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NEW RESULTS ON THE INNOVATIONS PROBLEM
FOR NON-LINEAR FILTERING*

by

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*Submitted to Stochastics.

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Abstract

Consider an observed stochastic process consisting of a signal with additive noise. Assume that the signal has finite energy and that the signal and noise are independent. In this paper we show, that under the above assumptions the innovations and observations σ -algebra are equal thereby proving a long-standing conjecture of Kailath.

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Introduction.

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space, $\mathcal{F} = (\mathcal{F}_t)$, $0 \leq t \leq 1$, a non-decreasing family of sub- σ -algebras and $W = (W_t, \mathcal{F}_t)$, $0 \leq t \leq 1$, a Wiener process. With a signal process, $\beta = (\beta_t, \mathcal{F}_t)$, and

$$(1) \quad y_t = \int_0^t \beta_s ds + W_t$$

as observations, the innovations problem is to determine whether $y = (y_t, \mathcal{F}_t)$ is adapted to the innovations process, $(v, \mathcal{F}_t^y)$. This process, whenever it exists (see, for example, [1]), is a Wiener process defined by the equation

$$(2) \quad v_t = y_t - \int_0^t \hat{\beta}_s ds$$

where $\hat{\beta}_t = E(\beta_t | y_s, 0 \leq s \leq t)$. The innovations problem, first posed by Kailath in 1967 and subsequently considered by Frost in his thesis [2] can be posed in probabilistic terms; namely, are the σ -algebras generated by these processes the same modulo null sets; i.e. is

$$\sigma\{y_s: s \leq t\} = \sigma\{v_s, s \leq t\} \pmod{P}?$$

In this paper, we show that in the form conjectured by Kailath [3] this problem has a positive solution. Our assumptions are that

(a) Signal and noise processes are independent

and

$$(b) \quad E\left(\int_0^1 \beta_s^2 ds\right) < \infty.$$

Our results generalise all known results on the innovations problem ([4], [3]). In [4] the signal process is assumed to be uniformly bounded. The proof given in [2] is incorrect (see [3]). This problem has also been considered by Benes [5] and Kallianpur [6] under slightly weaker hypotheses than ours. Their

proofs however appear to be incorrect. We have been informed that results similar to ours have been independently obtained by J.M.C. Clark and M.P. Ershov.

The problem considered here is a subclass of the more general innovations problem for stochastic differential equations ([7] Page 260). In this more general form, the innovations problem does not have (in general) a positive solution. A counter-example was given by Cirelson ([7], Page 150). In Cirelson's example no "filtering" takes place and thus it cannot be considered to be a counter-example to the innovations problem for non-linear filtering. Cirelson's example however can be modified to obtain examples where filtering does occur (cf. Beneš [8]). The proof presented in this paper utilizes the independence of the signal and noise processes in an essential way. Nevertheless we feel that the assumption of independence can be removed for a wide class of signal processes.

1. The Innovations Result under (a), (b).

Our proof consists of two parts: deriving a jointly measurable functional, $\gamma(s, x)$,

$$\gamma: [0,1] \times C[0,1] \rightarrow R$$

with the property that

$$\gamma(s, \beta(\omega)) = \hat{\beta}_s(\omega) \quad \lambda \times P - a.s.$$

(λ denotes Lebesgue measure on $[0,1]$), and then showing that any weak solution of the stochastic differential equation

$$(4) \quad d\xi_t = \gamma(t, \xi)dt + dv_t$$

is pathwise unique in the sense of Yamada and Watanabe [9]. It is a consequence of their work that (4) has a strong solution (in the sense of Ito) i.e. the observations are a functional of the innovations.

