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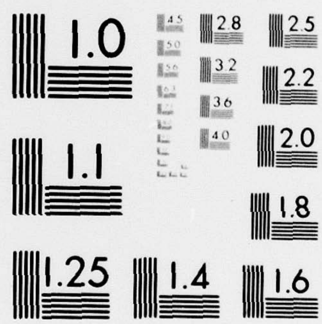

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DIVISION OF ENGINEERING AND WEAPONS

Report EW-8-79

Synthesis of Optimal Filters Using Microcomputers

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August 1979

ABSTRACT

This project deals with optimal synthesis of an RC high-pass ladder network. It shows how to obtain a network with the maximum possible gain and minimum total series capacitance. The current transfer function, source and load conductances are specified. Lagrange multipliers method is used to obtain optimization. The entire process is completed using a microcomputer which can handle a large number of poles and can perform the optimization in a very short time.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This project deals with optimal synthesis of an RC high-pass ladder network. It shows how to obtain a network with the maximum possible gain and minimum total series capacitance. The current transfer function, source and load conductances are specified. Lagrange multipliers method is used to obtain optimization. The entire process is completed using a microcomputer which can handle a large number of poles and can perform the optimization in a very short time.

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Although this project was carried out and completed in recent months, the idea was conceived several years ago while I was involved in a project funded by the U. S. Naval Academy Research Council (NARC). I would like to take this opportunity to express my appreciation.

Credit is due to Midshipman Richard A. Medley who spent many hours in his spare time to study the optimal synthesis and wrote the computer program.

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I. INTRODUCTION

Due to our increasing dependence on integrated circuit technology, optimal synthesis of filters has become more and more important. The size of the chip is directly proportional to the amount of resistance and capacitance contained within. Thus, by minimizing these values, one can minimize the cost of each chip. This method, however, does not necessarily reduce the overall cost of production. The man-hours involved in arriving at these optimum values may more than offset the savings realized by smaller chips. This is especially true as the complexity of the filter increases. To alleviate this problem, one can use a microcomputer to aid in design. This reduction in man-hours makes optimal synthesis economically desirable.

The optimal synthesis of a double-terminated high-pass RC filter is very tedious if done by hand. The whole process, however, can be done in only a fraction of the time using a microcomputer which has some form of BASIC. The program is limited only by the storage capacity and accuracy of the microcomputer.

II. SYNTHESIS PROCEDURES AND MICROCOMPUTER APPLICATION

To synthesize a double-terminated high-pass RC filter, as depicted in Fig. 1, one must first specify a high-pass current transfer function of the form:

$$I_{12}(s) = \frac{I_o}{I_s} = \frac{Hs^n}{(s + p_1)(s + p_2)\cdots(s + p_n)}$$

where the poles are simple and on the negative real axis. The source and load conductances must also be specified. From the current transfer function one can find the open-circuit impedance.

$$z_{12} = \frac{1}{g_\ell} I_{12}(s) = \frac{H}{g_\ell} \frac{s^n}{(s + p_1)(s + p_2)\cdots(s + p_n)}$$

This then can be expanded into partial fractions

$$z_{12} = \frac{H}{g_\ell}\left[1 + \frac{k_{12}^{(1)}}{s + p_1} + \frac{k_{12}^{(2)}}{s + p_2} + \cdots + \frac{k_{12}^{(n)}}{s + p_n}\right]$$

The residues of each pole can now be determined.

$$k_{12}^{(i)} = (s + p_i) \cdot \left.\frac{z_{12} g_\ell}{H}\right|_{s = -p_i} \qquad i = 1,2,3,\ldots n$$

The calculation of the residues of z_{12} takes place in lines 350-430 of the example program shown in the Appendix, and are contained in the list A(I).

If the network is compact then:

$$k_{11}^{(i)} \cdot k_{22}^{(i)} = |k_{12}^{(i)}|^2$$

where $k_{11}^{(i)}$ are the residues of z_{11} and $k_{22}^{(i)}$ are the residues of z_{22}.

