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DYNAMICS OF A MOVING MASS BEING ABRUPTLY STOPPED BY A CABLE

RW Buecher

December 1979

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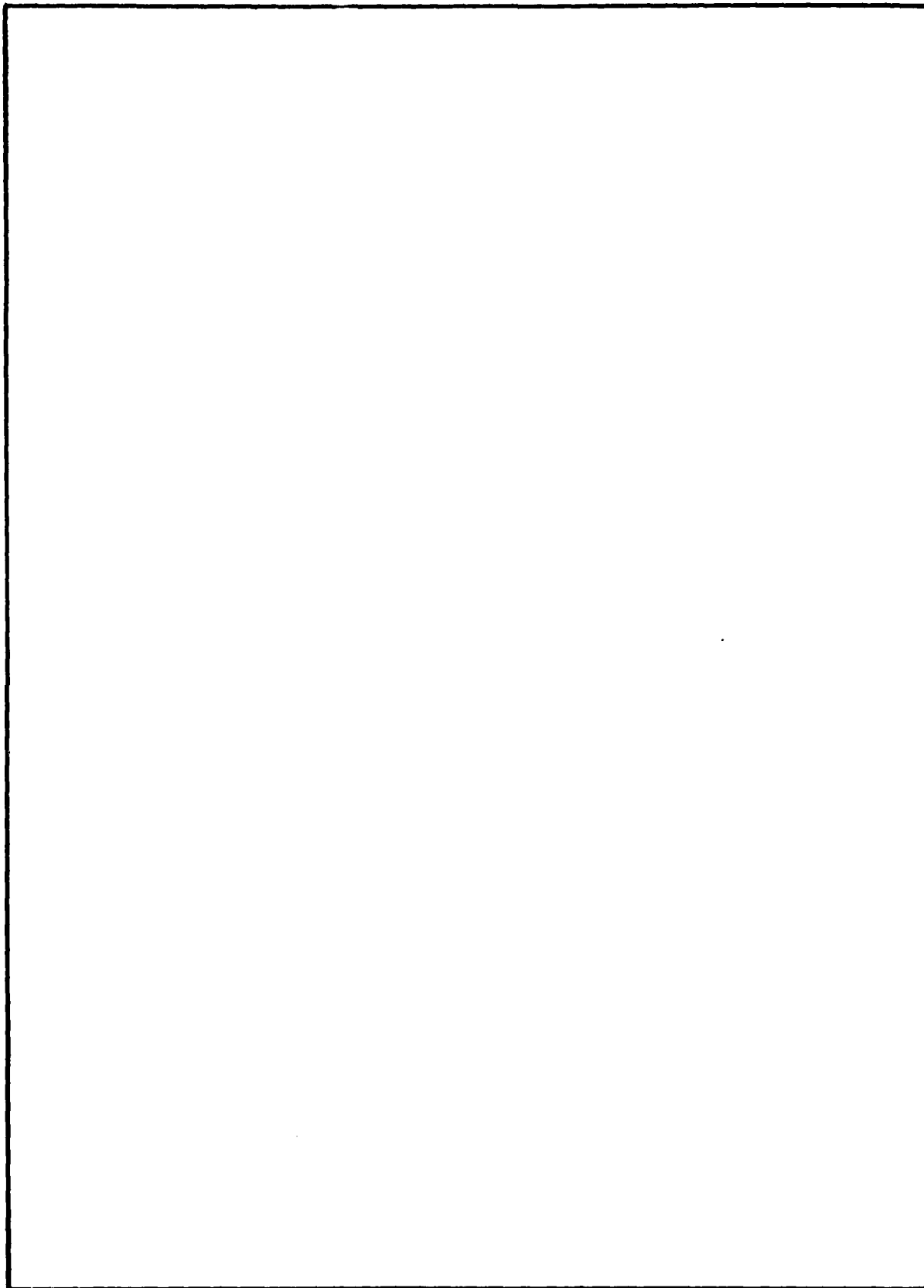
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INTRODUCTION

The use of fiber optics will allow cable systems to be developed which consist of long lengths of very small cable separated by masses which could be repeaters, sensors, or power supplies. One of the possible methods of deployment is to coil the cable and allow it to pull itself and the discrete masses out of a deployment vehicle. It is important to know what the tension in the cable will be as these masses are decelerated.

