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LOW COST ANTI-JAM DIGITAL DATA-LINKS TECHNIQUES INVESTIGATIONS

Telecommunication and Control Systems Laboratory
Department of Electrical Engineering
Texas A&M University
College Station, Texas 77843

Final Report for Phase III
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> This report documents the final phase of research under the subject contract. Previous results showed that the Minimum Probability of Error recursive detector for colored plus white noise, tracks the colored noise and subtracts it from the data. The present effort investigated the effects on optimum detector performance of carrier phase estimation. A good characterization of the effects was obtained.

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PREFACE

From 1971 through 1973, a new sampled-data processing technique for digital signals subject to colored multiplicative noise was developed and subsequently patented by the Principal Investigator, at NASA Langley Research Center. In 1974, a contract was issued by the Air Force Avionics Laboratory to determine if the same technique which provided processing gain against diffuse Doppler-spread multipath perturbations could be applied to anti-jam processing.

Anti-jam processing algorithms were produced under the 1974 contract, as well as a Monte Carlo simulation package for performance evaluation. Between 1976 and 1978, substantial evaluation of the algorithms was performed and documented, under an extension of the contract.

A final extension of the contract, through April 1979 served to support investigation of means for implementing carrier phase estimation and bit synchronization with the detection algorithms. This report documents those results and gives recommendations for further research.

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SECTION I

INTRODUCTION

This report documents further research under the subject contract whose previous results have been reported in [1,4]. The basic te .nical problem is that of optimum discrete-time recursive detection of binary signals subject to additive colored and white noise. Previous results showed that the Minimum Probability of Error detector is one which tracks the colored noise and subtracts it from the received data. The related question of identification of the statistics of the colored interfering process was extensively investigated in Reference 1.

The research effort, documented herein, was pointed toward several related questions. First, it was desired to investigate the problem of simulataneous estimation of the carrier phase references required by the coherent detection algorithm. It was desired to specifically determine the method for measuring phase and also the augmentation of the detection algorithm required to operate with imperfect phase estimates.

Next, it was desired to investigate the possibility of non-coherent detection with interference tracking, with application to Frequency-Shift-Keying and Differential Phase-Shift-Keying.

A third area of interest was to determine a method for obtaining bit synchronization for the interference-tracking detection algorithms. This would then lead to assembly of a complete algorithm for the so-called IDEI (Integrated Detection, Estimation, Identification) receiver.

Finally, it was desired to obtain Monte Carlo evaluation of the augmented detector, operating in an environment of colored plus white additive noise.

All of the desired areas are investigated below. An expected result is that the coherent detector performance is degraded when carrier phase is estimated from the received data. An unexpected result is that a non-coherent version of the interference-tracking detection algorithm does not exist.

Recommendations are given on further research which may lead to improved performance of the complete IDEI receiver.

SECTION II

COHERENT DETECTION WITH PHASE ESTIMATION

1. SIGNAL AND CHANNEL MODEL

Figure 1 shows the overall model of the signal channel and signal processor. A continuous-time signal, $s(t,m)$, is transmitted through the channel.

$$s(t,m) = A(t;m)\cos[\omega_c t + \phi(t;m)] \tag{1}$$

In (1), A( ) and ϕ( ) are the envelope and phase functions, respectively. m denotes a digital symbol, which in the present work is restricted to the binary alphabet, {0,1}. Any arbitrary signal waveform may be represented in the form of (1).

The signal is subjected to additive colored and white noise, as per the figure. Then the bandpass signal plus noise process is translated to baseband in two separate channels, using coherent product detection with sinusoidal reference signals which are in phase and in phase quadrature with the unmodulated carrier signal. Following the I-Q demodulation, the two low-pass signal components of the signal vector are sampled to produce a discrete-time vector. The discrete-time signal is then processed further to recover the message symbol decisions, $\hat{m}$.

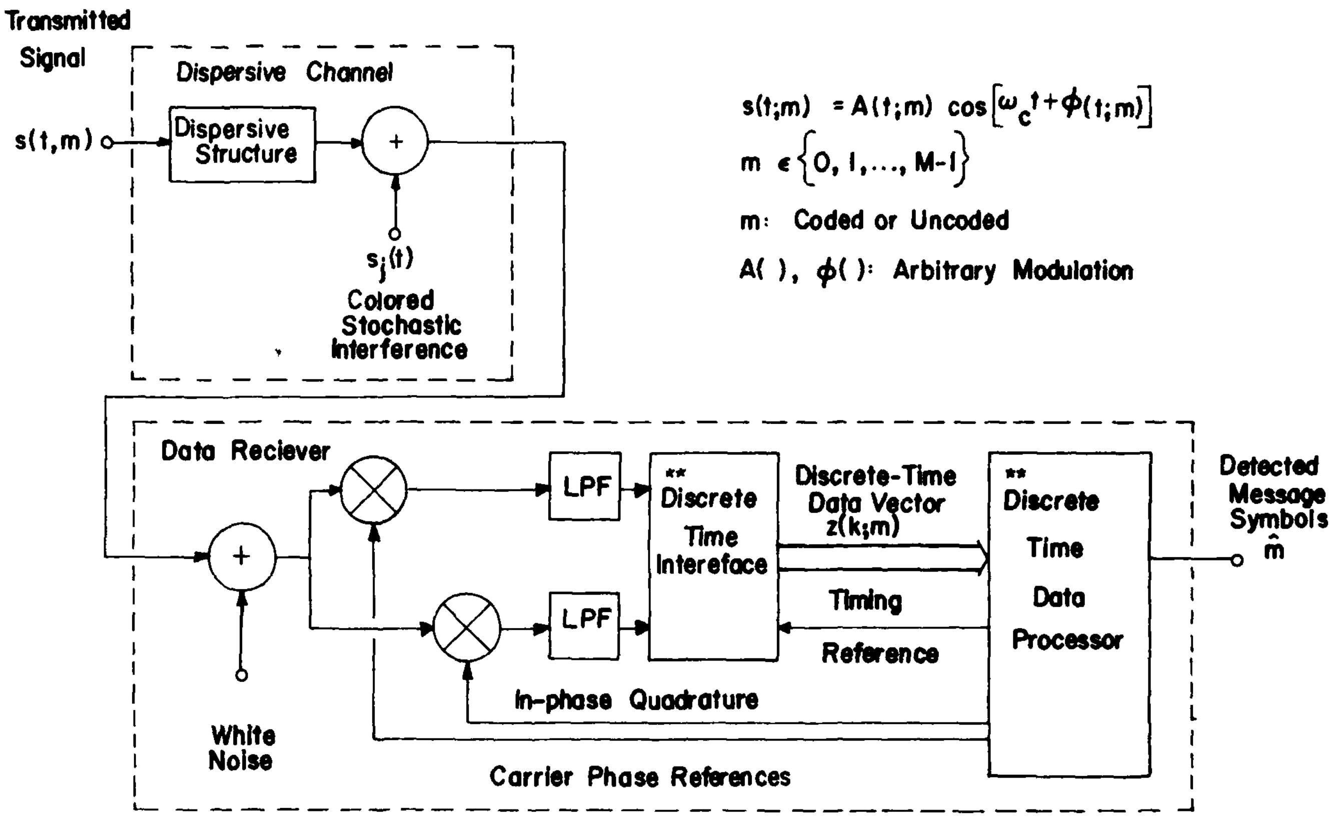
The I-Q product demodulators require reference sinusoids having precise phase references, matched to the phase (zero) of the unmodulated carrier signal. Since this phase is A Priori unknown, the phase reference must be provided by the signal processor, itself, by phase estimation from the received data vector. The reference phase, so produced, is generally a function of time, $\phi_0(t)$, as shown in Figure 2.

Since the signal phase is A Priori unknown, the signal model of (1) may be augmented with a random (or stochastic) phase term ϕ_s as

$$\begin{aligned} s(t,m) &= A(t;m)\cos[\omega_c t + \phi(t;m) + \phi_s] \\ &= s_i(t;m)\cos\omega_c t - s_q(t;m)\sin\omega_c t \end{aligned} \tag{2}$$

where

$$\begin{aligned} s_i(t;m) &= A(t;m)[\cos\phi(t;m)\cos\phi_s - \sin\phi(t;m)\sin\phi_s] \\ s_q(t;m) &= A(t;m)[\sin\phi(t;m)\cos\phi_s + \cos\phi(t;m)\sin\phi_s] \end{aligned} \tag{3}$$



$$s(t;m) = A(t;m) \cos\left[\omega_c t + \phi(t;m)\right]$$

$$m \epsilon \left\{0, 1, \ldots, M-1\right\}$$

m: Coded or Uncoded

A( ), ϕ( ): Arbitrary Modulation

**Binary Digital Processing, or Charge-Coupled Analog Processing

Figure 1. Physical Channel and Receiver Models

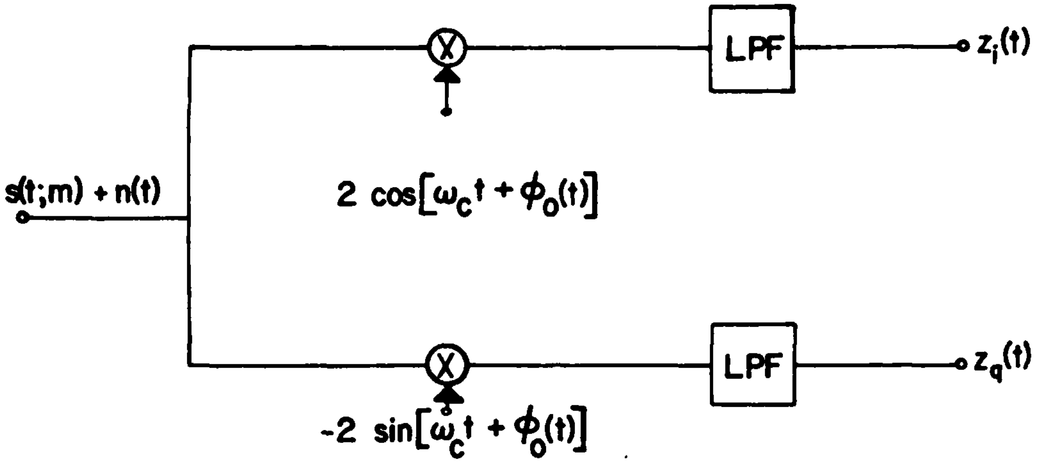

s(t;m) + n(t)
LPF
$z_i(t)$
$2\cos[\omega_c t + \phi_0(t)]$
LPF
$z_q(t)$
$-2\sin[\omega_c t + \phi_0(t)]$

Figure 2. I-Q Carrier Demodulator

are the in-phase and quadrature low-pass components of the band-pass $s(t;m)$.

The I-Q components of $s(t;m)$ form a vector

$$\begin{bmatrix} s_i(t;m) \\ s_q(t;m) \end{bmatrix} = \begin{bmatrix} \cos\phi_s & -\sin\phi_s \\ \sin\phi_s & \cos\phi_s \end{bmatrix} \begin{bmatrix} A(t;m)\cos\phi(t;m) \\ A(t;m)\sin\phi(t;m) \end{bmatrix} = \underline{s}(t;m) \tag{4}$$

Likewise, the additive colored and white noises may be written in terms of I-Q components as

$$\underline{y}(t) = \begin{bmatrix} y_i(t) \\ y_q(t) \end{bmatrix} \quad ; \quad \underline{n}(t) = \begin{bmatrix} n_i(t) \\ n_q(t) \end{bmatrix} \tag{5}$$

where $\underline{y}(t)$ is the low-pass I-Q colored interference vector and $\underline{n}(k)$ is the I-Q data vector, $\underline{z}(t)$ may then be written as

$$\underline{z}(t) = \underline{s}(t;m) + \underline{y}(t) + \underline{n}(t) \tag{6}$$

The problem of detecting the digital symbol, m, in the presence of colored noise, white noise, and unknown signal phase is essentially the problem of processing $\underline{z}(t)$ to make an optimum decision on m. This problem is analyzed in some detail below.

2. JOINT DETECTION WITH PHASE ESTIMATION

It is desired to reformulate the discrete-time recursive detection problem of [1] for the present case where the signal phase is unknown and time-varying. At this point it is still assumed that the symbol epoch, or timing, is known. The decision problem is based on processing the discretized I-Q data vector of (6). That is, a sequence of samples, $z(t_k)$ is processed recursively over the period of the binary symbol, m. Bit decision is made at the end of the symbol period. As in [1], decision-direction is to be used from symbol to symbol, in order to preclude a processor size which would grow exponentially with symbol sequence length.

The assumed data generating model is that of Figure 3, wherein $\underline{z}(k)$, $\underline{n}(k)$, $\underline{s}(k;m)$, and $\underline{y}(k)$ are the sampled versions of $\underline{z}(t)$, $\underline{n}(t)$, $\underline{s}(t;m)$, and $\underline{y}(t)$, respectively, and k is sample number. The colored interference process, $\underline{y}(k)$, is generated from zero-mean, white, Gaussian, unit-variance noise (a two-vector), $\underline{W}(k)$, which is independent of the channel noise, $\underline{n}(k)$. The true structure of the $\underline{y}(k)$ generator is the set $\{\Gamma ,\Phi,\Lambda\}$ which may also be unknown. The problem of joint identification of $\{\Gamma,\Phi,\Lambda\}$ has been treated in Reference 1.

The decision on m is to be made according to the maximum A Posteriori Probability (MAP) strategy. That is, a decision statistic, S^1 is to be formed recursively from the set of all data samples, z(k), taken in sequence during the symbol period. Let $\underline{Z}(k)$ denote the 2-K vector of K samples of the I-Q data during the period.

$$\underline{Z}(k) = [\underline{z}(K), \underline{z}(K-1), \ldots, \underline{z}(1)]^T$$

$$= \begin{bmatrix} \underline{z}(K) \\ \hline \underline{Z}(K-1) \end{bmatrix} \qquad (7)$$

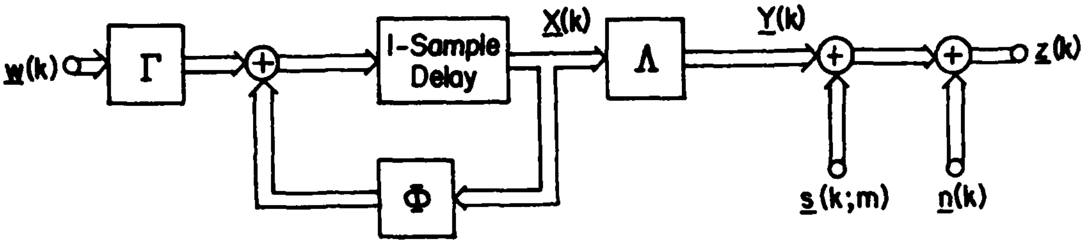


Figure 3. Data Generating Model

The MAP decision statistic is the probability

$$S^1(K,m) = p(m|\underline{Z}(K)) \qquad (8)$$

The decision rule is that the detected symbol, $\hat{m}$, is that one for which $S^1(K,\hat{m})$ is maximum.

Assuming that the A Priori probability of transmitted symbols, p(m), is known, maximization of $S^1(K,m)$ is obtained by just maximizing the Maximum Likelihood (ML) statistic, S(K,m), where

$$S^1(K,m) = \frac{p(m)}{p(\underline{Z}(K))} \cdot p(\underline{Z}(K)|m)$$

$$S(K,m) = p(\underline{Z}(K)|m) \qquad (9)$$

Now, the signal, $\underline{s}(k;m)$, is a function of an unknown phase process, $\phi_s(k)$, as per Eq. (4). Thus, define a K-vector, $\underline{\phi}(K)$, as

$$\underline{\Phi}(K) = [\phi_\delta(K), \phi_\delta(K-1), \ldots, \phi_\delta(1)]^T$$

$$= \begin{bmatrix} \phi_\delta(K) \\ \hdashline \underline{\Phi}(K-1) \end{bmatrix} \tag{10}$$

The unknown phase process, $\underline{\Phi}(K)$, is imbedded in the problem by using the composite detection approach, as

$$p(\underline{Z}(K)|m) = \int\int\cdots\int p(\underline{Z}(K), \underline{\Phi}(K)|m) d\phi_\delta(K)\cdots d\phi_\delta(1) \tag{11}$$

The ML decision statistic, S(K,m) is to be generated in recursive form. Thus, the argument of the integral in (11) is manipulated to obtain a recursive form.

We have

$$\begin{aligned} &p(\underline{Z}(K), \underline{\Phi}(K)|m) = \\ &\quad = p(\underline{z}(K), \underline{Z}(K-1), \phi_\delta(K), \underline{\Phi}(K-1)|m) \\ &\quad = p(\underline{z}(K), \phi_\delta(K)|\underline{Z}(K-1), \underline{\Phi}(K-1), m) \cdot \\ &\quad\quad p(\underline{Z}(K-1), \underline{\Phi}(K-1)|m) \\ &\quad = p(\underline{z}(K), \phi_\delta(K)|\underline{Z}(K-1), \underline{\Phi}(K-1), m) \cdot \\ &\quad\quad p(\underline{\Phi}(K-1)|\underline{Z}(K-1), m) \cdot p(\underline{Z}(K-1)|m) \end{aligned} \tag{12}$$

Then,

$$\begin{aligned} &p(\underline{Z}(K)|m) \\ &\quad = \int\int\cdots\int p(\underline{z}(K), \phi_\delta(K)|\underline{Z}(K-1), \underline{\Phi}(K-1), m) \cdot \\ &\quad\quad p(\underline{\Phi}(K-1)|\underline{Z}(K-1), m) \cdot p(\underline{Z}(K-1)|m) d\phi_\delta(K)\cdots d\phi_\delta(1) \end{aligned} \tag{13}$$

and

$$S(K,m) = S(K-1, m)Q(K) \tag{14}$$

where

$$\begin{aligned} Q(K) = \int\int\cdots\int &p(\underline{z}(K)|\phi_\delta(K), \underline{\Phi}(K-1), \underline{Z}(K-1), m) \cdot \\ &p(\phi_\delta(K)|\underline{\Phi}(K-1), \underline{Z}(K-1), m) \cdot p(\underline{\Phi}(K-1)|\underline{Z}(K-1), m) \cdot \\ &d\phi_\delta(K)\cdots d\phi_\delta(1) \end{aligned} \tag{15}$$

Now, let us define $\hat{\underline{\Phi}}(\ell)$ to be the conditional-mean estimate of $\underline{\Phi}(\ell)$, given the data $\underline{z}(k)$ for $k = 1,2,\ldots,\ell$, and given the symbol, m. Then, $\hat{\underline{\Phi}}(\ell)$ maximizes $p(\underline{\Phi}(\ell)|\underline{Z}(\ell), m)$. Now, it is assumed that the gradients of $p(\underline{z}(K)|\phi_\delta(K), \underline{\Phi}(K-1), \underline{Z}(K-1), m)$ and of $p(\phi_\delta(K)|\underline{\Phi}(K-1), \underline{Z}(K-1), m)$, with respect to $\phi_\delta(K-1),\ldots, \phi_\delta(1)$, evaluated in the neighborhood of $\hat{\underline{\Phi}}(K-1)$, are sufficiently small so that the approximation may be made

$$Q(K) \simeq \int p(\underline{z}(K)|\phi_\delta(K), \hat{\underline{\Phi}}(K-1), \underline{Z}(K-1), m) \cdot p(\phi_\delta(K)|\underline{Z}(K-1), \hat{\underline{\Phi}}(K-1), m)d\phi_\delta(K) \qquad (16)$$

This approximation says that the functions $p(\underline{z}(K)|(\ ))$ and $p(\phi\ (K)\ (\ ))$, viewed as functions of the $\phi_\delta(K-1), \ldots, \phi_\delta(1)$, are sufficiently "flat" that $p(\underline{\Phi}(K-1)|(\ ))$ appears as a multi-dimensional delta function, centered at the co-ordinates, $\hat{\phi}_\delta(K-1),\ldots, \hat{\phi}_\delta(1)$. The multiple integral then simply evaluates the argument at those coordinates, analagous to "sifting" with a delta function.

Physically, the approximation means the following. If a sufficiently accurate conditional-mean estimate may be obtained for the phase process, $\phi_\delta(1),\ldots,\phi_\delta(K-1)$, then the density, $p(\underline{\Phi}(K-1)|\underline{Z}(K-1),m)$, will have a very small variance about the mean estimate. Thus, the density $p(\underline{\Phi}(K-1)|(\ ))$ will be so highly concentrated that the densities, $p(\underline{z}(K)|(\ ))$ and $p(\phi_\delta(K)|(\ ))$ will be flat by comparison. Thus, the accuracy of the approximation depends entirely on the availability of a very good phase estimate.

