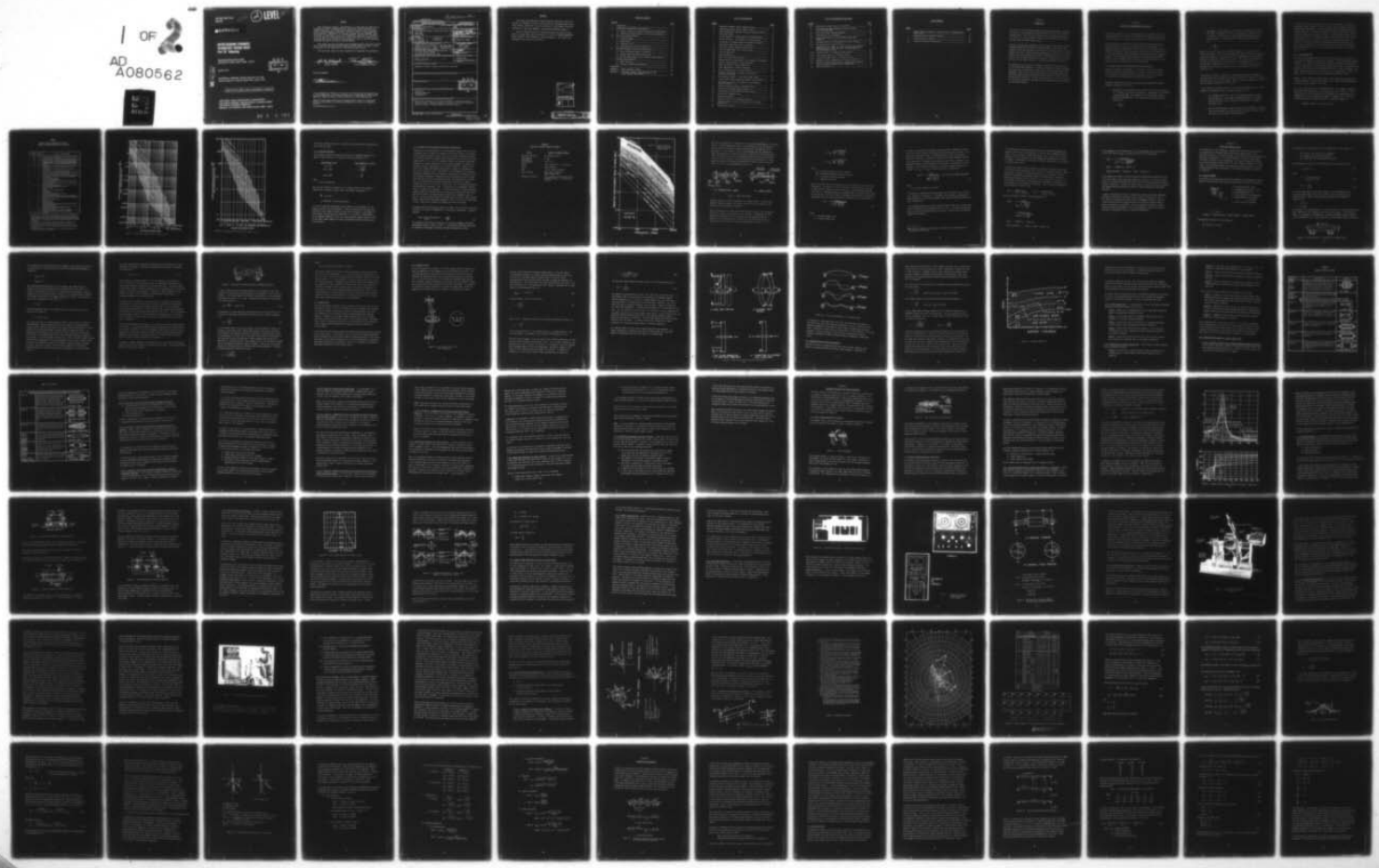
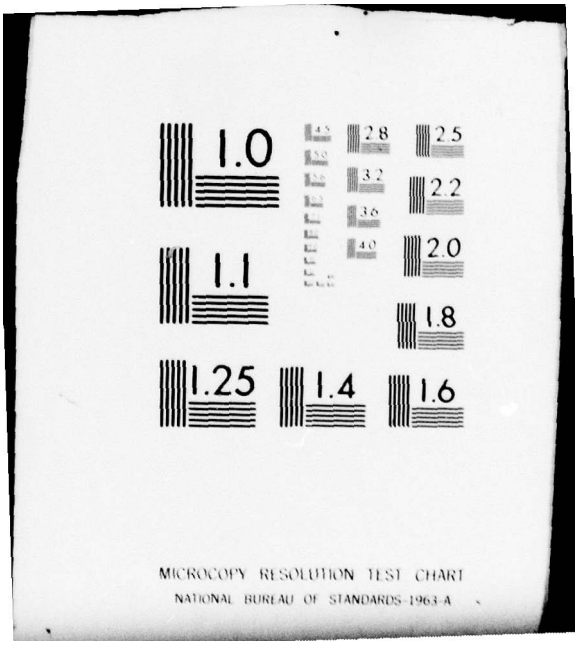
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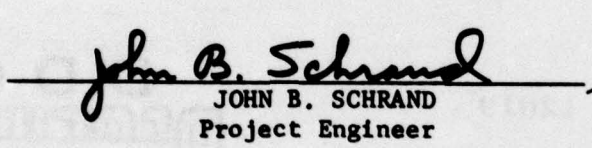
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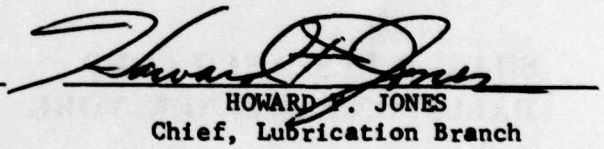
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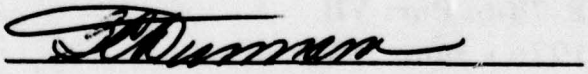


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This report summarizes the current technology in balancing rigid and flexible rotors. Balancing machines, balancing criteria and techniques, and the influence of shaft flexibility are discussed.

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FOREWORD

This report was prepared by Shaker Research Corporation under USAF Contract No. AF33615-76-C-2038. The contract was initiated under Project 3048, "Fuels, Lubrication, and Fire Protection", Task 304806, "Aerospace Lubrication", Work Unit 30480685, "Rotor-Bearing Dynamics Design."

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SECTION I

INTRODUCTION

As a result of the trend toward higher speed rotating machinery, balancing of rotors is receiving greater attention during both the rotor design and testing phases. Since the force developed due to unbalance increases as the square of the rotor speed, the need for greater attention to balance for high speed operation becomes obvious.

As rotor size and speed increases, the rotor tends to become more flexible. Unbalance magnitude and location for these flexible rotors become very important since they represent the driving force that can excite the flexible modes of the rotor. Current balancing technology, therefore, is concerned with not only the magnitude and location of unbalance forces but the flexibility of the rotating system and its supports. Balancing is thus an integral part of the overall subject of "Rotor-Bearing Dynamics Technology".

The purpose of this volume is to summarize the current state-of-the-art in rotor balancing technology. In order to describe the balance techniques in current use, a distinction is made between rigid body and flexible body balancing. Each of these rotor operating modes are discussed with their associated balancing approaches and available machines or techniques that are applied for balancing. The text has been prepared as a basic reference for the practicing engineer appended with the necessary analysis techniques for the design engineer.

SECTION II

ESTABLISHING REQUIREMENT FOR BALANCING

A number of approaches exist in establishing the need, degree and type of balance required for a machine. The approach often depends on the status of the machine; i.e.: design phase, test phase or field operation phase.

In the design phase, guidelines from both literature and experience are generally used as a basis for specifying balancing requirements. The permissible degree of unbalance is then verified by calculating the rotor response to unbalance. For testing and field operation, the vibration level of the machine during operation generally dictates the need for improved balance. The following two sections discuss available information and guides that are currently used as criteria for establishing both need and degree of balance recommended for rotating machinery.

2.1 Design Criteria for Establishing Balance Requirements

During the machinery design phase it is necessary to prevent excessive vibration amplitudes during operation. In order to provide some guidelines regarding recommended balance requirements, it is first necessary to review the general nomenclature associated with unbalance.

Two general descriptive terms are currently used to describe the degree of unbalance existing in a rotor,i.e.:

1. Unbalance may be considered as the result of a mass (m) concentrated at a distance (r) from the rotating axis. The product mr is expressed in weight terms ($\frac{wr}{g}$); the unbalance (wr) is thus given in ounce inches (gram millimeters). The resulting force induced from this unbalance is:

$$F = \frac{wr}{g}\,\omega^2 \qquad (1)$$

2. Alternately, the unbalance in a rotor may be considered as shifting the center of gravity of the rotor a distance (e) from the center of rotation. The unbalance is then expressed in millionths of an inch (microns). The resulting force induced from this unbalance is:

$$F = \frac{We}{g}\omega^2 \qquad (2)$$

Unlike Equation (1) where w is an unbalance weight, W in Equation (2) is total rotor weight. The concept of an unbalance weight (w) is more convenient for specifying balance corrections since it represents the required weight addition or removal at a radius on the rotor. The concept of eccentricity (e) is often used to describe the capabilities of a balancing machine. A machine with a sensitivity of 50 μ in. (1.27 μ m) implies that a rotor may be balanced such that the rotational center and geometrical center will be within 50 μ in.

One additional concept is important in establishing balance requirements of rotors and that is the distinction between rigid rotors, flexible rotors and operation near rotor critical speeds. These concepts are discussed in Section 3.0 and the reader is referred to that section.

In the rigid body operation region of a rotor, two documents exist to provide guidance on acceptable levels of residual unbalance; i.e.:

1. ISO DOCUMENT 1940(1973). This is the present standard on rigid rotor balancing. It contains comprehensive charts on acceptable residual unbalance level criteria, and definitions of required balancing terms.

2. MIL-STD-167(1954) Basic balance quality criteria document. Contains charts and formulas for acceptance quality criteria of rigid rotors in terms of rotor weight and operating speed.

In addition to these two documents which will be discussed in subsequent paragraphs, balance levels for rigid rotors as well as flexible rotors can be determined analytically. This is accomplished by performing unbalance response analysis. The rotor is modeled as an elastic mass system supported on elastic supports as discussed in Part I, "Rotor-Bearing Dynamics Technology Design Guide." Unbalance excitations may be placed at critical mass locations along the rotor and the response to unbalance calculated. Unbalance limits may be selected from established acceptable dynamic force levels at the bearings to protect them from fatigue and from established shaft amplitude levels that insure that excessive rubbing in seal regions does not occur.

2.1.1 ISO 1940(1973)

This specification deals with rigid rotors and assumes that the larger the rotor mass the greater the permissible unbalance. Since permissible residual unbalance U is related to rotor mass m, specific unbalance e is defined as $e = U/m$. Therefore, e is the distance between mass center and rotating center. Experience indicates that the allowable limits for e varies inversely as the speed ω(radians per second) of the rotor over a limited range of speed. The speed range is dictated by the rotor grade as described in Table 1.

Figures 1 and 2 indicate the speed range of each grade and the permissible residual unbalance in metric and English units in terms of unbalance weight and radius per unit weight of the rotor.

Use of the Table and curves is straightforward. As an example, consider a 500 pound steam turbine rotor. From Table 1, this is a class G 2.5 rotor. From Figure 1 for turbine operation at 4000 RPM, the required balance is 0.00024 lb-in/lb with a range from 0.0002 to 0.0003 lb-in/lb. Utilizing the nominal value the balance per plane, for two plane balance is:

$$0.00024/2 \times 500 \times 16 = 1.96 \text{ ounce inch}$$

TABLE 1

BALANCE QUALITY GRADES FOR VARIOUS GROUPS OF REPRESENTATIVE RIGID ROTORS

Balance Quality Grade G	e_{ω}[1,2] mm/s	Rotor Types - General Examples
G 4 000	4 000	Crankshaft-drives[3] of rigidly mounted slow marine diesel engines with uneven number of cylinders[4].
G 1 600	1 600	Crankshaft-drives of rigidly mounted large two-cycle engines.
G 630	630	Crankshaft-drives of rigidly mounted large four-cycle engines. Crankshaft-drives of elastically mounted marine diesel engines.
G 250	250	Crankshaft-drives of rigidly mounted fast four-cylinder diesel engines[4].
G 100	100	Crankshaft-drives of fast diesel engines with six or more cylinders[4]. Complete engines (gasoline or diesel) for cars, trucks, and locomotives[5].
G 40	40	Car wheels, wheel rims, wheel sets, drive shafts. Crankshaft-drives of elastically mounted fast four-cylinder engines (gasoline or diesel) with six or more cylinders[4]. Crankshaft-drives for engines of cars, trucks, and locomotives.
G 16	16	Drive shafts (propeller shafts, cardan shafts) with special requirements. Parts of crushing machinery. Parts of agricultural machinery. Individual components of engines (gasoline or diesel) for cars, trucks, and locomotives. Crankshaft-drives of engines with six or more cylinders under special requirements.
G 6,3	6,3	Parts or process plant machines. Marine main turbine gears (merchant service). Centrifuge drums. Fans. Assembled aircraft gas turbine rotors. Fly wheels. Pump impellers. Machine-tool and general machinery parts. Normal electrical armatures. Individual components of engines under special requirements.
G 2,5	2,5	Gas and steam turbines, including marine main turbines (merchant service). Rigid turbo-generator rotors. Rotors. Turbo-compressors. Machine-tool drives. Medium and large electrical armatures with special requirements. Small electrical armatures. Turbine-driven pumps.
G 1	1	Tape recorder and phonograph (gramophone) drives. Grinding machine drives. Small electrical armatures with special requirements.
G 0,4	0,4	Spindles, disks, and armatures of precision grinders. Gyroscopes.

1 $\omega = 2\pi n/60 \approx n/10$, if n is measured in revolutions per minute and ω in radians per second.

2 In general, for rigid rotors with two correction planes, one half of the recommended residual unbalance is to be taken for each plane; these values apply usually for any two arbitrarily chosen planes, but the state of unbalance may be improved upon at the bearings. (See 3.2 and 3.4). For disk-shaped rotors the full recommended value holds for one plane (see section 3).

3 A crankshaft-drive is an assembly which includes the crankshaft, a flywheel, clutch, pulley, vibration damper, rotating portion of connecting rod, etc. (see 3.5).

4 For the purposes of this International Standard, slow diesel engines are those with a piston velocity of less than 9 m/s; fast diesel engines are those with a piston velocity of greater than 9 m/s.

5 In complete engines, the rotor mass comprises the sum of all masses belonging to the crankshaft-drive described in Note 3 above.

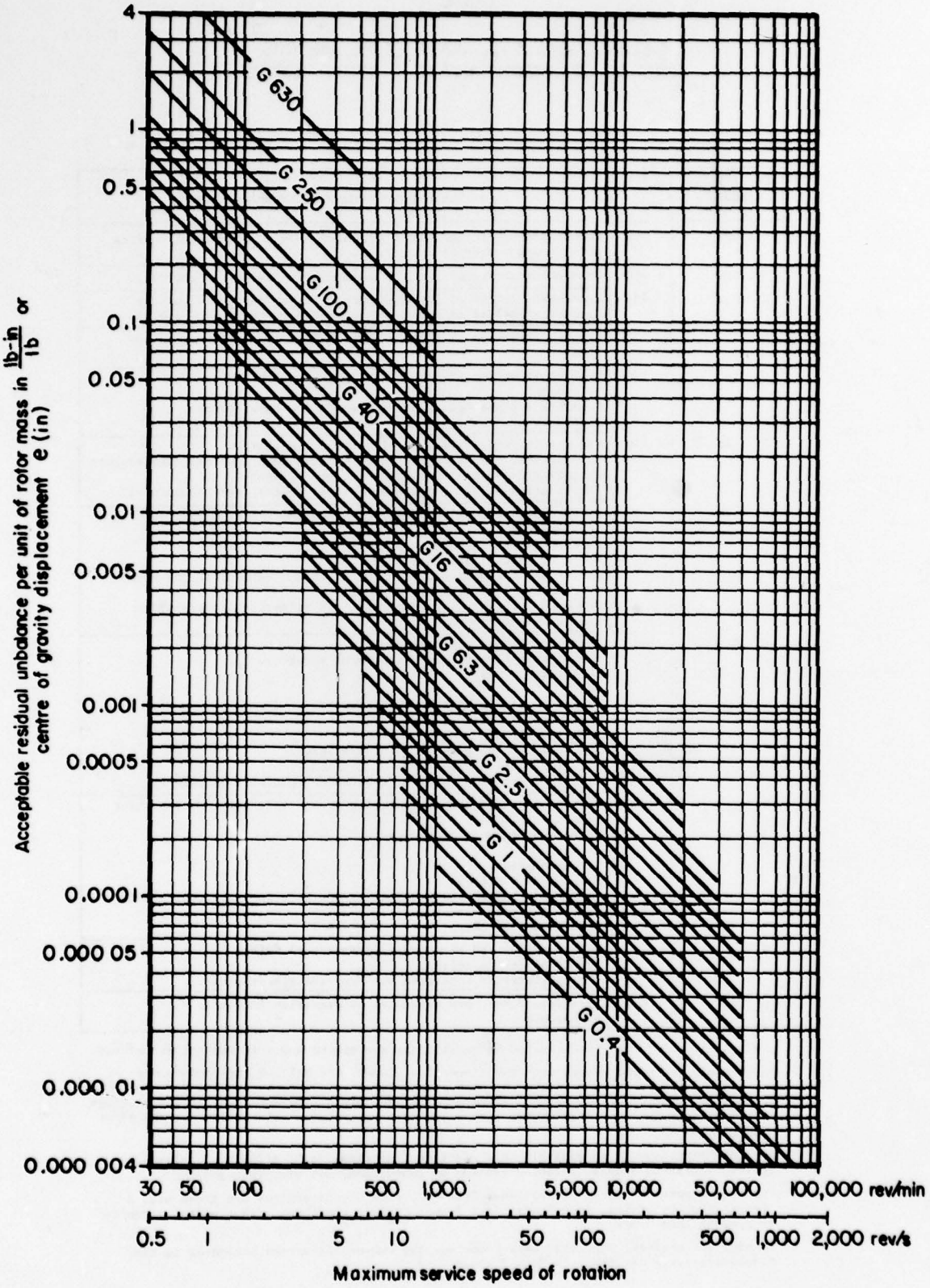


Figure 1 Permissible Balance Levels (English Units)

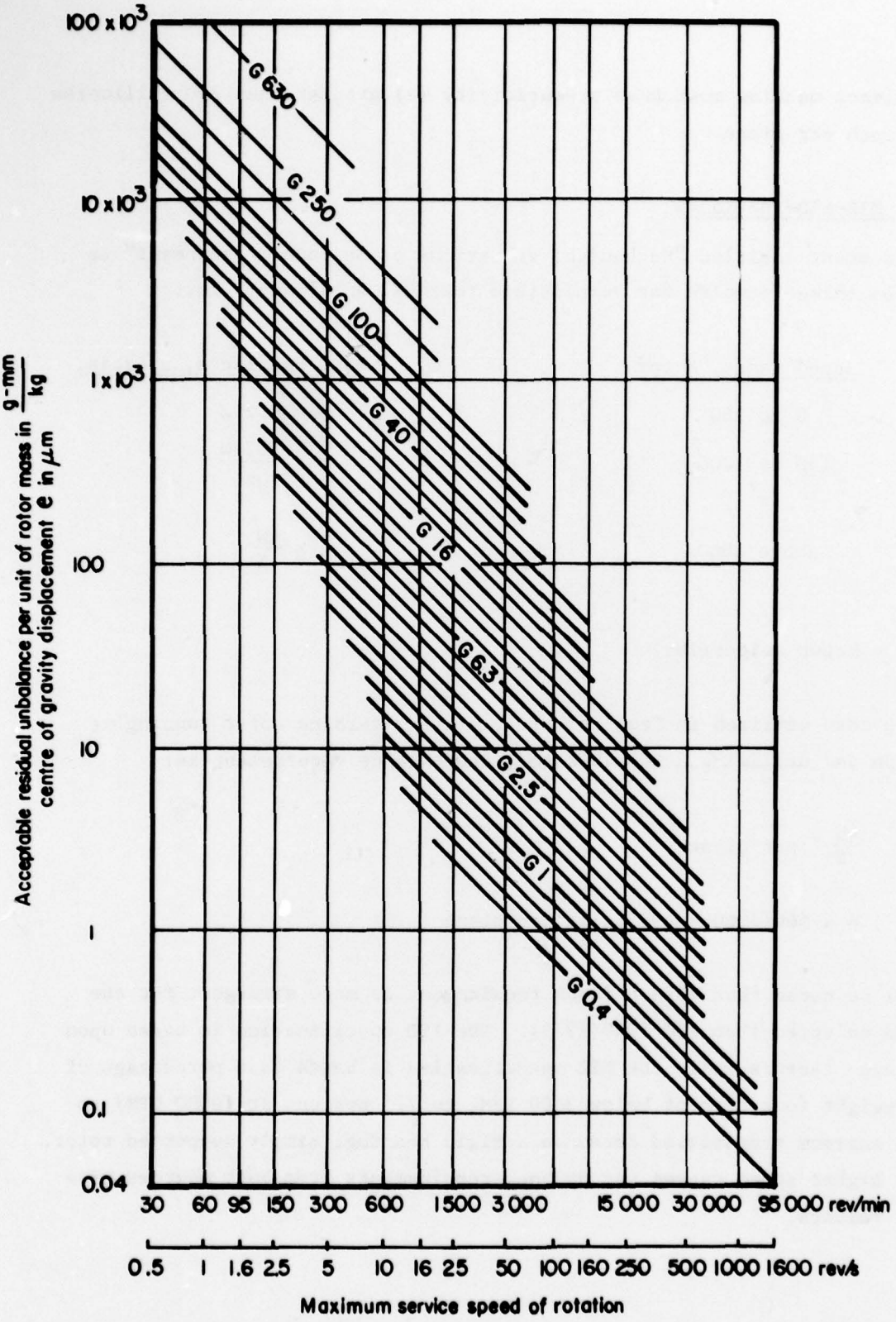


Figure 2 Permissible Balance Levels (Metric Units)

The balance machine must have a sensitivity (e) greater than 240 millionths of an inch per plane.

2.1.2 MIL-STD-167(1954)

This document entitled "Mechanical Vibrations of Shipboard Equipment" is based on three formulas for permissible residual unbalance; i.e.:

Speed Range, N rpm	Max. Unbalance, μ oz.in.
0 to 150	$\mu \leq 0.25W$
150 to 1000	$\mu \leq \frac{5630W}{N^2}$
above 1000	$\mu \leq \frac{4W}{N}$

where

W = Rotor weight(lbs)

For the case utilized in Section 2.1.1, a steam turbine rotor running at 4000 rpm and utilizing a 500 lb rotor, the balance requirement is:

$\frac{4W}{N}$ per plane

4 x 500/4000 = .50 oz-in per plane

It will be noted that this balance requirement is more stringent for the example selected than ISO 1940(1973). The ISO specification is based upon experience factors while the MIL specification is based on a percentage of rotor weight (one percent below 3600 RPM, to 7.5 percent at 10000 RPM) to be the maximum transmitted force on a rigid bearing, simply supported rotor. In the higher speed ranges the balance requirements from both sources give closer results.

2.2 Testing and Field Operation Criteria for Balancing

Vibration of rotating machinery is an undesirable characteristic that can ultimately lead to fatigue of components and machine failure if not maintained within reasonable limits. Any list of sources inducing vibration of a rotating machine is headed by mass unbalance of the rotor as the most common source. It is well to recognize however, that sources other than unbalance can induce vibration, it is not always possible to reduce the vibratory level by improvement in the balance condition. Table 2 lists some of the common vibrational sources and the range of frequencies at which they generally occur. Most guides for determination of acceptable vibrational amplitude levels of machines encompasses all sources of vibration including unbalance. Vibrational limits proposed by Catlin [1] and Blake [2] can be used for acceptable bearing cap vibrations due to unbalance. In operation of a machine, when casing vibrations at running frequencies exceed 0.2 in per sec (5.08 mm per sec) it is generally agreed that balancing is required. Fig. 3 illustrates the recommended vibration levels of common usage machinery. It should be pointed out when assessing balance condition by monitoring case or bearing cap vibrations that casing mass and stiffness can significantly influence the judgement levels of Fig. 3. For very massive and stiff housings such as found in high pressure barrel construction centrifugal compressor levels of 0.5 to 0.2 times those of Fig. 3 may be more suitable to adjudge the need for balance.

If shaft motions are measured, Equation (3) [Ref. 3] or 2.0 mils, (pk to pk) (whichever is less) is recommended for centrifugal compressors. A general approach

$$\text{Peak to Peak Displacement (mils)} = \frac{12000}{\text{rpm}} \qquad (3)$$

for determining the balance condition of a rotating assembly using shaft displacement probes is given in Ref. 4. Alternately, amplitude limits may be computed if knowledge of bearing stiffness properties is known.

TABLE 2

MACHINERY TYPICAL VIBRATORY SOURCES

Source	Dominant Frequency Range
Rotating Unbalance	N = (RPM of rotor)
Misalignment	N, 2N
Loose parts	2N
Bearing Whirl	.4N - .5N
Motors	Line frequency & 2 x line freq.
Gears	Mesh frequency
Rubs	.5N, N, high frequencies
Flow Induced	Blade pass frequency, Radom frequencies
Foundation Induced	Low frequencies (less than 10 Hz); Running frequency of neighboring machines

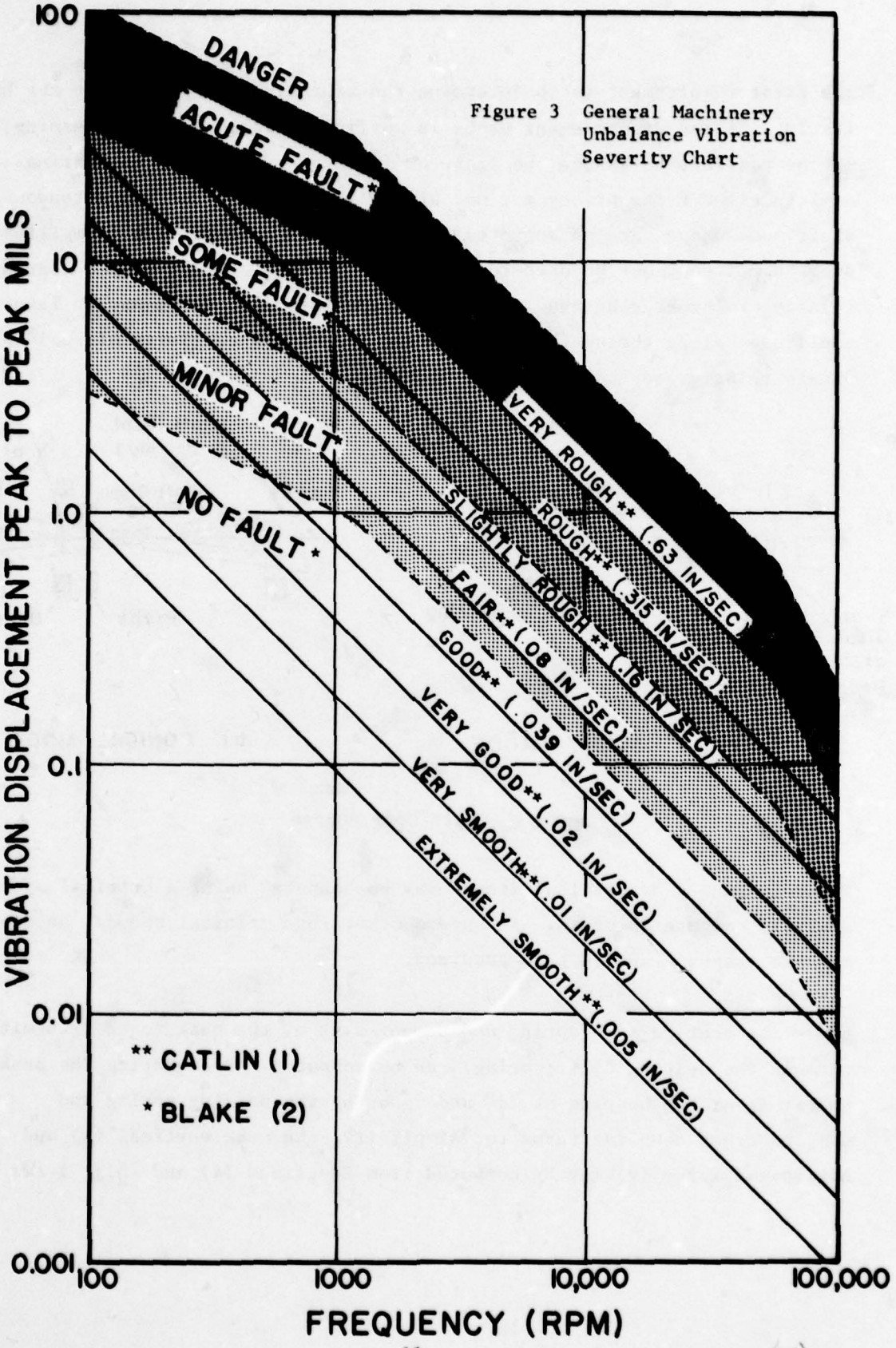
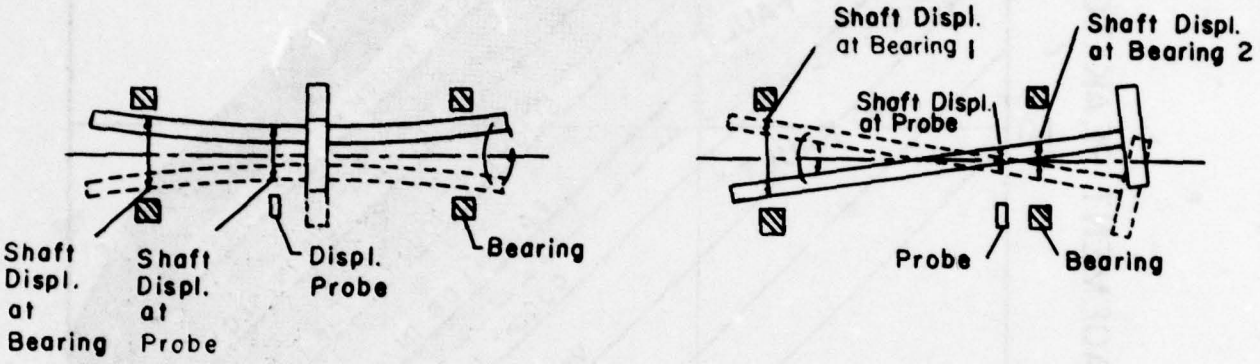


Figure 3 General Machinery Unbalance Vibration Severity Chart

The first requirement is to determine the vibrational amplitude at the bearing itself. If the displacement probe is sufficiently close to the bearing, it may be possible to assume the probe amplitudes are the same as bearing amplitudes. If the probes are not at the bearings and the instantaneous shaft mode shapes are as depicted in Fig. 4, the ratio of shaft amplitude to probe amplitude must be used to establish amplitude levels at the bearing relative to probe readings. The mode shapes are also necessary to insure amplitudes along the shaft do not exceed the clearance limits of seals of wheels relative to the amplitudes at the probe.



a) TRANSLATORY MODE

b) CONICAL MODE

Figure 4 Shaft Mode Shapes

The mode shapes at critical speeds may be computed using a critical speed analysis computer program. At speeds other than critical speeds, an unbalance response analysis is required.

Since the bearing is a spring and damper system, the peak force transmitted through the bearing film (spring) can be computed. Designating the peak amplitude at the bearing as "d" and ignoring the bearing spring and damping cross coupling terms for simplicity, the peak vertical (x) and horizontal force (y) may be computed from Equations (4) and (5); i.e.:

$$F_x = K_{xx}\, d \sqrt{1 + \left(\frac{\omega B_{xx}}{K_{xx}}\right)^2} \tag{4}$$

$$F_y = K_{yy}\, d \sqrt{1 + \left(\frac{\omega B_{yy}}{K_{yy}}\right)^2} \tag{5}$$

where

K_{xx} = vertical bearing stiffness (lbs/in)

K_{yy} = horizontal bearing stiffness (lbs/in)

ω = rotating frequency (rad/sec)

B = damping (lb-sec/in)

This is the peak cyclic force imposed on the bearing due to the peak displacement d. Utilization of this cyclic force on the bearing is required to calculate the fatigue life of the bearing. For babbit-lined bearings with .020 in. (.508 mm) thickness or greater babbit, a fatigue stress limit of 125 psi (8.27×10^5 N/m^2) (load on bearing projected area) is suggested. Considering only the vertical direction, Equation (4) may be modified to:

$$d(pk) = \frac{125\ DL}{K_{xx} \sqrt{1 + \left(\frac{\omega B_{xx}}{K_{xx}}\right)^2}} \tag{6}$$

where

D = bearing diameter (in)

L = bearing length (in)

In order to calculate the allowable peak displacement, Equation (6) indicates that both stiffness and damping characteristics must be known. It is noted from the equation that as damping approaches zero, the equation approaches $125DL/K_{xx}$. (Although it appears that minimal damping is desirable, amplitudes of undamped systems approach infinity at resonance.) As damping increases, the transmitted force increases. Therefore, as a first estimate of the allowable amplitudes, a critically damped system can be assumed; i.e. $B_{xx} = 2 K_{xx} m$. Equation (6) may be rewritten as:

$$d(pk)^{*} = \frac{125DL}{K_{xx}\sqrt{1 + \left(\frac{2\omega}{\omega_n}\right)^2}} \quad \text{(for critically damped bearings)} \tag{7}$$

where

ω_n = natural frequency (rad/sec)

If operation is maintained 20 percent away from a critical, $\omega/\omega_n = 0.8$ or 1.2, equation (7) reduces to: $66.25DL/K_{xx}$ or $48.1DL/K_{xx}$, respectively. Many bearings are not critically damped and, therefore, these estimates are quite conservative.

If tilting-pad bearings are used, limiting fatigue loads on the pivots must also be considered utilizing Equation (4) or (5). If flexible bearing supports are used, these equations are also used to insure they have adequate dynamic load capacity.

---

*This limit is based on a rigid shaft critical and is conservative for flexible shaft criticals.

If rolling-element bearings are used in lieu of fluid-film bearings, the dynamic cyclic forces from Equations (4) and (5) may be combined with the static forces to compute the bearing fatigue life. In the case of these bearings, it may be assumed that the damping term approaches zero and may be ignored. Knowledge of the bearing stiffness characteristics is essential to compute these forces.

In these discussions, we have attempted to simplify the analysis procedure so that quick hand calculations can be applied to establish realistic amplitude levels, In order to illustrate the use of this approach, we will consider a 12,000 rpm centrifugal compressor supported on a fluid-film, babbit-lined bearing 1 inch (2.54 cm) I. D. x .5 inch (1.27 cm) long. It is desirable to determine the allowable shaft amplitude as measured from a probe adjacent to the bearing at 12,000 rpm. The stiffness and damping of the bearing in the vertical are:

K_{xx} = 47,000 lbs/in (8.23×10^6 N/m) ωB_{xx} = 239,000 lbs/in (4.185×10^7 N/m)

Since the bearing is babbit, utilizing Equation (6)

$$d(pk) = \frac{125DL}{K_{xx}\sqrt{1 + \left(\frac{\omega B_{xx}}{K_{xx}}\right)^2}}$$

$$\frac{125 \times 1 \times .5}{47{,}000\sqrt{1 + \left(\frac{239}{47}\right)^2}}$$

$d(pk)$ = .000256 in. (.00065 cm)

double amplitude = .00051 in. pk/pk (.001295 cm)

If we selected a 2 inch diameter by 1 inch long bearing, K_{xx} = 20,175 lbs/in (3.53 x 10^6 N/m) and ωB_{xx} = 352,500 lb/in (6.17 x 10^7 N/m), then:

$$d(pk) = \frac{125 \times 2 \times 1}{20{,}175 \sqrt{1 + (\frac{352{,}500}{20{,}175})^2}}$$

$$d(pk) = .000798 \text{ in. } (.002027 \text{ cm})$$

$$\text{double amplitude} = .001416 \text{ in. pk/pk } (.004054 \text{ cm})$$

These numbers are based on a plain cylindrical bearing and 60 pound (266.9 N) rotor with .002 in. per inch (cm/cm) bearing clearance. It is clear, however, when we are establishing amplitude limits based on bearing capability, bearing design is important. In order to insure either of these levels are satisfactory the computed mode shape must be reviewed to insure adequate seal and wheel clearance.

