeneous. However, in some cases, it is not possible or appropriate to try to remove these nonrandom components. An example could be occurrence or nonoccurrence of snowfall over a fairly large area. Since, in general, the frequency (unconditional probability) of snow diminishes toward the south, the transition probabilities cannot be the same for locations on a north-south line as for locations on an east-west line. It is useful to discuss some of the consequences of directionality, or (as used here)

a case where the TPM may be constant from east to west, but varies from south to north. No substantial results will be derived here, but some data handling methods will be discussed.

A simple way to consider inhomogeneity that causes a gradual trend over an area from north to south is to assume that simulation will take place only in "eastward" and "southward" directions starting from the "northwest" corner of the area. Simulation would not take place as in cases A or D shown in figure 3. Let the transition matrix for "eastward" transitions be P, and let the TPM for "southward" transitions be Q.

Assume that a "shortest path" is taken from (0,0) to (i,j), where lexicographical ordering is used, so i indicates the number of rows down ("south") from (0,0) and j indicates the number of columns to the right ("east") of (0,0). Also assume that a one-dimensional first-order method of generation is used. An example will show that there is a difference in transition probabilities even when there are only three transitions between (0,0) and (1,2). Let the transition probability matrices be

$$P = \begin{bmatrix} .56 & .44 \\ .33 & .67 \end{bmatrix} \text{ and } Q = \begin{bmatrix} .31 & .69 \\ .09 & .91 \end{bmatrix}.$$

The three possible cases are shown in figure 7.

The matrices for the transitions in figure 7 are

$$\text{CASE E} \qquad QPP = \begin{bmatrix} .422299 & .577701 \\ .410661 & .589339 \end{bmatrix}$$

$$\text{CASE F} \qquad PQP = \begin{bmatrix} .379036 & .620964 \\ .367398 & .632602 \end{bmatrix}$$

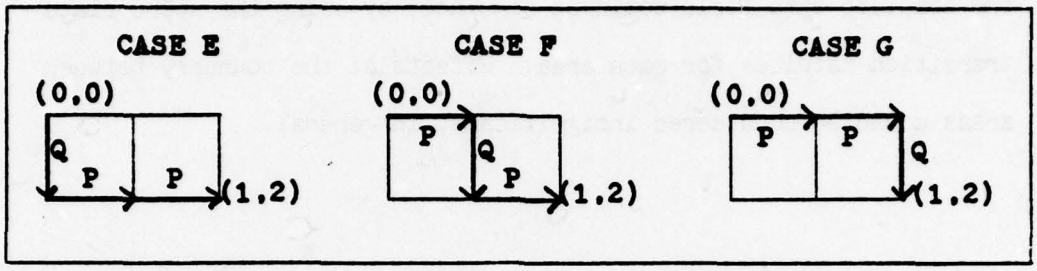

Figure 7. Three Possible Paths for a Two-Dimensional Simulation by One-Dimensional Processes

$$\text{CASE G} \qquad PPQ = \begin{bmatrix} .190936 & .809064 \\ .179298 & .820702 \end{bmatrix}$$

In general, the same transition probabilities do not result if the transition matrices P and Q are different, because matrix multiplication is not commutative. However, most of the available theory that can be applied to two-dimensional situations assumes homogeneity and this problem will not occur if the data is homogeneous. The data that is used as an example in chapter VII is very nearly homogeneous, so this problem will not be considered further in this thesis.

Another type of inhomogeneity that is frequently encountered is a fairly sharp division from one area to another. This could occur at a topographic boundary such as a coastline, or along a mountain range. Since attempting to handle this exactly using theory increases complexity greatly, in this thesis it will be assumed that investigations are confined to areas that are homogeneous. In case of an inhomogeneity, an analogous approach to that of Lloyd (1974) can be taken: to use a "generalized stationarity" that allows the entire field of interest to

be divided into areas that can be considered homogeneous. Transition matrices would be developed from the data for each separate area, and the complete data field could be simulated by using the appropriate transition matrices for each area. Effects at the boundary between areas could be considered insignificant, in general.

VI. Literature Survey - Simulations of Two-Dimensional Data Bases

Non-Markov Models

There have been very few attempts to simulate two-dimensional data bases. This section will discuss regression-type simulations, which are not exactly Markov in some cases. In a two-state case, the two models can be equivalent. As described in chapter V, all coordinates of points will assume a coordinate system in "lexicographical ordering."

Galbraith and Walley (1976, 15-2) describe a regression scheme where the expected value at a point depends only on the value at the two nearest points that are known. (For a simulation starting at the "northwest" corner and proceeding "eastward" and "southward", these two known points are immediately "north" and "west" of the unknown point). The unknown point is (i,j) and the known points are (i-1,j) and (i,j-1). Then $E(x_{ij} \mid \text{all predecessors}) = a + bx_{i-1,j} + cx_{i,j-1} + dx_{i-1,j}x_{i,j-1}$, where a, b, c, and d are found by linear regression methods. The interaction term may be deleted for simplicity. However, for a two-state chain, keeping the interaction term in will allow all transition probabilities in terms of the values at points (i,j-1) and (i-1,j) to be specified as follows:

$$P(1|0,0) = a = P(x_{ij} = 1 | x_{i,j-1} = 0,\ x_{i-1,j} = 0)$$

$$P(1|0,1) = a + c$$

$$P(1|1,0) = a + b$$

$$P(1|1,1) = a + b + c + d$$

To use the above equation for simulation of a two-state (0 or 1) Markov chain, either with or without the interaction term, the value of

$E(x_{ij})$ is computed and a random number between 0 and 1 is drawn. If the random number is greater than $1 - E(x_{ij})$, the state at (i,j) is 1 and otherwise it is 0.

Because of the limited (and fixed) number of parameters, this procedure cannot be used with more than two states, except as an approximation. A possible simulation method would be to compute $E(x_{ij})$, add an error term (also derived from the data) and use some procedure for converting the resulting value into one of the permitted states.

Welberry and Galbraith (1973) give examples of a two-state regression model for crystal growth (where 0 = absence of an atom, and 1 = presence of an atom). They assumed that there were no interactions, so that they could obtain some exact theoretical results.

These papers showed that a major problem is to statistically derive the expected value of the variable of interest, or equivalently, the stationary distribution. For a one-dimensional chain, the stationary distribution is obtained by solving VP = V and the expected value can be easily obtained from V. However, Galbraith and Walley (1976) showed that when an interaction term is included in the regression equation, there are always more unknowns than equations, so that the expected value cannot be found. In this thesis, the transition probabilities and other constants are found from the data by counting transitions, and the expected values and stationary distributions can be also obtained from the data by the same type of process.

Matalas (1967) describes a "multivariate weakly stationary generating process" that could be easily adapted to a two-dimensional data base. This method, since it was developed to handle a time series of several interrelated variables, would generate either a row or column

of data values simultaneously. The cross-correlations in and between the row or column being generated and the last row or column generated could be accounted for. However, this procedure could not handle a Markov field of order greater than two because correlations are considered only in two adjacent rows or columns.

Markov Simulations in Two Dimensions

The following suggested procedures differ from the "original vector Markov chain approach" (Pickard 1977) but they have Markov properties since the next computed value depends only on certain previously-computed data values and not on the entire process. The original vector approach, for all Markov chains, requires the input and output state spaces to be the same. For example, in a second-order Markov chain, overlapping states are defined, where the two-component vector state consisting of the most recent and present observations in sequence define the probability distribution for transition to the two-component vector state containing the present and next observations in order. If overlapping states are not required, the transition is to the single state consisting of the next observation. In this case, the transition probability matrix is not square, but is $N^2 \times N$. All of the elements of this matrix may be nonzero since there are no impossible transitions.

The following proposed two-dimensional Markov schemes will consist of some vector sequence of known observations that determine the probability distribution for the next observation. In all of these cases, two adjacent boundaries of the data field are generated by one-dimensional processes of some kind. The remaining points (including the other two boundaries) can be considered "interior points," which are generated by two-dimensional processes.

Galbraith and Walley (1976) consider a Markov-type simulation procedure analogous to their regression model with interaction terms. This approach was suggested earlier by Bartlett (1967), who called the process a "doubly stochastic Markov chain." In this procedure, after the top and left (or any two adjacent) boundaries are generated by one-dimensional processes, the "interior" data values are generated by rows (or columns). The transition matrix for transition to a point (i,j) is based on the state at point (i-1,j), immediately to the left, but a different transition matrix is used for each possible state at point (i,j-1), immediately above (i,j). This could be consolidated into one non-square matrix if the input states are the vector states corresponding to the observations to the left of and above each point.

These authors were not able to develop exact formulas for the expected value $E(x_{ij})$, where in a two-state binary chain, $E(x_{ij})$ is the proportion of the time that the state is 1, except in special cases. Galbraith and Walley (1976) numerically solved a conjectured general solution for $E(x_{ij})$ in a two-state case, and the numerical results closely approximated the conjectured values.

To simulate using this procedure, the total number of probabilities needed is N (unconditional probabilities) + N^2 (first-order Markov transition probabilities for generation of boundary values, assuming the same probabilities in both horizontal and vertical directions) + N^3 (conditional probabilities for interior points) = $N^3 + N^2 + N$. The number of independent probabilities is $N^3 - 1$.

Pickard (1977) developed a similar procedure, which generates a homogeneous Markov random field. His model is based on the joint distribution for the vertices of a unit square $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ where, for

example, A, B, and C are known and D is unknown. Then, for interior points, the probability distribution at D is based on the vector state (A,B,C). As in Galbraith and Wally's paper, his calculations apply only to two-state processes, although his procedures can be extended to (1) nonhomogeneous data bases, (2) more than two states, or (3) more than two dimensions. The number of probabilities required in this model is $N + N^2$ (as above) + N^4 (conditional probabilities for D, based on A, B, and C) = $N^4 + N^2 + N$, with $N^4 - N^3 + N^2 - 1$ independent probabilities.

Pickard also proved that the conditional probability at a point $(i + k, j + 1)$ based on the state at (i,k) is the same regardless of the path taken as long as it is one of the shortest paths. The arguments used to prove this also led to an explicit formula for the correlation, which is geometric when there are only two states.

Another Possible Procedure - "Method 1"

The following suggested simulation procedure is illustrated by the diagram in figure 8. Where an arrow is shown, the value at the head of the arrow is determined by first-order one-dimensional Markov processes based on the value at the tail of the arrow. Where a square is shown, the values at the two lower corners are obtained from the probability distribution based on the values at the two upper corners of the square. This procedure requires an $N^2 \times N^2$ matrix, plus a first-order matrix, and a vector of unconditional probabilities, so the number of probabilities requires is the same as in Pickard's (1977) model. This method of simulation will be called "Method 1." A possible advantage of this procedure is that fewer random numbers would need to be generated to produce a data field of a given size.

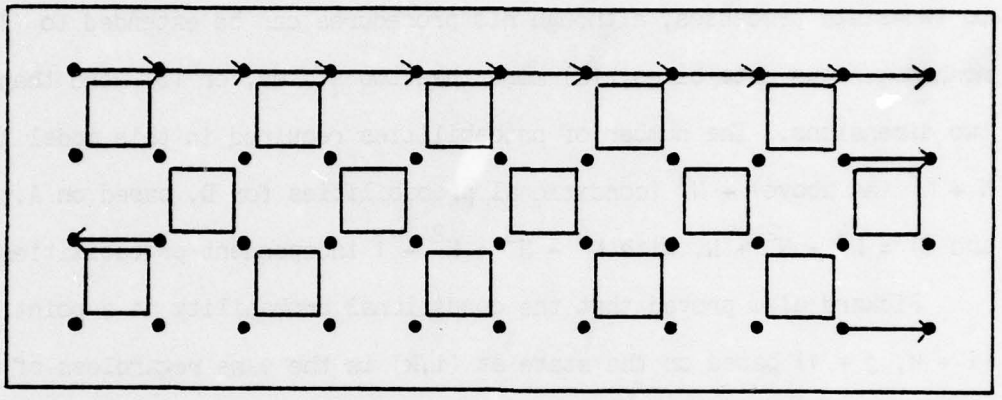

Figure 8. Simulation Scheme for "Method 1"

VII. Examples and Discussion of Two-Dimensional Data Base Simulations

Data Base: Selection, Analysis, and Characteristics

Since no published applications of Markov processes to a two-dimensional data base have been found, although the theory has been established, it was decided that an example would demonstrate the usefulness of the proposed methods, and reveal if there are any significant operational problems. The computer calculations were found to be quite simple, and none of the programs required more than about ten seconds on a CYBER 175 computer to calculate transition probabilities.

The data base chosen for this example was obtained from weekly maps of temperature departures from the 1941-70 normal, published in issues of the Weekly Weather and Crop Bulletin (USDA). A series of maps was selected for analysis, consisting of the 15 weekly maps starting with the first week (Monday through Sunday) ending in December, for the seven winters 1972-73 to 1978-79. A grid with 12 rows and 15 columns was superimposed on an area of the eastern and central United States (see figure 9). The 180 variables were always considered in lexicographical order, with the rows considered from top to bottom and the columns considered from left to right. In the computer programs, the variables were considered as a single sequence of 180 variables, but in other cases the variables were denoted by a letter denoting the row and a number indicating the column (such as C12), or by a pair such as (r,c), where r indicates the row and c indicates the column. Under this notation, the vector pair is the same as a 90 degree clockwise rotation of a Cartesian coordinate system.

The temperature departures were read from the map at each grid

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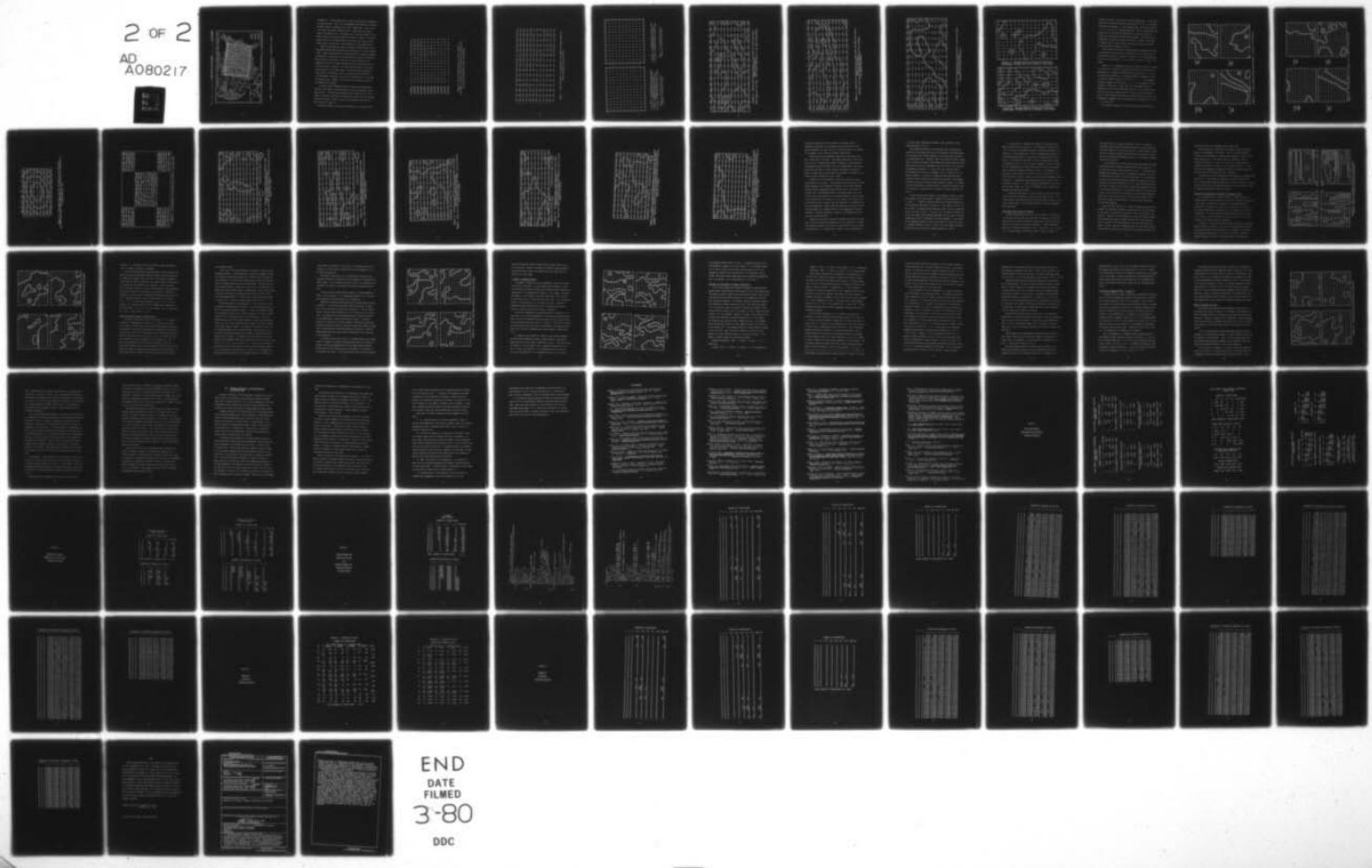

END
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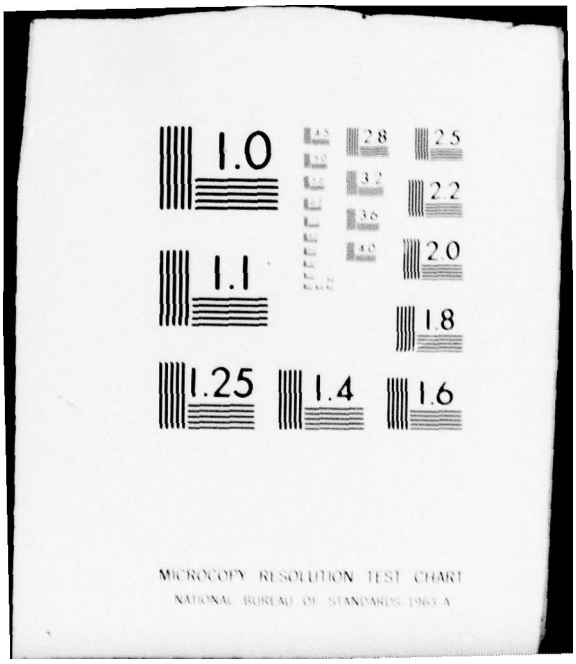
1.0
1.1
1.25
1.4
1.6
2.8
3.2
3.6
4.0
2.5
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2.0
1.8
MICROCOPY RESOLUTION TEST CHART
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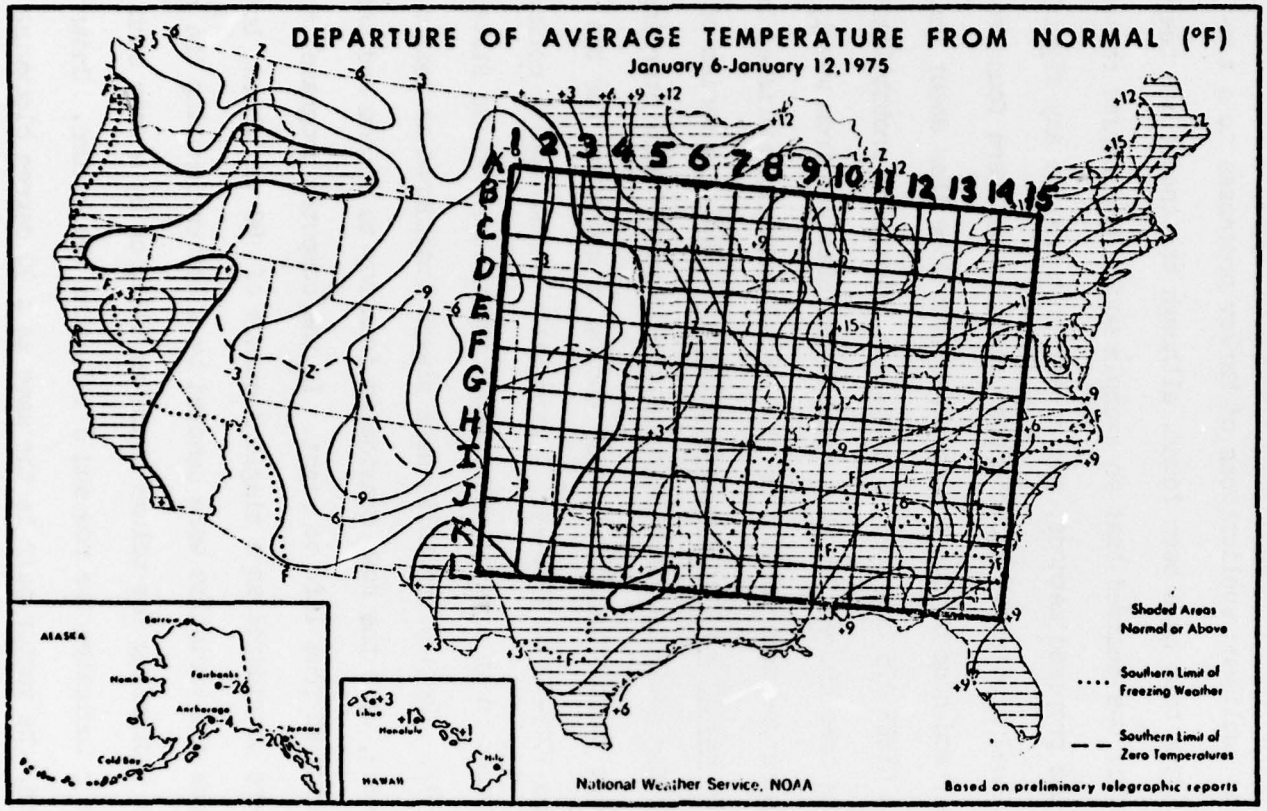

Figure 9 - Typical Weather Map of Weekly Temperature Departures from Normal, with Superimposed Grid

87

intersection. It was assumed that the area was sufficiently homogeneous for these analyses. Most of the inhomogeneity was removed by converting the data values to differences from normal temperatures. With a total of 105 maps, there were 18900 data points. These data values were stored on a computer file. For example, the stored data values for the period covered by the map in figure 9 are shown in figure 10. The line number consists of two digits each for the month, day, year, and row number, where the date given is the ending date of the weekly period.