This paper investigates what occurs when a moving mass is abruptly stopped by a cable.

DERIVATION OF EQUATIONS

INITIAL TENSION

Changes in tension in the cable are propagated down the cable at the speed of sound in the cable, C . Therefore, for an increment of time, Δt , after the cable becomes taut, only a length of cable equal to $C\Delta t$ is affected by the moving mass. The speed of sound in a cable is given by:

$$(1) \quad C = \left(\frac{Y}{\rho} \right)^{1/2}$$

where Y = Young's modulus

ρ = Mass density

Think of the cable as being made up of many small lumps of mass, Δm , separated by small springs of length $C\Delta t$ and spring constant K , figure 1. At the end of the first Δt , the mass, M , moving at velocity, V_0 , will have moved a distance $V_0\Delta t$ without affecting the first cable mass. At the end of Δt , the tension in the first spring is:

$$(2) \quad T_{\Delta t} = K(V_0 \cdot \Delta t)$$

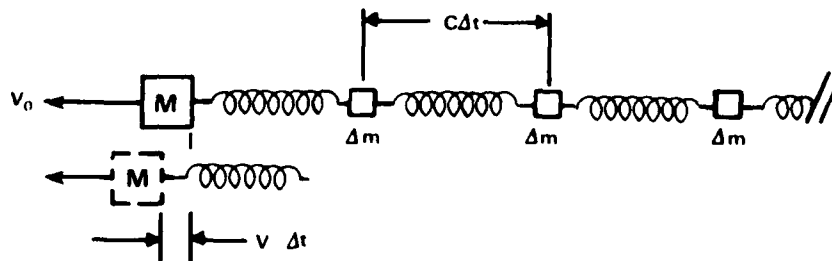


Figure 1. Discrete spring-mass system.

The spring constant is given by:

$$(3) \quad K = \frac{AY}{C \cdot \Delta t}$$

where: A = cross sectional area of cable

Putting equation 3 into equation 2,

$$(4) \quad T_{\Delta t} = \frac{AY}{C} V_o$$

The term Δt has been eliminated. Thus as $\Delta t \rightarrow 0$ and the discrete spring-mass system approaches a continuous cable, the cable tension instantaneously reaches an initial value, T_o , which is independent of the mass of the object being stopped.

$$(5) \quad T_o = \frac{AY}{C} V_o$$

MAXIMUM ALLOWABLE VELOCITY

Since Young's modulus is defined as stress/strain, σ/τ , and $\sigma = T_o/A$, the mass velocity at which the cable will break is:

$$(6) \quad V_{\text{break}} = C (\tau)_{\text{break}}$$

ONE-DIMENSIONAL WAVE EQUATION

It appears that the mass, in coming to a rather abrupt stop at the end of the cable, introduces a tension pulse into the cable which is propagated down its length at velocity C .

If the cable is long enough so that the mass is nearly stopped before the leading edge of the tension pulse reaches the end of the cable, the behavior of the cable can be described by a solution to the one-dimensional wave equation. The one-dimensional wave equation is:

$$(7) \quad \frac{\partial^2 U}{\partial x^2} = \frac{1}{C^2} \frac{\partial^2 U}{\partial t^2}$$

where U is a function of x and t which represents the displacement of an element of the cable from its original position.

At time equal zero, the mass on the end of the cable is moving at V_0 . Thus, the initial conditions are:

$$(8) \quad \left. \frac{\partial U}{\partial t} \right|_{\substack{x=0 \\ t=0}} = -V_0$$

$$(9) \quad U(0,0) = 0$$

The boundary condition at the mass is found by applying Newton's second law. The force exerted by the cable on the mass is:

$$(10) \quad F_{\text{mass}} = AY \left. \frac{\partial U}{\partial x} \right|_{x=0}$$

The acceleration of the mass due to this force is:

$$(11) \quad F_{\text{mass}} = M \left. \frac{\partial^2 U}{\partial t^2} \right|_{x=0}$$

