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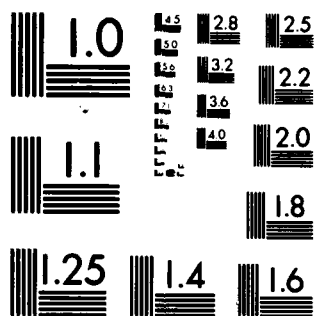
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Nonlinear Transfer of Kinetic Energy in the Atmosphere
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A review of the literature on nonlinear transfer of kinetic energy in the atmosphere using one and two dimensional spectral techniques are presented. The numerical programs for the conversion of energy have been written and tested. It was found that the hemispheric meteorological data from the AFGWC and from ETAC have serious gaps that hinder the progress of nonlinear kinetic energy transfer calculations. It is suggested to introduce an interpolation technique which will formally fill the data gaps, or,		

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alternatively, to acquire new data sets without such gaps. Energy transfer calculations can then be successfully completed.

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NONLINEAR TRANSFER OF KINETIC ENERGY IN THE ATMOSPHERE USING ONE- AND TWO-DIMENSIONAL SPECTRAL REPRESENTATION

A preliminary remark: Most of the work on this research was performed by Major Frank H. Bower, AFGWC, to whom inquiries about the scientific content should be addressed.

A b s t r a c t

The review of the literature on this subject has been accomplished. The equations and computer programs for the conversion of energy have been written and tested on the computer. This research has shown that the hemispheric meteorological data from the AFGWC and from ETAC have serious gaps that hinder the progress of this task. For future work it is suggested to introduce an interpolation technique which will formally fill the data gaps, or, alternatively, to acquire new data sets without such gaps. Then the calculation of energy transfers can be accomplished.

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1. Introduction

The equations describing atmospheric motion form a nonlinear system. The motions range from the planetary scale to the microscale. The kinetic energy of large-scale atmospheric motion is transferred between the scales of motion by the nonlinear interaction of atmospheric waves. The purpose of this research is to study two formulations of the nonlinear transfer of kinetic energy between the larger scales of motion using a data set which only recently has become available and which has not been used for this purpose.

Many studies of kinetic energy transfer have concentrated on the total eddy kinetic energy by utilizing the governing equations in a physical space formulation. A great deal also can be learned by looking at the growth and decay of the individual scales of eddies. It is thus necessary to convert the governing equations to spectral form or to the domain of scale or wave number. Two methods, which will be described in Section 3, have been proposed to study the nonlinear transfer of kinetic energy in the spectral domain.

Spectral models increasingly are being tested for use in numerical weather prediction (Machenhauer and Daley, 1972; Bourke, 1974; Bourke et al., 1977) and extended range prediction (Baede and Hansen, 1977). Recent studies with general circulation models have examined the model-generated kinetic energy using spectral analyses (Stone et al., 1977; Baker et al., 1977, 1978). Observational spectral studies of atmospheric kinetic energy increasingly are needed on at least a hemispheric scale for monitoring and evaluating the efficiency of numerical weather prediction models and general circulation models. Despite the work of many researchers, fully hemispheric spectral statistics, and to a lesser extent their analysis and interpretation, are still only poorly known.

Lack of a single hemispheric analysis scheme has hampered previous studies of the nonlinear transfer of kinetic energy. Past researchers used only a middle-latitude analysis, a tropical analysis, or meshed the two analyses to obtain fully hemispheric data coverage. Separate middle-latitude and tropical analyses overlap in areal extent but do not agree. Meshing of the two analyses could introduce extraneous waves which would contaminate the statistics on the nonlinear transfer of kinetic energy.

The National Meteorological Center, in 1974, and the Air Force Global Weather Center, in 1976, began using the Flattery analyses scheme (Flattery, 1970; Bergman et al., 1974). This analysis scheme is very different from the previous analysis schemes and gives one analysis for the entire hemisphere. This research will use data from the Flattery analysis scheme for computation of the nonlinear transfer of kinetic energy.

2. Present status of the question

Two methods have been proposed for the study of energetics in the domain of wave number. Saltzman (1957) proposed a one-dimensional wave number formulation and derived energy equations which govern the behavior of the individual scales. Dependent variables along a latitude circle are expressed in terms of a Fourier series. The wave number represents the number of waves around the latitude circle.