After the data base was stored, statistical characteristics of the data were examined using the Statistical Package for the Social Sciences (SPSS). These included the mean, standard deviation, skewness, maximum, and minimum for each grid point, and are shown in figures 14 through 18. While there are variations on every map, the data obviously has an organized structure, and there are meteorological explanations for most of the variations. Actually, this seven-year period may not have been typical because of the large number of extremes recorded (especially exceptionally cold winters), and even a typical seven-year period is not long enough to describe the statistical characteristics of the data with high accuracy. However, this data base is sufficiently realistic for demonstration purposes.

To produce a more uniform data base, the data values were standardized to the number of standard deviations from the average for each grid location. The seven-year average was used rather than the 1941-70 normal, since many locations were consistently colder than normal during these seven winters. Figure 11 shows the standardized data obtained from the data in figure 10.

For Markov analysis, the data was categorized into three and five

```
01127501   -6 -3  1  6  9  9  8  9  9  8 12 13 13 14 14
01127502   -6 -4  0  4  6  7  8  9 10  9 11 13 13 14 13
01127503   -6 -3 -1 -1  2  6  7  8 10  9 11 12 13 13 12
01127504   -7 -3 -2 -1  0  3  7  8  9 12 12 13 13 12 10
01127505   -5 -5 -4 -3  0  2  5  8 11 15 14 12 11  9  8
01127506   -4 -4 -3 -1  1  2  4  5  7  9 11 11 10  9  6
01127507   -3 -3 -2 -1  1  2  4  5  6  8 10 10  8  5  5
01127508   -3 -2 -1 -1  2  4  5  6  7  8  8  6  4  3  7
01127509   -6 -3 -1  1  4  3  5  7  7  6  5  6  5  3  8
01127510   -3 -2  0  2  4  2  3  5  5  5  6  7  6  6  9
01127511    0 -1  0  2  3  1  2  3  4  5  7  9  6  7 10
01127512    4  1  1  1  0 -1  3  4  5  6  9 10  9 10  9
```

Figure 10 - Data Field Corresponding to Grid in Figure 9.
The number at the Left of Each Line Consists of Two Digits Each
for the Month, Ending Date, Year, and Row Number.

-.44	-.20	.18	.71	1.08	1.12	1.00	1.16	1.22	1.17	1.60	1.65	1.62	1.70	1.71
-.41	-.26	.09	.50	.73	.81	.95	1.06	1.23	1.31	1.44	1.63	1.60	1.68	1.57
-.39	-.12	.02	.05	.34	.71	.88	.96	1.20	1.28	1.40	1.53	1.46	1.49	1.50
-.47	-.05	.02	.08	.15	.40	.87	1.03	1.15	1.42	1.42	1.48	1.41	1.36	1.29
-.33	-.15	-.09	-.04	.17	.34	.69	1.04	1.33	1.64	1.54	1.29	1.23	1.01	1.03
-.36	-.22	-.07	.13	.37	.47	.66	.78	.93	1.06	1.21	1.21	1.14	1.05	.77
-.25	-.20	-.02	.13	.38	.50	.67	.78	.88	.98	1.16	1.21	1.02	.68	.65
-.21	-.06	.08	.13	.47	.70	.76	.83	.96	1.04	1.06	.90	.64	.52	.95
-.61	-.16	.09	.32	.71	.67	.78	.89	.93	.85	.81	.95	.80	.44	1.06
-.34	-.09	.21	.47	.76	.55	.55	.70	.72	.78	.92	1.02	.90	.88	1.23
-.04	-.03	.23	.52	.64	.43	.52	.57	.67	.83	1.04	1.36	.97	1.10	1.40
.71	.32	.42	.41	.30	.17	.68	.76	.83	.92	1.29	1.52	1.41	1.56	1.32

Figure 11 - Data from Figure 10, Standardized to the Number of Standard Deviations from the Average for Each Grid Point

3	3	3	4	4	4	4	4	4	4	5	5	5	5	3
3	3	3	3	4	4	4	4	4	4	4	5	5	5	5
3	3	3	3	3	4	4	4	4	4	4	5	4	4	5
3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
3	3	3	3	3	3	4	4	4	5	5	4	4	4	4
3	3	3	3	3	3	4	4	4	4	4	4	4	4	4
3	3	3	3	3	4	4	4	4	4	4	4	4	4	4
3	3	3	3	3	4	4	4	4	4	4	4	4	4	4
2	3	3	3	4	4	4	4	4	4	4	4	4	3	4
3	3	3	3	4	4	4	4	4	4	4	4	4	4	4
3	3	3	4	4	3	4	4	4	4	4	4	4	4	4
4	3	3	3	3	3	4	4	4	4	4	5	4	5	4

1: At least 1.5 standard deviations below normal
2: 1.5 to .5 standard deviations below normal
3: Within .5 standard deviation of normal
4: .5 to 1.5 standard deviations above normal
5: At least 1.5 standard deviations above normal

Figure 12 - Data Values from Figure 11, Expressed in Terms of Five States

2	2	2	2	3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	3	3	3	3	3	3	2	2
2	2	2	2	2	2	3	3	3	3	3	3	2	2	3
2	2	2	2	2	2	3	3	3	3	3	3	3	2	3
2	2	2	2	3	2	2	2	2	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	3	3	3	3	3	3
2	2	2	2	2	2	2	3	3	3	3	3	3	3	3

1: At least .75 standard deviation below normal
2: Within .75 standard deviation of normal
3: At least .75 standard deviation above normal

Figure 13 - Data Values from Figure 11, Expressed in Terms of Three States

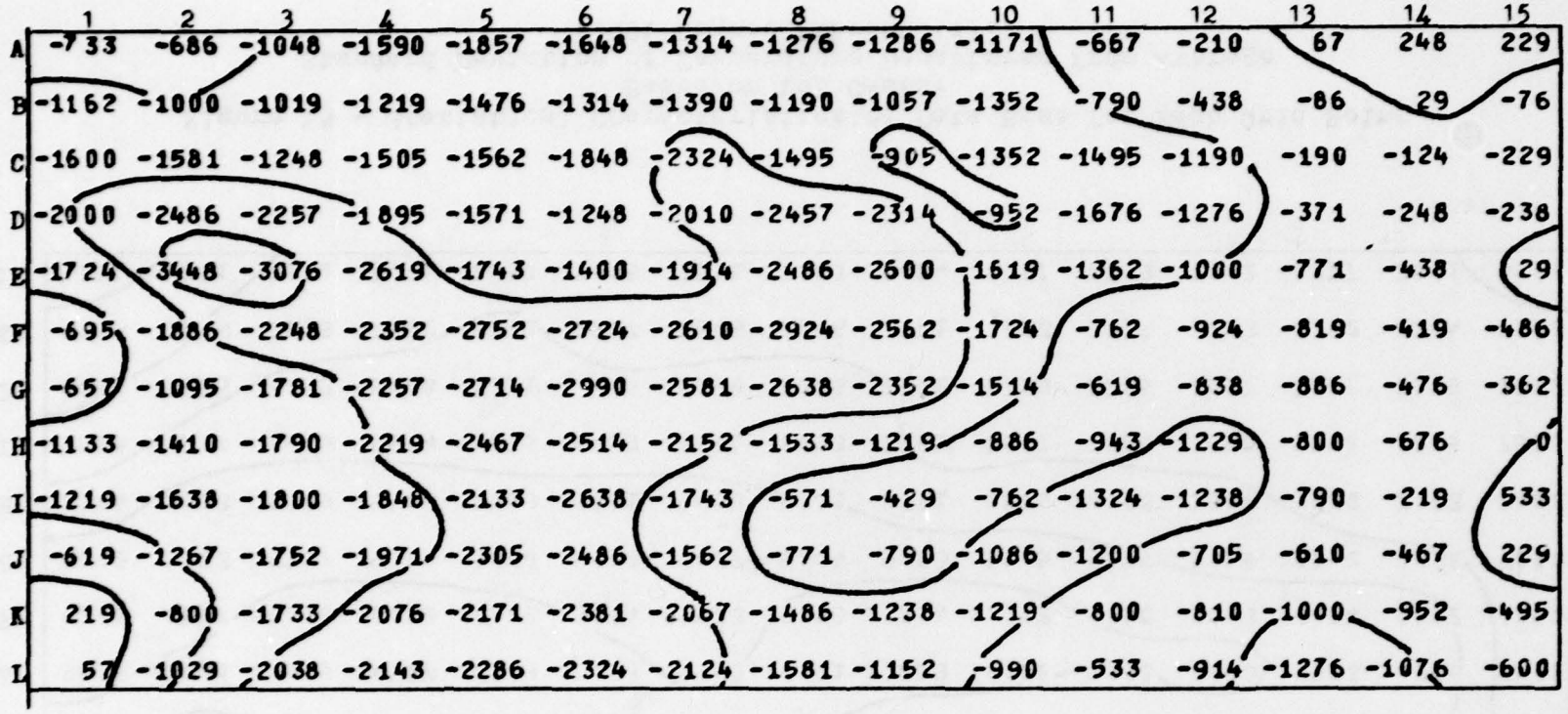

Figure 14 - Statistical Characteristics of Data Base for Each Grid Point, Based on 105 Cases: Mean Departure from 1941 - 1970 Normal Temperature (.001 Degree Fahrenheit).

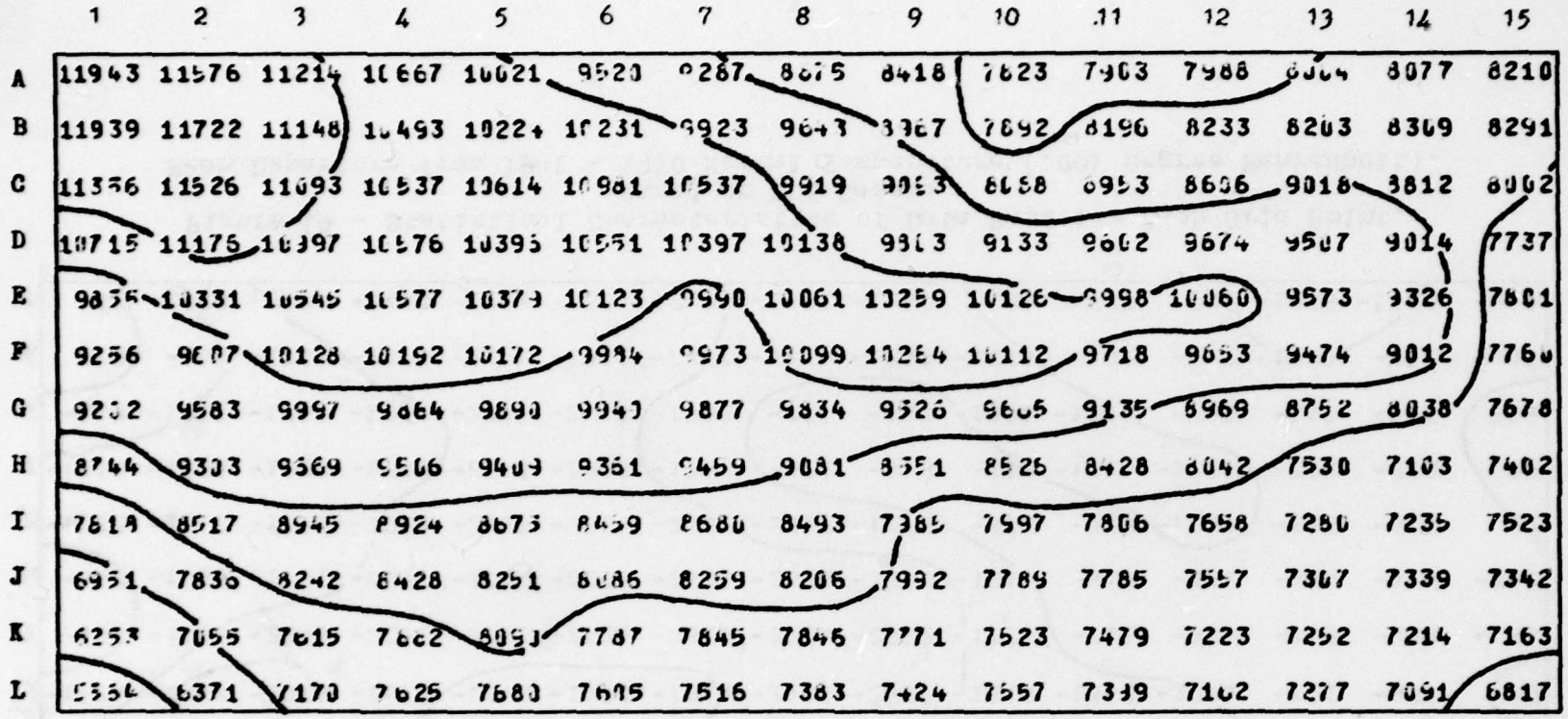

Figure 15 - Statistical Characteristics of Data Base for Each Grid Point, Based on 105 Cases: Standard Deviation of Temperature Departures from Average (.001 Degree Fahrenheit).

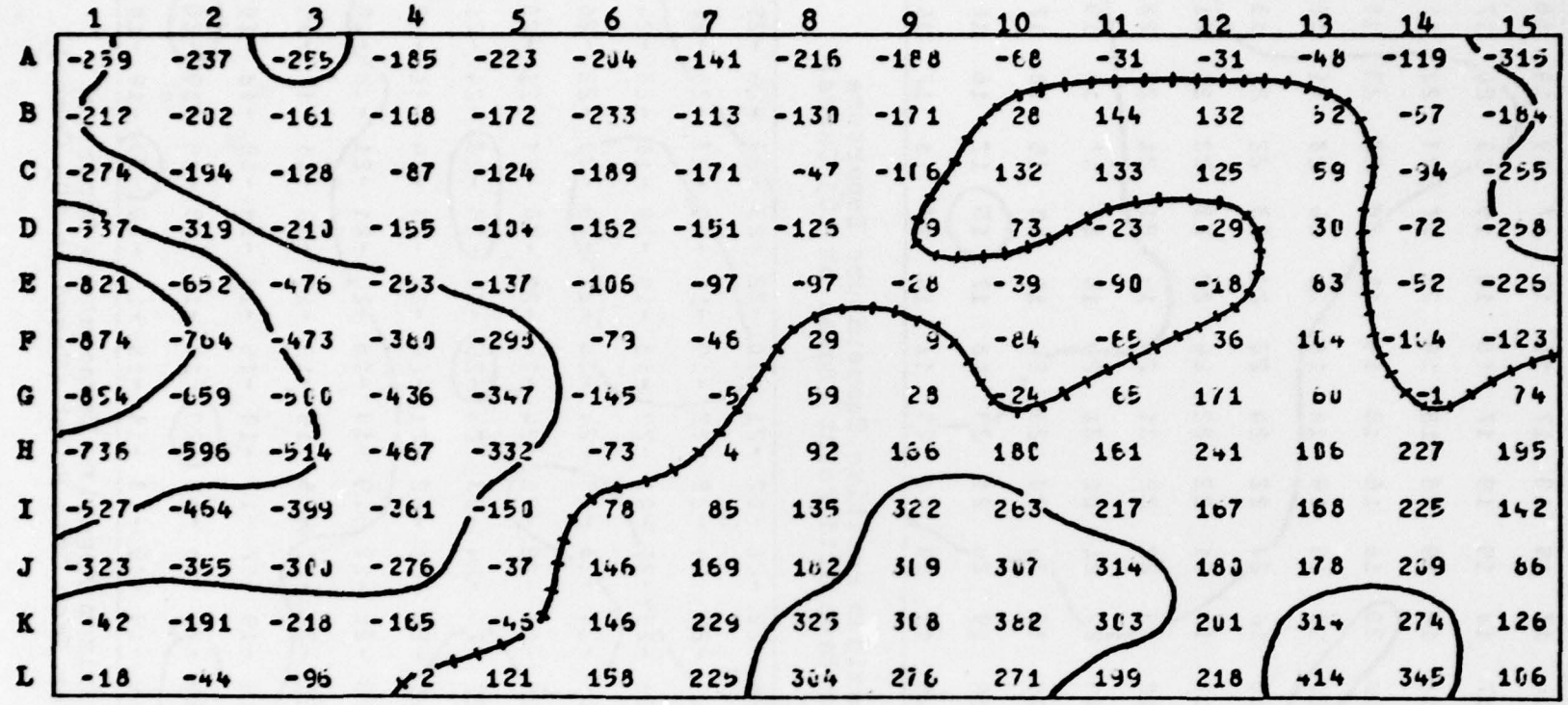

Figure 16 - Statistical Characteristics of Data Base for Each Grid Point, Based on 105 Cases: Skewness (.001 Degree Fahrenheit).

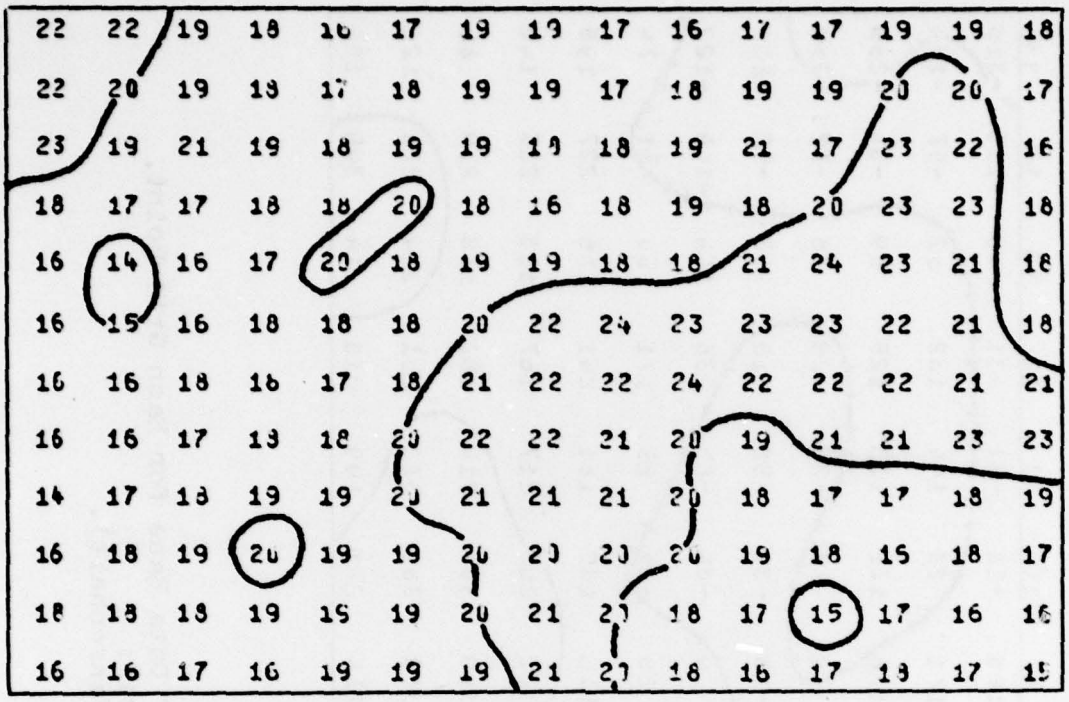

Figure 17 - Maximum Positive Temperature Departure from Normal for Each Grid Point (Based on 105 Cases)

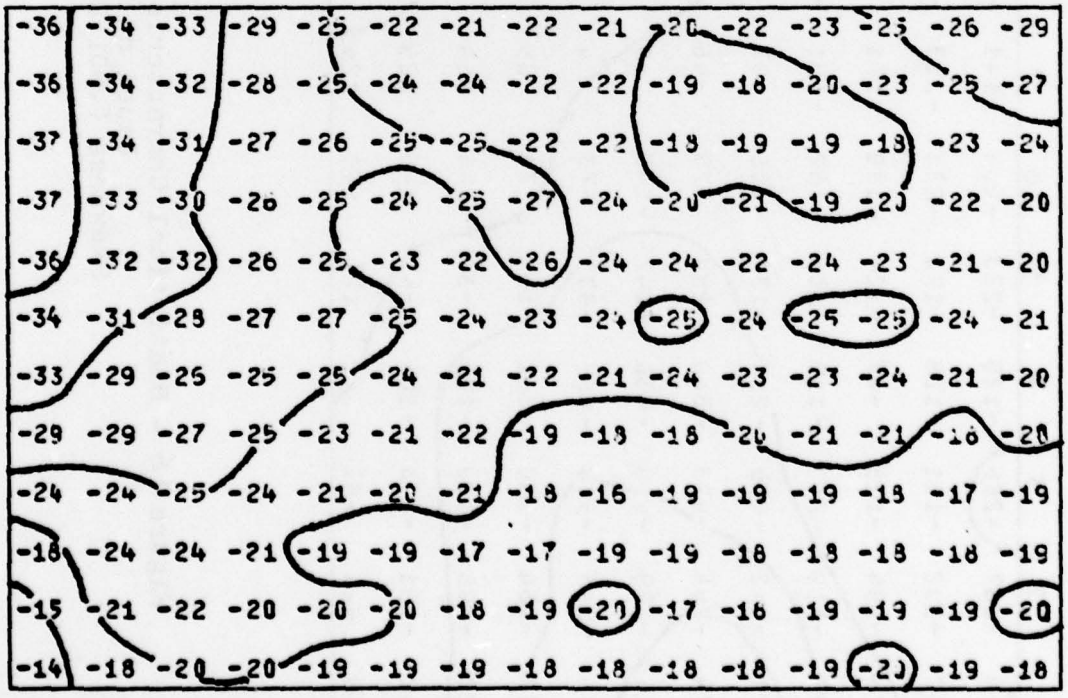

Figure 18 - Maximum Negative Temperature Departure from Normal for Each Grid Point (Based on 105 Cases)

states, with state 1 indicating the coldest temperatures. In the three-state case, the divisions between states were at -.75 and +.75 standard deviations from average. In the five-state case, the divisions were at -1.5, -.5, +.5, and +1.5 standard deviations. Most of the data analysis was carried out for both the three-state and five-state cases, although only five-state simulations were performed. The five-state and three-state data fields obtained from the data in figure 10 are shown in figures 12 and 13 respectively, and additional typical real data fields are shown in figures 19 and 20.