Similarly, now define $\hat{\phi}_\delta(\ell)$ to be the one-stage conditional-mean prediction of $\phi_\delta(K)$, given the previous data, $\underline{Z}(\ell-1)$, the previous conditional mean estimate, $\hat{\underline{\Phi}}(\ell-1)$, and the symbol, m. As previously, assume that $\hat{\phi}_\delta(\ell)$ is sufficiently accurate so that $p(\underline{z}(\ell)|(\ ))$ is flat, by comparison, in the neighborhood of $\hat{\phi}_\delta(\ell)$. This, then, yields the final approximation

$$Q(K) \simeq p(\underline{z}(K)|\hat{\phi}_\delta(K), \hat{\underline{\Phi}}(K-1), \underline{Z}(K-1), m) \qquad (17)$$

The recursive decision statistic is then

$$S(K,m) = \overset{K}{\underset{k=1}{\pi}} Q(k)$$

$$= \overset{K}{\underset{k=1}{\pi}} p(\underline{z}(k)|\hat{\phi}_s(k), \hat{\underline{\Phi}}(k-1), \underline{Z}(k-1), m) \qquad (18)$$

It is seen from (17) and (18) that the recursive detector must form the conditional probability function, $p(\underline{z}(k)|\hat{\phi}_s(k), \hat{\underline{\Phi}}(k-1), \underline{Z}(k-1), m)$, at each sample time (number) k. Moreover, operating in parallel with the decision circuitry, and furnishing recursive phase estimates to it, is a conditional-mean phase estimator-predictor. The estimator produces the estimates

$$\hat{\phi}_s(k) = E\{\hat{\phi}_s(k)|\hat{\underline{\Phi}}(k-1), \underline{Z}(k-1), m\}$$
$$\hat{\underline{\Phi}}(k) = E\{\underline{\Phi}(k)|\underline{Z}(k), m\} \qquad (19)$$

The problem of conditional-mean estimation of the phase of a sinusoid in Gaussian noise is a non-linear estimation problem without a known general solution. However, the first-order approximate solution is known and is a phase-locked loop [2]. The closely related approximate Maximum A Posteriori Probability estimator is also a phase-locked loop [3]. Given the symbol, m, and, hence, the corresponding signal waveform, $s(t;m)$, the bandpass received data, $z(t)$, consists of a sine wave of unknown (random) phase, imbedded in additive colored plus white Gaussian noise. Thus, the available solution to the estimation problem indicated by (14) is the decision-directed phase-locked loop. Note that the PLL is only the approximate solution to (19) for the case where the phase-estimation error is quite small. Thus, the optimality of the detection algorithm of (18) will depend on the phase estimation accuracy which may be realized in practice using the PLL.

3. THE I-Q DATA MODEL WITH PHASE ESTIMATION

In order to proceed with the detection and phase estimation algorithms, the discrete-time I-Q data generation model must be extended beyond that of equation (6) and Figure 3. Under the assumption that the I-Q demodulating reference sinusoid phases are estimated, the model changes somewhat. Let the physical model be shown in Figure 4.

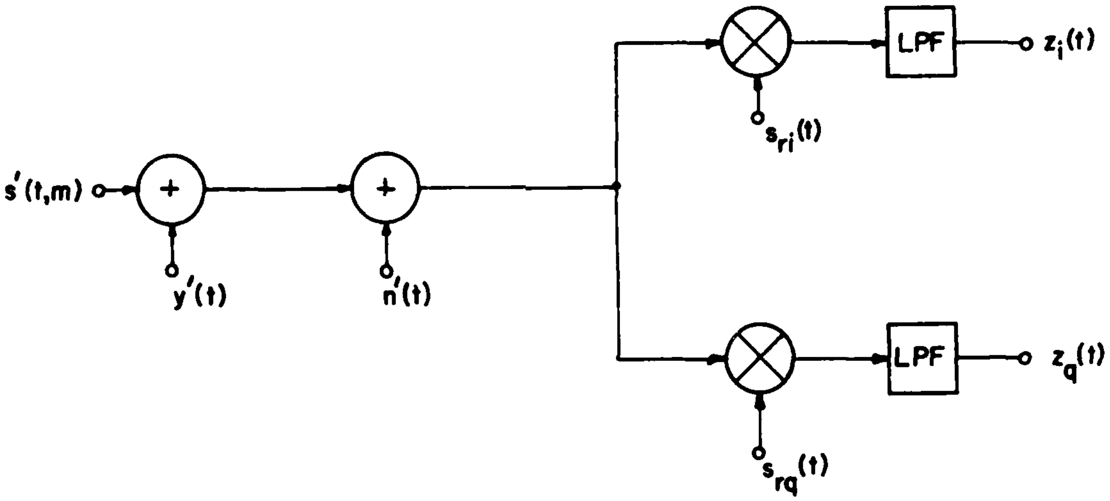


Figure 4. Data Model

In Figure 4, the transmitted signal with unknown phase is

$$s'(t,m) = A\cos[\omega_c t + \phi(t,m) + \phi_s(t)] \qquad (20)$$

where $\phi(t,m)$ is the angle modulation waveform, containing the symbol, m. The unknown, possibly time-varying, phase term is $\phi_s(t)$. The additive, zero-mean, Gaussian colored and white noises are respectively,

$$y'(t) = y_i'(t)\cos\omega_c t - y_q'(t)\sin\omega_c t$$

$$n'(t) = n_i'(t)\cos\omega_c t - n_q'(t)\sin\omega_c t \qquad (21)$$

where the i and q subscripts denote "in-phase" and "quadrature" low-pass components, respectively.

The product detector reference sinusoids are

$$s_{ri}(t) = 2\cos[\omega_c t + \hat{\phi}_s(t)]$$

$$s_{rq}(t) = -2\sin[\omega_c t + \hat{\phi}_s(t)] \qquad (22)$$

where $\hat{\phi}_{\delta}(t)$ is the phase estimate of $\phi_{\delta}(t)$, provided by the phase-locked loop. The usual problem of the phase-locked loop responding to the low frequency portion of the modulation $\phi(t,m)$ may be encountered, depending on the exact form of the modulation.

Now define,

$$\hat{\phi}_{\delta}(t) - \phi_{\delta}(t) \triangleq \varepsilon(t) \tag{23}$$

It may be shown that the low-pass I-Q data vector has the form

$$\begin{bmatrix} z_i(t) \\ z_q(t) \end{bmatrix} = \begin{bmatrix} \cos\varepsilon(t) & \sin\varepsilon(t) \\ -\sin\varepsilon(t) & \cos\varepsilon(t) \end{bmatrix} \begin{bmatrix} A\cos\phi(t,m) \\ A\sin\phi(t,m) \end{bmatrix} + \begin{bmatrix} y_i(t) \\ y_q(t) \end{bmatrix} + \begin{bmatrix} n_i(t) \\ n_q(t) \end{bmatrix} \tag{24}$$

where

$$\begin{bmatrix} y_i(t) \\ y_q(t) \end{bmatrix} = \begin{bmatrix} \cos\hat{\phi}_{\delta}(t) & \sin\hat{\phi}_{\delta}(t) \\ -\sin\hat{\phi}_{\delta}(t) & \cos\hat{\phi}_{\delta}(t) \end{bmatrix} \begin{bmatrix} y_i'(t) \\ y_q'(t) \end{bmatrix}$$

$$\begin{bmatrix} n_i(t) \\ n_q(t) \end{bmatrix} = \begin{bmatrix} \cos\phi_{\delta}(t) & \sin\phi_{\delta}(t) \\ -\sin\phi_{\delta}(t) & \cos\phi_{\delta}(t) \end{bmatrix} \begin{bmatrix} n_i'(t) \\ n_q'(t) \end{bmatrix} \tag{25}$$

With $\underline{n}'(t)$ white, Gaussian, zero-mean with variance, σ_r^2, then $\underline{n}(t)$ is also white, zero-mean, with variance σ_r^2. This is because the multiplying matrix is a rotation matrix. However, $\underline{n}(t)$ is not Gaussian, in general. For time periods which are short compared to the reciprocal bandwidth of $\hat{\phi}_{\delta}(t)$, $\underline{n}(t)$ appears approximately Gaussian. With $\underline{y}'(t)$ colored, Gaussian, zero-mean, with variance σ_y^2, $\underline{y}(t)$ is zero-mean with variance σ_y^2. $\underline{y}(t)$ is not Gaussian and may be of slightly greater bandwidth than $\underline{y}'(t)$, if the variation of $\hat{\phi}_{\delta}(t)$ is not small.

The new data model of (24) may be written in three equivalent forms, and in discrete time, as

$$z(k) = H[\hat{\phi}_{\delta}(k)]\{H[\phi_{\delta}(k)]\underline{\delta}(k;m) + \underline{y}'(k) + \underline{n}'(k)\} \qquad (26a)$$

$$\underline{z}(k) = H[\varepsilon(k)]\underline{\delta}(k;m) + \underline{y}(k) + \underline{n}(k) \qquad (26b)$$

$$\underline{z}(k) = H[k;m]\underline{\rho}(k) + \underline{y}(k) + \underline{n}(k) \qquad (26c)$$

where in (26)

$$H[\varepsilon(k)] = \begin{bmatrix} \cos\varepsilon(k) & \sin\varepsilon(k) \\ -\sin\varepsilon(k) & \cos\varepsilon(k) \end{bmatrix} \; ; \quad \underline{\delta}(k;m) = \begin{bmatrix} A\cos\phi(k;m) \\ A\sin\phi(k;m) \end{bmatrix}$$

$$H[k;m] = \begin{bmatrix} \cos\phi(k;m) & -\sin\phi(k;m) \\ \sin\phi(k;m) & \cos\phi(k;m) \end{bmatrix} ; \quad \underline{\rho}(k) = \begin{bmatrix} A\cos\varepsilon(k) \\ -A\sin\varepsilon(k) \end{bmatrix}$$

$$H[\phi_{\delta}(k)] = \begin{bmatrix} \cos\phi_{\delta}(k) & -\sin\phi_{\delta}(k) \\ \sin\phi_{\delta}(k) & \cos\phi_{\delta}(k) \end{bmatrix} \; ; \qquad (27)$$

$$H[\hat{\phi}_{\delta}(k)] = \begin{bmatrix} \cos\hat{\phi}_{\delta}(k) & \sin\hat{\phi}_{\delta}(k) \\ -\sin\hat{\phi}_{\delta}(k) & \cos\hat{\phi}_{\delta}(k) \end{bmatrix}$$

In (26b), the matrix H(k;m) is a function only of the signal. The vector, $\underline{\rho}(k)$, is a function only of the phase-tracking error process, $\varepsilon(k)$. Detection of m in the presence of $\underline{\rho}(k)$ is a multiplicative noise detection problem. The presence of the additive colored and white noise processes, $\underline{y}(k)$ and $\underline{n}(k)$, respectively, gives a compound detection problem, having multiplicative and additive colored noise.

The compound detection problem for multiplicative and additive colored Gaussian noise was solved in [4]. There it was found that the detector was one which tracked both the multiplicative and additive colored noises and attempted to remove them from the data, $\underline{z}(k)$. Although, in the present case, the various multiplicative and additive noises are not strictly Gaussian, the tracking detector may still be used. Note that when $\varepsilon(k)$ is small then $\underline{\rho}(k)$ is approximately

$$\underline{\rho}(k) \simeq A\begin{bmatrix} 1 \\ -\varepsilon(k) \end{bmatrix} \; ; \qquad |\varepsilon(k)| << 1 \qquad (28)$$

In this case, $\varepsilon(k)$, the phase tracking error, is Gaussian and $\underline{\rho}(k)$ is approximately Gaussian.

The final data generator diagram, corresponding to equations (26) is shown in Figure 5.

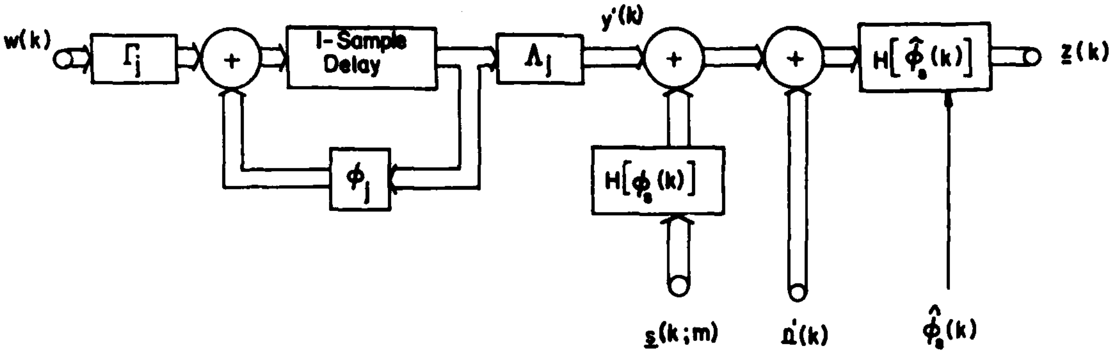


Figure 5. Data Generator Model for Phase Estimation

4. DETECTOR STRUCTURE AND ALGORITHMS

With the data generator given as in Figure 5, the tracking detector, with phase estimator, takes the form of Figure 6. In the detector, there are two decision-directed tracking filters, one implemented for the signal waveform corresponding to m=0, and the other for m=1. Each tracking filter is matched, in the Wiener sense, to both $\underline{\rho}(k)$, the multiplicative noise, and $\underline{y}(k)$, the additive noise. Thus, the detectors are implemented for the data, $\underline{z}(k)$, in the form of equation (26c). The tracking error waveforms, $\underline{\xi}(k;m)$, drive the decision circuitry which produces the decision on the received symbol as $\hat{m}$.

It was shown above that generally the phase estimator is decision-directed. However, a non-decision-directed phase estimator may be implemented if the transmitted signal possesses a residual unmodulated carrier component. This is shown as follows for a phase-shift-keyed signal.

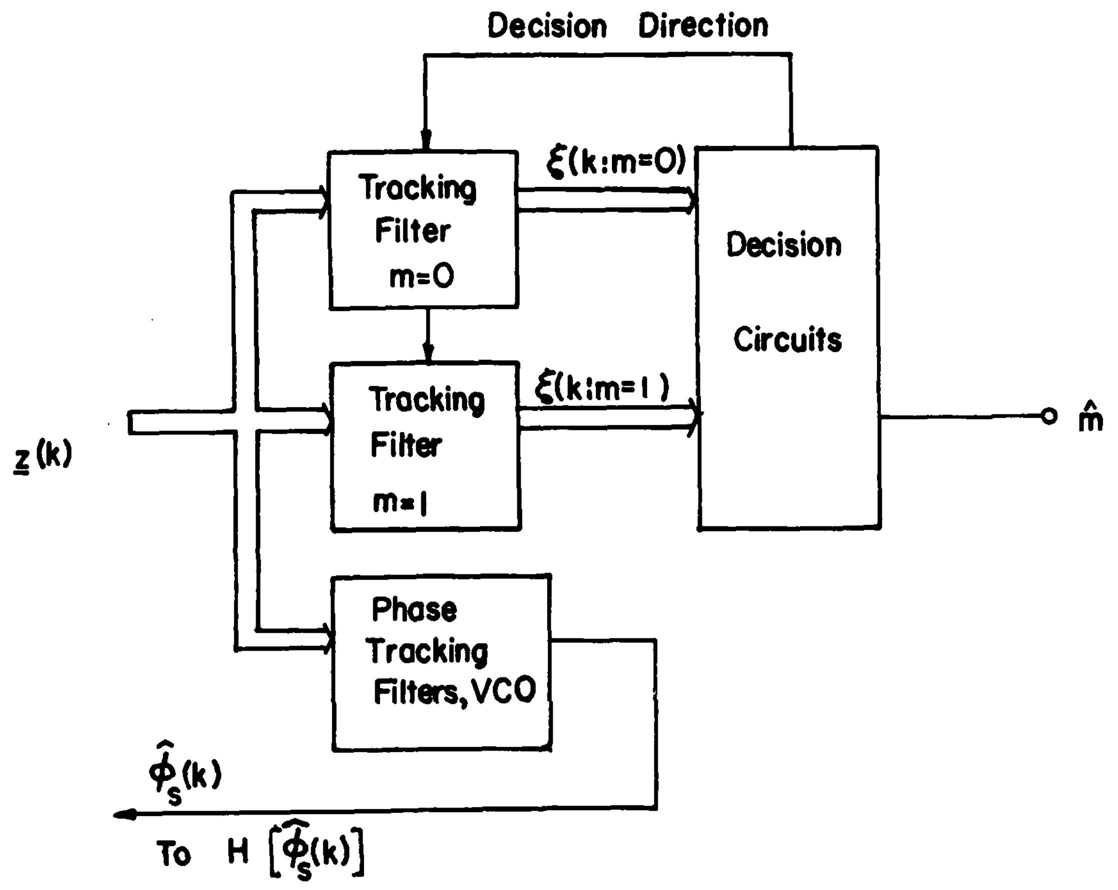


Figure 6. Compound Detector and Phase Estimation

Suppose that the signal phase term is

$$\phi(k;m) = \Delta\phi\cdot c(k;m) \;;\; c(k;m) = 1; \quad m=0$$
$$= -1; \quad m=1$$
$$0 < \Delta\phi < \pi/2 \qquad (29)$$

Then

$$\cos\phi(k;m) = \cos(\Delta\phi)$$
$$\sin\phi(k;m) = c(k;m)\cdot\sin(\Delta\phi) \qquad (30)$$

It follows that

$$H[k;m]\underline{\rho}(k) = A\cos(\Delta\phi)\begin{bmatrix}\cos\varepsilon(k)\\ -\sin\varepsilon(k)\end{bmatrix} + c(k;m)\cdot A\sin(\Delta\phi)\begin{bmatrix}\sin\varepsilon(k)\\ \cos\varepsilon(k)\end{bmatrix} \qquad (31)$$

From (31) it is seen that there is present in the received data an additive term proportional to $-\sin\varepsilon(k)$, which may be used to drive the phase estimator. Likewise, there is an additive term proportional to $\cos\varepsilon(k)$ which may be used to estimate A (coherent automatic gain control). The PSK waveform, $c(k;m)$, is present in both I-Q channels, due to the multiplicative process with components $\sin\varepsilon(k)$ and $\cos\varepsilon(k)$. Provided that the bandwidth of $c(k;m)$ is sufficiently wide and the closed-loop tracking bandwidth of the phase estimator is sufficiently small, the estimator can track phase in the presence of $c(k;m)$ without decision-direction.