Baer (1974) proposed the degree of the associated Legendre polynomial as a two-dimensional spectral index where the dependent variables are expressed in terms of spherical harmonics. This index incorporates two one-dimensional indices. Burrows (1975) used this approach to derive kinetic energy equations for the individual scales.

The equations of horizontal motion and the continuity equation in spherical coordinates are

$$\begin{aligned}\frac{\partial u}{\partial t} + \frac{u}{a \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{v}{a} \frac{\partial u}{\partial \phi} &= f v + u v \frac{\tan \phi}{a} - \frac{g}{a \cos \phi} \frac{\partial z}{\partial \lambda} - X \\ \frac{\partial v}{\partial t} + \frac{u}{a \cos \phi} \frac{\partial v}{\partial \lambda} + \frac{v}{a} \frac{\partial v}{\partial \phi} &= -f u - u^2 \frac{\tan \phi}{a} - \frac{g}{a} \frac{\partial z}{\partial \phi} - Y \\ \frac{1}{a \cos \phi} \frac{\partial u}{\partial \lambda} + \frac{1}{a} \frac{\partial v}{\partial \phi} - v \frac{\tan \phi}{a} &= 0\end{aligned}$$

In these equations the independent variables (λ, ϕ, p, t) represent longitude, latitude, pressure, and time, respectively. The dependent variables are u , v , and z , the west and south wind components, and the geopotential height of the isobaric surface, respectively; a is the radius of the earth; $f = 2\Omega \sin\phi$ is the coriolis parameter, where Ω is the earth's angular velocity; g is the gravitational acceleration; and X and Y are the eastward and northward components, respectively, of the frictional force per unit mass. These equations are used to derive the kinetic energy equations in the one- and two-dimensional formulations described in the following paragraphs. The above equations do not include the vertical advection terms. The reasons for this are explained in Section 4.

A. One-dimensional wave number formulation

The basis for Saltzman's formulation is that any of the meteorological dependent variables specified along a latitude circle may be written in terms of a Fourier representation

$$f(\lambda) = \sum_{l=-\infty}^{\infty} F(l) e^{il\lambda}.$$

The complex coefficients, $F(l)$, are given by

$$F(l) = \frac{1}{2\pi} \int_0^{2\pi} f(\lambda) e^{-il\lambda} d\lambda$$

and are the representation of $f(\lambda)$ in the wave number domain.

The spectral kinetic energy equation originally given by Saltzman (1957), has been modified (e.g., Yang, 1967) to apply to a limited latitudinal domain and is

$$\begin{aligned} \frac{\partial}{\partial t} K(l) = \frac{\partial}{\partial t} \int_{\phi_3}^{\phi_4} h(l) \cos \phi d\phi = & - \int_{\phi_3}^{\phi_4} \left\{ \frac{\cos \phi}{a} \frac{\partial}{\partial \phi} \left(\frac{\bar{u}}{\cos \phi} \right) \bar{\phi}_{uv}(l) \right. \\ & \left. + \frac{1}{a} \frac{\partial v}{\partial \phi} \bar{\phi}_{vv}(l) - \frac{\tan \phi}{a} \bar{v} \bar{\phi}_{uu}(l) \right\} \cos \phi d\phi \end{aligned}$$

$$\begin{aligned}
 & + \int_{\Phi_s}^{\Phi_N} \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \left\{ U_m \left[\frac{1}{a \cos \Phi} \psi_{uu_2}(m, l) + \frac{1}{a} \psi_{uv} \cos \Phi (m, l) \right. \right. \\
 & \quad \left. \left. - \frac{\tan \Phi}{a} \psi_{uv} (m, l) \right] + V_m \left[\frac{1}{a \cos \Phi} \psi_{uv_2}(m, l) \right. \right. \\
 & \quad \left. \left. + \frac{1}{a} \psi_{vv} \cos \Phi (m, l) - \frac{\tan \Phi}{a} \psi_{uu} (m, l) \right] \right\} \cos \Phi d\Phi \\
 & - \frac{1}{a} \left\{ \sum_{\substack{m=-\infty \\ m \neq 0}}^{\infty} \left[V_m \psi_{vv}(m, l) + U_m \psi_{uv}(m, l) \right] \cos \Phi \right\} \Big|_{\Phi_s}^{\Phi_N} \\
 & - \frac{g}{a} \left[\cos \Phi \Phi_{vz}(l) \right] \Big|_{\Phi_s}^{\Phi_N}
 \end{aligned}$$

