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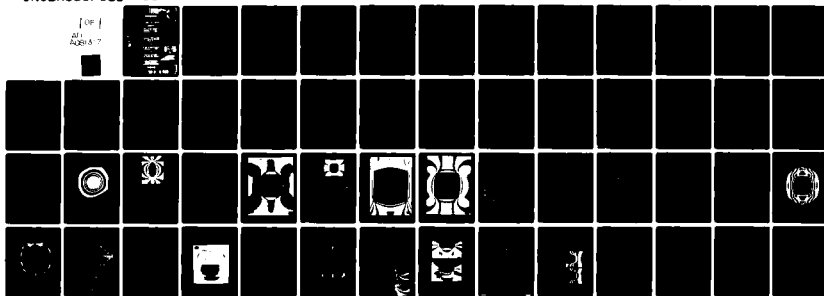
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10 A. J./Durelli K./Rajaiah

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LIGHTER AND STRONGER

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ABSTRACT

A new method has been developed that permits the direct design of shapes of two-dimensional structures and structural components, loaded in their plane, within specified design constraints and exhibiting optimum distribution of stresses. The method uses photoelasticity and requires a large field diffused light polariscope. Several problems of optimization related to the presence of holes in finite and infinite plates, subjected to uniaxial and biaxial loadings, are solved parametrically.

Some unexpected results have been found: 1) the optimum shape of a large hole in a bar of finite width, subjected to uniaxial load, is "quasi" square, but the transverse boundary has the configuration of a "bat"; 2) for the small hole in the large plate, a "barrel" shape has a lower s.c.f. than the circular hole and appreciably higher coefficient of efficiency; 3) the optimum shape of a tube, subjected to diametral compression, has small "hinges" and is much lighter and stronger than the circular tube. Applications are also shown to the design of dove-tails and slots in turbine blades and rotors, and to the design of star-shaped solid propellant grains for rockets.

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LIGHTER AND STRONGER

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Photoelasticity and moiré are used to analyze a three-dimensional rocket shape with a star shaped core subjected to internal pressure.
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A complete direct, full-field optical determination of dynamic stress distribution is illustrated. The method is applied to the study of flexural waves propagating in a urethane rubber bar. Results are compared with approximate theories of flexural waves.
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47. J. D. Hovanesian, Y. Y. Hung and A. J. Durelli, "New Optical Method to Determine Vibration-Induced Strains With Variable Sensitivity After Recording"--May 1978.
A steady-state vibrating object is illuminated with coherent light and its image is slightly misfocused in the film plane of a camera. The resulting processed film is called a "time-integrated specklegram." When the specklegram is Fourier filtered, it exhibits fringes depicting derivatives of the vibrational amplitude. The direction of the spatial derivative, as well as the fringe sensitivity may be easily and continuously varied during the Fourier filtering process. This new method is also much less demanding than holographic interferometry with respect to vibration isolation, optical set-up time, illuminating source coherence, required film resolution. etc.
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49. A. J. Durelli and K. Rajaiah, "Quasi-square Hole With Optimum Shape in an Infinite Plate Subjected to In-plane Loading"--January 1979. This paper deals with the optimization of the shape of the corners and sides of a square hole, located in a large plate and subjected to in-plane loads. Appreciable disagreement has been found between the results obtained previously by other investigators. Using an optimization technique, the authors have developed a quasi-square shape which introduces a stress concentration of only 2.54 in a uniaxial field, the comparable value for the circular hole being 3. The efficiency factor of the proposed optimum shape is 0.90, whereas the one of the best shape developed previously was 0.71. The shape also is developed that minimizes the stress concentration in the case of biaxial loading when the ratio of biaxiality is 1:-1.
50. A. J. Durelli and K. Rajaiah, "Optimum Hole Shapes in Finite Plates Under Uniaxial Load,"--February 1979. This paper presents optimized hole shapes in plates of finite width subjected to uniaxial load for a large range of hole to plate widths (D/W) ratios. The stress concentration factor for the optimized holes decreased by as much as 44% when compared to circular holes. Simultaneously, the area covered by the optimized hole increased by as much as 26% compared to the circular hole. Coefficients of efficiency between 0.91 and 0.96 are achieved. The geometries of the optimized holes for the D/W ratios considered are presented in a form suitable for use by designers. It is also suggested that the developed geometries may be applicable to cases of rectangular holes and to the tip of a crack. This information may be of interest in fracture mechanics.
51. A. J. Durelli and K. Rajaiah, "Determination of Strains in Photoelastic Coatings,"--May 1979. Photoelastic coatings can be cemented directly to actual structural components and tested under field conditions. This important advantage has made them relatively popular in industry. The information obtained, however, may be misinterpreted and lead to serious errors. A correct interpretation requires the separation of the principal strains and so far, this operation has been found very difficult. Following a previous paper by one of the authors, it is proposed to drill small holes in the coating and record the birefringence at points removed from the edge of the holes. The theoretical background of the method is reviewed; the technique necessary to use it is explained and two applications are described. The precision of the method is evaluated and found satisfactory in contradiction to information previously published in the literature.

52. A. J. Durelli and K. Rajaiah, "Optimized Inner Boundary Shapes in Circular Rings Under Diametral Compression,"--June 1979. Using a method developed by the authors, the configuration of the inside boundary of circular rings, subjected to diametral compression, has been optimized, keeping cleared the space enclosed by the original circular inside boundary. The range of diameters studied was $0.33 \leq ID/OD \leq 0.7$. In comparison with circular rings of the same ID/OD, the stress concentrations have been reduced by about 30%, the weight has been reduced by about 10% and coefficients of efficiency of about 0.96 have been attained. The maximum values of compressive and tensile stresses on the edge of the hole, are approximately equal, there are practically no gradients of stress along the edge of the hole, and sharp corners exhibit zero stress. The geometries for each ID/OD design are given in detail.

LIGHTER AND STRONGER

INTRODUCTION

In a simplified way, the development of the study of stress distributions in elastic materials can be reviewed by saying that it started with the determination of uniaxial, uniform states of stress (as for straight columns), continued with the study of discontinuities in very large members (as in the Kirsch problem), proceeded with the parametric study of discontinuities in finite plates (as in Howland problem and in Neuber's graphs) and ends now with the determination of optimum shapes, producing minimum stress values.

There is an essential change in the last step. The purpose is no more the determination of the stress associated with a prescribed geometry, but the determination of a geometry that will satisfy prescribed conditions. This last step means a change in design procedures. The design is no more conducted by analyzing shapes, and changing them, as a consequence of the analysis, until an acceptable shape is obtained, but by shaping a body that will give the desired shape. Actually, as it will be seen later, it may be more correct to say that the design and the analysis are conducted simultaneously.

A review of some of the previous contributions to the study of the problem has been made in ⁽¹⁾. The essential points of the proposed approach are presented in ⁽¹⁾ and in ⁽²⁾ where unusual aspects of the research development were brought up. The result of the optimization of the inside boundary of a tube subjected to diametral load, for all the range of OD/ID has been presented in ⁽³⁾. The result of the optimization of a quasi-square hole in a plate subjected to axial load, for all the range of ratios of width of plate to width of hole has been presented in ⁽⁴⁾ and ⁽⁵⁾.

A general review of the proposed method and of the results obtained so far will be presented in what follows, avoiding unnecessary repetitions, and some new applications will be presented.

THE OPTIMIZATION PROBLEM

Consider a circular hole in a finite plate under uniaxial tension as an illustration of the subject to be dealt with in this paper. The stress distribution around the hole is characterized by a stress concentration in the neighborhood of the transverse axis, high stress gradients, poor utilization of material around the hole, and a reversal of the sign of the stresses in the neighborhood of the longitudinal axis (Fig. 1). Several methods, such as reinforcement of the edge of the hole⁽⁶⁾, introduction of secondary holes⁽⁷⁾, enlargement of the plate near the hole⁽⁸⁾ etc. have been proposed to reduce the stress concentration. Methods like the variable thickness reinforcement of the edge have also been adopted to obtain more uniform stress distribution around the periphery of the hole⁽⁹⁾. All these methods involve either addition of more material to decrease stress concentration and to get more uniform stress distribution or introduction of another discontinuity with only the objective of reducing the stress concentration. However, there does not seem to be available any method which provides optimum shapes for discontinuities in stressed fields, and that can simultaneously 1) reduce the stress concentration, 2) obtain a uniform stress distribution around the discontinuity even in the presence of reversal of stress around the hole, and 3) decrease the weight of the plate. It is proposed that changing the shape of the discontinuity can achieve this situation and photoelasticity can be used as a tool for this purpose.

It is well known in literature that a circular hole in a circular plate is an optimum shape when subjected to a uniform pressure either on the inner or outer boundary or both (Fig. 2). Some time back, it was shown by the first author that for a hole in a biaxial stress field, a uniform stress at all points on the boundary is achieved when the hole is of elliptical

shape and the major and minor axes are in the ratio of the principal stresses, all axes being parallel and perpendicular to one another⁽¹⁰⁾. Such a hole was found to be of an optimum shape (Fig. 3). Optimization of the field, however, was not the original objective of the study of these problems, important in their own way. The same results for the elliptic holes were arrived at recently from analytical considerations by Bjorkman and Richards⁽¹¹⁾. In these cases, there is no reversal of stress at the hole boundary.

Optimization of the shape of hole discontinuities in general stress fields, with the exception of Heywood^{(12),(13)}, does not appear to have attracted the attention of engineers and scientists till recently. This could be due to the difficulties involved in the analytical and experimental techniques in optimizing boundaries subjected to stresses of opposite sign. Heywood⁽¹³⁾ attempted to arrive at optimum shapes for holes and fillets from large deformation experiments on thin rubber models subjected to tensile loads. Based on this study, he arrived at a barrel-shaped configuration as optimum for a hole in an infinite plate under tension. However, his technique is not general. It can be used only for tensile load applications and satisfaction of specified constraints does not seem possible. There is no proof either that the large deformed shape corresponds to the optimum shape in infinitesimal deformations.

OPTIMIZATION PROCESS

Sometime ago, the first author used the concept of an ideal fillet, defined it as a fillet without stress concentration and related it photo-elastically to the coincidence of the boundary with an isochromatic fringe. Some references can be found in a book⁽¹⁴⁾, reports and early papers^{(15),(16),(17)}

The same concept can be extended to the optimization of hole shapes with the basic difference that on the boundary of a hole, stress can change sign depending on the applied loading and the shape of the hole. Optimization has to be conducted for both the tensile and compressive segments of the hole boundary. The optimization process involves the removal of material from the low stress portions of the hole boundary of a photoelastic model till an isochromatic fringe coincides with the boundary both on the tensile and compressive segments.

Following the above approach, optimization of hole shapes was carried out in finite and infinite plates under uniaxial tension ^{(1),(4)}, in infinite plates under equal biaxial loading of opposite sign ⁽⁵⁾, in circular rings under diametral compression ⁽³⁾ and in plates under bending. Further work on the optimization of external boundaries in finite plates with optimized holes and under uniaxial tension has also been carried out.

METHOD USING PHOTOELASTICITY

The basic method of optimization using photoelastic models has already been explained in ^{(1),(3),(4)} and ⁽⁵⁾. Here, some technical aspects will be emphasized

Models are made of materials easy to machine and relatively free from time-edge effects. Homalite-100 would appear to be the most suitable. It should be emphasized that it would be difficult to use this method in a lens polariscope where the field is small and the model cannot be seen directly through the analyzer at the time the operator works on it. A large field diffused light polariscope is ideal for this purpose. Sometimes it may be practical to wear a binocular magnifier with a set of polarizer and quarter-wave plates attached to each of its lenses.

The model should be as large as possible and the operator should be able to remove material conveniently from the edges, particularly at the corner regions. Material from the low stress regions around the hole or fillet

is removed after the model has been loaded. When a small amount of material has to be removed from the edges, a good filing set, such as a tool and die makers file, is required. For removal of large amounts of material, a portable light weight router can be used.

The transformation of the shape from the original design to the optimum design can be done in a shorter or longer time depending on the length of the low stress region involved. As the operator removes material by filing off zones of low stress, the order of the fringe at the zone of high stress decreases. When both its inner and outer boundaries are to be optimized, due to the mutual dependency of the stresses on the two boundaries, it may be necessary to work on both the boundaries alternately. In a large field diffused light polariscope, the operator can watch the transferring of fringes as the filing takes place until the moment when one single fringe coincides with the boundary of the model. The improved design always brings the stress concentration factor (s.c.f.) down, reduces the weight of the component and increases the strength.

CRITERIA

The definition of the problem requires the specification of the constraints imposed by the design. These constraints dictate the maximum amount of material that may be removed from the low stress regions. If the functional requirements permit appreciable changes in design, one can reach several answers to the optimization problem.

It was proposed in an earlier paper⁽¹⁾ and used in others^{(3), (4)} and ⁽⁵⁾, that the degree of optimization be evaluated quantitatively by a coefficient of efficiency.

$$k_{eff} = \frac{1}{S_2 - S_0} \left\{ \frac{\int_{S_0}^{S_1} \sigma_t^+ ds}{\sigma_{all}^+} + \frac{\int_{S_1}^{S_2} \sigma_t^- ds}{\sigma_{all}^-} \right\}$$

where σ_{all} represents the maximum allowable stress (the positive and negative superscripts referring to tensile and compressive stresses, respectively), S_0 and S_1 are the limiting points of the segment of boundary subjected to tensile stresses and S_1 and S_2 are the limiting points of the segment of boundary with compressive stresses.

The coefficient of efficiency k_{eff} shows how efficiently the material at the hole boundary has been utilized for the given field. k_{eff} equal to one would mean that the stress levels are constant both on the tensile and compressive regions around the hole. The closer k_{eff} is to unity, the more efficient the design is. The coefficient indicates also how far the design is from the limit beyond which the design cannot be further improved.

The above criterion will be used to evaluate the different optimized hole shapes and fillets presented here.

USE OF OTHER STRESS ANALYSIS METHODS

In principle, it is possible to use other methods than photoelasticity to optimize the geometry of holes and fillets in stress fields. Figure 18 is an example of the isopachic pattern in the optimized ring. Since isochromatics and isopachics coincide at free boundaries, the same approach described above can be followed. However, the idea would not be practical since it is appreciably more difficult to work with the holographic set up than with the polariscope

It was pointed out previously⁽¹⁸⁾, that the distance between a free boundary and an immediate isostatic is inversely proportional to the value of the stress at the boundary. Figure 4 shows the isostatic pattern obtained using brittle coatings, along a fillet contour. This property would require that the first isostatic,

next to the optimized boundary, be parallel to it. Again, although it is possible to use this property, the idea is not practical since a new brittle coating test should be conducted after each change in shape.

It is also possible to use finite element methods to optimize configurations of plates loaded in their plane. The relative merit of this approach and the approach used here has been discussed elsewhere⁽¹⁹⁾.

APPLICATIONS

The optimization method will now be illustrated with several applications and some of the important conclusions will be highlighted.

