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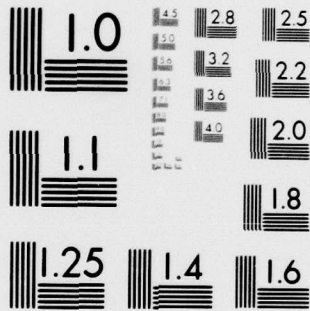
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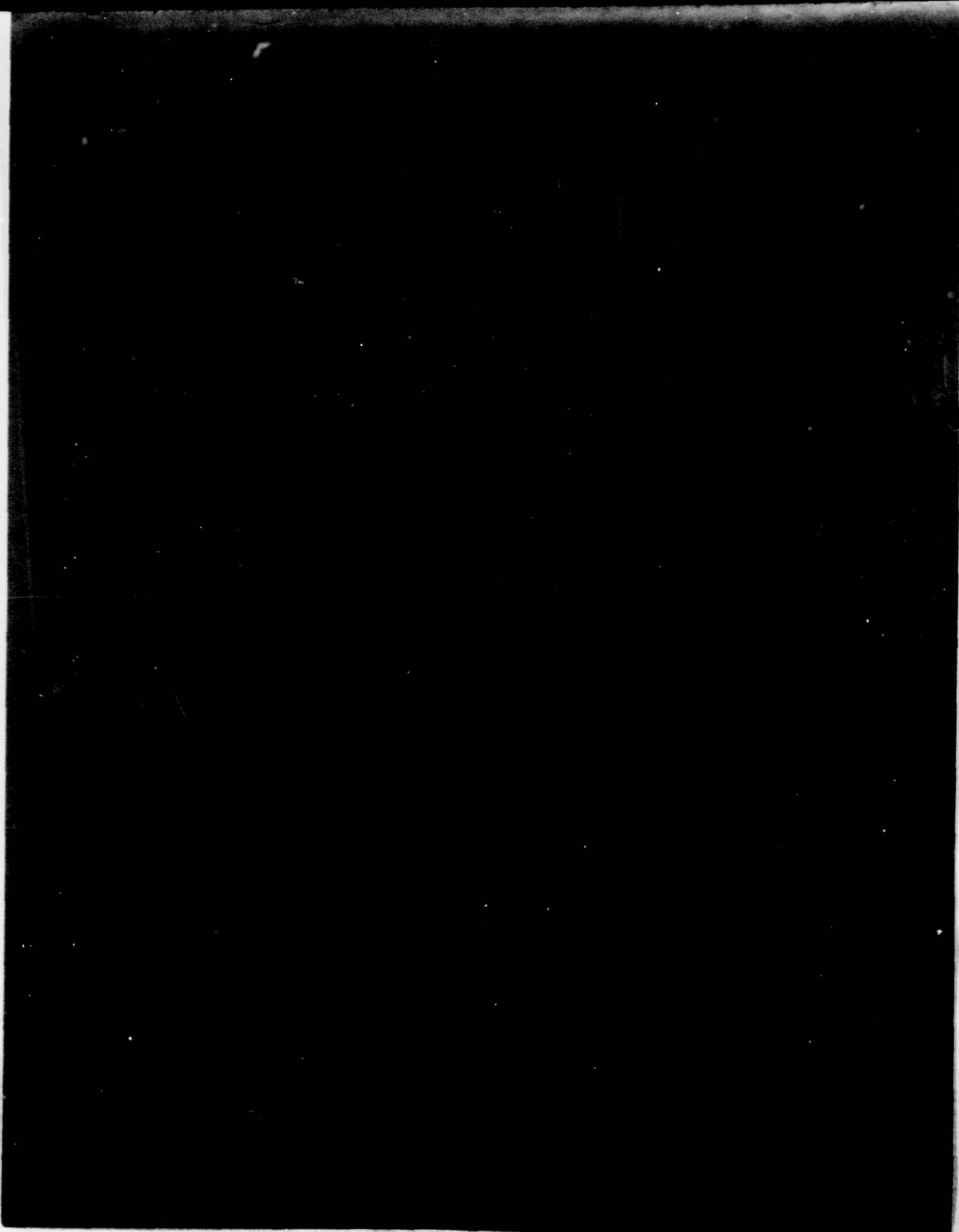
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