where $\Phi_{fg}(l) = [F(l)G(-l) + F(-l)G(l)]$ represents the correlation of two functions, and $\psi_{fg}(m, l) = [F(l-m)G(-l) + F(-l-m)G(l)]$ represents part of the triple correlation product. $K(l)$ is the kinetic energy in wave number l ; a is again the radius of the earth; $\bar{u}$ and $\bar{v}$ are the mean zonal and meridional wind speeds. Independent variables as sub-scripts show differentiation with respect to that variable.

The transform pairs are $f(l): u v z$
 $F(l): U V A.$

The first integral on the right-hand-side of the equation represents the conversion from the zonal kinetic energy into the kinetic energy of the l th component through the work of Reynolds stresses against the zonal velocity gradients. This is called a wave-zonal interaction.

The second integral represents the transfer of kinetic energy of other wave components into that of the l th component through the nonlinear wave interaction. Summation of this term over all wave numbers is zero since the nonlinear interaction does not create or destroy energy, but only redistributes it between the scales of motion. This is called a wave-wave interaction.

The third and fourth terms are fluxes. The third term is the flux of eddy kinetic energy arising from the nonlinear interaction among the waves. The fourth term represents the work done by the pressure force at the boundaries.

The symbol $\phi_{fg}(l)$ also can be written as $\sum_{n=1}^{\infty} \phi_{fg}(l) = \overline{f'g'}$. Accordingly, the expression for the spectrum of the meridional transport of any quantity f is given by $\phi_{fv}(l)$.

Several authors have studied the effects of specific terms in the above formulation on the maintenance of the kinetic energy of the individual scales. Saltzman (1970) reviewed previous studies. There are two primary studies dealing with the wave-wave and wave-zonal interactions. Both of these studies used only the horizontal wind components in middle-latitude regions. The general results from these studies will be discussed below.

Saltzman and Teweles (1964) used nine years of 500 mb data to derive statistics on the rate of transfer of kinetic energy in the wave-wave and wave-zonal interactions. They used geostrophic winds and included up to wave number $l=15$. In the mean, kinetic energy is transferred from the eddies to the mean flow with maximum contribution from wave numbers 2 and 7. The minimum contribution is from wave number 4.

Wave numbers 1, 3, 4, and 11-15 gain kinetic energy from the wave-wave interaction with wave numbers 2 and 5-10. Saltzman and Teweles (1964) first suggested that the loss by wave number 2 is replenished by a large forced conversion of available potential energy on this scale associated with the ocean-continent structure. The loss by wave numbers 5-10 is replenished by a conversion from available potential energy resulting from baroclinic instability. This wave number band is the one for

which baroclinic instability is the largest and corresponds to the cyclone-scale waves to both larger and smaller waves is in general agreement with the theoretical results of Fjørtoft (1953).

Yang (1967) used one year of data at eight pressure levels from 1000mb through 100 mb to study the vertical variations of the interactions. He showed that the conversion to zonal kinetic energy reaches a maximum at the jet stream level. The contribution of the long waves increases with height to the jet stream level whereas the contribution from the short waves reaches a maximum slightly below jet stream level. For the wave-wave interaction at low levels, the medium and short waves provide kinetic energy to the long waves. However, throughout most of the troposphere the medium waves provide kinetic energy to both long and short waves.

The previously discussed results apply to middle-latitudes and are much different from the interactions in the tropics. Kanamitsu et al., (1972) concluded from 200 mb data that the ultralong waves $\ell=1,2$ are important in the kinetic energy budget for the tropics. A thermally direct east-west vertical circulation (Krishnamurti, 1971) due to land-ocean heating contrasts evidently generates kinetic energy in wave numbers 1 and 2.

For wave-zonal interactions in the tropics $\ell=1$ provides the greatest amount of energy. All waves $\ell \geq 4$ draw energy from the zonal flow with a peak at about wave number 8. For wave-wave interactions $\ell=1,2$ lose energy to all other waves.