The correlation structure of the data was also examined using SPSS. There are many different ways of displaying the relationships, and a few examples are shown in figures 21 through 28. All correlations were based on the original data, not on the data as reduced to three or five states.

Figure 21 is a correlogram, showing the average correlation throughout the grid for all points no more than four rows or columns from an arbitrary point. Basically, the table value in position (i,j) is the autocorrelation coefficient, or average correlation, between all pairs of points with grid positions (r,c) and (r + i, c + j). For example, the second number in the third row (i = -2, j = -3) represents an autocorrelation coefficient of .8567, which is the average correlation between all pairs of points where the first point is two grid intervals north and three grid intervals west of the other. Since the second point is three intervals east and two intervals south of the first, the correlogram is symmetric about the origin and the correlation at i = 2, j = 3 is also .8567.

The correlation contours are nearly circular ellipses, with the

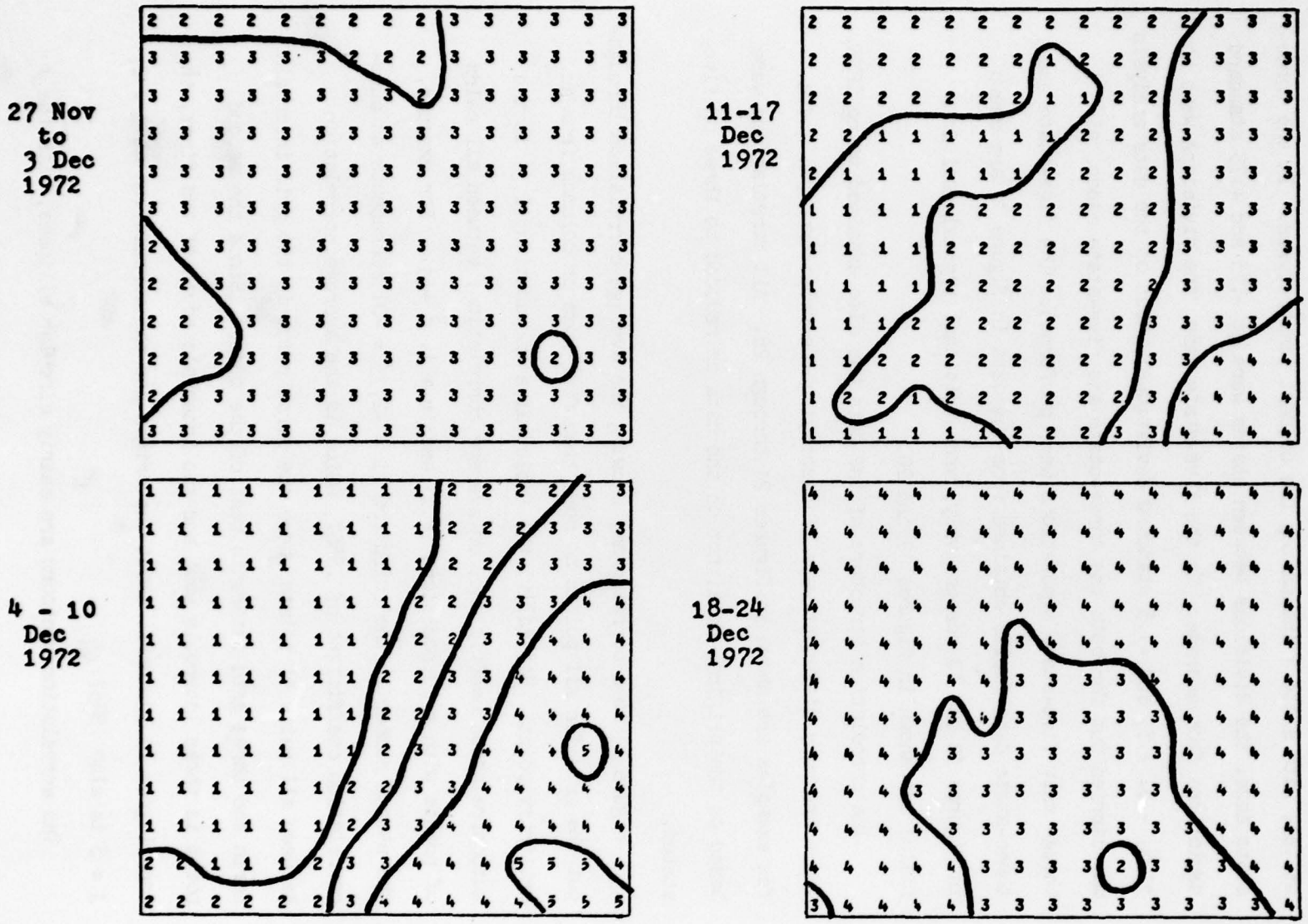

Figure 19 - Typical Weekly Gridded Data Fields, in Terms of Five States

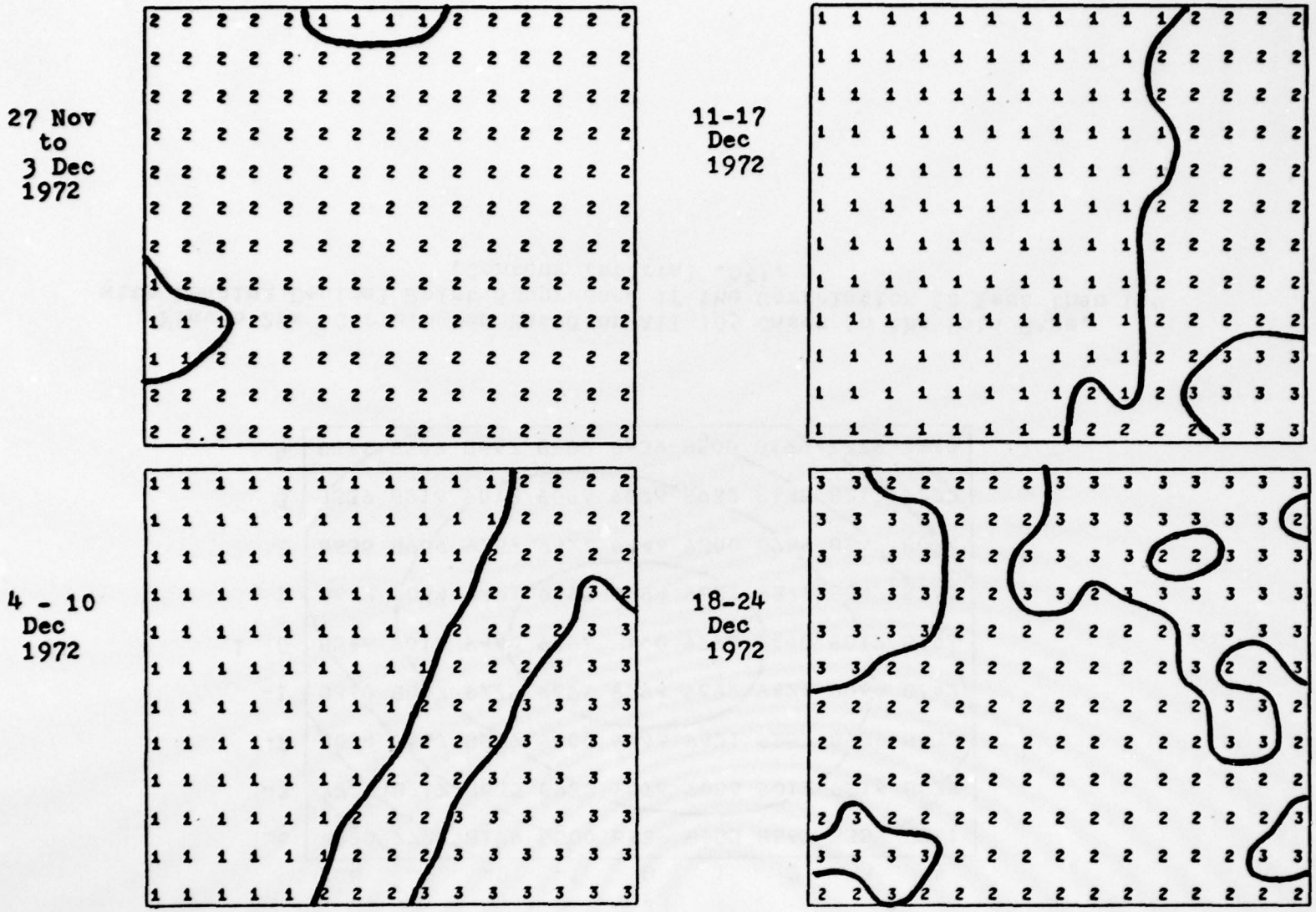

Figure 20 - The Same Data as in Figure 19, in Terms of Three States

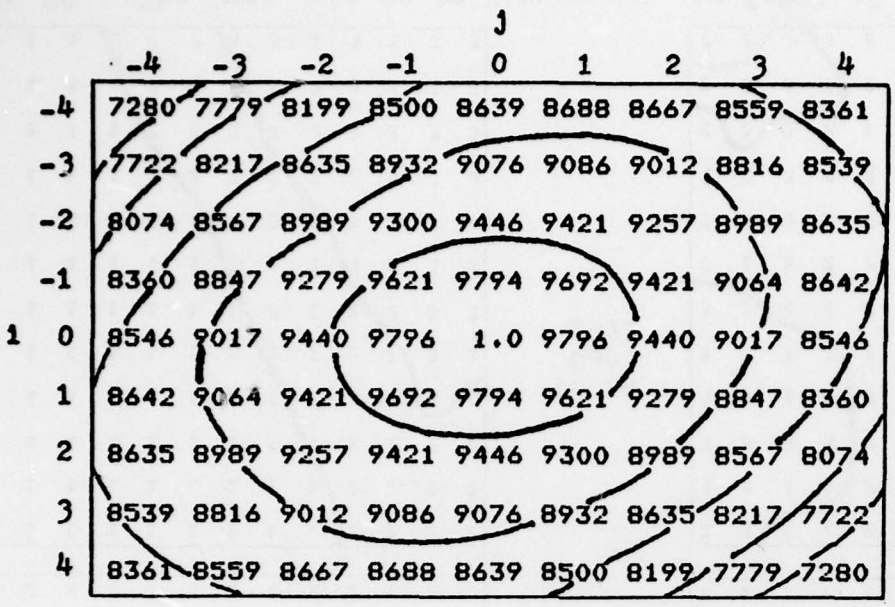

Figure 21: Correlogram Based on All 105 Cases in the Data Base, with Leading Decimal Point Suppressed if the Correlation is Less than 1.0 (Contour Interval .05).

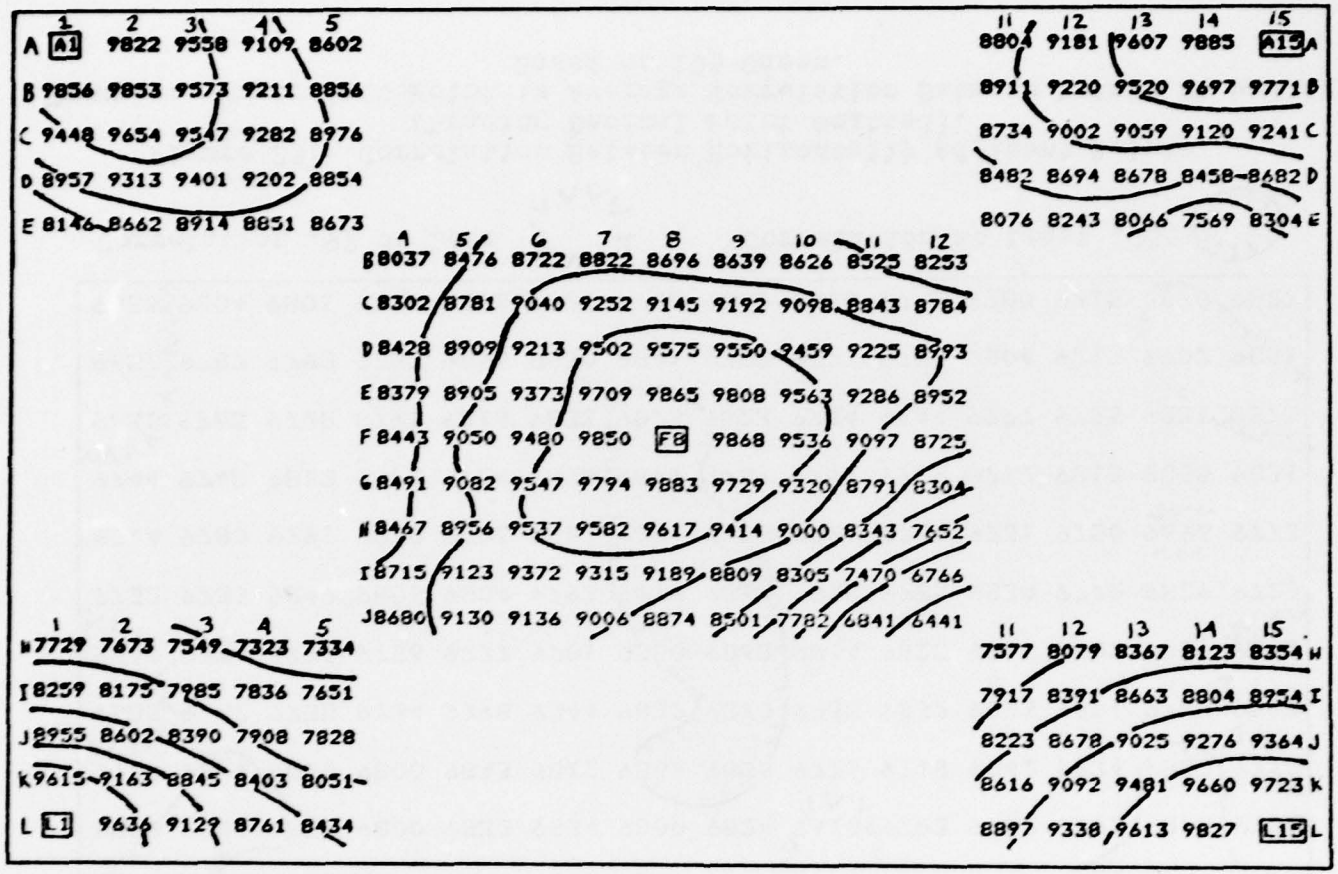

Figure 22 - Correlograms for Selected Grid Points in the Data Base

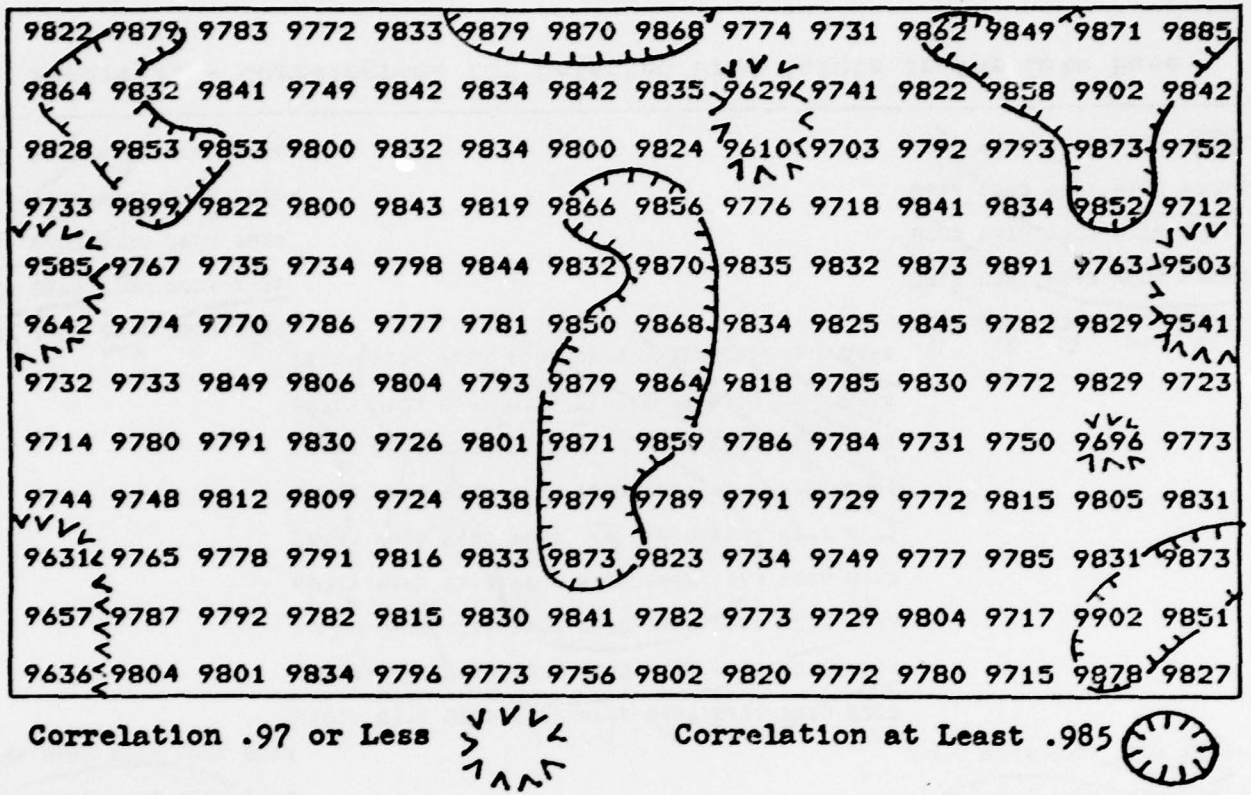

Figure 23: Correlation Between Horizontally Adjacent Points (Leading Decimal Point Omitted).
Example: Upper Left Point is Average Correlation Between Points A1 and A2, Based on 105 Cases.

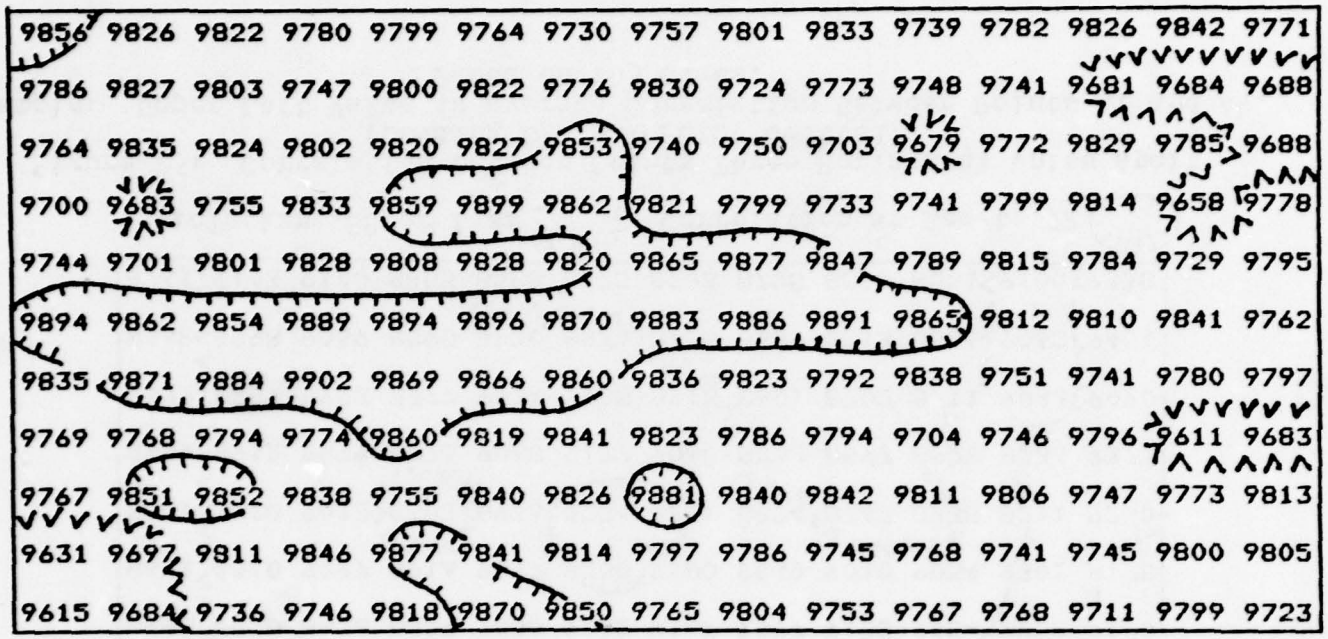

Figure 24: Correlation Between Vertically Adjacent Points (Leading Decimal Point Omitted).

Example: Upper Left Point is Average Correlation Between Points A1 and B1, Based on 105 Cases.

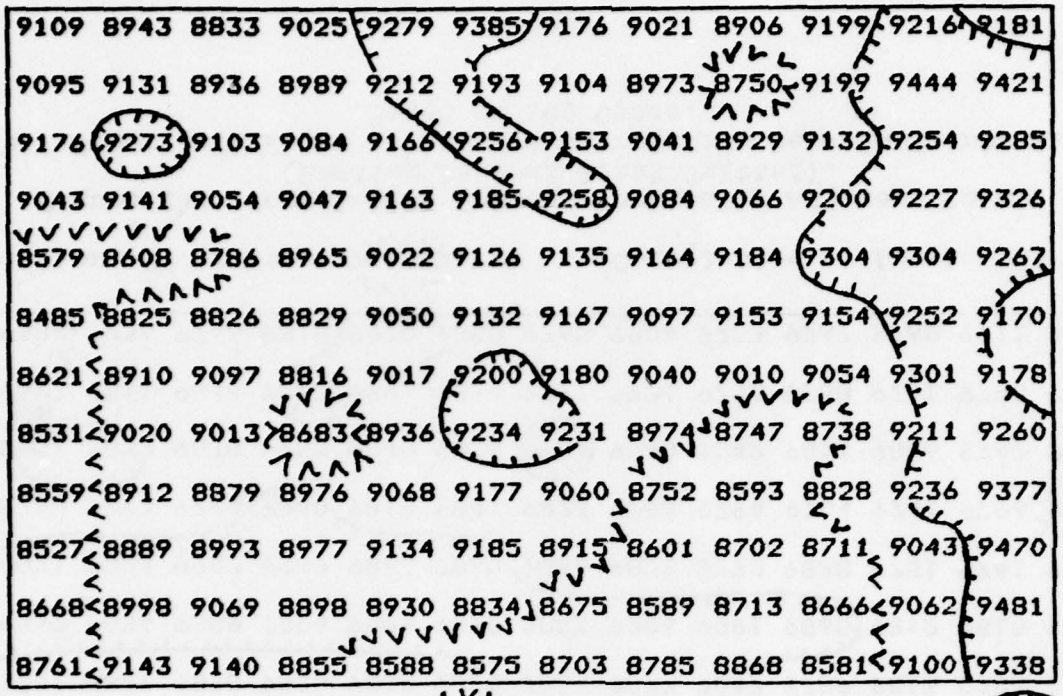

Figure 25: Correlation Between Points Three Horizontal Units Apart (Leading Decimal Point Omitted).
Example: Upper Left Value is Average Correlation Between Points A1 and A4, Based on 105 Cases.