Let $|k_{12}^{(i)}| = \alpha_i$, $k_{11}^{(i)} = x_i$ and $k_{22}^{(i)} = \frac{\alpha_i^2}{x_i}$

From the previously defined quantities z_{11} has the form:

$$z_{11} = \frac{H}{g_\ell}\left[1 + \frac{x_1}{s+p_1} + \frac{x_2}{s+p_2} + \cdots + \frac{x_n}{s+p_n}\right]$$

Also z_{22} has the form:

$$z_{22} = \frac{H}{g_\ell}\left[1 + \frac{\alpha_1^2/x_1}{s + p_1} + \frac{\alpha_2^2/x_2}{s + p_2} + \cdots + \frac{\alpha_n^2/x_n}{s + p_n}\right]$$

These two equations must satisfy the following constraints.

$$z_{11}\Big|_{s=0} = \frac{1}{g_s}$$

$$z_{22}\Big|_{s=0} = \frac{1}{g_\ell}$$

By using these constraints and the Lagrange multipliers method, one can obtain the following quadratic equation:

$$X^2 + \frac{1}{\sum_{i=1}^{n} \frac{\alpha_i}{p_i}} \left[1 - \frac{g_\ell}{g_s}\right] X - \frac{g_\ell}{g_s} = 0$$

Once X is known, it can be used to obtain the residues of z_{11} since $x_i = X\alpha_i$. The value of X and the residues of z_{11} are calculated in lines 440-490 of the example program with C3 containing X and the X(I) containing the residues of z_{11}.

By letting s = 0 and substituting the expression for X into z_{11}, one can obtain an equation for H.

$$H = \frac{g_\ell}{g_s} \frac{1}{1 + X \cdot \sum_{i=1}^{n} \frac{\alpha_i}{p_i}}$$

H is evaluated in line 500.

Once the residues of z_{11} and H have been determined, one must return the partial fraction expansion of z_{11} to a single fraction before a continued fraction expansion can be performed. To do this requires that one be able to expand a polynomial from its factored form. Consider the function.

$$Y(s) = (s + p_1)(s + p_2)\cdots(s + p_n)$$

$$Y(s) = s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n$$

where the coefficients a_i must be determined. It can be shown that the coefficients a_i are a sum of products following the form:

$$a_1 = p_1 + p_2 + \cdots + p_n$$

$$a_2 = p_1p_2 + p_1p_3 + \cdots + p_{n-1}p_n$$

$$a_3 = p_1p_2p_3 + p_1p_2p_4 + \cdots + p_{n-2}p_{n-1}p_n$$

$$\vdots$$

$$a_n = p_1 \cdot p_2 \cdot p_3 \cdots p_n$$

In evaluating these coefficients one must ensure that the number of FOR-NEXT loops is independent of the number of poles to maintain program flexibility.

To do this, consider the following truth table:

p_3	p_2	p_1
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

A FOR-NEXT loop can be used to form the product of p_1 and p_n for each row. When the pole is low, however, it is skipped and the counter is not incremented. The product is then summed into a list where the position is determined by the value of the counter when the product is completed. Thus F(1) corresponds to a_1, F(2) corresponds to a_2 and so forth. To generate the truth table one can use a sinusoid with the frequency depending on the particular column. When the sinusoid is negative, the pole is considered high. When it is positive the pole is low. Lines 520-810 perform the function of combining the partial fraction expansion. Once z_{11} is recombined, one can use an algorithm to obtain a Cauer 2 expansion of the admittance function. The algorithm is shown below.

$$Y_{11} = \frac{a_{13}s^2 + a_{12}s + a_{11}}{a_{23}s^2 + a_{22}s + a_{21}}$$

$$\begin{array}{lll} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & \\ a_{41} & a_{42} & \\ a_{51} & & \\ a_{61} & & \end{array}$$

where $a_{31} = \dfrac{a_{21} \cdot a_{12} - a_{11} \cdot a_{22}}{a_{21}}$

$$a_{32} = \frac{a_{21} \cdot a_{13} - a_{11} \cdot a_{23}}{a_{21}}$$

$$\vdots$$

The element values are given by:

$$e_1 = \frac{a_{11}}{a_{21}}$$

$$e_2 = \frac{a_{21}}{a_{31}}$$

$$\vdots$$

Line 980-1020 perform the Cauer 2 continued fraction expansion using the array S(J,I).