Hole in a Large Plate: Uniaxial Loading

The isochromatic pattern for an optimized hole of width D in a large plate of width W under uniaxial loading ($D/W = 0.14$) is presented in Fig. 5. The hole has a s.c.f. of only 2.54 compared to 3 for a circular hole leading to a 15% reduction in s.c.f. (Unless stated otherwise, s.c.f. are referred to the stress averaged over the net area. In this case, the figure 3, referred to the gross area, is probably convenient to designers).

Hole in a Large Plate: 1 : -1 Biaxial Loading

The isochromatic pattern for a double-barrel shaped hole in a large plate ($D/W = 0.16$) under biaxial loading of equal and opposite sign is shown in Fig. 6. In this case, there is a 10% reduction in s.c.f. compared to a circular hole.

Hole in a Finite Plate: Uniaxial Loading

The isochromatic patterns for two typical optimized holes in a finite plate under uniaxial loading are shown in Fig. 7 and 8. The stress distributions around the optimized holes and the corresponding circular holes are shown in Fig. 9 and the s.c.f. in the two cases along with percentage increase in hole area and k_{eff} are given in Fig. 10. It is seen that the s.c.f. have been

significantly reduced in the case of optimized holes with the largest decrease occurring for large holes. Simultaneously, the hole size also has increased significantly leading to a lighter plate. The increase in strength/weight ratio is quite enormous. The optimized hole geometries have been fitted with straight lines and arcs of circles and the information is given in Figs. 11 and 12. The information obtained for a square hole could be extended for rectangular holes for certain ranges of hole sizes and is shown in Fig. 13.

It is interesting to note that sharp corners are occurring in optimized holes with only zero stress at those points.

Hole in a Circular Ring under Diametral Compression

The isochromatic patterns for two typical optimized holes in a circular ring under diametral compression are shown in Figs. 14 and 15. The stress distribution around the optimized and the corresponding circular holes are presented in Fig. 16. The information on s.c.f. weight and k_{eff} are given in Fig. 17. Once again, it is seen that the increase in strength/weight ratio is substantial for the optimized ring. The k_{eff} is about 0.95 in all optimized rings while it is only about 0.61 in all circular rings. The isopachic pattern for an optimized ring with ID/OD = 0.7 is presented in Fig. 18. It is seen that isopachics also follow the optimized boundaries as may be expected. For this ID/OD ratio, the external boundary has also become an optimum boundary as the inner boundary became an optimum. Once again, the optimized hole geometries were fitted with straight lines and arcs of circles and the data is shown in Fig. 19.

Fracture tests were also conducted on the optimized and circular rings and a typical fracture pattern is in Fig. 20. Fracture in the optimized ring does not start at the hinges. The material used in this case was the polymer Homalite 100.

INDUSTRIAL APPLICATIONS

Three attempts have been made at optimizing holes and fillets for industrial applications: one to improve the attachment of a turbine blade to the turbine rotor, and two to improve the star-shape configuration of solid propellant grains. In what follows, these attempts will be evaluated using the coefficient of efficiency as a criterium, and two more related applications will be described.

a) Dove-tail

The stress distribution on the fillet of the blade is shown in Fig. 21. There is an appreciable decrease in the s.c.f., and an increase in the coefficient of efficiency. However, the coefficient of efficiency is only 0.76 suggesting that the fillet shape could be further changed to approach the optimum.

An attempt was also made to optimize the slot of the turbine rotor part of the blade dove-tail attachment. Two loading conditions were considered, a radial pulling (corresponding to the centrifugal force acting on the blade) and a tangential force. This type of problem is a further complication in the process of optimization since the optimum shape will not be the same for both loading conditions. The process was applied independently to each loading condition. Figure 22 shows the isochromatics for both designs, corresponding to the tangential force. The stresses so obtained, combined with those obtained in a similar manner for the case of the radial load, and the results obtained for other ratios of both loads, are shown in Fig. 23. The curve corresponding to the ratio 2.3 between tangential σ_t and radial σ_r , which was the design specification, gives uniform stress distribution along the boundary.

b) Star-shapes

Figure 24 shows a comparison between the stresses at two fillets located at the bottom of the straight side ray of a star, perforating a disk subjected to uniform pressure.

The similar problem, when the tip of the star rays was expanded with a triangular end as shown in Fig. 25, was the object of an early parametric study. Using the idea presented in ⁽¹⁰⁾, it was decided to give to the tip elliptic configurations. These ellipses, however, lying on a circle rather than on a straight line, had to be distorted following the procedure shown in Fig. 25. The idea seemed reasonable since the tops of the rays are essentially subjected to tangential and radial approximately uniform loadings. The parameters to be considered are: 1) the number of rays, 2) the web fraction or amount of material beyond the top of the perforation, 3) the ratio between the perimeter of the solid disk, and the perimeter of the perforation at the point of the major axis of the ellipse. The results of several tests are shown in Fig. 26, from which the width of the basic tip a and the optimum b/a can be obtained. The star-shaped configuration with angular sided rays, shown in ⁽¹⁾ has a coefficient of efficiency of 0.91 and seems very close to optimum.

PRECISION OF THE MEASUREMENTS

From experience in previous determinations conducted using the same optical method and similar materials and loading devices, it is estimated that fractional orders of birefringence can be measured in the polariscope with a reproducibility of about ± 0.01 of a fringe. This precision can be improved when readings are repeated. Systematic errors at edges can be appreciable when measurements are taken several days after the machining of the specimens, particularly in summer when moisture content in laboratories may be high.

Most of the measurements reported were taken immediately after machining. When that was not the case, the residual effect was measured independently and subtracted.

The precision increases with the level of the response. It is lower for large plates with small holes (small D/W). Since it is desirable to make holes relatively large to permit the machining of the edges with precision, the plates have to be large, the loading capacity of the frame is soon approached and the average stress is relatively low. The precision is higher for large holes (large D/W) when 5 or 6 fringe orders can easily be obtained. It is estimated that for the last case, the precision of results obtained from individual specimens was of the order of 2% or 3% of the average stress. In most of the papers to which a reference is made here, however, problems have been solved parametrically and curves have been drawn through the points corresponding to the individual tests. This operation increases appreciably the precision.

No estimate is being made of the error made when circles are matched to the experimentally obtained shapes, and their radii and centers are determined. Here also, the values have been obtained parametrically for a large range of D/W and curves drawn through the points. It is believed that little error is associated with this part of the operation.

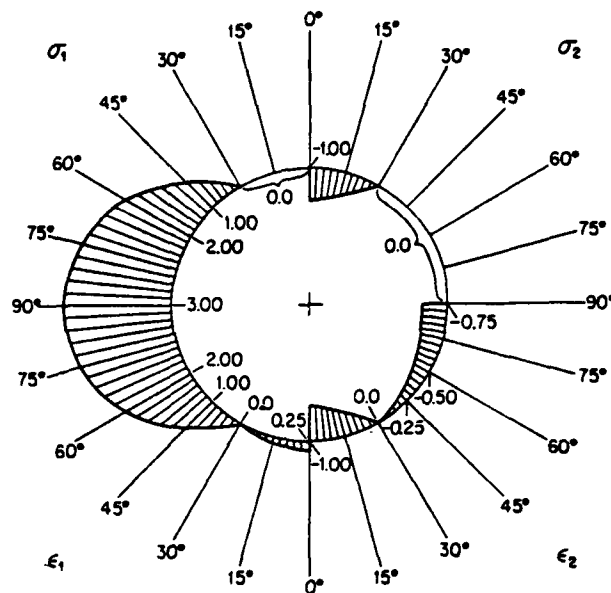
ACKNOWLEDGMENTS

Some of the photoelastic tests and optimization of shapes related to the star perforated disk were conducted by R. Lake, and some of those related to the slot in the turbine wheel dove-tail were conducted by W. F. Riley. Some of the results reported here have been taken from research programs supported financially by the Office of Naval Research and by the National Science Foundation. Appreciation is expressed here to the monitors of the programs, N. Perrone, N. Basdekas and C. Astill. The reproduction of the manuscript was made by P. Baxter

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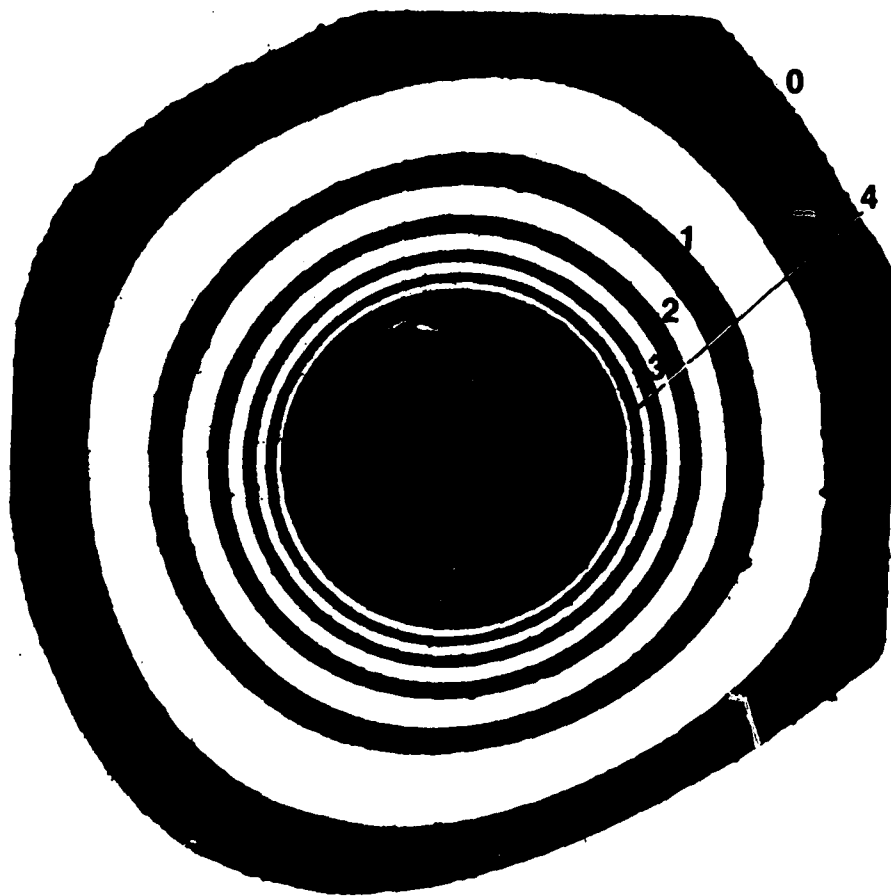
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FIG. 1 DISTRIBUTION OF STRESSES AND STRAINS AT THE CIRCULAR BOUNDARY OF AN INFINITE PLATE, WITH A CIRCULAR HOLE, UNDER A UNI-DIMENSIONAL LOAD. (KIRSCH'S SOLUTION) $\nu = 0.25$.



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FIG. 2 OPTIMUM SHAPE OF A HOLE IN A PLATE SUBJECTED TO IN-PLANE ISOTROPIC STATE OF STRESS. (ISOCHROMATICS)

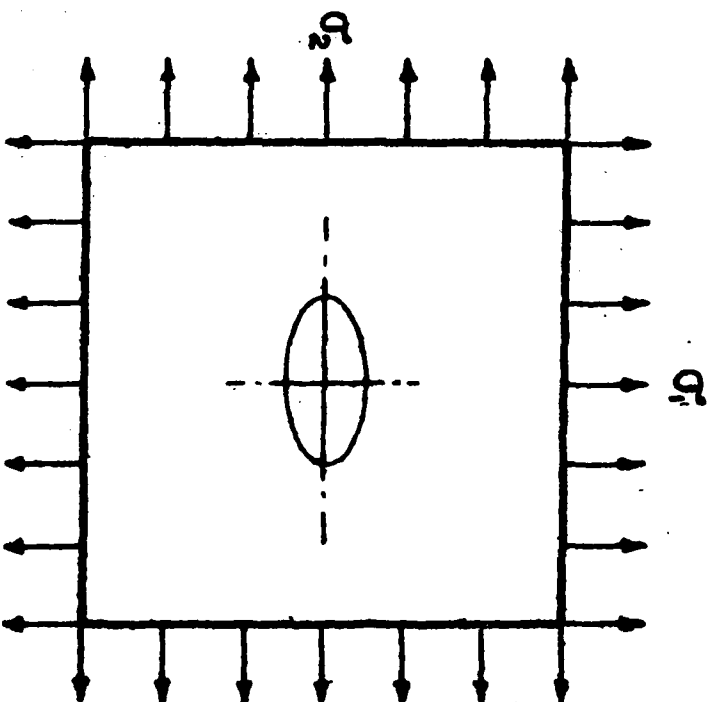
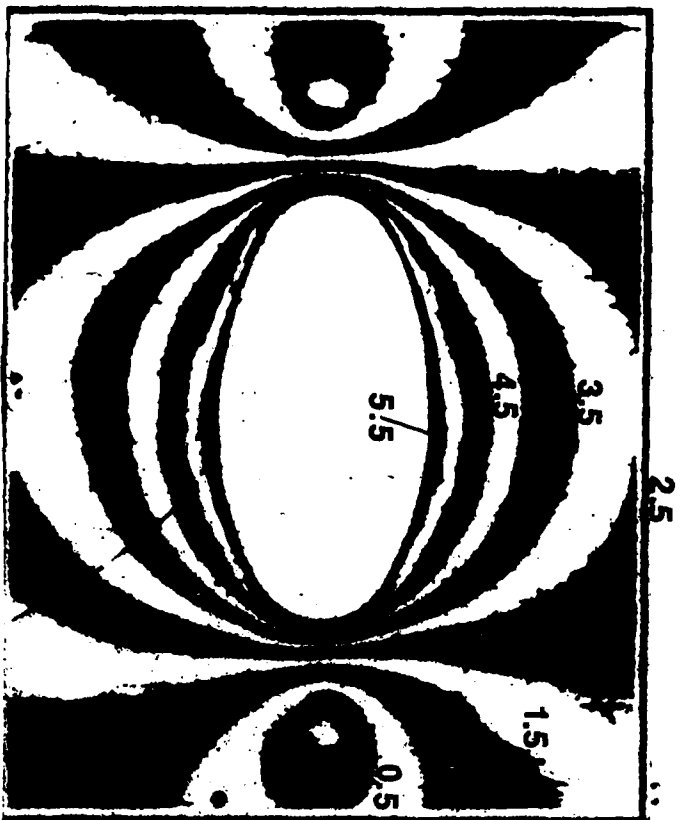
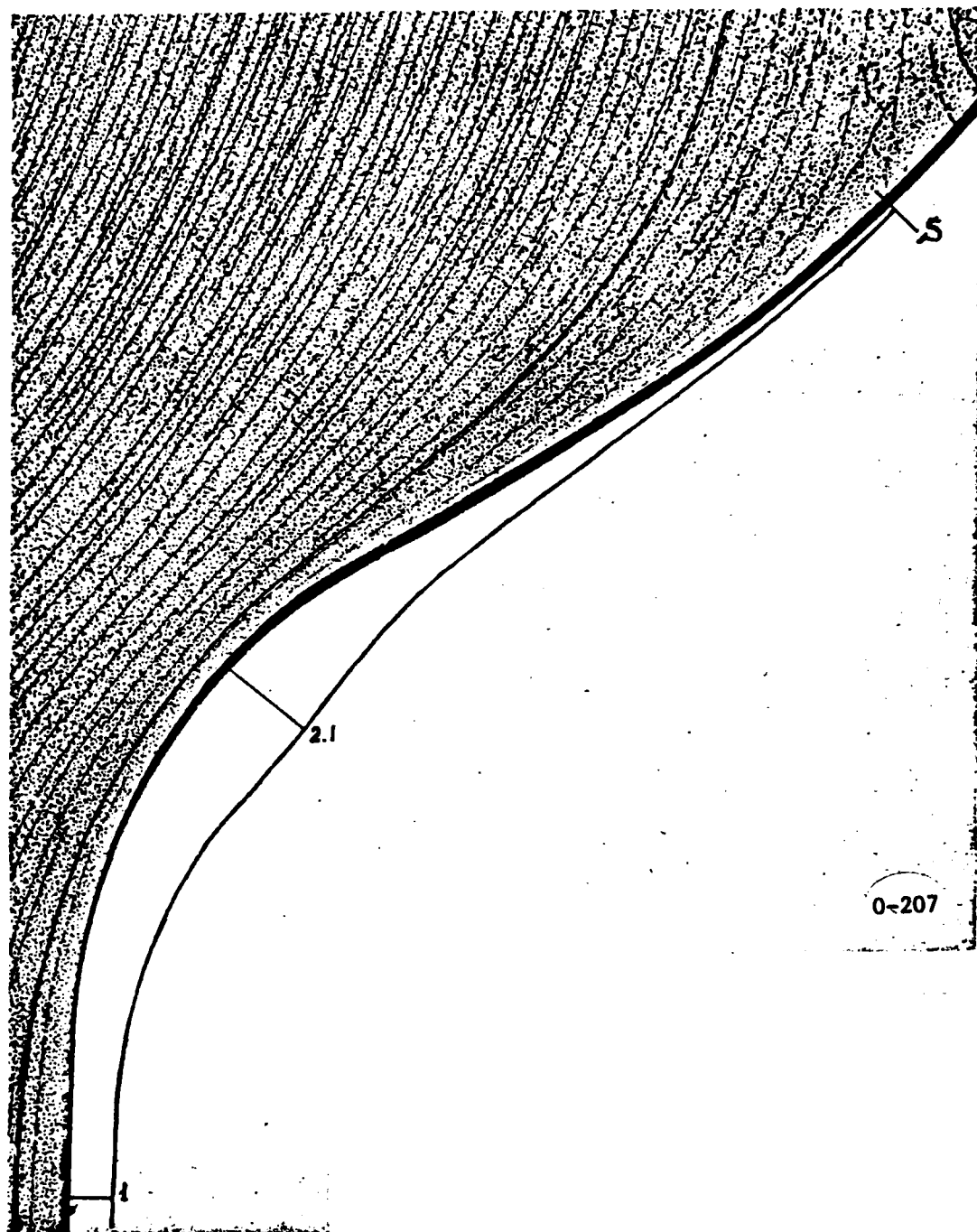