Under (a), (b), we may apply the results of Kallianpur and Striebel [10] to show that for $0 \leq t \leq 1$,

$$(5) \quad \hat{\beta}_t(\omega) = \frac{\int_{\hat{\Omega}} \beta_t(\hat{\omega}) \exp\left(\int_0^t \beta_s(\hat{\omega}) dy_s(\omega) - \frac{1}{2} \int_0^t \beta_s^2(\hat{\omega}) ds\right) dP(\hat{\omega})}{\int_{\hat{\Omega}} \exp\left(\int_0^t \beta_s(\hat{\omega}) dy_s(\omega) - \frac{1}{2} \int_0^t \beta_s^2(\hat{\omega}) ds\right) dP(\hat{\omega})} \quad \text{-- } \omega \text{ a.s.}$$

Replacing y_s by x_s for $x \in C[0,1]$, we arrive at $\gamma(t,x)$. The joint measurability of $\gamma(t,x)$ rests upon the measurability in $(t,\hat{\omega},x)$ of the functional, (or stochastic integral),

$$\int_0^t \beta_s(\hat{\omega}) dx_s \equiv \langle D\beta(\hat{\omega}), x \rangle_t$$

which represents a Gaussian random variable with respect to Wiener measure on $C[0,t]$ whenever $\beta(\hat{\omega})$ is in $L_2[0,t]$, for $t \leq 1$. The operator, D , is unitary from $L_2[0,1]$ onto C' , the Hilbert Space of continuous function with square integrable derivatives, and

$$Df(s) = \int_0^s f(r) dr.$$

(Further discussion is given by Kuo [11].)

Our hypotheses (a), (b) guarantee that the innovations $\{v_t\}$ can be constructed [1] and so, (4) is satisfied by the observations. To show that any weak solution to (4) is pathwise unique, we will need the following lemmas.

Lemma 1. Let

$$\rho(t,x,\hat{\omega}) = \exp\left(\int_0^t \beta_s(\hat{\omega}) dx_s - \frac{1}{2} \int_0^t \beta_s^2(\hat{\omega}) ds\right)$$

and

$$g(t,x) = \int_{\hat{\Omega}} \rho(t,x,\hat{\omega}) dP(\hat{\omega}).$$

Then

$$(a) \quad \mu_W \{x: \sup_{0 \leq t \leq 1} g(t,x) < \infty\} = 1$$

and

$$(b) \mu_W \{x: \inf_{0 \leq t \leq 1} g(t,x) > 0\} = 1.$$

where μ_W is Wiener measure on $C[0,1]$.

Proof: Recall that

$$P\{\hat{\omega}: \int_0^1 \beta_s^2 \hat{\omega}(\hat{\omega}) ds < \infty\} = 1,$$

and for each such $\hat{\omega}$, the process

$$\{\rho(t, W(\omega), \hat{\omega}), F_t^W\}$$

is a (right) continuous martingale. Consequently, $\{g(t, W(\omega)), F_t^W\}$ is a right continuous martingale. For let $\lambda(W)$ be a bounded F_s^W -measurable random variable, $s < t$.

Then

$$\begin{aligned} E_P[\lambda(\omega) \cdot g(t, W)] \\ &= \int_C \lambda(x) g(t, x) d\mu_W(x) \\ &= \int_{\hat{\Omega}} \left[\int_C \lambda(x) \rho(t, x, \hat{\omega}) d\mu_W(x) \right] dP(\hat{\omega}) \\ &= \int_{\hat{\Omega}} \left[\int_C \lambda(x) \rho(s, x, \hat{\omega}) d\mu_W(x) \right] dP(\hat{\omega}) \\ &= \int_C \lambda(x) g(s, x) d\mu_W(x), \end{aligned}$$

from which we obtain

$$E_P(g(t, W) | F_s^W) = g(s, W) - \omega \text{ a.s.}$$

Moreover, since the family of sub- σ -algebra, $\{F_s^W; 0 \leq s \leq 1\}$ is continuous, and

$$E_p(g(t,W)) = 1, \quad 0 \leq t \leq 1,$$

it follows that $\{g(t,W), F_t^W\}$ has a right continuous version [Thm. 3.1, 7].

Thus, we conclude that

$$P\{\omega: \sup_{0 \leq t \leq 1} g(t,W(\omega)) < \infty\} = 1$$

$$\text{since } P\{\omega: \sup_{0 \leq t \leq 1} g(t,W(\omega)) > \lambda\} \leq 1/\lambda$$

for $\lambda > 0$. This gives (a).

For (b), note that $g(t,x)$ is the Radon-Nikodym derivative, $\frac{d\mu_y}{d\mu_W}(t,x)$ where μ_y is the measure induced on $C[0,1]$ by the observations process, y . Since $\mu_y \sim \mu_W$, the proof is analogous to Lemma 6.5 [7].