A number of additional sources of vibrational limits of both case vibrations and shaft vibrations are available and are listed in references 5 thru 47. The reader is cautioned that many of these recommended limits are based on overall vibration and only generalization can be made on balance requirements for machinery in operation. It must further be recognized as illustrated with shaft measurements, that the measurement location on casings is important in assessing results of vibration measurements.

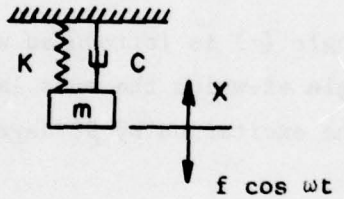
SECTION III

RIGID AND FLEXIBLE ROTORS

In discussions of rotor balancing, a distinction is made in both balancing requirements and techniques between rigid and flexible rotors. It is necessary therefore, to review the need for this distinction and some guides for defining the distinction. It is also necessary to review critical speeds of rotors as related to rotor balancing. The reader is referred to "Part I: Rotor-Bearing Dynamics Design Guide" for a more complete discussion of these areas. The following sections are presented in sufficient detail to permit an understanding of the relationship of rotor-bearing dynamics as related to balancing.

3.1 Critical Speeds

The concept of critical speed can best be understood by considering a single degree of freedom elastically supported mass system as depicted in Figure 5.



K = Stiffness (lb/in) (N/m)
C = Damping (lb sec/in) (N sec/m)
x = Displacement (in) (m)
F = Force (lbs) (N) (instantaneous)
m = Supported mass (lb sec^2/in)
(N sec^2/m)
f = Force (lbs) (N) (applied)

$F = f \cos \omega t$

Figure 5 Forced Vibration - Single Degree of Freedom System

The equation of motion for this system is:

$$m\ddot{x} + C\dot{x} + Kx = f \cos \omega t \qquad (8)$$

From inspection of equation (8) we can recognize each of the terms; i.e.:

$m\ddot{x}$ = inertia term (acceleration dependent)
$C\dot{x}$ = damping term (velocity dependent)
Kx = stiffness term (displacement dependent)

The instantaneous solution for displacement (x) from equation (8) is:

$$x = X\cos(\omega t - \alpha) \tag{9}$$

where

$$X = \frac{f}{\sqrt{(K-m\omega^2)^2 + C^2\omega^2}} \tag{10}$$

$$\tan \alpha = \frac{C\omega}{K-m\omega^2} \tag{11}$$

It is noted from equation (9) that an angle (α) is introduced which is called the phase angle. This is the angle at which the mass lags the excitation force. When the mass lags the excitation by 90 degrees in equation (11):

$$K-m\omega^2 = 0$$

or $$\omega = \sqrt{\frac{K}{m}}$$

This condition is termed resonance. As will be shown, in rotating systems, this same condition is more commonly called the critical speed. If the system of Figure 5 is expanded to two degrees of freedom, the simple system of Figure 6 can be representative of a shaft supported on bearings that contain flexibility.

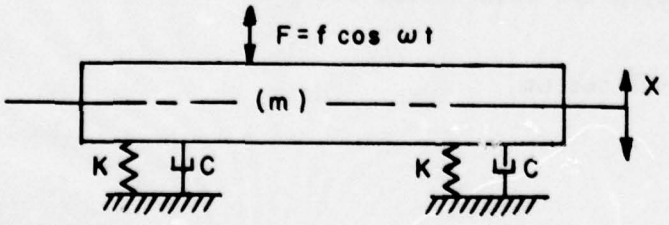


Figure 6 Forced Vibration - Two Degrees of Freedom System

If we observe the vertical motion only of Figure 6, and consider vibration in a plane parallel to the centerline, the vertical stiffness and damping are defined as:

$$K_{Total} = 2K$$

$$C_{Total} = 2C$$

The instantaneous displacement (x) is the same as the simple system of equation (9) previously discussed with the substitution of 2K and 2C for K and C in equations (10) and (11). To relate these discussions to rotating systems, we can consider the shaft of Figure 6 as a rotating member supported on bearings with stiffness and damping properties (K and C). The excitation (F) is due to unbalance in the rotor; i.e.:

$$F = mr\omega^2$$

Since the unbalance force is a rotating vector in the vertical plane only the force will vary as:

$$F = (mr\omega^2)\ \cos\omega t$$

It is thus seen that our simple system of Figure 6 can be used to provide an understanding of natural frequencies (critical speeds) of rotating members. We have already observed that the critical speed occurs when $2K-m\omega^2=0$. At this condition the system displacement response lags the excitation by 90^{o}. In consideration of balancing, this implies that the location of the unbalance force vector is 90 degrees ahead of the peak vibration amplitude as measured on the shaft. In any balancing operation near or above a critical speed, balance location from displacement or force measurements on the shaft or pedestal will not correctly indicate the location of unbalance unless the phase lag angle is known. Above the critical, the phase angle increases to 180 degrees. Further discussion of the phase angle and its importance is discussed under Section 4.

A second consideration regarding balancing and critical speeds is the shaft amplitude at resonance. Returning to equations (9) and (10) at resonance, the amplitude is:

$$x = \frac{f}{C\omega} \cos(\omega t - \alpha) \qquad (12)$$

The amplitude is dependent upon force (f) and damping (C) at resonance (ωC). If damping in the bearings is small as commonly encountered in rolling element bearings, amplitudes will become very large. This necessitates a much higher degree of balance (reduction of f in Equation (12)) if operation close to a critical is anticipated. Through this simple example, the importance of shaft critical speeds on balancing limits and methods of locating unbalance force is readily seen.

The simple system of Figure 6 can be utilized to illustrate two additional considerations. Only the vertical vibration was discussed in illustrating the influence of resonance on balance considerations. If the horizontal stiffness and damping differ from the vertical, the critical speed and amplitude response in the horizontal direction will also differ. This is discussed more thoroughly in Part I. In consideration of balancing, it is necessary for the reader to recognize that the support characteristics in both planes must be considered.

The second consideration in Figure 6, is to briefly review the angular motion of the rotor. In the previous discussions we considered only motion parallel to the shaft centerline. The type of unbalance that would excite this mode of vibration is a rotor with center of gravity centered between supports and an unbalance mass located in the plane of the CG. A single plane correction at the CG could be used to correct this condition.

If however, a moment unbalance existed about the CG, the motion of the shaft would be conical with one end out of phase with the other. The excitation mode would be illustrated in Figure 7.

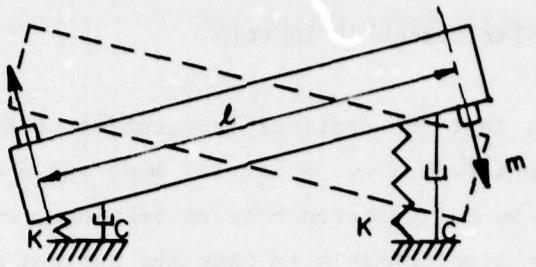


Figure 7 Two Degree of Freedom System with Moment Excitation

It may be observed that the system restraint is an angular one for this condition. The angular restoring moment of the springs is: $K\ell^2/2$ (in-lbs/radian). The inertial moment of the shaft is a function of the transverse mass moment of inertia; i.e., $I_T\ddot{\theta}$ The equation of motion for this system neglecting damping is:

$$I_T\ddot{\theta} + \frac{K\ell^2\theta}{2} = \ell mr\omega^2 \cos\omega t \tag{13}$$

The equation is in the same form as Equation (8) and all the previous discussions are applicable including the solution for the resonant frequency; i.e.:

$$\omega_n = \sqrt{\frac{K\ell^2}{2I_T}} \tag{14}$$

For a rotating system in the conical mode, the polar moment of inertia can introduce a gyroscopic effect which would influence the natural frequency of the system. Den Hartog, "Mechanical Vibrations," Fourth Edition, McGraw Hill Publishing Co., shows that if the shaft is whirling (or precessing) in the direction of spin, the gyroscopic inertia moment tends to make the amplitude of vibration smaller; thus, making the effective spring rate higher, and in this manner raising the natural frequency. This is the case corresponding to synchronous whirl as induced from rotating unbalance. Following through Den Hartog's derivation, we have

$$\omega_n = \sqrt{\frac{K\ell^2}{2(I_T - I_P)}} \tag{15}$$

where

I_p is the mass polar moment of inertia

It is seen, therefore, that two critical frequencies of the rotor system of Fig. 6 can be encountered, i.e.: a lateral mode and a conical mode. If the c.g. of the rotor is not centered between bearings, both modes can have conical motion. It is also possible to have the conical mode occur before the translatory mode. Considering rotor balance aspects, the importance is to recognize that these different critical speeds can be encountered and balancing requirements must recognize them if operation in proximity is anticipated. Since the Rotor-Bearing Dynamics Design Guide (Part I) discusses analysis of critical speeds, it is not the purpose here to provide a rigorous solution for determination of critical speeds but only to illustrate their importance in the area of rotor balancing.

3.2 Rigid Rotors

From the discussions presented in the previous section (3.1), the rotor was considered as rigid. The only influence the rotor contributed to resonance (or critical speed) was the mass or inertia characteristics of the rotor. The flexibility in the system occurred in the bearings and/or bearing supports. Unbalance forces under these conditions are due to any mass eccentricity from the rotational center of the rotating system. The magnitude of this rotating mass eccentricity can be increased by the deflection of the support system and amplified by operation in the vicinity of a critical speed.

For the most part balancing machines and balancing specifications are directed toward rigid rotors. The designer as well as the user of rotating machinery should recognize this characteristic when specifying techniques. The following section (3.3) discusses flexible rotors and the differences in unbalance amplitudes introduced by flexibility. Section 3.4 outlines the approach to distinguish rigid-rotors from flexible rotors.

3.3 Flexible Rotors

We have discussed the occurrence of critical speeds and their influence on phase and amplitude of the unbalance force as reacted by the rotor. The critical speeds were attributed to the flexibility of the rotor support (bearings and pedestal) and the mass and inertia properties of the rigid rotor. If we now reverse the rolls and make the bearings rigid and the rotor flexible, natural frequencies of the rotor due to its elastic properties and mass and inertia characteristics may be calculated. To illustrate the influence of shaft flexibility, consider the single disc of mass (m) symmetrically located between two bearings as illustrated in Figure 8.

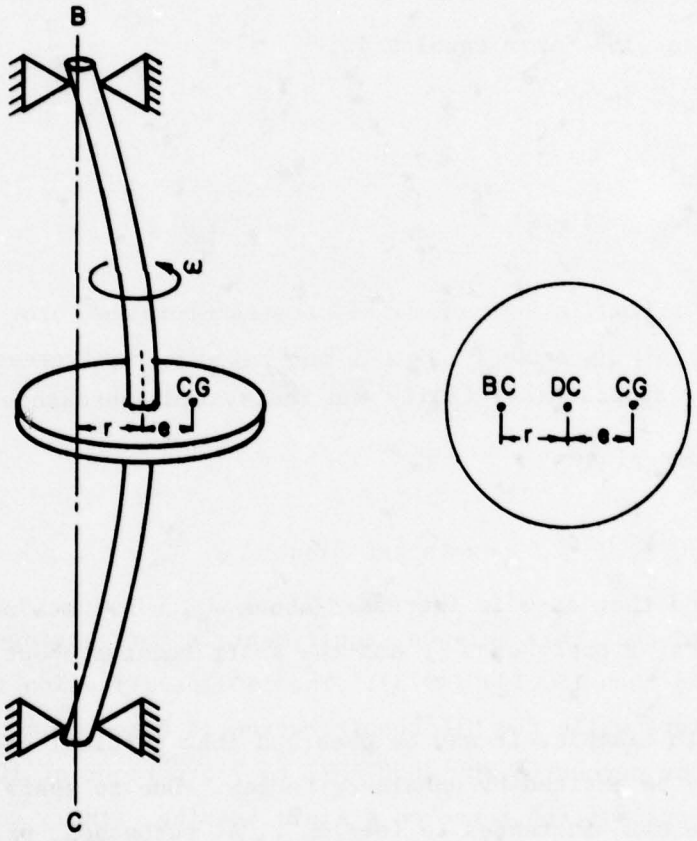


Figure 8 Synchronous Shaft Whirl Due to Unbalance

With the shaft rotating at constant angular speed, ω, the disc center is displaced a distance (r) from the bearing centerline due to the eccentricity (e) of the disc center of gravity (CG) to the disc center. Ignoring gravity and friction forces, the disc is under the influence of two forces; i.e., the centrifugal force $me\omega^2$ and restoring force (due to the shaft stiffness) Kr. The centrifugal force, however, increased due to shaft deflection and becomes $m\omega^2(e+r)$ or:

$$K_{shaft}\, r = m\omega^2(e+r) \tag{16}$$

Solving Equation (16) for r results in:

$$r = \frac{em\omega^2}{K-m\omega^2} \tag{17}$$

When $K = m\omega^2$, r approaches infinity and the system approaches resonance at:

$$\omega_n = \sqrt{\frac{K}{m}}$$

It will be noted that as ω is increased above ω_n, r becomes negative. For very high speeds, r approaches -e and the shaft rotates about its CG.

From this simple example, it may be observed that critical speeds due to shaft flexibility may be excited by unbalance forces. Due to shaft flexibility, the unbalance force $em\omega^2$ increases to $(e+r)m\omega^2$. At resonance, frictional damping due to windage tends to control the amplitude. Obviously, damping is not large for this condition and finer balance is required. If damping is considered to be of viscous nature, shaft amplitude may be defined as:

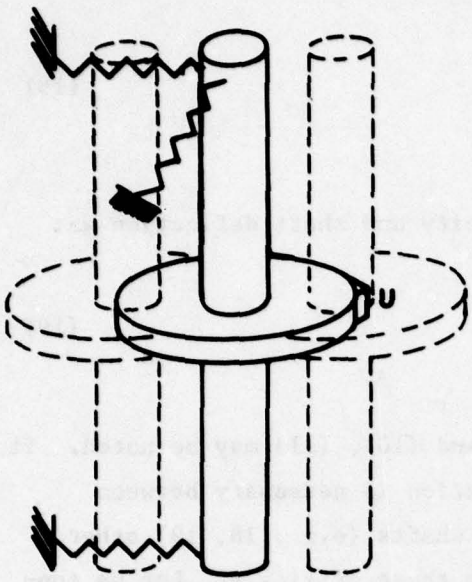
$$r = \frac{m\omega^2 e}{\sqrt{(K-m\omega^2)^2 + (C\omega)^2}} \tag{18}$$

and the phase angle between the mass eccentricity and shaft deflection as:

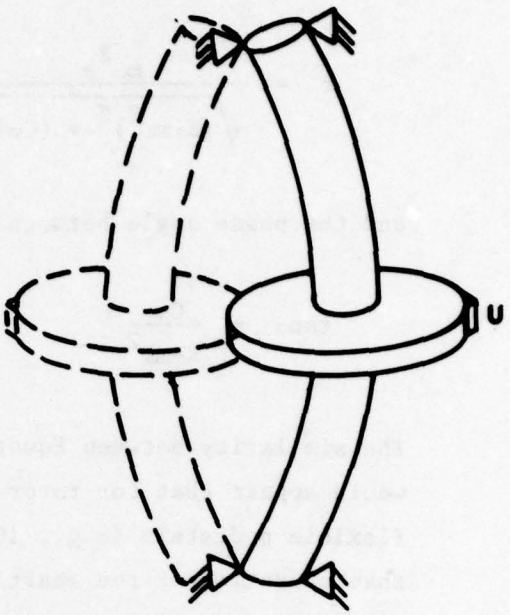
$$\tan\alpha = \frac{C\omega}{K-m\omega^2} \tag{19}$$

The similarity between Equations (18), (19), and (10), (11) may be noted. It would appear that for rotor balance no distinction is necessary between flexible pedestals (e.g., 10, 11) or flexible shafts (e.g., 18, 19) other than operation of the shaft below or away from these criticals. Let us consider the case of operation through a critical speed. Figure 9 illustrates two cases under discussion. A single plane unbalance (U) is illustrated at the disc. For the rigid body critical (9a) it is possible to correct unbalance at two planes ($U_c/2$) (9c) or at the single center plane. For the flexible body, the unbalance condition would become worse if two outboard planes as shown in Figure 9c were used for correction. The resulting moment would induce greater deflections of the shaft. It becomes obvious that recognition of shaft flexibility for selecting balance planes is extremely important.

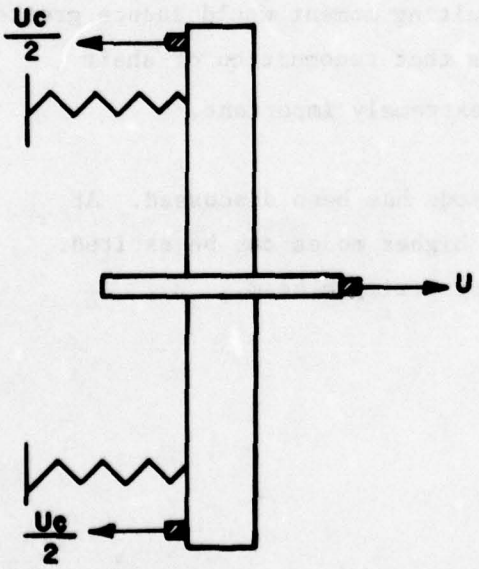
For flexible shafts only the first flexible mode has been discussed. At higher frequencies of unbalance excitations, higher modes can be excited. Figure 10 illustrates the first four modes for a simple beam.



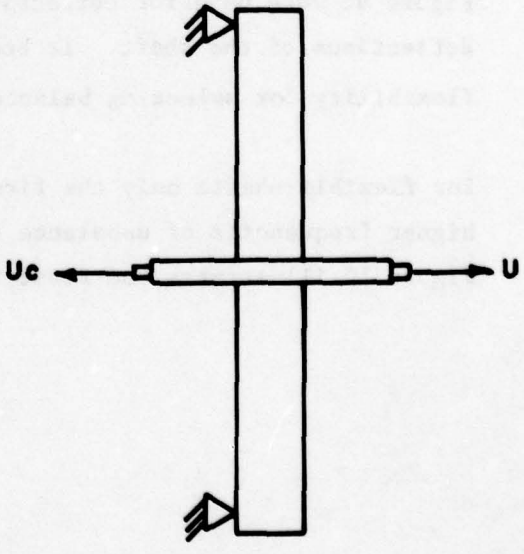
a) RIGID BODY CRITICAL



b) FLEXIBLE BODY CRITICAL



c) TWO PLANE CORRECTION OF RIGID BODY UNBALANCE



d) CORRECTION OF FLEXIBLE BODY UNBALANCE

Figure 9 Rigid and Flexible Balancing Differences

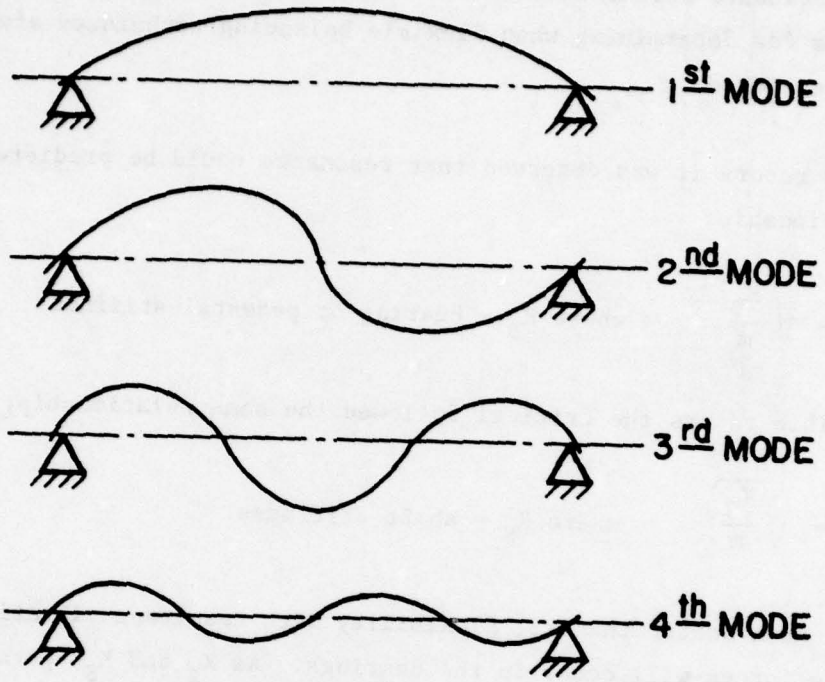


Figure 10 Critical Speed Modes of a Flexible Rotor

If operation is anticipated above the first two modes and in proximity of the third mode, rigid body two-plane balancing of a rotor can often lead to destructive moments in the shaft for one or more of these higher flexible modes. Balancing of these rotors are discussed in a later section. It is only necessary in this discussion for the reader to recognize the need for different techniques to balance flexible bodies as compared to rigid body rotors.

3.4 Classification of Rotor Flexibility

Sections 3.1 and 3.2 discussed rigid rotors with flexible supports, and Section 3.3 covered flexible rotors with rigid supports. Obviously, a

range exists where flexibility of both supports and rotor can be encountered. In this section we shall develop the critical speed map to identify the mode of shaft operation. Through the use of such a map and the proposed International Standard for Mechanical Balancing of Flexible Rotors (44), some guidelines for determining when flexible balancing techniques are required will be developed.

For rigid rotors it was observed that resonance could be predicted utilizing the relationship:

$$\omega = \sqrt{\frac{K_B}{m}} \qquad \text{where } K_B = \text{bearing or pedestal stiffness}$$

For flexible rotors the critical followed the same relationship; i.e.:

$$\omega = \sqrt{\frac{K_S}{m}} \qquad \text{where } K_S = \text{shaft stiffness}$$

If K_B is much softer than K_S, flexibility with resultant vibration due to unbalance forces will occur in the bearings. As K_B and K_S approach each other in magnitude, they act as series springs in the system and the critical speed is defined as:

$$\omega_n = \sqrt{\frac{K_B K_S}{m(K_B+K_S)}} \; ; \qquad K_{total} = \frac{K_B K_S}{K_B+K_S}$$

Utilizing a simple rotor as an illustrative example, the critical speed of a rotor may be plotted as a function of bearing stiffness as illustrated in Figure 11. The first four critical speeds are plotted in this figure. In the lower left section of the figure, operation is considered rigid body for the first two criticals. Flexible body operation occurs for operation near or above the first two criticals to the extreme right. When the critical speeds are dictated by bearing flexibility only (first two criticals at lower left), the log-log plot is a straight line with a slope of one-half. When the first two criticals are due to shaft flexibility, the plot is a

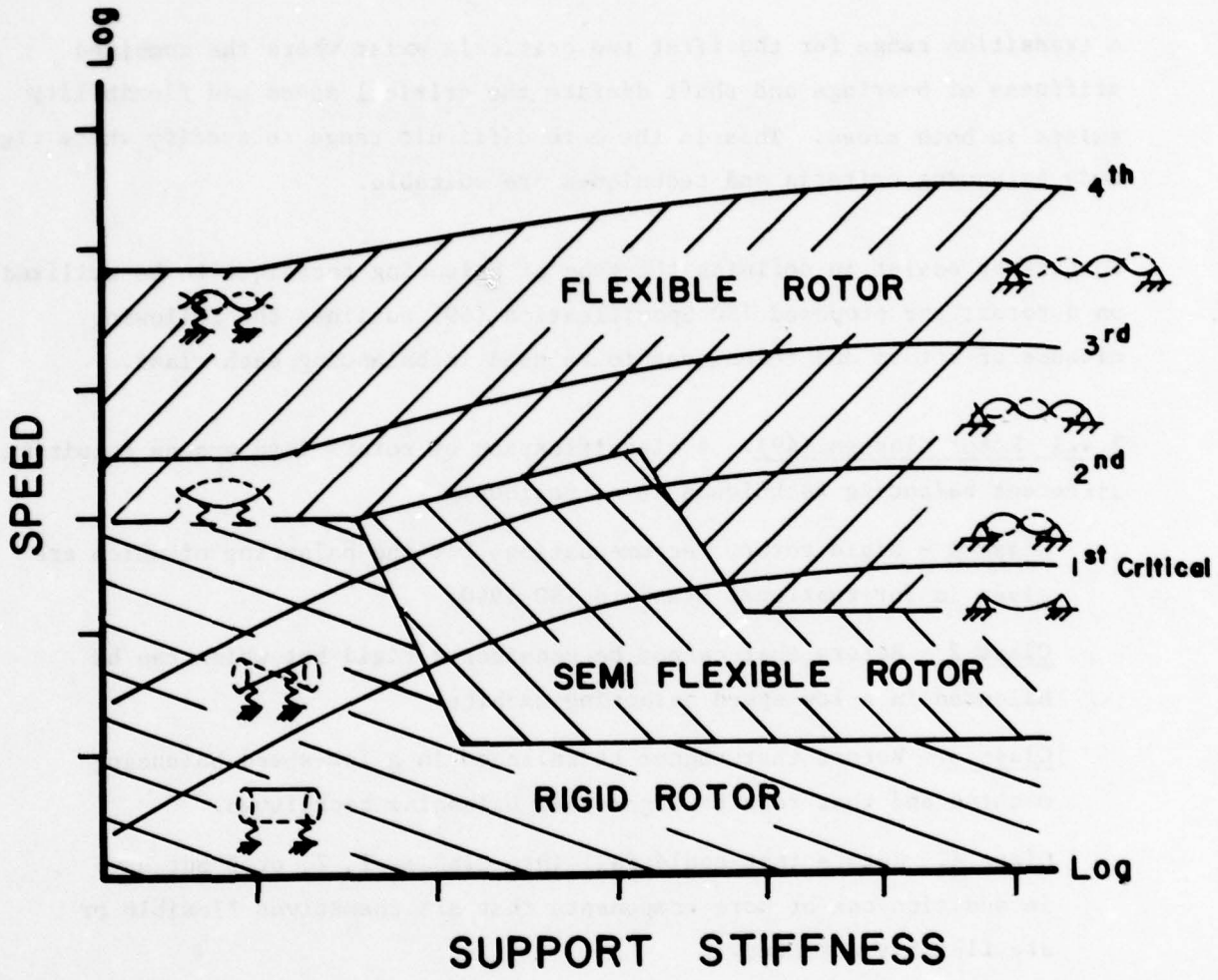


Figure 11 Critical Speed Map

horizontal line since the bearings are sufficiently stiff that only the shaft dictates the critical speeds. At high-speed operation above the first two criticals, shaft flexibility always exists.

A transition range for the first two criticals exist where the combined stiffness of bearings and shaft dictate the critical speed and flexibility exists in both areas. This is the more difficult range to specify where rigid body balancing criteria and techniques are suitable.

To further assist in defining the type of balancing technique to be utilized on a rotor, the proposed ISO Specification (49) outlines the following classes of rotors and techniques to be used in balancing each class.

3.4.1 Rotor Classes (49). A classification of rotors into groups requiring different balancing techniques is as follows:

Class 1 - Rigid rotor, recommendations for the balancing of which are given in International Standard ISO 1940.

Class 2 - Rotors that cannot be considered rigid but which can be balanced in a low-speed balancing machine.

Class 3. Rotors that cannot be balanced in a low-speed balancing machine and that require high-speed balancing techniques.

Class 4. Rotors that could fall into Classes 1, 2, or 3 but have in addition one or more components that are themselves flexible or are flexibly attached.

Class 5. Rotors that could fall into Class 3 but for some reason--e.g., economy--are balanced only for one speed of operation.

3.4.2 Subdivision of Class 2 Rotors (49). Rotors where the axial distribution of unbalance is known:

Class 2a - Rotors with a single transverse plane of unbalance; e.g., a single mass on a light flexible shaft whose unbalance can be neglected.

Class 2b. Rotors with two transverse planes of unbalance; e.g., two masses on a light shaft whose unbalance can be neglected.

Class 2c. Rotors with more than two transverse planes of unbalance.

Class 2d. Rotors with uniformly distributed or linearly distributed unbalance.

Class 2e. Rotors consisting of a rigid mass of significant axial length supported by light flexible shafts, whose unbalance can be neglected.

Rotors where the axial distribution of unbalance is not known:

Class 2f. Symmetrical rotors with two end correction planes whose maximum speed does not significantly approach second critical speed, whose service speed range does not contain first critical, and with controlled initial unbalance.

Class 2g. Symmetrical rotors with two end correction planes and a central correction plane whose maximum speed does not significantly approach second critical speed and with controlled initial unbalance.

Class 2h. Unsymmetrical rotors with controlled initial unbalance treated in a similar manner to class 2f.

This classification is also shown in Table 3. It should be recognized that the subdivision of class 2 rotors is intended to serve as an illustration showing the many reasons why rotors can often be balanced satisfactorily at low speed as rigid rotors even though they are flexible. Some rotors will fit into more than one category of the subdivision.

3.4.3 Balancing Procedures for Class 2 Rotors (49)

3.4.3.1 Class 2a - Rotors with a Single Transverse Plane of Unbalance. If the initial unbalance is known to be wholly contained in the transverse plane and the balancing is also carried out in this plane, then the rotor will be balanced for all speeds.

TABLE 3

CLASSIFICATION OF ROTORS

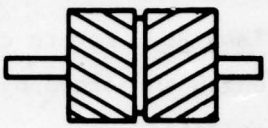
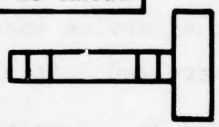
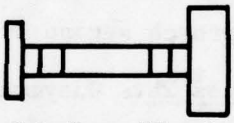
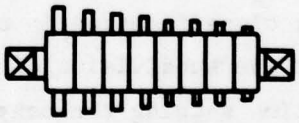
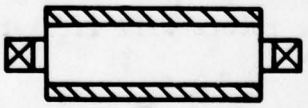
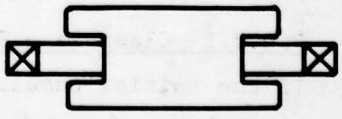
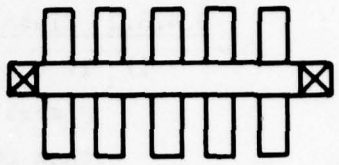
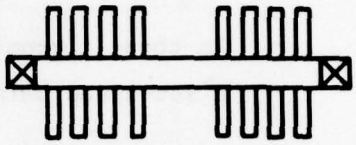
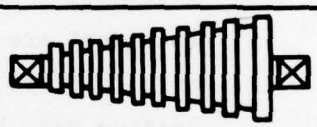
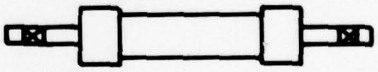
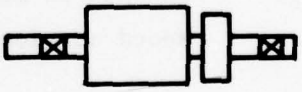
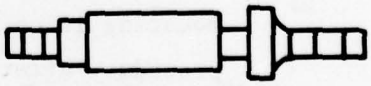
CLASS OF ROTOR	DESCRIPTION	EXAMPLE
CLASS 1 ROTOR RIGID ROTORS	A rotor is considered rigid when its unbalance can be corrected in any two (arbitrarily selected) planes and, after that correction, its unbalance does not significantly change at any speed up to maximum service speed.	 Gear Wheel
CLASS 2 ROTORS QUASI-FLEX ROTORS	A rotor that cannot be considered rigid but that can be balanced in a low speed balancing machine.	
Rotors Where the Axial Distribution of Unbalance is Known		
CLASS 2A	A rotor with a single transverse plane of unbalance, e.g., single mass on a light shaft whose unbalance can be neglected.	 Grinding Wheel
CLASS 2B	A rotor with two axial planes of unbalance, e.g., two masses on a light shaft whose unbalance can be neglected.	 Grinding Wheel With Pulley
CLASS 2C	A rotor with more than two transverse planes of unbalance.	 Compressor Rotor
CLASS 2D	A rotor with uniformly distributed unbalance.	 Printing Press Roller
CLASS 2E	A rotor consisting of a rigid mass of significant axial length supported by a flexible shaft whose unbalance can be neglected.	 Computer Memory Drum

TABLE 3 (Continued)

	Rotors Where Axial Distribution of Unbalance is Not Known	
CLASS 2F	A symmetrical rotor, with two end correction planes, whose maximum speed does not significantly approach second critical speed, whose service speed range does not contain first critical speed and with controlled initial unbalance.	 Centrifugal Pump
CLASS 2G	A symmetrical rotor with two end correction planes and a central correction plane whose maximum speed does not significantly approach second critical speed; and with a controlled initial unbalance.	 High Speed Centrifugal Pump
CLASS 2H	An unsymmetrical rotor with controlled initial unbalance treated in a similar matter to Class 2F rotor.	 T.P. Atear Turbine Rotor
CLASS 3 ROTORS FLEXIBLE ROTORS	A rotor that cannot be balanced in a low speed balancing machine and that requires high speed balancing techniques.	 Cenerator Rotor
CLASS 4 ROTORS SPECIAL FLEXIBLE ROTORS	A rotor that could fall into Classes 1, 2 or 3, but has in addition one or more components that are themselves flexible or are flexibly attached.	 Rotor With Centrifugal Switch
CLASS 5 ROTORS SINGLE SPEED FLEXIBLE ROTORS	A rotor that could fall into Class 3 but for some reason, e.g. economy is balanced only for a single service speed.	 Synchronous Motor

In these circumstances, the unbalance can be corrected in a low-speed balancing machine as effectively as at service speed, assuming adequate sensitivity of the balancing machine.

3.4.3.2 Class 2b - Rotors with Two Transverse Planes of Unbalance.

1) If the unbalance is known to be contained in two transverse planes and the balancing is also carried out in these planes, then the shaft will be balanced for all speeds.
2) Can be balanced as 2c.
3) Can be balanced as 2f.

In these circumstances the unbalance can be measured and corrected in a low-speed balancing machine as effectively as at service speed.

3.4.3.3 Class 2c - Rotors with More Than Two Transverse Planes of Unbalance. When a rotor is composed of more than two complete components that are distributed axially, it is likely that there will be more than two transverse planes of unbalance. A satisfactory state of balance may, however, be achieved by balancing in a low-speed balancing machine provided that rigid rotor operation is intended.

It is important to recognize that the assembly process may produce changes in the shaft runout that may subsequently change during high-speed service.

In some cases where a gross unbalance may occur in a single component it may be considered advantageous to balance this separately before mounting it on the rotor in addition to carrying out the balancing procedure after it is mounted.

3.4.3.4 Class 2d - Rotors with Uniformly Distributed or Linearly Distributed Unbalance. If by reason of design or method of construction a rotor has unbalances that are distributed uniformly along the entire length of the rotor (e.g., tubes), it may be possible by a suitable

axial disposition of two balancing planes to achieve satisfactory running over the entire speed range by balancing in a low-speed balancing machine.

It is likely that the optimum disposition of the two balancing planes to give the best overall running conditions can only be determined by experimentation on a number of rotors of similar type. The method is based on the probability that rotors of such design will have a similar axial distribution of unbalance.

