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STABILITY OF BETA LIMITED THERMONUCLEAR BURN.(U)

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The burn stability of a thermonuclear reacting plasma is examined under the assumption that a burn equilibrium exists due to the rapid increase of loss rate with plasma beta once a critical beta value is exceeded. It is found that perturbations about equilibrium generally result in a rapidly damped exponential decay and a second slow root which can be either growing (unstable) or damped. In the case where the slow root is growing, the possibility that it can be stabilized by feedback control of the rate at which neutral gas is fed into the system is considered.

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STABILITY OF BETA LIMITED THERMONUCLEAR BURN

Plasma heating by fusion products increases with temperature. Thus, once the ignition point is exceeded, in order to establish a thermonuclear burn equilibrium, it is necessary that plasma losses increase sufficiently rapidly with temperature. In this note we examine the implications of the hypothesis that such an equilibrium exists due to a rapid increase in losses with plasma beta once a critical value of beta is exceeded. For example, in tokamaks such a critical beta might result due to the onset of magnetohydrodynamic instabilities. Since generally accepted theoretical models for anomalous plasma loss in such devices as tokamaks are not available, we shall try to make as few assumptions as possible regarding the dependences of loss rates on plasma parameters except for the existence of a critical beta.

We utilize the following zero dimensional model equations for a deuterium-tritium mixture,

$$\dot{N}_\alpha = - \gamma_N^\alpha N_\alpha + \beta N_D N_T \ , \tag{1a}$$

$$\dot{N}_D = - \gamma_N^D N_D + S_D - \beta N_D N_T \ , \tag{1b}$$

$$\dot{N}_T = - \gamma_N^T N_T + S_T - \beta N_D N_T \ , \tag{1c}$$

Note: Manuscript submitted November 21, 1979.

$$\dot{E} = - \gamma_E^e N_e T - \gamma_E^D N_D T - \gamma_E^T N_T T - \gamma_E^\alpha N_\alpha T + \beta E_\alpha N_D N_T \ , \quad (1d)$$

where N_σ is the particle number of species σ ($\sigma = \alpha$, D, T, e for alpha particles, deuterons, tritons, and electrons), γ_N^σ is the loss rate of particles for species σ, γ_E^σ is the energy loss rate for species σ, $\beta(T)$ is the deuterium-tritium thermonuclear rate coefficient, E_α is the alpha particle creation energy, $E = (N_e + N_D + N_T + N_\alpha)T$ is the thermal particle energy content of the plasma, T is the temperature which has been taken to be equal for all particles (this is approximately true if the equilibration times are sufficiently short compared to the confinement time), and the dot denotes time derivitive. Ohmic heating has been neglected in Eq. (1d) since its contribution is small at the burn point. The quantity S_σ denotes the rate at which particles are supplied from an external neutral gas input (the effect of recycling of particles transported to the edge may be absorbed in the coefficient γ_N^σ).

Assuming $\gamma_N^D = \gamma_N^T$ and $S_D = S_T$, Eqs. (1b) and (1c) imply that $(N_D - N_T) \sim \exp(-\int^t \gamma_N^D \, dt)$. Thus under these circumstances it is reasonable to take $N_D = N_T$ and eliminate Eq. (1c). Assuming $N_\alpha << N_e, N_D$, neglecting the term $\beta N_D N_T$ on the right hand side of (1b), and utilizing the neutrality condition, $N_e = N_D + N_T + 2N_\alpha$, we obtain from Eqs. (1)

$$\dot{N}_D = - \gamma_N^D N_D + S_D \ , \quad (2a)$$

$$\dot{T} = - \left(\gamma_e + \frac{S_D}{N_D}\right) T + \beta e_\alpha N_D \ , \quad (2b)$$

where $e_\alpha \equiv E_\alpha/4$ and $\gamma_e \equiv \frac{1}{2}\gamma_E^e + \frac{1}{4}\gamma_E^D + \frac{1}{4}\gamma_E^T - \gamma_N^D$. In addition, we shall allow for feedback control of the external gas puffing rate, S_D. We assume that, based upon sensing of the particle number, N_D, the gas supply rate is adjusted,

$$\dot{S}_D = -\nu\,[S_D - F(N_D)] \;, \tag{2c}$$

where ν^{-1} is a time constant. This time constant reflects the fact that the control system will have a finite response time, as well as the fact that a particle supplied at the edge will take a finite time to diffuse into the reacting core plasma region. The latter time is probably of the order of $\left(\gamma_N^D\right)^{-1}$ and sets a upper limit on ν. For simplicity we shall, in what follows, neglect any T or N_D dependence of ν.