Saltzman and Fleisher (1960) noted that the same wave number may actually represent vastly different physical scales at widely separated latitudes. The two-dimensional spectral index formulation, to be discussed next, attempts to overcome this ambiguity and to more adequately represent the two-dimensional nature of the large-scale atmospheric flow.

B. Two-dimensional spectral index formulation

The surface spherical harmonics, $Y_{\ell n}(\lambda, \mu) = P_{\ell n}(\mu) \exp(i\ell\lambda)$, are also appropriate functions for a spectral expansion of meteorological variables. The series expansion of a scalar variable can be written as

$$A(\lambda, \mu) = \sum_{\ell=-L}^L \sum_{n=1}^N A_{\ell n} Y_{\ell n}(\lambda, \mu).$$

The coefficients in this expansion are given by

$$A_{\ell n} = \frac{1}{4\pi} \int_{-1}^1 \int_0^{2\pi} A(\lambda, \mu) Y_{\ell n}^*(\lambda, \mu) d\lambda d\mu.$$

The associated Legendre polynomials are given by

$$P_{\ell n}(\mu) = \sqrt{\frac{(2n+1)(n-\ell)!}{(n+\ell)!}} \frac{(1-\mu^2)^{\ell/2}}{2^n n!} \left(\frac{d}{d\mu}\right)^{(n-\ell)} (\mu^2-1)^n.$$

In the above equations $\mu = \sin\phi = \sin(\text{latitude})$; L and N are the truncation planetary wave number and planetary index, respectively; $Y_{\ell n}^*(\lambda, \mu) = P_{\ell n}(\mu) \exp(-i\ell\lambda)$ is the complex conjugate of $Y_{\ell n}(\lambda, \mu)$; and $P_{-\ell n}(\mu) \equiv P_{\ell n}(\mu)$ (Platzman, 1962). For the spherical harmonics, $n-\ell$ gives the number of zeroes between the poles. Baer (1972) proposed the use of the degree, n , of the associated Legendre polynomial as a two-dimensional scale index or planetary index.

The full set of expansion coefficients can be found only when the meteorological variables are known over both hemispheres. Since meteorological variables for only half the atmosphere will be used, it is necessary to make the further assumption that the fields are either symmetric (even parity expansion) or antisymmetric (odd parity expansion) about the equator. Symmetry is defined by $n-\ell$ even and antisymmetry by $n-\ell$ odd.

Burrows (1975, 1976) divided the wind components into rotational (R) and divergent (D) parts given by $u = u_R(\text{even}) + u_D(\text{odd})$, $v = v_R(\text{odd}) + v_D(\text{even})$ where the even/odd refer to the symmetry/antisymmetry. The procedure for separation of the wind components is to find the rotational part of the

These equations are symbolic because it is understood that series are to be inserted for wind components and height. For example, the MA1 term in the above equation would be written

$$\left[\frac{\partial}{\partial t} K(u_{\ell n}) \right]_{MA1} = \frac{(2-\delta_{0n})}{a} R_{\ell} u_{\ell n} \int_{-1}^1 \int_0^{2\pi} \left[- \sum_{\ell_1=-L}^L \sum_{n_1=|k_1|}^N V_{\ell_1 n_1} y_{\ell_1 n_1} \right]_{\ell_1+n_1, \text{ even}} \\ \cdot \left[\sum_{\ell_2=-L}^L \sum_{n_2=|k_2|}^N U_{\ell_2 n_2} \sqrt{1-u^2} \frac{d}{du} (y_{\ell_2 n_2}) \right]_{\ell_2+n_2, \text{ odd}} e^{-i\ell_3 \lambda} p_{\ell_3 n_3} d\lambda du_{\ell_3+n_3, \text{ odd}}$$

There are similar equations for $\frac{\partial}{\partial t} [K(u_{\ell n})]$, $\frac{\partial}{\partial t} [K(v_{\ell n})]$, and $\frac{\partial}{\partial t} [K(\psi_{\ell n})]$. In the above equations, the symbols below the terms stand for zonal and meridional advection, sphericity terms, coriolis, pressure gradient, and "friction remainder", respectively. $\delta_{0n} = \begin{cases} 0, & n \neq 0 \\ 1, & n = 0 \end{cases}$ is the Kronecker delta. The nonlinear interaction among the scales of motion is given by the sum of advection and sphericity terms.