FIG. 3 THE ELLIPSE IS THE OPTIMUM SHAPE OF A HOLE IN A PLATE SUBJECTED TO IN-PLANE STRESSES WHEN THE RATIO OF THE SEMIAXES IS THE SAME AS THE RATIO OF THE PRINCIPAL STRESSES.



0-207

FIG. 4 DETAIL OF THE FILLET. THE INVERSE OF THE DISTANCE BETWEEN THE ISOSTATIC AND THE BOUNDARY IS PROPORTIONAL TO THE STRESS ON THE BOUNDARY. S IS THE SINGULAR POINT.

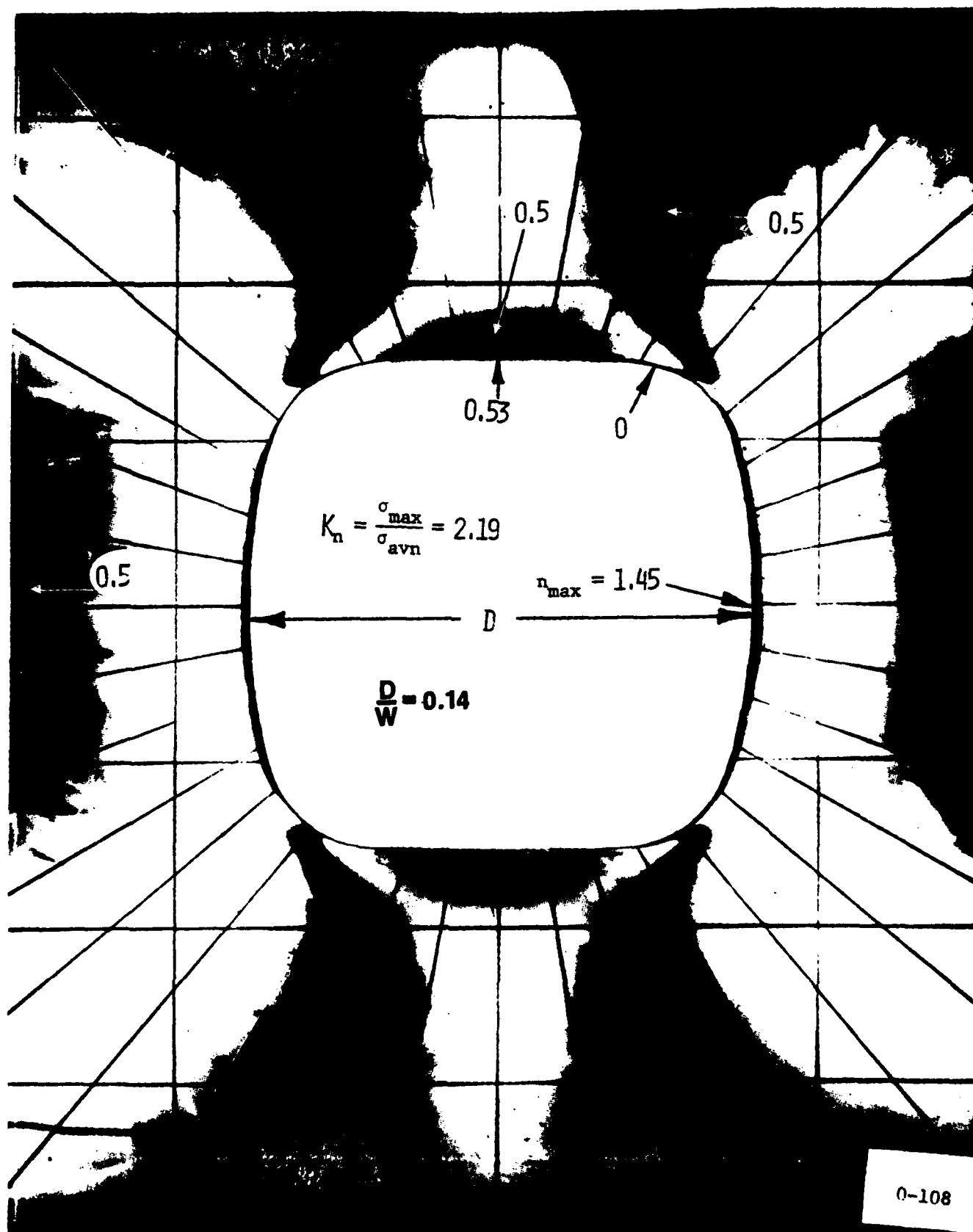


FIG. 5 STRESSES AT THE BOUNDARY OF AN OPTIMIZED QUASI-SQUARE HOLE IN LARGE PLATE ($\frac{D}{W} = 0.140$) SUBJECTED TO UNIAXIAL LOAD

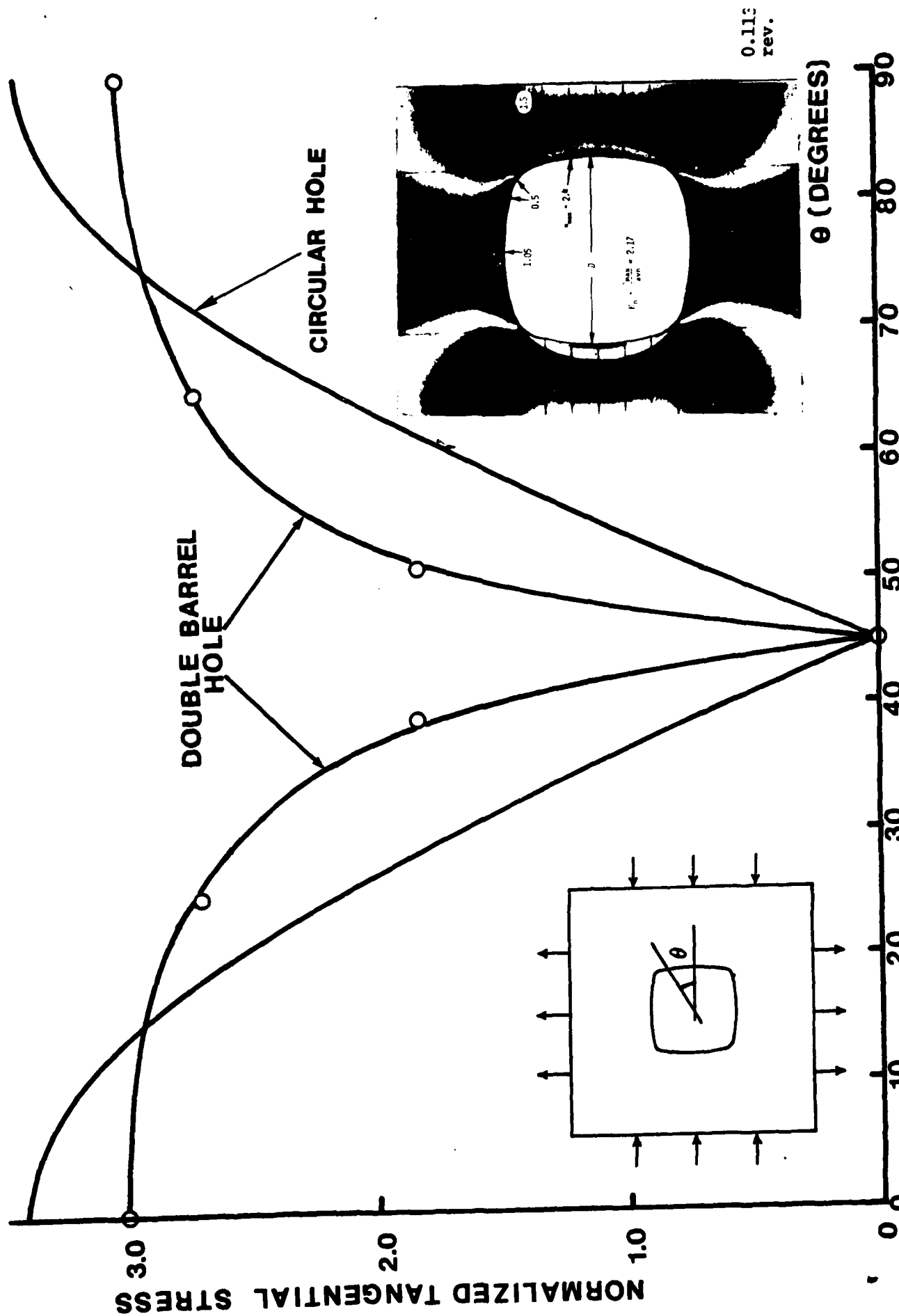


FIG. 6 DISTRIBUTION OF STRESS AROUND A DOUBLE BARREL HOLE IN A LARGE PLATE SUBJECTED TO BIAXIAL LOADING OF 1:-1 (PURE SHEAR)

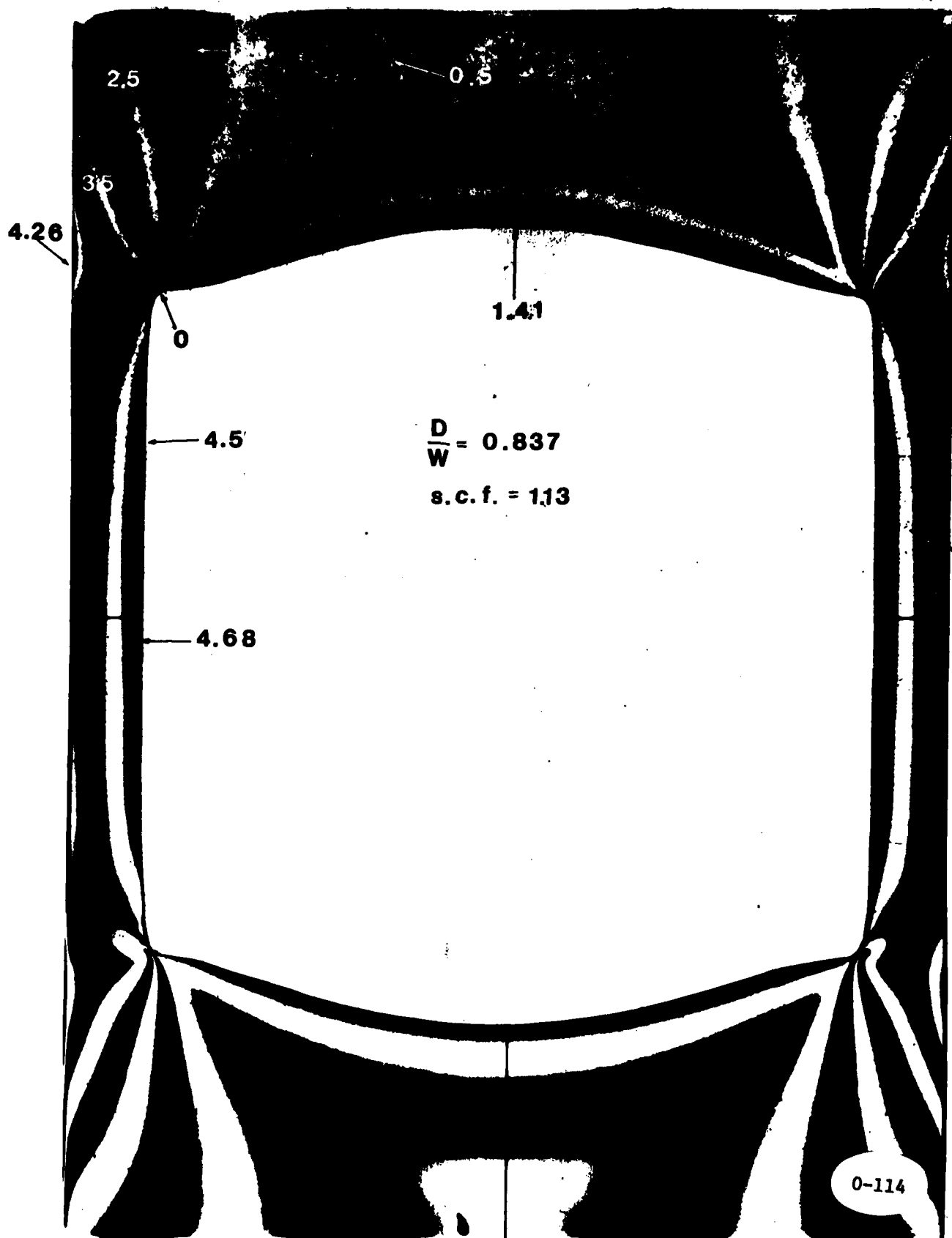


FIG. 7 OPTIMUM SHAPE OF A HOLE IN A FINITE PLATE SUBJECTED TO AXIAL LOADING ($D/W = 0.837$).