It is shown below that for a simple rotor system (comprising a single span rotor with no overhangs, uniform mass, and flexibility and operating speeds significantly below second critical speed) the optimum position for the two balancing planes is 22 per cent of the bearing span inboard of each bearing.

If rotors of this type (i.e., with uniform or linear distribution of unbalance) have balancing planes in the middle and at their ends, it is possible to balance the rotors satisfactorily by three-plane balancing which may be carried out in a low-speed balancing machine.

To do this it is necessary to assess what proportion of the total initial unbalance is to be corrected at the center plane. A method of making this assessment is given in Appendix E of Reference [49] for rotors which satisfy the following conditions:

1) Single span rotor with end bearings
2) Uniform mass distribution with no overhangs
3) Uniform bending flexibility of the shaft along its length
4) Continuous service speeds below and not significantly approaching second critical speed.

For such rotors Appendix E shows how the unbalance correction required at the center plane can be found directly from the initial dynamic unbalance measured at the two end correction planes.

3.4.3.5 Class 2e - Rotors with a Rigid Mass. If the unbalance in the rotor is known to be contained wholly within a substantially rigid section of the rotor and the balancing is carried out also within this section, then the unbalance will be more in all modes.

Such a rotor which has a rigid core and flexibility derived solely from flexible shafts can be balanced on a low-speed balancing machine provided the unbalance corrections are carried out at correction planes located within the rigid core section of the rotor.

3.4.3.6 Class 2f - Symmetrical Rotors with Controlled Initial Unbalance (Two Correction Planes). When a rotor is composed of separate components which are individually balanced before assembly as outlined in Section 5, a satisfactory state of balance may be achieved in a low-speed balancing machine provided the initial unbalance of the completed rotor does not exceed specified tolerances.

For such rotors the axial distribution and magnitude of the unbalance of the complete assembly will not be known. Since the maximum speed of this class of rotor does not significantly approach the second critical speed, the most unfavorable case that will occur with a given distribution of unbalance is when the individual contributions of the assembled components to the resultant unbalance lie in the same radial plane. It is then possible to estimate the maximum initial unbalance of the assembly that may be corrected in two correction planes that will result in satisfactory running conditions.

This type of analysis can therefore be carried out for any rotor whose initial unbalance consists of a distribution of small unbalances. The analysis does, however, require realistic data on shaft and bearing flexibilities.

3.4.3.7 Class 2g - Symmetrical Rotors with Controlled Initial Unbalance (Three Correction Planes). For rotors that conform to the requirements in Section 3.4.3.6 but that have in addition a third central correction

plane, then, provided that it is possible to hold the initial unbalance of the complete rotor to within twice the permissible initial unbalance arrived at in Section 3.4.3.6, the rotor may be balanced on a low-speed balancing machine as a rigid rotor--correcting a portion of the initial unbalance at the central plane and the remainder at the two end planes.

NOTE: Experience has shown that between 30 percent and 60 percent of the initial static unbalance should be placed in the central plane.

3.4.3.8 Class 2h - Unsymmetrical Rotors with Controlled Initial Unbalance. For rotors that do not conform to the configuration defined in Section 3.4.3.6; e.g., as regards symmetry or overhangs, it may be possible to carry out a similar estimation to that given in Appendix C [49] and hence to arrive at the maximum permissible unbalance that may be corrected satisfactorily at any given correction planes.

However, in extreme cases, the permissible initial unbalance arrived at in this way may be too small to make this method of balancing practicable and in these cases some other method of balancing the rotor will have to be adopted.

3.4.4 Balancing Procedures for Class 3 Rotors. Various procedures can be applied to rotors in this class, and three of these are outlined for guidance in Appendix A [49]. It should be emphasized that only the principles of the techniques are described in the Appendix and that they may require some refinement in practice.

Most of the possible methods of balancing class 3 rotors depend on the fact that the deflection of the rotor is the sum of the deflection components in its principal modes and that the distribution of the local center of gravity displacement can be similarly expressed in terms of modal components. The vibration in each mode is caused by the corresponding modal component of unbalance. Moreover, the vibration in the vicinity of a critical speed is predominately in the associated principal mode. If the component of

unbalance in a particular mode is reduced by a number of discrete balance masses, then the corresponding modal component of vibration is similarly reduced. If the modal components of unbalance are corrected in the ways described in Appendix A, then it is possible to achieve smooth running of the rotor up to any desired service speed.

For balancing purposes, the rotor can be mounted on any suitable bearings. If, however, the bearings are chosen so that the support conditions simulate those on site, then it is possible that the degree of subsequent on-site balancing required will be significantly reduced.

Transducers are positioned to measure bearing vibration or bearing force or rotor vibration, and they should be capable of measuring the amplitude of the once-per-revolution component of the vibration together with the phase angle relative to some fixed but arbitrary angular reference on the rotor. Alternatively, it is possible to use a measuring system which resolves the synchronous vibration into X and Y components.

It is desirable that the transducers should be of such a type that they do not undergo resonant vibration of themselves or their suspension at all test speeds.

If the rotor to be balanced has an overhang of appreciable mass that would normally be supported when installed on site, it may be desirable to provide an additional bearing to support this during the balancing operation.

3.4.5 Balancing Procedures for Class 4 Rotors. Rotors in this class may have a basic shaft and body construction that would fall into classes 1, 2, or 3. In addition, they will have one or more components that are either flexible or are flexibly mounted as that the balance of the whole system may change with change of speed.

Rotors in this class may fall into either of two categories:

1) Rotors whose balance changes continuously with change of speed; e.g., rubber bladed fans.

2) Rotors whose balance changes up to a certain speed and remains constant above that speed; e.g., rotors of single phase induction motors that carry a centrifugal starting switch.

It is sometimes possible to balance these rotors with counterbalances of similar characteristics. If not, the following procedures should be used.

Rotors that fall into category 1) above should be balanced in a balancing machine at the specified balance speed.

Rotors that fall into category 2) above may be balanced at any speed above that at which the balance ceases to change.

NOTE: It may be possible by careful design and location of the flexible components to minimize or counterbalance their effects, but it should be appreciated that rotors in this class are likely to be in balance at one speed only or within a limited range of speed.

3.4.6 Balancing Procedures for Class 5 Rotors. Some rotors that are flexible and pass through or approach one or more critical speed on their way up to full speed may sometimes be required to be in balance for one speed only (usually service speed). In general, rotors that fall into this class fulfill one or more of the following conditions:

1) The acceleration and deceleration up to and from full speed is so rapid that the amplitude of vibration at the critical speeds will not build up beyond acceptable limits.
2) The damping of the system is sufficiently high to suppress vibrations at the critical speeds to acceptable limits.
3) The rotor is supported in a sufficiently elastic environment to prevent serious vibrations being transmitted.
4) A high level of vibration at the critical speeds is acceptable.
5) A rotor runs at full speed for such long periods that otherwise unacceptable starting conditions can be accepted in this case.

A rotor that fulfills any of the conditions above should be balanced in a high-speed balancing machine or equivalent facility at the speed at which it is determined that the rotor should be in balance.

If the rotor falls into category 3) above, it is especially important that the stiffness of the balancing machine bearings should imitate site conditions sufficiently closely to ensure that at the balancing speed the predominant modes are the same as those which will be experimental on site.

Some consideration should be given to the axial balance weight distribution. Based on the probability that similar rotors will have similar unbalance distributions, it may be possible to choose optimum axial positions for the balancing planes and two planes may be sufficient. This may produce a minimum residual unbalance in the lower modes and thus minimize the vibrations when running through critical speeds.

SECTION IV

BALANCING MACHINES (RIGID BODY BALANCING)

The basic process of balancing is one in which the mass in a rotating body is altered to eliminate vibrations at the rotor support bearings. It is the function of balancing machines to indicate the amount and angular location of the correction to be applied. Measurements of unbalance can either be static (force type) or dynamic (moment type). In dynamic balancing machines either force or moment unbalance or combination of force and moment can be measured. Corrections to only one plane of a rotating body can only correct for force unbalance (static balance) while two planes are required for correcting moment unbalance.

4.1 Static Balancing Machine Principles

The simplest form of static balancing is accomplished by placing a component on level, parallel knife edges or rollers as shown on Figure 12.

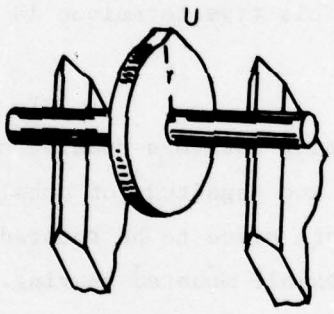


Figure 12 Static Balancing

The unbalance weight (U) causes the piece to rotate due to the gravity induced moment (U_r) until the heavy weight is at the bottom. Correction is made by adding weight on the top of the work piece and rotating 90 degrees until there is no tendency to roll.

Static balance is often applied to large thin discs where moment unbalance is necessarily small because of the small moment arms that can be developed in thin discs. The common method of balancing is to mount the work piece on

an adapter which contains a ball or pivot that sits on a flat level surface as illustrated in Figure 13. The heavy part of an unbalanced work piece

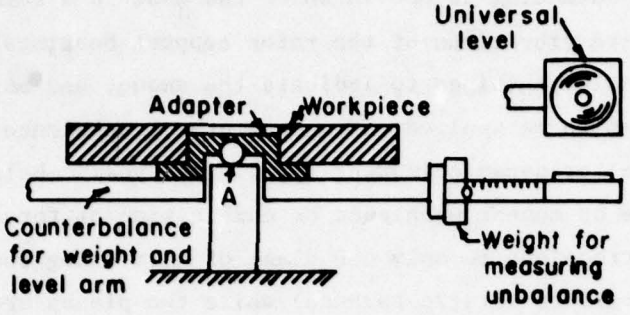


Figure 13 Ball Point Type Static Balancer

will cause the assembly to tilt downward as detected on a universal level. The unbalance may be measured by rotation of the work piece until the heavy side is opposite the counter weight. The counter weight is slid outward until the assembly is level. This type technique is often used for static automotive wheel balance.

Rotating types of static balancing machines usually utilize similar means of measuring the angular location and magnitude of unbalance as dynamic balancing machines. Commonly, the work piece to be rotated is supported by a rigid mounted bearing and a flexible mounted bearing. The rigid bearing acts as a fulcrum and the amplitude of vibration of the flexible supported bearing indicates the magnitude of unbalance.

4.2 Dynamic Balancing Machine Principles

All dynamic balancing machines necessitate rotation of the work piece and permit selection of two balance planes for force and moment correction. As discussed in Section 3.1, the unbalance forces can excite resonances of the rotor-bearing system which can influence both the observed magnitude and amplitude of vibration. This is an important consideration in balancing machines since the work piece and the balance machine supports represent a

mass-spring system that is subject to resonance. Most measurement systems either detect shaft motions or bearing vibrations; such resonances can significantly influence readings from a balance machine.

Since dynamic balancing machines permit selection of two planes along the work piece axis for balance correction, corrective weights applied to one plane can influence amplitudes at the other plane. A means is required in the machine to separate the effective unbalance read in one plane of correction from the influence of the effective unbalance in the other plane of correction. This is generally termed "plane separation," and is a basic requirement in dynamic balancing machines.

In order to provide exact and understandable information regarding the amount and angular location of unbalance, it is necessary for a balancing machine to provide accurate measurements. When vibration in terms of displacement, velocity, or acceleration is utilized as the sensing parameter for measuring unbalance, it is important that the information detected is due to unbalance and not unwanted frequencies. The balance machine, therefore, must consider a means of separating vibrational information induced from unbalance from vibrational information induced from other sources such as the drive motor and operating machines in close proximity to the balance machine.

From the foregoing discussion, three areas have been identified as basic principles requiring consideration in balance machines; i.e.:

a) Resonance of the work piece - balance machine system
b) Plane separation
c) Sensor signal conditioning

Each of these areas are discussed in the following sections.

4.2.1 Machine Classification by Operating Proximity to a Resonance. Rigid body critical speeds were identified in Section 3.1 as resonances of the rotor (as a nonflexible member) on its support bearings. Figure 6 depicts a rotor supported on flexible supports as would be encountered in a balance

machine. For motion parallel to the shaft axis, the system can be reduced to the simple single-degree-of-freedom system of Figure 5 where stiffness and damping properties are defined as the combined stiffness and damping of both bearings. Equation (8) defined the equations of motion and their solution was given in Equations (9), (10), and (11). The solution may be nondimensionalized and plotted to provide a visual indication of relevance of resonance to the balance machine. Defining:

a) $r = \omega/\omega_n$ (Frequency ratio:running frequency/natural frequency)

b) $\mu = xK/F$ (Force ratio:output force/input force)

c) $\zeta = C/2\sqrt{Km}$ (Critical damping ratio)

The results are plotted on Figure 14. The upper plot illustrates the vibrational force amplification for operation up to and through the first system resonance for different values of damping. The lower plot illustrates the phase angle between the unbalance forcing function and the system response function. It will be noted that if balancing was conducted at resonance ($r = 1$), considerable amplification of reaction to unbalance at the balance machine pickups would occur if damping was minimized. This would improve sensitivity to the measurements to permit a high degree of balance accuracy. Resonant type balance machines are available that take advantage of this fact. It will be observed from the lower plot of Figure 14, at resonance, the phase angle is 90 degrees. The signal at the pickup will lag the unbalance by 90 degrees. As damping is minimized, the change in phase angle is very sensitive to speed. Therefore, accurate determination of unbalance location is often difficult in a resonant balance machine.

If operation is conducted well below resonance and damping is minimized, phase angle measurement accuracy is enhanced. There is no phase shift at this condition, therefore, direct measurement of amplitude and phase are possible. Balance machines designed for operation in this range are classified hard bearing balance machines. They are limited in operating speed in order to avoid phase angle errors as resonance is approached.

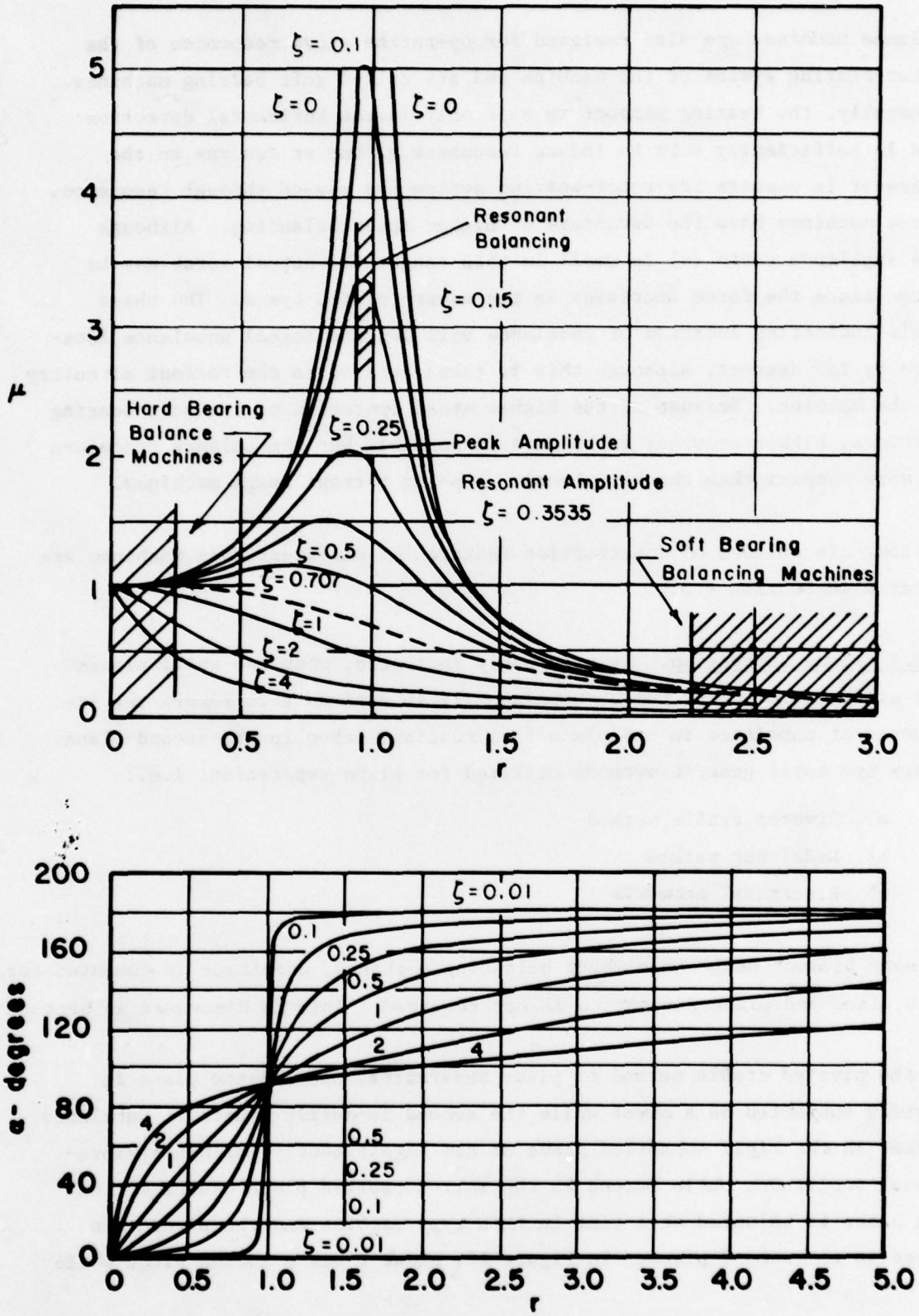


Figure 14 Balance Machine Classification by Proximity to Resonance

Balance machines are also designed for operation above resonance of the rotor-bearing system of the machine and are called soft bearing machines. Generally, the bearing support is soft only in the horizontal direction and is sufficiently soft to induce resonance at one or two rpm so the operator is unaware the rotor-bearing system has passed through resonance. These machines have the advantage of higher speed balancing. Although the amplitude ratio (u) is small in this range, the actual force may be large since the force increases as the square of the speed. The phase angle indicating location of unbalance will lag the actual unbalance location by 180 degrees, although this is taken care of in the readout circuitry of the machine. Because of the higher speed operation of the soft bearing machines, higher accuracy in balance is possible but the balance procedure is more complex than the hard bearing type in current usage machines.

Further discussions on construction features of different type machines are covered in Section 4.3.

4.2.2 Plane Separation. As previously indicated, when two axial planes are utilized to remove moment unbalance it is desirable to remove the influence of unbalance in one plane from readings taken in the second plane. There are three general methods utilized for plane separation; i.e.:

a) Pivoted cradle method
b) Nodal bar method
c) Electrical networks

On some present usage hard mount balancing machines, unbalance is computed for each plane and plane separation is not required. This is discussed in Section 4.3.

In the pivoted cradle method of plane separation, one balance plane is rigidly supported on a pivot while the second is softly mounted. Unbalance forces in the rigid supported plane do not significantly influence vibrational amplitudes while forces in the soft supported plane causes motion. One plane is balanced at a time in this type machine and the pivot then moved to the second plane. In Figure 15, plane B has a locked pivot while

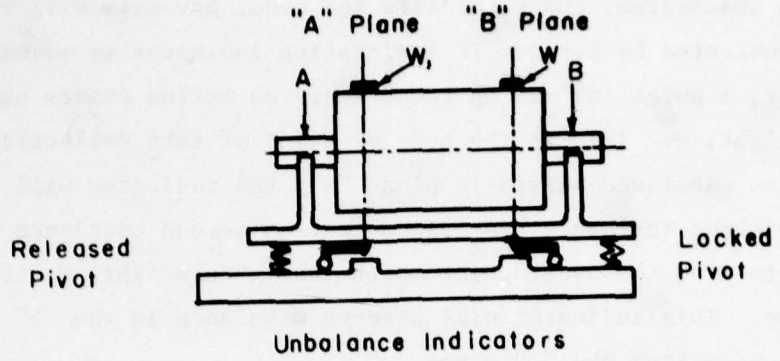


Figure 15 Pivoted Cradle Method of Plane Separation

plane A is released and may vibrate due to unbalance weight, W_1. Reversing the procedure and locking plane A permits measurements in plane B.

This type plane separation has been utilized in resonant type balancing machines to enhance sensitivity.

The nodal bar method is illustrated in Figure 16. When an unbalance weight, W, is placed in the "R" plane, the rotor will vibrate in a conical mode. If

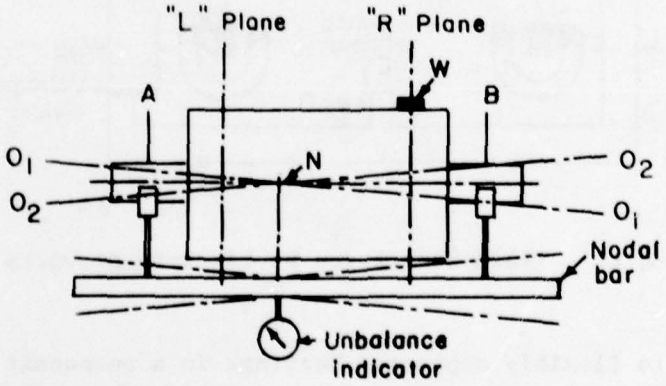


Figure 16 Nodal Bar Method of Plane Separation

the nodal bar is attached rigidly to the bearing support, it will also vibrate in the same mode as the rotor. If only one degree of freedom is

permitted in the system, the rotor axis and nodal bar axis will vibrate in the mode illustrated by O_1-O_2. If a vibration indicator is moved along the nodal bar, a point (N) can be found where no motion exists due to the unbalance weight, W. This is the node or point of zero deflection of the system. If an unbalance exists in plane "L", the indicator will detect the motion without influence from plane "R." A second unbalance indicator can be used to find the nodal point on the bar when weight, W, is moved to the "L" plane. This indicator will observe unbalance in the "R" plane without influence from the "L" plane.

The most common plane separation approach used in current-day balancing machines is by electrical nulling networks. If an unbalance weight is added to one plane, its influence in the second plane can be nulled electrically. One such scheme utilizes a voltage divider as illustrated in Figure 17. In this illustration, unbalance motion is detected by moving

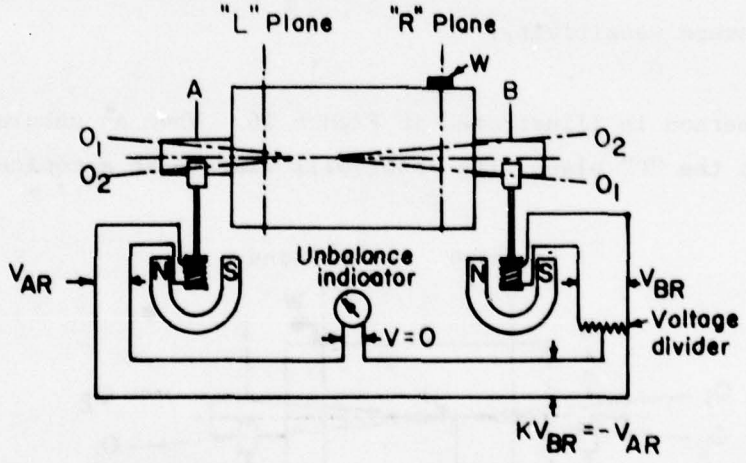


Figure 17 Plane Separation by Electric Networks

coils attached to flexibly supported bearings in a permanent field. The motions at B from an unbalance weight in plane "R" are greater than at A. The resulting voltage V_{BR} is impressed across the voltage divider which is adjusted to the voltage level of pickup A (V_{AR}) and used to null the voltage to zero. A similar network is used at the other end to separate that plane.

4.2.3 Sensor Signal Conditioning. In order to enhance balance machine sensitivity and accuracy, the signal from the unbalance measuring pickup generally requires conditioning. Velocity type sensors are in common usage although displacement and accelerometer sensors have been used.

Despite the type of sensor used, signals usually require amplification. The amplifier is most often a function of the type pickup used. If mechanical devices are used, such things as dial indicators may be applied as the amplifier. For electrically generated signals voltage amplifiers are common. In optical systems, optical amplifiers use beams of light as levers.

In addition to amplifying signals, in high sensitivity vibration pickups with electrical outputs it is desirable to filter out unwanted frequencies. Since the pickup is anchored to some portion of the balancing machine, vibratory signals other than those due to unbalance may also be detected. Two techniques are in common usage to eliminate unwanted signals from the vibration pickup; i.e.: band pass filtering and watt hour meter filtering. Band pass filters can be further classified as hand tuneable or automatic tracking.

The band pass filter is basically a narrow window that passes only frequencies in the range of the window opening and rejects all other frequencies. The width of the window opening is called the band width and is measured at the 3 db attenuation point. Figure 18 is the filter shape of a standard octave filter which is one full octave in bandwidth. The center frequency of the filter is at f_o. In this filter frequencies of 0.5 f_o are attenuated approximately 25 db or 17.8 to 1 reduction in amplitude. If the band pass filter is hand tuneable, the center frequency f_o may be shifted with a tuning knob. In balancing machines, the tuneable filter is set at running speed to pass vibrations at running speed and exclude others. If the center frequency is not adjusted properly on running speed, not only are the unbalance amplitude signals attenuated, but the phase is shifted due to the filter.

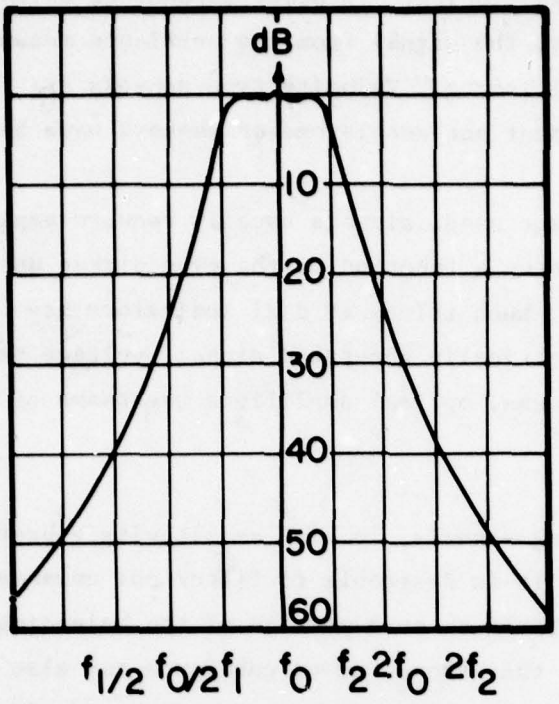


Figure 18 Filter Characteristic of Octave Filter

It is possible to control the filter center frequency automatically by driving the filter with a speed signal tachometer. These filters are termed tracking filters. Because a phase shift can be introduced if the center frequency of the filter differs from running frequency, two channel tracking filters with matched phase characteristics are often utilized. With this arrangement, both filters are driven by the running frequency and one filter utilized with the unbalance pickup and the other utilized to filter the once-per-revolution signal. Any phase shift introduced by the filter will be the same for the once-per-revolution reference and the unbalance signal. Both signals can then be fed to a phase meter for more accurate phase measurement.

The moving coil watt hour meter technique utilizes the principle that it measures only when the frequency of the voltage input signal and current input signal are the same. The unbalance sensor supplies a voltage to the meter whose amplitude changes with unbalance amplitude level. A speed

signal of constant current and once per revolution repetition rate of the rotor is also applied to the meter. Only voltages and currents having the same frequency combine in the meter which records their product ($W = EI\cos\theta$) where θ is the phase angle. Figure 19 illustrates the average and instantaneous torque developed in the watt meter for various phase angles between

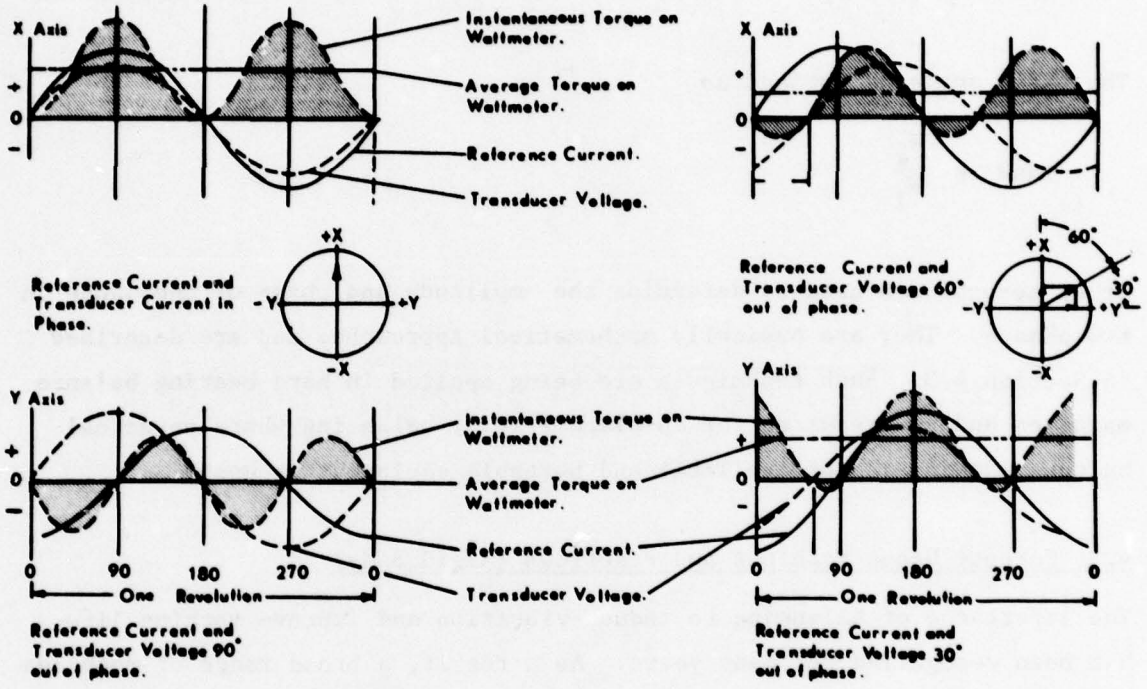


Figure 19 Wattmeter Measurement of Power from Product of Current and Voltage

vibration and the reference rpm current signal. When the phase angle between unbalance and reference is 90 degrees, the average torque in the meter is 0. If a second coil is used, utilizing the same unbalance signal but a reference current signal 90 degrees phase spaced from the first signal, the unbalance amplitude and phase can be determined while taking advantage of the filtering characteristics of the meter.

With a vertical and horizontal reference current speed pickup, watt meter average output is:

$$W_1 = EI\cos\theta$$

$$W_2 = EI\cos(\frac{\pi}{2} - \theta) = EI\sin\theta$$

The unbalance is proportional to

$$U \approx \sqrt{W_1^2 + W_2^2} = EI$$

The phase angle is defined as

$$\tan\theta = \frac{W_2}{W_1}$$

Other methods are used to determine the amplitude and phase of unbalance in two planes. They are basically mathematical approaches and are described in Section 4.3. Such techniques are being applied in hard bearing balance machines and are required for in-place (field) balancing where an actual balance machine is not available and portable equipment is used.

4.3 Current Usage Machines and Practices (Rigid Body)

The importance of balancing to reduce vibration and improve machine life has been recognized for many years. As a result, a broad range of machines have been developed in the past--some of which are still in use. Prior to 1930, measurements were observed by mechanical movement. During the 1930 to 1950 period, electrical measurement and readout type machines were developed. With the advances in computer developments, the current trend is digitizing vibration data and computer analysis of balance condition. In balancing of flexible rotors, this approach is receiving more attention.

In addition to general usage balance machines, a number of machines have been developed for specialty balancing and high production balance and are dedicated to a specific application. In the previous section (4.2) general features of balancing machines have been reviewed. In this section, some of the general purpose machines in current use are reviewed. Machines are discussed from the viewpoint of describing the balance practices applicable

to the basic machine types; i.e.: rigid bearing machines, resonant bearing machines, and soft bearing machines.

4.3.1 Basic Considerations. Despite the type of balancing machine in use, several basic considerations apply. The first requirement of the machine is to provide a signal that is indicative of the unbalance level. Since unbalance produces a force to the pedestal at a repetition rate equal to rotational speed, this is the only signal of importance for the balance operation. In the previous section, filters were discussed as a means of eliminating undesirable frequencies. The design of the machine is also important to minimize these influences. The method of driving the part to be balanced can introduce undesirable vibration. The method of mounting the balance machine (machine foundation) can also serve to isolate the machine from external vibrations to assist in insuring accurate signals for balancing. Finally, the method of supporting the rotor in the balance machine can be of importance. Each of these areas in the machine can introduce undesirable signals to the vibration pickup sensor. If we designate unwanted vibration data as noise and unbalance vibration as the signal, the sensitivity of the balance machine is a function of the signal to noise ratio. As the noise is reduced, smaller unbalance amplitudes can be detected and corrected for. The degree of attention to be given to the noise sources is a function of the balance accuracy that is desired.

Most current usage of fine balancing machines are rated for a balance sensitivity of 50 to 100 microinches, although 25 microinches is not uncommon. This is the resulting eccentricity between the mass center and rotating center. Finer balance down to 5 microinches has been accomplished in balancing machines. The ability of a balance machine to perform its specified capability will depend on the signal to noise ratio. For fine balancing, the manufacturer will recommend that the machine be mounted on a seismic mass (concrete pedestal) which is isolated from the building structure. If the drive motor operates at the same speed as the driven part, motor unbalance vibrations can be detected by the pickup which cannot be filtered out. If the rotor is belt driven, any pulley speed that is synchronous with running

will not be filtered out. Other belt noise may also be detected. Noise from the support bearings themselves, if rollers or sliding supports are used, may influence readings.

There is no simple rule for selecting the type supports, mounting, and drive suitable for the balance machine. It is important for the rotating machinery designer and user of the balance machine to recognize the limitations of the balance machine when specifying the balance requirements and in using a balancer to meet these requirements.

The pickups used to detect unbalance in each plane along the rotor axis are generally mounted to detect motion in one direction only. In horizontal mounted machines, the sensing direction is generally horizontal. The support stiffness characteristics of the balance machine usually differ in the horizontal and vertical directions. The plane of the pickup is the plane that dictates the machine type. In resonant machines, resonance is in the pickup plane and not the normal plane. Horizontal soft bearing machines contain soft pedestal stiffness in the pickup direction.

4.3.2 Hard Bearing Machines. As previously discussed (Section 4.2.1), hard bearing balance machines operate well below the resonance of the rotor mass-pedestal system. Although the nomenclature of hard bearing is used, it is the bearing mount or pedestal stiffness that is used as a support spring. The bearing itself is stiffer than the support. Figure 20 illustrates one method of designing the desired support stiffness characteristics. Both pedestals are calibrated in the factory prior to installation.