Lemma 2. Let

$$\alpha(t,x) = \int_{\hat{\Omega}} \left[\int_0^1 \beta_s^2(\hat{\omega}) ds \right] \cdot \rho(t,x,\hat{\omega}) dP(\hat{\omega}).$$

Then

$$\mu_W \{x: \sup_{0 \leq t \leq 1} \alpha(t,x) < \infty\} = 1.$$

Proof: The process

$$\{\alpha(t,W(\omega)), F_t^W\} \text{ is a right continuous martingale and for } 0 \leq t \leq 1, \\ E_p(\alpha(t,W)) = \int_{\hat{\Omega}} \left(\int_0^1 \beta_s^2(\hat{\omega}) ds \right) dP(\hat{\omega}).$$

Lemma 3. Let

$$m(t,x) = \int_{\hat{\Omega}} |\beta_t(\hat{\omega})|^2 \cdot \rho(t,x,\hat{\omega}) dP(\hat{\omega}).$$

Then

$$\mu_W \{x: \int_0^1 m(t,x) dt < \infty\} = 1.$$

Proof: Observe that

$$\int_{\hat{\Omega}} \left[\int_0^1 m(t, W(\omega)) dt \right] dP(\omega) = \int_0^1 \left[\int_{\hat{\Omega}} |\beta_t(\hat{\omega})|^2 dP(\hat{\omega}) \right] dt < \infty.$$

We return to the problem of comparing two weak solutions ξ_0, ξ_1 of (4), assuming that ξ_0, ξ_1 are both defined on the space (Ω, F, P) . Moreover, we may assume that μ_{ξ_0}, μ_{ξ_1} are each absolutely continuous with respect to Wiener measure. The proof of Proposition 1 [9] remains valid with respect to the restricted class of solutions ξ which satisfy the condition

$$P \left[\int_0^1 (\gamma(t, \xi))^2 dt < \infty \right] = 1.$$

See, in particular, pg. 161 [9]. From Lemmas 1,3, it follows that μ_{ξ_0}, μ_{ξ_1} are equivalent to Wiener measure since

$$\mu_W \{x: \int_0^1 (\gamma(t, x))^2 dt < \infty\} = 1.$$

Thus, for $i = 0, 1$, we conclude that

$$\sup_{0 \leq t \leq 1} g(t, \xi_i(\omega)) < \infty$$

$$\inf_{0 \leq t \leq 1} g(t, \xi_i(\omega)) > 0$$

$$\sup_{0 \leq t \leq 1} \alpha(t, \xi_i(\omega)) < \infty$$

$$\text{and} \quad \int_0^1 m(t, \xi_i(\omega)) dt < \infty$$

- ω a.s.

Theorem 1. If ξ_0, ξ_1 are weak solutions of (4), then

$$\sup_{0 \leq t \leq 1} |\xi_0(t, \omega) - \xi_1(t, \omega)| = 0 \quad P - a.s.$$

Proof. On $[0, 1] \times C[0, 1]$, define

$$\begin{aligned} L(t, \omega) &= |\gamma(t, \xi_0) - \gamma(t, \xi_1)| = \left| \frac{d(\xi_0 - \xi_1)}{dt} \right| \\ &= \left| \frac{f(t, \xi_0) - f(t, \xi_1)}{g(t, \xi_0)} + f(t, \xi_1) \left(\frac{1}{g(t, \xi_0)} - \frac{1}{g(t, \xi_1)} \right) \right| \end{aligned}$$

Then

$$(6) \quad (L(t, \omega))^2 \leq K(\omega) [(f(t, \xi_0) - f(t, \xi_1))^2 + (f(t, \xi_1))^2 (g(t, \xi_0) - g(t, \xi_1))^2]$$

$$\text{where } K(\omega) > \max \left(\frac{1}{\inf_{0 \leq t \leq 1} (g(t, \xi_0))^2}, \frac{1}{\inf_{0 \leq t \leq 1} (g(t, \xi_0))^2 \cdot \inf_{0 \leq t \leq 1} (g(t, \xi_1))^2} \right)$$