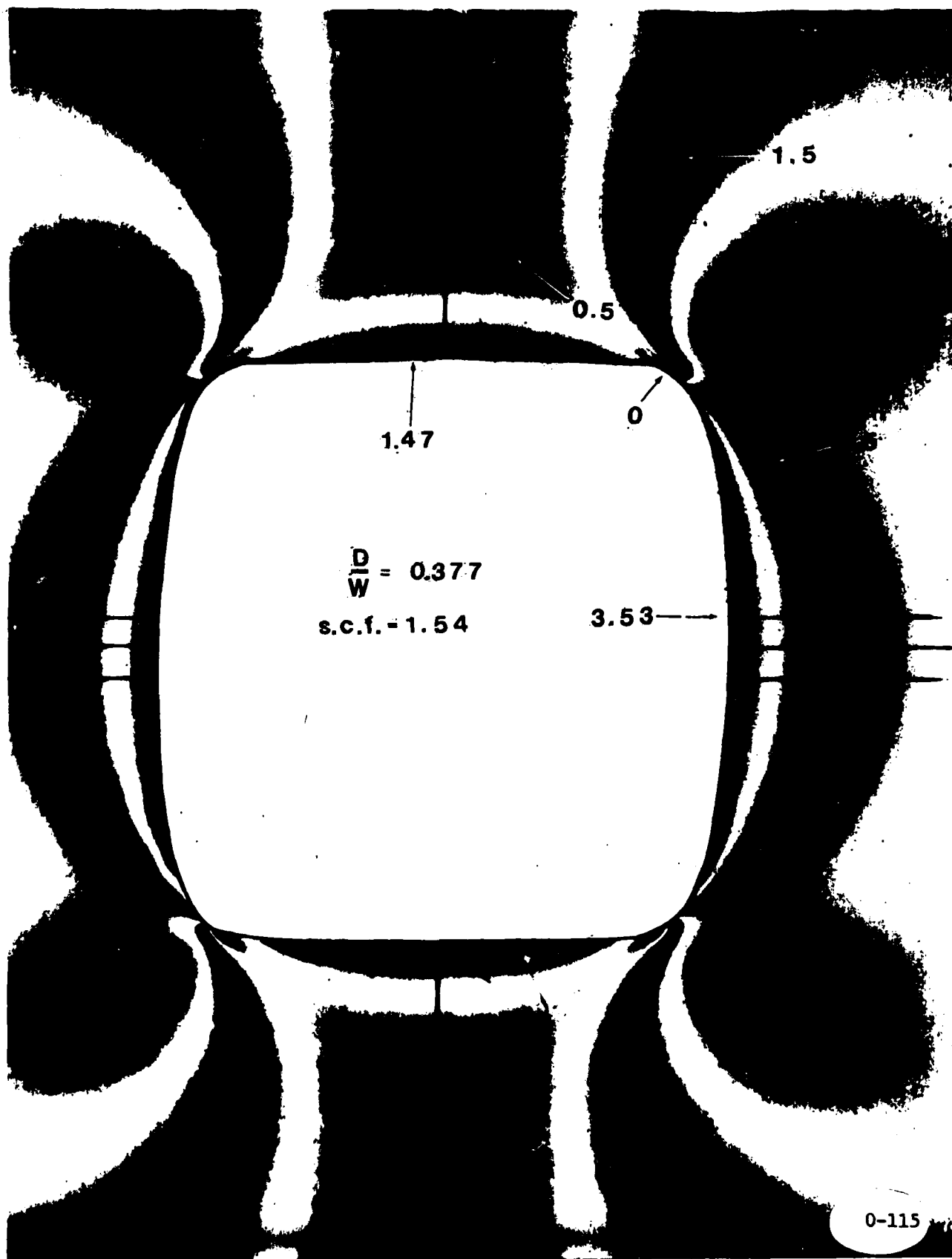


FIG. 8 OPTIMUM SHAPE OF A HOLE IN A PLATE SUBJECTED TO AXIAL LOADING ($D/W = 0.377$).

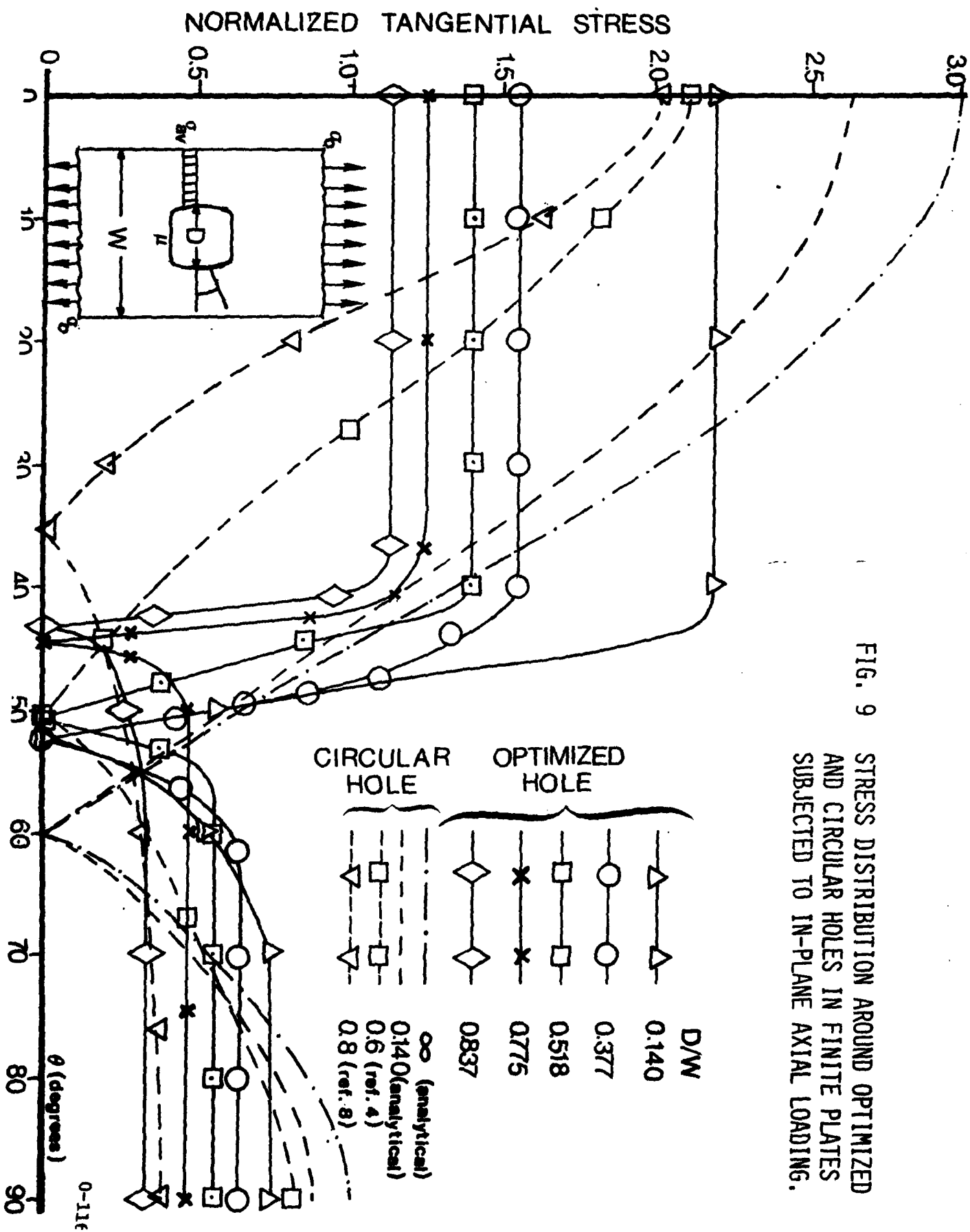


FIG. 9 STRESS DISTRIBUTION AROUND OPTIMIZED AND CIRCULAR HOLES IN FINITE PLATES SUBJECTED TO IN-PLANE AXIAL LOADING.

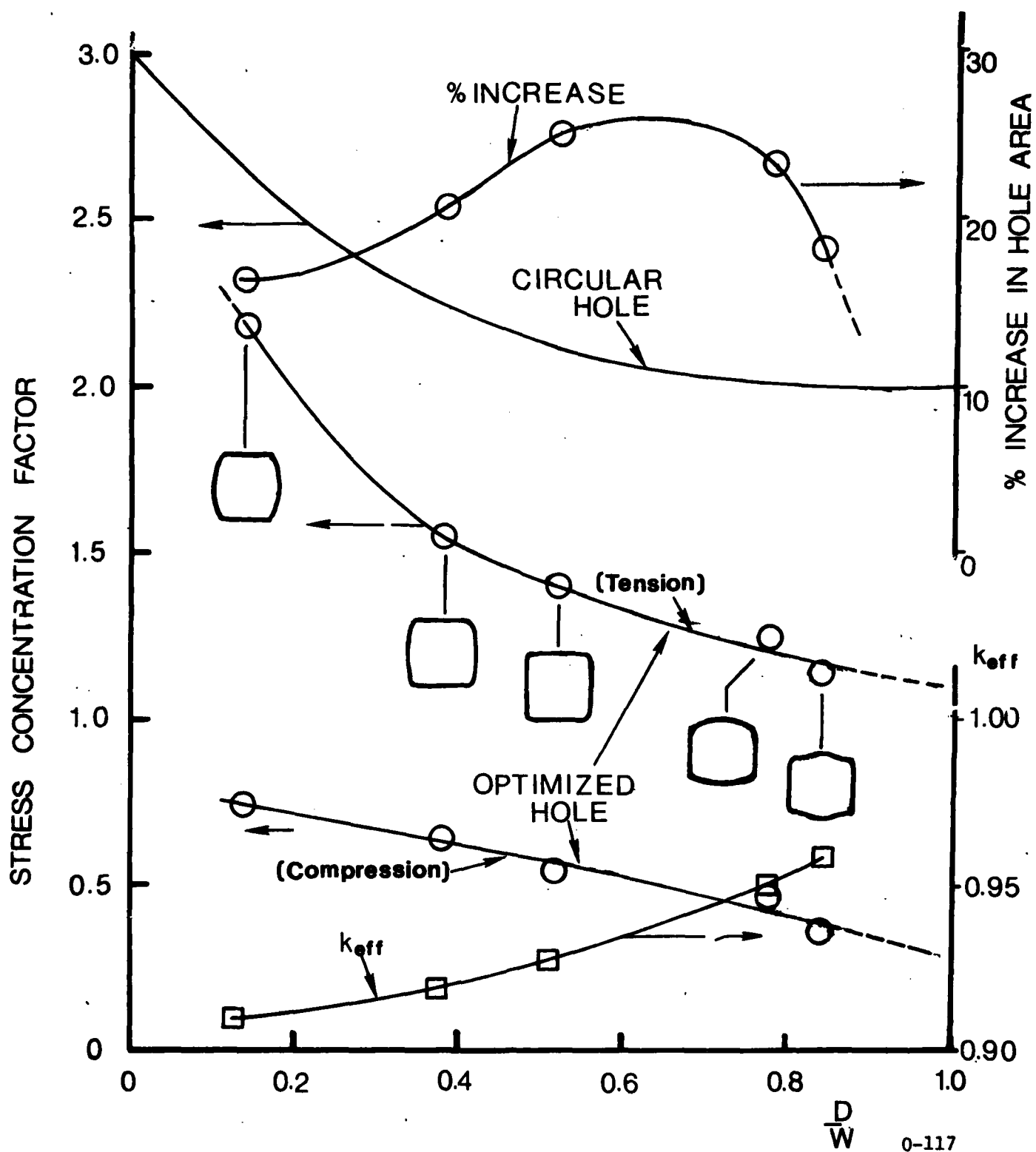


FIG. 10 STRESS CONCENTRATION FACTORS FOR OPTIMIZED HOLES IN FINITE PLATES SUBJECTED TO IN-PLANE AXIAL LOADING.