INCLINED ELLIPTICAL ORBIT AND ASSOCIATED
SATELLITE FIELD OF VIEW

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Group 61

TECHNICAL NOTE 1979-62

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CONTENTS

I.	INTRODUCTION	1
II.	GENERAL CONSIDERATIONS	2
III.	RESULTS	4
IV.	DETAILED MATHEMATICS	8

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I. INTRODUCTION

Satellites operating in the inclined elliptical orbits are used to cover the polar regions. The inclined synchronous altitude circular orbits have a range of 19,323 NM (35,805 km) and a field of view (the angle subtended by earth at the satellite) of 17.2° . However, a 12-hour Molniya Orbit has a varying range, and the field of view also varies as the satellite moves along the orbit. The purpose of this Technical Note is to place the range and field of view in evidence.

II. GENERAL CONSIDERATIONS

A. Only the inclined elliptical 12-hour orbits are considered. The inclination between the plane of the orbit and the plane of the equator describes the orientation of the orbit, and does not affect the range nor the field of view of the satellite.

B. The general rules used are Kepler's three laws applied to the orbital flight. The first law says that the orbit of the satellite is an ellipse with earth at one focus. The second law says that the area swept by the line joining satellite and earth in unit time is a constant. The third law says that the square of the period of the satellite is proportional to the cube of the semi-major axis of the orbit.

C. Figure 1 illustrates the geometry of the orbit, where R is the radius of the earth, $R=3441.66\text{NM}$; R_1, R_3 are the distances from the satellite to the edges of the earth, $R_1=R_3$; R_2 is the distance from the satellite to the center of the earth; $\theta_1, \theta_2, \theta_3$ are angles between the major axis and R_1, R_2, R_3 , respectively; α is the angle subtended by earth at the satellite; a is semi-major axis; h_a is the apogee altitude and h_p is the perigee altitude.

D. We want to plot the variation of the satellite field of view α , the satellite altitude (R_2-R), and the distance from the satellite to the edge of the earth R_1 , as a function of time since apogee passage.

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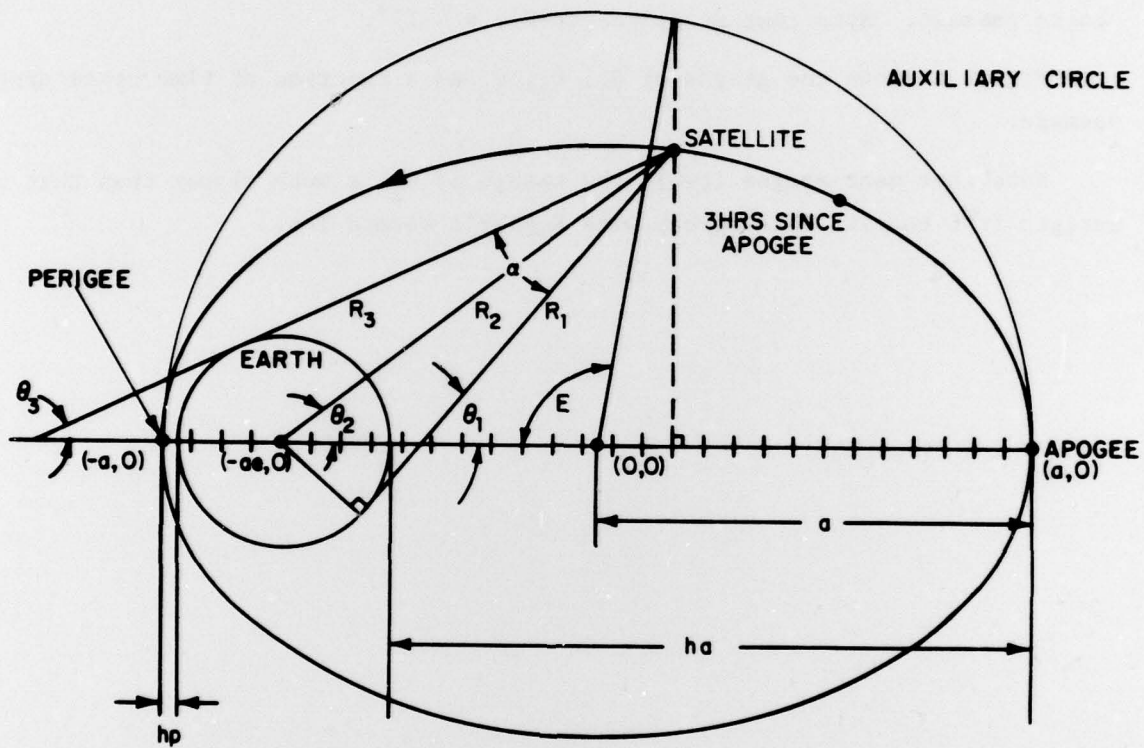


Fig. 1. Geometry of orbit.