Combining equations 10 and 11, the boundary condition is:

$$(12) \quad AY \left. \frac{\partial U}{\partial x} \right|_{x=0} = M \left. \frac{\partial^2 U}{\partial t^2} \right|_{x=0}$$

The solution obtained is:

$$(13) \quad U(x,t) = \frac{V_0 MC}{AY} \left[\begin{array}{c} \frac{AY}{e^{MC^2}} (x-Ct) \\ -1 \end{array} \right]$$

The force on the mass is found by putting equation 13 into equation 10.

$$(14) \quad F(0,t) = \frac{V_0 AY}{C} e^{-\frac{AY}{MC} t}$$

It can be seen that the initial value of the force on the mass is the same as that obtained in equation 5. As the mass is slowed down, cable tension at the mass decreases. After an amount of time, Δt , such that the mass is, for all practical purposes, stopped, the tension distribution in the cable would look like figure 2.

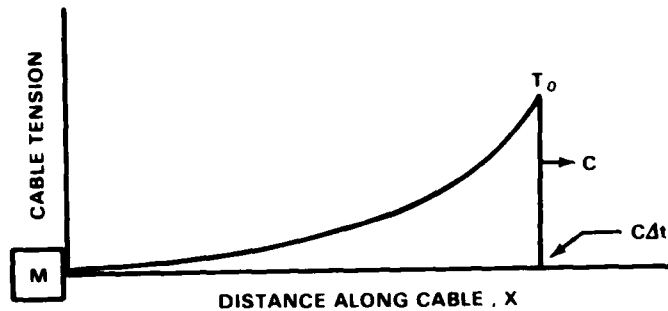


Figure 2. Tension distribution along the cable.

All of the kinetic energy of the mass is being carried away by the tension pulse. One-half of it is in the form of elastic potential energy and the other half is kinetic energy of the cable. The velocity distribution in the cable is expressed by the same exponential function as the tension distribution.

Tension in the cable is not a function of the mass of the object being stopped. However, if the mass were ten times larger, it would take ten times longer to stop it, ten times more cable would be involved in containing the tension pulse, and the cable would stretch ten times farther.

It is interesting to note that the safety factor of a cable cannot be increased by making the cable larger. Generally, the ultimate strength of a cable is a linear function of its cross sectional area. However, equation 14 indicates that the tension is also a linear function of cable cross sectional area. Thus, as the cable is made stronger, the force it exerts on the mass gets larger and the safety factor remains the same.

Cable tension can be reduced by making the cable springier (reduce Y) and less dense (reduce ρ).

COMPOSITE CABLES

Composite cables are fabricated of many different materials and are not properly characterized by terms such as Young's modulus or mass density. These terms are more appropriate to uniform wires or rods made of a single material. Characteristics of composite cables which can be measured are tension per unit tensile strain, E , and mass per unit length, m . Using these quantities, the speed of sound in a cable is:

$$(15) \quad C = \left(\frac{E}{\bar{m}} \right)^{1/2}$$

The function for cable tension at the mass, equation 14, becomes:

$$(16) \quad F(O,t) = V_0 (Em)^{1/2} e^{-\frac{(Em)^{1/2}}{M} t}$$

TENSION REFLECTIONS

When the leading edge of the tension pulse reaches the ocean bottom, or the ground, or a previously deployed mass, some of the energy will be reflected back along the cable, causing a reinforcement and increase in the cable tension.

Consider the case where the cable previously considered has the end opposite the moving mass attached to a very massive object. As the tension pulse moves down the cable, it is at first unaffected by the fact that the opposite end is fixed (figure 3a). After the leading edge of the tension pulse has been reflected at the fixed end, reinforcement of the tension will occur (figure 3b). Upon reaching the moving mass, the leading edge of the pulse will again be reflected, resulting in a further tension reinforcement, an increase in the force acting on the mass, and a more rapid deceleration of the mass (figure 3c).