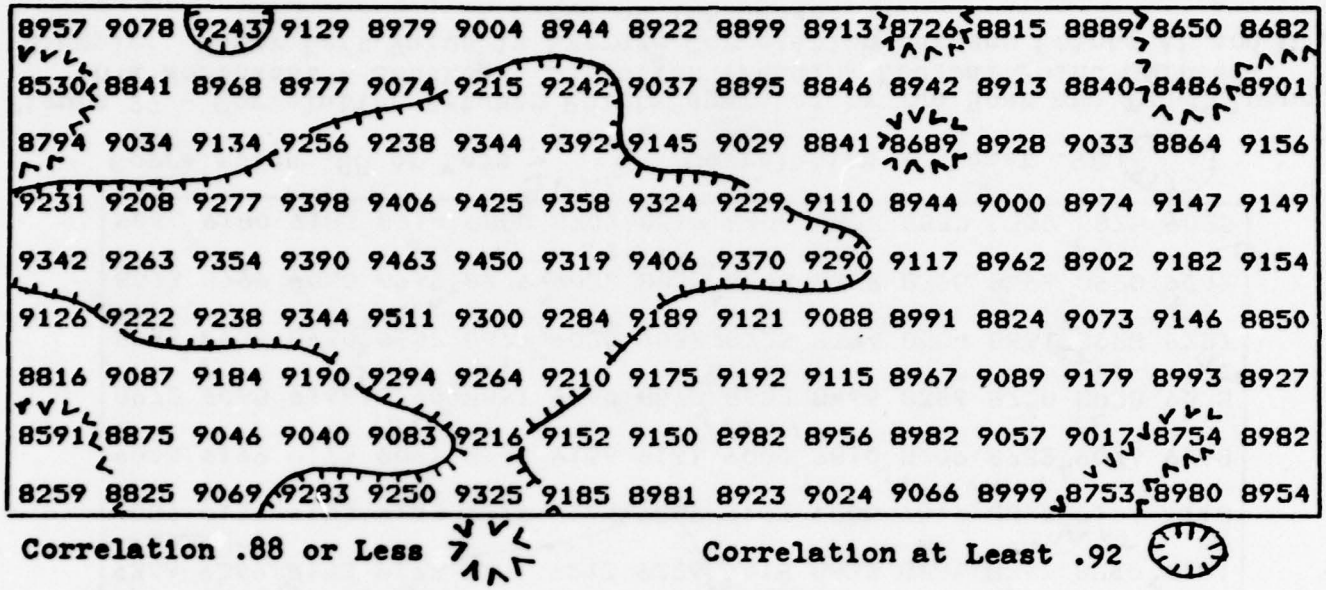

Figure 26: Correlation Between Points Three Vertical Units Apart (leading Decimal Point Omitted).

Example: Upper Left Value is Average Correlation Between Points A1 and D1, Based on 105 Cases.

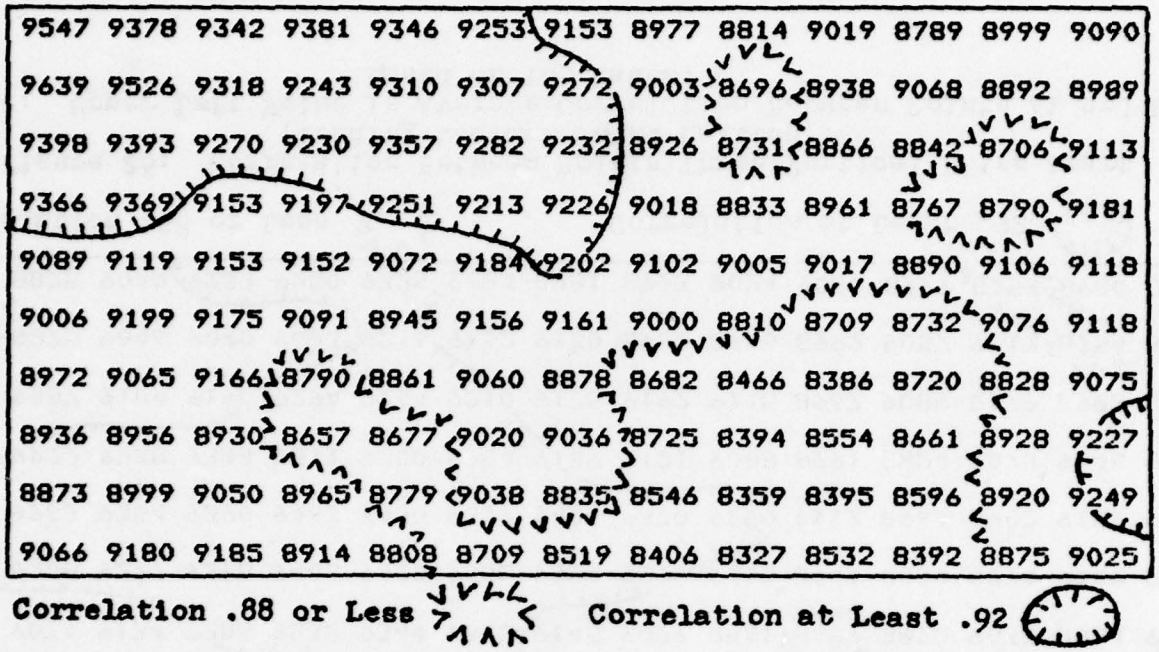

Figure 27 - Correlation Between Points Separated by Two Rows and Two Columns in a Northwest - Southeast Direction (Leading Decimal Point Omitted). Example: Upper Left Value is Average Correlation Between Points A1 and C3, Based on 105 Cases.

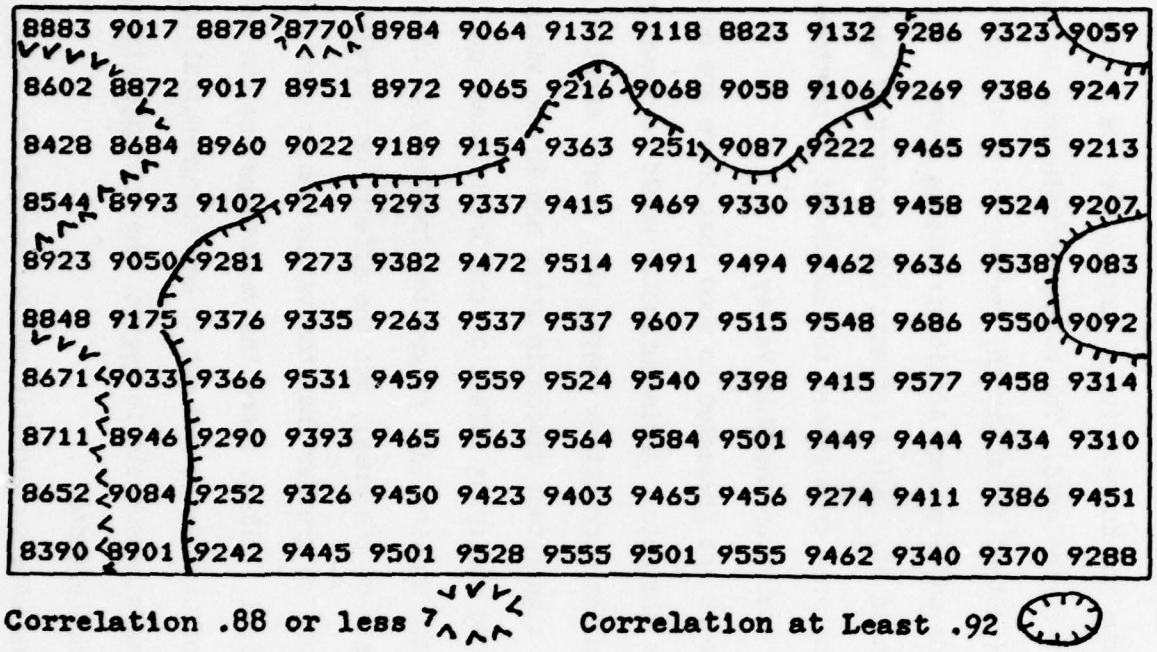

Figure 28 - Correlation Between Points Separated by Two Rows and Two Columns in a Northeast - Southwest Direction (Leading Decimal Point Omitted). Example: Upper Left Value is Average Correlation Between Points A3 and C1, Based on 105 Cases.

long axes oriented mostly from southwest to northeast, which is approximately parallel to the average winter wind direction in the middle atmosphere (the level that most strongly influences the movement of air masses) in the central and eastern United States.

Figure 22 shows the correlation structure for the corner points and a center point of the grid. The higher correlations to the southeast of point A1 (western North Dakota) correspond to the prevailing northwesterly winds and frequent Arctic air masses there. The pattern around point F8 (near St. Louis) reflects the tendency of many air masses to reach the area from either the southwest or northwest, and to leave the area by moving up the Ohio Valley.

Figures 23 to 28 are presented to show that there are no major discontinuities in the data. A discontinuity would be shown by a fairly sudden decrease in the correlation coefficient over a short distance. Possibly the most significant discontinuities are in the vicinity of Michigan (due to the modifying effect of the Great Lakes) and on the eastern slope of the Rockies (due to greater frequency of chinook winds in the western parts of the area), but the effects on correlation are fairly small and are no greater than the variations in correlation over the lower Mississippi Valley, where the variations are due to the prevailing movement of air masses and not to any topographical features. Therefore, it appears reasonable to neglect the inhomogeneities since they have only minor effects.

Figures 23 and 24 show correlations between points about 100 miles apart (with all correlations exceeding .95), and figures 25 to 28 show the correlation between points about 300 miles apart (with all correlations exceeding .8). As mentioned in chapter V, the autocorrelations

in the data field considered by Patankar (1954) and Whittle (1954) ranged from about .55 to .2.

The transition structure of the real data will be briefly discussed here even though the details of the process of obtaining transition matrices and a further discussion of the results will be given in following sections. In general, when five states were used, no cases where a transition skipped a state between horizontally adjacent points were found, and only two cases were found (from state 3 to 5, one case in Oklahoma on the map of 1 - 7 March 1976, and the other case in North Carolina on the map of 31 December 1973 to 6 January 1974) in a vertical direction. Of course, a larger data base may reveal more instances of such transitions, but based on the nature of the temperature data used, such occurrences should always be unusual. Any transition where the states in two horizontally or vertically adjacent points are not equal or consecutive will be called an "unusual transition of type 1."

It also was found that unequal or nonconsecutive states in diagonally adjacent points were unusual. Examples of such combinations are $\begin{matrix} 5 & \\ & 3 \end{matrix}$ and $\begin{matrix} & 4 \\ 2 & \end{matrix}$. To develop transition matrices for Pickard's method, all of the possible combinations of states in a "unit square," such as $\begin{matrix} 1 & 2 \\ 1 & 1 \end{matrix}$, are counted. By examining these frequency counts, only four cases were found, one each of $\begin{matrix} 1 & 2 \\ 2 & 3 \end{matrix}$, $\begin{matrix} 3 & 2 \\ 2 & 1 \end{matrix}$, $\begin{matrix} 3 & 2 \\ 4 & 3 \end{matrix}$, and $\begin{matrix} 3 & 4 \\ 4 & 5 \end{matrix}$. The cases of $\begin{matrix} 4 & 3 \\ 4 & 5 \end{matrix}$ and $\begin{matrix} 4 & 3 \\ 5 & 5 \end{matrix}$, and the two cases of $\begin{matrix} 3 & 4 \\ 5 & 5 \end{matrix}$ actually represented the two cases of "unusual transitions of type 1" mentioned in the last paragraph. If the states at two diagonally adjacent points are not equal or consecutive, and if no "unusual transition of type 1" occurs in the unit square containing the points, this is called an "unusual transition of type 2."

It is also useful to determine the number of occurrences of unequal and nonconsecutive states in a "unit diamond," an area such as $\begin{smallmatrix} & 2 & \\ 1 & & 2 \end{smallmatrix}$. This will be discussed more fully in describing "Method 2" of simulation. Other orientations of such a diamond-shaped area will not be considered here because of the process of simulation used in this thesis. If the states at the ends of the unit diamond are not equal or consecutive, and if no type 2 or type 1 transition occurs in either of the unit squares that contains three points from the unit diamond, this is called an "unusual transition of type 3." For example, $\begin{smallmatrix} & 3 & \\ 4 & & 2 \end{smallmatrix}$ would be an unusual transition of type 3, while $\begin{smallmatrix} & 4 & \\ 5 & & 3 \end{smallmatrix}$ would be an unusual transition of type 1. Based on the data (see the frequency counts in Appendix E), there were eight unusual transitions of type 3, plus five more of types 2 or 1. Many of these occurrences were on two consecutive maps, those of 5-11 and 12-18 February 1979, although three geographical areas were involved.

To summarize, three specific types of "unusual transitions" were defined and their occurrences in the actual data were counted. Since these "unusual transitions" were rare in the real data, a realistic simulation method based on this data base should not frequently generate such transitions.

First-Order Markov Transition Matrices

FORTRAN programs were written to count the number of transitions in both horizontal and vertical directions, for three and five states. These transition matrices and some additional data (which will be discussed in this section) are shown in appendix A. Where three states were used, no unusual transitions of type 1 (from state 1 to 3, or from 3 to 1 in horizontally or vertically adjacent points) occurred. As

mentioned before, two unusual transitions of type 1 occurred in a north-south direction when the data was divided into five states, but none were observed in a horizontal direction. In both cases, the horizontal and vertical transition matrices were quite similar, with most transition probabilities agreeing within .02. No consistent pattern of deviations was noticed.

In each program, the unconditional equilibrium probabilities were calculated from the transition probability matrix by solving VP = V, and requiring the sum of the probabilities to be 1. (This was done by expressing the problem in terms of simultaneous equations, and using the IMSL subroutine LEQT1F to solve the equations). The actual unconditional and calculated equilibrium probabilities were quite similar, although a few values differed by over .02. When comparing horizontal with vertical transitions, the actual probabilities agreed better than the calculated probabilities. No consistent pattern of deviations was noticed in this case either, although the cold states were frequently calculated to occur more often than they actually occurred, and the warm states usually were calculated to occur less often than they actually were observed.

Since the states were set up in terms of standard deviations from average, the theoretical frequency of each state could be calculated. In all cases, the middle state occurred a little less frequently than expected. Generally, both the warmest and coldest states were slightly more frequent than expected, except that when five states were used, state 5 occurred a little less frequently than the theoretical frequency. The fact that most extremes occurred more often than expected is verified by the fact that the kurtosis in most areas was lower than

the expected value for a standard normal distribution.

The reversibility of each matrix was checked to help verify homogeneity or inhomogeneity of the data. Only the five-state vertical transition matrix showed any difference when reversed, at least to the eighth decimal point. This matrix cannot be reversible, since a transition from state 3 to state 5 (from north to south) becomes a transition from state 5 to state 3 in a northward direction. In this case, both the reversed and reversible transition probability matrices are shown, and the differences are .002 or smaller.

To verify the intuitive supposition that simulating a two-dimensional data base by a one-dimensional method will produce unrealistic results, a demonstration was performed which is shown in figure 29. Either the left side of the data base was simulated, and then the rows, or the top edge was simulated, and then the columns.

Transition Matrices Based on Galbraith and Walley's Method

All configurations of states in the form $_{I}{}^{J}_{K}$ were counted by a FORTRAN program to develop transition matrices for Galbraith and Walley's (1976) method. As mentioned in chapter VI, this method bases the probability distribution for state K on the combination of states (I,J). The transition matrices are presented for both three and five states in appendix B. Only three "unusual transitions" were found, one case of $_{4}{}^{3}_{2}$ (type 2), and one case each of $_{4}{}^{3}_{5}$ and $_{5}{}^{3}_{5}$ (type 1), since only a portion of the unit square was considered.

A sample of four simulations performed using Galbraith and Walley's method is shown in figure 30. The top and left borders were generated using the first-order transition matrices. In general, the simulated data fields are fairly good, and it would be possible to postulate realistic weather conditions that would generate such temperature

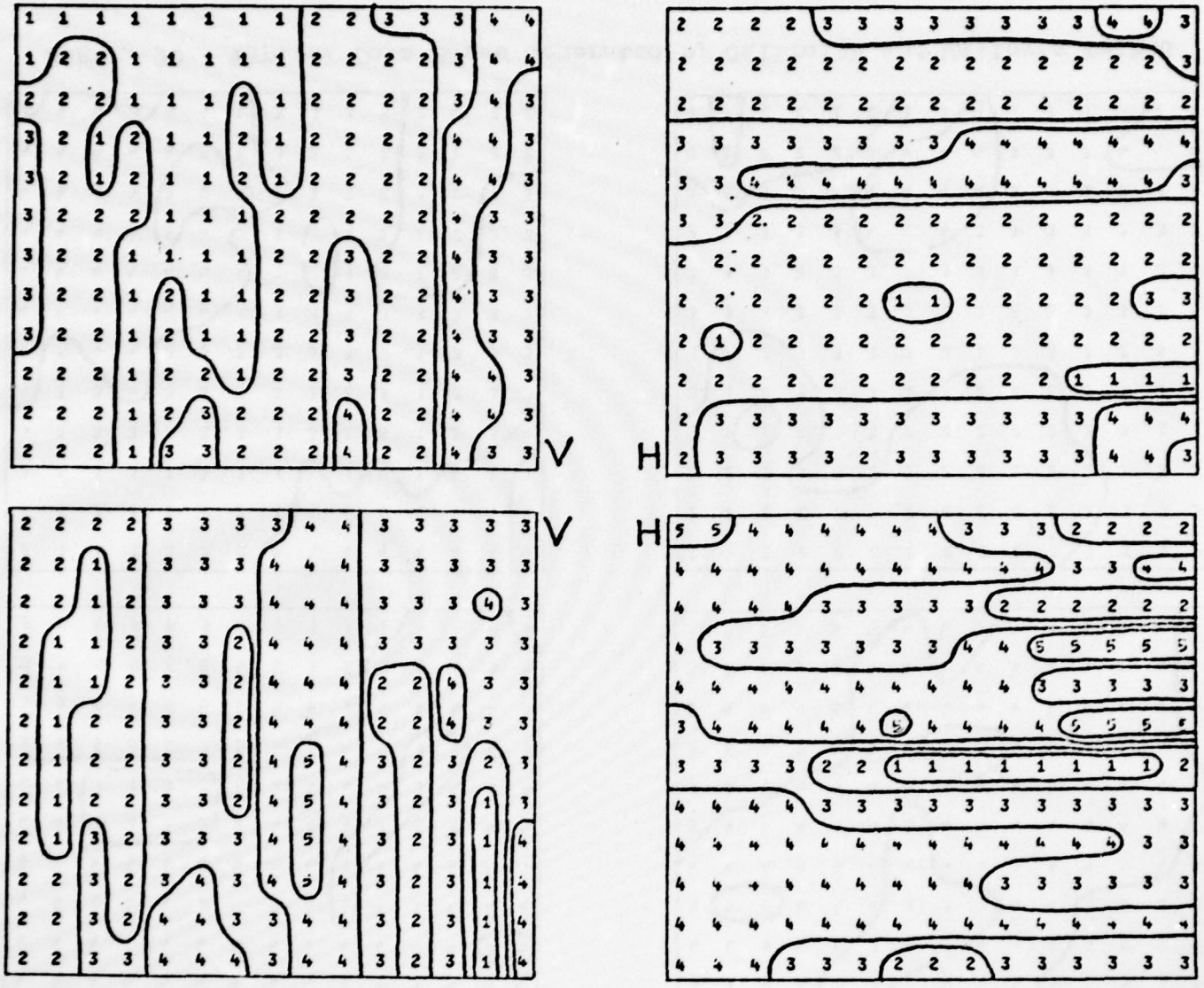

Figure 29 - Typical Data Bases Generated by First-Order Vertical (V) and Horizontal (H) Markov Processes

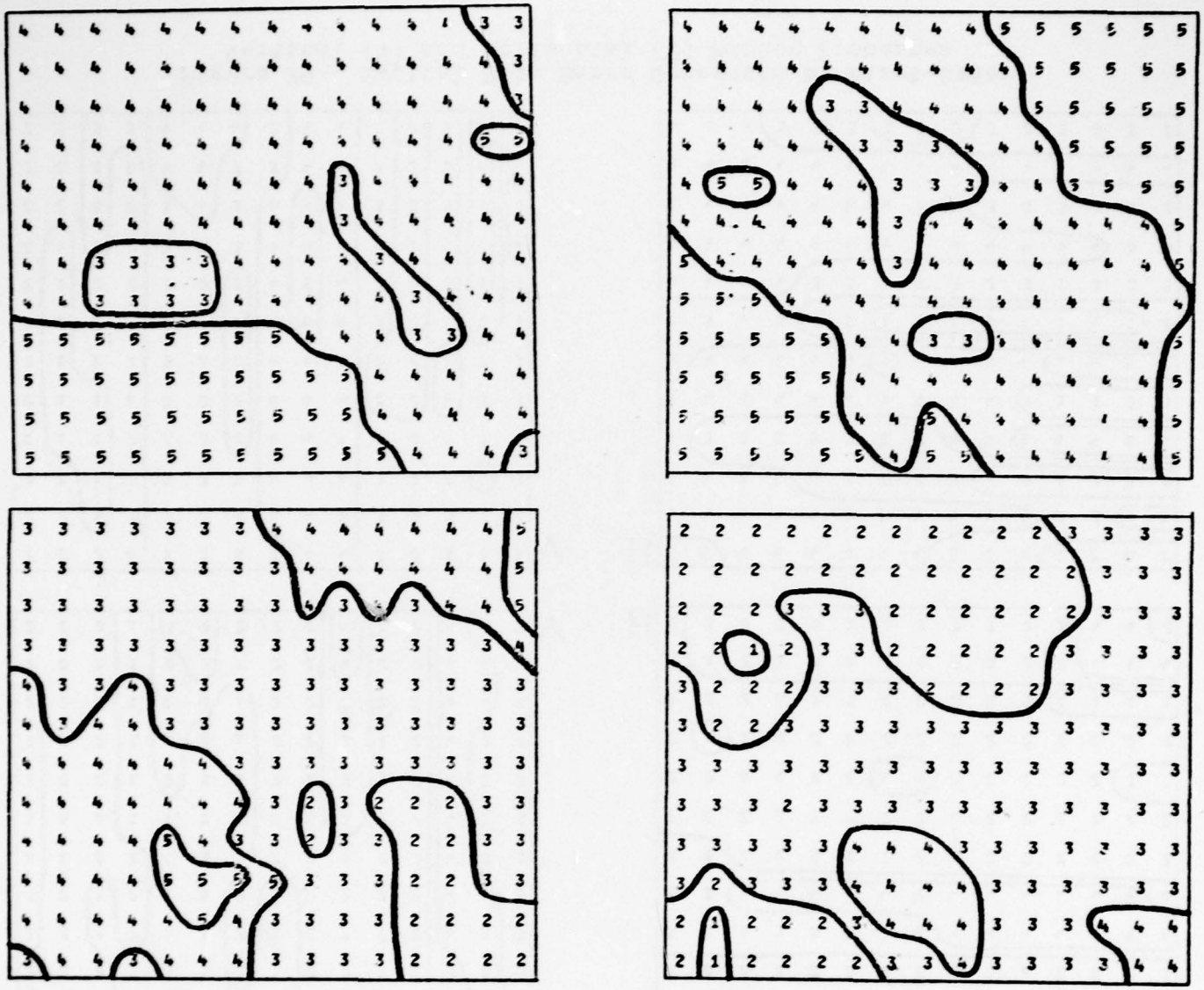

Figure 30 - Typical Data Bases Generated by Galbraith and Walley's Method

patterns. So, this method could be considered a fairly good method, and it is simple to perform on a computer.