Finally, the element values are printed in a convenient form along with the network gain.

III. EXAMPLES

A. Consider the following current transfer function with a load conductance of 5 mhos and a source conductance of 1 mho.

$$\frac{I_o}{I_s} = \frac{Hs^3}{(s+1)(s+2)(s+3)}$$

Solving for z_{12} yields:

$$z_{12} = \frac{H}{5}\left[1 + \frac{\frac{-1}{2}}{s+1} + \frac{8}{s+2} + \frac{\frac{-27}{2}}{s+3}\right]$$

Knowing the residues of z_{12}, one can solve for X and H.

$$X^2 + \frac{(1-5)}{9} X - 5 = 0$$

$$X = 2.47$$

$$H = 5 \cdot \frac{1}{1 + 2.47 \cdot 9} = .2155$$

With X known one can now determine the residues of z_{11} and thus the expression for z_{11}.

$$z_{11} = .0431\left[1 + \frac{1.234}{s+1} + \frac{19.75}{s+2} + \frac{33.34}{s+3}\right]$$

$$z_{11} = \frac{.0431s^3 + 2.598s^2 + 8.448s + 6}{s^3 + 6s^2 + 11s + 6}$$

z_{11} can now be expanded into a continued fraction using an algorithm for Cauer 2 as shown in Table I.

TABLE I

	6	11	6	1
1				
	6	8.448	2.598	.0431
2.351				
	2.552	3.402	.9569	
5.694				
	.4482	.3475	.0431	
.3148				
	1.424	.7118		
11.53				
	.1235	.0431		
.5734				
	.2153			
5				
	.0431			

The completed circuit is shown in Figure 2.

The entire synthesis process can be done using the micro-computer program shown in the Appendix. One must enter 3 for the number of poles, 1 for the source conductance, 5 for the load conductance, and each of the poles 1, 2, 3 entered separately. From this data the program prints out the value for each circuit element and the gain of the circuit.

B. Consider the following current transfer function with a load and source conductance of 1 mho.

$$\frac{I_o}{I_s} = \frac{Hs^5}{(s + 1)(s + 4)(s + 9)(s + 16)(s + 25)}$$

Solving for z_{12}

$$z_{12} = H\left[1 + \frac{-115.7 \times 10^{-6}}{s + 1} + \frac{.2709}{s + 4} + \frac{-13.18}{s + 9} + \frac{92.47}{s + 16} + \frac{-134.6}{s + 25}\right]$$

Knowing the residues of z_{12}, one can solve for X and H

$$X^2 + \frac{(1 - 1)}{12.7} X - 1 = 0$$

$$X = 1$$

$$H = 1 \cdot \frac{1}{1 + 12.7} = 7.303 \times 10^{-2}$$

With X known one can now determine the resiudes of z_{11} which in this case are the absolute value of the residues of z_{12}.

$$z_{11} = .07303 \left[1 + \frac{115.7 \times 10^{-6}}{s + 1} + \frac{.2709}{s + 4} + \frac{13.18}{s + 9} + \frac{92.47}{s + 16} + \frac{134.6}{s + 25} \right]$$

$$z_{11} = \frac{.07303s^5 + 21.58s^4 + 678.1s^3 + 6537s^2 + 20280s + 14400}{s^5 + 55s^4 + 1023s^3 + 7645s^2 + 21076s + 14400}$$

y_{11} can now be expanded into a continued fraction using an algorithm for Cauer 2 as shown in Table II

TABLE II

	14400	21076	7645	1023	55	1
1						
	14400	20281	6537	678.1	21.58	.0730
18.11						
	795.3	1108	344.9	33.42	.9269	
3.525						
	225.6	292.6	72.93	4.792	.0730	
2.957						
	76.32	87.85	16.53	.6695		
2.322						
	32.86	24.05	2.813	.0730		
1.027						
	23.00	10.00	.5000			
2.322						
	13.78	2.300	.0730			
2.957						
	4.660	.3304				
3.524						
	1.322	.0730				
18.11						
	.0730					
1						
	.0730					

The completed circuit is shown in Figure 3.