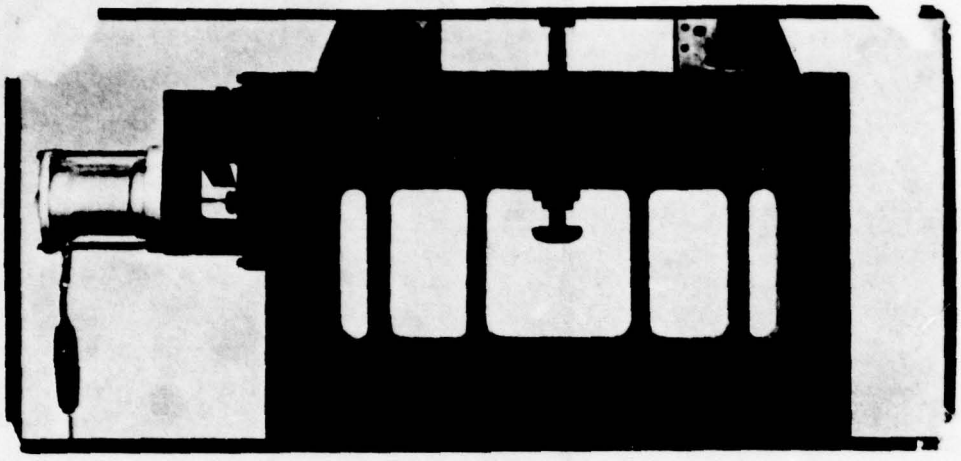
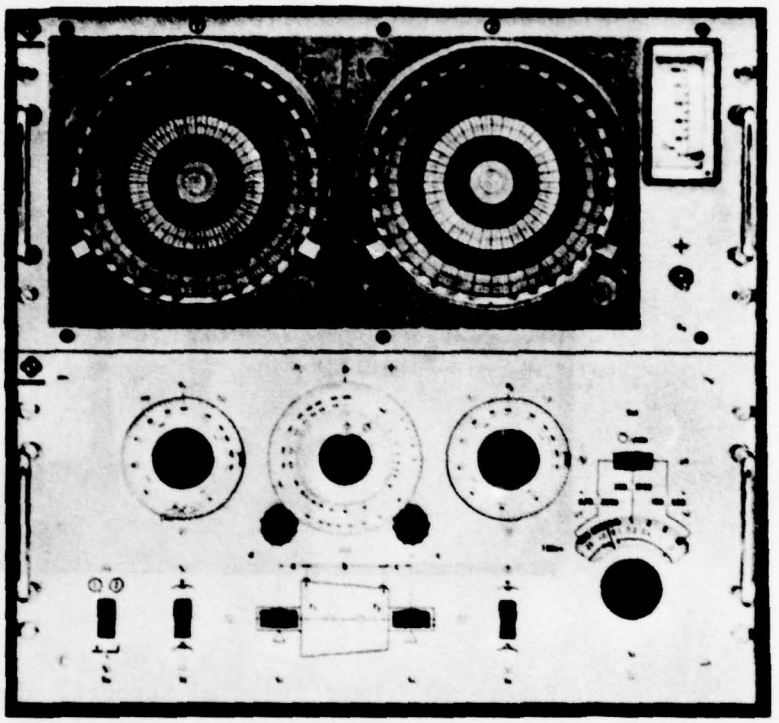
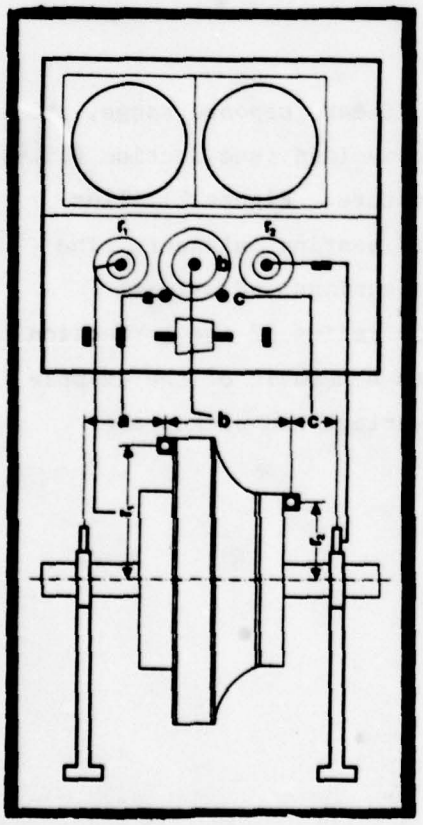


Figure 20 Hard Pedestal Support. Patented Schenck Design.

Since operation is conducted below resonance in a linear response range, the need for trial weights for plane separation can be avoided (see Section 4.3.4). The methods currently utilized are analytical in nature. Figure 21 illustrates the control console for a current usage hard bearing balancer. The setup procedure is to dial in the location of the bearings and balance planes (dimension a, b, and c in this figure). The radius of the correction plane is also dialed into the console. A plan view schematic of the example of Figure 21 is illustrated in Figure 22. The bearings and pickup are

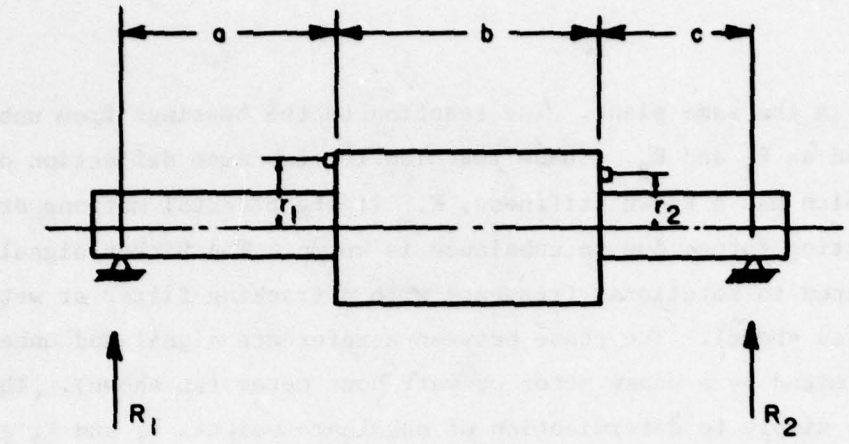


CONSOLE

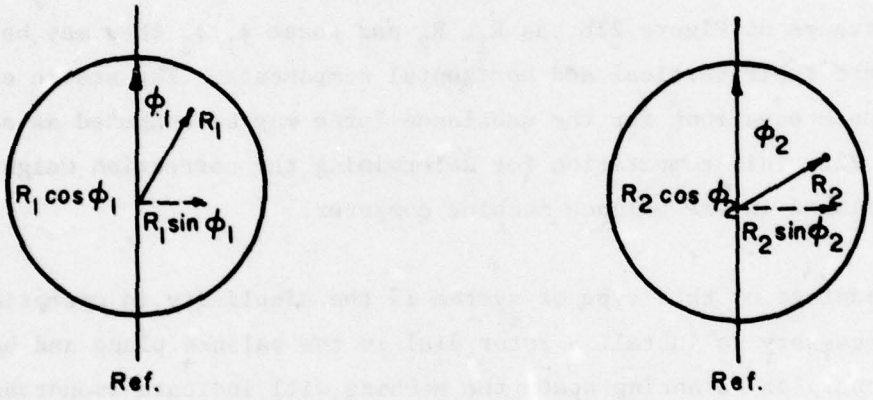


SCHEMATIC
OF
CONSOLE

Figure 21 Schenck Hard Bearing Balancing Machine Console Panel



a) Unbalance Schematic



b) Unbalance Pickup Response

Correction wights = U_1, U_2 (lbs. or Newtons)

Horizontal: $U_{1H} r_1 \frac{\omega^2}{g} + U_{2H} r_2 \frac{\omega^2}{g} = R_1 \sin \phi_1 + R_2 \sin \phi_2$

$$a\left[U_{1H} r_1 \frac{\omega^2}{g}\right] + [a + b]\left[U_{2H} r_2 \frac{\omega^2}{g}\right] - [a + b + c] R_2 \sin\phi_2 = 0$$

Vertical: $U_{1V} r_1 \frac{\omega^2}{g} + U_{2V} r_2 \frac{\omega^2}{g} = R_1 \cos\phi_1 + R_2 \cos\phi_2$

$$a\left[U_{1V} r_1 \frac{\omega^2}{g}\right] + [a + b]\left[U_{2V} r_2 \frac{\omega^2}{g}\right] - [a + b + c] R_2 \cos\phi_2 = 0$$

$$U_1 = \sqrt{U_{1H}^2 + U_{1V}^2}$$

$$U_2 = \sqrt{U_{2R}^2 + U_{2V}^2}$$

Figure 22 Calculation of Correction Weight for Hard Bearing Balancing Machine

located in the same plane. The reaction to the bearings from unbalance are indicated as R_1 and R_2. These reaction forces cause deflection of the pedestal which has a known stiffness, K. If the pedestal motions are known, the reaction forces due to unbalance is known. The pickup signal, therefore, is filtered to rotational frequency with a tracking filter or watt meter method (as shown). The phase between a reference signal and unbalance signal is determined by a phase meter or watt-hour meter (as shown). The problem resolves simply to determination of unbalance weights U_1 and U_2 given an input of a, b, c, r_1, r_2 and determination of R_1, R_2 and phase ϕ_1 and ϕ_2 by measurement. If the pickup output is assumed to occur as shown in the polar traces of Figure 22b, as R_1, R_2 and phase ϕ_1 ϕ_2 they may be broken down into their vertical and horizontal components. The static equilibrium, rigid beam equations for the unbalance force may be computed as shown on Figure 22. This computation for determining the correction weights is accomplished in the balance machine computer.

The advantage of this type of system is the simplicity in operation. It is only necessary to install a rotor dial in the balance plane and bearing locations. At balancing speed the machine will indicate amount and location of unbalance with no other preliminary adjustments.

4.3.3 Resonant Bearing Machines. Resonant machines utilize the amplification that occurs at resonance of the rotor-bearing system (with the rotor operating as a rigid body) for balancing.

For different rotor sizes and mass distributions, the resonant frequency of the system will differ. Therefore, a means of adjusting the support stiffness is required to insure resonance in the normal operating speed range of the balancer.

One type of resonant machine utilizes an overhead belt drive as shown in Figure 23. Plane separation in this machine is accomplished by locking one pedestal and balancing in the other plane. Locking one pedestal produces a

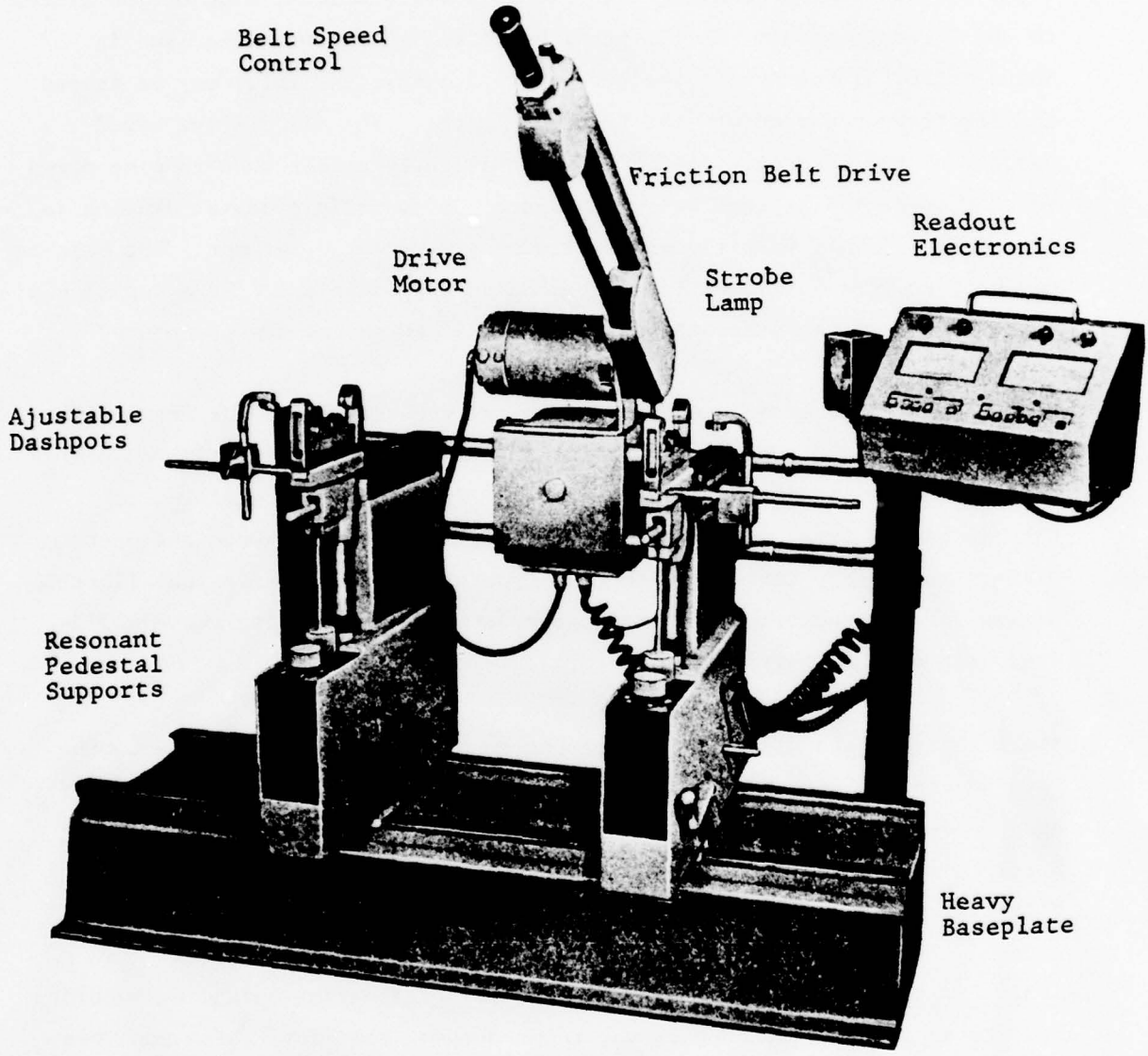


Figure 23 Resonant Balancing Machine
(Stewart Warner)

rigid support at that bearing that forces the vibrational node of the system to the locked bearing. This type of plane separation was discussed in Section 3.0. To determine the resonant frequency, the rotor may be tapped and the frequency read off the frequency meter. For fixed drive speed operation, the resonance may be adjusted slightly higher than running speed so that operation is just below resonance. Adjustable pedestal damping is available to limit vibrational amplitudes at running speed until the balance has been reduced sufficiently. After balancing one plane, the pedestal can be released and the alternate pedestal locked to repeat the procedure.

Since phase angle is dependent upon proximity to resonance and the amount of damping, a trial weight is applied at a known angular location to determine the amplitude and phase response. Phase angle, therefore, is established by trial and error in this specific machine. Once the operator becomes reasonably familiar with this type of machine, despite uncalibrated amplitudes and the trial and error procedure of establishing phase angle lag, balancing is quite straight forward.

The advantages and disadvantages of the resonant balancing machine to the hard bearing machine pertain to the specific machines and is not necessarily applicable to the basic principle of hard bearing versus resonant bearing. Specific designs of each machine exist that can improve on speed, simplicity, and accuracy of balancing.

4.3.4 Soft Bearing Balance Machine. Since soft bearing balancing machines operate above the resonance of the rotor support bearings, these machines are generally the highest speed balancers. As long as rotor operation remains in the rigid body range, high-speed balancing may be utilized. On small rigid rotors, it is often possible to utilize air to spin the rotor and balance to speeds of 50,000 rpm and higher. Most standard machines of this type are motor-belt driven and run at lower speeds in the range of 500 to 3,000 rpm. Providing the electronics (speed pickup, unbalance pickup, and filtering system) has the frequency response, the higher speeds are

possible if a means of driving to the higher speeds is available. In hard bearing machines, higher speeds than rated speed pushes operation close or into resonance--influencing phase and amplitude readings. Resonant balancing is limited to the resonant speed range of the pedestal. The soft bearing machine operates above resonance which permits higher speeds, although care must be exercised not to operate into other resonant modes of the machine or rotor.

Of the plane separation techniques, electrical nulling is the most common approach used in soft bearing machines. Unlike the hard bearing machine that utilizes analytical techniques to determine unbalance in each plane, electrical nulling or nodal plane separation requires applying an unbalance weight in each balance plane, one at a time, and separating its influence from the other plane. Therefore, initial setup of the machine is more time consuming than on the other type machines discussed. The general procedure used for plane separation is to balance a part by trial and error to a reasonable level. An unbalance weight (at least an order of magnitude larger than the residual unbalance in the rotor) is added to one balance plane. Its influence on the other plane is nulled back to the original residual unbalance reading of that plane. The weight is then removed and placed in the second balance plane. Its influence on the first plane is nulled back to the residual unbalance of that plane using the nulling potentiometer. Once the plane separation procedure is complete, any number of rotors of the same configuration may be balanced without further adjustment. The process, however, must be repeated for each different rotor configuration. When this machine is used for low quantity production balancing, it is desirable to have a master well-balanced rotor (of the configuration to be balanced) to use for plane separation setup and occasional checkout of the machine.

Since operation of the machine is above resonance, the unbalance pickup lags the unbalance location by 180 degrees. As long as the phase lag is a constant, it can be readily handled in the machine to invert the phase. As discussed in Section 4.3.5, it is well to recognize that inherent phase differences exist between different type pickups; i.e., peak velocity lags

peak displacement by 90 degrees and peak acceleration lags peak displacement by 180 degrees. These differences, however, are readily handled in a balance machine circuit.

Each of the different type balance machines require a phase mark on the rotor to relate to the location of unbalance. The common method is by means of photo-optics. A black mark is painted on a reflective part of the rotor and a reflective light source detects the change in reflectivity. A signal is thus available to indicate each time the black mark passes the photo cell. The peak value of the unbalance pickup signal indicates when the unbalance signal is at its maximum value. The angle between the peak signal from the unbalance pickup and the phase mark on the rotor establishes the phase angle. The method of readout may be through a phase meter, watt-hour meter, or by strobe light. In the case of a strobe light, the light will flash each time the unbalance amplitude from the pickup is at a maximum. The repetition rate of the light is equal to the rotational speed of the rotor. When the strobe light is flashed on the rotor, it appears to stand still; and the point observed will be the heavy spot (unbalance location) on the rotor. The light must be directed in the proper plane relative to the pickup to indicate the unbalance location. If the strobe light is fixed permanently in one plane on the machine, then the question of positioning does not arise. If the light can be moved, the proper location can be established by adding an unbalance weight to one plane of a balanced rotor and positioning the light to flash at that location. Using a strobe light eliminates the need for a speed pickup for phase information.

Figure 24 illustrates a typical soft bearing machine. This particular machine utilizes wattmeter filtering, electrical plane separation, and a coupling drive. Belt drives are quite common for general purpose balancing while coupling drives are often used for machines dedicated to production balancing of specific type rotors.

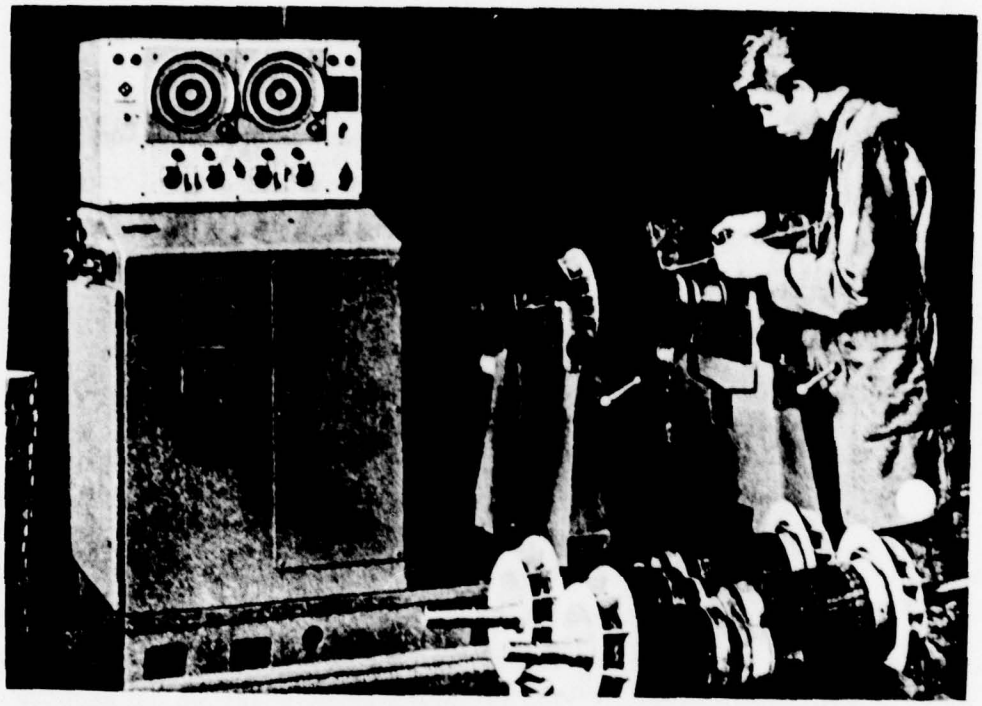


Figure 24 Schenck Soft Pedestal Balancing Machine

4.3.5 In-Place (Field) Balancing. It is often found desirable to balance some machinery after complete assembly in its housing. A number of reasons exist for balancing as an assembly, some of which are listed below.

a) After balancing of a complete rotor in a balancing machine, it may have to be disassembled for reassembly in its own housing; thus losing some of its original balance.

b) Modifications of a rotor after assembly may necessitate touchup balance.

c) Finer balance than was possible in the balance machine (due to speed limitations of the balance machine) may be required. In-place balancing permits balancing at operating speed.

d) After operation of a machine in the field, balance may have degraded necessitating rebalance.

e) Balance changes may occur due to thermal effects on machines in actual service, necessitating balancing after a machine rotor reaches a stable operating temperature.

When balancing an assembled rotor, the advantages of calibrated supports for the rotor as in a balance machine are no longer available. Plane separation techniques that are built into balancing machines are also not available. The principles of the balance machines are still required, however, including vibration pickups, a phase reference, signal amplification and filtering, and a method of reading phase angle. In some current usage balance machines, the electronics are portable and the sensors used for in-place balancing may be fed into the electronics of the balancing machine. In this section, we will consider balancing without the normal balancing machine electronics in respect to plane separation. The other features of balance machines, such as signal amplification and filtering, are found in a number of portable field instruments that are designed for in-place and field balancing. This type of equipment is considered for in-place balancing.

The first requirement for in-place balancing is to instrument the rotor to be balanced. Velocity or accelerometer sensors are commonly used and mounted on each bearing cap. One sensor is mounted at each bearing. It

is desirable to mount both pickups in the same plane to minimize difficulties with phase angles. With this instrumentation, a strobe light is the common method to measure phase angle. The strobe light is driven from the peak signal of the vibration pickup and is flashed on an exposed section of the rotating member. If the section that is observed with the strobe light is marked with numbers around its circumference, when the light flashes the number that continuously appears identifies a location on the shaft. In order to insure that the vibration detected is at running speed, a band-pass filter is used to filter vibration to running speed. This completes the instrumentation requirements for in-place balancing. The machine may be brought up to speed and the amplitude and strobe reading taken for each of the two sensors. After shutdown, a trial weight is added to one balance plane of a recorded magnitude radius and angular position relative to the markings of the rotor. The machine is returned to operating speed and the amplitude and strobe reading taken for both pickups. One further run is required, placing a known weight in the second balance plane at a known location angularly and readings taken. Utilizing these three sets of readings, the unbalance corrections for each plane can be computed or determined graphically. If the shaft was marked in 30 degree increments at the location of the strobe light, the strobe readings will be angular measurements of location of maximum amplitude of the pickup signal. Since the calculation procedure entails changes in phase angle due to placing of known unbalance weights, phase shifts due to the filter, angular location of the strobe light relative to the pickups, and the phase relation due to the type of pickup is no longer important--providing they are constant for all three sets of readings. If balancing is done by trial and error, these phase relations must be considered.

Before discussing analysis techniques, it is well to review the use of displacement probes to obtain the required measurements for computing unbalance. In many machines, no portion of the rotating assembly is exposed and, therefore, the use of a strobe light is no longer feasible. A reference mark within the machine is required for this condition. A phase meter or scope display is required to determine phase when the strobe light cannot be utilized. The noncontacting (proximeter type) probe has been commonly

used to establish a reference mark by viewing a flat spot or keyway on the shaft. Similarly, this type probe has been utilized to monitor shaft motions and detect the balance condition of the rotor. When two such probes are installed in the same plane but 90 degrees angularly spaced, they may be displayed on a CRT screen in a polar display to indicate the shaft orbit. If the phase mark from the reference mark proximeter is used to drive the Z axis of an oscilloscope, a reference mark can be indicated on the orbit to indicate the angle between probe and reference mark. The angle can be estimated by eye from oscilloscope trace.

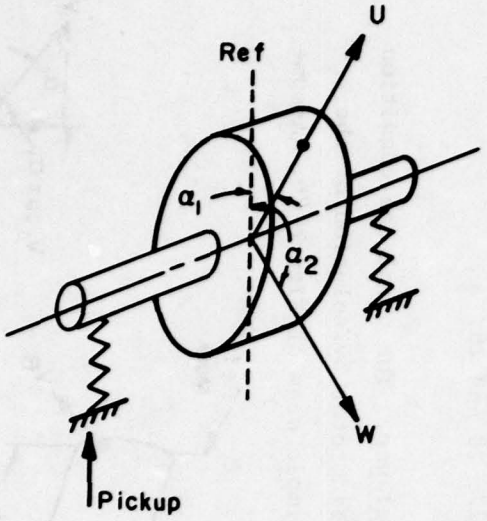
Included in current available field balancing equipment are dual-channel tracking filters and digital phase meters, as well as hand tunable band-pass filters.

4.3.6 Two-Plane Unbalance Determination. It was indicated for in-place balancing that three measurements are required of the pickup peak amplitude and phase from a reference point on the rotor operating at the same speed; i.e.:

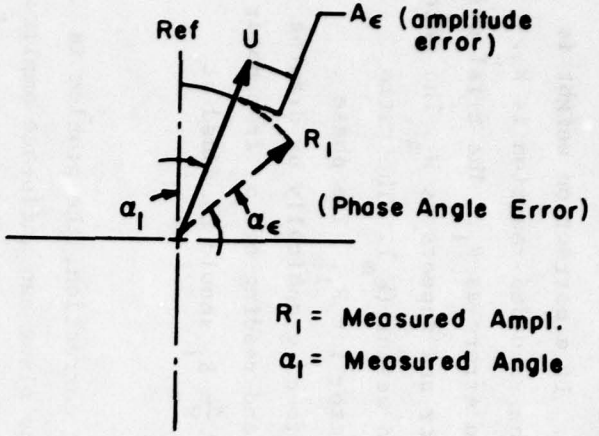
1) As is.
2) With known unbalance weight added in one balance plane at known location.
3) With known unbalance weight added in second balance plane at known location.

This generates sufficient information to permit a graphical determination of unbalance or an analytical determination. The following two sections discuss each approach.

4.3.6.1 Graphical Determination of Unbalance. In order to review a graphical technique for determination of unbalance, we shall consider a single plane balancing situation. Figure 25a illustrates the example under consideration. A pickup is installed at the bearing cap and the disc rotated at constant speed. The measured output from the

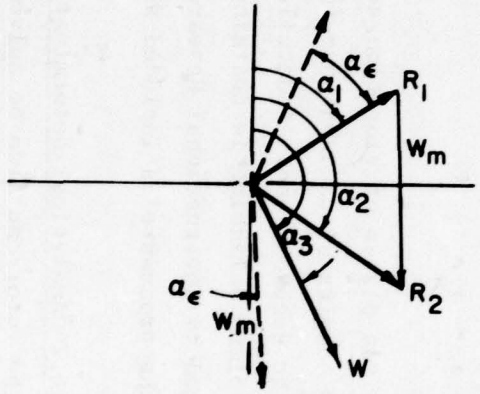


a) Single Plane Unbalance



b) Measurement Errors

1. Plot amplitude and phase R_1 and α_1
2. Add trial weight W at α_3 and plot
3. Plot reaction to trial weight R_2 and α_2
4. Construct W_m



c) Graphical Solution From Trial Weight W

5. Phase error is α_ε
6. Amplitude error is W/W_m
7. Unbalance amplitude = $R_1(W/W_m)$
8. Unbalance location = $\alpha_1 - \alpha_\varepsilon$

Figure 25 Determination of Amplitude and Phase Errors

pickup is assumed to contain an amplitude error and phase angle error (Figure 25b). If a velocity pickup is used, the output amplitude will be in terms of velocity while the unbalance u is in g[illegible] (oz). This is considered part of the amplitude error. If a correction weight is added (W) at α_2 as shown in Figure 25c, the recorded reaction is R_2. Vector R_2 has the same phase and amplitude errors as R_1. The trial weight also has the same amplitude and phase error and appears as W_m. The vector joining vectors R_1 and R_2 is equivalent to weight (W_m). The ratio of W to W_m is the amplitude correction factor for R_1. The phase angle error for correction weight can be found graphically by drawing a parallel line to W_m through the origin and reading off α_4 from polar coordinate paper. A correction weight of $\frac{W}{W_m} R_1$ should be added at $\alpha_3 + \alpha_2 - \alpha_4$.

When two planes of vibration are used for correction, the problem is more complex since unbalance forces in one plane can influence amplitudes in the second plane. A detailed solution for vector calculations for two plane balancing is contained in Application Report No. 327, published by International Research and Development Corporation. A copy of the procedure is included as Figures 27, 28 and 29.

4.3.6.2 Analytical Determination of Unbalance. The unbalance condition of the rotor can also be solved for a rigid rotor by solution of the equations for equilibrium. Consider a simple beam of Figure 26. Assume the unbalance condition as illustrated.

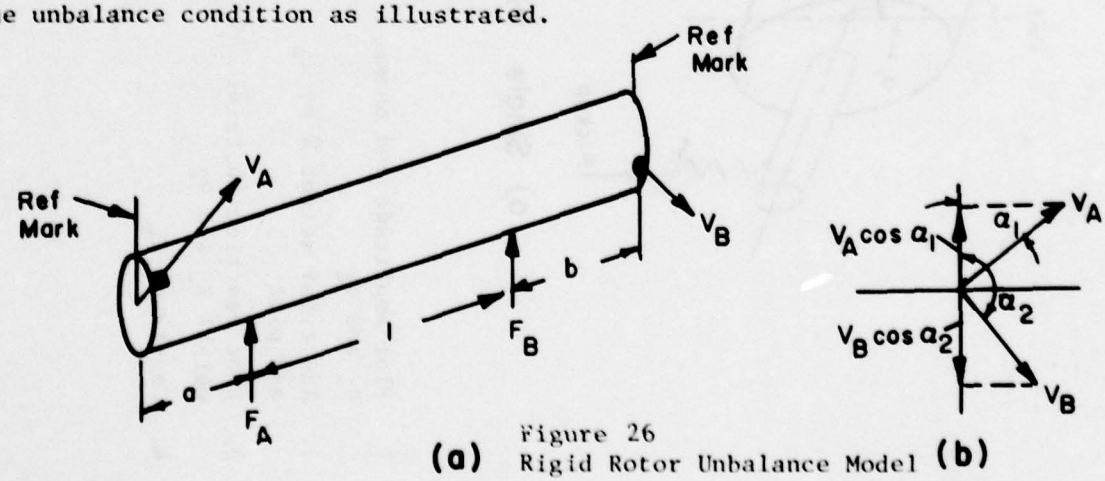


Figure 26
Rigid Rotor Unbalance Model

In the following instructions phase angles are recorded in the odd numbered item column. The amount of vibration and weights is recorded in the even numbered item column.

I Operate the rotor at the balancing speed and observe and record as Items 1, 2, 3, and 4; amount of phase, near and Far End. (Note: phase angles should increase in the clockwise direction).

II Stop the machine and add a trial weight at the Near End correction plane. Record the amount of the weight and its position in degrees clockwise from the reference mark. Items 5 and 6.

III Operate the rotor at the balancing speed and observe and record as Items 7, 8, 9, and 10.

IV Stop the machine and REMOVE the Near End trial weight. Add a trial weight at the Far End correction plane. Record the amount of the weight and its position in degrees clockwise from the reference mar. Items 11 and 12.

V Operate the rotor at the balancing speed and observe and record Items 13, 14, 15, and 16.

VI Construct vectors N, F, N_2, F_2, N_3 and F_3 by drawing the line at the phase angle whose length is scaled to length in mils vibration. Suggested scale is 1 mil = 1/2 inch.

VII Construct vectors A, B, ∝A, ∝B. Sample: To construct vector A instructions are shown ($N \cdot N_2$). This means connect the end of vector N to vector N_2. The angle of vector A is found by projecting a line parallel to A through the center in the direction from N to N_2. This is the phase angle of vector A. Measure the length of vector A to the same scale at which the other vectors were constructed. Items 17 through 24.

VIII Do the calculations as indicated for Items 25 through 32. Construct vectors ∝N and ∝F, then vectors C and D. Items 33 through 36. Calculate Items 37 and 38 as shown.

IX Construct the Unity Vector to be called 1.0 unit long at zero degrees. Items 39 and 40. Suggested scale is 1.0 unit = 5 mils = 2.5 inches. Construct vector ∝B. Items 37 and 38. Construct vector E and scale E and ∝B to same scale as Unity Vector. Items 41 and 42.

X Calculate Items 43 through 54.

XI Items 51 and 52 are the amount and position for the balance weight for the Near End correction plane, Items 53 and 54 the Far End.

XII Before making the final balance corrections it is desirable to make a graphic check of the solution. This is done as follows:

Construct vectors ∅A Items 43 and 44, and ∅B Items 45 and 46.
Calculate vector ∅∝B Amount = 50 x 24, Angle = 49 + 23.
Calculate vector ∅∝A Amount = 48 x 22, Angle = 47 + 21.
Construct vectors ∅∝B and ∅∝A.
Construct vector X (∅A + ∅∝B) should be equal and opposite to vector N.
Construct vector Y (∅B + ∅∝A) should be equal and opposite to vector F.

XIII Make the balance corrections as indicated in Step XI. Be sure the trial weight added in Step IV has been removed. The balance correction must be added at the same radius as the trial weights were placed.

XIV Operate the rotor and observe if the balance corrections reduced the vibration to an acceptable amount. If further corrections are required use new data observed as Items 1 through 4. Use the Items marked by an asterisk and recalculate Items 29 through 36, and 43 through 54. Do not disturb previous corrections.

NOTE: The more precisely the phase angles are measured and the weights added at the exact angle indicated, the more satisfactory will be the results.

During the calculations the results may include negative angles and angles larger than 360°. An angle of -35° is converted to a positive angle by subtracting the angle from 380° : 380° - 35° = 325°. An angle of 473° is converted to one less than 360 by subtracting 360° from the angle. 473° - 360° = 113°.