Only five-state simulations were performed, to better evaluate the results and reveal operational problems connected with the simulations. One immediately apparent result was that it was possible to generate transitions that were not observed in the original data, and that unusual transitions were generated quite frequently. With this method, mostly "unusual transitions of type 2" were generated. When developing the transition matrix, if there were no occurrences of a particular combination of input states, the probabilities were all specified to be zero. The method chosen to deal with the problem in this case is to set K = (I + J)/2, (using the configuration $_{I}{}^{J}_{K}$), if (I + J)/2 is an integer, and to randomly select from the integer values immediately above and below (I + J)/2 if it is not an integer. Then $_{4}{}^{2}$ would lead to $_{4}{}^{2}_{3}$ and $_{4}{}^{1}$ would lead to $_{4}{}^{1}_{3}$ or $_{4}{}^{1}_{2}$.

Transition Matrices Based on Pickard's Method

To calculate the transition matrix for Pickard's (1977) method, all occurrences of combinations of states in a unit square were counted. In this section, $_{I}{}^{J}_{L}{}^{K}$ will denote the states in the respective positions of the square. The computer program which performed these calculations is shown in appendix C (for the five-state case), along with both three-state and five-state transition matrices. Also, the "cumulative transition probability matrix" is shown for the five-state case. Such a matrix would probably be the easiest way to perform a simulation. A random number between 0 and 1 would be drawn, and the first value of the cumulative transition probability matrix which equals or exceeds the random number drawn is the state which would be selected for the data

point being simulated.

In this case, the computer program is included to demonstrate the procedures involved. The modifications to adapt this program to determine transition probability matrices for other simulation methods may not be extensive. The input data states would be stored in a data file which is attached under the local file name STATES. Four rows of data points would be stored in each line of the file, in the format 60I1, so three lines would be used to store a complete case. The 180 data values in each case are read in sequence and are stored in the array INPUT, with index numbers INPUT (1) to INPUT (180). The state value recorded at position (r,c) is stored in array position INPUT (15*(r - 1) + c).

The matrices used in this program are (1) ITPM, the counts of all occurrences of combinations of (I,J,K,L), in the configuration $\begin{matrix} J & K \\ I & L \end{matrix}$, (2) TPM, the transition probability matrix, (3) CTPM, the cumulative transition probability matrix, and (4) ISUM, the matrix of row sums in ITPM (the sum of all occurrences of each I,J,K combination). Since the FORTRAN version used at AFIT allows arrays of no more than three dimensions, the I and J state combinations are stored in the first dimension of the array, under the name IJ, according to the formula on line 19 in the computer program, with 25 possible values. The variables LI, LJ, LK, and LL are the array locations of the INPUT (state) variables that compose the unit square. For example, the states observed in the unit square at the northwest corner of the grid are stored in INPUT array locations LI = 16, LJ = 1, LK = 2, and LL = 17. Then, NK is the state at the point LK and NL is the state at point LL, with NIJ being a combination of the states at points LI and LJ. Lines 21 to 23 increment the appropriate array locations in ITPM and ISUM and add 1 to ITOTAL, the

total number of transitions. Since only 14 unit squares can be contained in a row of a grid 15 points wide, line 24 increments LI by 1 at the end of each row.

The section from lines 27 to 37 calculates the transition and cumulative transition probabilities from the number of transitions and the row sums. If no occurrences of a particular (I,J,K) combination are observed, all transition and cumulative transition probabilities are zero.

The last section contains PRINT statements. Variable ROWEND in lines 64 to 66 is either 1 or 0, depending on whether or not there are any occurrences of the particular (I,J,K) combination.

Examples of simulations performed using Pickard's method are shown in figure 31. The real data had eight unusual transitions of types 1 and 2, since the whole unit square was considered, in 105 data fields. However, Pickard's method generated 14 such combinations in six simulations. The simulations look acceptable and are at least as realistic as those generated by Galbraith and Walley's method. It would be difficult to determine if Pickard's method is sufficiently better than Galbraith and Walley's method to justify the additional computer storage space involved except by performing additional simulations. However, either method uses very little computer time because of the simplicity of the operations.

No boundary effects were noticed. If boundary effects occur, there would be a tendency to generate unusual patterns near the top and left borders (which were simulated first by first-order Markov processes), and the patterns would settle down to more realism in the interior of the simulated data field. Since Pickard proved that his method produces

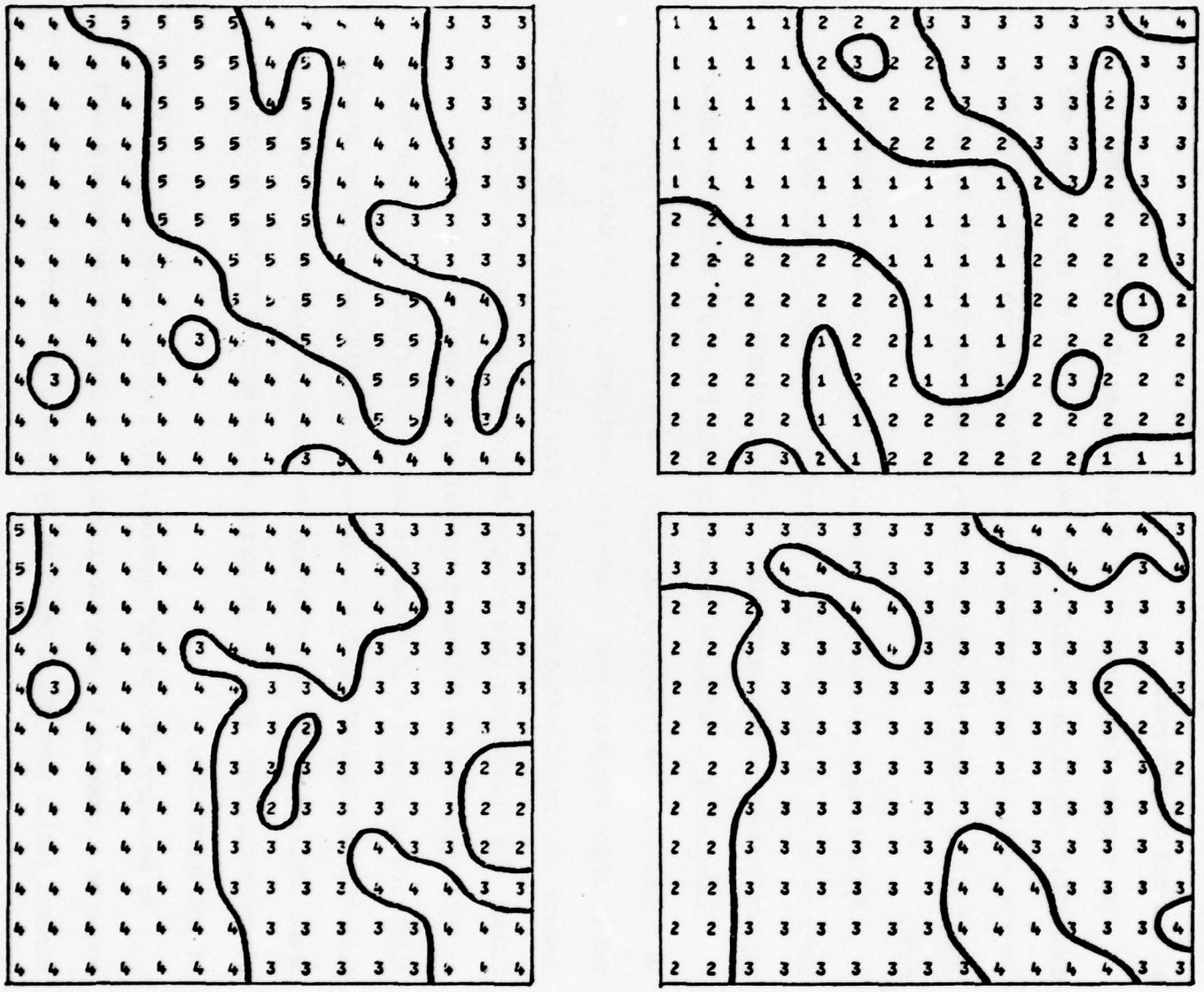

Figure 31 - Typical Data Bases Generated by Pickard's Method

stationary homogeneous Markov random fields, boundary effects would not be expected. Similarly, Galbraith and Walley's method appeared to produce stationary fields, even though they did not deal with this explicitly in their paper.

A "Method 1" Transition Matrix

The number of transitions used to determine a transition probability matrix for Method 1 can be obtained from the number of transitions counted for Pickard's method, since Pickard's method requires the frequencies of all combinations of states in a unit square. Denoting the states in a unit square by $\begin{matrix} J & K \\ I & L \end{matrix}$, the vector state (J,K) would be the input state, and (I,L) would be the output state. For a five-state process, this would give a 25 x 25 transition matrix. Transition probabilities were developed only for the five-state case, and the transition probability matrices are given in appendix D.

Simulations generated by this method are shown in figure 32. In each case, the top row was generated by a first-order horizontal Markov chain, and the unit squares defining the input and output states alternated in position from row to row as in figure 8. Since the first-order horizontal Markov chain was found to be reversible, the same transition matrix could be used to generate extra values needed at either end of a row.

In comparison with Galbraith and Walley's and Pickard's methods, the data fields simulated by Method 1 appear deficient, basically by generating far too many unusual transitions of type 1. This could occur, for example, in the row under a row of consecutive values of state 4. One (4,4) pair could generate a (4,3) immediately below it, and the next pair could generate a (5,4) immediately below it, so the set of

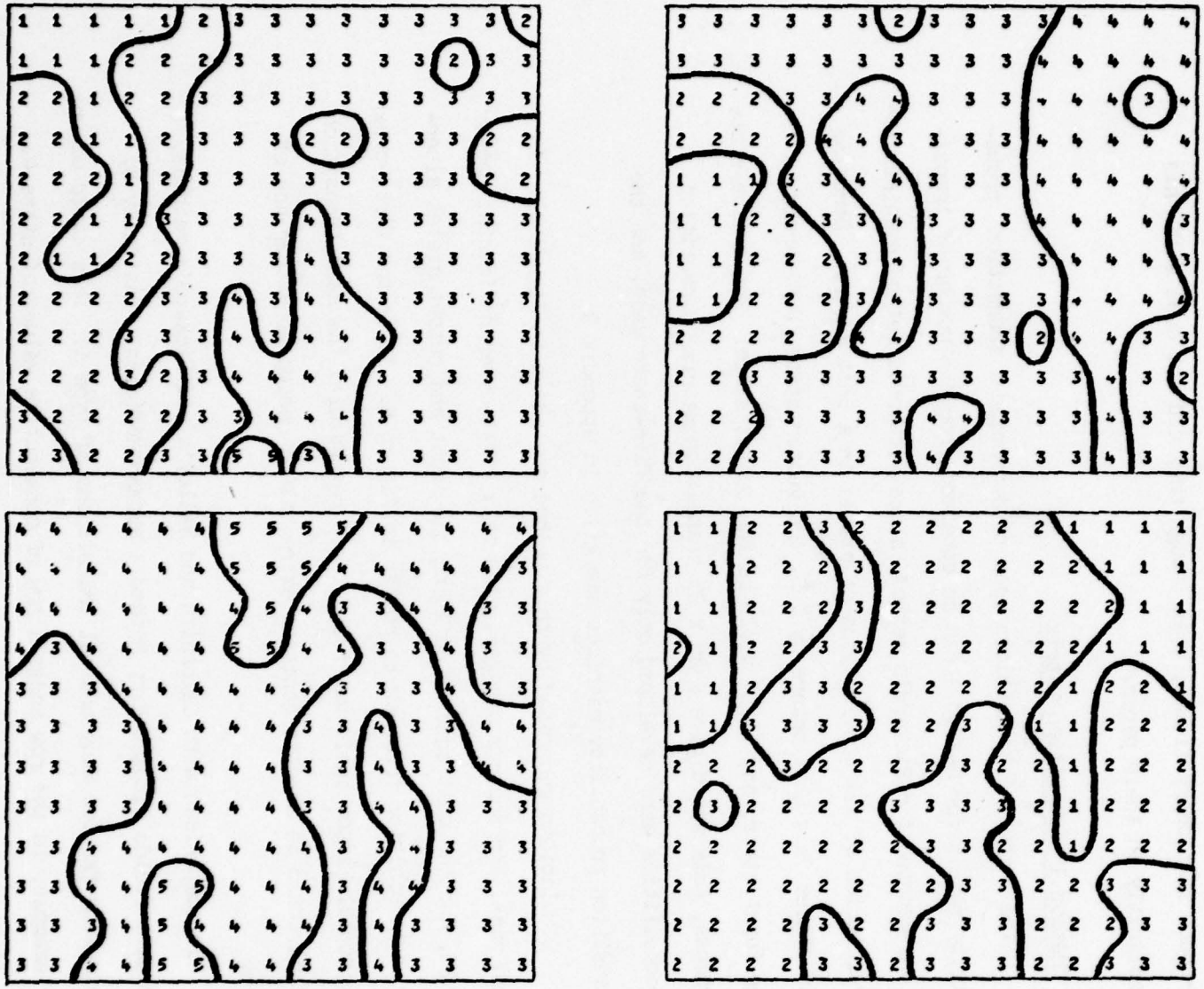

Figure 32 - Typical Data Bases Generated by "Method 1"

four generated values would be (4,3,5,4). A sequence such as (3,3,4,4) could generate a sequence of (2,2,5,5) below it. If an unusual transition of type 1 occurs, the two states involved, such as (3,5), are averaged to specify the points below them. If the average is not an integer, the integer values immediately below and above the average are used. For example, a (2,5) would generate a (3,4) below it.

Discussion of the Problem of "Unusual Transitions"

All of the two-dimensional simulation methods discussed here generated unusual transitions much more frequently than in the original data. The number of unusual transitions of each type was counted in the simulations produced by each method discussed so far, regardless of how the data values were simulated. For example, Galbraith and Walley's method could generate an unusual transition of type 3 even though the simulation of a data value does not consider a complete unit diamond. Type 2 transitions were counted in all unit squares, and type 3 transitions were counted in all unit diamonds. However, if a type 2 transition was found in either unit square that contained three points of a unit diamond, the transition was counted as type 2 rather than type 3. For example, the six points $\begin{matrix} 4 & 3 & 4 \\ 5 & 4 & 4 \end{matrix}$ contain two unit squares, $\begin{matrix} 4 & 3 \\ 5 & 4 \end{matrix}$ and $\begin{matrix} 3 & 4 \\ 4 & 4 \end{matrix}$, as well as a unit diamond $\begin{matrix} & 3 & \\ 5 & & 4 \end{matrix}$. One unusual transition of type 2 would be counted in this structure. Also, if a type 1 transition is found in a unit square, the unusual transition will be counted only as type 1. The number of unusual transitions found was as follows:

Galbraith and Walley: Type 1 - 7, Type 2 - 9, Type 3 - 1 in 6 simulations.

Pickard: Type 1 - 2, Type 2 - 12, Type 3 - 11, in 6 simulations.

Method 1: Type 1 - 10, Type 2 - none, Type 3 - 15, in 4 simulations.

Real data: Type 1 - 2, Type 2 - 2, Type 3 - 8, in 105 data fields.

These unusual transitions occur frequently in simulations due to the fact that a Markov chain does not account for correlation beyond the range of the data points used as input states. An unusual transition of type 2 could be generated easily, for example, by Galbraith and Walley's method as follows. A combination of $\begin{smallmatrix} & 3 \\ 3 & \end{smallmatrix}$ would generate a structure of $\begin{smallmatrix} & 3 \\ 3 & 4 \end{smallmatrix}$ with a probability of .0324. Assuming that the probabilities in a row are the same as first-order horizontal transition probabilities given in appendix A, the 3 in the top row would be followed by a 2 with a probability of .0679. This would give a diamond-shaped structure of $\begin{smallmatrix} & 3 & 2 \\ 3 & 4 & \end{smallmatrix}$, with a joint probability of (.0324)(.0674) = .00213, assuming that these transitions are independent and given that the $\begin{smallmatrix} & 3 \\ 3 & \end{smallmatrix}$ combination occurred.

With the proper assumptions, it is possible to calculate the probability of each different possible unusual transition of type 1 or 2. For the following discussion, assume that no unusual transitions of either type 1 or 2 occurred in the original data, so that any occurrence of either type in simulated data would be spurious. Also, assume that Galbraith and Walley's method is to be used for simulation. Then, no unusual transitions of any kind will be generated within the triangle of three numbers involved in a single transition, such as $\begin{smallmatrix} & 2 \\ 2 & 3 \end{smallmatrix}$. However, an unusual transition of type 2 may be generated in a unit diamond. The 12 possible combinations which start with a normal transition but generate an unusual transition of type 2 are as follows: $\begin{smallmatrix} 2 & 3 \\ 1 & 1 \end{smallmatrix}$, $\begin{smallmatrix} 3 & 4 \\ 2 & 2 \end{smallmatrix}$, $\begin{smallmatrix} 4 & 5 \\ 3 & 3 \end{smallmatrix}$, $\begin{smallmatrix} 2 & 1 \\ 3 & 3 \end{smallmatrix}$, $\begin{smallmatrix} 3 & 2 \\ 4 & 4 \end{smallmatrix}$, $\begin{smallmatrix} 4 & 3 \\ 5 & 5 \end{smallmatrix}$, $\begin{smallmatrix} 2 & 1 \\ 2 & 3 \end{smallmatrix}$, $\begin{smallmatrix} 3 & 2 \\ 3 & 4 \end{smallmatrix}$, $\begin{smallmatrix} 4 & 3 \\ 4 & 5 \end{smallmatrix}$, $\begin{smallmatrix} 2 & 3 \\ 2 & 1 \end{smallmatrix}$, $\begin{smallmatrix} 3 & 4 \\ 3 & 2 \end{smallmatrix}$ and $\begin{smallmatrix} 4 & 5 \\ 4 & 3 \end{smallmatrix}$. If the method of dealing with an unusual transition of type 2 is to have

the next simulated value be the average of the two states involved in the unusual transition, no unusual transitions of type 1 could be generated. If there were any real unusual transitions of type 1 or 2 in the data from which transition probabilities were determined, some more extreme combinations could be generated, such as $_{4}^{3}{}_{5}^{2}$ or $_{5}^{3}{}_{5}^{2}$. Both of these could be generated from the data set used as an example in this chapter, since the left three numbers in each unit diamond are combinations which were observed in the actual data.

It is desirable to see if the probability of unusual transitions can be calculated on an unconditional basis. For example, the unconditional probability of $_{3}^{3}{}_{4}^{2}$ is equal to the conditional probability of $_{3}^{3}{}_{4}^{2}$; given that $_{3}^{3}$ occurs, multiplied by the unconditional probability of an occurrence of $_{3}^{3}$. Actually, this problem is not analytically solvable at present, so it will be assumed here that the unconditional probability of any $_{I}^{K}$ combination is equal to the probability of that combination in the actual data. This can be found from the row sums of the number of transitions counted for Galbraith and Walley's method as in appendix B. For the example above, the probability of a $_{3}^{3}$ is the row sum where I = 3 and J = 3, divided by the total number of transitions, which is 5035/16170 = .3114. Multiplying by the conditional probability given above, the unconditional probability of $_{3}^{3}{}_{4}^{2}$ is (.3114)(.00213) = .000684. If this is repeated for each of the 12 possible unusual transitions of type 2, the total unconditional probability of generating an unusual transition of type 2 is .00827. The six simulations performed using Galbraith and Walley's method contained 14 x 11 x 6 = 924 points simulated by that method. Nine of these were unusual transitions of type 2 (all of the type 1 transitions were

generated due to such transitions actually existing in the data), so the frequency of generation was 9/924 = .00974. Only two cases of an unusual transition of type 2 were found in the data (excluding transitions of type 1), so the real probability would be about 2/16170 = .00012.