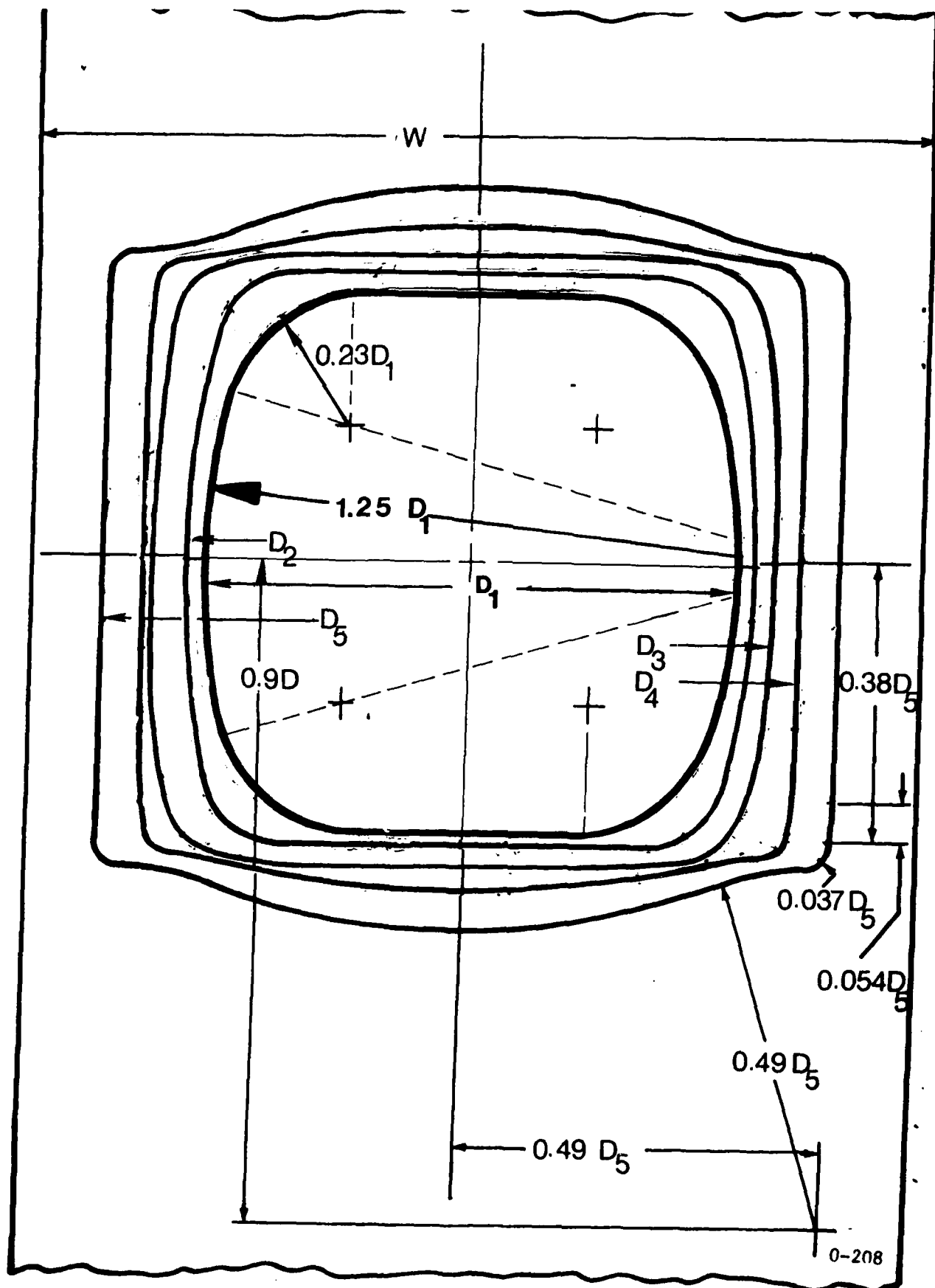


FIG. 11 OPTIMIZED GEOMETRY OF A QUASI-SQUARE HOLE IN A FINITE PLATE SUBJECTED TO IN-PLANE AXIAL LOADING (OTHER DIMENSIONS IN FIG. 10)

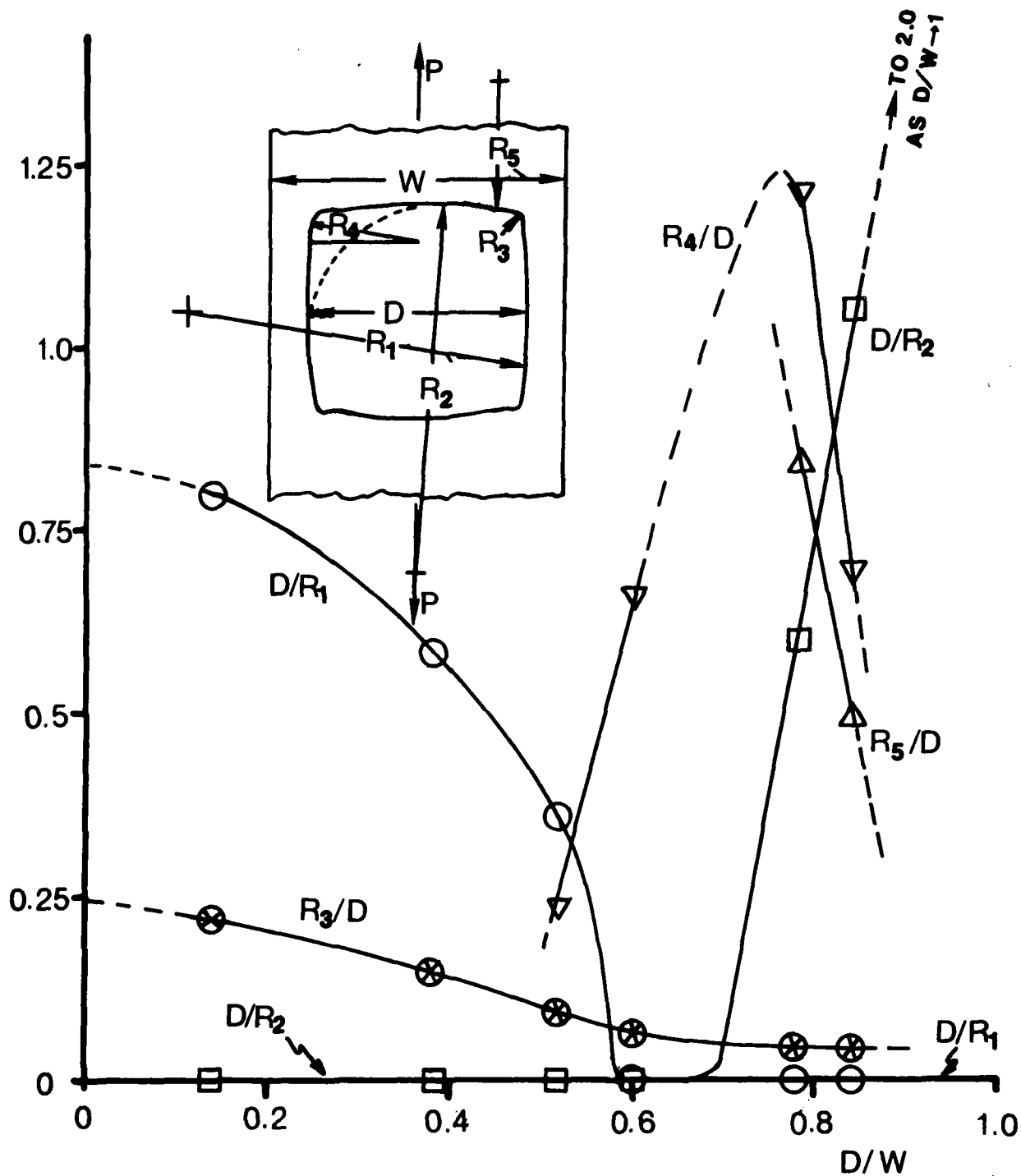


FIG. 12 RADII OF ELEMENTS OF THE HOLES PRODUCING OPTIMUM DISTRIBUTION OF STRESS IN FINITE PLATES, SUBJECTED TO IN-PLANE AXIAL LOADING.

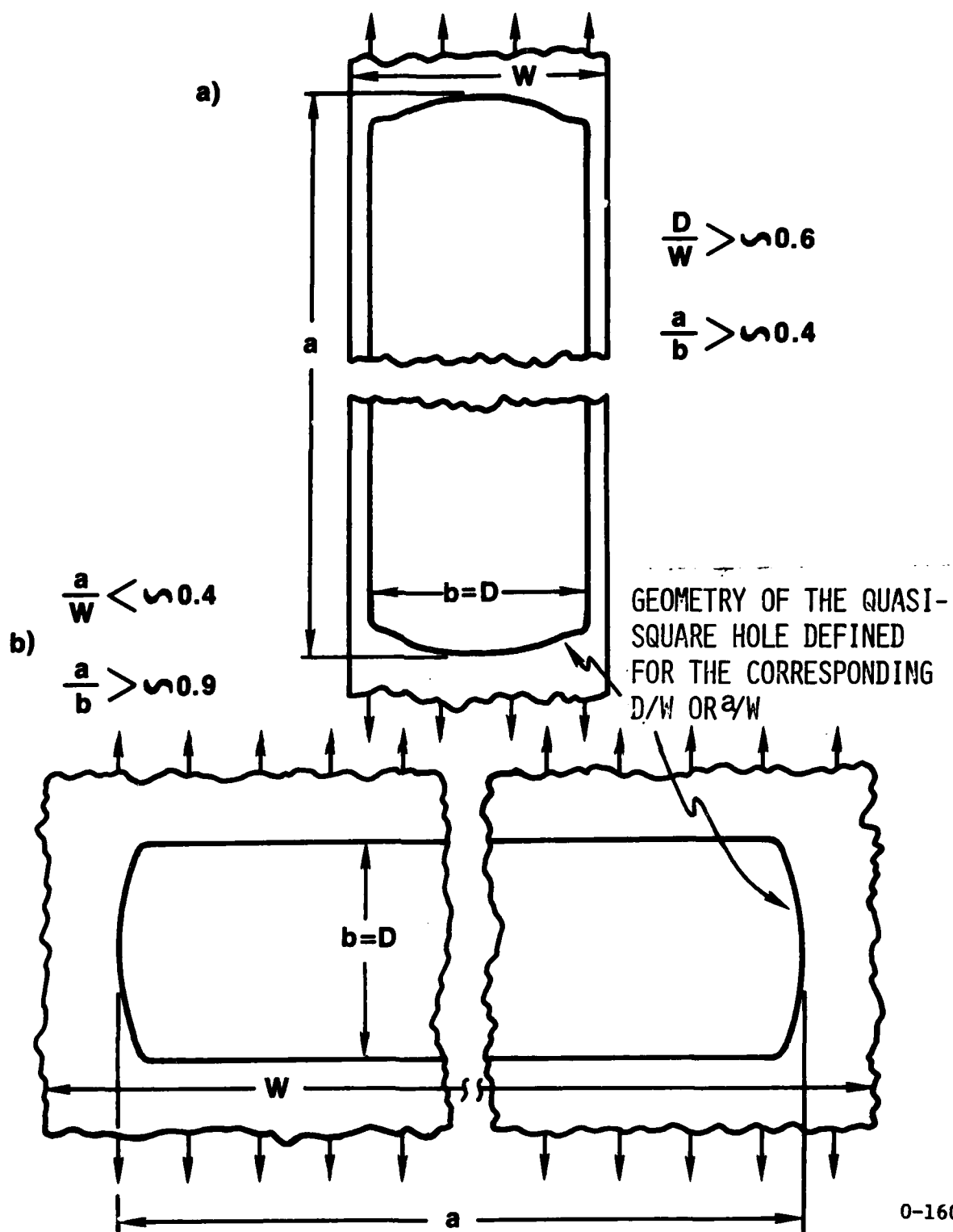


FIG. 13 OPTIMUM SHAPE FOR RECTANGULAR HOLES IN PLATES SUBJECTED TO IN-PLANE AXIAL LOADING. (GENERALIZATION OF THE QUASI-SQUARE OPTIMUM SHAPE)

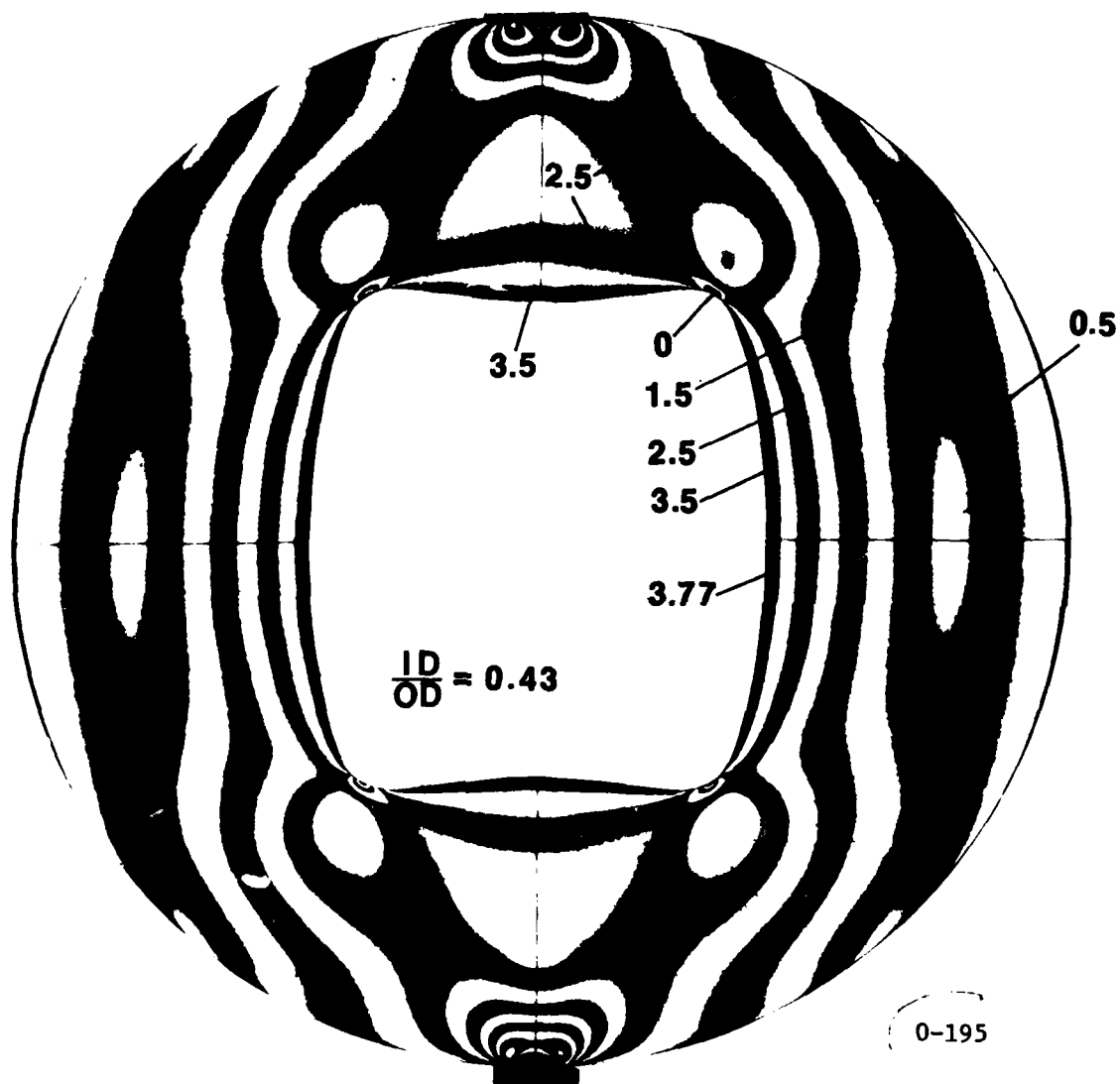


FIG. 14 ISOCHROMATICS IN A CIRCULAR RING WITH OPTIMIZED INNER BOUNDARY, WHEN SUBJECTED TO DIAMETRAL COMPRESSION $ID/OD = 0.43$.