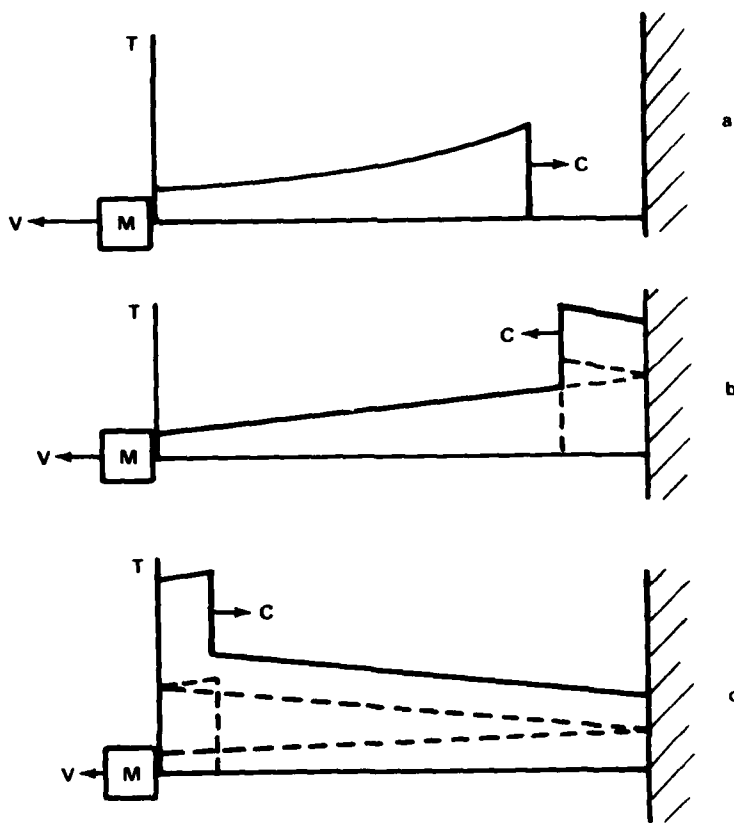


Figure 3. Cable tension distribution after reflection.

SIMULATION

A sequence of calculations was devised and put on a programmable calculator to provide an iterative solution for the cable tension and movement of the mass with a reflected tension pulse.

The time increment is equal to the time it takes the leading edge of the tension pulse to travel one-half the length of the cable.

$$(17) \quad \Delta t = \frac{L}{2C}$$

The tension being propagated down the cable from the mass at any time, t , is equal to the tension due to the movement of the mass plus the tension which is being reflected at the mass.

$$(18) \quad T(t) = V(Em)^{1/2} + T(t-4\Delta t)$$

Propagation of the tension pulse down the cable, its reflection at the fixed end, and its propagation back up the cable to the moving mass are accounted for by keeping track of the four previous values for the tension at the mass and shifting them with each time step.

The force on the mass at any time is:

$$(19) \quad F(t) = V(Em)^{1/2} + 2T(t-4\Delta t)$$

From the principle that impulse is equal to change of momentum:

$$(20) \quad M \Delta V_m = F \Delta t$$

Thus, if we assume that the force on the mass is constant over the time increment, the velocity of the mass at the end of the time increment, V_e , is:

$$(21) \quad V_e = V_s + \frac{F}{M} \Delta t$$

where V_s = velocity of mass at start of time increment.

Equations 18-21 can be iterated to determine the dynamics of the cable and mass.

EXPERIMENTAL VERIFICATION

DESCRIPTION OF TESTS

In order to determine whether the equations derived above have any basis in reality, an experiment was performed in which a massive object was stopped by a long, thin cable.

A cable 0.1 inch in diameter and 2,117 feet long was laid out on an airport taxiway. One end was tied to a stake which was driven into the ground, and the other end was attached to a strain gauge load cell which was mounted on a wheeled cart containing 300 pounds of lead. A strip chart recorder monitored the output of the load cell.

Tests were performed by backing up the cart and then pushing it forward by hand until just before the cable came taut. Speed of the cart was determined by timing the cart's last ten feet of travel before it came to the end of the cable.

Cable characteristics were $E = 3.92 \times 10^4$ lbs and $m = 3.23 \times 10^{-4}$ slugs/ft. The velocity of sound in the cable was therefore 11,016 ft/sec.