There are several possible methods of dealing with this problem, all of them arbitrary. Actually, using the transition matrix in cases where an unusual transition is simulated that matches one observed in the real data may still not be realistic, since the transition probabilities may be based on only one case. For example, using Galbraith and Walley's method, an occurrence of $_5{}^3$ always led to $_5{}^3_5$, so one structure of $_5{}^{3\ 2}_{5}$ was generated, and also a long sequence of $_5{}^{3\ 3\ 3\ 3}_{5\ 5\ 5\ 5}$. So, even when a few unusual transitions are observed to actually occur, it may be best to neglect them in developing the transition probabilities and substitute an empirical procedure.

The empirical procedure chosen could be used in the simulation either when all transition probabilities are zero or when the transition meets criteria for being "unusual." Except when unusual transitions of type 2 or type 1 are allowed because of such occurrences in the real data, most or all of the unusual transitions that need to be dealt with are type 2.

One procedure is the "compromise" procedure described earlier in this chapter, where the input states are averaged to determine the output state. For example, a $_5{}^3$ would result in a structure of $_5{}^3_4$, and a $_4{}^1$ would lead to either $_4{}^1_2$ or $_4{}^1_3$.

Another procedure would be to prohibit such transitions, and "back up" and arbitrarily select another alternative from the last transition that led to the "unusual transition." For example, if a

sequence results in $\begin{matrix} 3 & 3 & 2 \\ 3 & 4 & \end{matrix}$, the $\begin{matrix} & & 2 \\ 4 & & \end{matrix}$ is an unusual transition of type 2. The program would "back up" and replace the 4 by a 3 or a 2. While this procedure was not used in the simulations presented in this thesis, so that the problem of "unusual transitions" could be demonstrated, it should produce more realistic data fields then the "compromise" procedure, which leaves the "unusual transitions" in the simulated data fields.

An Additional Suggested Method - "Method 2"

It appears that most unusual transitions are generated because the state at the point immediately northeast of the data value to be simulated (when simulating by rows) is not considered. Since Pickard's and Galbraith and Walley's methods produced similar results, it appears that the point northwest of the point to be simulated provides mostly redundant information since its state value was already used in the simulation of the previous point. Therefore, a slight variation of Pickard's method is suggested, where state L is to be simulated based on the values of (I,J,K) in a unit diamond of the form $\begin{matrix} & J & \\ I & & K \\ & L & \end{matrix}$. On the right end of each row, the value of K is not available, so Galbraith and Walley's method would be used to simulate the last point in each row. This method will be referred to as "Method 2."

Only a few changes in the PICKARD computer program (appendix C) were needed to develop transition matrices for Method 2. In line 13, the DO statement goes from 1 to 13, since only 13 transitions per row can be counted. In line 24, LI = LI + 2 because two transitions must be skipped at the end of each row. In line 18, the array locations of LJ and LK are changed to LJ = LI - 14 and LK = LI - 13.

The transition matrices are given in appendix E, and examples of simulations produced by this method are shown in figure 33. As in previous discussion, an unusual transition of type 3 is defined as a case where I and K in a unit diamond are unequal and nonconsecutive states. This method generated 15 unusual transitions of type 3 in six simulations, plus one transition of type 2. No unusual transitions of type 1 were generated, in contrast to all of the other simulation methods. Seven of the type 3 transitions matched sequences which occurred in the real data. Based on a subjective examination of the simulations, the data fields produced by Method 2 appear to be at least as realistic as those generated by either Pickard's or Galbraith and Walley's methods.

General Guidelines for Users

This chapter has described several examples of applications of two-dimensional simulation methods. As a summary of this chapter, the basic procedures used will be reviewed and the steps involved will be described, in general. Because of the differences in data and requirements from one project to another, this section will not be a detailed user's guide but will simply describe some of the factors to be considered.

It will be assumed that the data needed has been selected and is available. The data should be reduced to a small number of states that are relevant to the simulation. Also, it will be assumed that one of the two-dimensional simulation methods (such as "Method 2") has been chosen as the desired procedure to use for the simulation.

After the data is collected, it is best to look at the characteristics of the data (1) by plotting some or all of the data fields, (2) by examining statistical characteristics at each grid point, such as

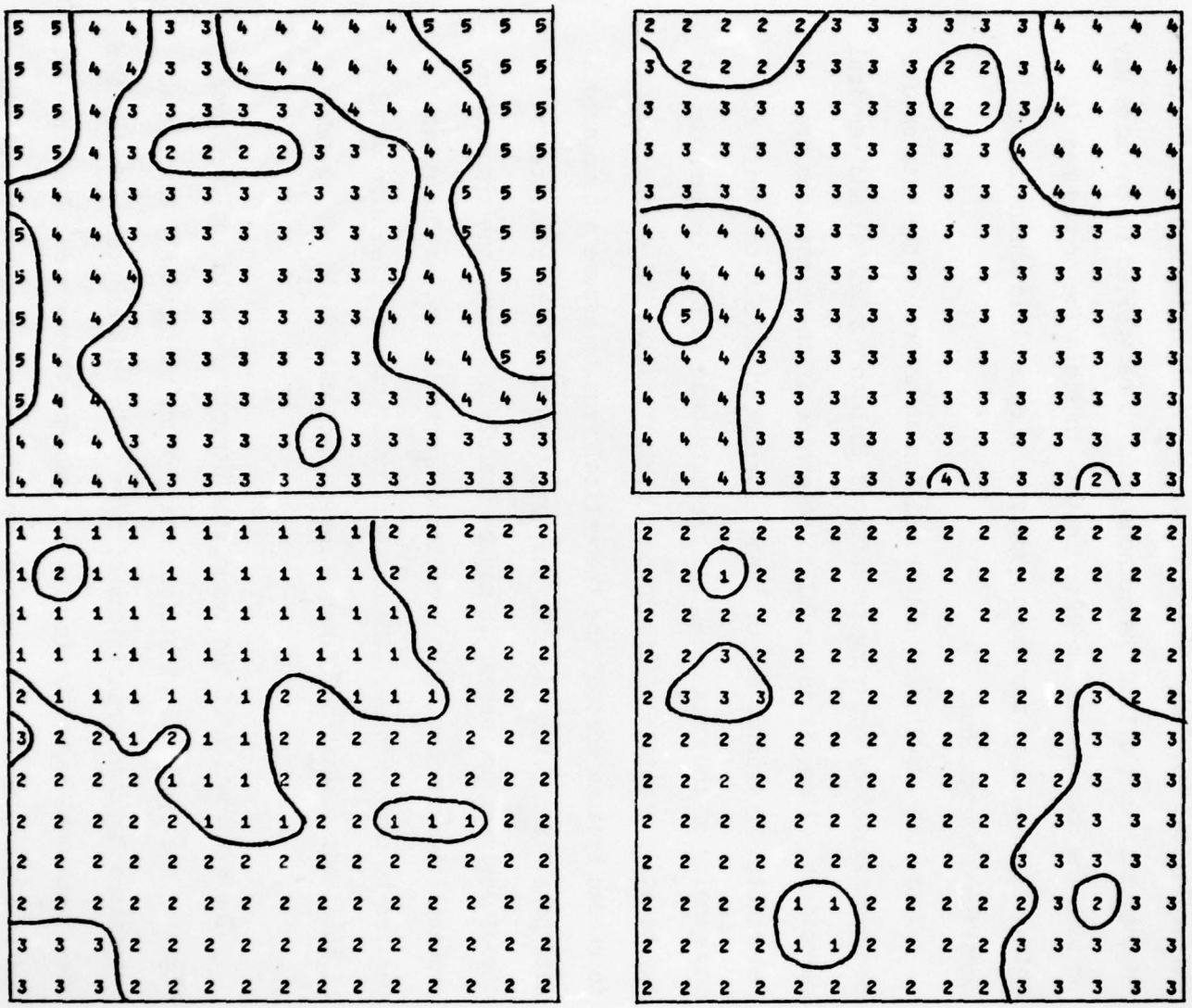

Figure 33 - Typical Data Bases Generated by "Method 2"

mean, standard deviation, skewness, and extremes, (3) by examining the correlation structure of the data, and (4) by subjectively applying meteorological knowledge to see if there are significant discontinuities in the data. Not all of the methods need to be applied in the same way or with the same detail as shown in this thesis. The procedures in this section will be assumed to apply to a data field which can be considered homogeneous.

Next, the one-dimensional transition structure of the data should be investigated by counting transitions in both horizontal and vertical directions. Also, it is useful to determine if these matrices are reversible. Comparing all of these matrices will help to verify the homogeneity of the data base. If the data base is not homogeneous, it may be still possible to simulate it realistically, but extra caution is needed because most of the theoretical results assume a homogeneous data base. Since two edges of the data base are to be simulated by one-dimensional processes, it may be useful to express the transition probabilities as cumulative transition probabilities, as described earlier in this chapter. The unconditional probabilities also should be obtained, so that the first point of each data field may be generated.

The examination of the transition structure should include investigating the types of transitions that occur, particularly those that are called "unusual transitions of type 1" in this chapter. While, the one-dimensional Markov chain will not generate these more often than in the real data, the two-dimensional procedures may generate these transitions.

Assuming that the two-dimensional simulation method has been

chosen, the next step is to develop the transition probability matrix (and usually the cumulative transition probability matrix) for the two-dimensional process by counting transitions in the original data. If there are no occurrences of a particular sequence of states, all of the transition probabilities should be set to zero (or a suitable flag in the program should be set) so that the program will not attempt to divide by zero to get transition probabilities.

Reviewing this transition probability matrix would show if there are many "unusual transitions of type 2" (or possibly type 3 also, if "Method 2" is used to develop the matrix). The method to deal with "unusual transitions" should be decided on at this time. The occurrence of no transition probabilities greater than zero for a particular transition could be used as a flag to detect the generation of an "unusual transition" that did not occur in the original data, and possibly the program could have other flags to detect other "unusual transitions." While the "compromise method" of dealing with these transitions was used here for illustrative purposes, prohibiting the "unusual transitions" by having the program "back up" when one occurs may produce more realistic data fields.

The last step is to write a computer program to generate data fields, if desired. Generally, the simulation procedure will (1) select the state at the starting point, usually the upper left corner, (2) generate two boundaries, usually the left and top, by one-dimensional processes, and (3) generate the interior points of the grid by the chosen two-dimensional method, dealing with "unusual transitions" as set up in the program. This program could then be used to generate two-dimensional simulated data fields.

VIII. Summary, Conclusions, and Recommendations for Further Study

The basic objectives of this thesis were to summarize the use of Markov chains in one dimension, particularly as applied to weather data, and to extend the theory to two (or more) dimensions so that the advantages of Markov models could be used to produce realistic but computationally simple simulations of higher dimensional data fields.

A general discussion of the characteristics of real weather variables and observed weather data is first presented, to help familiarize users with those characteristics that cannot and those that can be realistically modeled by a Markov process. A brief summary of the theory of Markov and semi-Markov processes is given in the context of the characteristics of weather data. The results of a literature search of the topics of applications of Markov and semi-Markov processes, mainly to weather data, are reported.

To investigate the possibility of extending Markov concepts to higher dimensions, several topics become especially important when simulating homogeneous multi-dimensional data fields. The first topic is reversibility: Can the same transition probability matrix be used in all directions, assuming that the same probabilities exist in all directions in the real data? The second topic is invariance of generation: Will the probability distribution of states at one point, given the state at another point which has already been specified, be the same regardless of the sequence with which the points in between are simulated (such as by columns instead of by rows)? The third isse is correlation: How much of the correlation structure of the real data will be reproduced by the simulation method? The fourth topic discussed

concerns the consequences of inhomogeneity or directionality in the data.

A few papers about simulations of two-dimensional data bases have been published, although only one example (which was not based on a Markov process) has been found (Wellberry and Galbraith (1973)). The Markovian methods of Galbraith and Walley (1976) and Pickard (1977) were described. These methods are different from the typical vector representation of Markov processes. Another suggested procedure, called "Method 1," was described.

Because little work has been published in the area of applying the theory to actual examples, the proposed simulation methods were applied to a set of temperature data on a grid over the eastern and central United States. Galbraith and Walley's method as well as Pickard's method produced realistic data fields, with the exception that they generated certain combinations of points (such as states 4 and 2 in diagonally adjacent points) much more frequently than these combinations were observed in the real data. "Method 1" produced data fields that were not as realistic as those generated by the other methods. A variation on Pickard's method, called "Method 2," greatly reduced the frequency of these "unusual transitions," without increasing the complexity of the model.

Three conclusions can be drawn from this exercise: (1) Pickard's method, Galbraith and Walley's method, and "Method 2" produce realistic data fields, although certain combinations of data values may be produced more frequently in the simulations than in the actual data. (2) Any method which does not use all of the closest data points which have already been simulated as input states will probably generate

more of these "unusual transitions" than a method which does consider all of the adjacent points. (3) Methods of the complexity used here as examples are quite feasible for economical computer simulation. However, the complexity increases rapidly either as the number of input points increases or the number of different states increases. In such cases, regression methods (possibly using dummy variables and some of the interaction terms) may be more appropriate than Markov methods.

Some areas for further investigation may be suggested from this thesis:

(1) In a theoretical area, it would be desirable to prove or disprove the homogeneity of the simulation schemes (other than Pickard's) discussed here, when a homogeneous data base is used to develop the transition probabilities.

(2) It would be interesting to investigate a data base for which the "unusual transitions" as discussed here are not infrequent. In such a case, would the various simulation methods continue to generate these "unusual transitions" more frequently than in the actual data, and would the simulations be realistic? A possible technique to evaluate a proposed simulation method would be to generate a large number of cases, develop transition matrices from the simulated data, and compare the matrices with those obtained from the real data.

(3) When a data field is not homogeneous, it is divided into areas that are considered homogeneous and separate transition matrices are used in each area. It would be useful to investigate if this causes any theoretical problems such as "boundary effects."

(4) The application of these methods to a data field that is fundamentally inhomogeneous (such as the probability of icing,

considering a data field with a considerable north-south extent, or the probability of snow, or almost any data field in a mountainous area) could be investigated to see if reasonable results can be produced.

(5) Statistical testing procedures suitable for testing the goodness of fit of a two-dimensional simulation procedure should be developed. While these may be similar to the statistical tests used for one-dimensional goodness of fit, other factors may need to be considered, such as the impact of "unusual transitions" that can be generated even if they do not occur in the original data.

Bibliography

Akaike, H. "A New Look at the Statistical Model Identification." IEEE Transactions on Automatic Control, AC-19 (6): 716-723 (December 1974).

Anderson, T.W. and Leo A. Goodman. "Statistical Inference About Markov Chains." Annals of Mathematical Statistics, 28 (1): 89-110 (March 1957).

Bartlett, M.S. "Inference and Stochastic Processes." Journal of the Royal Statistical Society A, 130 (3): 457-474 (1967).

-----. "The Frequency Goodness of Fit Test for Probability Chains." Proceedings of the Cambridge Philosophical Society, 47 (1): 86-95 (January 1951).

Besag, Julian. "Spatial Interaction and the Statistical Analysis of Spatial Pattern." Journal of the Royal Statistical Society B, 36 (2): 192-236 (1974).

Brooks, C.E.P. and N. Carruthers. Handbook of Statistical Methods in Meteorology. London: Her Majesty's Stationery Office, 1953 (413 pp).

Caskey, J.E., Jr. "A Markov Chain Model for the Probability of Precipitation Occurrence in Intervals of Various Lengths." Monthly Weather Review, 91 (6): 298-301 (June 1963).

Chin, Edwin H. "Modeling Daily Precipitation Occurrences with a Markov Chain." Water Resources Research, 13 (6): 949-956 (December 1977).

Cooke, D.S. "The Duration of Wet and Dry Spells at Moncton, New Brunswick." Quarterly Journal of the Royal Meteorological Society, 79 (342): 536-538 (October 1953).

Denny, J.L., C.C. Kisiel, and S.J. Yakowitz, "Procedures for Determining the Order of Dependence in Streamflow Records." Water Resources Research, 10 (5): 947-954 (October 1974).

Elderton, W. Palin. "Tables for Testing the Goodness of Fit of Theory to Observation." Biometrika, 1 (2): 155-163 (January 1902).

Feller, William. An Introduction to Probability Theory and Its Applications, Volume 1 (Third Edition). New York: John Wiley and Sons, 1968.

Feyerherm, A.M. and L.D. Bark. "Goodness of Fit of a Markov Chain Model for Sequences of Wet and Dry Days." Journal of Applied Meteorology, 6 (5): 770-773 (October 1967).

-----. "Statistical Methods for Persistent Precipitation Patterns." Journal of Applied Meteorology, 4 (3): 320-328 (June 1965).

Gabriel, K.R. and J. Neumann. " Markov Chain Model for Daily Rainfall Occurrences at Tel Aviv." Quarterly Journal of the Royal Meteorological Society, 88 (375): 90-95 (January 1962).

Galbraith, R.F. and D. Walley, "On a Two-Dimensional Binary Process." Journal of Applied Probability, 13 (3): 548-557 (September 1976).

Gates, P. and H. Tong. "On Markov Chain Modeling to Some Weather Data." Journal of Applied Meteorology, 15 (11): 1145-51 (November 1976).

Green, J.R. "A Generalized Probability Model for Sequences of Wet and Dry Days." Monthly Weather Review, 98 (3): 238-241 (March 1970).

-----. "A Model for Rainfall Occurrence." Journal of the Royal Statistical Society B, 26 (2): 345-353 (1964).

-----. "Two Probability Models for Sequences of Wet or Dry Days." Monthly Weather Review, 93 (3): 155-156 (March 1965).

Haan, C.T., D.M. Allen, and J.O. Street. "A Markov Chain Model of Daily Rainfall." Water Resources Research, 12 (3): 443-449 (June 1976).

Jorgensen, Donald L. "Persistency of Rain and No-Rain Periods During the Winter at San Francisco." Monthly Weather Review, 77 (9): 303-307 (November 1949).

Katz, R.W. "Estimating the Order of a Markov Chain: Another Look at the Tel Aviv Rainfall Data." Preprints for the Sixth Conference on Probability and Statistics in the Atmospheric Sciences of the American Meteorological Society. Abstracted in Bulletin of the American Meteorological Society, 60 (5): 590 (May 1979).

Lameiro, Gerard F. and Maurice C. Bryson. "The Effect of Markovian and Discrete-Climate Assumptions on the Performance Analysis of Solar Heating Systems." Journal of Applied Meteorology, 17 (3): 386-389 (March 1978).

Lee, Stephen Mead. An Approach to Modeling Weather Data by Means of Markov and Semi-Markov Chains. PhD Thesis. Arizona State University, 1973 (244 pp) (Dissertation Abstracts order number 73-20436).

Lloyd, E.H. "What Is, and What Is Not, a Markov Chain?" Journal of Hydrology, 22 (6): 1-28 (June 1974).

Longley, R.W. "The Length of Dry and Wet Periods." Quarterly Journal of the Royal Meteorological Society, 79 (342): 520-527 (October 1953).

Lowry, William P. and Donald Guthrie. "Markov Chains of Order Greater Than One." Monthly Weather Review, 96 (11): 798-801 (November 1968).

Matalas, N.C. "Mathematical Assessment of Synthetic Hydrology." Water Resources Research, 3 (4): 937-945 (1967).

Medhi, J. "A Markov Chain Model for the Occurrences of Dry and Wet Days." Indian Journal of Meteorology, Hydrology, and Geophysics, 27 (4): 431-435 (October 1976).

Mendenhall, William and Richard L. Scheaffer, Mathematical Statistics with Applications. North Scituate, Mass.: Duxbury Press, 1973 (649 pp).

Merrill, Kenneth O. A Semi-Markov Weather Model. MS Thesis. Wright-Patterson AFB, Ohio: Air Force Institute of Technology, March 1974 (126 pp) (AFIT/GSA/SM/74-8, AD-778 873).

Miller, Marvin E. and C.R. Weaver, Monthly and Annual Precipitation Probabilities for Selected Locations in Ohio. Wooster, Ohio: Ohio Agricultural Research and Development Center Research Bulletin 1017, March 1969 (44 pp).

Miller, Robert G., editor. Selected Topics in Statistical Meteorology. USAF, Air Weather Service Technical Report AWS TR 77-273, July 1977 (164 pp).

Newnham, E.V. "The Persistence of Wet and Dry Weather." Quarterly Journal of the Royal Meteorological Society, 42 (179): 153-162 (July 1916).

Nunn, Walter R. and Anthony M. Desiderio. Semi-Markov Processes: An Introduction. Arlington, Virginia: Center for Naval Analyses, July 1977 (30 pp) (CRC-335, AD-A 046 158).

Patankar, V.N. "The Goodness of Fit of Frequency Distributions Obtained from Stochastic Processes." Biometrika, 41 (3 and 4): 450-462 (December 1954).

Pickard, D.K. "A Curious Binary Lattice Process." Journal of Applied Probability, 14 (4): 717-731 (December 1977).

Sakamoto, Clarence M. Markov Chain Models for Probability of Hot and Cool Day Sequences and Hot Spells in Nevada, Environmental Science Services Administration report ESSA TR EDS-9, March 1970 (26 pp) (PB 193 221).

Schwarz, Gideon. "Estimating the Dimension of a Model," Annals of Mathematical Statistics, 6 (2): 461-464 (March 1978).

Srikanthan, R. and T.A. McMahon. "Review of Lag-One Markov Models for Generation of Annual Flows." Journal of Hydrology, 37 (4): 1-12 (April 1978).