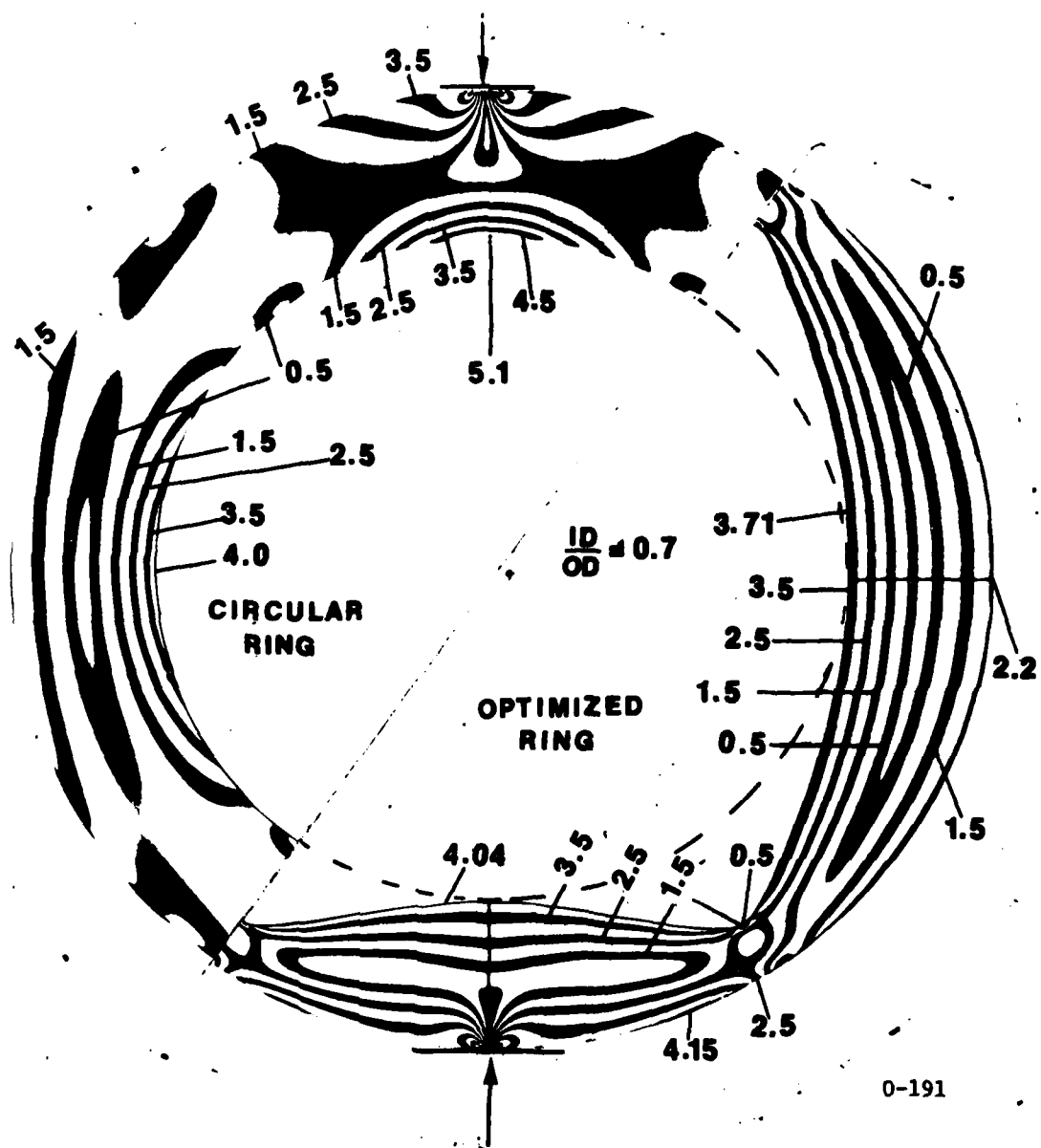


FIG. 15 LIGHT FIELD ISOCHROMATICS IN CIRCULAR AND OPTIMIZED RINGS, SUBJECTED TO THE SAME LOAD

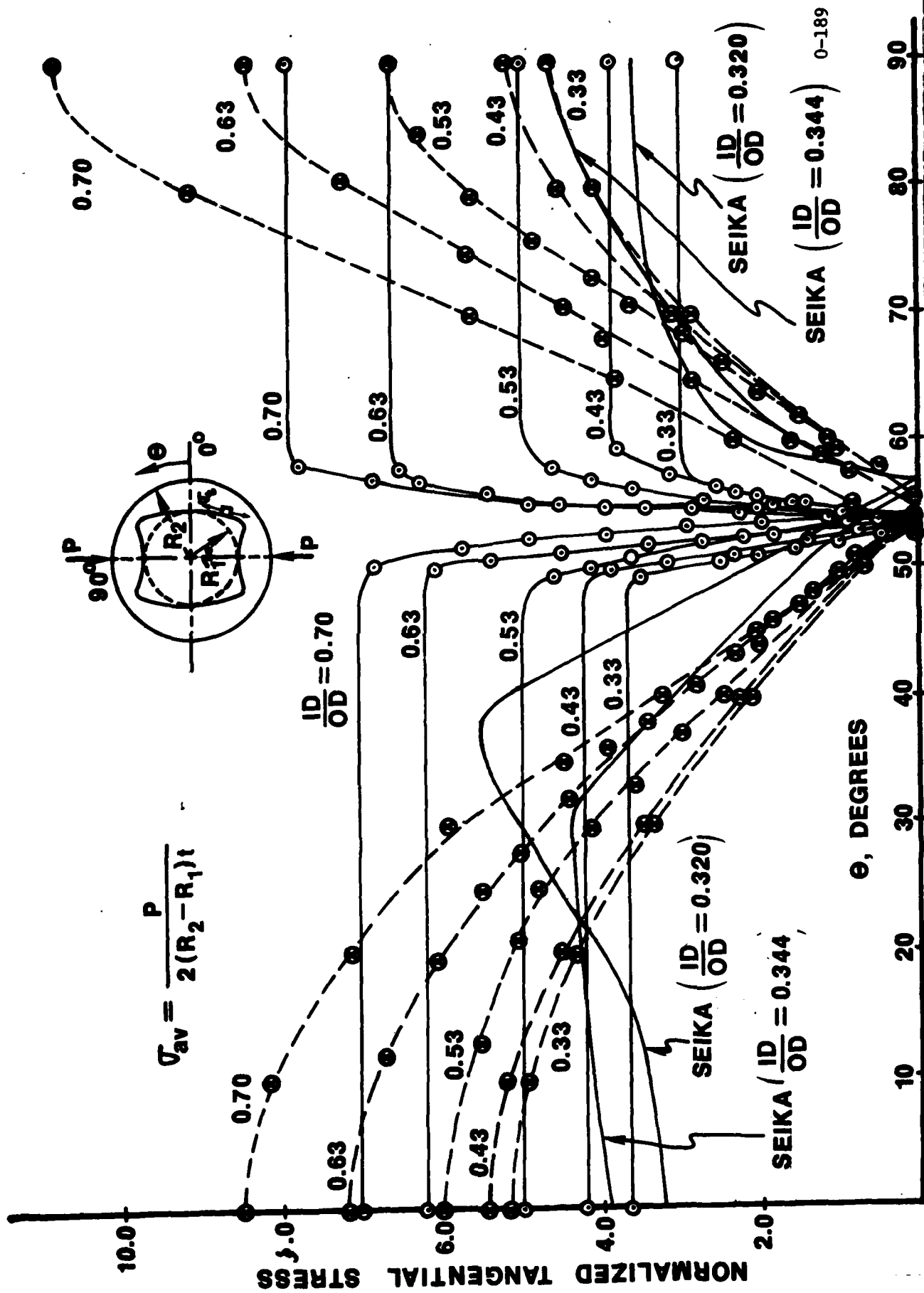


FIG. 16 STRESS DISTRIBUTION ALONG THE EDGE OF THE OPTIMIZED INNER BOUNDARY OF CIRCULAR RINGS SUBJECTED TO DIAMETRAL COMPRESSION.

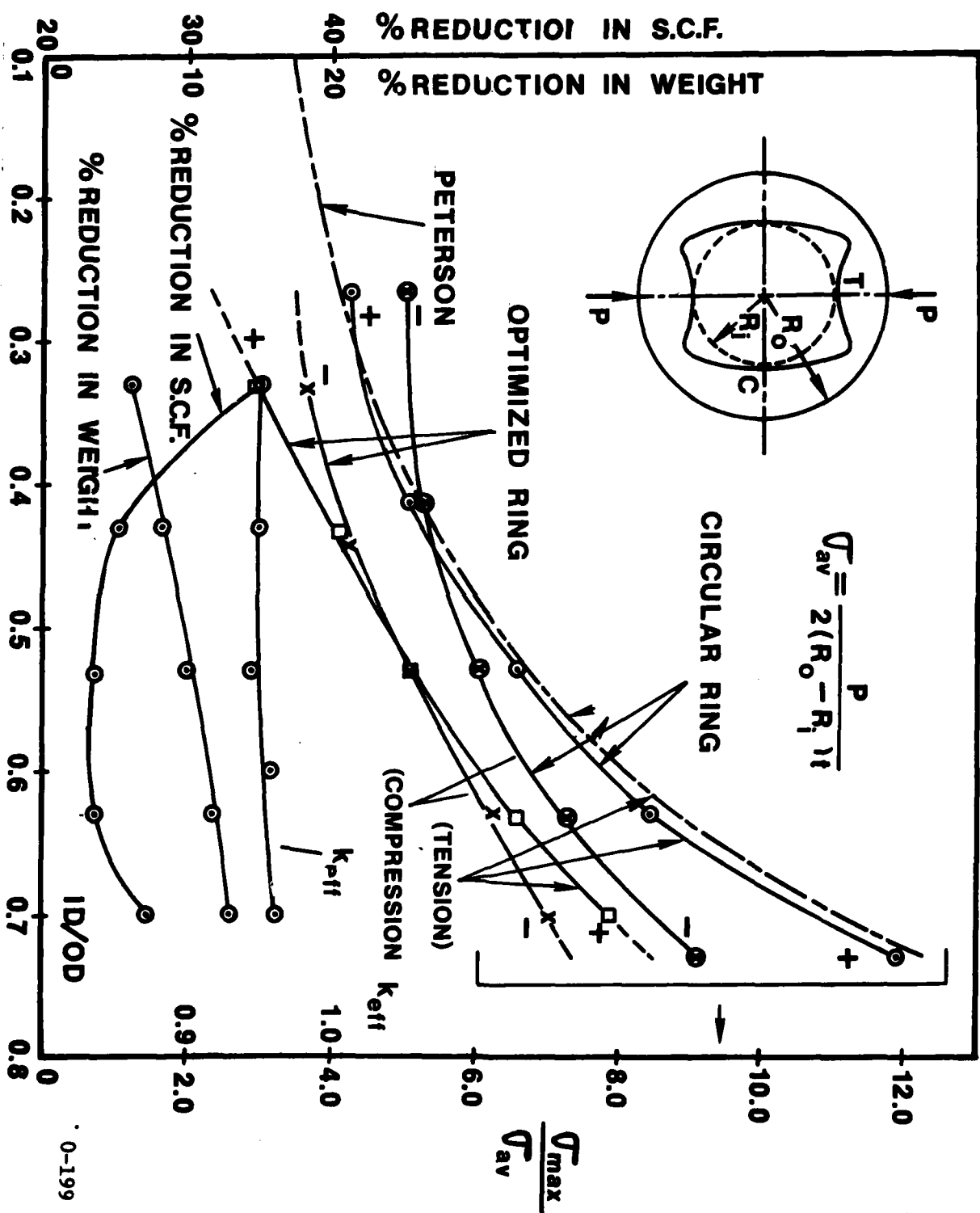


FIG. 17 STRESS DISTRIBUTION ALONG THE EDGE OF THE OPTIMIZED INNER BOUNDARY OF CIRCULAR RINGS SUBJECTED TO DIAMETRAL COMPRESSION.