For $0 \leq u \leq 1$, we write

$$\begin{aligned} (7) \quad \int_0^u L^2(t, \omega) dt &\leq K(\omega) \left[\int_0^u \left[\int_{\hat{\Omega}} |\beta_t(\hat{\omega})| |\rho(t, \xi_0, \hat{\omega}) - \rho(t, \xi_1, \hat{\omega})| dP(\hat{\omega}) \right]^2 dt \right. \\ &\quad \left. + \int_0^u (f(t, \xi_1))^2 \cdot \left[\int_{\hat{\Omega}} |\rho(t, \xi_0, \hat{\omega}) - \rho(t, \xi_1, \hat{\omega})| dP(\hat{\omega}) \right]^2 dt \right] \end{aligned}$$

and because e^x is convex, it follows for all $\hat{\omega}$, t , that

$$\begin{aligned}
 (8) \quad & |\rho(t, \xi_0, \hat{\omega}) - \rho(t, \xi_1, \hat{\omega})| \\
 & \leq \frac{1}{2} |\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})| \cdot \left| \int_0^t \beta_s(\hat{\omega}) d(\xi_0 - \xi_1)(\omega) \right| \\
 & = \frac{1}{2} |\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})| \cdot \left| \int_0^t \beta_s(\hat{\omega}) L(s, \omega) ds \right|
 \end{aligned}$$

Applying Hölder's inequality to the last integral term in (8), and bringing out $(\int_0^t (L(s, \omega))^2 ds)^{1/2}$, yields

$$(9) \quad \int_0^u (L(t, \omega))^2 dt \leq K(\omega) \int_0^u \left[\int_0^t (L(s, \omega))^2 ds \right] \cdot \psi(t, \omega) dt \quad - \omega \text{ a.s.}$$

where

$$\begin{aligned}
 \psi(t, \omega) = & \left[\int_{\hat{\Omega}} |\beta_t(\hat{\omega})| \cdot \frac{(\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega}))}{2} \cdot \left(\int_0^t \beta_s^2(\hat{\omega}) ds \right)^{1/2} dP \right]^2 \\
 & + \left[\int_{\hat{\Omega}} \beta_t(\hat{\omega}) \rho(t, \xi_1, \hat{\omega}) dP \right]^2 \\
 & \left[\int_{\hat{\Omega}} \left(\frac{\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})}{2} \right) \cdot \left(\int_0^t \beta_s^2(\hat{\omega}) ds \right)^{1/2} dP(\hat{\omega}) \right]^2.
 \end{aligned}$$

By showing that $\psi(t, \omega)$ is an integrable function of t - ω a.s., one may then iterate (9) and conclude that for $0 \leq u \leq 1$,

$$\int_0^u (L(t, \omega))^2 dt = 0 \quad \omega - \text{a.s.}$$

Hence $\sup_{0 \leq t \leq 1} |\xi_0(t, \omega) - \xi_1(t, \omega)|$

$$\leq \int_0^1 (L(t, \omega))^2 dt = 0 \quad \omega - \text{a.s.}$$

and we have established path-wise uniqueness for weak solutions of (4).

To see that $\psi(t, \omega)$ is integrable in t , ω - a.s, apply Hölder's inequality to the first term to obtain,

$$\begin{aligned}
 (10) \quad & \left[\int_{\hat{\Omega}} |\beta_t(\hat{\omega})| \left| \frac{\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})}{2} \right| \left(\int_0^t \beta_s^2(\hat{\omega}) ds \right)^{1/2} dP(\hat{\omega}) \right]^2 \\
 & \leq \left(\int_{\hat{\Omega}} |\beta_t^2(\hat{\omega})| \left| \frac{\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})}{2} \right| dP(\hat{\omega}) \right) \cdot \\
 & \quad \left(\int_{\hat{\Omega}} \left[\int_0^t \beta_s^2(\hat{\omega}) ds \right] \left(\frac{\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})}{2} \right) dP(\hat{\omega}) \right) \\
 & \leq \int_{\hat{\Omega}} |\beta_t^2(\hat{\omega})| \left(\frac{\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})}{2} \right) dP(\hat{\omega}) \\
 & \quad \sup_{0 \leq t \leq 1} \int_{\hat{\Omega}} \left[\int_0^1 \beta_s^2(\hat{\omega}) ds \right] \cdot \left(\frac{\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})}{2} \right) dP(\hat{\omega})
 \end{aligned}$$