Figure 27 Balancing Procedure

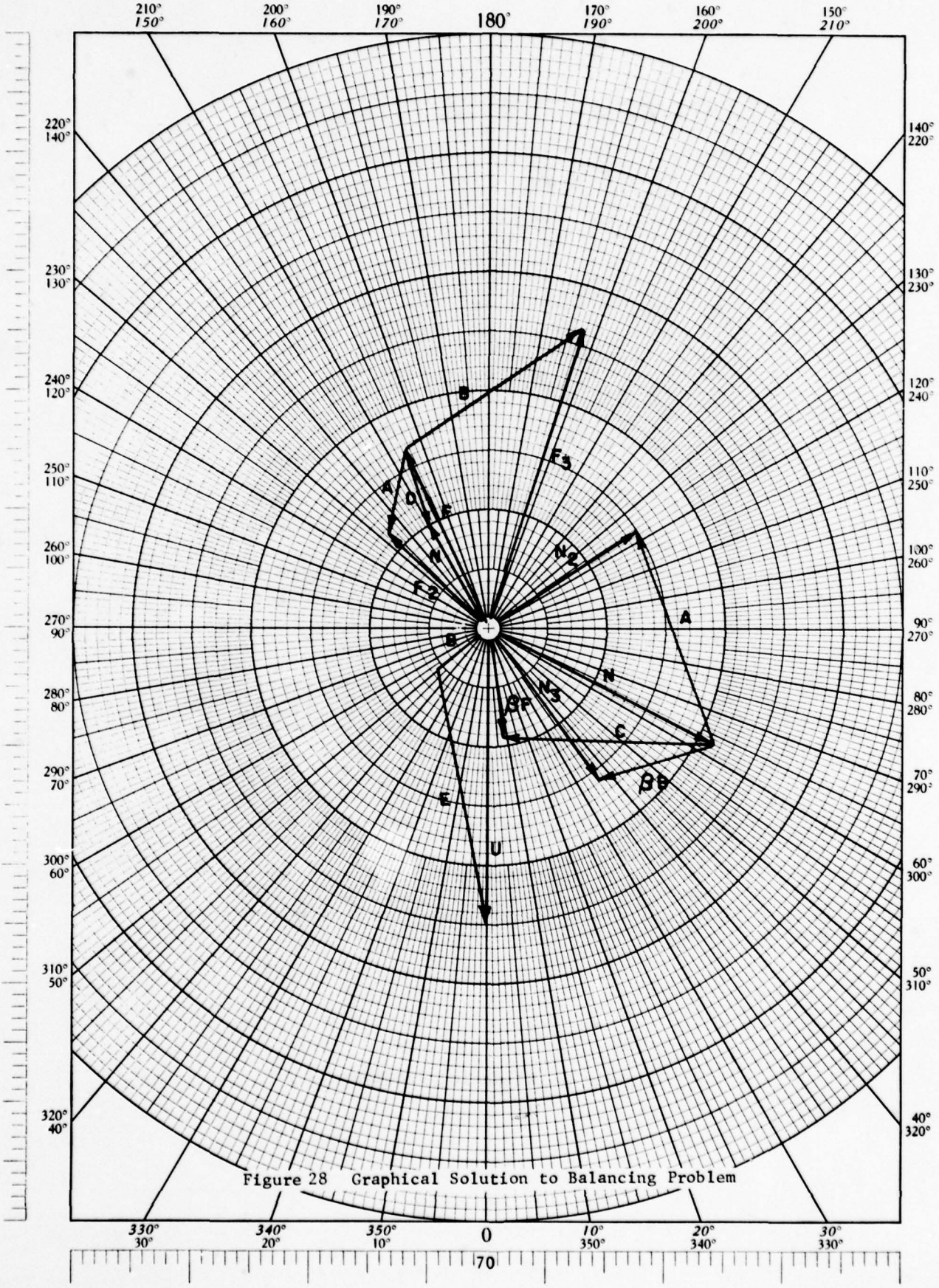


Figure 28 Graphical Solution to Balancing Problem

ROTOR: _____ SPEED: _____ TRIAL SHOT NO'S: _____

DATE: _____ CALCULATED BY: _____ CHECKED BY: _____

OPERATION	ROTOR CONDITION & RUN NO. OR CALCULATION PROCEDURE		SYM	ITEM NO.	PHASE ANGLE	ITEM NO.	VIBRATION READING / SCALE / MILS
I	Zero Rotor - TS #1		N	1	63°	2	8.6 x 1 = 8.6
			F	3	293°	4	6.5 x 1 = 6.5
II	Near End Trial Weight		W_{NT}	5°	272°	6°	10 (OZ)
III	Trial Shot - Near End TS #2		N_2	7	127°	8	5.9 x 1 = 5.9
			F_2	9	229°	10	4.5 x 1 = 4.5
IV	Far End Trial Weight		W_{FT}	11°	180°	12°	12 (OZ)
V	Trial Shot - Far End TS #3		N_3	13	36°	14	5.2 x 1 = 6.2
			F_3	15	152°	16	10.4 x 1 = 10.4
VI	$A = N_2 - N$ $(N - N_3)$		A	17°	201°	18°	7.6
	$B = F_2 - F$ $(F - F_3)$		B	19°	124°	20°	7.3
VII	$\alpha A = F_2 - F$ $(F - F_3)$		αA	21	350°	22	2.9
	$\beta B = N_2 - N$ $(N - N_3)$		βB	23	256°	24	4.2
VIII	25 - 21 - 17	26 - 22 - 18	α	25°	149°	26°	.392
	27 - 23 - 19	28 - 24 - 20	β	27°	162°	28°	.575
	29 - 25 - 1	30 - 26 - 2	αN	29	212°	30	3.29
	31 - 27 - 3	32 - 28 - 4	βF	31	9°	32	3.74
	$C = \beta F - N$ $(N - \beta F)$		C	33	269°	34	7.15
	$D = \alpha N - F$ $(F - \alpha N)$		D	35	20°	36	3.3
	37 - 25 - 27	38 - 26 - 28	$\alpha\beta$	37	311°	38	.22 UNITS
IX	Unity Vector - Plot		U	39	0°	40	1.0 UNITS
	$E = U - \alpha\beta$ $(\alpha\beta - U)$		E	41°	11°	42°	.87 UNITS
X	43 - 33 - 41	44 - 34 - 42	θA	43	257°	44	9.21
	45 - 35 - 41	45 - 36 - 42	θB	45	9°	46	3.8
	47 - 43 - 17	48 - 44 - 18	θ	47	56°	48	1.09
	49 - 45 - 19	50 - 46 - 20	θ	49	245°	50	.52
XI	51 - 5 - 47	52 - 6 - 48	W_{NC}	51	214° cw	52	10.8 (OZ)
	53 - 11 - 49	54 - 12 - 50	W_{FC}	53	265° cw	54	6.24 (OZ)
XII	Graphic Check of Results						
XIII	Make Weight Corrections						
XIV	Additional Corrections						

DETAILED CALCULATIONS

(25) = 21 - 17 360°-201° = 149°	(26) = 22 + 18 2.9÷7.6 = .382	(27) = 23 - 19 285°-124° = 162°	(28) = 24 + 20 4.2÷7.3 = .575	(29) = 25 + 1 149°+63° = 212°	(30) = 26 x 2 3.9x8.6 = 3.29	(31) = 27 + 3 162°+255° = 8°	(32) = 28 x 4 .575x6.5 = 3.74
(37) = 25 + 27 149°+162° = 311°	(38) = 26 x 28 .392x.575 = .22	(43) = 33 - 41 269°-11° = 257°	(44) = 34 - 42 7.15÷.97 = 9.21	(45) = 35 - 41 20°-11°=9°	(46) = 36 - 42 3.3÷.87 = 3.8	(47) = 43 - 17 257°-201° = 56°	(48) = 44 + 18 9.21÷7.5 = 1.09
	(49) = 45 - 17 9°-124° = -115° or 245°	(50) = 46 - 20 3.8÷7.3=.52	(51) = 5 - 47 270°-65°=214°	(52) = 6 x 48 10x1.03 = 10.8	(53) = 11 - 49 265°-245° = 20° or 360°-65°=295°	(54) = 12 x 50 12x.52 = 6.24	

Figure 29 Typical Example - Vector Calculations for Two-Plane Balancing

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The unbalance forces U_1 and U_2 can be broken into the vertical and horizontal components as illustrated in Figure 26b. The reaction to the unbalance forces occur at the bearing pedestals and are designated as F_A and F_B. Considering only the vertical components of unbalance, the sum of the forces and moments must be zero for equilibrium; i.e.:

$$F_A + F_B + U_A \cos\alpha_1 - U_B \cos\alpha_2 = 0 \tag{20}$$

$$aU_A \cos\alpha_1 + \ell F_B - (\ell+b)\, U_B \cos\alpha_2 = 0 \tag{21}$$

Similarly, the horizontal forces and moments must balance. If the force is measured at the bearing, we would like to determine the values and phase angles of U_A and U_B in both the vertical and horizontal directions without knowledge of the values a, b, and ℓ. Only the equations for vertical forces will be discussed since the horizontal force solution is the same procedure. The force measurements at the bearings are initially reduced to the vertical and horizontal components. F_A and F_B above refer to the vertical components.

Solving Equations (20 and (21) for F_A and F_B yields

$$F_A = -\left(\frac{a+\ell}{\ell}\right) U_A \cos\alpha_1 - \frac{b}{\ell} U_B \cos\alpha_2 \tag{22}$$

$$F_B = \frac{a}{\ell} U_A \cos\alpha_1 + \frac{b+\ell}{\ell} U_B \cos\alpha_2 \tag{23}$$

Let

$$C_1 = \frac{a}{\ell}$$

$$C_2 = \frac{b}{\ell}$$

Equations (22) and (23) may be rewritten:

$$F_A = -(C_1 + 1)\, U_A \cos\alpha_1 - C_2\, U_B \cos\alpha_2 \qquad (24)$$

$$F_B = C_1\, U_A \cos\alpha_1 + (C_2 + 1)\, U_B \cos\alpha_2 \qquad (25)$$

If an unbalance weight is added to balance plane U_A and the vertical component is identified as T, Equations (24) and (25) can be rewritten:

$$F_{A_1} = -(C_1 + 1)\,(U_A \cos\alpha_1 + T) - C_2\, U_B \cos\alpha_2 \qquad (26)$$

$$F_{B_1} = C_1\,(U_A \cos\alpha_1 + T) - (C_2 + 1)\, U_B \cos\alpha_2 \qquad (27)$$

The unbalance weight is then added to plane containing U_B and Equations (24) and (25) for the vertical component T are:

$$F_{A_2} = -(C_1 + 1)\, U_A \cos\alpha_1 - C_2\,(U_B \cos\alpha_2 - T) \qquad (28)$$

$$F_{B_2} = C_1\, U_A \cos\alpha_1 + (C_2 + 1)\,(U_B \cos\alpha_2 - T) \qquad (29)$$

We have six equations with two unknown unbalance forces and two unknown coefficients (C_1 and C_2). Combining equations:

$$(24)\text{-}(26) \quad F_A - F_{A_1} = (C_1 + 1)T; \quad (C_1 + 1) = \frac{F_A - F_{A_1}}{T}$$

$$(25)\text{-}(27) \quad F_B - F_{B_1} = C_1 T \quad ; \quad C_1 = \frac{F_B - F_{B_1}}{T}$$

$$(25)\text{-}(29) \quad F_B - F_{B_2} = -(C_2 + 1)T; \quad (C_2 + 1) = -\frac{F_B - F_{B_2}}{T}$$

$$(26)\text{-}(28) \quad F_{A_1} - F_{A_2} = -C_2 T \quad ; \quad C_2 = -\frac{F_{A_1} - F_{A_2}}{T}$$

The coefficients C_1 and C_2 can be eliminated from Equations (24) and (25) to permit determination of $U_A \cos\alpha_1$ and $U_B \cos\alpha_2$ in terms of measured forces at the bearings. In a similar manner, the horizontal components $U_A \sin\alpha_1$ and $U_B \cos\alpha_2$ may be determined in terms of the horizontal reaction forces at the bearings. The actual unbalance in plane U_A is:

$$U_A = \sqrt{(U_A \cos\alpha)^2 + (U_A \sin\alpha)^2}$$

$$\tan\alpha = \frac{U_A \sin\alpha}{U_A \cos\alpha}$$

It has been assumed that the measurements at the bearings were in force terms. Actual measurements, however, are generally in terms of acceleration or velocity at the pedestal. Figure 30 is a typical installation of a velocity pickup in the vertical plane of a bearing cap.

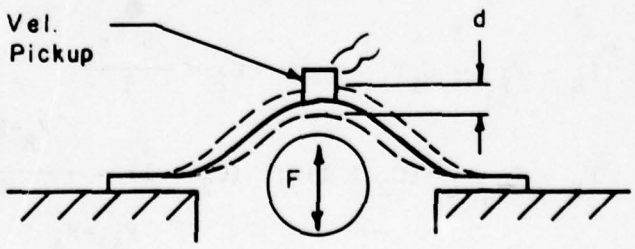


Figure 30 Bearing Cap Vibration

The reaction force at the bearing is transmitted to the cap causing a deflection of the cap. The peak-to-peak value of the deflection is represented by (d) where: $d/2 = F/K$. K is the stiffness of the bearing cap and is an unknown constant. The resultant peak velocity for the peak force is

$Vel = \pi fd$ f = rotational frequency (hz) and is a constant if the balancing speed is constant

rewriting:

$$V = \frac{\pi fF}{K} \quad \text{or} \quad F = \frac{KV}{\pi f}$$

if $Q = \frac{F}{\pi f}$; $F = QV$

The peak velocity measured by the pickup is directly proportional to the peak driving force. All reaction forces F_A, F_B, F_{A_1}, etc. when using velocity as the measurement parameter are: QV_A, QV_B, QV_{A_1}, etc. If we rewrite Equation (24) substituting the values of the coefficients C_1, C_2, $C_1 + 1$, $C_2 + 2$ and substituting velocity for force, we have

$$QV_A = -\left[\frac{QV_A - QV_{A_1}}{T}\right] U_A \cos\alpha_1 + \left[\frac{QV_{A_1} - QV_{A_2}}{T}\right] \cos\alpha_2 \qquad (30)$$

Factoring Q we have

$$V_A = -\left[\frac{V_A - V_{A_1}}{T}\right] U_A \cos\alpha_1 + \left[\frac{V_{A_1} - V_{A_2}}{T}\right] \cos\alpha_2 \qquad (31)$$

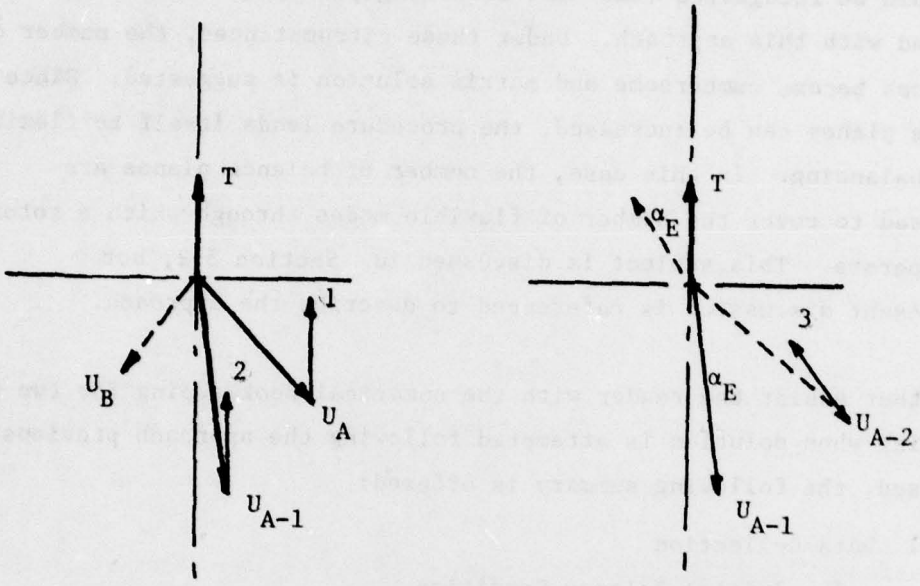
We, therefore, may use velocity measurements directly for determination of the unbalance force.

The question of phase relations between unbalance location, pickup response, and phase errors must also be accounted for when calculating unbalance reaction. All of these phase shifts (i.e., through pickup, through filters and amplifiers) are assumed constant throughout the balancing procedure for operation at the same balancing speed.

Phase correction for single plane balancing was discussed in Section 4.3.6.1. In Figure 25b, phase angle error was described as α_E. This phase error (α_E) represents a single plane unbalance condition. As noted previously, the second plane also influences the phase angle. In Figure 31a, an unbalance reaction vector at one sensor location is shown as U_A with no phase error. When a trial weight is added (vector T), the result is to shift U_A to point 1. The reaction at end A to unbalance in plane B is U_B. The results at end A is vector U_{A-1}. Again adding the trial weight (T), shifts vector U_{A-1} to point 2. The magnitude and direction of the shaft vector is not altered by the influence of end B. The phase error may thus be determined as illustrated on Figure 25 for a single plane unbalance. The phase error is shown on Figure 31b, illustrating the similarity to a single unbalance plane graphical solution. Both amplitude error and phase angle error may differ for each plane.

In a similar manner these errors may be determined at the second balance plane.

To summarize the analytical approach, the beam equation for reaction could be solved for one unknown balance condition and two trial weight conditions resulting in six equations. Solution of the equations results in coefficients, in terms of measured reactions, representative of the rotor dimensions. Solution for the unbalance forces producing these reactions could then be solved. In addition, if a constant phase angle error exists, it can also be computed. Solution of the equations are more straight forward utilizing matrix algebra; but in the simple case, solution by substitution was considered helpful to the reader in visualizing the approach.



U_A Unbalance A end

U_B Unbalance B end

T Trial weight

U_{A-1} Apparent unbalance A end due to influence B end

U_A to 1 Influence on A end from trial weight (ignoring B end)

U_{A-1} to 2 Influence on A end from trial weight (including influence of B end)

U_{A-2} Shift in location of U_{A-1} due to phase error

U_{A-2} to 3 Influence of trial weight

Figure 31 Correcting for Phase Errors in Two Planes

It should be recognized that more balancing planes than two could be utilized with this approach. Under these circumstances, the number of equations become cumbersome and matrix solution is suggested. Since balance planes can be increased, the procedure lends itself to flexible shaft balancing. In this case, the number of balance planes are increased to cover the number of flexible modes through which a rotor will operate. This subject is discussed in Section 5.2, but the present discussion is referenced to describe the approach.

To further assist the reader with the numerical bookkeeping for two plane balancing when solution is attempted following the approach previously discussed, the following summary is offered:

A) Data Collection

1. Initial Balance Condition

   A end x, units @ α_1 phase (from ref.)
   B end y, units @ α_2 phase

   Phase angles may include constant phase error

2. Trial Weight Added to Plane A (weight = T)

   A end x_2 units @ α_3 phase
   B end y_2 units @ α_4 phase

3. Trial Weight (T) Added to Plane B

   A end x_3 units @ α_5 phase
   B end y_3 units @ α_6 phase

B) Vertical and Horizontal Reaction Components (F) to Unbalance (U)

	Plane A	Plane B
1. Vertical :	$F_A = x_1\cos\alpha_1$	$F_B = y_1\cos\alpha_2$
	$F_{A_1} = x_2\cos\alpha_3$	$F_{B_1} = y_2\cos\alpha_4$
	$F_{A_2} = x_3\cos\alpha_5$	$F_{B_2} = y_3\cos\alpha_6$
2. Horizontal:	$F_C = x_1\sin\alpha_1$	$F_D = y_1\sin\alpha_2$
	$F_{C_1} = x_2\sin\alpha_3$	$F_{D_1} = y_2\sin\alpha_4$
	$F_{C_3} = x_3\sin\alpha_5$	$F_{D_2} = y_3\sin\alpha_6$

C) Coefficients

1. Vertical:

$$C_1 = \frac{F_B - F_{B_1}}{T} \quad ; \quad (C_1+1) = \frac{F_A - F_{A_1}}{T}$$

$$C_2 = -\frac{F_{A_1} - F_{A_2}}{T}; \quad (C_2+1) = \frac{F_{A_1} - F_{A_3}}{T}$$

2. Horizontal:

$$C_{1H} = \frac{F_D - F_{D_1}}{T} \quad ; \quad (C_{1H}+1) = \frac{F_C - F_{C_1}}{T}$$

$$C_{2H} = -\frac{F_{C_1} - F_{C_2}}{T}; \quad (C_{2H}+1) = -\frac{F_{C_1} - F_{C_3}}{T}$$

D) Unbalance Components

1. Vertical Component

End A: $$U_A\cos\alpha_1 = \frac{C_2 U_B\cos\alpha_2 - F_A}{-(C_1+1)}$$

End B: $$U_B\cos\alpha_2 = \frac{F_B C_2}{C_1(F_A+C_2) + (C_1+1)(C_2+1)}$$

2. Horizontal Component

End A: $U_A \sin\alpha_1 = \dfrac{C_{2H}U_B\sin\alpha_2 - F_C}{-(C_{1H}+1)}$

End B: $U_B \sin\alpha_2 = \dfrac{F_D C_{2H}}{C_{1H}(F_C+C_{2H}) + (C_{1H}+1)(C_{2H}+1)}$

E) Unbalance

End A: $U_A = \sqrt{(U_A\cos\alpha_1)^2 + (U_A\sin\alpha_1)^2}$

$U_B = \sqrt{(U_B\cos\alpha_1)^2 + (U_B\sin\alpha_1)^2}$

F) Phase of Unbalance

1. End A: $\tan\alpha_1 = \dfrac{U_A\sin\alpha_1}{U_A\cos\alpha_1}$

2. End B: $\tan\alpha_2 = \dfrac{U_B\sin\alpha_2}{U_B\cos\alpha_2}$

G) Phase Correction

1. End A: $\alpha_{E-A} = \alpha_3 - \cos^{-1}\dfrac{x_2^2 + T_1^2 - x_1^2}{2x_2T_1}$

where: $T_1^2 = x_2^2 + x_1^2 - 2x_1x_2\cos(\alpha_1-\alpha_3)$

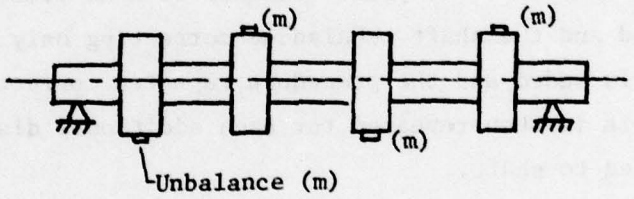
2. End B: $\alpha_{E-B} = \alpha_6 - \cos^{-1}\dfrac{y_3^2 + T_2^2 - y_1^2}{2y_3T_2}$

where: $T_2^2 = y_3^2 + y_1^2 - 2y_3y_1\cos(\alpha_2-\alpha_6)$

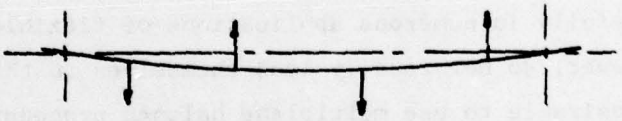
SECTION V

FLEXIBLE ROTOR BALANCING

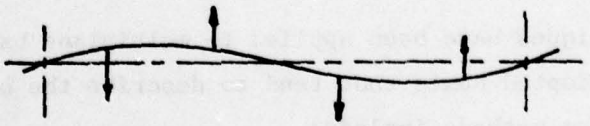
Section 3.3 discussed the need for different balance techniques when considering operation of rotors at speeds above one or more of their flexible resonant speeds (critical speeds). The shape that a flexible rotor will assume at a natural frequency of the bearing-rotor system is termed the mode shape. Figure 10 of Section 3.3 illustrates the first four mode shapes of a simple beam. If unbalance forces exist in a flexible rotating member such that their phase orientations tend to induce a particular mode shape, operation through that mode will induce large amplitudes. To illustrate this point consider the rotor of Figure 32 with four discs and two bearings.



a) Rotor



b) First Bending Mode



c) Second Bending Mode

Figure 32 Four Plane Unbalance Operating Through the Shaft Second Flexible Mode

If each disc contains equal unbalance as shown, the system is in static balance and the resultant moments do not seriously influence the first bending mode but provide substantial driving components for the second mode. It is evident, that internal moments are introduced into the shaft due to the balance condition of each disc. To remove all these unbalanced forces obviously necessitates four balance planes, one at each disc.

One approach commonly used to improve balance is to individually balance each disc prior to installation on the shaft. Final balance is accomplished by two plane touch-up balance on the assembly. Since individual disc balancing requires a balancing mandrel, where concentricities must be maintained accurately and disc attachment is important, a technique called stack balancing has often been applied to eliminate the need for a mandrel. In this case, the shaft is two plane balanced with no discs. The first disc is then installed and the shaft rebalanced correcting only on the disc. The second disc is added and the procedure repeated correcting only on the second disc. This is then repeated for each additional disc or rotating component attached to shaft.

These procedures of individual component balancing and stack balancing have been used successfully in numerous applications of flexible shaft assemblies. Some designs however, do not readily lend themselves to these techniques and it becomes desirable to use multiplane balance procedures (more than two planes).

A number of techniques have been applied to multiplane balancing of flexible rotors and have adopted names that tend to describe the basic procedure. The two most common methods include:

Modal Balancing (References 48 through 65)
Influence Coefficient Balancing (References 66 through 72)

There are a number of practical aspects involved with each of the balancing

methods that should be considered before attempting to utilize the procedures. Firstly, it is necessary to operate the rotor to each of the critical speeds for which a balanced condition is desired. This implies that either in-place balancing of a rotor in its own housing is required or a high speed spin pit is needed. For equipment such as rotary compressors (axials or centrifugals) and turbines a vacuum chamber is desirable to reduce input drive power if a spin pit is utilized. If it is necessary to disassemble a rotor after balancing in order to install it into its housing, a high speed spin pit is an undesirable approach since disassembly and reassembly will influence balance. A second consideration is that of the bearing and bearing support characteristics used during balance. The rotor support characteristics in terms of stiffness and damping properties can influence the mode shape for the rotor at its critical speeds. If there is a significant difference in rotor support properties between a spin pit or high speed balance installation and the actual housing of the rotor, balanced vibration levels in the spin pit can differ from final installation levels. Finally, the complete balancing procedure can be quite lengthy since more than two balance planes are generally used and more than one balance speed (depending upon the number of criticals for which balance is desired) is required. Anyone having experienced the difficulties of accurately positioning balance weights on low speed rigid body balancing machines will recognize the difficulties involved in high speed flexible shaft balancing. Continuing efforts by experimenters utilizing digital computer techniques are actively engaged in minimizing some of these difficult problem areas.

5.1 Modal Balancing

A fundamental property of all oscillatory systems is the orthogonality relation for principal modes of oscillation [Ref. 73]. In balancing flexible rotors through n critical speeds or modes, this principle implies that amplitudes can be reduced for each mode independently without altering amplitudes at other modes given at least n balancing planes for correction.

Rotors can be corrected mode by mode without theoretically influencing other modes. This technique of balancing has been termed Modal balancing. The response of a rotor to unbalance at any speed is the sum of the responses of each individual mode. When operation of a rotor is conducted in close proximity to one mode, the response of all other modes are generally negligible unless two modes are in close proximity. In order to select balance weights for Modal balancing, it is necessary to know the mode shape for each critical speed (or mode) for which the shaft is to be balanced. This may be obtained by calculation or by measurement. To calculate the mode shape, a damped critical speed can be computed utilizing the RSVP program described in Part I or similar shaft dynamics programs. Measurements would require the use of proximeter sensors along the rotor length while operating the rotor through its resonant modes of interest. Alternately the rotor could be subjected to a vibration resonant search to establish resonant frequencies and mode shape. This approach excludes the bearing dynamic influences and gyroscopic stiffening effects however, and may lead to some inaccuracies.

It should be noted that the vibrational amplitudes of flexible rotors at each mode is a sum of the amplitudes of all modes of the rotor. Theoretically, for zero deflection it would be necessary to balance each mode in an infinite number of planes. This is true for any flexible balancing technique. Since an infinite number of planes is not practical, one relys on the fact that the amplitude influence of other modes are minimal when balancing one mode. This implies that to balance the first flexible mode a single plane is required, to balance the second mode two planes are required and three planes for the third mode. This results in using the number of balance planes equal to the number of modes to be balanced or n planes. Most investigators agree that the precision of balance can be improved by increasing the number of balancing planes at the expense of added complexity and time. Therefore, more than n balancing planes are often used by experimenters. In modal balancing as a minimum n planes are considered desirable for balancing n modes.

In order to acquaint the reader with the modal balancing technique, a simple flexible beam is considered containing three unbalance forces. Operation through three modes is desired. The calculated mode shapes are as shown in Figure 33. Let the balance planes be selected as stations 2, 4 and 5 and the unknown unbalance forces located at stations 1, 3 and 6.

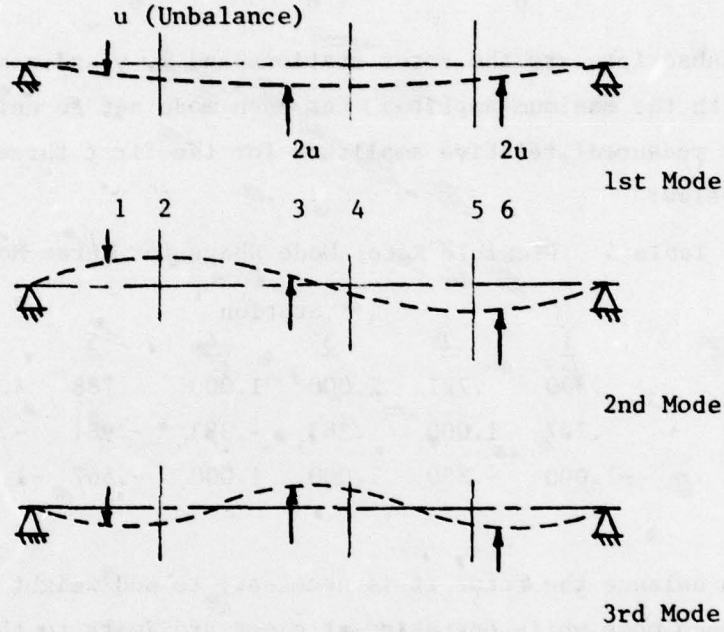


Figure 33 Three Plane Modal Balancing Example

Since it is desirable to operate the rotor through three modes, a minimum of three balance planes are required. A further discussion of the number of balance planes; i.e.; n or n + 2 is given in Reference [61]. It is also noted that the unbalance forces were all selected in one plane while in most situations they may fall in different planes since they are vectiorial. They may, however, be resolved into two planes that are normal and each plane treated separately and finally superimposed on each other. Therefore, the single plane was selected for illustration purposes.

The mode shape for each mode may be described as:

1st Mode	2nd Mode	3rd Mode
x_1	y_1	z_1
x_2	y_2	z_2
x_3	y_3	z_3
x_4	y_4	z_4
x_5	y_5	z_5
x_6	y_6	z_6

where the subscripts are the rotor stations and x, y and z are relative amplitudes with the maximum amplitude for each mode set to unity. The calculated (or measured) relative amplitude for the first three modes are tabulated below:

Table 4 Flexible Rotor Mode Shape for Three Modes

	Station					
Mode	1	2	3	4	5	6
x	.390	.721	1.000	1.000	.788	.647
y	.707	1.000	.383	-.383	-.981	-.981
z	-1.000	-.850	1.000	1.000	-.567	-1.000

In order to balance the rotor it is necessary to add weight at each balancing plane for each mode while operating at close proximity to the mode. For each mode, the added weights are selected to insure the potential energy in the flexed shaft goes to zero. The potential energy in the shaft for the first mode due to the unbalance at stations 1, 3 and 6 is:

$$U_B = \left[\frac{U_1 r_1}{g}\omega^2\right] x_1 + \left[\frac{U_3 r_3}{g}\omega^2\right] x_3 + \left[\frac{U_6 r_6}{g}\omega^2\right] x_6 \qquad (32)$$

where: U = unknown unbalance

r = radius of unbalance

g = gravitational units

ω = rotational speed (rad/sec)

To set the energy equal to zero when adding weights at station 2, 4 and 5,

$$U_B = \left[\frac{U_2 r_2}{g}\,\omega^2\right] x_2 + \left[\frac{U_4 r_4}{g}\,\omega^2\right] x_4 + \left[\frac{U_5 r_5}{g}\,\omega^2\right] x_5 \tag{33}$$

Substituting F for Ur results in:

$$U_B = F_2 x_2 + F_4 x_4 + F_5 x_5 \tag{34}$$

In selecting F_2, F_4 and F_5, it is required that they not effect the next two modes; i.e.:

$$F_2 y_2 + F_4 y_4 + F_5 y_5 = 0 \tag{35}$$

$$F_2 z_2 + F_4 z_4 + F_5 z_5 = 0 \tag{36}$$

Rewriting all three equations:

$$F_2 x_2 + F_4 x_4 + F_5 x_5 = U_B \tag{37}$$

$$F_2 y_2 + F_4 y_4 + F_5 y_5 = 0 \tag{38}$$

$$F_2 z_2 + F_4 z_4 + F_5 z_5 = 0 \tag{39}$$

For the second mode equations 37, 38 and 39 are:

$$\begin{aligned} \text{EQ } 37 &= 0 \\ \text{EQ } 38 &= U_B \\ \text{EQ } 39 &= 0 \end{aligned} \tag{40}$$

Finally for the third mode:

$$\begin{aligned} \text{EQ } 37 &= 0 \\ \text{EQ } 38 &= 0 \\ \text{EQ } 39 &= U_B \end{aligned} \tag{41}$$

These equations (37, 38, 39, set 40 and set 41) can be rewritten using the relative mode shapes of Table 4.

$$.721\ F_2 + 1.0\ F_4 + .788\ F_5 = U_B$$

$$1.000\ F_2 - .383\ F_4 - .981\ F_5 = 0 \quad \text{Mode 1}$$

$$-.85\ F_2 + 1.00\ F_4 - .567\ F_5 = 0$$

Solution to these equations result in:

$$F_2 = .428\ U_B$$

$$F_4 = .501\ U_B$$

$$F_5 = .241\ U_B$$

$$\frac{F_2}{F_4} = .85$$

$$\frac{F_4}{F_4} = 1.0$$

$$\frac{F_5}{F_4} = .48$$

The magnitude of the unbalance correction is not known but the ratio of weight additions for the first mode without influencing the second and third modes are as indicated above. Whatever weight is required at station 4, 85 percent of this value is added in phase at station 2 and 48 percent in phase at station 5. Balancing must be done in close proximity to the mode to insure the mode shape closely approximates that mode. In a similar manner, the three Equations (40) may be solved for correction weights for the second mode and Equations (41) for the third mode. In each case, the absolute magnitude of the corrections are unknown but their relative amplitude and phase relationship for a single plane are determined.

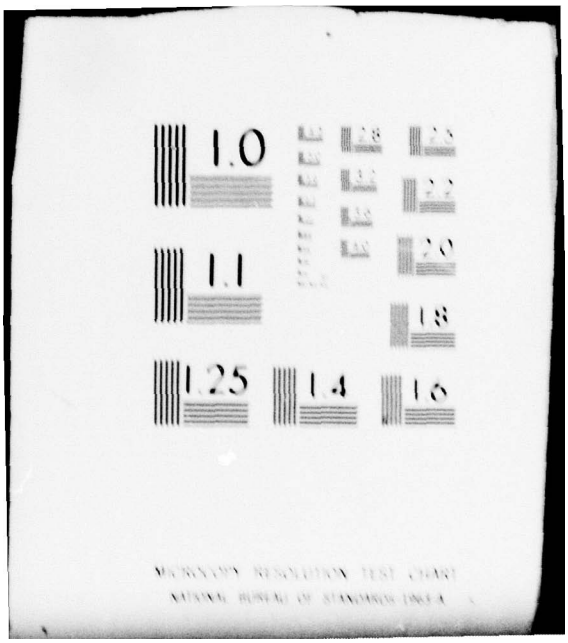
In this simplified example only the vertical components of unbalance were considered. In actual practice, unbalance amplitudes would be complex

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MICROCOPY RESOLUTION TEST CHART
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numbers including amplitude and phase. The number of balance planes is generally dictated by the number of criticals (N) through which operation is desired. Bishop [51] suggests N balance planes and Kellenberger [62] suggest (N+2) planes for greater accuracy. Balancing has been accomplished by trial and error with knowledge of the mode shape to be balanced, or numerically following an approach similar to that previously discussed. With accurate knowledge of the shaft mode shape for each balance mode, one pickup could be used for measurement of amplitude if not placed on a node. Most investigators have used two or more to verify mode shapes. Modal balancing has proved difficult when two modes are in close proximity to each other and for non-axial symmetrical rotors such as those with large overhung weights. A number of practical considerations are also of importance when applying modal balancing techniques. Since balance is accomplished in close proximity to a resonant mode, phase angles of unbalance location as seen by the sensor or pickup used, will lag the actual location by as much as 90 degrees. Location of the proper angular correction plane will be a trial and error process. If non-shaft contacting displacement probes are used for measurement of vibration, care must be exercised to minimize runout at the probe which introduces errors. Further discussion regarding modal balancing and number of balance planes is provided in Appendix A.

5.2 Influence Coefficient Balancing

Unlike the modal balancing techniques, influence coefficient balancing does not require prior determination of rotor stiffness characteristics or bearing support characteristics. To simply illustrate the technique consider a flexible rotor with flexible supports of Figure 34.