Each decision-directed tracking filter in Figure 6 is of the form of Figure 7. In the figure, the inner loop, composed of elements G_ρ, Φ_ρ, Λ_ρ, and $H[k;m]$, track the multiplicative process, $\underline{\rho}(k)$. The elements $\{G_\rho, \Phi_\rho, \Lambda_\rho\}$ are the elements of a Wiener filter in Kalman canonical form, matched to $\underline{\rho}(k)$. $H[k,m]$ contains the signal waveform elements, as in (27). The outer loop tracks the additive colored interference, $\underline{y}(k)$. The elements, $\{G_j, \Phi_j, \Lambda_j\}$, are those of a Wiener filter matched to $y(k)$. The filter algorithms are

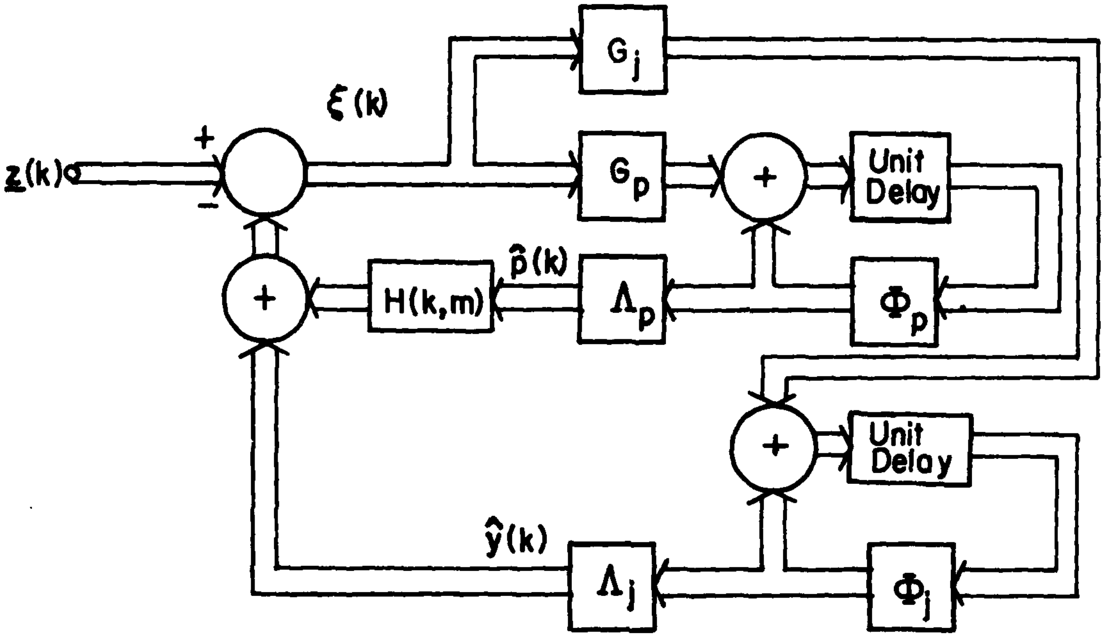


Figure 7. Tracking Filter

$$\underline{\rho}(k) = \Lambda_\rho \hat{\underline{x}}_\rho(k|k-1)$$

$$\hat{\underline{x}}_\rho(k|k-1) = \Phi_\rho[\hat{\underline{x}}_\rho(k-1|k-2) + G_\rho \underline{\xi}(k-1)]$$

$$\hat{\underline{y}}(k) = \Lambda_j \hat{\underline{x}}_j(k|k-1)$$

$$\hat{\underline{x}}_j(k|k-1) = \Phi_j[\hat{\underline{x}}_j(k-1|k-2) + G_j \underline{\xi}(k-1)]$$

$$\underline{\xi}(k) = \underline{z}(k) - [H(k;m)\hat{\underline{\rho}}(k) + \hat{\underline{y}}(k)] \qquad (32)$$

It is seen from Figure 7 and (32) that the $\hat{\underline{y}}(k)$ filter and $\hat{\underline{\rho}}(k)$ filter are uncoupled, except for that coupling inherent in the pseudo-innovations, $\underline{\xi}(k)$. Filter design consists of selecting the two sets of parameters $\{G_j, \Phi_j, \Lambda_j\}$ and $\{G_\rho, \Phi_\rho, \Lambda_\rho\}$. The selection is based on either real-time identification of $\underline{y}(k)$ and $\underline{\rho}(k)$, as per [1], or on an ad hoc worst case design. The ad hoc design, while not optimum, would, under conditions discussed in [1], produce acceptable results.

5. THE PHASE ESTIMATOR

From equations (26c) and (31) we may write an expression for the (continuous-time) data vector, as seen by the phase estimator, as

$$\underline{z}(t) = A'\begin{bmatrix}\cos\varepsilon(t)\\ -\sin\varepsilon(t)\end{bmatrix} + \underline{\eta}(t) \qquad (33)$$

In (33), $\underline{\eta}(t)$ is the total noise process due to $\underline{y}(t)$, $\underline{n}(t)$, and c(t:m). For the bandwidth of $\underline{y}(t)$ and the band-rate of c(t;m) sufficiently great with respect to the closed loop bandwidth of the phase estimator, the noise process, $\underline{\eta}(t)$, will appear white to the phase estimator.

It is seen that the problem of deriving the phase reference, $\hat{\phi}_\Delta(t)$, which is an accurate estimate of the residual carrier phase, $\phi_\Delta(t)$, is that of minimizing $\varepsilon(t)$ in the presence of the unknown amplitude, A', and noise, $\underline{\eta}(t)$. This is, essentially, a phase-locked loop problem. Under the assumption that $\underline{\eta}(t)$ is white and Gaussian, the solution is the classical phase-locked loop.

Note that the usual problem of unknown signal amplitude, A', is present. There are two classical solutions. One is to use the Q-channel only, for phase estimation, with an ideal pre-limiter to remove dependence on A'. The other solution is to also use the I-channel to estimate A' and to then control the gain of the Q-channel. An extension of the second method is shown in Figure 8.

In Figure 8, the Q-channel waveform, $z_q(t)$ is processed by a "Loop Filter" with low frequency gain, H(0), to produce an estimate of the term, (-A'sinε(t)), weighted by H(0). The I-channel waveform, $z_i(t)$, is processed by a low-pass filter with unit low frequency gain to procuce an estimate of the term, A'cosε(t). The two filter output terms are then divided point-wise in a digital divider to provide an estimate of (-tanε(t)), weighted by H(0). The latter estimate then drives the Voltage-Controlled-Oscillator (VCO) to produce the reference phase, $\hat{\phi}_\Delta(t)$. It can be seen from the defining equation (23) for $\varepsilon(t)$ that the mechanization of Figure 8 causes $\hat{\phi}_\Delta(t)$ to track $\phi_\Delta(t)$.

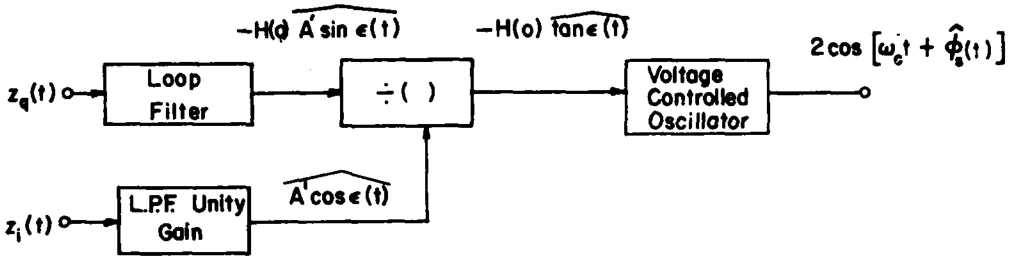

$-H(o)\,\widehat{A'\sin\epsilon(t)}$
$-H(o)\,\widehat{\tan\epsilon(t)}$
$2\cos\left[\omega_c t + \hat{\phi}_s(t)\right]$
$z_q(t)$
Loop Filter
$\div(\ )$
Voltage Controlled Oscillator
$z_i(t)$
L.P.F. Unity Gain
$\widehat{A'\cos\epsilon(t)}$

Figure 8. Tangent Phase-Locked Loop

The usual phase-locked loop generates a tracking error voltage proportional to $(-\sin\varepsilon(t))$. The present implementation provides a tracking error proportional to $(-\tan\varepsilon(t))$, which will yield higher loop gain for a large tracking error, $\varepsilon(t)$. However, the main reason for using the "Tangent-Loop" mechanization is to obtain the automatic gain control feature in the cancellation of the unknown amplitude, A'.

The design of the loop parameters, notably the loop filter, is performed by assuming linear operation of the loop. That is, when $\varepsilon(t)$ is small, say less than 12° in magnitude, then the approximation holds

$$\tan\varepsilon(t) \simeq \sin\varepsilon(t) \simeq \varepsilon(t) \tag{34}$$

Then, the overall system operates as a linear servo-mechanism for phase, or as a linear phase-locked loop.

In the usual implementation, the Loop Filter in the quadrature channel is implemented with one finite zero of transmission and one finite, nonzero, pole. The pole frequency, zero frequency, and low-frequency gain, H(0), are set to realize the desired closed-loop noise bandwidth, static phase error for VCO frequency offset, and second order dynamic response. The low pass filter in the I-channel is set for the same zero and pole frequencies as for the Q-channel Loop Filter, but with unit low-frequency gain.

Note that for the PLL to operate properly, the signal to noise ratio must be large in the closed-loop equivalent noise bandwidth of the loop, itself The PLL bandwidth is to be maintained small enough to just accomodate the dynamics of the received signal phase, $\phi_\delta(t)$, due to Doppler effects on the transmission link. For the case where the incident noise is dominated by colored interference, such as jamming, the loop perfc ιιance will be affected by that portion of the colored interference falling within the (narrow) loop bandwidth.

6. THE LOOP FILTER MECHANIZATION

The continuous-time version of the Loop Filter is characterized by the transfer function

$$H(s) = K[\frac{s-z}{s-p}] \tag{35}$$

where K, z, and p are real, with z and p being negative. Let $\mu(t)$ and z(t) denote the filter input and output, respectively. A state variable representation is set up, using the single filter state, x(t), as

$$\dot{x}(t) = px(t) + \mu(t)$$
$$z(t) = K(p-z)x(t) + K\mu(t) \tag{36}$$

The filter is converted to discrete time by driving it with an ideal sampler and zero-order hold circuit and observing the output only at sampling instants, $t = t_k$ for k = 1,2,3,.... The differential equation of (36) is then solved between the kth and (k+1)st sampling times as

$$x((k+1)T) = \exp[p((k+1)T-kT)] \cdot x(kT)$$
$$+ \int_{kT}^{(k+1)T} \exp[p(k+1)T - \tau]W(\tau)d\tau \tag{37}$$

where

$$W(t) = \mu(kT); \quad kT \leq t < (k+1)T \tag{38}$$

and T is the sampling interval. The differential equation solution then yeilds the governing difference equation (discrete-time) for the filter as

$$x(k+1) = \phi x(k) + \gamma \mu(k)$$

$$z(k) = K(p-z)x(k) + K\mu(k)$$

$$\phi = \exp(pT) \quad : \quad \gamma = 1/p(\phi-1) \qquad (39)$$

The Loop Filter constants, K, z, p, are set according to specifications on the linearized closed-loop transfer function for phase. The VCO output phase, $\hat{\phi}_{\delta}(t)$ is given by

$$\hat{\phi}_{\delta}(t) = \int\{-[\phi_{\delta}(t) - \hat{\phi}_{\delta}(t)]*h(t)\}dt \qquad (40)$$

or

$$\hat{\Phi}_{\delta}(s) = -\frac{[\Phi_{\delta}(s) - \hat{\Phi}_{\delta}(s)]\cdot H(s)}{s} \qquad (41)$$

where H(s) is the Loop Filter transfer function given in (35).

The closed-loop transfer function for the PLL is then

$$G(s) = \frac{\hat{\Phi}_{\delta}(s)}{\Phi_{\delta}(s)} = \frac{H(s)}{s + H(s)} \qquad (42)$$

Substituting for H(s) yields

$$G(s) = \frac{K(s-z)}{s^2 + (K-p)s - Kz} = \frac{K(s-z)}{s^2 + 2\delta\omega_n s + \omega_n^2} \qquad (43)$$

where δ and ω_n are the classical damping ratio and resonant frequency for a second-order servo system.

The Loop Filter low frequency gain, H(0), is given by

$$H(0) = \lim_{s\to 0} K\left(\frac{s-z}{s-p}\right) = K\,\frac{z}{p} \qquad (44)$$

For most PLL designs the following assumptions hold

$$-z \ll H(0)$$

$$-p \ll K \qquad (45)$$

Thus, by equating like terms in the denominator of (43)

$$K \simeq 2\delta\omega_n$$

$$-Kz = \omega_n^2 \qquad (46)$$

Now, it may be shown that the one-sided closed-loop noise bandwidth, in Hz, for G(s) is [5]

$$B_n = \frac{K}{4}\left[\frac{K-z}{K-p}\right] \simeq \frac{K-z}{4} = \frac{\omega_n}{8\delta}\left[1 + 4\delta^2\right] \qquad (47)$$

Thus,

$$K = \left[\frac{16\ \delta^2}{1 + 4\ \delta^2}\right] \cdot B_n$$

$$z = -\left[\frac{4}{1 + 4\ \delta^2}\right] \cdot B_n \qquad (48)$$

For loop dynamic stability, the damping ratio is set as

$$\delta = 1/\sqrt{2} \qquad (49)$$

Then

$$K = 8/3\ B_n$$

$$z = -\ 4/3\ B_n = -K/2 \qquad (50)$$

The Loop Filter pole frequency, p, is generally set as small as possible in magnitude. This is because p affects the "static phase error" when tracking with a fixed Doppler offset in the received frequency. In order to hold the loop in lock when the input phase $\phi_\delta(t)$ has a constant first derivative requires a constant driving voltage into the VCO and hence a constant phase error, $\varepsilon(t)$. Thus,

$$\frac{d}{dt}\hat{\phi}\ (t) \triangleq \Delta\omega = -\ H(0)\ \tan\varepsilon_{sp} \qquad (51)$$

where ε_{sp} is the static phase error for a Doppler offset, $\Delta\omega = 2\pi\Delta f$. The d.c. gain of the loop filter is

$$H(0) = K \frac{z}{p} = \frac{32}{9} \frac{B_n^2}{|p|} \tag{52}$$

For desired small values of static phase error

$$2\pi\Delta f = \frac{32}{9} \frac{B_n^2}{|2\ f_p|} \cdot \varepsilon_{sp} \tag{53}$$

where f_p is the Hertz value of -p. Thus,

$$f_p = \frac{32}{9(2\pi)^2} \frac{B_n^2\ \varepsilon_{sp}}{\Delta f} \tag{54}$$

Equation (54) gives the relation between the various quantities and f_p.

Thus, the design equations for the quadrature channel loop filter are

$$K_q = 8/3\ B_n$$

$$z = -4/3\ B_n \qquad ;\ \text{quadrature filter} \tag{55}$$

$$p = - \frac{}{18} \frac{B_n^2\ \varepsilon_{sp}}{\Delta f}$$

where ε_{sp} is static phase error in radians for a Doppler offset of Δf Hertz and a closed loop noise bandwidth of B_n Hz.

For the inphase filter, the same pole, p, and zero, z, are used, but the d.c. gain is reduced to unity to give a filter gain constant

$$K_i = p/z \qquad :\ \text{inphase filter} \tag{56}$$

The block diagram of the phase estimator is given in Figure 9. In the figure, the discrete time version of the VCO (phase integrator) is represented by

$$\hat{\phi}_\delta(k+1) = \hat{\phi}_\delta(k) + T/2[v(k+1) + v(k)] \tag{57}$$

where v(k) is the VCO input.

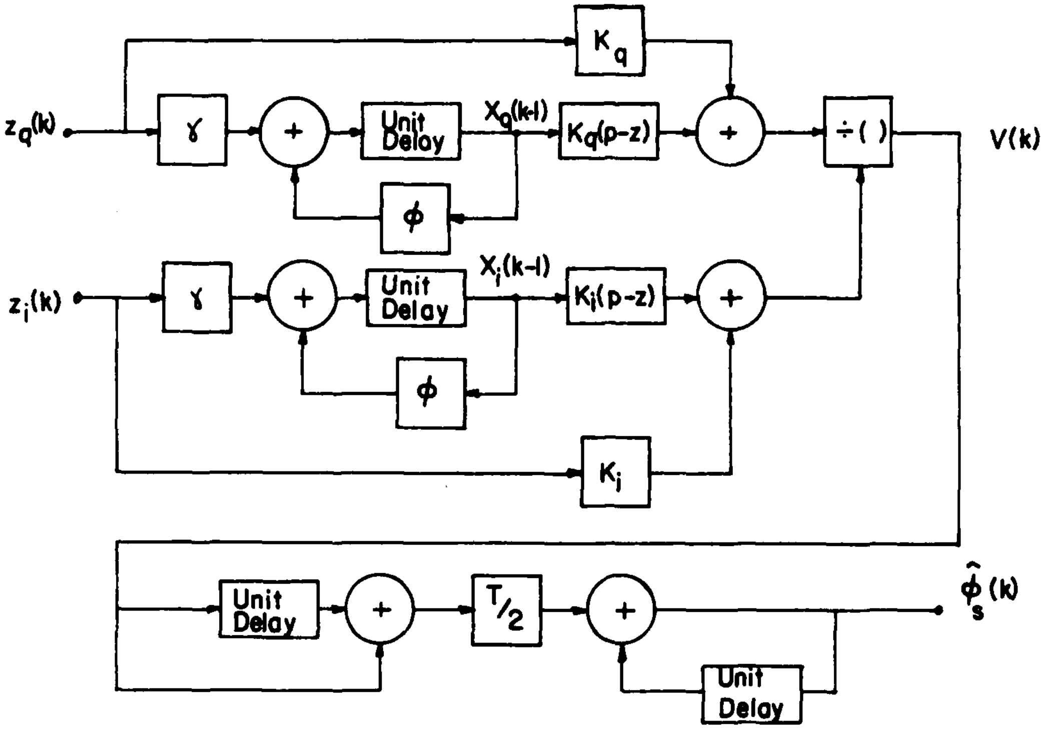


Figure 9. Discrete-Time Phase Estimator

SECTION III

ON THE EXISTANCE OF NON-COHERENT TRACKING DETECTORS

It is desired now to determine if a non-coherent version of the tracking detector exists. In [1] the non-coherent version of the standard FSK detector for white noise was derived. The approach for the tracking detector will be similar. An unknown constant phase term will be introduced into the formulation of the detection problem. Then, the detection statistic will be averaged with respect to the unknown phase. Up to this point, the procedure is the same as was followed in II.2. That is, the problem is that of composite detection for unknown phase. In II.2 there existed a solution of the composite detection problem which produced a phase estimator as part of the detector. In the present formulation, the phase estimator solution is purposely rejected and no attempt is made to take advantage of possible phase information. Rather the unknown phase is defined to be uniformly distributed over the interval, $[0, 2\pi]$, and to be a constant random variable over the time interval of the signal symbol. Then it is to be determined whether averaging the decision statistic over phase produces a sufficient statistic for detection.

The unknown phase enters the problem as per Figure 2, where now $\phi_0(t)$ is defined to be constant over the symbol interval, which is also the processing time. Also $\phi_0(t)$ is uniformly distributed as

$$\begin{array}{lll} \phi_0(t) = \phi & : & 0 \leq t \leq T \\ & & \\ p(\phi) = 1/2\pi & : & 0 \leq \phi \leq 2\pi \\ \quad = 0 & & \text{otherwise} \end{array} \tag{58}$$

The discrete time data model, $\underline{z}(k)$, is essentially that of (26a) where $\hat{\phi}_s(k) = \phi_0$ and $\phi_s(k) = 0$. Thus,

$$\underline{z}(k) = H(\phi)[\underline{s}(k) + \underline{y}(k) + \underline{n}(k)] \tag{59}$$

where $\underline{s}(k)$ is the transmitted signal, $\underline{y}(k)$ is the colored interference, and $\underline{n}(k)$ is the white noise.

The detection statistic, $S(K)$, is formed recursively from the $\underline{z}(k)$, and is the Maximum A Posteriori Probability function, $p(m|z(K))$, where $\underline{Z}(K)$ is the 2K partitioned vector,

$$\underline{Z}(K) = [\underline{z}(K), \underline{z}(K-1), \ldots, \underline{z}(1)]^T \qquad (60)$$

The quantity, m, is the signal digit, which for the binary case is either 0 or 1. Under the assumption that the transmitted digits, m, are equally distributed (p(m) = 1/2; m=0,1), the MAP statistic is equivalent to the Maximum-Likelihood (ML) statistic, $p(\underline{Z}(K)|m)$. Thus, S(K) is obtained by averaging the joint density on $\underline{Z}(K)$ and ϕ, given m.

$$S(K) = p(\underline{Z}(K)|m) = \int_0^{2\pi} p(\underline{Z}(K), \phi|m)d\phi$$

$$= \int_0^{2\pi} \frac{1}{2\pi} p(\underline{Z}(K)|m, \phi)d\phi \qquad (61)$$

The conditional density, $p(\underline{Z}(K)|m, \phi)$ is

$$p(\underline{Z}(K)|m,\phi) = \prod_{k=1}^{K} p(\underline{z}(k)|\underline{Z}(k-1), m, \phi) \qquad (62)$$

Now, $p(\underline{z}(k)|\underline{Z}(k-1), m, \phi)$ is Gaussian, under the definition that $\underline{y}(k)$ and $\underline{n}(k)$ are Gaussian, and is given by

$$p(\underline{z}(k)|\underline{Z}(k-1), m, \phi) =$$

$$= \frac{1}{2\pi\sigma_\nu^2} \exp[- \frac{1}{2\sigma_\nu^2} (\underline{z}(k) - \hat{\underline{z}}(k|k-1, m, \phi))^T(\underline{z}(k) - \hat{\underline{z}}(k|k-1, m, \phi))] \qquad (63)$$

In (63), σ_ν^2 is the steady-state Innovations variance and $\hat{\underline{z}}(k|k-1, m, \phi)$ is the recursive estimate of the kth data sample, given all the data up through the (k-1)st sample. This one-sample predictive estimate is obtained from the Kalman-form filter of Figure 10. In the figure, the quantities, $\{\Phi, \Lambda, G\}$, are the appropriate Kalman (Wiener) filter parameters for tracking $\underline{y}(k)$, the colored interference, in the presence of $\underline{n}(k)$, the white noise.