Once again, the process can be done using a microcomputer in a similar fashion for Example A.

IV. CONCLUSION

As evidenced by the preceding examples, the use of microcomputers greatly reduces the time required to optimally synthesise a filter. One is required to know only the source conductance, load conductance and each pole to obtain the circuit gain and the value of each circuit element. In addition to decreased time, the microcomputer also provides a greater degree of reliability over long-hand methods. This is expecially true as the filter becomes more complex.

Reduced time and greater reliability makes the use of a microcomputer essential in the optimal design of filters.

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[4] R. A. Stein and A. I. A. Salama, "Resistance and Capacitance Minimization in Low-Pass RC Ladder Networks," IEEE Transactions on Circuits and Systems, pp. 27-31, January 1975.

Appendix

```
100  PRINT "WOULD YOU LIKE A DESCRIPTION OF THE PROGRAM?"
110  PRINT "ENTER 1 FOR YES, 0 FOR NO"
120  INPUT V
130  IF V=0 THEN 220
140  DIM R(11),F(11),T(21)
150  DIM S(22,11)
160  PRINT "THIS PROGRAM SYNTHESIZES AN OPTIMAL
170  PRINT  HIGH-PASS FILTER.  FROM THE CURRENT
180  PRINT  TRANSFER FUNCTION ONE MUST ENTER EACH
190  PRINT  POLE, THE SOURCE CONDUCTANCE, AND THE
200  PRINT  LOAD CONDUCTANCE.  (THE POLES ARE
210  PRINT  ENTERED AS POSITIVE NUMBERS.)"
220  LET E=C=K=0
230  LET B=1
240  LET P=3.141592645
250  PRINT "INPUT THE NUMBER OF POLES (1<N<10)"
260  INPUT N
270  PRINT "INPUT THE SOURCE CONDUCTANCE"
280  INPUT G1
290  PRINT "INPUT THE LOAD CONDUCTANCE"
300  INPUT G2
310  FOR I=1 TO N
320  PRINT "INPUT P";I
330  INPUT L(I)
340  NEXT I
342  REM THE FOLLOWING STEPS CALCULATE THE RESIDUCES OF Z12 WHICH
344  REM ARE THEN USED TO CALCULATE THE RESIDUES OF Z11
350  FOR I=1 TO N
360  FOR J=1 TO N
370  IF J=1 THEN 390
380  LET B=B*L(I)/*(L(J)-L(I))
390  NEXT J
400  LET A(I)=ABS (B*L(I))
410  LET B=1
420  LET C=C+A(I)/L(I)
430  NEXT I
440  LET C1=(1-G2/G1)/C
450  LET C2=SQR(C1*C1+4*G2/G1)
460  LET C3=(C2-C1)/2
470  FOR I=1 TO N
480  LET X(I)=A(I)*C3
490  NEXT I
492  REM WITH THE RESIDUES OF Z11 KNOWN, ONE CAN CALCULATE THE GAIN H
500  LET H=G2/G1/(1+C3*C)
502  REM THE FOLLOWING STEPS ARE USED TO PUT THE PARTIAL FRACTION
504  REM EXPANSION OF Z11 BACK INTO ONE FRACTION
510  LET D=2^N
520  FOR J=1 TO D-1
530  FOR I=1 TO N
```

```
540  IF SIN(P*(J+.001)*2/2^I)>0 THEN 570
550  LET E=E+1
560  LET B=B*L(I)
570  NEXT I
580  LET F(E)=F(E)+B
590  LET E=0
600  LET B=1
610  NEXT J
620  LET D=D/2
630  FOR J=1 TO N
640  FOR I=1 TO N-1
650  I<>J THEN 670
660  LET K=K+1
670  LET K=K+1
680  LET M(I)=L(K)
690  NEXT I
700  LET K=0
710  FOR 0=1 TO D-1
720  FOR I=1 TO N-1
730  IF SIN(P*(0+0.001)*2/2 I) 0  THEN 760
740  LET E=E+1
750  LET B=B*M(I)
760  NEXT I
770  LET Q(J,E)=Q(J,E)+B*X(J)
780  LET E=0
790  LET B=1
800  NEXT 0
810  NEXT J
820  LET R(1)=F(1)
830  FOR I=1 TO N
840  LET R(1)=R(1)+X(I)
850  NEXT I
860  FOR J=1 TO N
870  FOR I=1 TO N
880  LET R(J+1)=R(J+1)+Q(I,J)
890  NEXT I
900  LET R(J+1)=R(J+1)+F(J+1)
910  NEXT J
920  LET S(1,N+1)=1
930  LET S(2,N+1)=H/G2
940  FOR I=1 TO N
950  LET S(1,N+1-1)=F(I)
960  LET S(2,N+1-I)=R(I)*H/G2
970  NEXT I
972  REM FINALLY A CAUER 2 EXPANSION IS DONE
980  FOR J=1 TO 2*N
990  FOR I=1 TO N
1000 LET S(J+2,I)=(S(J+1,1)*S(J,I+1)-S(J,1)*S(J+1,I+1))/S(J+1,1)
1010 NEXT I
1020 NEXT J
1030 PRINT "THE ELEMENT VALUES ARE:"
1040 FOR I=1 TO 2*N+1
1050 LET T(I)=S(I,1)/S(I+1,1)
```

```
1060 LET U=2*INT(I/2)
1070 IF U=I THEN 1110
1080 LET K=K+1
1090 PRINT "G";K;"=";T(I)
1100 GO TO 1130
1110 LET E=E+1
1120 PRINT "C";E;1/T(I)
1130 NEXT I
1140 PRINT
1150 PRINT "THE GAIN OF THIS FILTER IS";H
1160 END
```