Integrating this product over t gives

$$\begin{aligned}
 (11) \quad & \sup_{0 \leq t \leq 1} \int_{\hat{\Omega}} \left[\int_0^1 \beta_s^2(\hat{\omega}) ds \right] \cdot \left(\frac{\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})}{2} \right) dP(\hat{\omega}) \\
 & \cdot \int_0^u \left[\int_{\hat{\Omega}} |\beta_s^2(\hat{\omega})| \left| \frac{\rho(t, \xi_0, \hat{\omega}) + \rho(t, \xi_1, \hat{\omega})}{2} \right| dP(\hat{\omega}) \right] dt. < \infty \quad \omega - \text{a.s.}
 \end{aligned}$$

by Lemmas 2,3.

The second term of $\psi(t, \omega)$ is handled analogously using Lemmas 1,2,3.

This completes the proof.

Thus, the observations process $\{y_t\}$, $0 \leq t \leq 1$, is the (unique) strong solution satisfying (4) under the restriction that

$$P\left(\int_0^1 \gamma(t, \xi)^2 dt < \infty\right) = 1.$$

Final Remarks.

Let us rewrite equation (2) as

(12) $v = (I - N)y$, where N is a non-linear operator from $C[0,1;\mu_y]$ into $C[0,1;\mu_y]$. Under assumptions (a) and (b) we have shown that an inverse operator $(I + \hat{N})$ exists such that P - a.s.

$$(13) \quad y = (I + \hat{N})v.$$

Moreover, if $\pi_t : C[0,1] \rightarrow C[0,1]$ denotes the truncation operator defined by

$$(14) \quad (\pi_t x)(s) = \begin{cases} x_s, & 0 \leq s \leq t \\ 0, & \text{otherwise} \end{cases},$$

then

$$(15) \quad (I + \hat{N})(\pi_t v) = \pi_t y, \quad \text{that is, the operator } (I + \hat{N}) \text{ is } \underline{\text{causal}}.$$

Our results in this paper suggest the investigation of causal-invertibility of non-linear causal operators on abstract Wiener Spaces (in the sense of Gross) using methods of stochastic integration and martingales. Such an investigation would also be of importance in the theory of stochastic stability of feedback systems.

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REFERENCES

- [1] M. Fujisaki, G. Kallianpur and H. Kunita, *Stochastic differential equations for the nonlinear filtering problem*, Osaka J. Math. 9 (1972), 19-40.
- [2] P. Frost, *Estimation in continuous time nonlinear systems*, Dissertation, Stanford University, Stanford, California, June 1968.
- [3] P.A. Frost and T. Kailath, *An innovations approach to least-squares estimation--Part III: nonlinear estimation in white gaussian noise*, IEEE Trans. on Automatic Control AC-16 no. 3 (1971), 217-226.
- [4] J.M.C. Clark, *Conditions for the one-to-one correspondence between an observation process and its innovations*, Tech. Report no. 1, Imperial College, London, England, 1969.
- [5] V.E. Āenes, *Extension of Clark's innovations equivalence theorem to the case of signal z independent of noise, with $\int_0^T z_s^2 ds < \infty$ a.s.*, Mathematical Programming Study 5 (1976), 2-7.
- [6] G. Kallianpur, *A linear stochastic system with discontinuous control*, Prof. of International Symposium on Stochastic Differential Equations, Kyoto 1972, edited by K. Ito, Wiley Interscience, 1978, pp. 127-140.
- [7] R. Lipster and A. Shirayev, *Statistics of random processes, I, general theory*, Springer-Verlag, New York, 1977.
- [8] V.E. Āenes, *Nonexistence of strong nonanticipating solutions to stochastic DEs: implications for functional DEs, filtering and control*, Stochastic Processes and Their Applications 5 no. 3 (1977), 243-263.
- [9] T. Yamada and S. Watanabe, *On the uniqueness of solution of stochastic differential equation*, J. Math. Kyoto Univ. 11 (1971), 155-167.
- [10] G. Kallianpur and C. Striebel, *Estimation of stochastic processes: arbitrary system processes with additive white noise error*, Ann. Math. Stat. 39 (1968), 785-801.
- [11] H.H. Kuo, *Gaussian measures in banach spaces*, Lecture Notes in Mathematics 463, Springer-Verlag, Berlin, 1975.