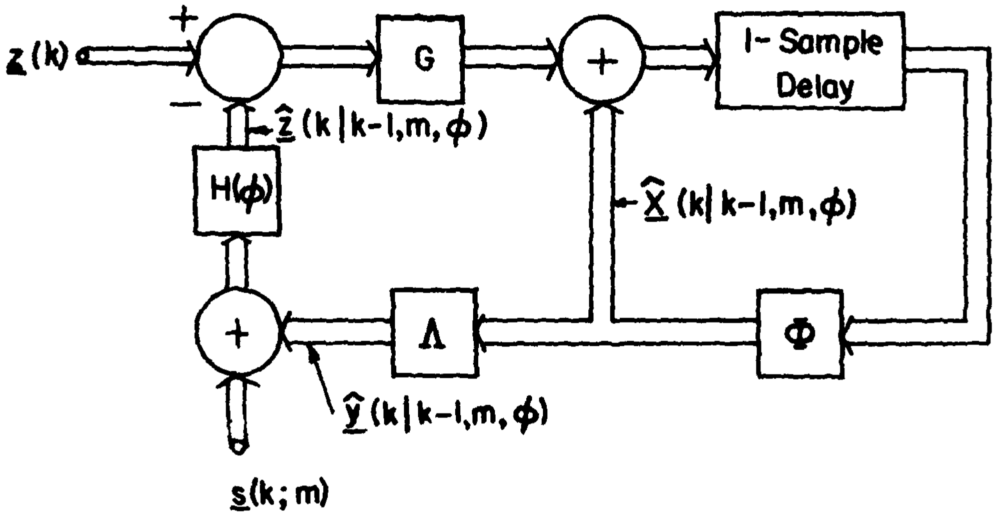

$\underline{z}(k)$
+
−
G
+
1- Sample Delay
$\hat{\underline{z}}(k|k-1,m,\phi)$
$H(\phi)$
$\hat{\underline{X}}(k|k-1,m,\phi)$
+
Λ
Φ
$\hat{\underline{y}}(k|k-1,m,\phi)$
$\underline{s}(k;m)$

Figure 10. Kalman Filter

The filtering algorithms are

$$\hat{\underline{z}}(k|k-1) = H(\phi)[\underline{s}(k) + \hat{\underline{y}}(k|k-1) = H(\phi)\underline{s}(k) + H(\phi)\Lambda\Phi\hat{\underline{x}}(k-1)$$

$$\hat{\underline{x}}(k) = \Psi(\phi)\hat{\underline{x}}(k-1) + \underline{\mu}(k,\phi);\ \Psi(\phi) = [I-GH(\phi)\Lambda]\Phi$$

$$\underline{\mu}(k,\phi) = G[\underline{z}(k) - H(\phi)\underline{s}(k)] \tag{64}$$

The solution to (64) at the kth sample is given by

$$\hat{\underline{z}}(k|k-1,\ m,\ \phi) = H(\phi)[\underline{s}(k;m) + \Lambda\Phi[\Psi^{k-1}(\phi)\hat{\underline{x}}(0) + \sum_{i=1}^{k-1} \Psi^{i-1}(\phi)\underline{\mu}(k-i),\phi)]] \tag{65}$$

It is seen at this point that any hope of averaging $p(\underline{Z}(K)|m,\phi)$ over ϕ is futile due to the internal dependency of $\hat{\underline{z}}(k|k-1,\ m,\ \phi)$ on ϕ. That is, it is the feedback dependency of the estimate $\hat{\underline{y}}(k|k-1,\ m,\ \phi)$ upon ϕ which defeats the prospect of averaging over ϕ.

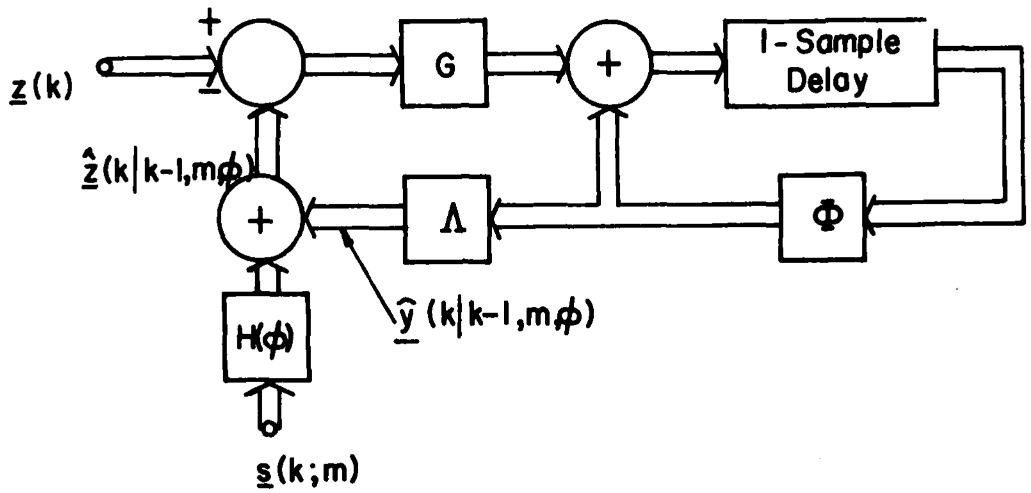

+
$\underline{z}(k)$
−
G
+
1-Sample Delay
$\hat{\underline{z}}(k|k-1,m\phi)$
+
Λ
Φ
H(φ)
$\hat{\underline{y}}(k|k-1,m\phi)$
$\underline{s}(k;m)$

Figure 11. Kalman Filter

There is a second possibility for a noncoherent implementation. The term, $H(\phi)\underline{y}(k)$, in the data model of (59) is not strictly Gaussian, but does have the same first and second moments as $\underline{y}(k)$, since $H(\phi)$ is unitary. Also, since ϕ is constant over a symbol period, $H(\phi)\underline{y}(k)$ has the same short-term spectral properties as $\underline{y}(k)$. Thus, the data form may be redefined as

$$\underline{z}(k) = H(\phi)\underline{s}(k;m) + \underline{y}(k) + \underline{n}(k) \quad ; \quad 1 \leq k \leq K \qquad (66)$$

where $\underline{y}(k)$ and $\underline{n}(k)$ have replaced $H(\phi)\underline{y}(k)$ and $H(\phi)\underline{n}(k)$, respectively. In (66), $\underline{y}(k)$ and $\underline{n}(k)$ are taken as Gaussian.

The resulting Kalman estimator for $\hat{\underline{z}}(k|k-1, m, \phi)$, corresponding to the data model of (66) is as in Figure 11.

The filtering algorithms now are

$$\hat{\underline{z}}(k|k-1, m, \phi) = H(\phi)\underline{s}(k;m) + \hat{\underline{y}}(k|k-1, m, \phi)$$

$$= H(\phi)\underline{s}(k;m) + \Lambda\Phi\hat{\underline{x}}(k-1)$$

$$\hat{\underline{x}}(k) = \Psi\hat{\underline{x}}(k-1) + \underline{\mu}(k,\phi) \ ; \ \Psi = (I-G\Lambda)\Phi$$

$$\underline{\mu}(k;\phi) = G[\underline{z}(k) - H(\phi)\underline{s}(k;m)] \qquad (67)$$

The solution to (67) is

$$\hat{\underline{z}}(k|k-1,\ m,\ \phi) = H(\phi)\underline{\delta}(k;m) + \Lambda\Phi[\Psi^{k-1}\hat{\underline{x}}(0) + \sum_{i=1}^{k-1} \Psi^{i-1}\underline{\mu}(k-i\ ,\phi)] \tag{68}$$

Now, (68) is somewhat of an improvement over (65) in that Ψ is no longer a function of ϕ. Unfortunately, $\underline{\mu}(\ )$ is still dependent on ϕ and this causes the dependency of $\hat{\underline{z}}(k|k-1,\ m,\ \phi)$ on ϕ to be internal because of the feedback structure of the filter. Thus, averaging $p(\underline{Z}(K)|m,\phi)$ over ϕ is still not feasible.

The *argument* of the exponent of $p(\underline{z}(k)|\underline{Z}(K-1),\ m,\ \phi)$ in (63) is

$$\begin{aligned} \text{Arg} = {} & (\underline{z}(k) - \hat{\underline{y}}(k|k-1,\ m,\ \phi))^T(\cdot) + \underline{\delta}^T(k;m)\underline{\delta}(k;m) \\ & - 2\underline{\delta}^T(k;m)H^T(\phi)[\underline{z}(k) - \hat{\underline{y}}(k|k-1,\ m,\ \phi)] \end{aligned} \tag{69}$$

This argument is of the same form as is encountered in the standard non-coherent FSK detector problem [1], except that $(\underline{z}(k) - \hat{\underline{y}}(k|k-1,\ m,\ \phi))$ has replaced $\underline{z}(k)$. Were it not for the fact that $\hat{\underline{y}}(k|k-1,\ m,\ \phi)$ is an explicit function of ϕ, as in (67), then the averaging over ϕ would be exactly the same as in the FSK problem. Unfortunately, there seems to be no further recourse to the problem at this point.

SECTION IV

SIMULATION RESULTS

A Monte-Carlo simulation program was written to obtain error-rate results for coherent detection with phase estimation. The detecto algorithm which was implemented was that detailed in Section II.4. The program realized the compound detector and phase estimator of Figure 6 where the tracking filters were of the form given in Figure 7. The phase estimator was the Tangent Phase-locked loop shown in Figure 9.

In order to reduce simulation run times, the Monte Carlo program, documented in [4], was not modified for present use. Rather, an entirely new program was written. In the new program, the data generator, shown in Figure 5, was reduced from three states, as in [4], to one state. This resulted in the Kalman filters also having one state in each branch shown in Figure 7. Since computation load increases exponentially with state size, a considerable savings was made. All that was lost was some flexibility in modeling the additive colored noise process. For the purposes of the present work, the one-state model was sufficient.

It was desired to test the compound detector and phase estimator in a realistic but stressful environment. Thus, a phase-locked loop noise bandwidth of 2.5 Hz was chosen as being as small as could likely be realized in a reasonable implementation. It was desired to run the phase-locked loop at 0.3 radians r.m.s., phase error, or less. Thus, it was necessary to relate the various simulation parameters, such as E/N_o, colored noise bandwidth, etc., to the phase-locked loop signal to noise ratio.

Letting J denote the power of the colored process, $\underline{y}(k)$, (in bandpass form) and B_J the one-sided equivalent noise bandwidth of the low-pass I-Q process, an eqivalent white bandpass spectral density, N_J, for the colored process is defined by

$$J = N_J \cdot 2\,B_J \tag{70}$$

Then, the total equivalnet white noise spectral density is

$$N_T = N_o + N_J \tag{71}$$

where N_0 is the density of the incident additive white receiver noise.

The symbol energy, E, in the received signal is related to total signal power, S, and symbol period, T, by

$$E = S \cdot T \cdot L_M(\Delta\phi) \tag{72}$$

where $L_M(\Delta\phi)$ is the "modulation loss" factor given by

$$L_M(\Delta\phi) = \sin^2(\Delta\phi) \tag{73}$$

where $\Delta\phi$ is phase deviation in radians for the phase-shift keyed signal. Thus,

$$S = \frac{E}{L_M(\Delta\phi) \cdot T} \tag{74}$$

From (70) and (74) results

$$\frac{S}{J} = \frac{E}{L_M(\Delta\phi) \cdot T \cdot N_J \cdot 2B_J} \tag{75}$$

Now,

$$(\frac{E}{N_0})N_0 = E = (\frac{S}{J}) \cdot L_M(\Delta\phi) \cdot (2\,\frac{B_J}{R}) \cdot N_J \tag{76}$$

where R = 1/T is symbol rate. Thus,

$$N_J = \frac{(E/N_0)}{L_M(\Delta\phi) \cdot (\frac{S}{J})} \cdot (\frac{R}{2B_J}) \cdot N_0 \tag{77}$$

and

$$N_T = [1 + \frac{(E/N_0)}{L_M(\Delta\phi) \cdot (\frac{S}{J})} \cdot (\frac{R}{2B_J})]N_0 \tag{78}$$

It is desired to compute the ratio of residual carrier power to total white noise power in the Loop-noise bandwidth (one-sided), B_N. The residual carrier power, S_C is

$$S_C = L_C(\Delta\phi)S = \frac{L_C(\Delta\phi)}{L_M(\Delta\phi)}\,\frac{E}{T} = \frac{R\ E}{\tan^2(\Delta\phi)} \tag{79}$$

where $L_C(\Delta\phi)$ is "carrier loss" given by

$$L_C(\Delta\phi) = \cos^2(\Delta\phi) \tag{80}$$

The desired signal to noise ratio is

$$\left.\frac{S_C}{N}\right|_{B_N} = \frac{S_C}{N_T B_N} = \frac{(R/B_N)\cdot(E/N_o)}{\tan^2(\Delta\phi)[1 + \dfrac{(E/N_o)}{L_M(\Delta\phi)\cdot(\frac{S}{J})}\cdot(\frac{R}{2B_J})]} \tag{81}$$

Note that when the equivalent white spectral density of the colored interfering process is much larger than the receiver white noise spectral density, then (81) reduces to

$$\left.\frac{S_C}{N}\right|_{B_N} \simeq 2\cos^2(\Delta\phi)\cdot(\frac{B_J}{B_N})\cdot(\frac{S}{J}) \tag{82}$$

The loop phase error variance, under the assumption that the loop is operating linearly for phase (large loop signal to noise ratio), is given by

$$\sigma_\phi^2 = \frac{1}{\left.\frac{S_C}{N}\right|_{B_N}} \quad \text{radians}^2 \tag{83}$$

and, from (83) and (81)

$$\sigma_\phi^2 = \tan^2(\Delta\phi)\left[\frac{1}{(R/B_N)\cdot(E/N_o)} + \frac{\frac{1}{2}(B_N/B_J)}{L_M(\Delta\phi)\cdot(\frac{S}{J})}\right] \tag{84}$$

Figure 12 shows simulation results for the case of narrow-band interference for binary phase-shift-keying (PSK). The equivalent square bandwidth of the colored interference process is 275 HZ. The signal symbol rate is 2500 baud. Thus the "bandwidth to bit-rate ratio" is BW/BR = 0.109. This is the same case for which extensive previous results were reported.

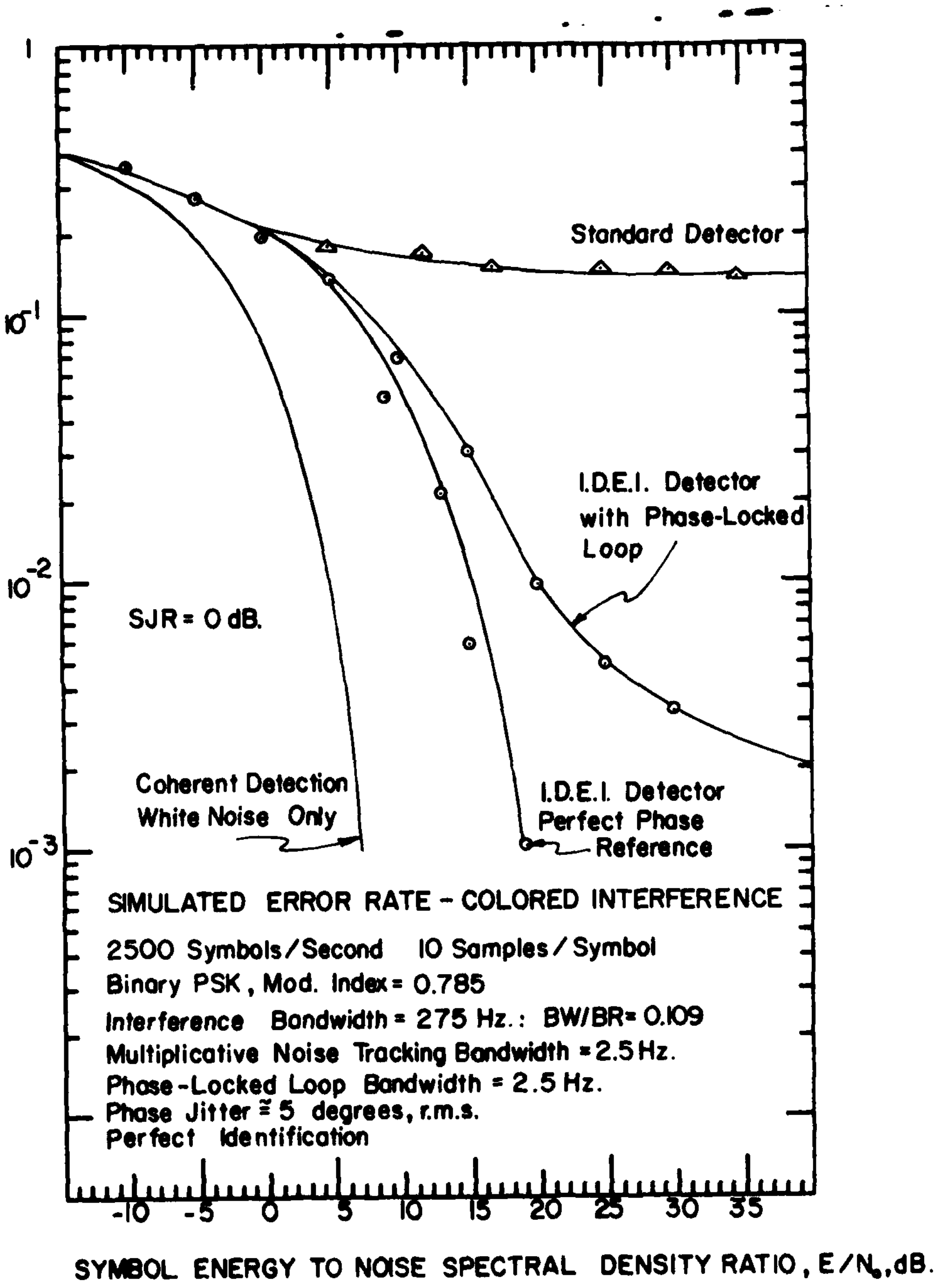


Figure 12. Simulation Results

For the case of Figure 12, the ratio of signal power to additive colored noise is unity, or zero dB. A phase-locked loop is implemer⁺ed, as described in Sections II.5. and II.6. The loop noise bandwidth is 2.5 Hz, being the smallest assumed to be practical for this case. The detector employs both additive noise tracking and multiplicative noise tracking with the latter matched to a multiplicative noise bandwidth of 2.5 Hz. Perfect identification is assumed for the colored additive noise.

From (84), the predicted value of loop phase jitter is determined to be 5.4° r.m.s. The actual r.m.s. values recorded in the simulation were between 1.7° and 9.6° for runs up to 1500 symbols in length. The loop was observed to always be in lock, slipping no cycles during any run.

The results plotted in Figure 12 include the reference graphs of coherent PSK detection for white noise only, and IDEI detection with perfect phase reference. Also, is shown the behavior of the standard discrete-time matched filter detector. The matched filter is seen to saturate at an error rate of 0.14, as usual [1]. The IDEI detector is seen to yield a convex error rate curve for $-10 \text{ dB} \leq E/N_0 \leq 20 \text{ dB}$. However, for $20 \text{ dB} < E/N_0$, the slope of the error rate curve becomes much less steep. Although the error-rate continues to decrease for increasing E/N_0, the rate of decrease is not as good as was obtained for "pure" multiplicative noise in [4].

The implications (or "cause") of the change in slope of the error rate curve for $20 \text{ dB} < E/N_0$ are, at present, unknown. Clearly, there is a transition at $E/N_0 \simeq 20$ dB for the case shown. It has been observed in the past that such transitions may be due to the breakdown of basic modeling assumptions on which the "optimum" detector is founded. One such questionable assumption which is suspect here is that the multiplicative noise process, due to carrier-tracking phase error, is Gaussian. Also, it may be that the phase-tracking detection algorithm is subject to an irreducible error-rate, as detailed in [8]. It is noted that the IDEI detector for multiplicative noise has not previously shown such an irreducible error.

In conclusion, this simulation for the SJR = 0 dB case shows that much of the performance measured previously for perfect phase is lost, when a standard phase-locked loop is used in parallel with the IDEI detector. It is recalled that this implementation is not the true optimum, for two reasons. One is the Gaussian multiplicative noise approximation. The second is that the phase-tracking loop is external to the detector and, thus, does not take advantage of the colored noise tracking capability of the detector itself. It may well be that a more optimum implementation will result by imbedding the phase-estimation algorithm within the detector itself.