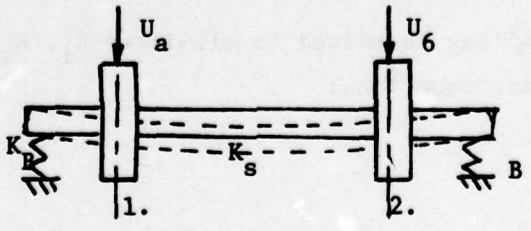


Figure 34 Flexible Rotor with Unbalance

If an unbalance force (U_a) is applied to the shaft at location 1, the deflection x_1 at this location will be a sum of the flexible shaft deflection due to K_s (shaft stiffness) and the bearing deflection due to K_B (bearing stiffness). The characteristics of the shaft-bearing system may be represented by R_1 and the deflection can be written as:

$$x_1 = U_a R_1$$

Adding a second unbalance force (U_b) to location 2 adds additional deflection at location 1. Identifying the shaft-bearing system deflection characteristics at 1 due to a force at 2 of R_2, the total deflection at x_1 for the first mode becomes:

$$x_1 = U_a R_1 + U_b R_2 \tag{42}$$

Similarily the deflection at x_2 for the first mode due to U_a and U_b unbalances is:

$$x_2 = U_a R_3 + U_b R_4 \tag{43}$$

If a trial weight T is added at location 1, the deflections at locations 1 and 2 become:

$$y_1 = (U_a + T) R_1 + U_b R_2$$

$$y_2 = (U_a + T) R_3 + U_b R_4 \tag{44}$$

Moving the trial weight to location 2 results in the third set of deflections; i.e.:

$$z_1 = U_a R_1 + (U_b + T) R_2$$

$$z_2 = U_a R_3 + (U_b + T) R_4 \tag{45}$$

Equations 42 through 45 may be solved to eliminate R_1, R_2, R_3 and R_4, resulting in the following equations:

$$x_1 = U_a \frac{(y_1 - x_1)}{T} + U_b \frac{(z_1 - x_1)}{T}$$

$$x_2 = U_a \frac{(y_2 - x_2)}{T} + U_b \frac{(z_2 - x_2)}{T} \qquad (46)$$

The coefficients of U_a and U_b are called the influence coefficients. The balancing problem involves reducing x_1 and x_2 to zero. This is accomplished by solving the two equations for U_a and U_b. As noted, no knowledge of shaft and bearing properties were required to obtain the coefficients.

A higher balancing speed can be selected, and a new set of influence coefficients determined in the same manner for balancing the higher mode. Where balancing of several modes are desired, several balance speeds can be selected and influence coefficients for each speed determined. A least squares best fit for balance over the speed range has been used [69] for this situation. This has been termed a least squares influence coefficient method in contrast to an exact point method. A further variation [71] proposed using linear programming to optimize the balance of rotors operating through several bending critical speeds.

In the simple example shown, angularity of unbalance location was ignored and only a single plane of balance considered for illustrative purposes. Only two balance and measurement planes were also used. In practice, the number of balance planes are generally greater than two. In experimental verification of the influence coefficient technique, balance planes have been reported ranging from 4 to 6 and measurement planes from 1 to 7. Tonneson [74] concluded that the minimum amplitude attainable is a function of the number of balance planes, their axial location, the speed range and the number of measurement transducers and their axial location.

In practice, influence coefficient balancing is accomplished using matrix algebra and computer solution. The procedure, however, follows the same

steps as that used in illustrating the approach. If we assume N balance planes and n measuring planes, the rotor is run to balancing speed and amplitude and phase measurements taken at each of the n measuring planes. The rotor is then stopped and a trial weight (T) added to the first balance plane N_1 and readings taken at each measuring plane. The first set of influence coefficients may be calculated from:

$$C_{ij} = \frac{\text{whirl amplitude with trial wgt.} - \text{whirl amplitude without trial wgt.}}{\text{trial weight}} \tag{47}$$

$$C_{ij} = a^{i}_{ij} - a^{o}_{ij}$$

where C_{ij}; a^{i}_{ij} and a^{o}_{ij} are complex numbers

The trial weight is then moved to the second balance plane, third plane---N plane with influence coefficients calculated for each condition. An influence coefficient matrix [A] can be constructed which relates the rotor whirl amplitudes a^o (a n row vector) to original unbalance U^o (a N column vector); i.e.:

$$\left\{a^{o}\right\} = [A] \left\{U^{o}\right\}$$

The original rotor unbalance U^o may be determined by multiplying the original whirl amplitude a^o by the inverse matrix [A]; i.e.:

$$\left\{U^{o}\right\} = [A]^{-1} \left\{a^{o}\right\}$$

The correction weight W^o_c is equal to the negative value of the unbalance. Since U^o is a complex number, the absolute value of unbalance is:

$$|U^{o}| = \sqrt{(U^{o}_{i})^{2}_{r} + (U^{o}_{i})^{2}_{i}} \quad ; \tan \phi = \frac{U^{o}_{i}}{U^{o}_{r}}$$

Further discussion regarding influence coefficient balancing may be found in References 69 through 72. Experiments conducted with this technique are discussed in References 74 through 80. Appendix B discusses the least squares error method of balancing and includes a computer program listing.

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APPENDIX A

MODAL BALANCING THEORY

By

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L.E. Barrett
Dr. P.E. Allaire

A.1 BACKGROUND

The balancing problem has been approached by two different schools of thought: those who view the rotor as a series of point masses and those who treat the rotor as a continuous elastic body. The treatment of the rotor as a continuum has led to the modal concept pioneered by Bishop and Parkinson. In this method, the rotor amplitude is expressed as a power series function of the system undamped eigenvalues. Bishop shows that the general unbalance distribution may be expressed in terms of modal unbalance eccentricities. The rotor amplitude of motion near a critical speed is thus primarily affected by that particular modal unbalance distribution while the higher order modes have little influence on the lower critical speed response. They then state that the rotor should be balanced mode by mode by placing proper weights at the antinodes. Several excellent papers have been published by Kellenberger of Brown Boveri who has treated the balancing problem of both continuously distributed and local unbalances and discusses the problem of balancing in N or N + 2 planes.

A.2 FLEXIBLE ROTOR BALANCING IN N + B PLANES

Den Hartog, in his paper on "The Balancing of Flexible Rotors" states that if a rotor consists of a straight weightless shaft with N concentrated masses along its length and supported in B bearings with an arbitrary unbalance distribution, then it can be balanced perfectly by placing small correction weights in N + B planes along the length of the rotor.

He also states that if the flexible rotor is balanced on rigid support, then the balance of the rotor so obtained will not be a function of the bearing impedances. That is, the introduction of support flexibility and damping to the rotor system will not materially effect the rotor unbalance response if the N + B plane method of balancing is used.

It should be noted, however, that the inverse condition is not true. If a rotor is balanced on soft supports, such as with a standard flexible mount balancing machine, then the rotor may not necessarily be in balance when run in the actual machine in which the support stiffness values are

substantially higher than the balance machine. Therefore, it can be concluded that balancing on the hard bearing support balancing machines is preferable to balancing on soft support machines.

The N + B concept of Den Hartog is extended to multimass flexible rotors and he states that nearly perfect balance at all speeds can be obtained by balancing in N + B planes, where N now means the number of rotor critical speeds in the speed range from zero to four times the maximum service speed of the machine.

To demonstrate the N + B method of balancing, consider Figure A-1 which represents a single mass rotor with an arbitrary unbalance of $u_k = m_k e_k$.

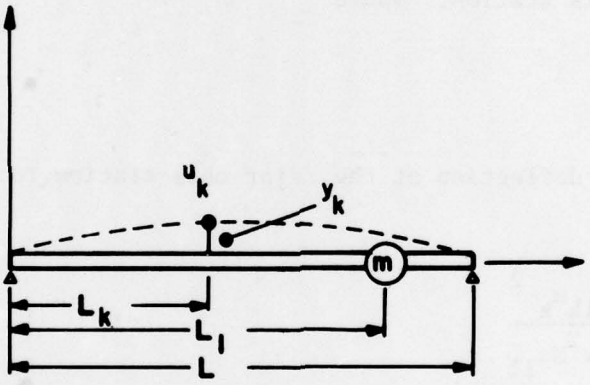


Figure A-1 Single Modal Mass Rotor with Arbitrary Point Mass Unbalance

The equation of motion in y - z plane for the major mass stations can be obtained by the influence coefficient method as follows

$$y_1 = \alpha_{11} P_1 + \alpha_{1k} P_k \qquad \text{(A-1)}$$

where

α_{11} = deflection of station 1 due to a unit force at 1
α_{1k} = deflection at station 1 due to a unit force at station k
P_k = sum of external and inertial forces acting at station 1

110

and

$$P_1 = M\ddot{Y}_1 = M\omega^2 y_1$$

for synchronous motion neglecting external damping or other external forces at the major mass stations

$$P_k = m_k(y_k + e_k)\omega^2 = \omega^2(u_k + m_k y_k)$$

The deflection at y_1 is given by

$$y_1 = \omega^2\{M y_1 \alpha_{11} + (u_k + m_k y_k)\alpha_{1k}\} \qquad \text{(A-2)}$$

where $u_k = m_k e_k$.

It is assumed that the balance correction weight m_k is small in comparison to the major mass station. Hence

$$\frac{m_k y_k}{m y_1} << 1$$

Solving for the deflection at the major mass station for a series of unbalances u_k

$$y_1 = \frac{\Sigma \alpha_{1k} u_k \omega^2}{1-\omega^2 M \alpha_{11}} \qquad \text{(A-3)}$$

If the amplitude at the major mass station y_1 is to be zero at all speeds, then it is apparent that the balancing requirement is

$$\Sigma\ \alpha_{1k}\ u_k = 0 \qquad \text{(A-4)}$$

In addition to reducing the amplitude of motion at the major mass station, it is also desired to reduce the forces transmitted to the bearings. The bearing force reactions are given by

$$F_{b1} + F_{b2} = \omega^2\{M y_1 + \Sigma\ m_k(y_k + e_k)\} = 0 \qquad \text{(A-5)}$$

Since the unbalance masses m_k are small in comparison to M and the shaft

deflection y_k is of the order of mils of deflection, whereas the distance e_k at which the unbalance masses are acting may be several inches, the vanishing of the bearing forces requires that

$$\omega^2(m\, y_1 + m_k\, e_k) = 0 \qquad \text{(A-6)}$$

If the balance criterion of Equation (A-4) is met so that the motion at the major mass station is zero, then the vanishing of the sum of the bearing forces requires that

$$\sum_{k=1}^{n} u_k = 0 \qquad \text{(A-7)}$$

This is recognized as simply the first requirement for rigid body balancing.

The third balancing requirement is obtained by summing moments about the first bearing

$$F_{b2} \cdot L = \omega^2\{m\, y_1\, L_1 + \Sigma\, m_k\, L_k(y_k + e_k)\} = 0 \qquad \text{(A-8)}$$

This reduces to the requirement that

$$\Sigma\, L_k u_k = 0 \qquad \text{(A-9)}$$

In summary, the requirements for flexible rotor balancing may be stated as two equations of rigid body balance plus a flexible rotor balance requirement

$$\left.\begin{array}{ll} \text{a.} \quad \Sigma\, u_k = 0 & \\ \text{b.} \quad \Sigma\, L_k\, u_k = 0 & \end{array}\right\} \text{rigid rotor balance} \qquad \text{(A-10)}$$
$$\text{c.} \quad \Sigma\, a_{1k}\, u_k = 0 \quad \text{flexible rotor}$$

With these equations we can now better understand the argument behind the N and N + 2 plane method of balancing as presented by Kellenberger and developed somewhat along the line of Den Hartog but in a more elaborate fashion. Kellenberger states that the N + 2 plane method with a two bearing machine should always be superior to the N plane method, and that rigid

body balancing should first be done. Bishop and Parkinson argue that only N planes are necessary and that rigid body balancing should not be done. According to their line of reasoning, everyone should throw away their rigid body balancing machines and use the modal technique which they have developed. Unfortunately, in all of their many papers on the subject they do not inform us as to how they actually calculate the modal balance weights except by trial and error.

If the N plane method alone is used to balance either the single mass model of Figure A-1 or a multimass rotor of Figure A-2 to pass through the first

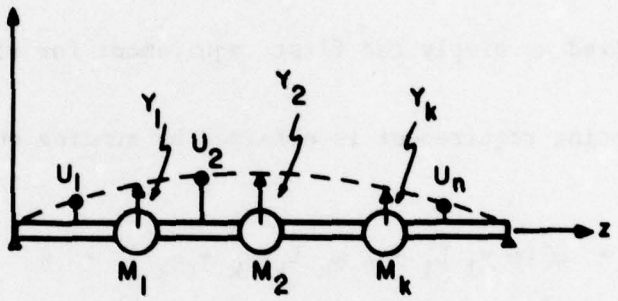


Figure A-2 Multimass Rotor with Distributed Point Mass Unbalance

critical speed, then only one balance correction weight is needed to reduce the amplitude at the major mass station or shaft antinode to zero. The balance correction u_{b1} placed at the major mass station is given by

$$u_{b1} = -\frac{1}{\alpha_{11}} \Sigma\alpha_{1k}\, u_k \qquad \text{(A-11)}$$

Although the amplitude at the major mass station has been reduced to zero, the transmitted bearing forces are nonvanishing. In order to eliminate the transmitted bearing forces due to unbalance as well as reduce the rotor amplitude of motion while passing through the first critical speed, two additional balance planes are required. Let u_{b2} and u_{b3} be two additional balance correction weights placed on the rotor.

The balance correction weights are given by

$$a.\quad u_{b1} + u_{b2} + u_{b3} = -\Sigma u_k = R_1$$

$$b.\quad L_1 u_{b1} + L_2 u_{b2} + L_3 u_{b3} = -\Sigma L_k u_k = R_2$$

$$c.\quad u_{b1} + \frac{\alpha_{12}}{\alpha_{11}} u_{b2} + \frac{\alpha_{13}}{\alpha_{11}} u_{b3} = -\frac{1}{\alpha_{11}} \Sigma\alpha_{1k} u_k = R_3 \qquad \text{(A-12)}$$

A.3 BALANCING IN A TEST FACILITY WITHOUT TRIAL WEIGHTS

The values of u_k components are unknown and can never be exactly calculated for a complex multimass system. However, it should be noted that the first two quantities on the right hand side can be measured with a rigid rotor balancing machine.

In the simplified derivation presented, no damping effect of the bearings or on the rotor was taken into account. The influence of damping will alter the rotor amplitude and phase angle relationships from the simple expression given in Equation (A-3). However, several companies are developing elaborate spin pit test facilities in which they can run the rotor in a vacuum and mount the rotor on ball bearings. In this way, the damping forces acting on the rotor are negligible and equations equivalent to Equation (A-3) are valid.

By placing a noncontacting probe near the center of the rotor which should correspond to the major balancing plane number 1, the rotor may be balanced from the observation of the rotor amplitude of motion without the use of trial weights.

Equation (A-3) may be rewritten in the form

$$y_1 = \frac{\Sigma\, \alpha_{1k}\, u_k \omega^2}{1 - f_1^2} \qquad \text{(A-13)}$$

where $f_1 = \frac{\omega}{\omega_{cr1}}$ = critical speed ratio on rigid supports

ω_{cr1} = rotor first critical speed on rigid supports

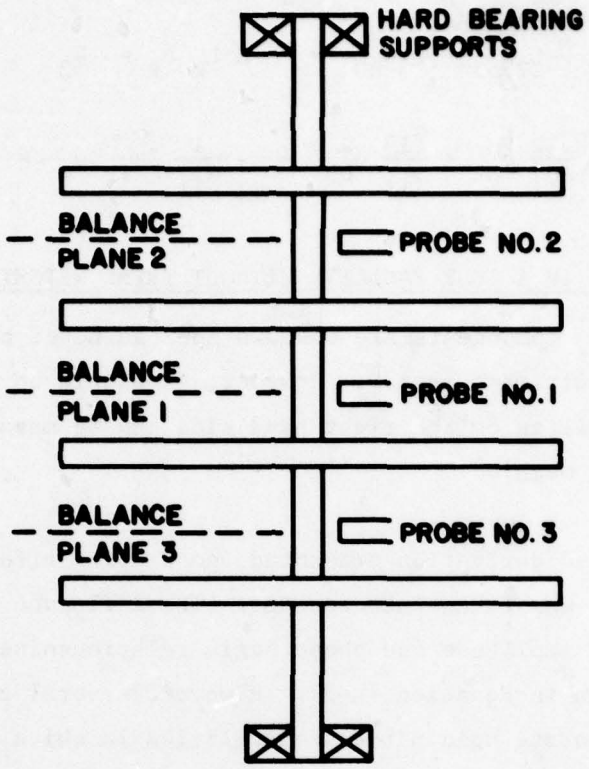


Figure A-3 Vertical Spin Pit Balance Facility

The rotor first critical speed may be experimentally determined by placing the rotor on fixed end supports and electromagnetically exciting it to determine the response frequency. This works well on long thin rotors where gyroscopic effects due to shaft rotation do not materially change the synchronous critical speed.

The right hand side of Equation (A-12) for R_3 is given by

$$R_3 = -\frac{1}{\alpha_{11}} \Sigma \alpha_{1k} u_k = -m_1 y_1 \frac{(1-f_1^2)}{f_1^2} \qquad \text{(A-14)}$$

The value of M_1 for a multimass rotor is now interpreted as the modal mass and will be shown to be given by

$$M_1 = \int_0^L \rho\phi_1(z)^2 dz = \Sigma\, m_i(z)\phi_{1i}(z)^2 \qquad \text{(A-15)}$$

where $\phi_1(z)$ = mode of shape of rotor for first critical speed.

The balance correction weights for three plane balancing for the first critical speed is given by

$$\begin{bmatrix} u_{b1} \\ u_{b2} \\ u_{b3} \end{bmatrix} = -\begin{bmatrix} 1 & 1 & 1 \\ L_1 & L_2 & L_3 \\ 1 & \frac{\alpha_{12}}{\alpha_{11}} & \frac{\alpha_{13}}{\alpha_{11}} \end{bmatrix}^{-1} \begin{bmatrix} \Sigma\, u_k \\ \Sigma\, L_k\, U_k \\ \frac{w_1 y_1 (1-f_1^2)}{g f_1^2} \end{bmatrix} \qquad \text{(A-16)}$$

where u_{b1} are expressed in lb-in.

To summarize the three plane method without trial weights, the following steps are taken:

1. Determine $\Sigma\, u_k$ and $\Sigma\, L_k\, u_k$ by a standard soft mount bearing machine. However, do not rigid body balance the rotor.
2. Calculate the rotor critical speed from a computer code with rigid supports or experimentally measure the rotor natural frequency by exciting it while placed on knife edge supports.
3. Place a static load at the major balance station 1 and measure the deflections at stations 2 and 3. From these deflections calculate the ratio of α_{12}/α_{11} and α_{13}/α_{11}.
4. Calculate the rotor modal weight W_1 from the critical speed computer code. If this is not available, take $W_1 = W_{total}/2$.
5. Place the rotor in the spin pit facility and record the rotor amplitude and phase y_1.
6. Calculate the balance correction weights u_{b1} and place on the rotor.
7. Rerun the rotor with the balance correction weights added and refine the balance by the influence coefficient method if the desired balance is not achieved.

A.4 FLEXIBLE ROTOR MODAL RESPONSE

The dynamical equations of motion of the multimass rotor may be written in matrix form where the rotor is composed of N mass stations

$$(m)_N \ddot{(y)}_N + (c)_N \dot{(y)}_N + (k)_N (y)_N = \omega^2 (u) e^{i\omega t} \tag{A-17}$$

The N mass station system has N critical speeds, however, we are only interested in the first j critical speed values such that

$$4\omega_{max} < \omega_{cj}$$

For each value of critical speed there is a corresponding mode shape

$$\phi_j(y_j) \text{ or } \phi_{ji}$$

It is assumed that the rotor amplitude can be expressed as a series in terms of the critical speed mode shapes as follows

$$\begin{bmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_N \end{bmatrix} = q_1 \begin{bmatrix} \phi_{11} \\ \phi_{12} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{1N} \end{bmatrix} + q_2 \begin{bmatrix} \phi_{21} \\ \phi_{22} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{2N} \end{bmatrix} + \ldots + q_j \begin{bmatrix} \phi_{j1} \\ \phi_{j2} \\ \cdot \\ \cdot \\ \cdot \\ \phi_{jN} \end{bmatrix} \tag{A-18}$$

The displacements at any station k are given by

$$y_k = \sum_{i=1}^{j} q_i \phi_{ik} \tag{A-19}$$

The equations of motion may be expressed in the series form by

$$\sum_{i=1}^{j} \ddot{q}_i [M][\phi_i] + \sum_{i=1}^{j} \dot{q}_i [c][\phi_i] + \sum q_i [k][\phi_i] = \omega^2 [u] \tag{A-20}$$

The equations may be uncoupled by multiplying by the kth mode and using the

principal of orthogonality of the modes. It is also assumed that damping may also be approximately treated if the damping is either small or is proportional to the [k] or [M] matrix, or if the complete complex mode shapes are used.

Multiplying the transpose vector $[\phi_j]^T$, we obtain

$$[\phi_k]^T[M][\phi_i]\ddot{q}_i + [\phi_k]^T[c][\phi_i]\dot{q}_i + [\phi_k]^T[k][\phi_i]q_i = \omega^2[\phi_k]^T[u] \qquad \text{(A-21)}$$

From orthogonality we obtain

$$[\phi_k]^T[M][\phi_i] = \delta_{ik}M_k$$

m_k = modal mass for the kth natural frequency

We obtain j uncoupled equations of the form

$$m_k\ddot{q}_k + c_k\dot{q}_k + K_kq_k = \omega^2[\phi_k]^T[u]e^{i\omega t} \qquad \text{(A-22)}$$

By dividing the equation by the modal mass M_k, the resulting equation can be obtained

$$\ddot{q}_k + 2\omega_k\xi_k\dot{q}_k + \omega_k^2 q_k = E_k e^{i\omega t} \qquad \text{(A-23)}$$

where

w_k = kth natural frequency

ξ_k = kth mode damping = $\dfrac{[\phi_k]^T[c][\phi_k]}{2M_k\omega_k}$

E_k = modal unbalance eccentricity = $\dfrac{[\phi_k]^T[u]}{M_k}$

The solution of q_k is given by

$$q_k = \frac{\omega^2 E_k}{\omega^2 - \omega_k^2 + 2i\omega_k\xi_k\omega} \qquad \text{(A-24)}$$

Let

$$f_k = \frac{\omega}{\omega_k}$$

then

$$q_k = \frac{E_k f_k^2}{1 - f_k^2 + 2i\xi_k f_k} = e_k A_k (f_k) \qquad \text{(A-25)}$$

Notice now that the modal multiplication factor q_k appears to be in similar form to the amplitude equation for the single mass rotor.

The motion at any station y is given by

$$[y] = E_1 A_1 [\phi_1] + E_2 A_2 [\phi_2] + \ldots + E_j A_j [\phi_j]$$

Examination of the amplitude response of the rotor at first glance would appear to be expressed as an infinite sum of all of the various modal components. However, most rotating machinery usually operates only through several critical speeds. Examination of the modal amplification factors shows that the higher modes become vanishingly small and do not have to be considered.

For example, consider the modal amplification factors for a uniform rotor operating at 95 percent of the first critical speed and $\omega_2/\omega_1 = 4$, $\omega_2/\omega_3 = 9$. The modal amplification factors are given by (neglecting damping).

$$A_1 = \frac{.95^2}{1 - .95^2} = 9.26$$

$$A_2 = \frac{\left(\frac{.95}{4}\right)^2}{1 - \left(\frac{.95}{9}\right)^2} = 0.06 \frac{A_2}{A_1} = 0.006$$

$$A_3 = \frac{\left(\frac{.95}{9}\right)^2}{1 - \left(\frac{.95}{4}\right)^2} = 0.01 \frac{A_3}{A_1} = 0.0001$$

Therefore, if the rotor is operated close to the first critical speed, the amplitude of motion will be predominantly due to the first mode distributions. The second mode will contribute only 1 or 2 percent to the rotor

motion and the third mode will hardly be felt at all.

This then has led to the concept of modal balancing where the modal unbalance E_1 is the first corrected while running near the first critical and then the rotor speed is increased to near the second critical speed where E_2 is then corrected. It can also be seen that if a rotor is balanced by the conventional influence coefficient method near the first critical speed, then due to slight errors in the measurement process only the E_1 modal component will be accurately balanced. If the rotor is operated near a higher critical speed, there can still remain a substantial unbalance in the higher modes to excite the rotor.

A.5 FLEXIBLE ROTOR MODAL BALANCING

The most general case of distributed unbalance u(z) may be expanded in terms of the modal eccentricity components in terms of the mode shapes

$$u(z) = \Sigma E_k m(z) \phi(z)$$

where E_k is given by

$$\int_0^L u(z) \phi_k(z) dz = M_k E_k$$

To balance out the rotor first critical speed using a three plane balance procedure

$$u_1 = E_1 m_1 \phi_{11} + E_2 m_2 \phi_{21}$$
$$u_2 = E_1 m_2 \phi_{12} + E_2 m_2 \phi_{22}$$
$$u_3 = E_1 m_3 \phi_{13} + E_2 m_2 \phi_{23}$$

where m_1 are the effect masses at the balance stations. The effective masses may be computed from the critical speed mode shape and values of modal mass by

$$M_1 = m_1 \phi_{11}^2 + m_2 \phi_{12}^2 + m_3 \phi_{13}^2$$
$$M_2 = m_1 \phi_{21}^2 + m_2 \phi_{22}^2 + m_3 \phi_{23}^2$$

The effective mass stations should also satisfy the rotor orthgonality conditions

$$0 = m_1 \phi_{11} \phi_{21} + m_2 \phi_{12} \phi_{22} + m_3 \phi_{13} \phi_{23}$$

The unbalance can now be expressed as the sum of the two modal components

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_1 + \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_2$$

The first set of unbalances will excite the rotor first mode and the second set will only excite the second critical speed.

In the combined modal-influence coefficient method of balancing near the first or higher order critical speed, a modal unbalance distribution is used rather than a single value. The trial unbalance distribution for the first mode is given by

$$(u_1)_T = U_T \begin{bmatrix} 1 \\ \dfrac{m_2\phi_{12}}{m_1\phi_{11}} \\ \dfrac{m_3\phi_{13}}{m_1\phi_{11}} \end{bmatrix} \qquad \text{(A-26)}$$

This trial weight distribution is placed on the shaft and the rotor amplitude is measured. The trial unbalance eccentricity is given by

$$E_{1T} = \frac{u_T}{M_1} \left[\phi_{11} + \frac{m_2\phi_{12}^2}{m_1\phi_{11}} + \frac{m_3\phi_{13}^2}{m_1\phi_{11}} \right] \qquad \text{(A-27)}$$

The rotor amplitude of motion at any point along the rotor is given by

$$(y)_T = (E_1 + E_{1T})\, A_1(\phi_1) + E_2 A_2(\phi_2) + \ldots + E_j A_j(\phi_j) \qquad \text{(A-28)}$$

Subtracting the original measured amplitude at station k from the trial run, we obtain

$$y_{Tk} - y_k = E_{1T} A_1 \phi_{k1} \tag{A-29}$$

The complex first modal amplification factor is determined by

$$A_1 = \frac{1}{N} \sum_{k=1}^{N} \left(\frac{y_{Tk}-y_k}{E_{1T}\phi_{ik}}\right) \tag{A-30}$$

where N is the number of stations at which measurements are made. If the first probe is assumed to be near the maximum rotor amplitude, then the modal unbalance is given by

$$E_1 = \frac{y_1}{A_1\phi_{11}} = \left(\frac{y_1}{y_{T1}-y_1}\right) E_{1T} \tag{A-31}$$

The modal balance correction E_{1b} is placed opposite the unbalance eccentricity

$$E_{1b} = - E_1$$

The correction balance weights for the first critical speed are simply given by ratio of rotor amplitude at the center station 1 by

$$\begin{bmatrix} u_{11} \\ u_{21} \\ u_{31} \end{bmatrix}_b = \left(\frac{y_1}{y_1-y_{T1}}\right) \begin{bmatrix} u_{11} \\ u_{21} \\ u_{31} \end{bmatrix}_T \tag{A-32}$$

APPENDIX B

MULTIMASS FLEXIBLE ROTOR BALANCING BY THE LEAST SQUARES ERROR METHOD

By

A.B. Palazzolo
E.J. Gunther

The computer program included in this Appendix was written and experimentally verified by A.B. Palazzolo.

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B.1 BACKGROUND OF INFLUENCE COEFFICIENT BALANCING

The method of balancing a flexible rotor may be roughly divided into two classifications: the modal method and the influence coefficient method.

The influence coefficient method as applied to two plane rotor balancing was first described by Thearle in 1934. The fundamental procedure that Thearle used was to determine the rotor response at two locations by applying arbitrary trial weights at each of the selected planes and then measuring the rotor response.

By this procedure, a set of four influence coefficients was determined in which the balance weights could be calculated in order that the amplitude be reduced to zero at the two measurement planes and for the given speed.

At the time that Thearle developed this procedure, a semianalytic graphical method was used to calculate the two plane balance corrections. This procedure still has wide application in industry for two plane trim balance. With the advent of the small programmable portable calculators, this method has been formalized for numerical computation. The basic procedure of Thearle may be expanded into a three or multi plane balancing procedure in which the number of balancing planes is equal to the number of probe or vibration pickup stations. This method is known as the exact point speed influence coefficient method and has been elaborated on by Tessarzik and Badgley.

When the basic influence coefficient method of balancing is employed, then for practical considerations it is necessary to use a computer as graphical or hand calculations are unwielding and time consuming.

The increasingly easy accessibility of large digital computers that perform complex arithmetic have made sophisticated balancing techniques more and more practical. The Least Squares Method is one of these which was proposed by Goodman in 1964 and extended to include complex arithmetic by Lund in 1971.

The solution of the rotor unbalance components for an N plane exact point influence coefficient procedure requires the inversion of an N x N complex matrix or a 2N x 2N real matrix for the proper determination of the rotor unbalance correction weights and their corresponding phase angles.

It has been demonstrated that the application of the exact point influence coefficient method will satisfactorily balance out the rotor motion of a multimass system at a given speed. However, the method will not necessarily insure that the rotor will be adequately balanced over a given speed range, particularly if several high amplitude critical speeds are encountered in the operating speed range.

The reason for this includes numerous factors such as the assumed linearity of the system, shaft runout and bow, and the number of balance planes selected.

The procedure using a least squared error fit of the vibration data over a number of speeds, as developed by Goodman and Lund, removes some of the difficulties encountered with the exact point method.

For a discrete multimass rotor system, the total rotor response may be expressed as follows:

$$\{Z\} = [\alpha_u]\{e_u\} + [\alpha_r]\{\delta_r\} + [\alpha_m]\{\delta_m\} + \{\delta_o\}$$

The total rotor response assuming a linear system is composed of the responses due to mass unbalance eccentricity e_u at the various stations, shaft bow $\{\delta_r\}$ unbalance couples such as caused by skewed discs, and electrical or mechanical runout vectors $\{\delta_o\}$ which may be constant or speed dependent but do not dynamically excite the system.

When a trial weight is placed on the rotor (e_T) the new rotor response is given by

$$\{Z_T\} = \{Z\} + [\alpha_u]\{e_T\}$$

Hence, when the original vector Z is subtracted and divided by the trial weight then

$$[\alpha_u] = \frac{\{Z_T\}-\{Z\}}{\{e_T\}}$$

Thus, the influence coefficient method can only determine the mass center unbalance coefficients $[\alpha_u]$. At very low speed, the influence coefficient matrix $[\alpha_r]$ is unity and the rotor amplitude of motion is given by

$$\{Z\} = \{\delta_r\} + \{\delta_o\} = \{\delta\}$$

It is difficult, if not impossible, to determine what combination of shaft bow and constant runout the low speed motion is composed of. Therefore, as an approximate balancing procedure the low speed motion will be subtracted from the original rotor motion at speed; assume that

$$\{Z\} = [\alpha_u]\{e_u\} + \{\delta\} + (\varepsilon \text{ error})$$

where ε represents the error function due to assuming that the δ amplitude function remains constant.

In the least squares error procedure the number of planes N does not necessarily equal the number of amplitude measurements. The total number of measurements M is equal to the number of probes L times the number of speeds, K.

By using the least squared error procedure which is described in the next section, the total error function S_M is minimized over a given speed range.

The limitation of this procedure is that it requires the use of a computer to perform the complex matrix operations and that large amounts of vibration data must be recorded. These limitations, however, can be somewhat overcome through the use of a time sharing computer and/or a mini data acquisitions system for data reduction and processing.

A short chronology, Table B-1, has been constructed to provide a general

TABLE B-1

INFLUENCE COEFFICIENT BALANCING

Type	Developed By	Balance Criterion	Balance Planes Required
Single Plane	THEARLE, General Electric 1934	After one correction weight added require that a single measurement become zero. ONE TRIAL WEIGHT RUN IS REQUIRED TO CALCULATE ONE INFLUENCE COEFFICIENT.	1
Two Plane		After two correction weights added require that two measurements become zero. TWO TRIAL WEIGHT RUNS ARE REQUIRED TO CALCULATE FOUR INFLUENCE COEFFICIENTS.	2
Exact Point Speed	LUND RIEGER TESSARZIK BADGLEY, Mechanical Tech. Inc., 1967, 1971	After N correction weights added require that N measurements become zero. N TRIAL WEIGHT RUNS ARE REQUIRED TO CALCULATE NxN INFLUENCE COEFFICIENTS	N
Least Squares	GOODMAN, General Electric 1964 LUND, Mechanical Tech. Inc. 1972	After N correction weights added require that the sum of the squares of the resulting measurement amplitudes be a minimum, taking into consideration that the total number of measurements is greater than the number of balance planes. Number of measurements = number of probes x number of speeds = M N TRIAL WEIGHT RUNS ARE REQUIRED TO CALCULATE MxN INFLUENCE COEFFICIENTS	N

Measurement: Probe reading at certain speed

Response : Velocity or displacement reading

outline of the influence coefficient balancing method. Notice that the Exact Point Speed and Least Squares Methods utilize the same number of balance planes, however, the Least Squares Method consumes more response data in order to get a better idea of the trends of the system.

In the event that the bowed rotor effect contributes significantly to the dynamic response of the rotor one may reference the balancing work by Parkinson, Jackson, and Bishop or that by Nicholas, Gunter, and Allaire in this area.