III. RESULTS

Figure 2 plots satellite altitude ($R_2 - R$) and the distance from the satellite to the edge of the earth R_1 (or R_3), versus time since apogee passage.

Figure 3 shows the change of field of view α as a function of time since apogee passage. Note that at apogee ($t=0$), $\alpha \sim 17^\circ$.

Figure 4 shows the graphs of θ_1 , θ_2 , θ_3 as a function of time since apogee passage.

Note that near apogee ($t=0$), the change of θ_2 is much slower than that near perigee ($t=6$ hour). This agrees with Kepler's second law.

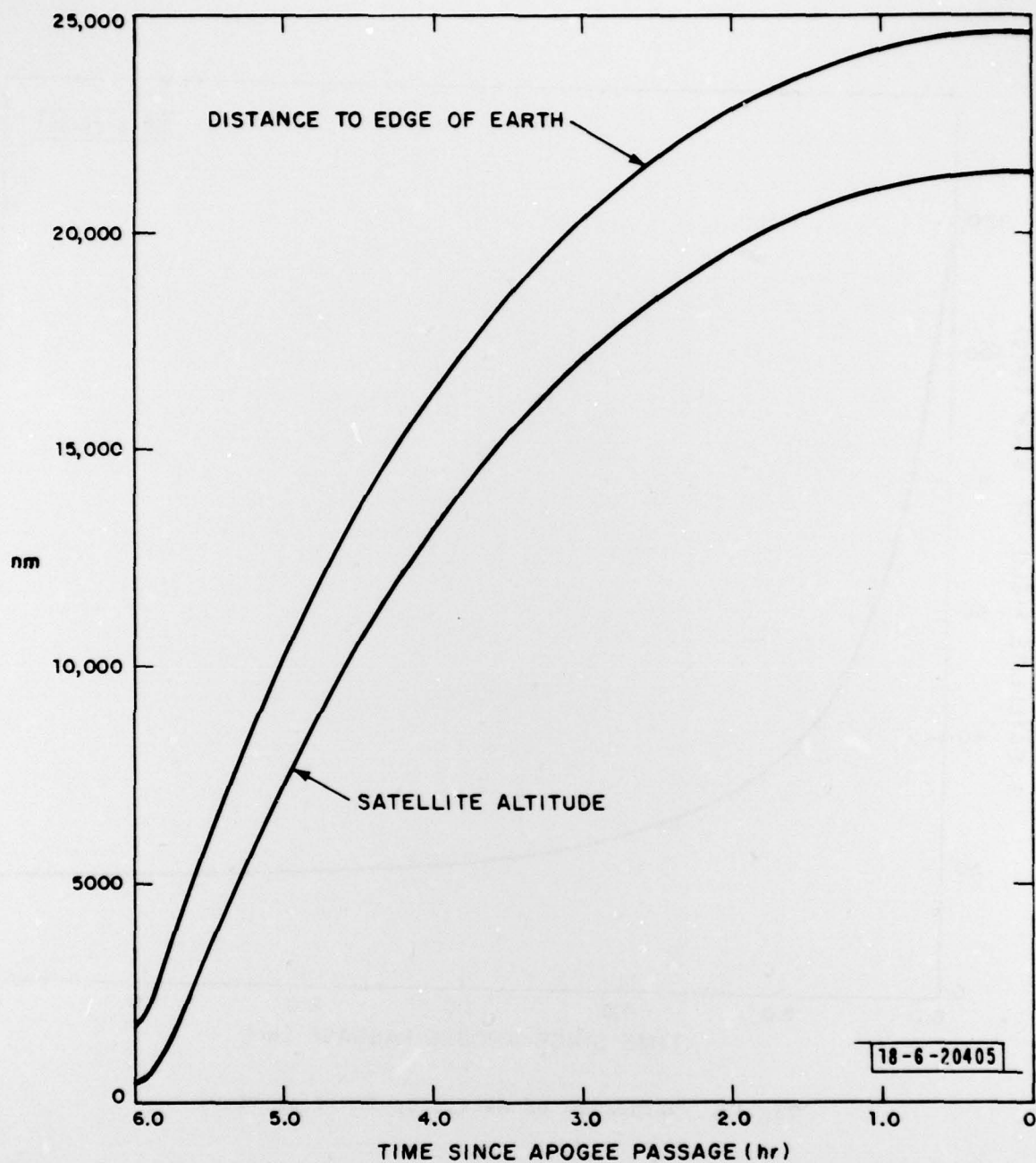


Fig. 2. Variations of satellite altitude and distance from satellite to the edges of the earth.

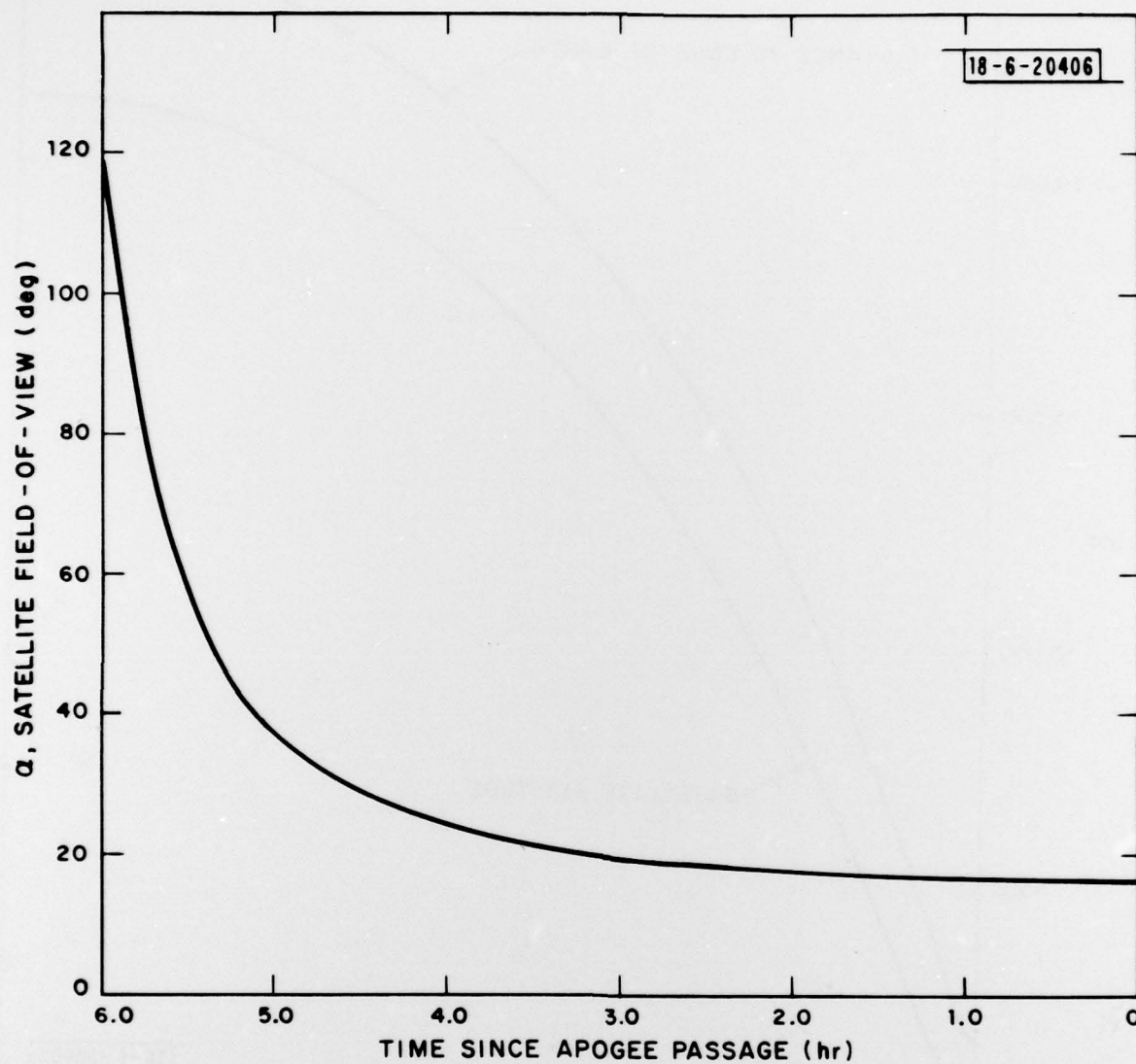


Fig. 3. Variation of satellite field of view.