SECTION V

COMPLETE RECEIVER ALGORITHMS

1. A PROPOSED BIT SYNCHRONIZATION ALGORITHM

So far in the investigation of IDEI detection, it has been assumed that bit timing information is available. This is important for the detector in terms of setting the start and stop times of the computation which produces the decision statistic, S(K). However, now the synchronization problem is finally examined.

Many practical bit synchronizers are based on the "Delay-lock Loop," [6, 7]. This technique applies to any coherent signalling scheme, but is generally used for phase-shift-keying (PSK). Generally, the implementation uses two signal cross-correlators driven with time-staggered signal reference waveforms. The correlator outputs are time-staggered versions of the noisy signal autocorrelation function. By subtracting the staggered autocorrelation functions, a tracking error function is produced which drives the reference genereator into bit synchronism with the receiver signal.

The key to the functioning of the delay-lock bit synchronizer is the production of a signal (from the correlator output) which is a positive, even function whose maximum occurs when the reference generator is in synchronization with the received bit. Those positive even functions (autocorrelation functions) also happen to be the sufficient statistics for detection for the standard detectors which use delay-lack bit synchronization.

In the IDEI detector, the sufficient statistic for detection is the pseudo-innovations process, or noise tracking error. It was seen in [1] that there was associated with the statistic a function which was positive, with minimum value occuring for perfect identification of the required noise statistics. With "positive" or "negative" identification errors (in the sense of Figures 36 and 43 of [1]), the function value increased. The function was the variance of the noise tracking error.

Now, it is conjectured that the IDEI tracking error variance, which is necessarily positive, will be minimum for the reference signal, $s(k;n)$, exactly synchronized with the received bit. It is also conjectured that the variance will increase as the reference, $s(k;n)$, becomes unsynchronized, regardless of whether $\underline{\delta}(k; n)$ leads or lags the received bit. If this conjecture proves true, then it is a simple matter to use the reciprocal of the tracking error variance in the same fashion that the Delay-Lock Loop uses the autocorrelation function, to form a synchronization tracking error function.

2. THE COMPLETE ALGORITHM

The complete IDEI algorithm (excluding identification) can be postulated as follows, for binary signalling. See Figure 13. Two IDEI detectors, with imbedded phase estimators are implemented, one with early waveform reference signals and one with late. Each detector contains two tracking filters of the form of Figure 7. In each detector are produced the detection statistics, S_0 and S_I, which are the tracking error variances, conditioned on the two different received symbols, m=0 and m=1, respectively. In each detector,symbol decision is made as usual. Based on the symbol decision, the assumed correct tracking error variances, $\hat{S}_e$ and $\hat{S}_\ell$, are produced by the early and late detectors, respectively. The reciprocal of each variance is taken and the results subtracted to form a "Synch Control" driving signal, which is filtered with suitable gain and time constant. A modulo-2 adder is implememted to determine if the decisions in the early and late detectors do not result in the same detected symbol. If not, the synch. control signal is inhibited, and synch is maintained as previously. Decision-directed reinitialization of the filters is carried out in the usual manner, independently in the early and late detectors.

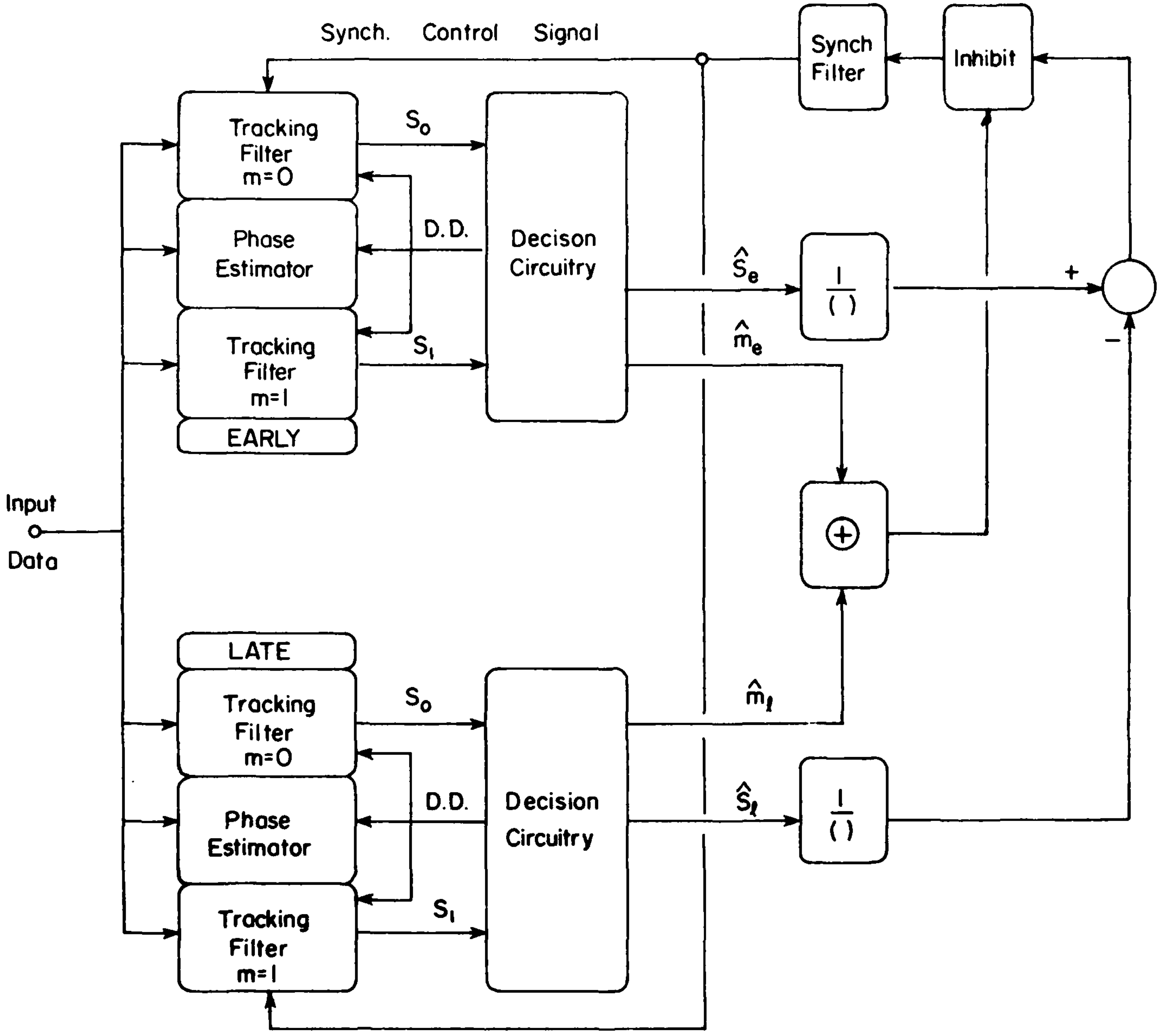


Figure 13. Complete Algorithm Diagram.

SECTION VI

CONCLUSIONS

The research documented in this report has yielded several interesting results. These are summarized below in the order of the governing tasks in the Contract Statement of Work.

Task 4.

The IDEI (interference-tracking) detection algorithms were extended to include provision of the required carrier phase reference through phase tracking. A separate phase-locked Loop was implemented, processing the received data in parallel with the detection algorithm, itself. The detection algorithm was augmented to track the multiplicative noise resulting from the phase reference variations, as well as tracking the colored additive noise.

Task 5.

It was shown analytically that a non-coherent version of the IDEI detection algorithm does not exist. This result is due to the feedback structure inherent in the IDEI tracking filter. The internal dependency of the detection statistic on the unknown phase makes it impractical to carry out the phase averaging necessary to obtain a non-coherent type algorithm.

Task 6.

Based on the result of Task 5, a non-coherent IDEI detector for Differential Phase Shift Keying is also impractical of derivation.

Task 7.

A bit synchronization technique was proposed, based on the Early-Late method. This bit synchronization scheme then led to the postulation of a complete receiver algortihm including interference-tracking, phase estimation, and bit synchronization. A block diagram of the algorithm was given.

Task 8.

The Monte Carlo simulation routine used and reported previously [1, 4] was restructured and re-written. The routine was simplified considerably and was augmented to accomodate the new detection and phase-tracking algorithms. The chief reason for this effort was to achieve shorter run times in line with restrictions imposed by the ASD Computer Facility (CDC-6600).

The performance of the IDEI detector with phase tracking was evaluated. It was found that the performance was considerabley degraded over previous results for perfect phase references. Two possible causes for the degradation were discussed.

In summary, further research on the IDEI algorithms is recommended in the following areas. Most importantly, a method of phase estimation should be sought wherein the phase estimator is imbedded in the interference tracking filter. The purpose is to reduce the effects of the large additive colored noise upon the phase estimator. Rather than tracking phase in parallel with the colored noise tracking filters, phase should be tracked after the colored noise has been removed from the data. Secondly, further effort should be devoted to optimizing the multiplicative noise tracking filter for the non-Gaussian perturbations produced by the phase variations. Finally, the proposed bit synchronization algorithm should be studied and evaluated.