B.2 LEAST SQUARES BALANCING THEORY

The analysis presented here assumes that the rotor bearing system shown in Figure B-1 may be modeled as a linear multi-degree of freedom system. Then the steady state response of the rotor, at any particular degree of freedom (location along the shaft), to a distribution of single frequency harmonic forces acting at the rotor stations is

$$X_i e^{i\omega t} = \left(\sum_{k=1}^{n} F_k \alpha_{ik}\right) e^{i\omega t} \qquad \text{(B-1)}$$

where

X_i = coefficient of ith response (amplitude, phase)

k = dummy index

n = number of degrees of freedom used to model rotor

ω = forcing frequency (rad/sec)

F_k = complex coefficient of harmonic force acting on the kth degree of freedom

$$^{*}\alpha_{ik} = \sum_{r=1}^{2n} \frac{q_{ri}\ \mu_{rk}}{a_r(i\omega - p_r)} \qquad \text{(B-2)}$$

= theoretical complex "influence coefficient" for the case where the response is a displacement

* Formula derived by considering the response of a multi-degree of freedom system with asymmetric property matrices to a harmonic force distribution.

q_{ri} = ith component of rth system mode shape

μ_{rk} = kth component of rth system adjoint mode shape

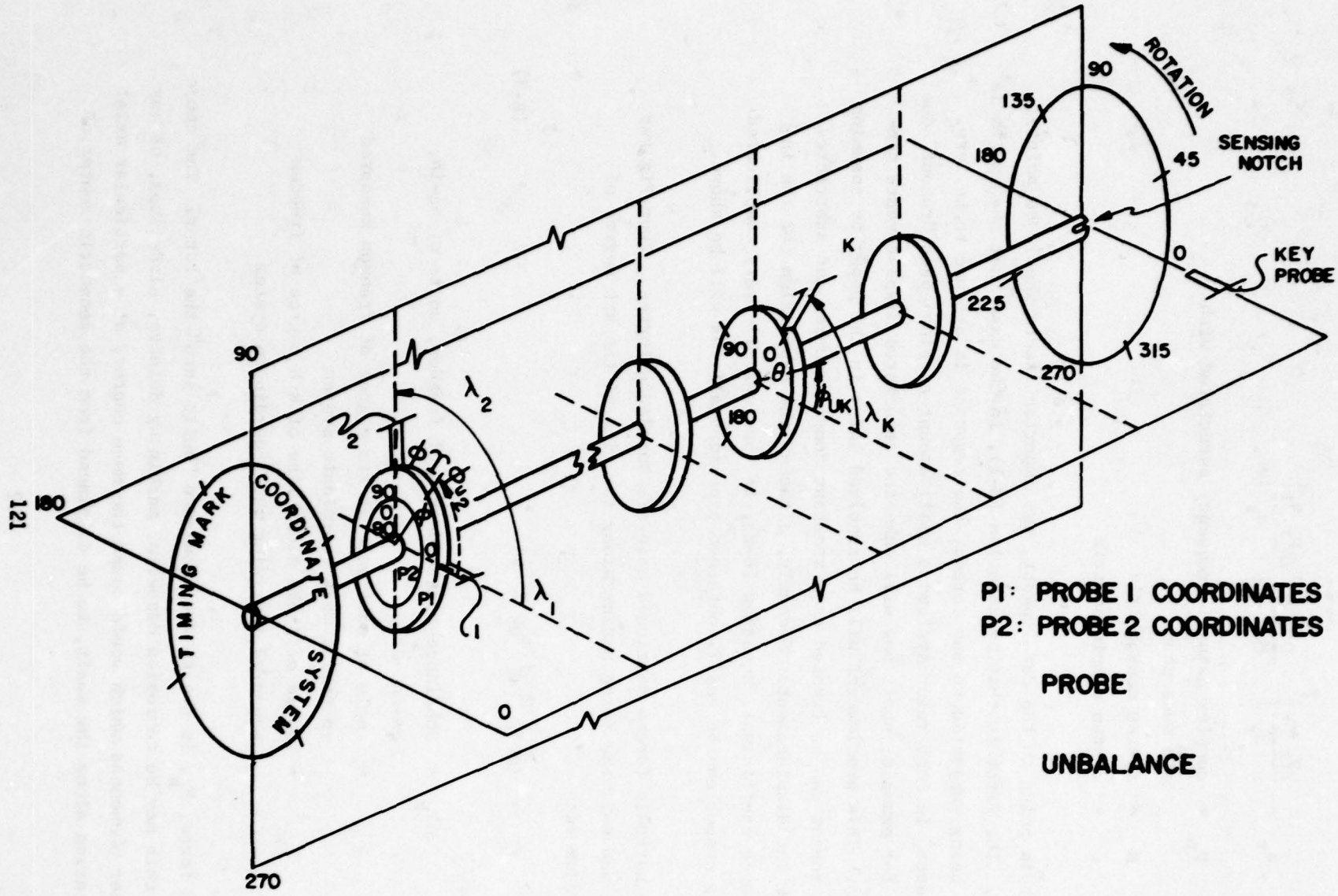


Figure B-1 Mathematical-Disc Model of a Rotor Bearing System Showing Unbalance Planes, Probe Locations, a Timing Mark, and Probe Coordinate Systems

$$a_r = \left\{\begin{matrix} p_r \mu_r \\ \hline \mu_r \end{matrix}\right\}^T \left[\begin{matrix} 0 & \vdots & M \\ \hline M & \vdots & C \end{matrix}\right] \left\{\begin{matrix} p_r q_r \\ \hline q_r \end{matrix}\right\}$$

p_r = complex natural frequency associated with rth mode shape

M = system mass matrix

C = system damping matrix

At this point in the development, two important facts should be pointed out. The first is that by Equation (B-1), in the case that the system is not being subjected to any forces, the response is assumed to be zero. However, in most rotor systems a small amount of slow speed "runout" due to, for example, shaft bow will make the zero force displacements non-zero. This predicament will be resolved later in the paper by assuming the runout can be treated as a constant complex number and subtracted from the displacement. Secondly, although the expression for the influence coefficient, Equation (B-2), is theoretically very complicated this number can be easily obtained experimentally as will be shown.

The harmonic forces mentioned here are unbalance forces. From Figures B-1 and B-2, the force acting along the line of the kth degree of freedom is:

$$F_k = (me)_k \omega^2 e^{i(\phi_{uk} - \lambda_k)} \qquad \text{(B-3)}$$

where

$(me)_k$ = unbalance at kth degree of freedom, units of oz-in, gm-in, etc.

ϕ_{uk} = angle of unbalance at kth degree of freedom measured in timing mark coordinate system

λ_k = angle of probe along line of kth degree of freedom measured in timing mark coordinate system

The force, F_k, is due to the unbalance "built into" the rotor. The cause of this may be corrosion deposits, machining defects, blade loss, or any other phenomena which would cause the mass center, at a particular axial location along the shaft, to be displaced from the geometric center of

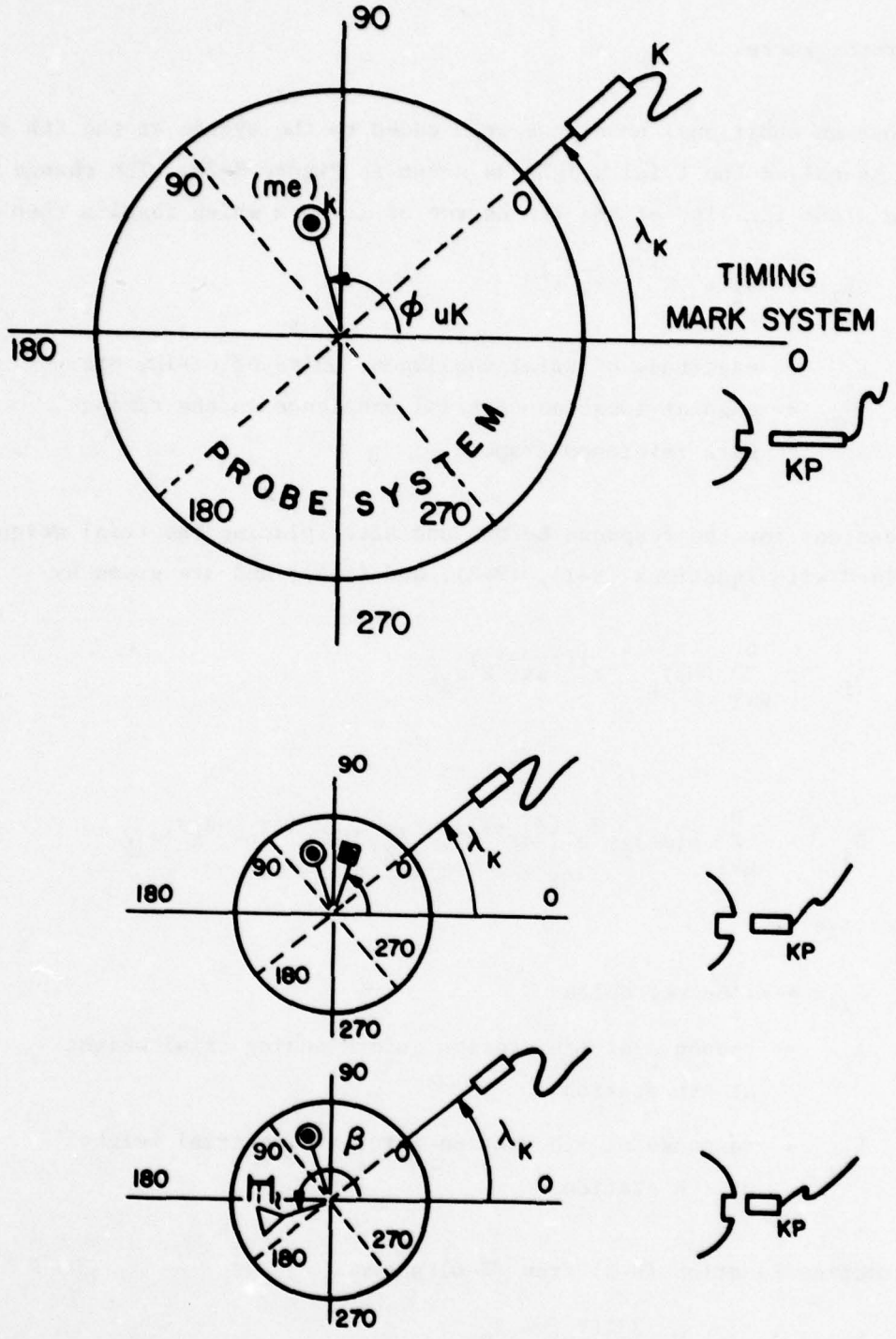


Figure B-2

a) Unknown "Built In" Unbalance at Kth Station
b) Trial Weight Added to ℓth Balance Plane
c) Correction Weight Added to ℓth Balance Plane

* NOTE: Probes usually not located at balance planes, however, these figures are drawn for the general case.

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the rotor there.

Suppose an additional unbalance were added to the system at the ℓth station. This is called the trial weight as shown in Figure B-2b. The change in force along the line of the ℓth degree of freedom which results then is

$$\Delta F_\ell = U_\ell \omega^2 e^{i(\tau_{u\ell} - \lambda_\ell)} \tag{B-4}$$

U_ℓ = magnitude of trial unbalance, units of oz-in, etc.

$\tau_{u\ell}$ = angular location of trial unbalance in the timing mark reference frame

Expressions for the response before and after placing the trial weight are obtained with Equations (B-1), (B-3), and (B-4), and are given by

$$A_i = \sum_{k=1}^{n} (me)_k \omega^2 e^{i(\phi_{uk} - \lambda_k)} \alpha_{ik} \tag{B-5}$$

and

$$B_{i_\ell} = \sum_{k=1}^{n} \{(me)_k \omega^2 e^{i(\phi_{uk} - \lambda_k)} + \delta_{k\ell} U_k \omega^2 e^{i(\tau_{uk} - \lambda_k)}\} \alpha_{ik} \tag{B-6}$$

respectively.

$\delta_{k\ell}$ = Kronecker delta

A_i = response at ith station before adding trial weight at ℓth station

B_{i_ℓ} = response at ith station after adding trial weight at ℓth station

Subtracting Equation (B-5) from (B-6) shows

$$B_{i\ell} - A_i = U_\ell \omega^2 e^{i(\tau_{u\ell} - \lambda_\ell)} \alpha_{i\ell}$$

or

$$\alpha_{i\ell} = \frac{B_{i\ell} - A_i}{U_\ell \omega^2 e^{i(\tau_{u\ell} - \lambda_\ell)}} \tag{B-7}$$

Emphasis should again be noted that these results assume linearity between force and response of the system and the influence coefficient does not change with the additional trial weight. The response at the ith station may now be expressed by substituting Equations (B-3) and (B-7) into (B-1), as

$$X_i = \sum_{k=1}^{n} (me)_k \omega^2 e^{i(\phi_{uk}-\lambda_k)} \left(\frac{B_{ik}-A_i}{U_k \omega^2 e^{i(\tau_{uk}-\lambda_k)}} \right) \tag{B-8}$$

Though this summation extends over all n degrees of freedom modeled in the system, many of these may not have any unbalance forces modeled there. Examples of these would be probes monitoring base motion or a probe monitoring a location along the shaft where there is no balance plane. Then if N is the total number of balance planes, Equation (B-8) is rewritten

$$X_i = \sum_{k=1}^{N} (me)_k e^{i\phi_{uk}} \left(\frac{B_{ik}-A_i}{U_k e^{i\tau_{uk}}} \right) \tag{B-9}$$

Then redefining the influence coefficient as

$$\alpha_{ik} = \left(\frac{B_{ik}-A_i}{U_k e^{i\tau_{uk}}} \right) \tag{B-10}$$

and the unknown unbalance at the kth station as

$$G_k = (me)_k e^{i\phi_{uk}} \tag{B-11}$$

the response becomes

$$X_i = \sum_{k=1}^{N} G_k \alpha_{ik} \tag{B-12}$$

As mentioned earlier, the influence coefficient can be easily experimentally measured using Equation (B-10). The coefficient is then the change in response at station i due to adding a trial unbalance at station k divided by the trial unbalance. The units of unbalance are not important,

as long as they are consistent, since they cancel in the response Equation (B-9).

The subscript i has been used to this point as signifying the location where the response is measured; i.e., a probe location. In balancing the rotor, the location and the speed at which the response is measured actually define the measurement. So the subscript i should be considered a measurement label, where a specific measurement consists of probe location and speed. Consider a case when the response is measured at L probe locations and at K speeds at each location. Then the total number of measurements is M = K x L.

If a set of N correction unbalances were added to the balance planes, the resulting response at the ith measurement is, by Equation (B-12)

$$\varepsilon_i = \sum_{\ell=1}^{N} (G_\ell + T_\ell)\alpha_{i\ell} \qquad \text{(B-13)}$$

where

ℓ = dummy index

T_ℓ = $|T_\ell|\, e^{i\beta_\ell}$ see Figure B-2c, correction unbalance, $|T_\ell|$ measured in units consistent with those of the unknown and trial unbalances

This relationship again assumes linearity between the response and force due to the addition of the correction weights. It also assumes the addition of the correction weights does not alter the value of the influence coefficients.

Equation (B-13) may be rewritten

$$\varepsilon_i = A_i + \sum_{\ell=1}^{N} T_\ell \alpha_{i\ell} \qquad \text{(B-14)}$$

where

$$A_i = \sum_{\ell=1}^{N} G_\ell \alpha_{i\ell}$$ = original response at ith measurement before adding any additional weights, minus the synchronous component of slow speed runout, by Equations (B-12) and (B-5).

As mentioned previously, there are M measurements. Then Equation (B-14) represents a set of M equations which may be written in matrix notation as

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_M \end{bmatrix} = \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_M \end{bmatrix} + \begin{bmatrix} \alpha_{11} & \alpha_{12} & \cdots & \alpha_{1N} \\ \alpha_{21} & \alpha_{22} & \cdots & \alpha_{2N} \\ \cdot & & & \\ & \cdot & & \\ & & & \cdot \\ \alpha_{M1} & \alpha_{M2} & \cdots & \alpha_{MN} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ \cdot \\ \cdot \\ \cdot \\ T_N \end{bmatrix}$$

The organization of this equation is more clearly illustrated by the example in Figure B-3.

The sum over all M measurements of the magnitude squared of the residual responses after adding the correction weights is by Equation (B-14)

$$S_M = \sum_{i=1}^{M} |\varepsilon_i|^2 = \sum_{i=1}^{M} \left| A_i + \sum_{\ell=1}^{N} T_\ell \alpha_{i\ell} \right|^2 \qquad \text{(B-15)}$$

$$= \sum_{i=1}^{M} \left(A_i + \sum_{\ell=1}^{N} T_\ell \alpha_{i\ell} \right) \left(\bar{A}_i + \sum_{\ell=1}^{N} \bar{T}_\ell \bar{\alpha}_{i\ell} \right) \qquad \text{(B-16)}$$

where the bar denotes complex conjugate. The goal of the least squares balancing procedure is to minimize S_M by the choice of a special set of correction unbalances. Mathematically, these unbalances are calculated from the condition:

$$\frac{\partial S_M}{\partial \bar{T}_\ell} = 0 \quad (\ell = 1, \ldots, N) \qquad \text{(B-17)}$$

L = 3 = # of probe locations K = 3 = # of speeds

N = 2 = # of balance planes M = KxL = 9 = # of measurements

$$
\begin{bmatrix} \varepsilon_1, (p_1) \\ \varepsilon_2, (p_2) \\ \varepsilon_3, (p_3) \\ \varepsilon_1, (p_1) \\ \varepsilon_5, (p_2) \\ \varepsilon_6, (p_3) \\ \varepsilon_7, (p_1) \\ \varepsilon_8, (p_2) \\ \varepsilon_9, (p_3) \end{bmatrix}
\begin{matrix} \text{1st Speed} \\ \\ \\ \text{2nd Speed} \\ \\ \\ \text{3rd Speed} \\ \\ \end{matrix}
=
\begin{bmatrix} A_1 - A_{01}, (p_1) \\ A_2 - A_{02}, (p_2) \\ A_3 - A_{03}, (p_3) \\ A_1 - A_{01}, (p_1) \\ A_5 - A_{02}, (p_2) \\ A_6 - A_{03}, (p_3) \\ A_7 - A_{01}, (p_1) \\ A_8 - A_{02}, (p_2) \\ A_9 - A_{03}, (p_3) \end{bmatrix}
\begin{matrix} \text{1st Speed} \\ \\ \\ \text{2nd Speed} \\ \\ \\ \text{3rd Speed} \\ \\ \end{matrix}
+
\begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \\ \alpha_{31} & \alpha_{32} \\ \alpha_{41} & \alpha_{42} \\ \alpha_{51} & \alpha_{52} \\ \alpha_{61} & \alpha_{62} \\ \alpha_{71} & \alpha_{72} \\ \alpha_{81} & \alpha_{82} \\ \alpha_{91} & \alpha_{92} \end{bmatrix}
\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}
$$

p_1 → at probe 1

p_2 → at probe 2

p_3 → at probe 3

A_{oj} : low speed synchronous "runout" at jth probe

A_i : original complex response at ith measurement

Figure B-3 Example of Matrix Organization of Response Equations

where $\bar{T}_\ell$ is the complex conjugate of T_ℓ and the differentiation is partial (otherwise it would be improper as S_M is not an analytic function). The differentiation then shows

$$\sum_{i=1}^{M}\left(A_i + \sum_{\ell=1}^{N} T_\ell \alpha_{i\ell}\right)\bar{\alpha}_{i\ell} = 0 \qquad \text{(B-18)}$$

or

$$-\sum_{i=1}^{M} A_i \bar{\alpha}_{i\ell} = \sum_{i=1}^{M} \bar{\alpha}_{i\ell}\left(\sum_{\ell=1}^{N} T_\ell \alpha_{i\ell}\right) \qquad \text{(B-18a)}$$

There are N equations like Equation (18a) since S_M was minimized with respect to a particular correction unbalance, T_ℓ, and there are N of these. The full set of minimization equations can be written in matrix notation as

$$-\begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{21} & \cdots & \bar{\alpha}_{M1} \\ \bar{\alpha}_{12} & & \cdots & \bar{\alpha}_{M2} \\ & \cdots & & \\ \bar{\alpha}_{1N} & \bar{\alpha}_{2N} & \cdots & \bar{\alpha}_{MN} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \cdot \\ \cdot \\ \cdot \\ A_M \end{bmatrix} \qquad \text{(B-19)}$$

$$= \begin{bmatrix} \bar{\alpha}_{11} & \bar{\alpha}_{21} & \cdots & \bar{\alpha}_{M1} \\ \bar{\alpha}_{12} & & \cdots & \bar{\alpha}_{M2} \\ \cdot & & & \\ & \cdot & & \\ & & \cdot & \\ \bar{\alpha}_{1N} & \bar{\alpha}_{2N} & \cdots & \bar{\alpha}_{MN} \end{bmatrix} \times \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1N} \\ \alpha_{21} & \cdots & \alpha_{2N} \\ \cdot & & \\ & \cdot & \\ & & \cdot \\ \alpha_{M1} & \alpha_{M2} & \alpha_{MN} \end{bmatrix} \begin{bmatrix} T_1 \\ \cdot \\ \cdot \\ T_N \end{bmatrix}$$

or

$$-[\bar{\alpha}]^T \{A\} = [\bar{\alpha}]^T [\alpha] \{T\} \qquad \text{(B-20)}$$

Solving for the correction weights then

$$\{T\} = -\left[[\bar{\alpha}]^T [\alpha]\right]^{-1} [\bar{\alpha}]^T \{A\} \qquad \text{(B-21)}$$

The vector {T} contains as components N unbalances predicted to minimize S_M (see Equation B-15). These are complex numbers whose polar form indicate the magnitude and angular location of the correction unbalances

i.e. $T_\ell = |T_\ell| e^{i\beta_\ell}$, and see Figure B-2c

Section B.4 contains the computer program that utilizes this theory to predict correction weights. Along with the program is an input-output description so that the program may be applied to a practical balancing situation.

B.3 RESPONSE MEASUREMENT

Throughout this report the word "response" has been used to indicate a physically observable description of the rotor motion. The response may be an acceleration, velocity, or displacement since the form of the response Equation (B-9) is the same in each case. Regardless of the type of variable used, it must be a complex quantity described by an amplitude and phase angle. The amplitude, R, is easily seen to be the amplitude of the signal indicating the synchronous component of the quantity being measured. The phase angle is less easily understood since it involves grasping the idea of a clock or timing reference. A general mathematical expression may be written for the steady state synchronous response.

$$Xe^{i\omega t} = Re^{i(\omega t+\psi)} \qquad \text{(B-22)}$$

or

$$X = Re^{i\psi}$$

Though this is complex, as were all the quantities in this report, only its real part is observable. That is

$$X_{OBS} = R \cos(\omega t + \psi) \qquad \text{(B-23)}$$

Since in monitoring the rotor motion, it is of no concern when the motion

began, conventional units of time--i.e., seconds--have little significance and the beginning and end of a period of the synchronous motion become the timing reference.

An indicator is needed to signal the end of a period and the beginning of the next. Different schemes are available to perform this task and a clever one is shown in Figure B-1; i.e., the alignment of the sensing notch and key probe. This convention is repeatedly shown in Figures B-1, B-2, and B-4 to emphasize that the "zero" of the timing mark reference frame is in the direction of the key probe only when the notch is aligned with the probe. This point is very important since all unbalance angles are referenced in the direction of rotation from this "zero." Mathematically, this alignment occurs from Equation B-23 at

$$\omega t = 0, 2\pi, 4\pi, \text{ etc.}$$

The motion is still not completely defined though, and its description will only be complete when the fraction of the period between the positive amplitude being attained and the initiation of the period is known. From Equation B-23 this fraction is

$$\frac{(\psi/\omega)}{(2\pi/\omega)} = \frac{\psi}{2\pi}$$

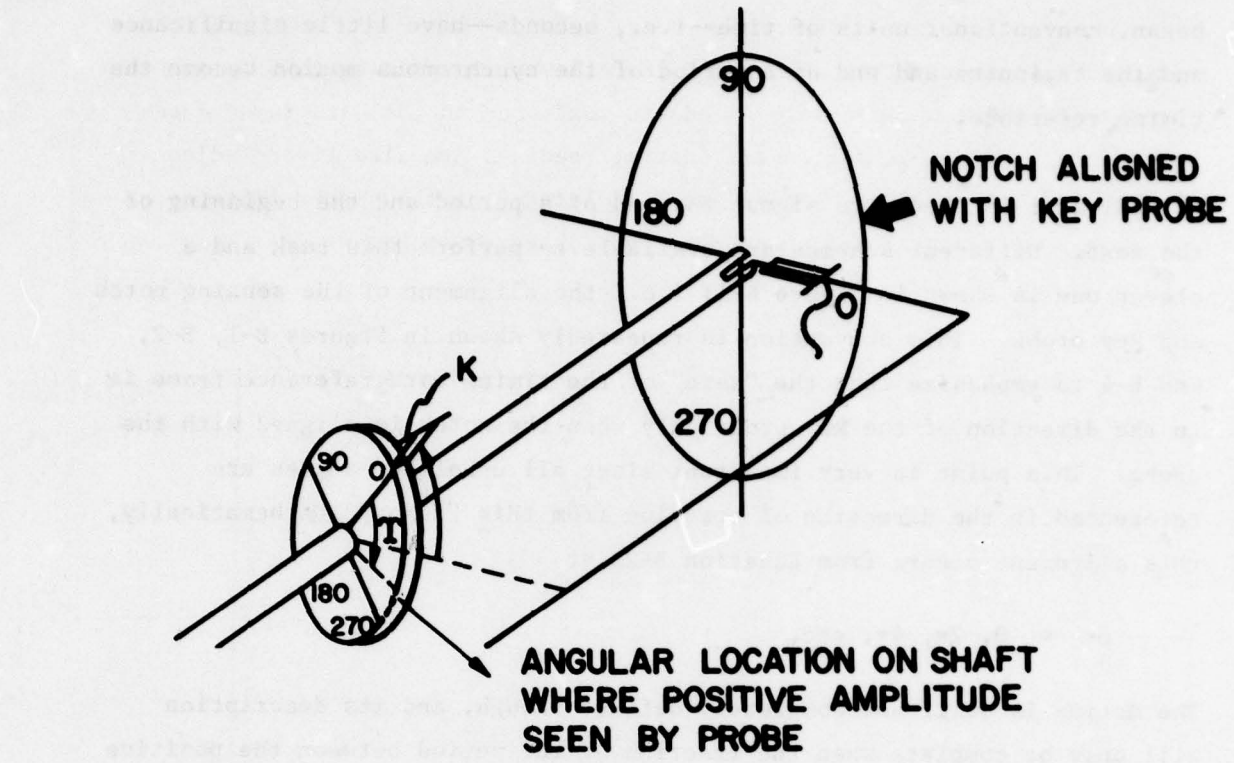
This provides the physical definition of the phase angle, ψ. Some instrumentation indicates the lag of the positive amplitude behind the initiation of a new period, then the response is mathematically described as

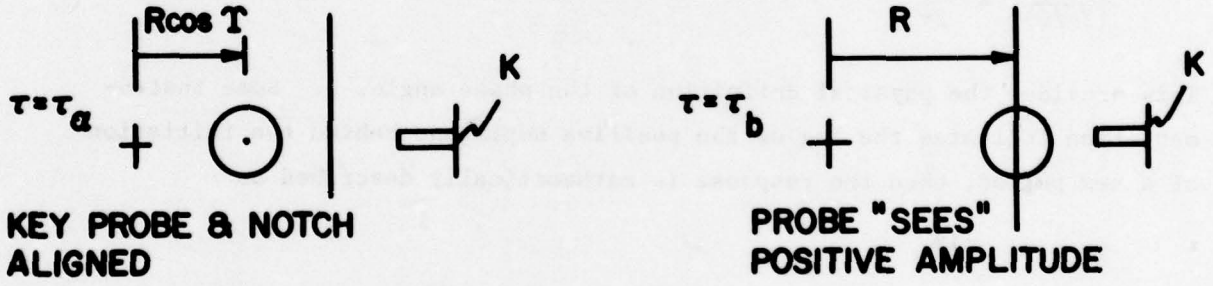
$$X = R\, e^{-i\psi}$$

Figure B-4 may aid in mathematically defining the response in order to use the Least Squares Balancing procedure.

B.4 LEAST SQUARES BALANCING COMPUTER PROGRAM

A large portion of the computer output is simply a listing of the data input for verification purposes. The slow roll and initial (without trial weights) readings are listed first. Note that the order of the initial reading





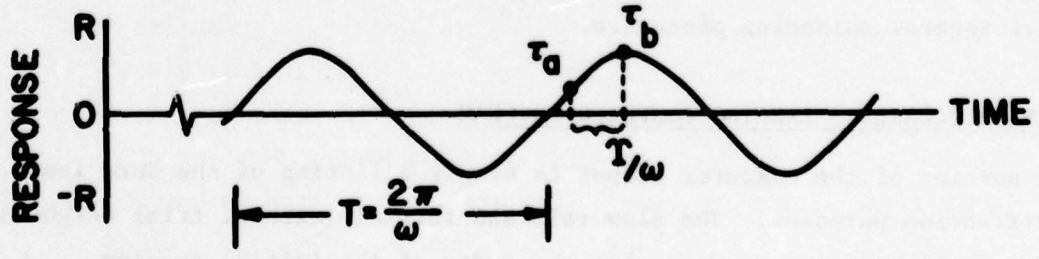


Figure B-4 Measuring Complex Response when Instrumentation Indicates Lag of Positive Amplitude Behind "Timing Mark"

printout is exactly that which must be followed in entering this data. The probes are the inner index to speeds as mentioned in the data input description. The runout subtracted initial readings are also listed adjacent to the initial readings. These are calculated from the vector filter peak to peak amplitude (A) and phase readings (ϕ) as shown below.

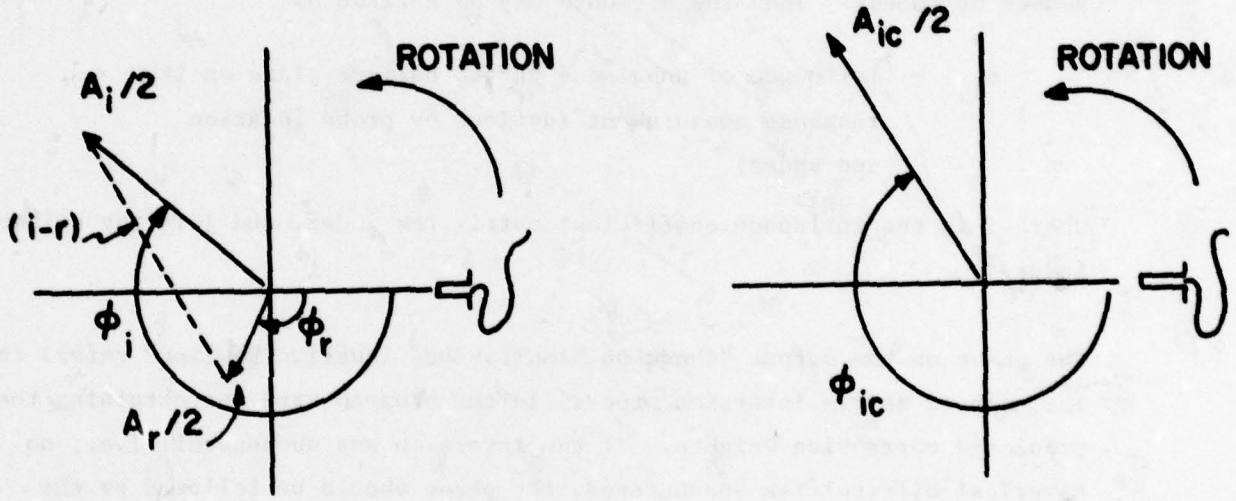


Figure B-5 Runout Correction

The quantities (A_i, ϕ_i) represent the initial readings and (A_r, ϕ_r) represent the slow roll readings. The diagram shows that the runout subtracted initial reading (A_{ic}, ϕ_{ic}) has the same phase angle convention as the vector filter; however, the amplitude represents the peak to peak value divided by 2. The sum of the squares of the initial reading amplitudes (not corrected for runout) are then listed for comparison with the same quantity after the correction weights have been added. This is followed by a listing of the response readings after trial weights have been added to various balance planes. Again, the ordering of this printout is exactly that which must be followed in entering this data as computer program input. As mentioned in the data input description, probe locations are inner indexed to measurement speeds, which are in turn inner indexed to the balance plane in which the trial weight is located. Adjacent to these

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readings are the same but with slow roll subtracted, as explained for the initial readings. The trial unbalances are then listed in magnitude and phase (convention explained in the main body of the report). The complex influence coefficient matrix is also listed with elements given by amplitude and phase. Note that the column dimension is equal to the number of balance planes, while the row dimension is equal to the number of measurements being examined; that is, the number of probe locations times the number of speeds. Then the elements may be defined by

α_{ij} = influence of unbalance at jth balance plane on ith response measurement (defined by probe location and speed)

where i is the influence coefficient matrix row index, and j is the column index.

The phase on the output "Check on Simultaneous Equation Routine" refers to the complex matrix inversion process in the program used for obtaining the predicted correction weights. If the inversion was successful; i.e., no numerical difficulties encountered, the phase should be followed by the same number as balance planes, lines, with a pair of zeros on each line. Though this will probably never occur, if these numbers become on the order of one-tenth or greater, try entering less data until the numbers return to zero.