Example A

```
FILTER

WOULD YOU LIKE A DESCRIPTION OF THE PROGRAM?
ENTER 1 FOR YES, 0 FOR NO
? 1

     THIS PROGRAM SYNTHESIZES AN OPTIMAL HIGH-
PASS FILTER.  FROM THE CURRENT TRANSFER FUNCTION
ONE MUST ENTER EACH POLE, THE SOURCE CONDUCTANCE,
AND THE LOAD CONDUCTANCE.  (THE POLES ARE
ENTERED AS POSITIVE NUMBERS.)
? 3
INPUT THE SOURCE CONDUCTANCE
? 1
INPUT THE LOAD CONDUCTANCE
? 5
INPUT P 1
? 1
INPUT P 2
? 2
INPUT P 3
? 3
THE ELEMENT VALUES ARE:
G 1 = 1.
C 1 = 0.425307
G 2 = 5.69406
C 2 = 3.17637
G 3 = 11.5297
C 3 = 1.74378
C 4 = 5.

THE GAIN OF THIS FILTER IS 0.215297
```

Example B

```
FILTER

WOULD YOU LIKE A DESCRIPTION OF THE PROGRAM?
ENTER 1 FOR YES, 0 FOR NO
? 0
INPUT THE NUMBER OF POLES (1<N<10)
? 5
INPUT THE SOURCE CONDUCTANCE
? 1
INPUT THE LOAD CONDUCTANCE
? 1
INPUT P 1
? 1
INPUT P 2
? 4
INPUT P 3
? 9
INPUT P 4
? 16
INPUT P 5
? 25
THE ELEMENT VALUES ARE:
G 1 = 1.
C 1 = 5.52264E-2
G 2 = 3.52454
C 2 = 0.338232
G 3 = 2.3223
C 3 = 0.973783
G 4 = 2.32244
C 4 = 0.338229
G 5 = 3.52454
C 5 = 5.52264E-2
G 6 = 1.

THE GAIN OF THIS FILTER IS 7.30256E-2
```