APPENDIX A

THE CLOSED-FORM ERROR-RATE PROGRAM

(This appendix contains listings of the newly written simulation program and the closed-form error-rate evaluation program reported previously.)

```
C  THIS IS MAIN PROGRAM FOR THE CLOSED-FORM ERROR RATE FOR
C  IMPERFECT IDENTIFICATION WHICH IS A EXTENSION OF PROGRAM YOONM5
C  ** REQUIRED SUBROUTINE **
C  (1) RES3   ; MAIN,DATA,INPUT1,INPUT2,PARALL,PREPAR
C  (2) CFERAT  ; CFERAT,WKFLT,ERF
C  (3) VTT     ; VTT,CAYLEY,GAUS
C  (4) EIGEN
C  (5) COMAT
C  REMARK
C      (1) CHECKING THE CLOSED-FORM ERROR RATE FOR PERFECT
C      IDENTIFICATION, SET IMODE 1 AVOIDING THE SAME EIGEN-VALUE
C      IN SUB. CAYLEY
C      (2) TO GET THE STEADY-STATE KALMAN GAIN, SET KSMAX 50-100
C      IN GENERAL.
C      (3) ESTIMATED TRANSITION MATRIX PHEER AND DPHEE ARE VARYED
C      IN SUBROUTINE INPUT1 AND ESTIMATED KALMAN GAIN GSTAR IS
C      VARYED IN SUBROUTINE INPUT2 EACH TIME.
C  PROGRAMMER
C      CHANG-JUNE YOON
C      ELECTRICAL ENGINEERING DEPT.
C      TEXAS A & M UNIVERSITY
C
      COMMON/ORDER/N,N2
      COMMON/SAMPLE/NSPB,TB,TBR
      COMMON/OPTION/NOS,AEST
      COMMON/RATIO/ENODB,ENODBR,SJRDB,SJRDBR
      COMMON/GDB/GN,GNR,GJ,GJR
      COMMON/WORNOW/IMODE,KSMAX,IOJ
      COMMON/FREQ/FZ,FP(3)
      COMMON/PARAM/GAMMA(6,2),PHEE(6,6),H(2,6),Q(2,2),R(2,2)
      COMMON/PARAMR/PHEER(6,6),DPHEE(6,6),GSTAR(6,2),BSTAR(2,2)
      CALL ASSIGN(5,'SY:RES3.DAT',11,'RDO','NC',1)
C
      CALL DATA
C
C     NOPTN1
C         1,NO CHANGE
C         2,CHANGE ENODB
C         3,CHANGE SJRDBR
C         4,CHANGE NSPB,GK
C     NOPTN2
C         1,NO CHANGE
C         2,CHANGE ENODB
C         3,CHANGE SJRDBR
C         4,CHANGE GK
C         5,CHANGE SJRDB,SJRDBR
C     IF NOPTN1,NOPTN2 IS 1, THEN NCASE1,NCASE2 IS 1 RESPECTIVELY
C     NCASE1  ; NUMBER OF CASE FOR NOPTN1
C     NCASE2  ; NUMBER OF CASE FOR NOPTN2
C     IPARAM
C         0,NO PRINT-OUT PARAMETERS AND STATISTICS IN INPUT1 AND INPUT2
C         1,PRINT-OUT
C     IGV
C         0,NO CALCULATION KALMAN GAIN FOR A CORRECT PARAMETERS INPUT1.
C         1,CALCULATION.
C
      READ(5,701) NOPTN1,NOPTN2,NCASE1,NCASE2,IPARAM,IGV
  701 FORMAT(6I5)
      READ(5,702) GK
  702 FORMAT(E15.6)
      DO 2000 II=1,NCASE1
      GO TO (1,2,3,4),NOPTN1
    1 GO TO 50
    2 READ(5,705) ENODB
```

```
      ENODBR=ENODB
      GO TO 50
    3 READ(5,705) SJRDBR
      GO TO 50
    4 READ(5,707) NSPB,GK
   50 CONTINUE
  705 FORMAT(E15.6)
  706 FORMAT(2E15.6)
  707 FORMAT(I5,E15.6)
C
      DO 1000 III=1,NCASE2
      GO TO (11,12,13,14,15),NOPTN2
   11 GO TO 60
   12 READ(5,705) ENODB
      ENODBR=ENODB
      GO TO 60
   13 READ(5,705) SJRDBR
      GO TO 60
   14 READ(5,705) GK
      GO TO 60
   15 READ(5,706) SJRDB,SJRDBR
   60 CONTINUE
C
      WRITE(6,650) NSPB,TB,ENODB,SJRDB,SJRDBR,GK,AEST
  650 FORMAT(2X,5HNSPB=,I5,2X,3HTB=,E13.6,2X,6HENODB=,E13.6,2X,
     16HSJRDB=,E13.6,2X,7HSJRDBR=,E13.6,2X,3HGK=,E13.6,2X,5HAEST=,E13.6)
      CALL INPUT1(IPARAM,IGV)
      CALL INPUT2(IPARAM,GK)
C
      CALL CFERAT(ERATCL)
C
      WRITE(6,651) ERATCL
  651 FORMAT(2X,26HCLOSED-FORM ERROR RATE IS ,30X,10H*********=,E13.6)
 1000 CONTINUE
      WRITE(6,751)
  751 FORMAT(5X,11HEND OF CASE,//)
 2000 CONTINUE
      STOP
      END
C
      SUBROUTINE DATA
      COMMON/ORDER/N,N2
      COMMON/SAMPLE/NSPB,TB,TBR
      COMMON/OPTION/NOS,AEST
      COMMON/RATIO/ENODB,ENODBR,SJRDB,SJRDBR
      COMMON/WORNOW/IMODE,KSMAX,IOJ
      COMMON/FREQ/FZ,FP(3)
C  N,N2 : SYSTEM ORDER
C  NOS  : (1) PSK, (2) FSK
C  AEST : SIGNAL MAGNITUDE IN SUB. REFGEN
C  IMODE: (1) DIAGONAL PHEE MATRIX AND PERFECT IDENTIFICATION
C         (2) DIAGONAL PHEE MATRIX
C         (3) GENERAL IMPERFECT IDENTIFICATION
C  KSMAX: MAXIMUM NUMBER OF ITERATION FOR STEADY-STATE KALMAN GAIN
C  IGV  : (0) NO CALCULATION FOR CORRECT KALMAN GAIN AND VINOV IN INPUT1
C         (1) CALCULATION FOR CORRECT KALMAN GAIN AND VINOV IN INPUT1
C  FZ,FP: ZERO,POLE FREQUENCY FOR LOW-PASS FILTER
      READ(5,600) N,N2
      READ(5,601) NSPB,TB,TBR
      READ(5,602) NOS,AEST
      READ(5,603) ENODB,ENODBR,SJRDB,SJRDBR
      READ(5,604) IMODE,KSMAX,IOJ
      READ(5,603) FZ,(FP(I),I=1,3)
  600 FORMAT(2I5)
  601 FORMAT(I5,2E15.6)
  602 FORMAT(I5,E15.6)
  603 FORMAT(4E15.6)
  604 FORMAT(3I5)
      RETURN
      END
C
      SUBROUTINE INPUT1(IPARAM, IGV)
C  TO GET THE REAL PARAMETERS AND STATISTICS GIVEN VALUES.
C  ALL WE NEED IN HERE ARE GAMMA, PHEE, H, R
C  GAIN AND VINOV ARE FOR REFERENCE
C  IF IGV : 0 - NO CALCULATION FOR CORRECT KALMAN GAIN AND VINOV
C         : 1 - CALCULATION FOR CORRECT KALMAN GAIN AND VINOV
C  THEREFORE GK ALWAYS SET 1..FOR PERFECT IDENTIFICATION.
      COMMON/ORDER/N, N2
      COMMON/SAMPLE/NSPB, TB, TBR
      COMMON/OPTION/NOS, AEST
      COMMON/RATIO/ENODB, ENODBR, SJRDB, SJRDBR
      COMMON/GDB/GN, GNR, GJ, GJR
      COMMON/WORNOW/IMODE, KSMAX, IOJ
      COMMON/FREQ/FZ, FP(3)
      COMMON/PARAM/GAMMA(6, 2), PHEE(6, 6), H(2, 6), Q(2, 2), R(2, 2)
      DIMENSION VINOV(2, 2), GAIN(6, 2)
      CALL PARALL(1., BN, GAMMA, PHEE, H, ENODB, SJRDB, GN, GJ, R, IGV
     1, GAIN, VINOV)
      IF(IPARAM. EQ. 0) GO TO 40
      WRITE(6, 610)
  610 FORMAT(2X, 32H*REAL PARAMETERS AND STATISTICS*, /)
      DO 20 I=1, N
      WRITE(6, 611) I, GAMMA(I, 1), I, PHEE(I, I), I, H(1, I)
  611 FORMAT(2X, 5HGAMD(, I1, 2H)=, E13. 6, 2X, 5HPHID(, I1, 2H)=, E13. 6,
     12X, 3HHT(, I1, 2H)=, E13. 6)
   20 CONTINUE
      IF(IGV. EQ. 0) GO TO 40
      WRITE(6, 615) GN
  615 FORMAT(/, 2X, 3HGN=, E13. 6)
      WRITE(6, 612)
  612 FORMAT(/, 2X, 5HGAIN=, 26X, 6HVINOV=)
      DO 25 I=1, N2
      IF(I. GT. 2) GO TO 30
      WRITE(6, 613) (GAIN(I, J), J=1, 2), (VINOV(I, J), J=1, 2)
  613 FORMAT(2X, 2E13. 6, 5X, 2E13. 6)
      GO TO 25
   30 WRITE(6, 614) (GAIN(I, J), J=1, 2)
  614 FORMAT(2X, 2E13. 6)
   25 CONTINUE
   40 CONTINUE
      WRITE(6, 617) BN
  617 FORMAT(/, 2X, 21HEQUIVALENT BANDWIDTH=, E13. 6, /)
      RETURN
      END
C
      SUBROUTINE INPUT2(IPARAM, GK)
C  THIS SUBROUTINE GSTAR AND DPHEE FOR DIFFERENT FILTER BANDWIDTH
C  THESE GSTAR AND DPHEE WITH GAMMA, PHEE, H, R ARE USED TO CALCULATE
C  RESIDUAL VARIANCE IN SUBROUTINE CFERAT AND VTT.
C  THEREFORE IGV ALWAYS SET 1 HERE.
      COMMON/ORDER/N, N2
      COMMON/RATIO/ENODB, ENODBR, SJRDB, SJRDBR
      COMMON/GDB/GN, GNR, GJ, GJR
      COMMON/WORNOW/IMODE, KSMAX, IOJ
      COMMON/PARAM/GAMMA(6, 2), PHEE(6, 6), H(2, 6), Q(2, 2), R(2, 2)
      COMMON/PARAMR/PHEER(6, 6), DPHEE(6, 6), GSTAR(6, 2), BSTAR(2, 2)
      DIMENSION GAMMAR(6, 2), RR(2, 2), VINOVR(2, 2)
      CALL PARALL(GK, BNR, GAMMAR, PHEER, H, ENODBR, SJRDBR, GNR, GJR, RR
     1, 1, GSTAR, VINOVR)
      DO 10 I=1, N2
      DO 10 J=1, N2
   10 DPHEE(I,J)=PHEE(I,J)-PHEER(I,J)
      IF(IPARAM.EQ.0) RETURN
      WRITE(6,600)
  600 FORMAT(/,2X,35HESTIMATED PARAMETERS AND STATISTICS,/)
      DO 20 I=1,N
      WRITE(6,601) I,GAMMAR(I,1),I,PHEER(I,I),I,H(1,I),I,DPHEE(I,I)
     1,I,QSTAR(I,1)
  601 FORMAT(2X,'GAMDR(',I1,')=',E13.6,2X,'PHIDR(',I1,')=',E13.6
     1,2X,'HTR(',I1,')=',E13.6,5X,'DPHEE(',I1,')=',E13.6,2X,'QSTAR('
     2,I1,')=',E13.6)
   20 CONTINUE
      WRITE(6,602) BNR
  602 FORMAT(/,2X,21HEQUIVALENT BANDWIDTH=,E13.6)
      WRITE(6,603) VINOVR(1,1)
  603 FORMAT(/,2X,12HVINOVR(1,1)=,E13.6)
      RETURN
      END
C
      SUBROUTINE PARALL(QK,BN,GAMMA,PHEE,H,ENODB,SJRDB,QN,QJ,R,IGV
     1,GAIN,VINOV)
      COMMON/ORDER/N,N2
      COMMON/OPTION/NOS,AEST
      COMMON/SAMPLE/NSPB,TB,TBR
      COMMON/WORNOW/IMODE,KSMAX,IOJ
      COMMON/FREQ/FZ,FP(3)
      DIMENSION FPR(3)
      DIMENSION GAMD(3),PHID(3),HT(3)
      DIMENSION GAMMA(6,2),PHEE(6,6),H(2,6),Q(2,2),R(2,2)
      DIMENSION PVPT(6,6),QTQ(6,6),VEST(6,6),VPRED(6,6),HVHT(2,2)
     1,VINOV(2,2),VINV(2,2),VPHT(6,2),GAIN(6,2),GH(6,6)
      REAL IMGH(6,6)
      PI=4.*ATAN(1.)
      DELPHI=.785
      DELMEG=DELPHI*2.*PI/TB
      IF(NOS.EQ.1) GO TO 1
      SUMF=0.0
      DO 2 K=1,NSPB
    2 SUMF=SUMF+(SIN((K-.5)*TB*DELMEG/NSPB))**2
      CONSTF=SQRT(SUMF)
      QN=CONSTF*10.**(-ENODB/20.)
      GO TO 3
    1 CONSTP=SQRT(NSPB/2.)*ABS(SIN(DELPHI))
      QN=CONSTP*10.**(-ENODB/20.)
    3 QJ=10.**(-SJRDB/20.)/SQRT(2.)
      R(1,1)=QN**2
      R(1,2)=0.0
      R(2,1)=0.0
      R(2,2)=QN**2
      FZR=QK*FZ
      DO 5 I=1,N
    5 FPR(I)=QK*FP(I)
      T=TB/NSPB
      CALL PREPAR(T,FZR,FPR,GAMD,PHID,HT,BN,IOJ)
      DO 10 I=1,N2
      DO 10 J=1,2
   10 GAMMA(I,J)=0.0
      DO 11 I=1,N
      GAMMA(I,1)=GAMD(I)
   11 GAMMA(I+N,2)=GAMD(I)
C   NEW WEIGHTED GAMMA MATRIX
      DO 12 I=1,N2
      DO 12 J=1,2
   12 GAMMA(I,J)=QJ*GAMMA(I,J)
      DO 15 I=1,N2
      DO 15 J=1,N2
   15 PHEE(I,J)=0.0
```

```
      DO 16 I=1,N
      PHEE(I,I)=PHID(I)
   16 PHEE(I+N,I+N)=PHID(I)
      DO 20 I=1,2
      DO 20 J=1,N2
   20 H(I,J)=0.0
      DO 21 I=1,N
      H(1,I)=HT(I)
   21 H(2,I+N)=HT(I)
C
      IF(IGV.EQ.0) RETURN
C  CALCULATE THE STEADY-STATE KALMAN-GAIN
      DO 32 I=1,N2
      DO 32 J=1,N2
   32 VEST(I,J)=0.0
      DO 35 KS=1,KSMAX
      CALL MABCT(PHEE,N2,N2,VEST,N2,PHEE,N2,PVPT,6,6,6,6,6,6,6,6)
      CALL MATMUL(2,GAMMA,N2,2,GAMMA,N2,GTG,6,2,6,2,6,6)
      CALL MATAS(1,PVPT,N2,N2,GTG,VPRED,6,6)
      CALL MABCT(H,2,N2,VPRED,N2,H,2,HVHT,2,6,6,6,2,6,2,2)
      CALL MATAS(1,R,2,2,HVHT,VINOV,2,2)
      CALL MATMUL(2,VPRED,N2,N2,H,2,VPHT,6,6,2,6,6,2)
      DET=VINOV(1,1)*VINOV(2,2)-VINOV(1,2)*VINOV(2,1)
      VINV(1,1)=VINOV(2,2)/DET
      VINV(1,2)=-VINOV(1,2)/DET
      VINV(2,1)=-VINOV(2,1)/DET
      VINV(2,2)=VINOV(1,1)/DET
      CALL MATMUL(1,VPHT,N2,2,VINV,2,GAIN,6,2,2,2,6,2)
      CALL MATMUL(1,GAIN,N2,2,H,N2,GH,6,2,2,6,6,6)
      DO 36 I=1,N2
      DO 36 J=1,N2
      IMGH(I,J)=-GH(I,J)
      IF(I.EQ.J) IMGH(I,J)=1.0-GH(I,J)
   36 CONTINUE
      CALL MATMUL(1,IMGH,N2,N2,VPRED,N2,VEST,6,6,6,6,6,6)
   35 CONTINUE
      RETURN
      END
      SUBROUTINE PREPAR(T,FZ,FP,GAMD,PHID,HT,BN,INOPT)
C     ****************************************************************
C     PREPAR: MODIFICATION SUBROUTINE ADDED TO SUBROUTINE INPUT
C             TO PERFORM PRE-CALCULATIONS OF FILTER PARAMETERS
C     *I/O PARAMETERS*
C     * INPUT *
C     T: SAMPLING TIME
C     FZ: ZERO FREQUENCY
C     FP: POLE PREQUENCIES (3)
C     INOPT: 1-DIGIT CODE FOR SELECTION OF REAL/COMPLEX ZERO
C            AND UNITY GAIN/VARIANCE FOR FILTER PARAMETER CALCULATIONS
C            =1,REAL ZERO,UNIT GAIN
C            =2,REAL ZERO,UNIT VARIANCE
C            =3,COMPLEX ZERO,UNIT GAIN
C            =4,COMPLEX ZERO,UNIT VARIANCE
C     * OUTPUT *
C     PHID: FILTER TRANSITION WEIGHTS(3)
C     GAMD: FILTER INPUT WEIGHTS(3)
C     HT: FILTER OUTPUT WEIGHTS(3)
C     BN: EQUIVALENCE NOISE BANDWIDTH
C     * INTERNAL FILTER PARAMETERS *
C     Z: ZERO FREQUENCY, IN RADIANS
C     P: POLE FREQUENCIES (3), IN RADIANS
C     R: RESIDUES(3)
C     RE: RESIDUES(3)
C     GAINK: GAIN CONSTANT
C     ****************************************************************
      DIMENSION FP(3),P(3),R(3),RE(3),PHID(3),GAMD(3),HT(3)
      PI=4. *ATAN(1. )
      IF(INOPT. GT. 2) GO TO 100
C     FREQUENCY CALCULATIONS
      Z=(-2. )*PI*FZ
      DO 1 I=1,3
    1 P(I)=(-2. )*PI*FP(I)
C     RESIDUE CALCULATIONS
      DO 5 I=1,3
      D=1.
      DO 10 J=1,3
      IF(I. EQ. J) GO TO 10
      D=D*(P(I)-P(J))
   10 CONTINUE
      R(I)=(P(I)-Z)/D
    5 CONTINUE
C     TRANSITION WEIGHTS
      DO 20 I=1,3
   20 PHID(I)=EXP(P(I)*T)
C     INPUT WEIGHTS
      DO 25 I=1,3
   25 GAMD(I)=(1. -PHID(I))*R(I)/(-P(I))
C     UNITY GAIN
      IF(INOPT. NE. 1) GO TO 30
      GAINK=P(1)*P(2)*P(3)/Z
      GO TO 35
   30 CONTINUE
C     UNITY VARIANCE
      SUM=0. 0
      DO 40 I=1,3
      DO 40 J=1,3
   40 SUM=SUM+GAMD(I)*GAMD(J)/(1. 0-PHID(I)*PHID(J))
      GAINK=1. /SQRT(SUM)
   35 CONTINUE
C   NEW WEIGHTED INPUT MATRIX
      DO 36 I=1,3
   36 GAMD(I)=GAINK*GAMD(I)
C     OUTPUT WEIGHTS
      DO 45 I=1,3
   45 HT(I)=1.
C     EQUIVALENT NOISE BANDWIDTH
      DO 50 I=1,3
      D=1.
      DO 55 J=1,3
      IF(I. EQ. J) GO TO 55
      D=D*(P(I)**2-P(J)**2)
   55 CONTINUE
      RE(I)=GAINK**2*(P(I)**2-Z**2)/(2. *P(I)*D)
   50 CONTINUE
      GO=GAINK*Z/(P(1)*P(2)*P(3))
      BN=(RE(1)+RE(2)+RE(3))/(2. *GO**2)
      RETURN
  100 CONTINUE
C     MODIFIED TRANSFER FUNCTION HAVING COMPLEX ZERO.
C     FREQUENCY CALCULATION.
C     Z**2=P(2)**2-2*P(1)**2,TO HAVE A JW-AXIS ZERO,Z SHOULD BE POSITIVE
      Z=-2. *PI*FZ
      P(1)=Z
      P(2)=SQRT(3. )*P(1)
      P(3)=-2. *PI*FP(3)
      FP(1)=P(1)/(-2. *PI)
      FP(2)=P(2)/(-2. *PI)
      FP(3)=P(3)/(-2. *PI)
C     RESIDUE CALCULATIONS
      DO 110 I=1,3
      D=1.
      DO 120 J=1,3
      IF(I.EQ.J) GO TO 120
      D=D*(P(I)-P(J))
  120 CONTINUE
      R(I)=(P(I)**2+Z**2)/D
  110 CONTINUE
C     TRANSITION WEIGHTS
      DO 125 I=1,3
  125 PHID(I)=EXP(P(I)*T)
C     INPUT WEIGHTS
      DO 130 I=1,3
  130 GAMD(I)=(1.-PHID(I))*R(I)/(1.0-PHID(I)*PHID(J))
C     UNITY GAIN
      IF(INOPT.NE.3) GO TO 135
      GAINK=2.*PI*FP(1)*FP(2)*FP(3)/FZ**2
      GO TO 140
  135 CONTINUE
C     UNITY VARIANCE
      SUM=0.0
      DO 150 I=1,3
      DO 150 J=1,3
  150 SUM=SUM+GAMD(I)*GAMD(J)/(1.0-PHID(I)*PHID(J))
      GAINK=1./SQRT(SUM)
  140 CONTINUE
C  NEW WEIGHTED INPUT MARTRIX
      DO 141 I=1,3
  141 GAMD(I)=GAINK*GAMD(I)
C     OUTPUT WEIGHT
      DO 155 I=1,3
  155 HT(I)=1.
C     EQIVALENT NOISE BANDWIDTH
      DO 160 I=1,3
      D=1.
      DO 170 J=1,3
      IF(I.EQ.J) GO TO 170
      D=D*(P(I)**2-P(J)**2)
  170 CONTINUE
      RE(I)=(-1.)*GAINK**2*(P(I)**2+Z**2)**2/(2.*P(I)*D)
  160 CONTINUE
      GO=(-1.)*GAINK*Z**2/(P(1)*P(2)*P(3))
      EN=(RE(1)+RE(2)+RE(3))/(2.*GO**2)
      RETURN
      END
C
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      SUBROUTINE CFERAT(ERATCL)
      EXTERNAL ERF
      COMMON/ORDER/N,N2
      COMMON/SAMPLE/NSPB,TB,TBR
      COMMON/WORNOW/IMODE,KSMAX,IOJ
      COMMON/PARAM/GAMMA(6,2),PHEE(6,6),H(2,6),Q(2,2),R(2,2)
      COMMON/PARAMR/PHEER(6,6),DPHEE(6,6),GSTAR(6,2),BSTAR(2,2)
      DIMENSION XEST1(6),XEST2(6),XPRED1(6),XPRED2(6)
      DIMENSION SIG1(2),SIG2(2),ES1(2),ES2(2)
      DIMENSION B1(2,300),VTTJ(2,2),VTILDA(300)
      DIMENSION VXX(6,6),TEMP(6,6),GTG(6,6),F(6,6),VXXT(6,6),VXXT1(6,6)
     1,VXXT2(6,6),VXTXT(6,6),VXTXT1(6,6),VXTXT2(6,6),VXTXT3(6,6)
     2,VXTXT4(6,6),VXTXT5(6,6),GRG(6,6),TEMP1(6,6)
      REAL IMGH(6,6)
C
      CALL MATMUL(2,GAMMA,N2,2,GAMMA,N2,GTG,6,2,6,2,6,6)
      CALL MATMUL(1,GSTAR,N2,2,H,N2,TEMP,6,2,2,6,6,6)
      DO 5 I=1,N2
      DO 5 J=1,N2
      IMGH(I,J)=-TEMP(I,J)
      IF(I.EQ.J) IMGH(I,J)=1.0-TEMP(I,J)
    5 CONTINUE
      CALL MATMUL(1,PHEER,N2,N2,IMGH,N2,F,6,6,6,6,6,6)
C
C   INITIALIZE VXX,VXXT AND VXTXT
      DO 6 I=1,N2
      DO 6 J=1,N2
      VXX(I,J)=0.0
      VXXT(I,J)=0.0
    6 VXTXT(I,J)=0.0
C
      DO 1 KS=1,KSMAX
C   VXX(K) = PHEE * VXX(K-1) * PHEE' + GAMMA * Q * GAMMA'
      CALL MABCT(PHEE,N2,N2,VXX,N2,PHEE,N2,TEMP,6,6,6,6,6,6,6,6)
      CALL MATAS(1,TEMP,N2,N2,GTG,VXX,6,6)
C   VXXT(K!K-1) = PHEE * VXXT(K-1!K-2) * F' + PHEE * VXX(K) * DPHEE'
C                      + GAMMA * Q * GAMMA'
      CALL MABCT(PHEE,N2,N2,VXXT,N2,F,N2,VXXT1,6,6,6,6,6,6,6,6)
      CALL MABCT(PHEE,N2,N2,VXX,N2,DPHEE,N2,VXXT2,6,6,6,6,6,6,6,6)
      CALL MATAS(1,VXXT1,N2,N2,VXXT2,TEMP,6,6)
      CALL MATAS(1,TEMP,N2,N2,GTG,VXXT,6,6)
C   VXTXT(K+1!K) = F * VXTXT(K!K-1) * F' + 2. * DPHEE * VXXT(K!K-1) * F'
C                      + DPHEE * VXX(K) * DPHEE' + GAMMA * Q * GAMMA'
C                      + PHEER * GSTAR * R * GSTAR' * PHEER'
      CALL MABCT(F,N2,N2,VXTXT,N2,F,N2,VXTXT1,6,6,6,6,6,6,6,6)
      CALL MABCT(DPHEE,N2,N2,VXXT,N2,F,N2,VXTXT2,6,6,6,6,6,6,6,6)
      DO 15 I=1,N2
      DO 15 J=1,N2
   15 VXTXT3(I,J)=VXTXT2(J,I)
      CALL MABCT(DPHEE,N2,N2,VXX,N2,DPHEE,N2,VXTXT4,6,6,6,6,6,6,6,6)
      CALL MABCT(GSTAR,N2,2,R,2,GSTAR,N2,GRG,6,2,2,2,6,2,6,6)
      CALL MABCT(PHEER,N2,N2,GRG,N2,PHEER,N2,VXTXT5,6,6,6,6,6,6,6,6)
      CALL MATAS(1,VXTXT1,N2,N2,VXTXT2,TEMP,6,6)
      CALL MATAS(1,TEMP,N2,N2,VXTXT3,TEMP1,6,6)
      CALL MATAS(1,TEMP1,N2,N2,VXTXT4,TEMP,6,6)
      CALL MATAS(1,TEMP,N2,N2,GTG,TEMP1,6,6)
      CALL MATAS(1,TEMP1,N2,N2,VXTXT5,VXTXT,6,6)
    1 CONTINUE
C
      DO 25 I=1,N2
      XEST1(I)=0.0
      XEST2(I)=0.0
   25 CONTINUE
C   SIG1  :  ES(M=0,N=0)
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C SIG2  :  ES(M=0,N=1)
      A=0.0
      DO 20 K=1,NSPB
      CALL REFGEN(K,0,FTR0,GTR0,FRR0,GRR0)
      CALL REFGEN(K,1,FTR1,GTR1,FRR1,GRR1)
      SIG1(1)=FTR0-FRR0
      SIG1(2)=GTR0-GRR0
      SIG2(1)=FTR0-FRR1
      SIG2(2)=GTR0-GRR1
      CALL WKFLT(K,XEST1,XPRED1,SIG1,ES1)
      CALL WKFLT(K,XEST2,XPRED2,SIG2,ES2)
      A=A+(ES2(1)**2+ES2(2)**2-ES1(1)**2-ES1(2)**2)
      B1(1,K)=ES1(1)-ES2(1)
      B1(2,K)=ES1(2)-ES2(2)
   20 CONTINUE
C  THE VII, INNOVATION VARIANCE, IS DECOUPLED,SO IS VTTJ SINCE
C  THE TEST SYSTEM IS DECOUPLED.
C  IF THE SYSTEM IS COUPLED, THEN THE EVALUATION OF MEAN AND VARIANCE
C  MUST BE MODIFIED.
      DO 30 J=1,NSPB
      L=J-1
      CALL VTT(L,F,VXTXT,VXXT,VTTJ)
      VTILDA(J)=VTTJ(1,1)
   30 CONTINUE
      B=0.0
      DO 35 J=1,NSPB
      DO 35 K=1,NSPB
      L=IABS(J-K)+1
      B=B+(B1(1,J)*B1(1,K)+B1(2,J)*B1(2,K))*VTILDA(L)
   35 CONTINUE
      IF(B.LE.0.) GO TO 50
      SIGMAB=SQRT(B)
      X=A/SIGMAB
      SUFX=X/(2.*SQRT(2.))
      ERATCL=0.5*(1.-ERF(SUFX))
      WRITE(6,600) A,SIGMAB,VTILDA(1),X
  600 FORMAT(2X,'MU=',E13.6,2X,'SIGMA=',E13.6,2X,'VTT(0)=',E13.6,
     12X,'MU/SIGMA=',E13.6)
      RETURN
   50 WRITE(6,601)
  601 FORMAT(2X,20HVARIANCE IS NEGATIVE)
      DO 60 I=1,NSPB
   60 WRITE(6,602) I,VTILDA(I),B1(1,I),B1(2,I)
  602 FORMAT(2X,7HVTILDA(,I3,2H)=,E13.6,2X,15HTRACKING ERROR=,2E15.6)
      RETURN
      END
      SUBROUTINE WKFLT(KS,XEST,XPRED,SIG,V)
      COMMON/ORDER/N,N2
      COMMON/PARAM/GAMMA(6,2),PHEE(6,6),H(2,6),Q(2,2),R(2,2)
      COMMON/PARAMR/PHEER(6,6),DPHEE(6,6),GSTAR(6,2),BSTAR(2,2)
      DIMENSION XEST(6),XPRED(6),SIG(2),V(2),ZHAT(2),GV(6)
      CALL MATVEC(PHEER,N2,N2,XEST,XPRED,6,6)
      CALL MATVEC(H,2,N2,XPRED,ZHAT,2,6)
      CALL VECAS(2,SIG,ZHAT,V,2)
      CALL MATVEC(GSTAR,N2,2,V,GV,6,2)
      CALL VECAS(1,XPRED,GV,XEST,6)
      RETURN
      END
      SUBROUTINE REFGEN(KS,M,FTR,GTR,FRR,GRR)
      COMMON/SAMPLE/NSPB,TB,TBR
      COMMON/OPTION/NOS,AEST
      TK=(KS-0.5)/NSPB
      TKRMOD=(TK-IFIX(TK))*TBR
      DELPHI=.785
      DELMEG=DELPHI*8.*ATAN(1.)/TB
      IF(NOS.NE.1) GO TO 1
      IF(M. EQ. 0) PHEETR=DELPHI
      IF(M. EQ. 1) PHEETR=-DELPHI
      GO TO 2
    1 IF(M. EQ. 0) PHEETR=DELMEG*TKRMOD
      IF(M. EQ. 1) PHEETR=-DELMEG*TKRMOD
    2 FTR=COS(PHEETR)
      GTR=SIN(PHEETR)
      FRR=AEST*COS(PHEETR)
      GRR=AEST*SIN(PHEETR)
      RETURN
      END
      FUNCTION ERF(X)
C     THIS IS AN APPROXIMATION OF ERROR FUNCTION HAVING
C     LESS THAN 1. 5E-7 ERROR AND ASSUMED X IS POSITIVE
C     ERROF FUNCTION IS SYMMETRIC
      P=0. 3275911
      A1=0. 254829592
      A2=-0. 284496736
      A3=1. 421413741
      A4=-1. 453152027
      A5=1. 061405429
      XX=ABS(X)
      T=1. /(1. +P*XX)
      ERF=1. -(A1*T+A2*T**2+A3*T**3+A4*T**4+A5*T**5)*EXP(-XX**2)
      IF(X. GE. 0. ) ERF=ERF
      IF(X. LT. 0. ) ERF=-ERF
      RETURN
      END
      SUBROUTINE VTT(JP, F, VXTXT, VXXT, VTTJ)
      COMMON/ORDER/N, N2
      COMMON/WORNOW/IMODE, KSMAX, IOJ
      COMMON/PARAM/GAMMA(6, 2), PHEE(6, 6), H(2, 6), Q(2, 2), R(2, 2)
      COMMON/PARAMR/PHEER(6, 6), DPHEE(6, 6), GSTAR(6, 2), BSTAR(2, 2)
      DIMENSION F(6, 6), VXTXT(6, 6), VXXT(6, 6), VTTJ(2, 2)
      DIMENSION VHT(6, 2), HVHT(2, 2), B1(6, 2), B2(6, 2), B3(6, 2)
     1, B4(6, 2), B5(2, 2), TEMP(6, 6), FL(6, 6)
      DIMENSION PHEEJ(6, 6), V(6, 6), V1(6, 6), V2(6, 6), V3(6, 6), V4(2, 2)
      IF(JP) 1, 2, 3
1     WRITE(6, 11)
   11 FORMAT(2X, 28HNEGATIVE J POWER IN VTT SUB. )
      RETURN
2     CALL MABCT(H, 2, N2, VXTXT, N2, H, 2, HVHT, 2, 6, 6, 6, 2, 6, 2, 2)
      CALL MATAS(1, HVHT, 2, 2, R, BSTAR, 2, 2)
      DO 4 I=1, 2
      DO 4 J=1, 2
    4 VTTJ(I, J)=BSTAR(I, J)
      RETURN
3     CONTINUE
C  [ V * H' - GSTAR * ( H * V * H' + R ) ]
      CALL MATMUL(2, VXTXT, N2, N2, H, 2, VHT, 6, 6, 2, 6, 6, 2)
      CALL MATMUL(1, GSTAR, N2, 2, BSTAR, 2, B1, 6, 2, 2, 2, 6, 2)
      CALL MATAS(2, VHT, N2, 2, B1, B2, 6, 2)
C  PHEER * [ V * H' - GSTAR * ( H * V * H' + R ) ]
      CALL MATMUL(1, PHEER, N2, N2, B2, 2, B3, 6, 6, 6, 2, 6, 2)
C  F = [ PHEER * ( I - GSTAR * H ) ]**(J-1)
      CALL CAYLEY(IMODE, F, JP-1, FL)
C  H * F**(J-1) * PHEER * [ V * H'  - B ]
      CALL MATMUL(1, FL, N2, N2, B3, 2, B4, 6, 6, 6, 2, 6, 2)
      CALL MATMUL(1, H, 2, N2, B4, 2, B5, 2, 6, 6, 2, 2, 2)
      IF(IMODE. EQ. 1) GO TO 100
C
C  SUM[ F**(I-1) * DPHEE * PHEE**(J-I) ]
      DO 5 I1=1, N2
      DO 5 I2=1, N2
    5 TEMP(I1, I2)=0. 0
      DO 10 I=1, JP
      L1=I-1
      L2=JP-I
      CALL CAYLEY(IMODE, F, L1, FL)
      CALL CAYLEY(IMODE, PHEE, L2, PHEEJ)
      CALL MATMUL(1, FL, N2, N2, DPHEE, N2, V1, 6, 6, 6, 6, 6, 6)
      CALL MATMUL(1, V1, N2, N2, PHEEJ, N2, V2, 6, 6, 6, 6, 6, 6)
      CALL MATAS(1, TEMP, N2, N2, V2, V, 6, 6)
      DO 15 II=1, N2
      DO 15 JJ=1, N2
      TEMP(II, JJ)=V(II, JJ)
   15 CONTINUE
   10 CONTINUE
C  SUM[ F**(I-1) * DPHEE * PHEE**(J-I) ] * VXXT
      CALL MATMUL(1, V, N2, N2, VXXT, N2, V3, 6, 6, 6, 6, 6, 6)
C  H * SUM[ F**(I-1) * DPHEE * PHEE**(J-I) ] * VXXT * H'
      CALL MABCT(H, 2, N2, V3, N2, H, 2, V4, 2, 6, 6, 6, 2, 6, 2, 2)
      CALL MATAS(1, B5, 2, 2, V4, VTTJ, 2, 2)
      RETURN
100   DO 110 I=1, 2
      DO 110 J=1, 2
      VTTJ(I, J)=B5(I, J)
  110 CONTINUE
      RETURN
      END
      SUBROUTINE CAYLEY(IMODE, F, L, FL)
C  THIS SUBROUTINE PRODUCE THE MATRIX HIGH POWERED USING
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C  CAYLEY-HAMILTON'S THEOREM TO REDUCE THE ERROR.
C  IMODE
C     (1) AND (2) : F IS DIAGONAL MATRIX SOTHAT IT HAS SAME EIGEN-VALUE.
C     THIS REQUIRE SPECIAL GAUS SUBROUTINE TO SOLVE THE LINEAR EQUATIONS.
C     (3)         : F IS GENERAL MATRIX AND HAS THE DISTINGUISHED
C      EIGEN-VLAUE
C  F          INPUT MATRIX TO BE MULTIPLIED BY HIGH POWER
C  FL         RESULTANT MATRIX
      COMMON/ORDER/N,N2
      COMPLEX EV(6),A1(6,6),B1(6),ALFA(6),FL1(6,6),CMPLX
      DIMENSION F(6,6),A(12,12),B(12),X(12),FL(6,6),FP(6,6,6)
      DIMENSION SF(6,6)
      IF(L) 1,2,3
1     WRITE(6,4)
    4 FORMAT(' NEGATIVE L IN SUB. CAYLEY')
      RETURN
2     DO 5 I=1,N2
      DO 5 J=1,N2
      FL(I,J)=0.0
      IF(I.EQ.J) FL(I,J)=1.
    5 CONTINUE
      RETURN
3     CONTINUE
      IF(L.NE.1) GO TO 7
      DO 6 I=1,N2
      DO 6 J=1,N2
      FL(I,J)=F(I,J)
    6 CONTINUE
      RETURN
7     CONTINUE
      IF(IMODE.NE.3) GO TO 150
      N4=N*4
      CALL EIGEN(F,N2,EV)
C  USING GAUSS ELIMINATION METHOD, COMPLEX MATRIX CONSISTED WITH
C  EIGENVALUES IS PARTITIONED.
      DO 20 I=1,N2
      DO 10 J=1,N2
   10 A1(I,J)=EV(I)**(J-1)
   20 B1(I)=EV(I)**L
      DO 40 I=1,N2
      DO 30 J=1,N2
      A(I,J)=REAL(A1(I,J))
      A(I,J+N2)=-AIMAG(A1(I,J))
      A(I+N2,J)=AIMAG(A1(I,J))
      A(I+N2,J+N2)=REAL(A1(I,J))
   30 CONTINUE
      B(I)=REAL(B1(I))
      B(I+N2)=AIMAG(B1(I))
   40 CONTINUE
      CALL GAUS(A,B,X,N4,IERROR)
C  GENERATE THE COEFFICIENTS OF CHARACTERISTIC FUNCTION
      DO 50 I=1,N2
      ALFA(I)=CMPLX(X(I),X(I+N2))
   50 CONTINUE
C  CAYLEY-HAMILTON'S THEOREM
      DO 70 I=1,N2
      DO 70 J=1,N2
      FP(I,J,1)=0.0
      IF(I.EQ.J) FP(I,J,1)=1.
   70 CONTINUE
      DO 75 I=1,N2
      DO 75 J=1,N2
      FP(I,J,2)=F(I,J)
   75 CONTINUE
      NM2=N-2
      IF(NM1) 90,90,95
   95 DO 80 NP=3,N
      DO 85 I=1,N2
      DO 85 J=1,N2
      FP(I,J,NP)=0.0
      DO 85 M=1,N2
      FP(I,J,NP)=FP(I,J,NP)+FP(I,M,NP-1)*F(M,J)
   85 CONTINUE
   80 CONTINUE
   90 CONTINUE
      DO 100 I=1,N2
      DO 100 J=1,N2
      FL1(I,J)=CMPLX(0.0,0.0)
  100 CONTINUE
      DO 110 NP=1,N
      DO 120 I=1,N2
      DO 120 J=1,N2
      FL1(I,J)=FL1(I,J)+ALFA(NP)*FP(I,J,NP)
  120 CONTINUE
  110 CONTINUE
      DO 130 I=1,N2
      DO 130 J=1,N2
      FL(I,J)=REAL(FL1(I,J))
  130 CONTINUE
      RETURN
150   CONTINUE
      DO 152 I=1,N
      DO 152 J=1,N
  152 SF(I,J)=F(I,J)
      CALL EIGEN(SF,N,EV)
      DO 220 I=1,N
      DO 210 J=1,N
  210 A1(I,J)=EV(I)**(J-1)
  220 B1(I)=EV(I)**L
      DO 240 I=1,N
      DO 230 J=1,N
      A(I,J)=REAL(A1(I,J))
      A(I,J+N)=-AIMAG(A1(I,J))
      A(I+N,J)=AIMAG(A1(I,J))
      A(I+N,J+N)=REAL(A1(I,J))
  230 CONTINUE
      B(I)=REAL(B1(I))
      B(I+N)=AIMAG(B1(I))
  240 CONTINUE
      CALL GAUS(A,B,X,N2,IERROR)
      DO 250 I=1,N
      ALFA(I)=CMPLX(X(I),X(I+N))
  250 CONTINUE
      DO 270 I=1,N
      DO 270 J=1,N
      FP(I,J,1)=0.0
      IF(I.EQ.J) FP(I,J,1)=1.
  270 CONTINUE
      DO 275 I=1,N
      DO 275 J=1,N
      FP(I,J,2)=SF(I,J)
  275 CONTINUE
      NM2=N-2
      IF(NM2) 290,290,295
  295 DO 280 NP=3,N
      DO 285 I=1,N
      DO 285 J=1,N
      FP(I,J,NP)=0.0
      DO 285 M=1,N
      FP(I,J,NP)=FP(I,J,NP)+FP(I,M,NP-1)*SF(M,J)
  285 CONTINUE
  280 CONTINUE
  290 CONTINUE
      DO 300 I=1,N
      DO 300 J=1,N
      FL1(I,J)=CMPLX(0.0,0.0)
  300 CONTINUE
      DO 310 NP=1,N
      DO 320 I=1,N
      DO 320 J=1,N
      FL1(I,J)=FL1(I,J)+ALFA(NP)*FP(I,J,NP)
  320 CONTINUE
  310 CONTINUE
      DO 330 I=1,N
      DO 330 J=1,N
      FL(I,J)=REAL(FL1(I,J))