From Eqs. (2a) and (2b) the equilibrium burn condition is

$$\bar{S}_D = \bar{\gamma}_N^D\,\bar{N}_D, \tag{3a}$$

$$e_\alpha\,\bar{\beta}\,\bar{N}_D = (\bar{\gamma}_e + \bar{S}_D/\bar{N}_D)\bar{T} \;, \tag{3b}$$

where the overbar denotes equilibrium. Linearizing Eqs. (2) for perturbations about this equilibrium of the form $\exp(p\bar{\gamma}_N^D t)$ we obtain

$$\begin{gathered}(p + 1 + \alpha_1)\;[p + (\gamma + 1)\;(1 - \alpha_5) + \gamma\alpha_4]\;(p + \nu) + \\ \alpha_2\alpha_6\nu - \alpha_2(p + \nu)\;(\gamma\alpha_3 - \gamma - 2) + \alpha_6\nu[p + (\gamma + 1)\;(1 - \alpha_5) + \gamma\alpha_4] = 0,\end{gathered} \tag{4}$$

where

$$\gamma = \bar{\gamma}_e/\bar{\gamma}_N^D,\ \nu = \bar{\nu}/\bar{\gamma}_N^D,\ \alpha_1 = (\bar{N}_D/\bar{\gamma}_N^D)\ \partial\gamma_N^D/\partial N_D,\ \alpha_2 = (\bar{T}/\bar{\gamma}_N^D)\ \partial\gamma_N^D/\gamma T,$$

$\alpha_3 = (\bar{N}_D/\bar{\gamma}_e)\ \partial\gamma_e/\partial N_D$, $\alpha_4 = (\bar{T}/\bar{\gamma}_e)\ \partial\gamma_e/\partial T$, $\alpha_5 = (\bar{T}/\bar{\beta})d\beta(T)/dT$, and $\alpha_6 = (\bar{N}_D/\bar{S}_D)\ dF(N_D)/dN_D$. In the case where there is no feedback and S_D is held constant independent of plasma parameters, Eq.(4) becomes ($\nu \to 0$)

$$(p + 1 + \alpha_1)\ \ [p + (\gamma + 1)\ \ (1 - \alpha_5) + \gamma\alpha_4] - \alpha_2(\gamma\alpha_3 - \gamma - 2) = 0. \tag{5}$$

We now consider a special case: there is a rapid increase of energy loss (γ_e) past the critical value of β, but the particle loss rate (γ_N^D) is relatively uneffected (this situation might result from the onset of magnetic field braiding in a tokamak).

In this case

$$\alpha_3 \sim \alpha_4 >> \alpha_1 \sim \alpha_2 \sim 1 \tag{6}$$

(Note that since the α terms are derivative of logarithms with respect to logarithms, much greater than inequalities, such as (6), are in actual situations likely to be marginal at best. Thus the case which we consider should be regarded as illustrative only.)

For this situation, Eqs. (5) and (6) show that without feedback there are two roots $p = p_1,\ p_2$,

$$p_1 \simeq -\gamma\alpha_4\ \ , \tag{7a}$$

$$p_2 \simeq (\alpha_2\alpha_3/\alpha_4) - (1 + \alpha_1). \tag{7b}$$

The p_1 root represents a rapidly damped solution ($-p_1 \gg |p_2|$). The root p_2 can be either damped or growing depending on the signs of α_2 and α_1 and the relative magnitudes of the α coefficients. We now consider the possibility of stabilizing the root p_2 by feedback (for the case $p_2 > 0$). Applying (6) and (4) we find that stability results if both

$$\nu > p_2 \bar{\gamma}_N^{D} \quad \text{and} \quad dF/dN_D > p_2 \bar{\gamma}_N^{D} , \tag{8}$$

and the burn is unstable otherwise. Thus we see that stabilization occurs if the system response time is less than the instability growth time, and if, in addition, the feedback is strong enough ($dF/dN_D > p_2\bar{\gamma}_N^{D}$). Since a lower limit on γ results from the finite time for a particle supplied at the edge to reach the reacting core, it is not clear that (8) can be satisfied.

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