The mass of the cart and lead was $M = 10.2$ slugs. The result of each of the series of tests was similar. For the particular test which will be used as an example, $V_0 = 10$ ft/sec.

ACCOUNTING FOR FRICTION

There are two factors which were not considered in the derivation of equations but which had a significant effect on the tests. These are friction in the wheels of the cart and friction of the cable sliding along the taxiway.

The force required to pull the cart was approximately 20 pounds. In order to drag the 2,117 feet of cable along the asphalt, a force of 16 pounds was required. Since the cart stretched the cable about $5\frac{1}{2}$ feet before it was stopped, the energy consumed by friction in the wheels was 110 ft-lbs, and about 28 ft-lbs was consumed by friction in the cable. These add up to a significant fraction of the original 510 ft-lbs of kinetic energy in the mass.

Friction in the cart's wheels is easily added to the simulation by adding the friction force to the force acting on the mass, equation 19. Friction in the cable is not as easily dealt with.

As the leading edge of the tension pulse moves along the cable, the cable goes from a standstill in front of the pulse to 10 ft/sec behind the leading edge. One would expect that friction on the cable would cause the magnitude of the leading edge to gradually decrease. The experimental results indicate that this is what happens. Several calculation methods intended to produce this effect were tried in the simulation but none of them were successful. In the end, the energy consumed by cable friction was simply accounted for by adding one-half the total cable friction force to equation 19.

RESULTS

Figure 4 shows the tension seen by the load cell on the cart compared to that predicted by the calculator simulation. The experimental results clearly show the initial instantaneous increase in cable tension and the effect of the leading edge of the tension pulse which is reflected between the mass and the fixed end of the cable. The fact that the predicted tension continues to bounce after the experimental tension has smoothed out is due to the lack of a proper method in the simulation for accounting for cable friction.

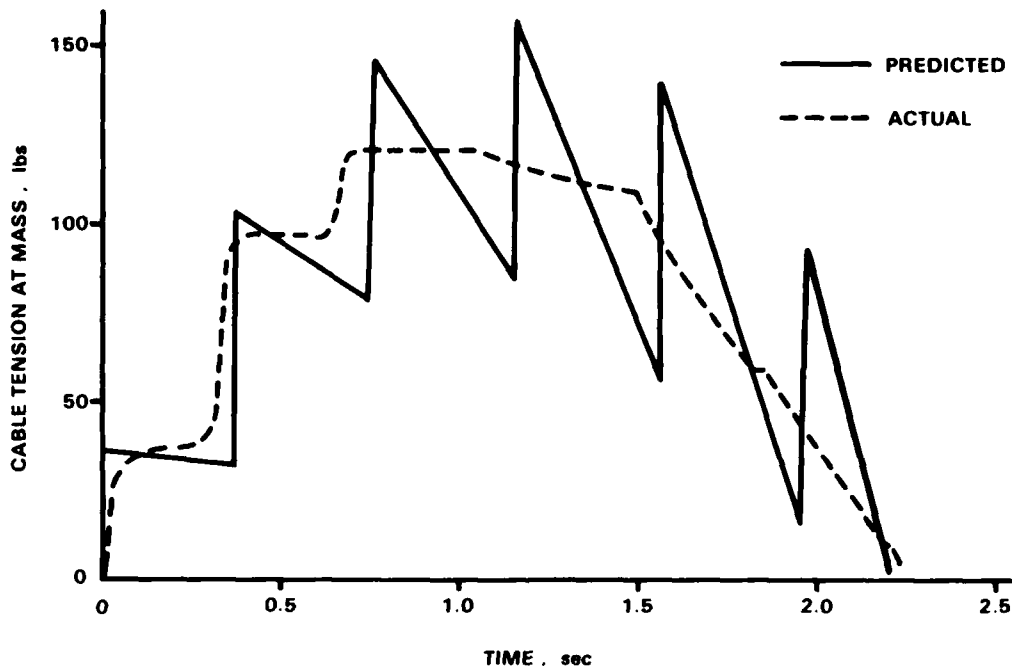


Figure 4. Predicted and actual cable tension at the mass.