      FL(I+N,J+N)=REAL(FL1(I,J))
      FL(I+N,J)=0.0
      FL(I,J+N)=0.0
  330 CONTINUE
      RETURN
      END
      SUBROUTINE GAUS(A,B,X,N,IERROR)
      DIMENSION A(12,12),B(12),X(12)
C
C  THIS SUBROUTINE IS IN 'NUMERICAL ANAYSIS' BY L.W.JOHNSON AND R.D.
C  RIESS ,1977 BY ADDISON-WESLEY PUB.CO.
C  SUBROUTINE GAUS USES GAUSS ELIMINATION (WITHOUT PIVOTING) TO SOLVE
C  THE SYSTEM AX=B. THE CALLING PROGRAM MUST SUPPLY THE MATRIX A, THE
C  VECTOR B AND AN INTEGER N (WHERE A IS (NXN). ARRAYS A AND B ARE
C  DESTROYED IN GAUS. THE SOLUTION IS RETURNED IN X AND A FLAG,IERROR,
C  IS SET TO 1 IF A IS NON-SINGULAR AND IS SET TO 2 IF A IS SINGULAR.
C  TO GET MORE ACCURATE SOLUTION, CALL SUBROUTINE RESCOR AFTER GAUS.
C
      NM1=N-1
      DO 5 I=1,NM1
C
C  SEARCH FOR NON-ZERO PIVOT ELEMENT AND INTERCHANGE ROWS IF NECESSARY.
C  IF NO NON-ZERO PIVOT ELEMENT IS FOUND, SET IERROR=2 AND RETURN
C
      DO 3 J=I,N
      IF(A(J,I).EQ.0.) GO TO 3
      DO 2 K=I,N
      TEMP=A(I,K)
      A(I,K)=A(J,K)
    2 A(J,K)=TEMP
      TEMP=B(I)
      B(I)=B(J)
      B(J)=TEMP
      GO TO 4
    3 CONTINUE
      GO TO 8
C
C  ELIMINATE THE COEFFICIENTS OF X(I) IN ROWS I+1,...,N
C
    4 IP1=I+1
      DO 5 K=IP1,N
      Q=-A(K,I)/A(I,I)
      A(K,I)=0.0
      B(K)=Q*B(I)+B(K)
      DO 5 J=IP1,N
    5 A(K,J)=Q*A(I,J)+A(K,J)
      IF(A(N,N).EQ.0.) GO TO 8
C
C  BACKSOLVE THR EQUIVALENT TRIANGULARIZED SYSTEM, SET IERROR=1,
C  AND RETURN
C
      X(N)=B(N)/A(N,N)
      NPI=N+1
      DO 7 K=1,NM1
      Q=0.
      NMK=N-K
      DO 6 J=1,K
    6 Q=Q+A(NMK,NP1-J)*X(NP1-J)
    7 X(NMK)=(B(NMK)-Q)/A(NMK,NMK)
      IERROR=1
      RETURN
    8 IERROR=2
      RETURN
      END
      SUBROUTINE EIGEN(A,N,EVALUE)
      DIMENSION A(6,6),RR(6),RI(6),IANA(36),AT(36)
      COMPLEX CMPLX,EVALUE(6)
      DO 6 I=1,N
      DO 7 J=1,N
      K=N*(I-1)
    7 AT(J+K)=A(I,J)
    6 CONTINUE
      CALL HSBG(N,AT,N)
      CALL ATEIG(N,AT,RR,RI,IANA,N)
      DO 5 I=1,N
      EVALUE(I)=CMPLX(RR(I),RI(I))
C      WRITE(6,500)
C  500 FORMAT(5X,'THE EIGENVALUE IS')
C      WRITE(6,600)
C  600 FORMAT(10X,'REAL ROOT',15X,'IMAG ROOT')
C      WRITE(6,700) RR(I),RI(I)
C  700 FORMAT (5X,E15.6,14X,E15.6)
  5   CONTINUE
      RETURN
      END
C        SUBROUTINE HSBG                                                HSBG  40
C        PURPOSE                                                        HSBG  60
C           TO REDUCE A REAL MATRIX INTO UPPER ALMOST TRIANGULAR FORM   HSBG  70
C        USAGE                                                          HSBG  90
C           CALL HSBG(N,A,IA)                                           HSBG 100
C        DESCRIPTION OF THE PARAMETERS                                  HSBG 120
C           N      ORDER OF THE MATRIX                                  HSBG 130
C           A      THE INPUT MATRIX, N BY N                             HSBG 140
C           IA     SIZE OF THE FIRST DIMENSION ASSIGNED TO THE ARRAY    HSBG 150
C                  A IN THE CALLING PROGRAM WHEN THE MATRIX IS IN       HSBG 160
C                  DOUBLE SUBSCRIPTED DATA STORAGE MODE.  IA=N WHEN     HSBG 170
C                  THE MATRIX IS IN SSP VECTOR STORAGE MODE.            HSBG 180
C                                                                       HSBG 190
C        REMARKS                                                        HSBG 200
C           THE HESSENBERG FORM REPLACES THE ORIGINAL MATRIX IN THE     HSBG 210
C           ARRAY A.                                                    HSBG 220
C                                                                       HSBG 230
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  HSBG 240
C           NONE                                                        HSBG 250
C                                                                       HSBG 260
C        METHOD                                                         HSBG 270
C           SIMILARITY TRANSFORMATIONS USING ELEMENTARY ELIMINATION     HSBG 280
C           MATRICES, WITH PARTIAL PIVOTING.                            HSBG 290
C                                                                       HSBG 300
C        REFERENCES                                                     HSBG 310
C           J.H. WILKINSON - THE ALGEBRAIC EIGENVALUE PROBLEM -         HSBG 320
C           CLARENDON PRESS, OXFORD, 1965.                              HSBG 330
C                                                                       HSBG 340
C        ...............................................................HSBG 350
C                                                                       HSBG 360
      SUBROUTINE HSBG(N,A,IA)                                           HSBG 370
      DIMENSION A(36)
      L=N                                                               HSBG 400
      NIA=L*IA
      LIA=NIA-IA
C
C        L IS THE ROW INDEX OF THE ELIMINATION
C
   20 IF(L-3) 360,40,40
   40 LIA=LIA-IA
      L1=L-1
      L2=L1-1
C
C        SEARCH FOR THE PIVOTAL ELEMENT IN THE LTH ROW
C
      ISUB=LIA+L
      IPIV=ISUB-IA
      PIV=ABS(A(IPIV))
      IF(L-3) 90,90,50
   50 M=IPIV-IA
      DO 80 I=L,M,IA
      T=ABS(A(I))
      IF(T-PIV) 80,80,60
   60 IPIV=I
      PIV=T
   80 CONTINUE
   90 IF(PIV) 100,320,100
  100 IF(PIV-ABS(A(ISUB))) 180,180,120
C
C        INTERCHANGE THE COLUMNS
C
  120 M=IPIV-L
      DO 140 I=1,L
      J=M+I
      T=A(J)
      K=LIA+I
      A(J)=A(K)
  140 A(K)=T
C
C        INTERCHANGE THE ROWS
C
      M=L2-M/IA
      DO 160 I=L1,NIA,IA
      T=A(I)
      J=I-M
      A(I)=A(J)
  160 A(J)=T
C
C        TERMS OF THE ELEMENTARY TRANSFORMATION
C
  180 DO 200 I=L,LIA,IA
  200 A(I)=A(I)/A(ISUB)
C
C        RIGHT TRANSFORMATION
C
      J=-IA
      DO 240 I=1,L2
      J=J+IA
      LJ=L+J
      DO 220 K=1,L1
      KJ=K+J
      KL=K+LIA
  220 A(KJ)=A(KJ)-A(LJ)*A(KL)
  240 CONTINUE
C
C        LEFT TRANSFORMATION
C
      K=-IA
      DO 300 I=1,N
      K=K+IA                                                            HSBG1070
      LK=K+L1                                                           HSBG1080
      S=A(LK)                                                           HSBG1090
      LJ=L-IA                                                           HSBG1100
      DO 280 J=1,L2                                                     HSBG1110
      JK=K+J                                                            HSBG1120
      LJ=LJ+IA                                                          HSBG1130
  280 S=S+A(LJ)*A(JK)*1.0D0                                             HSBG1140
  300 A(LK)=S                                                           HSBG1150
C                                                                       HSBG1160
C        SET THE LOWER PART OF THE MATRIX TO ZERO                       HSBG1170
C                                                                       HSBG1180
      DO 310 I=L,LIA,IA                                                 HSBG1190
  310 A(I)=0.0                                                          HSBG1200
  320 L=L1                                                              HSBG1210
      GO TO 20                                                          HSBG1220
  360 RETURN                                                            HSBG1230
      END                                                               HSBG1240
C                                                                       ATEI  10
C     ..................................................................ATEI  20
C                                                                       ATEI  30
C        SUBROUTINE ATEIG                                               ATEI  40
C                                                                       ATEI  50
C        PURPOSE                                                        ATEI  60
C           COMPUTE THE EIGENVALUES OF A REAL ALMOST TRIANGULAR MATRIX  ATEI  70
C                                                                       ATEI  80
C        USAGE                                                          ATEI  90
C           CALL ATEIG(M,A,RR,RI,IANA,IA)                               ATEI 100
C                                                                       ATEI 110
C        DESCRIPTION OF THE PARAMETERS                                  ATEI 120
C           M      ORDER OF THE MATRIX                                  ATEI 130
C           A      THE INPUT MATRIX, M BY M                             ATEI 140
C           RR     VECTOR CONTAINING THE REAL PARTS OF THE EIGENVALUES  ATEI 150
C                  ON RETURN                                            ATEI 160
C           RI     VECTOR CONTAINING THE IMAGINARY PARTS OF THE EIGEN-  ATEI 170
C                  VALUES ON RETURN                                     ATEI 180
C           IANA   VECTOR WHOSE DIMENSION MUST BE GREATER THAN OR EQUAL ATEI 190
C                  TO M, CONTAINING ON RETURN INDICATIONS ABOUT THE WAY ATEI 200
C                  THE EIGENVALUES APPEARED (SEE MATH. DESCRIPTION)     ATEI 210
C           IA     SIZE OF THE FIRST DIMENSION ASSIGNED TO THE ARRAY A  ATEI 220
C                  IN THE CALLING PROGRAM WHEN THE MATRIX IS IN DOUBLE  ATEI 230
C                  SUBSCRIPTED DATA STORAGE MODE.                       ATEI 240
C                  IA=M WHEN THE MATRIX IS IN SSP VECTOR STORAGE MODE.  ATEI 250
C                                                                       ATEI 260
C        REMARKS                                                        ATEI 270
C           THE ORIGINAL MATRIX IS DESTROYED                            ATEI 280
C           THE DIMENSION OF RR AND RI MUST BE GREATER OR EQUAL TO M    ATEI 290
C                                                                       ATEI 300
C        SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED                  ATEI 310
C           NONE                                                        ATEI 320
C                                                                       ATEI 330
C        METHOD                                                         ATEI 340
C           QR DOUBLE ITERATION                                         ATEI 350
C                                                                       ATEI 360
C        REFERENCES                                                     ATEI 370
C           J.G.F. FRANCIS - THE QR TRANSFORMATION---THE COMPUTER       ATEI 380
C           JOURNAL, VOL. 4, NO. 3, OCTOBER 1961, VOL. 4, NO. 4, JANUARYATEI 390
C           1962.  J.H. WILKINSON - THE ALGEBRAIC EIGENVALUE PROBLEM -  ATEI 400
C           CLARENDON PRESS, OXFORD, 1965.                              ATEI 410
C                                                                       ATEI 420
C     ..................................................................ATEI 430
C                                                                       ATEI 440
      SUBROUTINE ATEIG(M,A,RR,RI,IANA,IA)                               ATEI 450
      DIMENSION A(36),RR(6),RI(6),PRR(2),PRI(2),IANA(36)
      INTEGER P,P1,Q                                                    ATEI 470
C                                                                       ATEI 480
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```
      E7=1.0E-8
      E6=1.0E-6
      E10=1.0E-10
      DELTA=0.5
      MAXIT=30
C
C        INITIALIZATION
C
      N=M
   20 N1=N-1
      IN=N1*IA
      NN=IN+N
      IF(N1) 30,1300,30
   30 NP=N+1
C
C        ITERATION COUNTER
C
      IT=0
C
C        ROOTS OF THE 2ND ORDER MAIN SUBMATRIX AT THE PREVIOUS
C        ITERATION
C
      DO 40 I=1,2
      PRR(I)=0.0
   40 PRI(I)=0.0
C
C        LAST TWO SUBDIAGONAL ELEMENTS AT THE PREVIOUS ITERATION
C
      PAN=0.0
      PAN1=0.0
C
C        ORIGIN SHIFT
C
      R=0.0
      S=0.0
C
C        ROOTS OF THE LOWER MAIN 2 BY 2 SUBMATRIX
C
      N2=N1-1
      IN1=IN-IA
      NN1=IN1+N
      N1N=IN+N1
      N1N1=IN1+N1
   60 T=A(N1N1)-A(NN)
      U=T*T
      V=4.0*A(N1N)*A(NN1)
      IF(ABS(V)-U*E7) 100,100,65
   65 T=U+V
      IF(ABS(T)-AMAX1(U,ABS(V))*E6) 67,67,68
   67 T=0.0
   68 U=(A(N1N1)+A(NN))/2.0
      V=SQRT(ABS(T))/2.0
      IF(T)140,70,70
   70 IF(U) 80,75,75
   75 RR(N1)=U+V
      RR(N)=U-V
      GO TO 130
   80 RR(N1)=U-V
      RR(N)=U+V
      GO TO 130
  100 IF(T)120,110,110
  110 RR(N1)=A(N1N1)
      RR(N)=A(NN)
      GO TO 130
  120 RR(N1)=A(NN)
      RR(N)=A(N1N1)
  130 RI(N)=0.0
      RI(N1)=0.0
      GO TO 160
  140 RR(N1)=U
      RR(N)=U
      RI(N1)=V
      RI(N)=-V
  160 IF(N2)1280,1280,180
C
C         TESTS OF CONVERGENCE
C
  180 N1N2=N1N1-IA
      RMOD=RR(N1)*RR(N1)+RI(N1)*RI(N1)
      EPS=E10*SQRT(RMOD)
      IF(ABS(A(N1N2))-EPS)1280,1280,240
  240 IF(ABS(A(NN1))-E10*ABS(A(NN))) 1300,1300,250
  250 IF(ABS(PAN1-A(N1N2))-ABS(A(N1N2))*E6) 1240,1240,260
  260 IF(ABS(PAN-A(NN1))-ABS(A(NN1))*E6)1240,1240,300
  300 IF(IT-MAXIT) 320,1240,1240
C
C         COMPUTE THE SHIFT
C
  320 J=1
      DO 360 I=1,2
      K=NP-I
      IF(ABS(RR(K)-PRR(I))+ABS(RI(K)-PRI(I))-DELTA*(ABS(RR(K))
     1    +ABS(RI(K)))) 340,360,360
  340 J=J+I
  360 CONTINUE
      GO TO (440,460,460,480),J
  440 R=0.0
      S=0.0
      GO TO 500
  460 J=N+2-J
      R=RR(J)*RR(J)
      S=RR(J)+RR(J)
      GO TO 500
  480 R=RR(N)*RR(N1)-RI(N)*RI(N1)
      S=RR(N)+RR(N1)
C
C         SAVE THE LAST TWO SUBDIAGONAL TERMS AND THE ROOTS OF THE
C         SUBMATRIX BEFORE ITERATION
C
  500 PAN=A(NN1)
      PAN1=A(N1N2)
      DO 520 I=1,2
      K=NP-I
      PRR(I)=RR(K)
  520 PRI(I)=RI(K)
C
C         SEARCH FOR A PARTITION OF THE MATRIX, DEFINED BY P AND Q
C
      P=N2
C     IPI=N1N2
      IF (N-3) 600,600,525
  525 IPI=N1N2
      DO 580 J=2,N2
      IPI=IPI-IA-1
      IF(ABS(A(IPI))-EPS) 600,600,530
  530 IPIP=IPI+IA
      IPIP2=IPIP+IA
      D=A(IPIP)*(A(IPIP)-S)+A(IPIP2)*A(IPIP+1)+R
      IF(D)540,560,540
  540 IF(ABS(A(IPI)*A(IPIP+1))*(ABS(A(IPIP)+A(IPIP2+1)-S)+ABS(A(IPIP2+2)
     1 )) -ABS(D)*EPS) 620,620,560
  560 P=N1-J
  580 CONTINUE                                                          ATEI1790
  600 Q=P                                                               ATEI1800
      GO TO 680                                                         ATEI1810
  620 P1=P-1                                                            ATEI1820
      Q=P1
      IF (P1-1) 680,680,650
  650 DO 660 I=2,P1
      IPI=IPI-IA-1                                                      ATEI1850
      IF(ABS(A(IPI))-EPS)680,680,660                                    ATEI1860
  660 Q=Q-1                                                             ATEI1870
C                                                                       ATEI1880
C        QR DOUBLE ITERATION                                            ATEI1890
C                                                                       ATEI1900
  680 II=(P-1)*IA+P                                                     ATEI1910
      DO 1220 I=P,N1                                                    ATEI1920
      II1=II-IA                                                         ATEI1930
      IIP=II+IA                                                         ATEI1940
      IF(I-P)720,700,720                                                ATEI1950
  700 IPI=II+1                                                          ATEI1960
      IPIP=IIP+1                                                        ATEI1970
C                                                                       ATEI1980
C        INITIALIZATION OF THE TRANSFORMATION                           ATEI1990
C                                                                       ATEI2000
      Q1=A(II)*(A(II)-S)+A(IIP)*A(IPI)+R                                ATEI2010
      Q2=A(IPI)*(A(IPIP)+A(II)-S)                                       ATEI2020
      Q3=A(IPI)*A(IPIP+1)                                               ATEI2030
      A(IPI+1)=0.0                                                      ATEI2040
      GO TO 780                                                         ATEI2050
  720 Q1=A(II1)                                                         ATEI2060
      Q2=A(II1+1)                                                       ATEI2070
      IF(I-N2)740,740,760                                               ATEI2080
  740 Q3=A(II1+2)                                                       ATEI2090
      GO TO 780                                                         ATEI2100
  760 Q3=0.0                                                            ATEI2110
  780 CAP=SQRT(Q1*Q1+Q2*Q2+Q3*Q3)                                       ATEI2120
      IF(CAP)800,860,800                                                ATEI2130
  800 IF(Q1)820,840,840                                                 ATEI2140
  820 CAP=-CAP                                                          ATEI2150
  840 T=Q1+CAP                                                          ATEI2160
      PSI1=Q2/T                                                         ATEI2170
      PSI2=Q3/T                                                         ATEI2180
      ALPHA=2.0/(1.0+PSI1*PSI1+PSI2*PSI2)                               ATEI2190
      GO TO 880                                                         ATEI2200
  860 ALPHA=2.0                                                         ATEI2210
      PSI1=0.0                                                          ATEI2220
      PSI2=0.0                                                          ATEI2230
  880 IF(I-Q)900,960,900                                                ATEI2240
  900 IF(I-P)920,940,920                                                ATEI2250
  920 A(II1)=-CAP                                                       ATEI2260
      GO TO 960                                                         ATEI2270
  940 A(II1)=-A(II1)                                                    ATEI2280
C        ROW OPERATION                                                  ATEI2300
C                                                                       ATEI2310
  960 IJ=II                                                             ATEI2320
      DO 1040 J=I,N                                                     ATEI2330
      T=PSI1*A(IJ+1)                                                    ATEI2340
      IF(I-N1)980,1000,1000                                             ATEI2350
  980 IP2J=IJ+2                                                         ATEI2360
      T=T+PSI2*A(IP2J)                                                  ATEI2370
 1000 ETA=ALPHA*(T+A(IJ))                                               ATEI2380
      A(IJ)=A(IJ)-ETA                                                   ATEI2390
      A(IJ+1)=A(IJ+1)-PSI1*ETA                                          ATEI2400
      IF(I-N1)1020,1040,1040                                            ATEI2410
 1020 A(IP2J)=A(IP2J)-PSI2*ETA                                          ATEI2420
 1040 IJ=IJ+IA                                                          ATEI2430
C                                                                       ATEI2440
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C       COLUMN OPERATION                                                ATEI2450
C                                                                       ATEI2460
      IF(I-N1)1080,1060,1060                                            ATEI2470
 1060 K=N                                                               ATEI2480
      GO TO 1100                                                        ATEI2490
 1080 K=I+2                                                             ATEI2500
 1100 IP=IIP-I                                                          ATEI2510
      DO 1180 J=Q,K                                                     ATEI2520
      JIP=IP+J                                                          ATEI2530
      JI=JIP-IA                                                         ATEI2540
      T=PSI1*A(JIP)                                                     ATEI2550
      IF(I-N1)1120,1140,1140                                            ATEI2560
 1120 JIP2=JIP+IA                                                       ATEI2570
      T=T+PSI2*A(JIP2)                                                  ATEI2580
 1140 ETA=ALPHA*(T+A(JI))                                               ATEI2590
      A(JI)=A(JI)-ETA                                                   ATEI2600
      A(JIP)=A(JIP)-ETA*PSI1                                            ATEI2610
      IF(I-N1)1160,1180,1180                                            ATEI2620
 1160 A(JIP2)=A(JIP2)-ETA*PSI2                                          ATEI2630
 1180 CONTINUE                                                          ATEI2640
      IF(I-N2)1200,1220,1220                                            ATEI2650
 1200 JI=II+3                                                           ATEI2660
      JIP=JI+IA                                                         ATEI2670
      JIP2=JIP+IA                                                       ATEI2680
      ETA=ALPHA*PSI2*A(JIP2)                                            ATEI2690
      A(JI)=-ETA                                                        ATEI2700
      A(JIP)=-ETA*PSI1                                                  ATEI2710
      A(JIP2)=A(JIP2)-ETA*PSI2                                          ATEI2720
 1220 II=IIP+1                                                          ATEI2730
      IT=IT+1                                                           ATEI2740
      GO TO 60                                                          ATEI2750
C       END OF ITERATION                                                ATEI2770
 1240 IF(ABS(A(NN1))-ABS(A(N1N2))) 1300,1280,1280                       ATEI2790
C                                                                       ATEI2800
C       TWO EIGENVALUES HAVE BEEN FOUND                                 ATEI2810
C                                                                       ATEI2820
 1280 IANA(N)=0                                                         ATEI2830
      IANA(N1)=2                                                        ATEI2840
      N=N2                                                              ATEI2850
      IF(N2)1400,1400,20                                                ATEI2860
C                                                                       ATEI2870
C       ONE EIGENVALUE HAS BEEN FOUND                                   ATEI2880
C                                                                       ATEI2890
 1300 RR(N)=A(NN)                                                       ATEI2900
      RI(N)=0.0                                                         ATEI2910
      IANA(N)=1                                                         ATEI2920
      IF(N1)1400,1400,1320                                              ATEI2930
 1320 N=N1                                                              ATEI2940
      GO TO 20                                                          ATEI2950
 1400 RETURN                                                            ATEI2960
      END                                                               ATEI2970
      SUBROUTINE MATMUL(IMOT,A,N,M,B,L,C,NA,MA,NB,MB,NC,MC)
      DIMENSION A(NA,MA),B(NB,MB),C(NC,MC)
C     A , B , C ARE GENERAL MATRIX
C     IF A X B =C, THEN IMOT IS 1
C     IF A X B'=C, THEN IMOT IS 2
      DO 1 I=1,N
      DO 1 J=1,L
      C(I,J)=0.0
      DO 1 K=1,M
      GO TO (2,3),IMOT
2     B1=B(K,J)
      GO TO 1
3     B1=B(J,K)
1     C(I,J)=C(I,J)+A(I,K)*B1
      RETURN
      END
      SUBROUTINE MATAS(IAOS,A,N,M,B,C,NA,MA)
      DIMENSION A(NA,MA),B(NA,MA),C(NA,MA)
C     IF A + B = C, THEN IAOS IS 1
C     IF A - B = C, THEN IAOS IS 2
      IF(IAOS.NE.1) GO TO 10
      DO 1 I=1,N
      DO 1 J=1,M
    1 C(I,J)=A(I,J)+B(I,J)
      RETURN
   10 DO 2 I=1,N
      DO 2 J=1,M
    2 C(I,J)=A(I,J)-B(I,J)
      RETURN
      END
      SUBROUTINE MATVEC(A,N,M,B,C,NA,MA)
      DIMENSION A(NA,MA),B(MA),C(NA)
      DO 1 I=1,N
      C(I)=0.0
      DO 1 J=1,M
1     C(I)=C(I)+A(I,J)*B(J)
      RETURN
      END
      SUBROUTINE VECAS(IAOS,A,B,C,N)
      DIMENSION A(N),B(N),C(N)
C     A , B , C ARE VECTORS
C     IF A + B = C, THEN IAOS IS 1
C     IF A - B = C, THEN IAOS IS 2
      IF(IAOS.NE.1)GO TO 10
      DO 1 I=1,N
    1 C(I)=A(I)+B(I)
      RETURN
   10 DO 2 I=1,N
    2 C(I)=A(I)-B(I)
      RETURN
      END
      SUBROUTINE MABCT(A,N,M,B,L,C,LL,D,NA,MA,NB,MB,NC,MC,ND,MD)
      DIMENSION A(NA,MA),B(NB,MB),C(NC,MC),D(ND,MD),AB(6,6)
      DO 10 I=1,N
      DO 10 J=1,L
      AB(I,J)=0.0
      DO 10 K=1,M
      AB(I,J)=AB(I,J)+A(I,K)*B(K,J)
   10 CONTINUE
      DO 20 I=1,N
      DO 20 J=1,LL
      D(I,J)=0.0
      DO 20 K=1,L
      D(I,J)=D(I,J)+AB(I,K)*C(J,K)
   20 CONTINUE
      RETURN
      END
```