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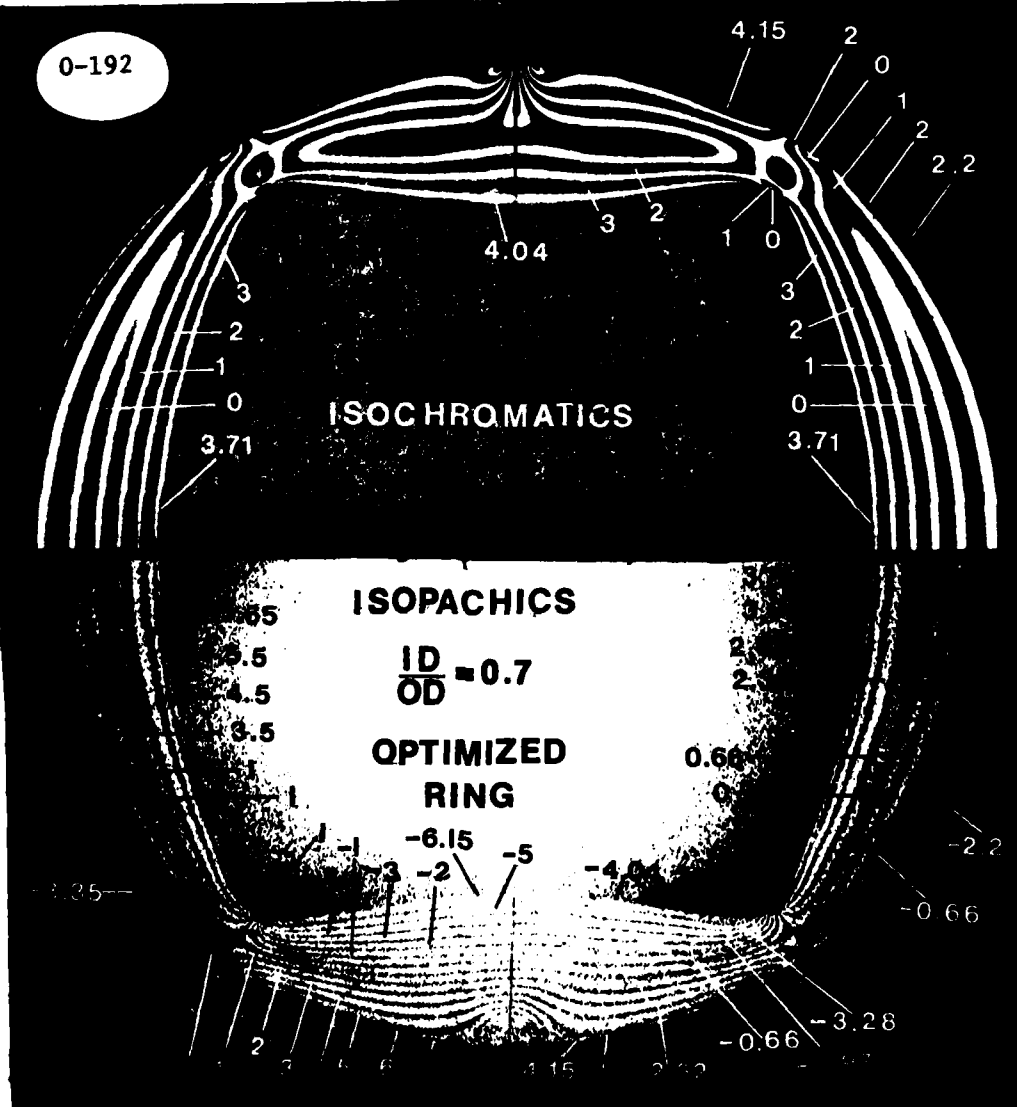
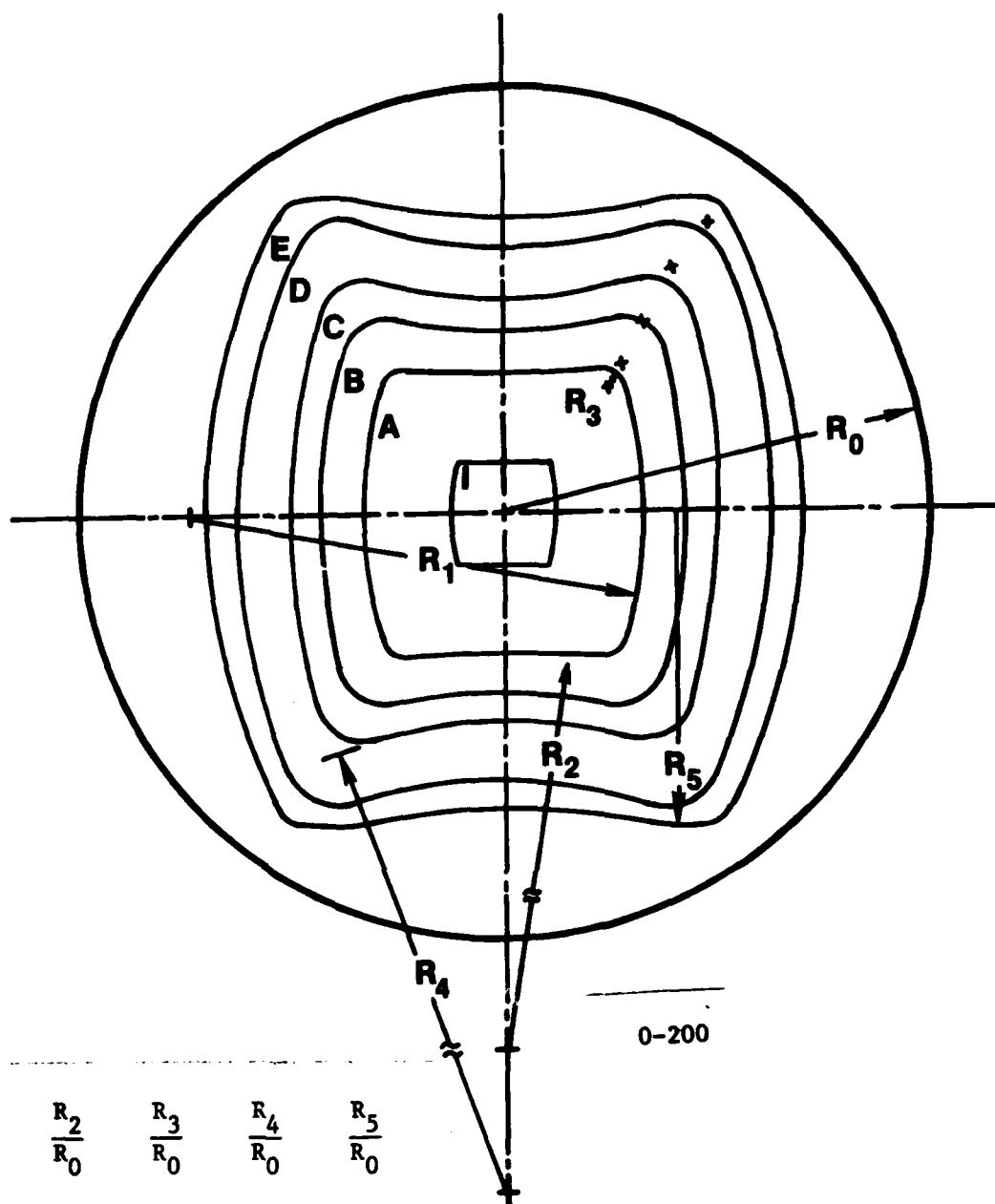
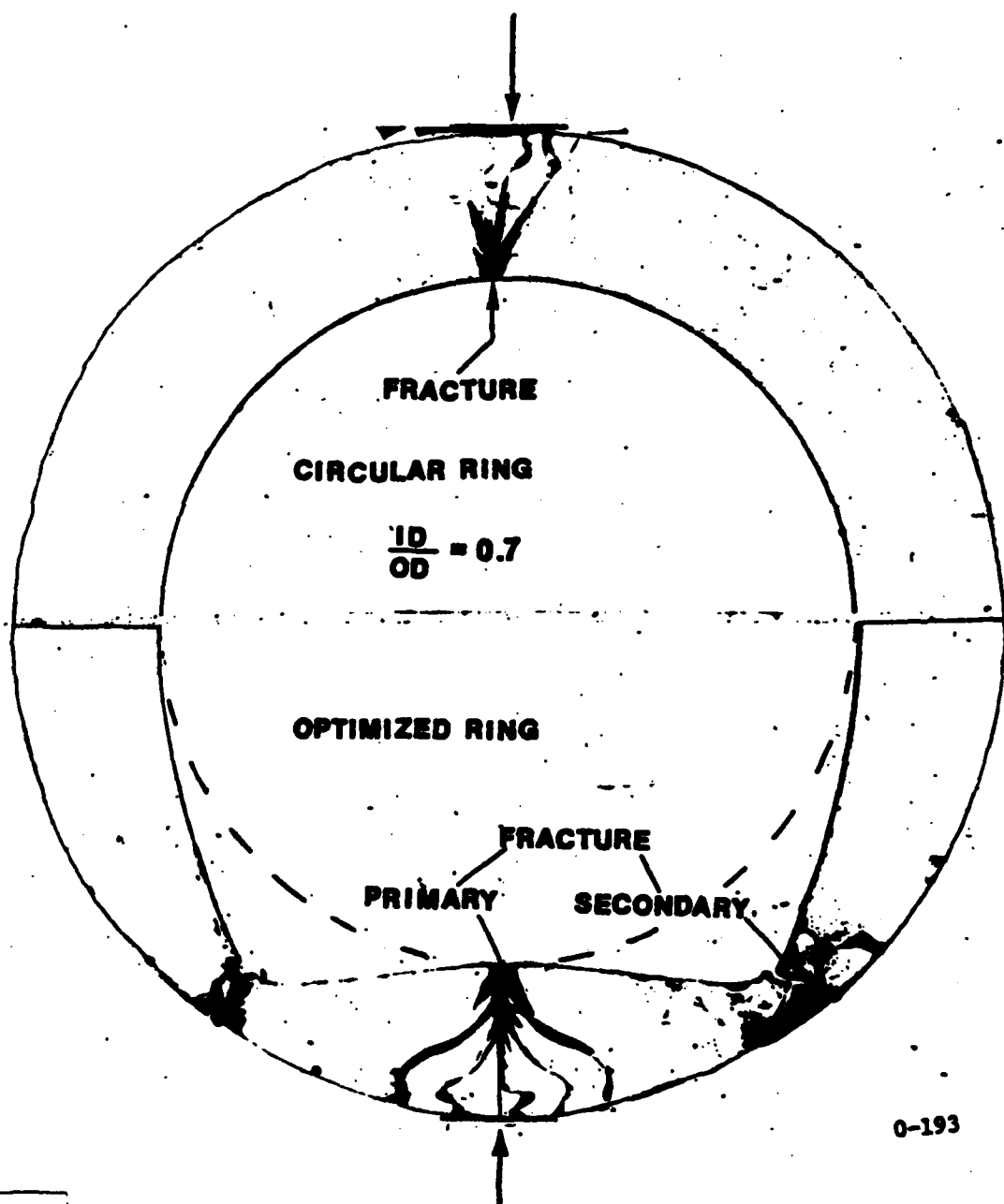


FIG. 18 DARK FIELD ISOCROMATICS FROM PHOTOELASTICITY AND ISOPACHICS FROM HOLOGRAPHIC INTERFEROMETRY FOR THE OPTIMIZED RING ($ID/OD = 0.7$). (ISOPACHICS ORDERS ON RIGHT SIDE, NORMALIZED).



	$\frac{ID}{OD}$	$\frac{R_1}{R_0}$	$\frac{R_2}{R_0}$	$\frac{R_3}{R_0}$	$\frac{R_4}{R_0}$	$\frac{R_5}{R_0}$
A	0.33	1.06	1.71	0.05	-	-
B	0.43	1.43	1.44	0.11	-	-
C	0.53	1.73	1.01	0.10	1.25	-
D	0.63	1.51	1.13	0.12	-	-
E	0.70	1.49	1.49	0.06	-	0.73
I	0.14	0.35	∞	0.05	-	- (Estimated)

FIG. 19 OPTIMUM SHAPE OF THE INNER BOUNDARY OF CIRCULAR RINGS
SUBJECTED TO DIAMETRAL COMPRESSION.



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FIG. 20 FRACTURE MECHANISMS IN THE CIRCULAR AND OPTIMIZED RINGS

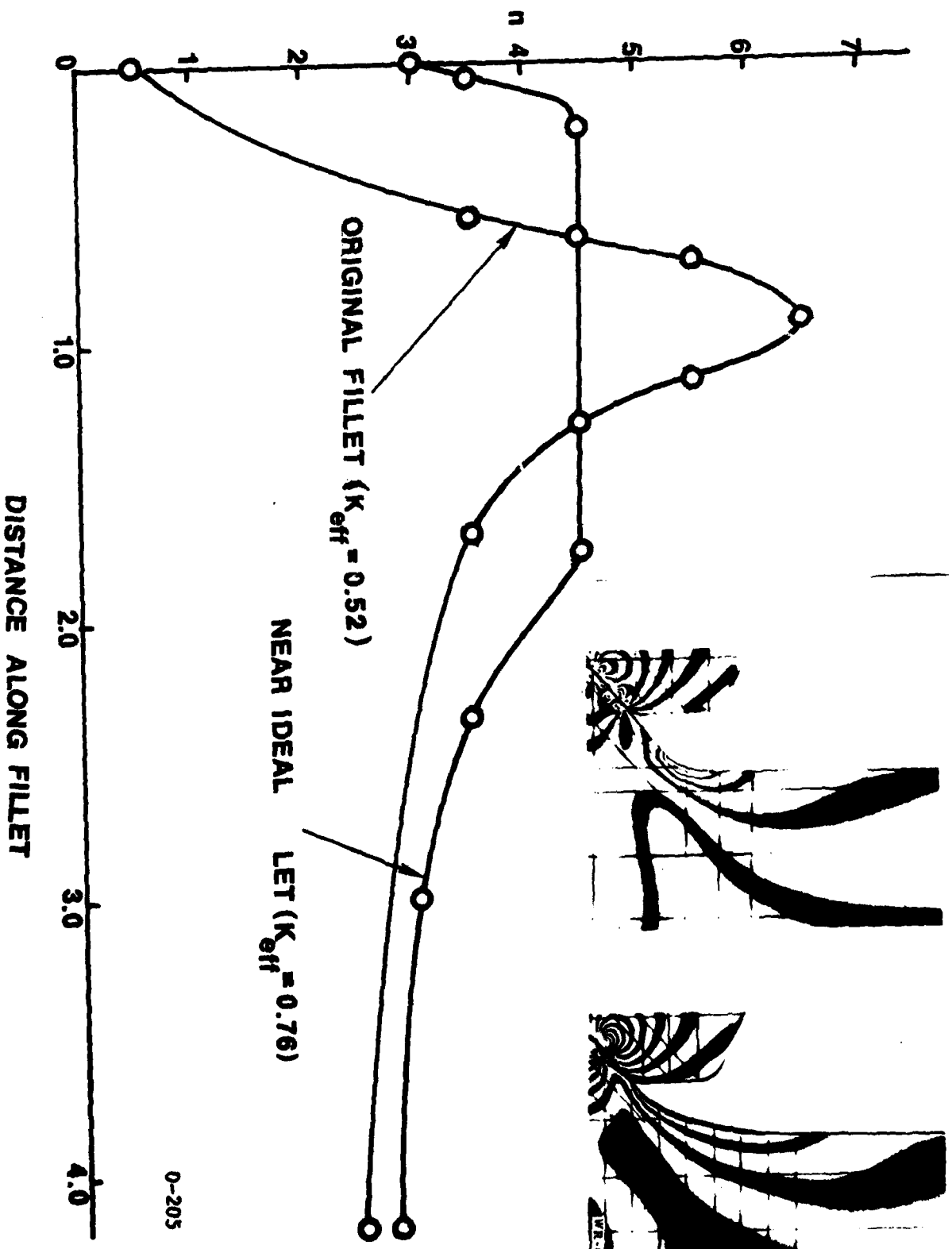
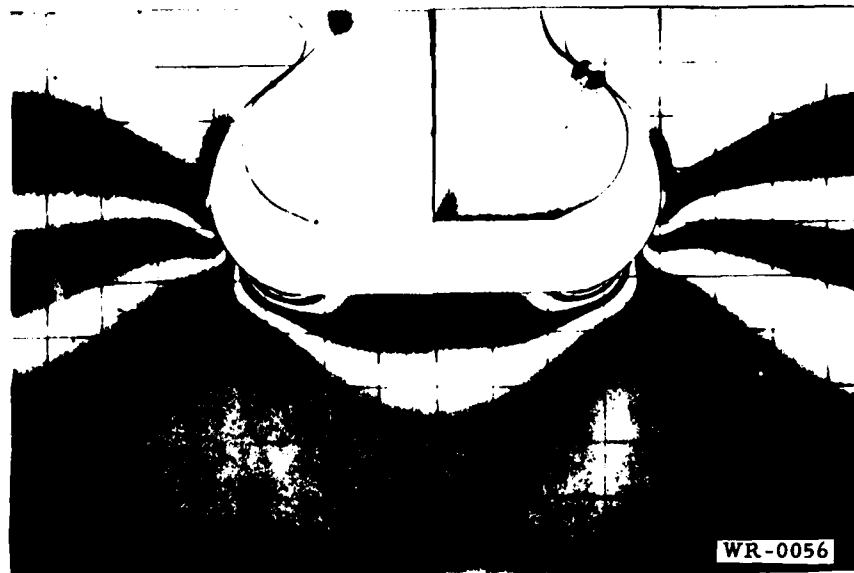


FIG. 21 FILLET AT THE DOVE TAIL JOINT BETWEEN BLADES AND ROTOR.