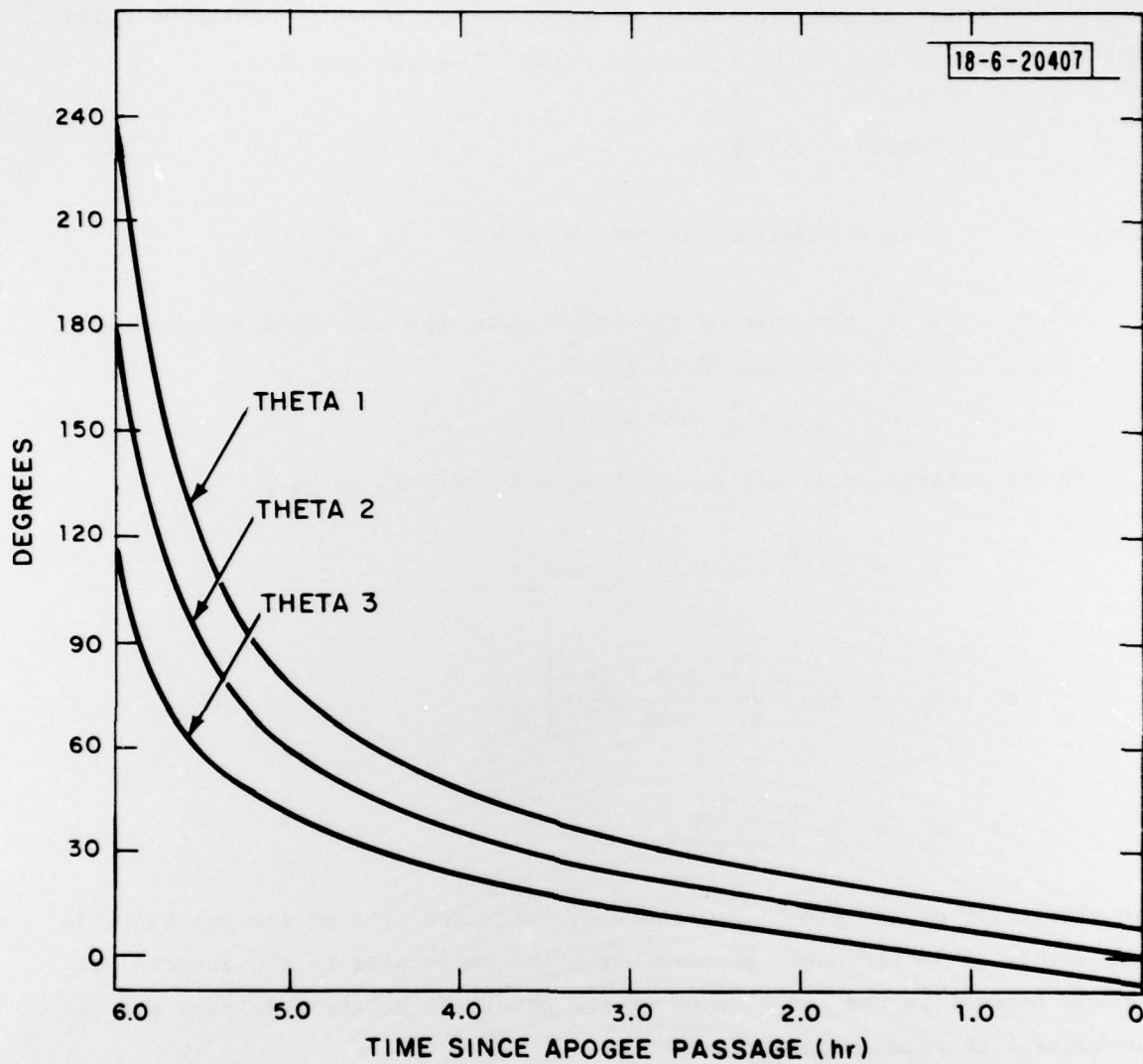


Fig. 4. Variations of the angles θ_1 , θ_2 and θ_3 .