Next on the output listing appears the predicted correction unbalances. These have the same units as the trial unbalances and are angularly referenced in the same manner. Finally, a list of the computer predicted residual responses (after adding correction weights) including the slow roll runout components is printed. This is followed by a printout of the sum of the squares of the residual response amplitudes since this is the quantity to be minimized in the process.

```
      PROGRAM LTSQAP(INPUT,OUTPUT)                                          00000000
C                                                                           00000100
C        PROGRAM FOR PLAIN LEASTS SQUARES PREDICTION OF BALANCE WEIGHTS     00000200
C        WRITTEN BY ALAN PALAZZOLO  4/20/77                                 00000300
C        REFERENCE LUND AND GOODMAN PAPERS ON BALANCING                     00000400
C                                                                           00000500
C        INPUT DESCRIPTION                                                  00000600
C                                                                           00000700
C        CARDS 1 ( 3 OF THESE )                                             00000800
C        HEADING CARDS ON PRINTOUT                                          00000900
C        INCLUDE UNITS OF UNBALANCE AND DISPLACEMENT ON THESE               00001000
C                                                                           00001100
C        CARD 2                                                             00001200
C        N (NUMBER OF UNBALANCE PLANES  MAX 5)  I10                         00001300
C        L (NUMBER OF PROBES  MAX 10)  I10                                  00001400
C        K (NUMBER OF SPEEDS  MAX 10)  I10                                  00001500
C                                                                           00001600
C        CARDS 3 ( L OF THESE )  RUNOUT AT PROBES                           00001700
C        AMPR : AMPLITUDE  P-P/2  F10.3                                     00001800
C        PHASER : PHASE LAG FROM VECTOR FILTER IN DEGREES  F10.3            00001900
C                                                                           00002000
C        CARDS 4 (K TIMES L OF THESE ) INITIAL PROBE READINGS               00002100
C        ENTER PROBE READINGS AT EACH SPEED                                 00002200
C        AMPB : AMPLITUDE  P-P/2  F10.3                                     00002300
C        PHASEB : PHASE LAG IN DEGREES  F10.3                               00002400
C                                                                           00002500
C        CARDS 5 ( N TIMES K TIMES L OF THESE ) PROBE DATA AFTER ADDING     00002600
C        TRIAL WEIGHTS                                                      00002700
C        PROBES ARE THE INNER INDEX TO SPEEDS WHICH IS THE INNER            00002800
C        INDEX TO UNBALANCE PLANES                                          00002900
C        AMPA : AMPLITUDE  P-P/2  F10.3                                     00003000
C        PHASEA : PHASE LAG IN DEGREES  F10.3                               00003100
C                                                                           00003200
C        CARDS 6 ( N OF THESE )  TRIAL WEIGHT DATA                          00003300
C        UAMP : MAGNITUDE OF TRIAL UNBALANCE  F10.3                         00003400
C        UANG : ANGLE IN DEGREES  F10.3                                     00003500
C                                                                           00003600
                                                                            --------
C        ALL UNBALANCE ANGLES REFERENCED TO DIRECTION OF KEY PROBE          00003700
```

```
C         AND IN THE DIRECTION OF ROTATION                                    00003800
C                                                                             00003900
C         ALL PROBE ANGLES ARE THE PHASE ANGLE LAG OF THE AMPLITUDE           00004000
C         SEEN BY THE PROBE BEHIND THE TIMING MARK                            00004100
C                                                                             00004200
C                                                                             00004300
C*************************************************************************** 00004400
      DIMENSION ICENT(3,8),AMPINF(5),ANGINF(5)                                00004500
      COMPLEX C(10),DUMMY,FINAL(10,10),SYST2(15,15),CC(10),                   00004600
     6 STORE(15,15),TRUNBAL(10),ALPHA(100,10),SUM,RUNOUT(10),CMPLX,           00004700
     6 DI(10,10),TEMP,CONJG,CHECKER,DAF(10,10,10),SUBRUN                      00004800
C         LIMIT 10 PROBES  10 SPEEDS   5 UNBALANCE PLANES                     00004900
C         N IS THE NUMBER OF UNBALANCE PLANES, L IS THE NUMBER OF PROBES      00005000
C         K IS THE NUMBER OF SPEEDS                                           00005100
      PRINT 66                                                                00005200
   66 FORMAT(1H1,*PROGRAM FOR PLAIN LEAST SQUARES PREDICTION*)                00005300
      PRINT 67                                                                00005400
   67 FORMAT(1HO,* OF CORRECTION UNBALANCES . WRITTEN BY ALAN PALAZZOLO        00005500
     64/28/77  REFERENCE LUND AND GOODMAN PAPERS*)                            00005600
      DO 81 J=1,3                                                             00005700
      READ 83,(ICENT(J,I),I=1,8)                                              00005800
   83 FORMAT(8A10)                                                            00005900
   81 CONTINUE                                                                00006000
      PRINT 77,((ICENT(J,I),I=1,8),J=1,3)                                     00006100
   77 FORMAT(1H-,3(10X,8A10/1HO))                                             00006200
      READ 1,N,L,K                                                            00006300
    1 FORMAT(3I10)                                                            00006400
      M=L*K                                                                   00006500
      PRINT 5,N,L,K,M                                                         00006600
    5 FORMAT(1H-,8X,*NUMBER OF BALANCE PLANES*,I3,5X,*NUMBER OF PROBES*,      00006700
     6 I3,5X,*NUMBER OF SPEEDS*,I3,5X,*NUMBER OF MEASUREMENTS TO MINIMIZ      00006800
     6E*,I3)                                                                  00006900
C         READ IN RUNOUT AMPLITUDE AND PHASE ( DEGREES) AT EACH PROBE         00007000
      PRINT 12                                                                00007100
   12 FORMAT(1H-,30X,*RUNOUT,AMPLITUDE...LAG ANGLE IN DEGREES*)               00007200
      DO 10 I=1,L                                                             00007300
      READ 11,AMPR,PHASER                                                     00007400
   11 FORMAT(2F10.3)                                                          00007500
```

```
      PRINT 13,AMPR,PHASER                                                  00007600
   13 FORMAT(1HO,25X,F10.3,5X,F10.3)                                        00007700
      PHASER=PHASER/57.2958                                                 00007800
      A1=AMPR*COS(PHASER)                                                   00007900
      A2=-AMPR*SIN(PHASER)                                                  00008000
      RUNOUT(I)=CMPLX(A1,A2)                                                00008100
   10 CONTINUE                                                              00008200
C        ENTER INITIAL PROBE DATA                                           00008300
      PRINT 15                                                              00008400
   15 FORMAT(1H-,5X,*INITIAL READINGS,AMPLITUDE...LAG ANGLE IN DEGREES*,    00008500
     615X,*SAME EXCEPT RUNOUT SUBTRACTED*)                                  00008600
      SUMSQB=0.0                                                            00008700
      DO 20 I=1,K                                                           00008800
C        NOMBER OF SPEEDS INDEX                                             00008900
      DO 22 J=1,L                                                           00009000
C        NUMBER OF PROBES INDEX                                             00009100
      READ 24,AMPB,PHASEB                                                   00009200
   24 FORMAT(2F10.3)                                                        00009300
      STPHB=PHASEB                                                          00009400
      PHASEB=PHASEB/57.2958                                                 00009500
      AB1=AMPB*COS(PHASEB)                                                  00009600
      AB2=-AMPB*SIN(PHASEB)                                                 00009700
      DI(I,J)=CMPLX(AB1,AB2)                                                00009800
C        SUBTRACT RUNOUT FROM INITIAL READINGS FOR PRINTOUT                 00009900
      SUBRUN=DI(I,J)-RUNOUT(J)                                              00010000
      D1=REAL(SUBRUN)                                                       00010100
C        PRINT THIS OUT JUST LIKE VECTOR FILTER                             00010200
      D2=-AIMAG(SUBRUN)                                                     00010300
           IF(D1.EQ.0.0.AND.D2.EQ.0.0)  D3=0.0                              00010400
           IF(D1.EQ.0.0.AND.D2.EQ.0.0) GO TO 42                             00010500
      D3=ATAN2(D2,D1)*57.2958                                               00010600
   42 CONTINUE                                                              00010700
      D4=CABS(SUBRUN)                                                       00010800
      IF(D3.LT.0.0) D3=D3+360.0                                             00010900
      PRINT 16,I,J,AMPB,STPHB,D4,D3                                         00011000
   16 FORMAT(1HO,5X,*SPEED*,I3,5X,*PROBE*,I3,10X,F10.3,5X,F10.3,            00011100
     610X,F10.3,5X,F10.3)                                                   00011200
      SUMSQB=SUMSQB+AMPB*AMPB                                               00011300
```

```
   22 CONTINUE                                                          00011400
   20 CONTINUE                                                          00011500
      PRINT 505,SUMSQB                                                  00011600
  505 FORMAT(1H-,*THE SUM OF THE AMPLITUDE SQUARES OF THE INITAL READING00011700
     6S IS *,F12.4)                                                     00011800
C        ENTER FINAL PROBE DATA                                         00011900
C        ASUMMING THIS ORDER  ...  PROBES , SPEEDS , UNBALANCE          00012000
      PRINT 27                                                          00012100
   27 FORMAT(1H-,5X,*READINGS AFTER TRIAL WEIGHT ADDED ,AMPLITUDE ...   00012200
     6LAG ANGLE IN DEGREES*,17X,*SAME EXCEPT RUNOUT SUBTRACTED*)        00012300
      DO 30 I1=1,N                                                      00012400
C        UNBALANCE PLANE INDEX                                          00012500
      DO 40 I2=1,K                                                      00012600
C        SPEED INDEX                                                    00012700
      DO 50 I3=1,L                                                      00012800
C        PROBE INDEX                                                    00012900
      READ 55,AMPA,PHASEA                                               00013000
   55 FORMAT(2F10.3)                                                    00013100
      STPHA=PHASEA                                                      00013200
      PHASEA=PHASEA/57.2958                                             00013300
      AA1=AMPA*COS(PHASEA)                                              00013400
      AA2=-AMPA*SIN(PHASEA)                                             00013500
      DAF(I1,I2,I3)=CMPLX(AA1,AA2)                                      00013600
C        SUBTRACT RUNOUT FROM THESE READINGS FOR PRINTOUT               00013700
      SUBRUN=DAF(I1,I2,I3)-RUNOUT(I3)                                   00013800
      D1=REAL(SUBRUN)                                                   00013900
C        PRINTOUT RUNOUT CORRECTED READING LIKE VECTOR FILTER           00014000
C        SUBRUN IS A REGULAR COMPLEX NUMBER. IN POLAR FORM MAKE ITS     00014100
C        ANGLE THE LAG OF THE POSITIVE AMPLITUDE BEHIND THE TIMING MARK 00014200
      D2=-AIMAG(SUBRUN)                                                 00014300
      IF(D1.EQ.0.0.AND.D2.EQ.0.0)  D3=0.0                               00014400
      IF(D1.EQ.0.0.AND.D2.EQ.0.0) GO TO 44                              00014500
      D3=ATAN2(D2,D1)*57.2958                                           00014600
   44 CONTINUE                                                          00014700
      D4=CABS(SUBRUN)                                                   00014800
       IF(D3.LT.0.0) D3=D3+360.0                                        00014900
      PRINT 35,I1,I2,I3,AMPA,STPHA,D4,D3                                00015000
   35 FORMAT(1HO,7X,*BALANCE PLANE*,I3,5X,                              00015100
```

```
     6*SPEED*,I3,5X,*PROBE*,I3,10X,F10.3,5X,F10.3,10X,F10.3,5X,F10.3)   00015200
  50 CONTINUE                                                           00015300
  40 CONTINUE                                                           00015400
  30 CONTINUE                                                           00015500
C       READ IN TRIAL WEIGHT LOCATIONS                                  00015600
     PRINT 72                                                           00015700
  72 FORMAT(1H-,10X,*TRIAL UNBALANCES , MAGNITUDE...ANGLE IN DEGREES ME00015800
     6ASURED IN THE DIRECTION OF ROTATION FROM THE KEY PROBE DIRECTION*)00015900
     DO 70 I=1,N                                                        00016000
     READ 75,UAMP,UANG                                                  00016100
  75 FORMAT(2F10.3)                                                     00016200
     PRINT 74,I,UAMP,UANG                                               00016300
  74 FORMAT(1HO,5X,*  BALANCE PLANE*,I3,10X,F10.3,5X,F10.3)             00016400
     UANG=UANG/57.2958                                                  00016500
     A1=UAMP*COS(UANG)                                                  00016600
     A2=UAMP*SIN(UANG)                                                  00016700
     TRUNBAL(I)=CMPLX(A1,A2)                                            00016800
  70 CONTINUE                                                           00016900
C       DEFINE INFLUENCE COEFFICIENT MATRIX                             00017000
     IP=1                                                               00017100
     JP=1                                                               00017200
     DO 80 I=1,M                                                        00017300
     DO 90 J=1,N                                                        00017400
     ALPHA(I,J)=(DAF(J,JP,IP)-DI(JP,IP))/TRUNBAL(J)                     00017500
  90 CONTINUE                                                           00017600
C       DEFINE THE PROBE NUMBER IP AND THE SPEED NUMBER                 00017700
C       JP FROM THE CASE NUMBER  I                                      00017800
     IP=IP+1                                                            00017900
     IF(IP.GT.L) IP=1                                                   00018000
     IF(IP.EQ.1) JP=JP+1                                                00018100
  80 CONTINUE                                                           00018200
     PRINT 93                                                           00018300
  93 FORMAT(1H-,*COMPLEX INFLUENCE COEFFICIENT MATRIX UNITS ARE UNITS O00018400
     6F     RESPONSE DIVIDED BY UNITS OF UNBALANCE,MAGNITUDE...PHASE IN 00018500
     6DEGREES*)                                                         00018600
     DO 97 I=1,M                                                        00018700
     DO 99 J=1,N                                                        00018800
     B1=AIMAG(ALPHA(I,J))                                               00018900
```

```
      B2=REAL(ALPHA(I,J))                                                00019000
      AMPINF(J)=CABS(ALPHA(I,J))                                         00019100
      IF(B1.EQ.0.0.AND.B2.EQ.0.0) ANGINF(J)=0.0                          00019200
      IF(B1.EQ.0.0.AND.B2.EQ.0.0) GO TO 60                               00019300
      ANGINF(J)=ATAN2(B1,B2)*57.2958                                     00019400
   60 CONTINUE                                                           00019500
      IF(ANGINF(J).LT.0.0) ANGINF(J)=ANGINF(J)+360.0                     00019600
   99 CONTINUE                                                           00019700
      PRINT 98,(AMPINF(J),ANGINF(J),J=1,N)                               00019800
   98 FORMAT(1HO,5(8X,F8.3,2X,F8.3))                                     00019900
   97 CONTINUE                                                           00020000
C        DEFINE THE CONJUGATE TRANSPOSE INFLUENCE COEFFICIENT MATRIX     00020100
C        POSTMULTIPLIED BY THE INFLUENCE COEFFICIENT MATRIX              00020200
      DO 100 I=1,N                                                       00020300
      DO 110 J=1,N                                                       00020400
      SUM=(0.0,0.0)                                                      00020500
      DO 115 I1=1,M                                                      00020600
      TEMP=ALPHA(I1,I)                                                   00020700
      TEMP=CONJG(TEMP)                                                   00020800
      SUM=SUM+ALPHA(I1,J)*TEMP                                           00020900
  115 CONTINUE                                                           00021000
      SYST2(I,J)=SUM                                                     00021100
  110 CONTINUE                                                           00021200
  100 CONTINUE                                                           00021300
C        DEFINE THE VECTOR INFLUENCE COEFFICIENT CONJUGATE TRANSPOSE     00021400
C        MATRIX TIMES THE RUNOUT SUBTRACTED ORIGINAL VECTOR              00021500
      DO 125 I=1,N                                                       00021600
      SUM=(0.0,0.0)                                                      00021700
      IP=1                                                               00021800
      JP=1                                                               00021900
      DO 130 I1=1,M                                                      00022000
      TEMP=ALPHA(I1,I)                                                   00022100
      TEMP=CONJG(TEMP)                                                   00022200
      SUM=SUM+TEMP*(DI(JP,IP)-RUNOUT(IP))                                00022300
C        DEFINE THE PROBE NUMBER IP AND THE SPEED NUMBER JP              00022400
C        FROM THE CASE NUMBER I1                                         00022500
      IP=IP+1                                                            00022600
      IF(IP.GT.L) IP=1                                                   00022700
```

```
      IF(IP.EQ.1) JP=JP+1                                                00022800
  130 CONTINUE                                                           00022900
      CC(I)=SUM                                                          00023000
  125 CONTINUE                                                           00023100
C         TRANSFER CC INTO THE LAST COLUMN OF SYST2                      00023200
      DO 140 I=1,N                                                       00023300
      SYST2(I,N+1)=CC(I)                                                 00023400
  140 CONTINUE                                                           00023500
C         SOLVE THE SIMULTANEOUS EQ. AND OBTAIN THE                      00023600
C         CORRECTION WEIGHTS , BUT FIRST STORE SYST2                     00023700
      MAT=N+1                                                            00023800
      DO 150 I=1,N                                                       00023900
      DO 160 J=1,MAT                                                     00024000
      STORE(I,J)=SYST2(I,J)                                              00024100
  160 CONTINUE                                                           00024200
  150 CONTINUE                                                           00024300
      CALL SOLVE(SYST2,C,N)                                              00024400
C         RESTORE SYST2 TO ITS ORIGINAL VALUE ( DESTROYED IN SOLVE)      00024500
      DO 170 I=1,N                                                       00024600
      DO 180 J=1,MAT                                                     00024700
      SYST2(I,J)=STORE(I,J)                                              00024800
  180 CONTINUE                                                           00024900
  170 CONTINUE                                                           00025000
      CALL SIMCHEK(SYST2,C,N)                                            00025100
      PRINT 200                                                          00025200
  200 FORMAT(1H-,10X,*CORRECTION UNBALANCES,MAGNITUDE...ANGLE IN DEGREES00025300
     6 MEASURED IN THE DIRECTION OF ROTATION FROM THE KEY PROBE DIRECTI00025400
     6N*)                                                                00025500
      DO 205 I=1,N                                                       00025600
      A1=REAL(C(I))                                                      00025700
      A2=AIMAG(C(I))                                                     00025800
      IF(A2.EQ.0.0.AND.A1.EQ.0.0) A3=0.0                                 00025900
      IF(A2.EQ.0.0.AND.A1.EQ.0.0) GO TO 62                               00026000
      A3=ATAN2(A2,A1)*57.2958                                            00026100
   62 CONTINUE                                                           00026200
      A4=CABS(C(I))                                                      00026300
      IF(A3.LT.0.0) A3=A3+360.0                                          00026400
      PRINT 210,I,A4,A3                                                  00026500
```

```
210 FORMAT(1HO,5X,*UNBALANCE PLANE *,I3,10X,*MAGNITUDE *,F10.3,10X,    00026600
   6 *ANGLE(DEG) *,F10.3)                                              00026700
205 CONTINUE                                                           00026800
    IP=1                                                               00026900
    JP=1                                                               00027000
    DO 300 I=1,M                                                       00027100
    SUM=(0.0,0.0)                                                      00027200
    DO 400 I1=1,N                                                      00027300
    SUM=SUM+ALPHA(I,I1)*C(I1)                                          00027400
400 CONTINUE                                                           00027500
    SUM=SUM+DI(JP,IP)                                                  00027600
    FINAL(JP,IP)=SUM                                                   00027700
    IP=IP+1                                                            00027800
    IF(IP.GT.L) IP=1                                                   00027900
    IF(IP.EQ.1) JP=JP+1                                                00028000
300 CONTINUE                                                           00028100
    PRINT 450                                                          00028200
450 FORMAT(1H-,20X,*RESIDUAL DISPLACEMENTS WITH CORRECTION WEIGHTS ADD00028300
   6ED INCLUDING RUNOUT*)                                              00028400
    SUMSQA=0.0                                                         00028500
    DO 475 I=1,K                                                       00028600
    PRINT 480,I                                                        00028700
480 FORMAT(1HO,5X,*SPEED *,I3)                                         00028800
    DO 490 J=1,L                                                       00028900
    A1=REAL(FINAL(I,J))                                                00029000
    A2=-AIMAG(FINAL(I,J))                                              00029100
    IF(A2.EQ.0.0.AND.A1.EQ.0.0) A3=0.0                                 00029200
    IF(A2.EQ.0.0.AND.A1.EQ.0.0) GO TO 64                               00029300
    A3=ATAN2(A2,A1)*57.2958                                            00029400
 64 CONTINUE                                                           00029500
    A4=CABS(FINAL(I,J))                                                00029600
    IF(A3.LT.0.0) A3=A3+360.0                                          00029700
    PRINT 500,J,A4,A3                                                  00029800
500 FORMAT(1HO,5X,*PROBE *,I3,10X,*MAGNITUDE *,F10.3,                  00029900
   610X,*LAG ANGLE (DEG) *,F10.3)                                      00030000
    SUMSQA=SUMSQA+A4*A4                                                00030100
490 CONTINUE                                                           00030200
475 CONTINUE                                                           00030300
```

```
      PRINT 510,SUMSQA                                                  00030400
  510 FORMAT(1H-,*THE SUM OF THE AMPLITUDE SQUARES OF THE RESIDUAL DISPL00030500
     6ACEMENTS IS *,F12.4)                                              00030600
      END                                                               00030700
```

```
      SUBROUTINE SOLVE(SYST2,C,NORDER)                                  00030800
C        THIS SUBROUTINE SOLVES THE SIMULTANEOUS EQUATIONS CONTAINED    00030900
C        IN MATRIX SYST2 BY DIAGONALIZATION AND BACK SUBSTITUTIONS      00031000
C        WITHOUT PIVOTING                                               00031100
C        THIS IS ONLY GOOD FOR SYSTEMS OF ORDER NORDER-1                00031200
C        SOURCE D.LI  2/77                                              00031300
      COMPLEX DUMMY,CONJG                                               00031400
      COMPLEX SYST2(15,15),C(10)                                        00031500
      NN=NORDER                                                         00031600
      MM=NORDER+1                                                       00031700
      DO 100 I=1,NN                                                     00031800
      K=I                                                               00031900
   15 IF(CABS(SYST2(K,I)).GT.0.000001) GO TO 20                         00032000
      K=K+1                                                             00032100
      IF(K-NN) 15,15,200                                                00032200
   20 IF(I-K) 40,60,200                                                 00032300
   40 DO 50 M=1,MM                                                      00032400
      DUMMY=SYST2(I,M)                                                  00032500
      SYST2(I,M)=SYST2(K,M)                                             00032600
   50 SYST2(K,M)=DUMMY                                                  00032700
   60 II=I+1                                                            00032800
      IF(II.GT.NN) GO TO 100                                            00032900
      DO 70 N=II,NN                                                     00033000
      IF(CABS(SYST2(N,I)).LT.0.000001) GO TO 70                         00033100
      SQ=(CABS(SYST2(I,I)))**2                                          00033200
      DUMMY=SYST2(N,I)*CONJG(SYST2(I,I))/SQ                             00033300
      DO 80 M=1,MM                                                      00033400
   80 SYST2(N,M)=SYST2(N,M)-SYST2(I,M)*DUMMY                            00033500
   70 CONTINUE                                                          00033600
  100 CONTINUE                                                          00033700
      GO TO 300                                                         00033800
  200 IF(I.EQ.NN)  PRINT 500                                            00033900
      IF(I.LT.NN) PRINT 510                                             00034000
  500 FORMAT(1H ,19HMATRIX HAS ZERO ROW)                                00034100
  510 FORMAT(1H ,22HMATRIX HAS ZERO COLUMN)                             00034200
C        THE ROW REMOVED FROM THE SYSTEM IN THE MAIN PROGRAM,           00034300
C        THE LAST ROW IN (LAMBDA*I-D)),WAS NOT REDUNDANT                00034400
C        THE DETERMINANT OF THE REMAINING SYSTEM IS ZERO                00034500
```

```
      STOP                                                              00034600
C        START BACK SUBSTITUTIONS                                       00034700
  300 I=NN                                                              00034800
  320 DUMMY=(0.0,0.0)                                                   00034900
      IF(I.EQ.NN)   GO TO 350                                           00035000
      JJJ=I+1                                                           00035100
      DO 330 J=JJJ,NN                                                   00035200
  330 DUMMY=DUMMY+SYST2(I,J)*C(J)                                       00035300
  350 SQ=(CABS(SYST2(I,I)))**2                                          00035400
      C(I)=-(DUMMY+SYST2(I,MM))*CONJG(SYST2(I,I))/SQ                    00035500
      I=I-1                                                             00035600
      IF(I) 400,400,320                                                 00035700
  400 RETURN                                                            00035800
      END                                                               00035900


C        THIS ROUTINE CHECKS SOLVE (COMPLEX SIMULTANEOUS EQ SOLVER)     00036000
      SUBROUTINE SIMCHEK(SYST2,C,NORDER)                                00036100
      COMPLEX CHECKER,SYST2(15,15),C(10)                                00036200
      N=NORDER+1                                                        00036300
      NN=N-1                                                            00036400
      PRINT 90                                                          00036500
   90 FORMAT(1HO,10X,*CHECK ON SIMULTANEOUS EQUATION ROUTINE*)          00036600
      DO 10 I=1,NN                                                      00036700
      CHECKER=(0.0,0.0)                                                 00036800
      DO 20 J=1,NN                                                      00036900
      CHECKER=CHECKER+SYST2(I,J)*C(J)                                   00037000
   20 CONTINUE                                                          00037100
      CHECKER=CHECKER+SYST2(I,N)                                        00037200
      PRINT 40,CHECKER                                                  00037300
   40 FORMAT(1H ,10X,2F15.5)                                            00037400
   10 CONTINUE                                                          00037500
      RETURN                                                            00037600
      END                                                               00037700
```

SAMPLE OUTPUT

PROGRAM FOR PLAIN LEAST SQUARES PREDICTION

OF CORRECTION UNBALANCES . WRITTEN BY ALAN PALAZZOLO 4/28/77 REFERENCE LUND AND GOODMAN PAPERS

FINAL PROJECT 3 PLANE BALANCE OF LAB ROTOR KIT

3 MASS ROTOR BALANCE SPEEDS 1200,1400,1500,1800,1900,6500,7000,7500

AMPLITIDE IN MILS , UNBALANCE IN GRAM INCHES A.B.P. 5/2/77

NUMBER OF BALANCE PLANES 3 NUMBER OF PROBES 3 NUMBER OF SPEEDS 8 NUMBER OF MEASUREMENTS TO MINIMIZE 24

RUNOUT,AMPLITUDE...LAG ANGLE IN DEGREES

.100	325.000
.150	125.000
1.500	290.000

INITIAL READINGS,AMPLITUDE...LAG ANGLE IN DEGREES			SAME EXCEPT RUNOUT SUBTRACTED	
SPEED 1 PROBE 1	.150	330.000	.051	339.815
SPEED 1 PROBE 2	.450	149.000	.319	160.031
SPEED 1 PROBE 3	1.500	295.000	.131	22.500
SPEED 2 PROBE 1	.150	335.000	.054	353.627
SPEED 2 PROBE 2	.750	155.000	.625	161.896
SPEED 2 PROBE 3	1.450	290.000	.050	110.000
SPEED 3 PROBE 1	.150	330.000	.051	339.815
SPEED 3 PROBE 2	1.000	160.000	.881	165.602

SPEED 3	PROBE 3	1.400	285.000	.161	159.199
SPEED 4	PROBE 1	.500	340.000	.404	343.671
SPEED 4	PROBE 2	1.500	320.000	1.645	318.648
SPEED 4	PROBE 3	3.500	310.000	2.152	323.789
SPEED 5	PROBE 1	.350	345.000	.258	352.609
SPEED 5	PROBE 2	.750	342.000	.874	336.075
SPEED 5	PROBE 3	2.650	311.000	1.360	334.276
SPEED 6	PROBE 1	.600	340.000	.504	342.943
SPEED 6	PROBE 2	.900	340.000	1.026	335.192
SPEED 6	PROBE 3	1.400	303.000	.343	43.338
SPEED 7	PROBE 1	.750	5.000	.676	10.453
SPEED 7	PROBE 2	1.250	10.000	1.320	4.090
SPEED 7	PROBE 3	1.700	286.000	.229	258.806
SPEED 8	PROBE 1	1.150	20.000	1.096	24.287
SPEED 8	PROBE 2	2.900	30.000	2.917	27.064
SPEED 8	PROBE 3	2.100	255.000	1.224	210.361

THE SUM OF THE AMPLITUDE SQUARES OF THE INITAL READINGS IS 52.8900

READINGS AFTER TRIAL WEIGHT ADDED , AMPLITUDE ...				LAG ANGLE IN DEGREES		SAME EXCEPT RUNOUT SUBTRACTED	
BALANCE PLANE 1	SPEED 1	PROBE 1		.150	337.000	.056	358.723
BALANCE PLANE 1	SPEED 1	PROBE 2		.400	145.000	.264	156.202
BALANCE PLANE 1	SPEED 1	PROBE 3		1.500	310.000	.521	30.000
BALANCE PLANE 1	SPEED 2	PROBE 1		.150	337.000	.056	358.723
BALANCE PLANE 1	SPEED 2	PROBE 2		.550	155.000	.427	165.122
BALANCE PLANE 1	SPEED 2	PROBE 3		1.450	305.000	.388	34.836
BALANCE PLANE 1	SPEED 3	PROBE 1		.150	340.000	.059	5.855
BALANCE PLANE 1	SPEED 3	PROBE 2		.700	157.000	.578	164.901
BALANCE PLANE 1	SPEED 3	PROBE 3		1.400	297.000	.203	52.914
BALANCE PLANE 1	SPEED 4	PROBE 1		.350	335.000	.252	338.949
BALANCE PLANE 1	SPEED 4	PROBE 2		1.500	321.000	1.645	319.559
BALANCE PLANE 1	SPEED 4	PROBE 3		2.900	311.000	1.593	330.720
BALANCE PLANE 1	SPEED 5	PROBE 1		.250	337.000	.154	344.779
BALANCE PLANE 1	SPEED 5	PROBE 2		.500	340.000	.629	332.135
BALANCE PLANE 1	SPEED 5	PROBE 3		2.250	309.000	.964	339.420
BALANCE PLANE 1	SPEED 6	PROBE 1		.800	350.000	.711	353.409
BALANCE PLANE 1	SPEED 6	PROBE 2		.950	340.000	1.076	335.415
BALANCE PLANE 1	SPEED 6	PROBE 3		1.100	285.000	.415	123.344
BALANCE PLANE 1	SPEED 7	PROBE 1		1.100	10.000	1.032	13.930
BALANCE PLANE 1	SPEED 7	PROBE 2		2.000	10.000	2.068	6.231
BALANCE PLANE 1	SPEED 7	PROBE 3		1.750	255.000	1.006	196.210
BALANCE PLANE 1	SPEED 8	PROBE 1		2.750	45.000	2.734	47.064
BALANCE PLANE 1	SPEED 8	PROBE 2		6.000	30.000	6.015	28.576

BALANCE PLANE	1	SPEED	8	PROBE	3	5.000	233.000	4.368	216.262
BALANCE PLANE	2	SPEED	1	PROBE	1	.150	205.000	.218	181.587
BALANCE PLANE	2	SPEED	1	PROBE	2	.350	90.000	.243	69.253
BALANCE PLANE	2	SPEED	1	PROBE	3	1.550	225.000	1.639	168.974
BALANCE PLANE	2	SPEED	2	PROBE	1	.100	235.000	.141	190.000
BALANCE PLANE	2	SPEED	2	PROBE	2	.500	105.000	.363	96.868
BALANCE PLANE	2	SPEED	2	PROBE	3	1.450	250.000	1.010	177.334
BALANCE PLANE	2	SPEED	3	PROBE	1	.050	225.000	.119	169.374
BALANCE PLANE	2	SPEED	3	PROBE	2	.600	115.000	.453	111.704
BALANCE PLANE	2	SPEED	3	PROBE	3	1.400	275.000	.391	177.823
BALANCE PLANE	2	SPEED	4	PROBE	1	.350	250.000	.338	233.405
BALANCE PLANE	2	SPEED	4	PROBE	2	1.100	145.000	.960	148.062
BALANCE PLANE	2	SPEED	4	PROBE	3	2.700	255.000	1.704	224.682
BALANCE PLANE	2	SPEED	5	PROBE	1	.300	325.000	.200	325.000
BALANCE PLANE	2	SPEED	5	PROBE	2	.300	160.000	.197	185.907
BALANCE PLANE	2	SPEED	5	PROBE	3	2.250	300.000	.816	318.627
BALANCE PLANE	2	SPEED	6	PROBE	1	.750	325.000	.650	325.000
BALANCE PLANE	2	SPEED	6	PROBE	2	.500	300.000	.650	301.153
BALANCE PLANE	2	SPEED	6	PROBE	3	1.200	298.000	.354	81.816
BALANCE PLANE	2	SPEED	7	PROBE	1	.800	348.000	.709	351.159
BALANCE PLANE	2	SPEED	7	PROBE	2	1.800	305.000	1.950	305.000
BALANCE PLANE	2	SPEED	7	PROBE	3	1.600	260.000	.808	191.865
BALANCE PLANE	2	SPEED	8	PROBE	1	1.500	50.000	1.495	53.822
BALANCE PLANE	2	SPEED	8	PROBE	2	3.000	5.000	3.078	2.581
BALANCE PLANE	2	SPEED	8	PROBE	3	3.000	240.000	2.338	210.558

BALANCE PLANE	3	SPEED	1	PROBE	1	.150	330.000	.051	339.815
BALANCE PLANE	3	SPEED	1	PROBE	2	.350	147.000	.218	161.917
BALANCE PLANE	3	SPEED	1	PROBE	3	1.500	302.000	.314	26.000
BALANCE PLANE	3	SPEED	2	PROBE	1	.150	339.000	.058	3.547
BALANCE PLANE	3	SPEED	2	PROBE	2	.450	150.000	.320	161.412
BALANCE PLANE	3	SPEED	2	PROBE	3	1.450	300.000	.262	35.964
BALANCE PLANE	3	SPEED	3	PROBE	1	.100	341.000	.028	63.000
BALANCE PLANE	3	SPEED	3	PROBE	2	.550	160.000	.436	171.389
BALANCE PLANE	3	SPEED	3	PROBE	3	1.400	299.000	.248	48.160
BALANCE PLANE	3	SPEED	4	PROBE	1	.350	333.000	.251	336.174
BALANCE PLANE	3	SPEED	4	PROBE	2	.600	310.000	.750	309.000
BALANCE PLANE	3	SPEED	4	PROBE	3	2.900	306.000	1.516	321.831
BALANCE PLANE	3	SPEED	5	PROBE	1	.250	335.000	.153	341.538
BALANCE PLANE	3	SPEED	5	PROBE	2	.150	320.000	.297	312.500
BALANCE PLANE	3	SPEED	5	PROBE	3	2.250	306.000	.908	333.096
BALANCE PLANE	3	SPEED	6	PROBE	1	.450	333.000	.351	335.271
BALANCE PLANE	3	SPEED	6	PROBE	2	.300	295.000	.448	298.329
BALANCE PLANE	3	SPEED	6	PROBE	3	1.600	312.000	.600	21.577
BALANCE PLANE	3	SPEED	7	PROBE	1	.450	355.000	.367	2.834
BALANCE PLANE	3	SPEED	7	PROBE	2	.050	18.000	.171	321.196
BALANCE PLANE	3	SPEED	7	PROBE	3	1.900	310.000	.710	356.288
BALANCE PLANE	3	SPEED	8	PROBE	1	.250	345.000	.160	357.364
BALANCE PLANE	3	SPEED	8	PROBE	2	.400	188.000	.358	209.934
BALANCE PLANE	3	SPEED	8	PROBE	3	2.100	330.000	1.354	15.396

TRIAL UNBALANCES , MAGNITUDE...ANGLE IN DEGREES MEASURED IN THE DIRECTION OF ROTATION FROM THE KEY PROBE DIRECTION

BALANCE PLANE 1	.350	90.000
BALANCE PLANE 2	.350	90.000
BALANCE PLANE 3	.350	90,000

COMPLEX INFLUENCE COEFFICIENT MATRIX UNITS ARE UNITS OF RESPONSE DIVIDED BY UNITS OF UNBALANCE,MAGNITUDE...PHASE IN DEGREES

.052	206.500	.760	92.500	0.000	0.000
.166	272.304	1.153	252.959	.288	294.051
1.119	237.500	5.000	98.659	.523	241.500
.015	204.000	.555	84.527	.030	203.000
.571	295.000	1.642	253.215	.869	287.594
1.082	242.500	2.834	90.000	.722	245.000
.075	205.000	.486	103.490	.158	140.208
.866	263.060	2.043	253.613	1.286	290.000
.836	249.000	.697	80.000	.975	248.000
.441	98.604	1.744	75.008	.453	94.384
.075	219.500	7.422	127.885	2.614	123.462
1.722	144.818	8.427	91.421	1.828	121.570
.309	86.239	.352	48.570	.321	82.304
.717	104.012	3.000	108.572	1.753	102.745
1.169	127.931	1.759	94.791	1.295	113.360
.668	253.516	.659	347.337	.465	90.323
.143	290.000	1.739	78.132	2.057	92.861
1.402	103.150	.656	119.921	.881	272.767
1.025	249.505	.670	351.320	.905	70.712

2.143	260.000	4.866	6.696	3.430	80.332
2.638	86.509	2.139	94.478	2.210	256.648
5.073	209.114	2.185	171.239	2.732	61.374
8.857	240.000	3.659	338.128	9.355	62.623
9.008	51.450	3.181	59.224	7.305	247.500

CHECK ON SIMULTANEOUS EQUATION ROUTINE

.00000	-.00000
.00000	-.00000
-.00000	0.00000

CORRECTION UNBALANCES,MAGNITUDE...ANGLE IN DEGREES MEASURED IN THE DIRECTION OF ROTATION FROM THE KEY PROBE DIRECTION

UNBALANCE PLANE	1	MAGNITUDE	.092	ANGLE(DEG)	122.573
UNBALANCE PLANE	2	MAGNITUDE	.112	ANGLE(DEG)	106.412
UNBALANCE PLANE	3	MAGNITUDE	.374	ANGLE(DEG)	100.384

RESIDUAL DISPLACEMENTS WITH CORRECTION WEIGHTS ADDED INCLUDING RUNOUT

SPEED	1				
PROBE	1	MAGNITUDE	.070	LAG ANGLE (DEG)	320.061
PROBE	2	MAGNITUDE	.226	LAG ANGLE (DEG)	133.595
PROBE	3	MAGNITUDE	1.141	LAG ANGLE (DEG)	291.454
SPEED	2				
PROBE	1	MAGNITUDE	.092	LAG ANGLE (DEG)	333.259
PROBE	2	MAGNITUDE	.218	LAG ANGLE (DEG)	146.209
PROBE	3	MAGNITUDE	1.334	LAG ANGLE (DEG)	294.491
SPEED	3				
PROBE	1	MAGNITUDE	.060	LAG ANGLE (DEG)	6.771
PROBE	2	MAGNITUDE	.245	LAG ANGLE (DEG)	170.168
PROBE	3	MAGNITUDE	1.472	LAG ANGLE (DEG)	299.163

*U.S.Government Printing Office: 1979 — 657-084/389