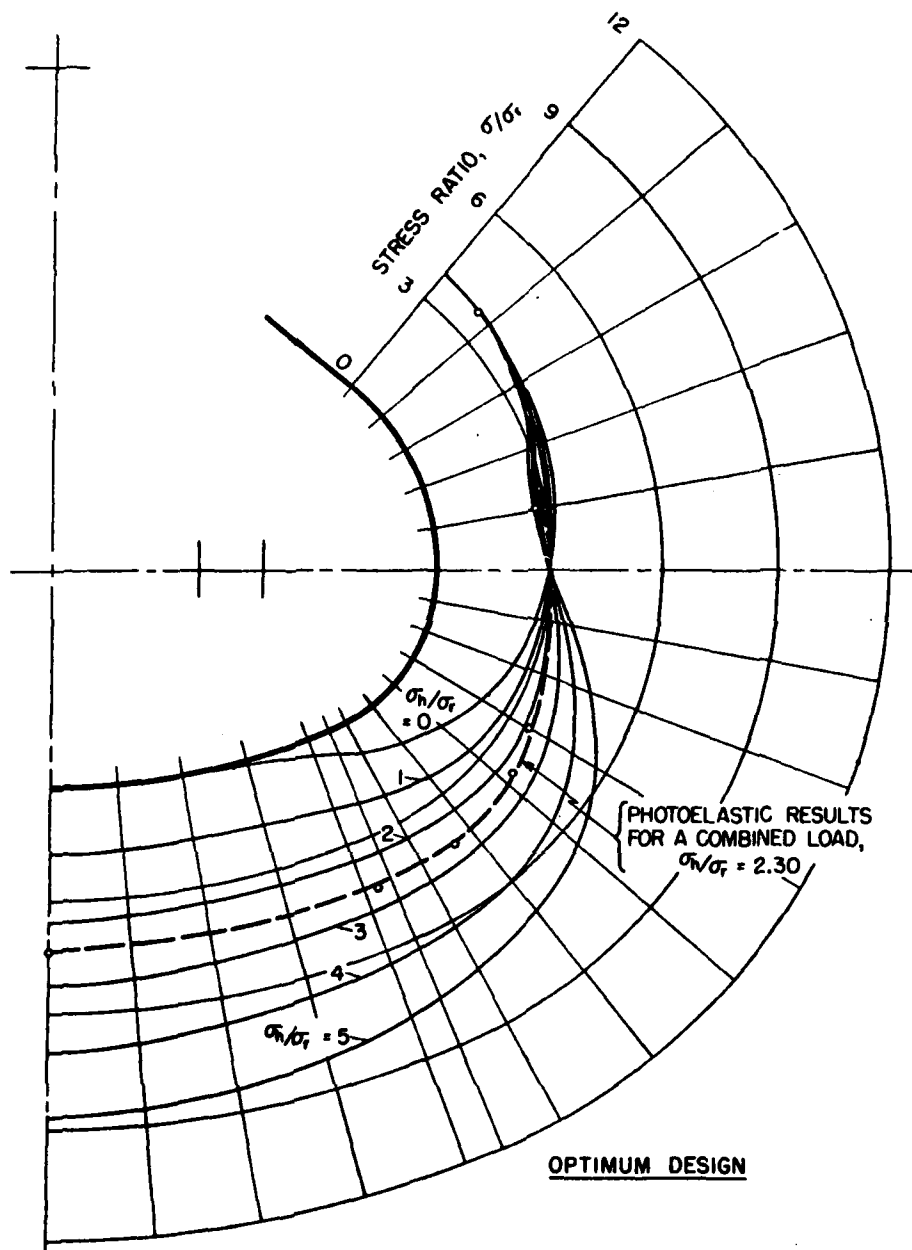
ORIGINAL DESIGN



REDESIGN

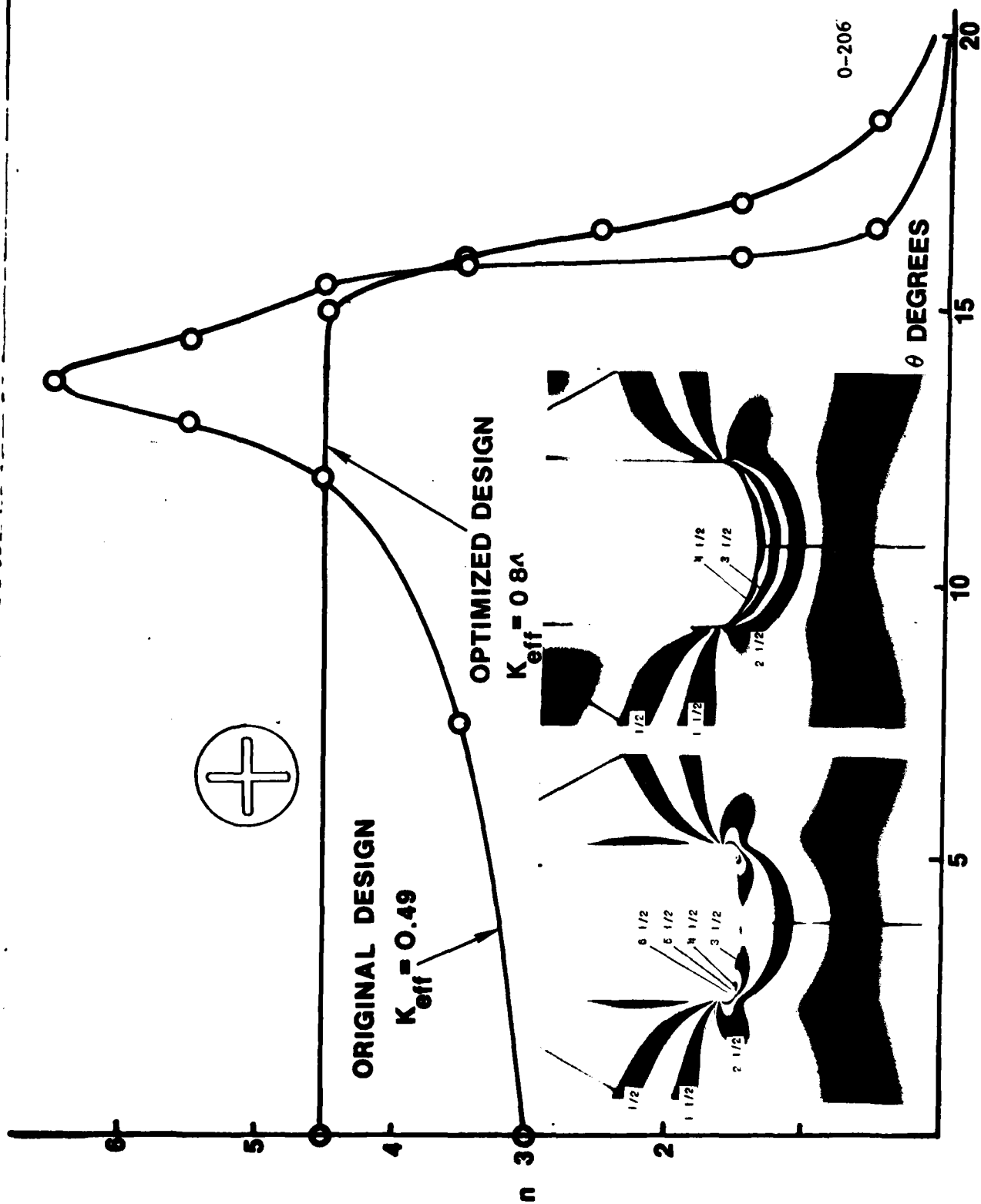
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FIG. 22 PHOTOELASTIC ISOCHROMATIC PATTERNS PRODUCED BY A TANGENTIAL LOADING IN THE ORIGINAL AND REDESIGNED MODELS OF THE SLOT OF A TURBINE ROTOR.

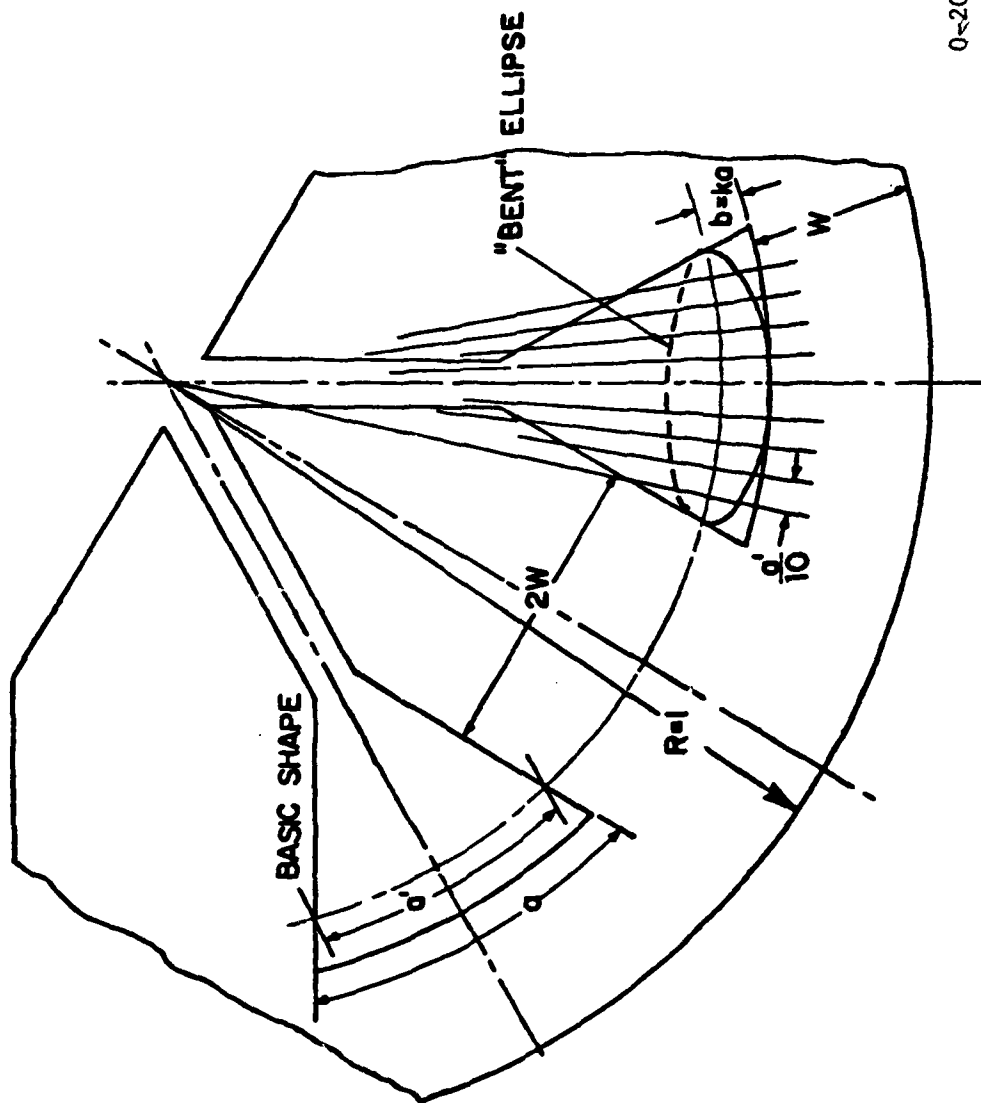


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FIG. 23 BOUNDARY STRESS DISTRIBUTIONS FOR VARIOUS RATIOS OF TANGENTIAL LOADING TO RADIAL BLADE LOADING IN THE REDESIGNED MODEL OF THE SLOT OF A TURBINE ROTOR.



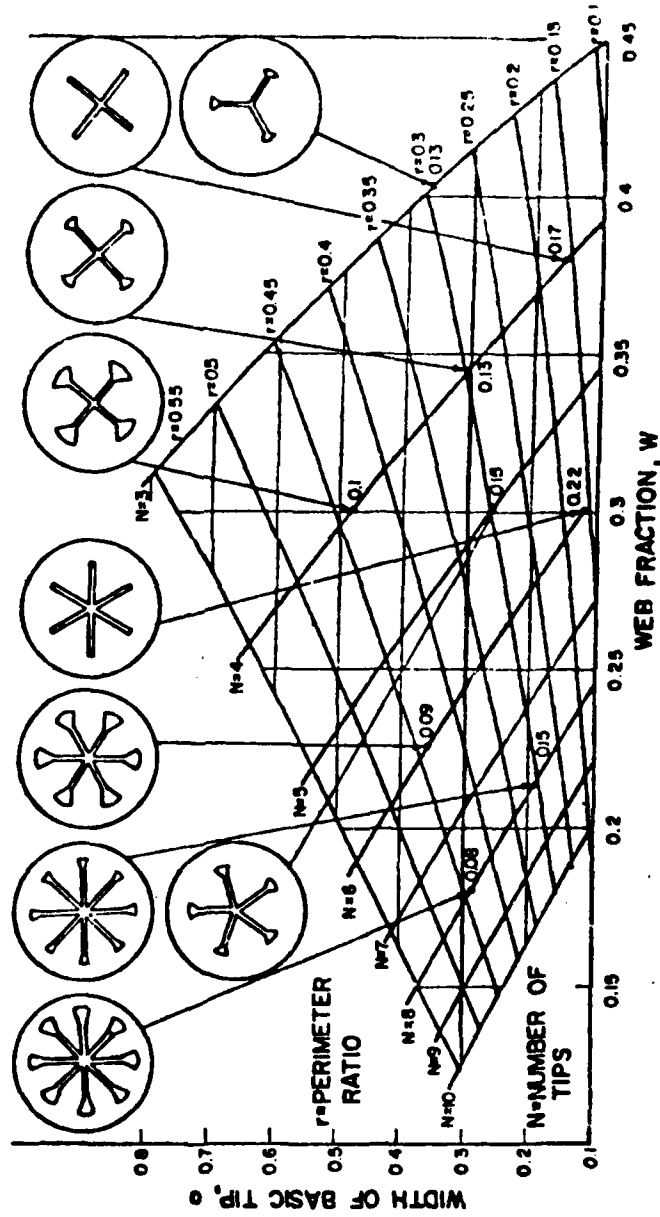
0 FIG. 24 BOTTOM OF THE RAYS OF STAR-SHAPED CONFIGURATION OF PROPELLANT GRAIN WITH STRAIGHT SIDED RAYS.



0-202

FIG. 25 GEOMETRY OF ENLARGED TIP OF RAYS IN STAR PERFORATED DISKS AND METHOD OF LAYING OUT THE TIP FORM.

THE VALUES GIVEN CORRESPOND TO OPTIMUM b/a FOR THE PARTICULAR GEOMETRY



0-203

FIG. 26 RELATIONSHIP BETWEEN THE GEOMETRIC PARAMETERS IN STAR PERFORATED DISKS WITH ENLARGED TIP OF RAYS.

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19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Optimization Photoelasticity Weight reduction Stress concentrations Propellant grains Turbine dove-tails		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A new method has been developed that permits the direct design of shapes of two-dimensional structures and structural components, loaded in their plane, within specified design constraints and exhibiting optimum distribution of stresses. The method uses photoelasticity and requires a large field diffused light polariscope. Several problems of optimization related to the presence of holes in finite and infinite plates, subjected to uniaxial and biaxial loadings, are solved parametrically.		

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Some unexpected results have been found: (1) the optimum shape of a large hole in a bar of finite width, subjected to uniaxial load, is "quasi" square, but the transverse boundary has the configuration of a "hat"; (2) for the small hole in the large plate, a "barrel" shape has a lower s.c.f. than the circular hole and appreciably higher coefficient of efficiency; (3) the optimum shape of a tube, subjected to diametral compression, has small "hinges" and is much lighter and stronger than the circular tube. Applications are also shown to the design of dove-tails and slots in turbine blades and rotors, and to the design of star-shaped solid propellant grains for rockets.