Sundararaj, N. and S. Ramachandra. "Markov-Dependent Geometric Models for Weather Spells and Weather Cycles: A Study." Indian Journal of Meteorology, Hydrology, and Geophysics, 26 (2): 221-226 (April 1975).

Tong, H. "Determinations of the Order of a Markov Chain by Akaike's Information Criterion." Journal of Applied Probability, 12 (3): 488-497 (September 1975).

United States, Department of Agriculture (Economics, Statistics, and Cooperative Service) and Department of Commerce (Environmental Data and Information Service, NOAA), Weekly Weather and Crop Bulletin, various weekly issues from 59 (49), 4 December 1972, to 66 (11), 13 March 1979.

United States, Department of Commerce, Environmental Science Services Administration, Environmental Data Service, Climatic Atlas of the United States, June 1968 (80 pp, 271 maps, 15 tables).

United States, Department of Commerce, National Oceanic and Atmospheric Administration, Environmental Data and Information Service, National Climatic Center. Daily Series, Synoptic Weather Maps, (Part I: Northern Hemisphere Sea Level and 500 Millibar Charts), 1940 to present (Issued by organizations with differing names during earlier portions of this period).

-----. Local Climatological Data for Columbus, Ohio, monthly issues from 1957 to 1978.

-----. Local Climatological Data for Dayton, Ohio, various monthly and annual issues from 1957 to 1978.

United States, Department of Commerce, Weather Bureau, Decennial Census of U.S. Climate, Climatic Summary of the United States: Supplement for 1951-1960 - Ohio, Climatography of the U.S. Number 86-29, 1964 (77 pp).

-----. Historical Weather Maps, 1899-1939.

Weiss, L.L. "Sequences of Wet or Dry Days Described by a Markov Chain Probability Model." Monthly Weather Review, 92 (4): 169-176 (April 1964).

Welberry, T.R. and R. Galbraith, "A Two-Dimensional Model of Crystal Growth Disorder." Journal of Applied Crystallography, 6 (1): 87-96 (1973).

Whittle, P. "On Stationary Processes in the Plane." Biometrika, 41 (3 and 4): 434-449 (December 1954).

Williams, C.B. "Sequences of Wet and Dry Days Considered in Relation to the Logarithmic Series." Quarterly Journal of the Royal Meteorological Society, 78 (335): 91-96 (January 1952).

Wiser, E.H. "Modified Markov Probability Models of Sequences of Precipitation Events." Monthly Weather Review, 93 (8): 511-516 (August 1965).

Yost, F.G. and E.A. Aronson, "A Markov Chain Model for Cloud Cover Sequences in Albuquerque." Albuquerque, New Mexico: Air Force Sandia Laboratories, October 1977 (11 pp) (SAND 77-1430).

APPENDIX A

First Order Markov Three-State and Five-State Transition Matrices

FIRST ORDER, THREE STATES, HORIZONTAL TRANSITIONS

NUMBER OF TRANSITIONS

I	J = 1	J = 2	J = 3	ROW SUM
1	3749	373	0	4122
2	386	8562	399	9347
3	0	409	3762	4171

TOTAL NUMBER OF TRANSITIONS 17640

TRANSITION PROBABILITY MATRIX

I	J = 1	J = 2	J = 3
1	.9095	.0905	.0
2	.0413	.9160	.0427
3	.0	.0981	.9019

UNCONDITIONAL PROBABILITIES CALCULATED FROM TPM

J = 1	J = 2	J = 3
.2412	.5286	.2301

ACTUAL PROBABILITIES

.2337	.5299	.2365

THEORETICAL PROBABILITIES

.2266	.5467	.2266

FIRST ORDER, THREE STATES, VERTICAL TRANSITIONS

NUMBER OF TRANSITIONS

I	J = 1	J = 2	J = 3	ROW SUM
1	3681	385	0	4066
2	366	8383	378	9127
3	0	439	3693	4132

TOTAL NUMBER OF TRANSITIONS 17325

TRANSITION PROBABILITY MATRIX

I	J = 1	J = 2	J = 3
1	.9053	.0947	.0
2	.0401	.9185	.0414
3	.0	.1062	.8938

UNCONDITIONAL PROBABILITIES CALCULATED FROM TPM

J = 1	J = 2	J = 3
.2336	.5515	.2149

ACTUAL PROBABILITIES

.2347	.5268	.2385

THEORETICAL PROBABILITIES

.2266	.5467	.2266

FIRST ORDER, FIVE STATES, HORIZONTAL TRANSITIONS

NUMBER OF TRANSITIONS

I	J=1	J=2	J=3	J=4	J=5	ROW SUM
1	1035	224	0	0	0	1259
2	209	3525	424	0	0	4158
3	0	449	5722	446	0	6617
4	0	0	474	3799	191	4464
5	0	0	0	165	977	1142

TOTAL NUMBER OF TRANSITIONS 17640

TRANSITION PROBABILITY MATRIX

I	J=1	J=2	J=3	J=4	J=5
1	.8221	.1779	.0	.0	.0
2	.0503	.8478	.1020	.0	.0
3	.0	.0679	.8647	.0674	.0
4	.0	.0	.1062	.8510	.0428
5	.0	.0	.0	.1445	.8555

UNCONDITIONAL PROBABILITIES CALCULATED FROM TPM

J=1	J=2	J=3	J=4	J=5
.0702	.2486	.3737	.2372	.0702

ACTUAL PROBABILITIES

.0714	.2357	.3751	.2531	.0647

THEORETICAL PROBABILITIES

.0668	.2417	.3829	.2417	.0668

FIRST ORDER, FIVE STATES, VERTICAL TRANSITIONS

NUMBER OF TRANSITIONS

I	J=1	J=2	J=3	J=4	J=5	ROW SUM
1	990	240	0	0	0	1230
2	235	3404	431	0	0	4070
3	0	431	5604	442	2	6479
4	0	0	488	3782	172	4442
5	0	0	0	172	932	1104

TOTAL NUMBER OF TRANSITIONS 17325

TRANSITION PROBABILITY MATRIX

I	J=1	J=2	J=3	J=4	J=5
1	.8049	.1951	.0	.0	.0
2	.0577	.8364	.1059	.0	.0
3	.0	.0665	.8649	.0682	.0003
4	.0	.0	.1099	.8514	.0387
5	.0	.0	.0	.1558	.8442

UNCONDITIONAL PROBABILITIES CALCULATED FROM TPM

J=1	J=2	J=3	J=4	J=5
.0716	.2421	.3854	.2404	.0605

ACTUAL PROBABILITIES

.0710	.2349	.3740	.2564	.0637

THEORETICAL PROBABILITIES

.0668	.2417	.3829	.2417	.0668

REVERSE TPM

I	J=1	J=2	J=3	J=4	J=5
1	.8049	.1951	.0	.0	.0
2	.0577	.8364	.1059	.0	.0
3	.0	.0665	.8649	.0685	.0
4	.0	.0	.1094	.8514	.0392
5	.0	.0	.0020	.1538	.8442

REVERSIBLE TPM

I	J=1	J=2	J=3	J=4	J=5
1	.8049	.1951	.0	.0	.0
2	.0577	.8364	.1059	.0	.0
3	.0	.0665	.8649	.0684	.0002
4	.0	.0	.1096	.8514	.0390
5	.0	.0	.0010	.1548	.8442

APPENDIX B

Galbraith and Walley

Three-State and Five-State

Transition Matrices

GALBRAITH AND WALLEY
THREE STATES

NUMBER OF TRANSITIONS

I	J	K = 1	K = 2	K = 3	ROW SUM
1	1	3241	118	0	3359
1	2	196	221	0	417
1	3	0	0	0	0
2	1	208	237	0	445
2	2	145	7369	146	7660
2	3	0	259	227	486
3	1	0	0	0	0
3	2	0	241	197	438
3	3	0	138	3227	3365

TOTAL NUMBER OF TRANSITIONS 16170

TRANSITION PROBABILITY MATRIX

I	J	K = 1	K = 2	K = 3
1	1	.9649	.0351	.0
1	2	.4700	.5300	.0
1	3	.0	.0	.0
2	1	.4674	.5326	.0
2	2	.0189	.9620	.0191
2	3	.0	.5329	.4671
3	1	.0	.0	.0
3	2	.0	.5502	.4498
3	3	.0	.0410	.9590

GALBRAITH AND WALLEY
FIVE STATES

NUMBER OF TRANSITIONS

I	J	K = 1	K = 2	K = 3	K = 4	K = 5	ROW SUM
1	1	822	67	0	0	0	889
1	2	128	141	0	0	0	269
2	1	102	161	0	0	0	263
2	2	91	2811	143	0	0	3045
2	3	0	240	242	0	0	482
3	2	0	250	253	0	0	503
3	3	0	159	4713	163	0	5035
3	4	0	0	308	246	0	554
4	2	0	0	1	0	0	1
4	3	0	0	291	245	1	537
4	4	0	0	146	3128	78	3352
4	5	0	0	0	112	94	206
5	3	0	0	0	0	1	1
5	4	0	0	0	97	86	183
5	5	0	0	0	53	797	850

TRANSITION PROBABILITY MATRIX

I	J	K = 1	K = 2	K = 3	K = 4	K = 5
1	1	.9246	.0754	.0	.0	.0
1	2	.4758	.5242	.0	.0	.0
2	1	.3878	.6122	.0	.0	.0
2	2	.0299	.9232	.0470	.0	.0
2	3	.0	.4979	.5021	.0	.0
3	2	.0	.4970	.5030	.0	.0
3	3	.0	.0316	.9360	.0324	.0
3	4	.0	.0	.5560	.4440	.0
4	2	.0	.0	1.0	.0	.0
4	3	.0	.0	.5419	.4562	.0019
4	4	.0	.0	.0436	.9332	.0233
4	5	.0	.0	.0	.5437	.4563
5	3	.0	.0	.0	.0	1.0
5	4	.0	.0	.0	.5301	.4699
5	5	.0	.0	.0	.0624	.9376

APPENDIX C

Pickard Three-State Transition Matrices and Computer Program and Transition Matrices for Five States

PICKARD
THREE STATES

NUMBER OF TRANSITIONS

I J K	L = 1	L = 2	L = 3	ROW SUM
1 1 1	3135	111	0	3246
1 1 2	83	118	0	201
1 2 1	106	7	0	113
1 2 2	113	103	0	216
2 1 1	101	111	0	212
2 1 2	10	128	0	138
2 2 1	107	126	0	233
2 2 2	135	7093	138	7366
2 2 3	0	109	131	240
2 3 2	0	148	8	156
2 3 3	0	150	96	246
3 2 2	0	109	99	208
3 2 3	0	3	127	130
3 3 2	0	132	98	230
3 3 3	0	135	3100	3235

TOTAL NUMBER OF TRANSITIONS 16170

TRANSITION PROBABILITY MATRIX

I J K	L = 1	L = 2	L = 3
1 1 1	.9658	.0342	.0
1 1 2	.4129	.5871	.0
1 2 1	.9381	.0619	.0
1 2 2	.5231	.4769	.0
2 1 1	.4764	.5236	.0
2 1 2	.0725	.9275	.0
2 2 1	.4592	.5408	.0
2 2 2	.0183	.9629	.0187
2 2 3	.0	.4542	.5458
2 3 2	.0	.9487	.0513
2 3 3	.0	.6098	.3902
3 2 2	.0	.5240	.4760
3 2 3	.0	.0231	.9769
3 3 2	.0	.5739	.4261
3 3 3	.0	.0417	.9583

```
      PROGRAM PICKARD (INPUT,OUTPUT,STATES,TAPE5=STATES)
      DIMENSION ITPM(25,5,5),TPM(25,5,5),CTPM(25,5,4),ISUM(25,5),
     +INPUT(200)
C  SET VALUES TO ZERO
      DO 12 IJ = 1, 25
      DO 12 K = 1, 5
      DO 11 L = 1, 5
      ITPM(IJ,K,L) = 0
      CTPM(IJ,K,L) = 0.
11    TPM(IJ,K,L) = 0.
12    ISUM(IJ,K) = 0
      ITOTAL = 0
C COUNT OCCURRENCES OF TRANSITIONS
      DO 52 II = 1, 105
      READ (5,1) (INPUT(I),I = 1, 180)
      LI = 15
      DO 49 I = 2, 12
      DO 48 J = 2, 15
      LI=LI+1  $  LJ=LI-15  $  LK=LI-14  $  LL=LI+1
      NIJ = 5*(INPUT(LI) - 1) + INPUT(LJ)
      NK = INPUT(LK)  $  NL = INPUT(LL)
      ITPM(NIJ,NK,NL) = ITPM(NIJ,NK,NL) + 1
      ISUM(NIJ,NK) = ISUM(NIJ,NK) + 1
48    ITOTAL = ITOTAL + 1
49    LI = LI + 1
52    CONTINUE
C  CALCULATE TRANSITION PROBABILITIES
      DO 55 IJ = 1, 25
      DO 55 K = 1, 5
      IF (ISUM(IJ,K) .EQ. 0) GO TO 55
      DO 54 L = 1, 5
      TPM(IJ,K,L) = FLOAT(ITPM(IJ,K,L))/FLOAT(ISUM(IJ,K))
      CTPM(IJ,K,1) = TPM(IJ,K,1)
      IF (L.EQ.1) GO TO 54
      IF (L.EQ.5) GO TO 54
      CTPM(IJ,K,L) = CTPM(IJ,K,L-1) + TPM(IJ,K,L)
54    CONTINUE
55    CONTINUE
      ..
```

```
C  PRINT STATEMENTS
      PRINT 58
      DO 64 IJ = 1, 25
      I = (IJ + 4)/5
      J = IJ - 5*I + 5
      DO 64 K = 1, 5
      IF ((IJ.EQ.11).AND.(K.EQ.1)) PRINT 58
      IF ((IJ.EQ.21).AND.(K.EQ.1)) PRINT 58
64    PRINT 70, I, J, K, (ITPM(IJ,K,L),L=1,5), ISUM(IJ,K)
      PRINT*,'  '
      PRINT*,'  TOTAL NUMBER OF TRANSITIONS IS  ',ITOTAL
      PRINT 71
      DO 72 IJ = 1, 25
      I = (IJ + 4)/5
      J = IJ - 5*I + 5
      DO 72 K = 1, 5
      IF ((IJ.EQ.11).AND.(K.EQ.1)) PRINT 71
      IF ((IJ.EQ.21).AND.(K.EQ.1)) PRINT 71
72    PRINT 73, I, J, K, (TPM(IJ,K,L), L = 1, 5)
      PRINT 80
      DO 82 IJ = 1, 25
      I =(IJ + 4)/5
      J = IJ - 5*I + 5
      DO 82 K = 1, 5
      IF ((IJ.EQ.11) .AND. (K.EQ.1)) PRINT 80
      IF ((IJ.EQ.21) .AND. (K.EQ.1)) PRINT 80
      ROWEND = 0.
      IF (CTPM(IJ,K,4).NE.0) ROWEND = 1.
      IF (TPM(IJ,K,5).NE.0) ROWEND = 1.
82    PRINT 73, I, J, K, (CTPM(IJ,K,L),L=1,4), ROWEND
1     FORMAT (60I1)
58    FORMAT (1H1,11X,'NUMBER OF TRANSITIONS',//,
     +'   I  J  K   L=1  L=2  L=3  L=4  L=5  ROW SUM',//)
70    FORMAT (1X,3I3,I6,4I5,I7)
71    FORMAT (1H1,14X,'TRANSITION PROBABILITY MATRIX',//,
     +'   I  J  K   L = 1  L = 2  L = 3  L = 4  L = 5',//)
73    FORMAT (1X,3I3,2X,5(1X,F6.4))
80    FORMAT (1H1,3X,'CUMULATIVE TRANSITION PROBABILITY MATRIX',/,
     +'   I  J  K   L = 1  L = 2  L = 3  L = 4  L = 5',//)
      STOP  $  END
..
```

NUMBER OF TRANSITIONS

I	J	K	L=1	L=2	L=3	L=4	L=5	ROW SUM
1	1	1	759	62	0	0	0	821
1	1	2	51	68	0	0	0	119
1	1	3	0	0	0	0	0	0
1	1	4	0	0	0	0	0	0
1	1	5	0	0	0	0	0	0
1	2	1	63	5	0	0	0	68
1	2	2	77	73	0	0	0	150
1	2	3	0	0	0	0	0	0
1	2	4	0	0	0	0	0	0
1	2	5	0	0	0	0	0	0
1	3	1	0	0	0	0	0	0
1	3	2	0	0	0	0	0	0
1	3	3	0	0	0	0	0	0
1	3	4	0	0	0	0	0	0
1	3	5	0	0	0	0	0	0
1	4	1	0	0	0	0	0	0
1	4	2	0	0	0	0	0	0
1	4	3	0	0	0	0	0	0
1	4	4	0	0	0	0	0	0
1	4	5	0	0	0	0	0	0
1	5	1	0	0	0	0	0	0
1	5	2	0	0	0	0	0	0
1	5	3	0	0	0	0	0	0
1	5	4	0	0	0	0	0	0
1	5	5	0	0	0	0	0	0
2	1	1	49	83	0	0	0	132
2	1	2	6	85	1	0	0	92
2	1	3	0	0	0	0	0	0
2	1	4	0	0	0	0	0	0
2	1	5	0	0	0	0	0	0
2	2	1	53	78	0	0	0	131
2	2	2	84	2597	134	0	0	2815
2	2	3	0	94	122	0	0	216
2	2	4	0	0	0	0	0	0
2	2	5	0	0	0	0	0	0
2	3	1	0	0	0	0	0	0
2	3	2	1	129	8	0	0	136
2	3	3	0	146	120	0	0	266
2	3	4	0	0	0	0	0	0
2	3	5	0	0	0	0	0	0
2	4	1	0	0	0	0	0	0
2	4	2	0	0	0	0	0	0
2	4	3	0	0	0	0	0	0
2	4	4	0	0	0	0	0	0
2	4	5	0	0	0	0	0	0
2	5	1	0	0	0	0	0	0
2	5	2	0	0	0	0	0	0
2	5	3	0	0	0	0	0	0
2	5	4	0	0	0	0	0	0
2	5	5	0	0	0	0	0	0

NUMBER OF TRANSITIONS

I	J	K	L=1	L=2	L=3	L=4	L=5	ROW SUM
3	1	1	0	0	0	0	0	0
3	1	2	0	0	0	0	0	0
3	1	3	0	0	0	0	0	0
3	1	4	0	0	0	0	0	0
3	1	5	0	0	0	0	0	0
3	2	1	0	0	0	0	0	0
3	2	2	0	118	117	0	0	235
3	2	3	0	4	161	0	0	165
3	2	4	0	0	0	0	0	0
3	2	5	0	0	0	0	0	0
3	3	1	0	0	0	0	0	0
3	3	2	0	132	136	0	0	268
3	3	3	0	155	4407	146	0	4708
3	3	4	0	0	138	130	0	268
3	3	5	0	0	0	0	0	0
3	4	1	0	0	0	0	0	0
3	4	2	0	0	0	0	0	0
3	4	3	0	0	145	17	0	162
3	4	4	0	0	170	116	0	286
3	4	5	0	0	0	0	0	0
3	5	1	0	0	0	0	0	0
3	5	2	0	0	0	0	0	0
3	5	3	0	0	0	0	0	0
3	5	4	0	0	0	0	0	0
3	5	5	0	0	0	0	0	0
4	1	1	0	0	0	0	0	0
4	1	2	0	0	0	0	0	0
4	1	3	0	0	0	0	0	0
4	1	4	0	0	0	0	0	0
4	1	5	0	0	0	0	0	0
4	2	1	0	0	0	0	0	0
4	2	2	0	0	0	0	0	0
4	2	3	0	0	0	0	0	0
4	2	4	0	0	0	0	0	0
4	2	5	0	0	0	0	0	0
4	3	1	0	0	0	0	0	0
4	3	2	0	0	1	0	0	1
4	3	3	0	0	148	120	0	268
4	3	4	0	0	8	127	1	136
4	3	5	0	0	0	0	0	0
4	4	1	0	0	0	0	0	0
4	4	2	0	0	0	0	0	0
4	4	3	0	0	143	125	1	269
4	4	4	0	0	138	2941	73	3152
4	4	5	0	0	0	51	59	110
4	5	1	0	0	0	0	0	0
4	5	2	0	0	0	0	0	0
4	5	3	0	0	0	0	0	0
4	5	4	0	0	0	60	4	64
4	5	5	0	0	0	61	35	95

NUMBER OF TRANSITIONS

I	J	K	L=1	L=2	L=3	L=4	L=5	ROW SUM
5	1	1	0	0	0	0	0	0
5	1	2	0	0	0	0	0	0
5	1	3	0	0	0	0	0	0
5	1	4	0	0	0	0	0	0
5	1	5	0	0	0	0	0	0
5	2	1	0	0	0	0	0	0
5	2	2	0	0	0	0	0	0
5	2	3	0	0	0	0	0	0
5	2	4	0	0	0	0	0	0
5	2	5	0	0	0	0	0	0
5	3	1	0	0	0	0	0	0
5	3	2	0	0	0	0	0	0
5	3	3	0	0	0	0	0	0
5	3	4	0	0	0	0	2	2
5	3	5	0	0	0	0	0	0
5	4	1	0	0	0	0	0	0
5	4	2	0	0	0	0	0	0
5	4	3	0	0	0	0	1	1
5	4	4	0	0	0	43	54	97
5	4	5	0	0	0	3	58	61
5	5	1	0	0	0	0	0	0
5	5	2	0	0	0	0	0	0
5	5	3	0	0	0	0	0	0
5	5	4	0	0	0	54	30	84
5	5	5	0	0	0	50	739	789