IV. DETAILED MATHEMATICS

A. The set of equations used to generate the plots is explained below^{*}.
By Kepler's third law, we have for the period T of the satellite

$$T(\text{sec}) = 2\pi \sqrt{\frac{a^3}{\mu}}$$

where a is semi-major axis of the orbit.

μ = the mass of the earth times the universal gravitational constant in ft^3/sec^2 .

$$= 1.40766 \times 10^{16} \text{ ft}^3/\text{sec}^2$$

By the definition of ellipse with eccentricity e , we have

$$R_2 = e\left(\frac{a}{e} - ae\right) + R_2 \cos \theta_2$$

$$\text{or } \theta_2 = \cos^{-1} \left[\frac{R_2 - a + ae^2}{eR_2} \right]$$

$$\text{and } e = \frac{h_a - h_p}{h_a + h_p + 2R}$$

For simplicity of the equations, the eccentric anomaly E of the satellite is introduced. E is the angle measured from the major axis to the outward projection (normal to the major axis) of the satellite on the auxiliary circle. The angle E is related to θ_2 by

$$\sin E = \frac{\sqrt{1-e^2} \sin \theta_2}{1 + e \cos \theta_2}$$

$$\cos E = \frac{e + \cos \theta_2}{1 + e \cos \theta_2}$$

^{*} Flight Performance Handbook for Orbital Operations, edited by R. W. Wolverson, Space Technology Laboratories, Inc. (Wiley, New York, 1961).

Using E, the following basic equation of the motion of the satellite can be derived from Kepler's second law,

$$R_2 = a(1 - e \cos E)$$

The time t since apogee passage can also be expressed in terms of E,

$$t = T/2 - (T/2\pi)(E - e \sin E) \quad .$$

B. Computations are carried out as follows. For a 12-hour orbit, period T=12 hours, therefore, from the equation

$$T(\text{sec}) = 2\pi\sqrt{\frac{a^3}{\mu}} \quad , \quad \mu = 1.40766 \times 10^6 \text{ ft}^3/\text{sec}^2$$

we get

$$12 \times 3600 = 2\pi\sqrt{\frac{a^3}{\mu}}$$

or

$$a = \left(\frac{12 \times 3600}{2\pi} \times \sqrt{\mu}\right)^{2/3} \text{ ft}$$

$$= 87304207.72 \text{ ft}$$

or

$$a = 14359.24 \text{ NM} \quad .$$

Note that

$$h_a + h_p + 2R = 2a$$

and

$$R = 3441.66 \text{ NM} .$$

Hence

$$h_a = 2a - 2R - h_p = 21835.16 - h_p$$

for a 12-hour orbit.

Therefore, for perigee altitude h_p varying from 200 NM to 400 NM, the apogee altitude h_a varies less than 1 percent. Choosing $h_p = 400$ NM, we can compute the apogee altitude h_a and hence the eccentricity e from the following equations;

$$h_a = 2a - 2R - h_p$$

$$e = \frac{h_a - h_p}{h_a + h_p + 2R} .$$

As the satellite travels from apogee to perigee, the eccentric anomaly E changes from 180° to 0° . Therefore, using the following equations,

$$R_2 = a(1 - e \cos E)$$

$$t = 12/2 - (12/2\pi)(E - e \sin E)$$

$$R_1 = \sqrt{R_2^2 - R^2}$$

we can plot the distance R_1 from the satellite to the edge of the earth, and the satellite altitude ($R_2 - R$) as a function of time t since apogee passage (see Fig. 2).

Also from the geometry of the orbit, we have

$$\alpha = 2 \sin^{-1}(R/R_2)$$

$$\theta_1 = \theta_2 + \alpha/2$$

$$\text{and } \theta_3 = \theta_2 - \alpha/2$$

where

$$\theta_2 = \cos^{-1} \left[\frac{R_2 - a + ae^2}{eR_2} \right] .$$

Therefore we can also plot the satellite field of view α and the three angles $\theta_1, \theta_2, \theta_3$ versus time t since apogee passage (see Figs. 3 and 4).

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