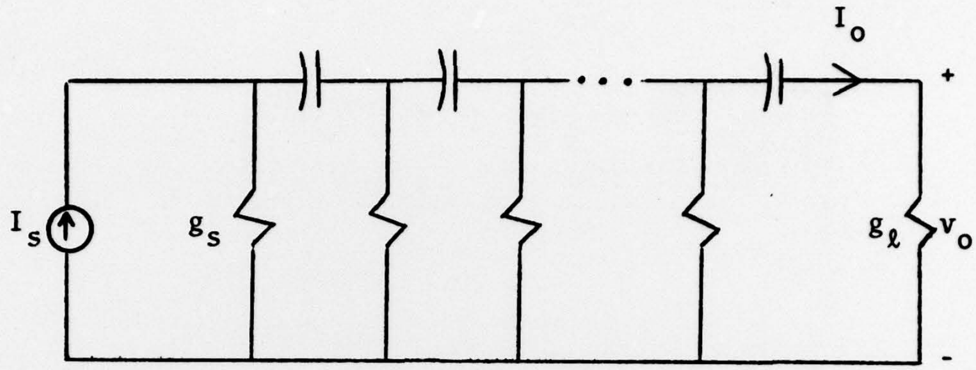

Figure 1

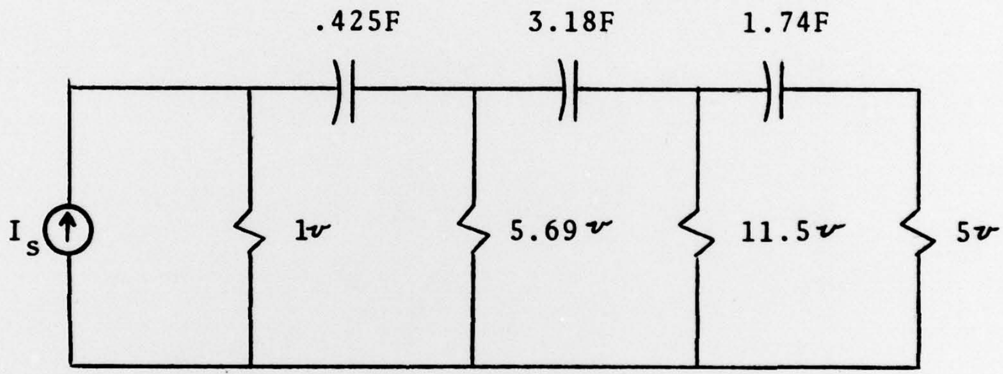

Figure 2

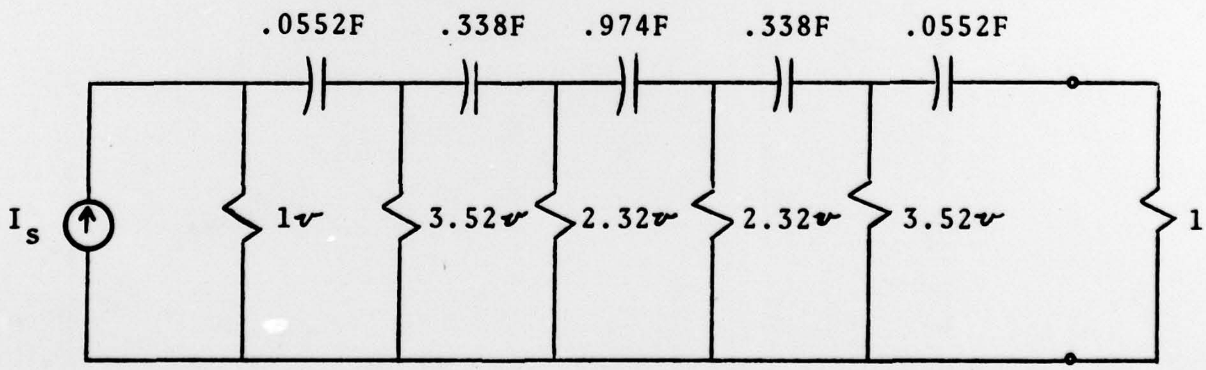

Figure 3

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