TOTAL NUMBER OF TRANSITIONS IS 16170

TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
1	1	1	.9245	.0755	0.0000	0.0000	0.0000
1	1	2	.4286	.5714	0.0000	0.0000	0.0000
1	1	3	1.0000	0.0000	0.0000	0.0000	0.0000
1	1	4	1.0000	0.0000	0.0000	0.0000	0.0000
1	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	1	.9265	.0735	0.0000	0.0000	0.0000
1	2	2	.5133	.4867	0.0000	0.0000	0.0000
1	2	3	1.0000	0.0000	0.0000	0.0000	0.0000
1	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	5	1.0000	0.0000	0.0000	0.0000	0.0000
1	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	5	1.0000	0.0000	0.0000	0.0000	0.0000
1	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	4	1.0000	0.0000	0.0000	0.0000	0.0000
1	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	2	1.0000	0.0000	0.0000	0.0000	0.0000
1	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	1	.3712	.6288	0.0000	0.0000	0.0000
2	1	2	.0652	.9239	.0109	0.0000	0.0000
2	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	2	1	.4046	.5954	0.0000	0.0000	0.0000
2	2	2	.0298	.9226	.0476	0.0000	0.0000
2	2	3	0.0000	.4352	.5648	0.0000	0.0000
2	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	2	.0072	.9348	.0580	0.0000	0.0000
2	3	3	0.0000	.5489	.4511	0.0000	0.0000
2	3	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	4	1.0000	0.0000	0.0000	0.0000	0.0000
2	4	5	1.0000	0.0000	0.0000	0.0000	0.0000
2	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	3	1.0000	0.0000	0.0000	0.0000	0.0000
2	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	5	0.0000	0.0000	0.0000	0.0000	0.0000

TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
3	1	1	1.0000	0.0000	0.0000	0.0000	0.0000
3	1	2	1.0000	0.0000	0.0000	0.0000	0.0000
3	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	2	0.0000	.5021	.4979	0.0000	0.0000
3	2	3	0.0000	.0242	.9758	0.0000	0.0000
3	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	3	2	0.0000	.4925	.5075	0.0000	0.0000
3	3	3	0.0000	.0329	.9361	.0310	0.0000
3	3	4	0.0000	0.0000	.5149	.4851	0.0000
3	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	3	0.0000	0.0000	.8951	.1049	0.0000
3	4	4	0.0000	0.0000	.5944	.4056	0.0000
3	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	3	2	0.0000	0.0000	1.0000	0.0000	0.0000
4	3	3	0.0000	0.0000	.5522	.4478	0.0000
4	3	4	0.0000	0.0000	.0588	.9338	.0074
4	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	3	0.0000	0.0000	.5316	.4647	.0037
4	4	4	0.0000	0.0000	.0438	.9331	.0232
4	4	5	0.0000	0.0000	0.0000	.4636	.5364
4	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	4	0.0000	0.0000	0.0000	.9375	.0625
4	5	5	0.0000	0.0000	0.0000	.6354	.3646

TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
5	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	4	0.0000	0.0000	0.0000	0.0000	1.0000
5	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	3	0.0000	0.0000	0.0000	0.0000	1.0000
5	4	4	0.0000	0.0000	0.0000	.4433	.5567
5	4	5	0.0000	0.0000	0.0000	.0492	.9508
5	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	4	0.0000	0.0000	0.0000	.6429	.3571
5	5	5	0.0000	0.0000	0.0000	.0634	.9366

CUMULATIVE TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
1	1	1	.9245	1.0000	1.0000	1.0000	1.0000
1	1	2	.4286	1.0000	1.0000	1.0000	1.0000
1	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	1	.9265	1.0000	1.0000	1.0000	1.0000
1	2	2	.5133	1.0000	1.0000	1.0000	1.0000
1	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	2	0.0406	0.0000	0.0000	0.0000	0.0000
1	3	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	1	.3712	1.0000	1.0000	1.0000	1.0000
2	1	2	.0652	.9891	1.0000	1.0000	1.0000
2	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	2	1	.4046	1.0000	1.0000	1.0000	1.0000
2	2	2	.0298	.9524	1.0000	1.0000	1.0000
2	2	3	0.0000	.4352	1.0000	1.0000	1.0000
2	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	2	.0372	.9420	1.0000	1.0000	1.0000
2	3	3	0.0000	.5489	1.0000	1.0000	1.0000
2	3	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	5	0.0000	0.0000	0.0000	0.0000	0.0000

CUMULATIVE TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
3	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	2	0.0000	.5021	1.0000	1.0000	1.0000
3	2	3	0.0000	.0242	1.0000	1.0000	1.0000
3	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	3	2	0.0000	.4925	1.0000	1.0000	1.0000
3	3	3	0.0000	.0329	.9690	1.0000	1.0000
3	3	4	0.0000	0.0000	.5149	1.0000	1.0000
3	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	3	0.0000	0.0000	.8951	1.0000	1.0000
3	4	4	0.0000	0.0000	.5944	1.0000	1.0000
3	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	3	2	0.0000	0.0000	1.0000	1.0000	1.0000
4	3	3	0.0000	0.0000	.5922	1.0000	1.0000
4	3	4	0.0000	0.0000	.0588	.9926	1.0000
4	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	3	0.0000	0.0000	.5316	.9963	1.0000
4	4	4	0.0000	0.0000	.0436	.9768	1.0000
4	4	5	0.0000	0.0000	0.0000	.4635	1.0000
4	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	4	0.0000	0.0000	0.0000	.9375	1.0000
4	5	5	0.0000	0.0000	0.0000	.6354	1.0000

CUMULATIVE TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
5	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	4	0.0000	0.0000	0.0000	0.0000	1.0000
5	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	3	0.0000	0.0000	0.0000	0.0000	1.0000
5	4	4	0.0000	0.0000	0.0000	.4433	1.0000
5	4	5	0.0000	0.0000	0.0000	.0492	1.0000
5	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	4	0.0000	0.0000	0.0000	.6429	1.0000
5	5	5	0.0000	0.0000	0.0000	.0634	1.0000

APPENDIX D

"Method 1"

Five-State

Transition Matrices

"METHOD 1" TRANSITION MATRIX

NUMBER OF TRANSITIONS

JK = INPUT STATE, IL = OUTPUT STATE

JK	IL	NUMBER	IL	NUMBER	IL	NUMBER	IL	NUMBER	TOTAL
11	11	759	12	62	21	49	22	83	953
12	11	51	12	68	21	6	22	85	
	23	1							211
21	11	63	12	5	21	53	22	78	199
22	11	77	12	73	21	84	22	2597	
	23	134	32	118	33	117			3200
23	22	94	23	122	32	4	33	161	381
32	21	1	22	129	23	8	32	132	
	33	136	43	1					407
33	22	146	23	120	32	155	33	4407	
	34	146	43	148	44	120			5242
34	33	138	34	130	43	8	44	127	
	45	1	55	2					406
43	33	145	34	17	43	143	44	125	
	45	1	55	1					432
44	33	170	34	116	43	138	44	2941	
	45	73	54	43	55	54			3535
45	44	51	45	59	54	3	55	58	171
54	44	60	45	4	54	54	55	30	148
55	44	61	45	35	54	50	55	739	885

TOTAL NUMBER OF TRANSITIONS 16170

"METHOD 1" TRANSITION MATRIX

TRANSITION PROBABILITIES

JK = INPUT STATE, IL = OUTPUT STATE

JK	IL	PROB.	IL	PROB.	IL	PROB.	IL	PROB.
11	11	.7964	12	.0651	21	.0514	22	.0871
12	11	.2417	12	.3223	21	.0284	22	.4028
	23	.0047						
21	11	.3166	12	.0251	21	.2663	22	.3920
22	11	.0241	12	.0228	21	.0262	22	.8116
	23	.0419	32	.0369	33	.0366		
23	22	.2467	23	.3202	32	.0105	33	.4226
32	21	.0025	22	.3194	23	.3391	32	.3243
	33	.3342	43	.0025				
33	22	.0279	23	.0229	32	.0296	33	.8407
	34	.0279	43	.0282	44	.0229		
34	33	.3399	34	.3202	43	.0197	44	.3128
	45	.0025	55	.0049				
43	33	.3356	34	.0394	43	.3311	44	.2894
	45	.0023	55	.0023				
44	33	.0481	34	.0328	43	.0390	44	.8320
	45	.0207	54	.0122	55	.0153		
45	44	.2982	45	.3450	54	.0175	55	.3392
54	44	.4054	45	.0270	54	.3649	55	.2027
55	44	.0689	45	.0395	54	.0565	55	.8350

APPENDIX E

"Method 2"

Five-State

Transition Matrices

NUMBER OF TRANSITIONS

I	J	K	L=1	L=2	L=3	L=4	L=5	ROW SUM
1	1	1	683	28	0	0	0	711
1	1	2	95	36	0	0	0	132
1	1	3	0	0	0	0	0	0
1	1	4	0	0	0	0	0	0
1	1	5	0	0	0	0	0	0
1	2	1	27	4	0	0	0	31
1	2	2	90	128	0	0	0	218
1	2	3	0	1	0	0	0	1
1	2	4	0	0	0	0	0	0
1	2	5	0	0	0	0	0	0
1	3	1	0	0	0	0	0	0
1	3	2	0	0	0	0	0	0
1	3	3	0	0	0	0	0	0
1	3	4	0	0	0	0	0	0
1	3	5	0	0	0	0	0	0
1	4	1	0	0	0	0	0	0
1	4	2	0	0	0	0	0	0
1	4	3	0	0	0	0	0	0
1	4	4	0	0	0	0	0	0
1	4	5	0	0	0	0	0	0
1	5	1	0	0	0	0	0	0
1	5	2	0	0	0	0	0	0
1	5	3	0	0	0	0	0	0
1	5	4	0	0	0	0	0	0
1	5	5	0	0	0	0	0	0
2	1	1	79	96	0	0	0	175
2	1	2	16	52	0	0	0	68
2	1	3	0	0	0	0	0	0
2	1	4	0	0	0	0	0	0
2	1	5	0	0	0	0	0	0
2	2	1	35	109	0	0	0	144
2	2	2	50	2302	60	0	0	2412
2	2	3	0	176	73	0	0	249
2	2	4	0	0	0	0	0	0
2	2	5	0	0	0	0	0	0
2	3	1	0	0	0	0	0	0
2	3	2	0	64	10	0	0	74
2	3	3	0	159	197	0	0	356
2	3	4	0	0	0	0	0	0
2	3	5	0	0	0	0	0	0
2	4	1	0	0	0	0	0	0
2	4	2	0	0	0	0	0	0
2	4	3	0	0	0	0	0	0
2	4	4	0	0	0	0	0	0
2	4	5	0	0	0	0	0	0
2	5	1	0	0	0	0	0	0
2	5	2	0	0	0	0	0	0
2	5	3	0	0	0	0	0	0
2	5	4	0	0	0	0	0	0
2	5	5	0	0	0	0	0	0

NUMBER OF TRANSITIONS

I	J	K	L=1	L=2	L=3	L=4	L=5	ROW SUM
3	1	1	0	0	0	0	0	0
3	1	2	0	0	0	0	0	0
3	1	3	0	0	0	0	0	0
3	1	4	0	0	0	0	0	0
3	1	5	0	0	0	0	0	0
3	2	1	0	5	0	0	0	5
3	2	2	0	216	148	0	0	364
3	2	3	0	19	84	0	0	103
3	2	4	0	0	0	0	0	0
3	2	5	0	0	0	0	0	0
3	3	1	0	0	0	0	0	0
3	3	2	0	70	225	0	0	295
3	3	3	0	79	3951	70	0	4100
3	3	4	0	0	229	78	0	307
3	3	5	0	0	0	0	0	0
3	4	1	0	0	0	0	0	0
3	4	2	0	0	0	0	0	0
3	4	3	0	0	86	26	0	112
3	4	4	0	0	195	199	0	394
3	4	5	0	0	0	0	0	0
3	5	1	0	0	0	0	0	0
3	5	2	0	0	0	0	0	0
3	5	3	0	0	0	0	0	0
3	5	4	0	0	0	0	0	0
3	5	5	0	0	0	0	0	0
4	1	1	0	0	0	0	0	0
4	1	2	0	0	0	0	0	0
4	1	3	0	0	0	0	0	0
4	1	4	0	0	0	0	0	0
4	1	5	0	0	0	0	0	0
4	2	1	0	0	0	0	0	0
4	2	2	0	0	1	0	0	1
4	2	3	0	0	0	0	0	0
4	2	4	0	0	0	0	0	0
4	2	5	0	0	0	0	0	0
4	3	1	0	0	0	0	0	0
4	3	2	0	0	2	0	0	2
4	3	3	0	0	255	171	0	426
4	3	4	0	0	17	52	1	70
4	3	5	0	0	0	0	0	0
4	4	1	0	0	0	0	0	0
4	4	2	0	0	0	0	0	0
4	4	3	0	0	68	204	1	273
4	4	4	0	0	65	2612	40	2717
4	4	5	0	0	0	96	34	130
4	5	1	0	0	0	0	0	0
4	5	2	0	0	0	0	0	0
4	5	3	0	0	0	0	0	0
4	5	4	0	0	0	40	14	54
4	5	5	0	0	0	63	78	141

NUMBER OF TRANSITIONS

I	J	K	L=1	L=2	L=3	L=4	L=5	ROW SUM
5	1	1	0	0	0	0	0	0
5	1	2	0	0	0	0	0	0
5	1	3	0	0	0	0	0	0
5	1	4	0	0	0	0	0	0
5	1	5	0	0	0	0	0	0
5	2	1	0	0	0	0	0	0
5	2	2	0	0	0	0	0	0
5	2	3	0	0	0	0	0	0
5	2	4	0	0	0	0	0	0
5	2	5	0	0	0	0	0	0
5	3	1	0	0	0	0	0	0
5	3	2	0	0	0	0	0	0
5	3	3	0	0	0	0	0	0
5	3	4	0	0	0	0	1	1
5	3	5	0	0	0	0	0	0
5	4	1	0	0	0	0	0	0
5	4	2	0	0	0	0	0	0
5	4	3	0	0	0	1	0	1
5	4	4	0	0	0	76	52	128
5	4	5	0	0	0	6	24	30
5	5	1	0	0	0	0	0	0
5	5	2	0	0	0	0	0	0
5	5	3	0	0	0	0	0	0
5	5	4	0	0	0	23	64	87
5	5	5	0	0	0	27	676	703

TOTAL NUMBER OF TRANSITIONS IS 15015

TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
1	1	1	.9606	.0394	0.0000	0.0000	0.0000
1	1	2	.7273	.2727	0.0000	0.0000	0.0000
1	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	1	.8710	.1290	0.0000	0.0000	0.0000
1	2	2	.4128	.5872	0.0000	0.0000	0.0000
1	2	3	0.0000	1.0000	0.0000	0.0000	0.0000
1	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	1	.4514	.5486	0.0000	0.0000	0.0000
2	1	2	.2353	.7647	0.0000	0.0000	0.0000
2	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	2	1	.2431	.7569	0.0000	0.0000	0.0000
2	2	2	.0207	.9544	.0249	0.0000	0.0000
2	2	3	0.0000	.7068	.2932	0.0000	0.0000
2	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	2	0.0000	.8649	.1351	0.0000	0.0000
2	3	3	0.0000	.4466	.5534	0.0000	0.0000
2	3	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	5	0.0000	0.0000	0.0000	0.0000	0.0000

TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
3	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	1	0.0000	1.0000	0.0000	0.0000	0.0000
3	2	2	0.0000	.5934	.4066	0.0000	0.0000
3	2	3	0.0000	.1845	.8155	0.0000	0.0000
3	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	3	2	0.0000	.2373	.7627	0.0000	0.0000
3	3	3	0.0000	.0193	.9637	.0171	0.0000
3	3	4	0.0000	0.0000	.7459	.2541	0.0000
3	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	3	0.0000	0.0000	.7679	.2321	0.0000
3	4	4	0.0000	0.0000	.4949	.5051	0.0000
3	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	2	0.0000	0.0000	1.0000	0.0000	0.0000
4	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	3	2	0.0000	0.0000	1.0000	0.0000	0.0000
4	3	3	0.0000	0.0000	.5986	.4014	0.0000
4	3	4	0.0000	0.0000	.2429	.7429	.0143
4	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	3	0.0000	0.0000	.2491	.7473	.0037
4	4	4	0.0000	0.0000	.0239	.9614	.0147
4	4	5	0.0000	0.0000	0.0000	.7385	.2615
4	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	4	0.0000	0.0000	0.0000	.7407	.2593
4	5	5	0.0000	0.0000	0.0000	.4468	.5532

TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
5	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	4	0.0000	0.0000	0.0000	0.0000	1.0000
5	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	3	0.0000	0.0000	0.0000	1.0000	0.0000
5	4	4	0.0000	0.0000	0.0000	.5938	.4063
5	4	5	0.0000	0.0000	0.0000	.2000	.8000
5	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	4	0.0000	0.0000	0.0000	.2644	.7356
5	5	5	0.0000	0.0000	0.0000	.0384	.9616

CUMULATIVE TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
1	1	1	.9606	1.0000	1.0000	1.0000	1.0000
1	1	2	.7273	1.0000	1.0000	1.0000	1.0000
1	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	1	.8710	1.0000	1.0000	1.0000	1.0000
1	2	2	.4128	1.0000	1.0000	1.0000	1.0000
1	2	3	0.0000	1.0000	1.0000	1.0000	1.0000
1	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
1	5	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	1	.4514	1.0000	1.0000	1.0000	1.0000
2	1	2	.2353	1.0000	1.0000	1.0000	1.0000
2	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	2	1	.2431	1.0000	1.0000	1.0000	1.0000
2	2	2	.0207	.9751	1.0000	1.0000	1.0000
2	2	3	0.0000	.7068	1.0000	1.0000	1.0000
2	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	2	0.0000	.8649	1.0000	1.0000	1.0000
2	3	3	0.0000	.4466	1.0000	1.0000	1.0000
2	3	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
2	5	5	0.0000	0.0000	0.0000	0.0000	0.0000

CUMULATIVE TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
3	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	1	0.0000	1.0000	1.0000	1.0000	1.0000
3	2	2	0.0000	.5934	1.0000	1.0000	1.0000
3	2	3	0.0000	.1845	1.0000	1.0000	1.0000
3	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	3	2	0.0000	.2373	1.0000	1.0000	1.0000
3	3	3	0.0000	.0193	.9829	1.0000	1.0000
3	3	4	0.0000	0.0000	.7459	1.0000	1.0000
3	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	4	3	0.0000	0.0000	.7679	1.0000	1.0000
3	4	4	0.0000	0.0000	.4949	1.0000	1.0000
3	4	5	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	4	0.0000	0.0000	0.0000	0.0000	0.0000
3	5	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
4	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	2	0.0000	0.0000	1.0000	1.0000	1.0000
4	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
4	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	3	2	0.0000	0.0000	1.0000	1.0000	1.0000
4	3	3	0.0000	0.0000	.5986	1.0000	1.0000
4	3	4	0.0000	0.0000	.2429	.9857	1.0000
4	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	4	3	0.0000	0.0000	.2491	.9963	1.0000
4	4	4	0.0000	0.0000	.0239	.9853	1.0000
4	4	5	0.0000	0.0000	0.0000	.7385	1.0000
4	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
4	5	4	0.0000	0.0000	0.0000	.7407	1.0000
4	5	5	0.0000	0.0000	0.0000	.4468	1.0000

CUMULATIVE TRANSITION PROBABILITY MATRIX

I	J	K	L = 1	L = 2	L = 3	L = 4	L = 5
5	1	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	4	0.0000	0.0000	0.0000	0.0000	0.0000
5	1	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	4	0.0000	0.0000	0.0000	0.0000	0.0000
5	2	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	3	4	0.0000	0.0000	0.0000	0.0000	1.0000
5	3	5	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	4	3	0.0000	0.0000	0.0000	1.0000	1.0000
5	4	4	0.0000	0.0000	0.0000	.5938	1.0000
5	4	5	0.0000	0.0000	0.0000	.2000	1.0000
5	5	1	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	2	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	3	0.0000	0.0000	0.0000	0.0000	0.0000
5	5	4	0.0000	0.0000	0.0000	.2644	1.0000
5	5	5	0.0000	0.0000	0.0000	.0384	1.0000

Vita

Captain Schroeder was born on 20 October 1951 in Columbus, Ohio. In 1973, he graduated with a B.A. in mathematics and physics from Capital University, located in Columbus, Ohio, and was commissioned through AFROTC at the same time. He was then sent to St. Louis University for training in the Air Force Basic Meteorology Program. His next assignment, prior to AFIT attendance, was at the Air Force Global Weather Central at Offutt AFB, Nebraska, first as a weather forecaster in the Northern Hemisphere Forecasting Section and later as a Systems Duty Officer, a shift supervisor of the computer installation and related facilities at AFGWC. His next assignment will be at Headquarters, Air Force Logistics Command at Wright-Patterson AFB, in Plans and Programs (AFLC/XR).

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In many cases, weather is only one of many inputs to a simulation model, so a realistic but simple weather simulation method should be included in the model. This paper has three major areas of concern: (1) A fairly extensive review of applications of one-dimensional Markov and semi-Markov chains to weather data simulations, (2) A consideration of factors involved in and methods that are appropriate for extending

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Markov concepts to simulating gridded data in two or more dimensions, and (3) Evaluation of the proposed methods in terms of realism and simplicity of application. A discussion of the general characteristics of real weather variables and observed weather data in the context of simulating weather as a stochastic process is also given.

The data base used for the example consisted of gridded weekly maps of temperature departures from normal in an area of the United States. For most analyses, the data was converted to five states, from state 1 (coldest) to state 5 (warmest). In the real data, it was rare to have an occurrence of unequal and nonconsecutive states in adjacent grid points. Such occurrences were called "unusual transitions," and one criterion for evaluating the realism of a weather simulation scheme was the frequency of generating such transitions.

Most of the proposed two-dimensional simulation methods produced data fields that were basically quite realistic, but they also produced "unusual transitions" more frequently than in the original data. An additional suggested method, no more complex than the other methods, was found to greatly reduce the frequency of these "unusual transitions" in the simulated data fields. In this method, the probability distribution of the state at any interior point of the grid depends on the states observed at the points immediately west, north, and northeast of each point, and the interior of the grid is simulated by rows (in a west-to-east direction). All of the proposed simulation methods are very easy to perform on a computer.