The pressure gradient term, in the above equations, does not generate kinetic energy. It does act to balance wind and mass fields which are thrown out of balance by turbulence and generation of kinetic energy from available potential energy. The generation of kinetic energy from available potential energy is not present in these equations because of the assumed parities for the wind and height fields. The pressure gradient force contributes only to the rotational wind components which are non-divergent and thus cannot show the generation of kinetic energy from available potential energy.

The coriolis force does no net work but affects the total kinetic energy in each n-scale. It should be noted that the coriolis term does

not enter into the equations based on grid point values. It does enter into the l -domain (one-dimensional wave number) equations for the individual components, but when the equations are added to give the total kinetic energy in each l -scale, the coriolis contribution is zero.

Burrows (1975, 1976) has made the only study of the nonlinear transfer of kinetic energy using the two-dimensional formulation. He used 15 days of August, 1970 data at eight pressure levels. He also compared his n -domain (two-dimensional index) results to the averages of Saltzman and Fleisher (1962) for middle-latitudes and to the results of Kanamitsu et al., (1972) for tropical latitudes. It should be noted that short-term averages of the nonlinear transfers may not be representative of longer term averages.

3. Prepared computer programs

The primary contribution of this research is an application of Burrow's two-dimensional formulation for the nonlinear transfer of kinetic energy to a longer period in a different year and involving objectively-analyzed data on a fully hemispheric grid. Saltzman's procedure is applied to the same data.

Data for August 1977 have been obtained for the whole Northern Hemisphere from AFGWC on magnetic tapes and, independently, from ETAC. The data, obtained by the Flattery scheme, are for the points on a numerical weather prediction grid extending to the equator at eight pressure levels from 1000 mb through 100 mb. For the purpose of using the present programs, the data are interpolated to a latitude/longitude grid by interpolation, using the interpolating polynomial through 12 nearest points.

After interpolation to a latitude/longitude grid, the wind and height for each pressure level are converted to the spectral form by the methods described in Section 2. The spectral coefficients are the basic variables for the remainder of the calculations.

For an initial analysis of the nonlinear transfer of kinetic energy based on data from the new analysis scheme it is advantageous to retain only those terms of major importance. The net effect of the nonlinear terms involving vertical velocity is one order of magnitude less than the major horizontal terms. In addition, vertical velocity is difficult to evaluate even with a large expenditure of resources. The terms involving vertical velocity are therefore excluded from the kinetic energy equations.

Each term in the kinetic energy equations is evaluated for each pressure level. The time derivatives, from the left-hand-side of the equations, is evaluated by centered finite time differences. The terms from the right-hand side of the equations are evaluated from values of height and wind at a single synoptic time, with the exception of the "friction remainder" term which is used to balance the equation. The "friction remainder" contains not only friction but also the contribution from unevaluated terms, errors in the evaluations, and errors arising from equating time-centered and instantaneous values. The unevaluated terms are the generation of kinetic energy by the conversion of available potential energy, the interaction with waves smaller than the resolved scale, and in the two-dimensional formulation the fluxes of kinetic energy due to cross-equatorial flow. Unresolved scales also influence the kinetic energy in the resolved scales due to aliasing.

The nonlinear terms in the two-dimensional formulation are evaluated by the transform method described by Machenhauer and Rasmussen (1972)

which eliminates the requirement for large numbers of interaction coefficients (Platzman, 1960). Very briefly, the transform method involves a conversion from the spectral domain to the physical domain, where the multiplication is performed, followed by a conversion back to the spectral domain. A Fast Fourier Transform program written by Singleton (1969) is modified and used for integration over longitude. Gauss-Legendre quadrature is used for integration over latitude.

A listing of all these programs is available from Major Frank Bower, AFGWC.

The results from the computation are not available since the data on the tapes from the National Weather Service and from ETAC have serious gaps. The transform methods, on the other hand, give reliable results only with complete data sets, without gaps. For this reason there will be a delay of a year or so until we find a suitable interpolation method or some other, more complete, data sets.

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