APPENDIX B

THE MONTE-CARLO SIMULATION PROGRAM

```
C     **************************************************************
C     THIS PROGRAMMING IS CALLED PHASET.  ITS MAIN PURPOSE IS TO
C     ESTIMATE THE UNKNOWN PHASE USING THE PHASE LOCKED LOOP HAVING
C     A VERY NARROW BANDWIDTH.
C       PROGRAMMER
C       CHANGJUNE YOON
C       TEXAS A & M UNIVERSITY
C       START JUNE, 1978
C     **************************************************************
      COMMON/SAMPLE/NSPB,TB
      COMMON/PHASE/PHEES,PHEEO
      COMMON/GDB/ENODB,SJRDB
      DIMENSION HM0(2,2),HM1(2,2),VEST0(4,4),VEST1(4,4),XEST0(4)
     1,XEST1(4),VARIN0(2,2),VARIN1(2,2),V0(2),V1(2)
      DIMENSION GAIN0(4,2),GAIN1(4,2)
      REAL MEAN
      LOGICAL*1 STRNG(8)
      INTEGER*4 JTIME
      CALL ASSIGN(5,'SY:PHASET.DAT',13,'RDO','NC',1)
      CALL INPUT
      READ(5,1) NOCASE,NPRNT
    1 FORMAT(2I5)
      DO 2000 NCASE=1,NOCASE
      READ(5,2) NOSYM,ENODB
    2 FORMAT(I5,E15.6)
      KSMAX=NOSYM*NSPB
      CALL INIT(XJI,XJQ,XEST0,XEST1,VEST0,VEST1,XPI,XPQ,VCO
     1,ERROR,ERRORS,MEAN,VARANS)
      CALL GTIM(JTIME)
      CALL TIMASC(JTIME,STRNG)
      WRITE(6,7272) (STRNG(II),II=1,8)
 7272 FORMAT(1X,'START TIME IS ',8A1)
      WRITE(6,50)
   50 FORMAT(6X,2HIB,5X,5HERROR,14X,6HERRATE,11X,6HERRORS,12X
     1,6HERRATS,12X,16HPHEEO IN DEGREES,5X,17HMEAN AND VARIANCE)
      DO 1000 KS=1,KSMAX
      CALL SIGNAL(KS,BB,SI,SQ)
      CALL RFI(KS,XJI,XJQ,YI,YQ)
      CALL DATA(SI,SQ,YI,YQ,ZI,ZQ)
      CALL VCOUT(KS,ZI,ZQ,XPI,XPQ,VCO,MEAN,VARANS)
      CALL REFGEN(KS,0,FTR0,GTR0,HM0)
      CALL REFGEN(KS,1,FTR1,GTR1,HM1)
      CALL KALMAN(KS,ZI,ZQ,HM0,VEST0,XEST0,GAIN0,VARIN0,DET0,V0)
      CALL KALMAN(KS,ZI,ZQ,HM1,VEST1,XEST1,GAIN1,VARIN1,DET1,V1)
      CALL COST(KS,V0,VARIN0,DET0,SUM0)
      CALL COST(KS,V1,VARIN1,DET1,SUM1)
      CALL STAND(KS,ZI,ZQ,SUMS,FTR0,GTR0,FTR1,GTR1
     1,AFSK0,AFSK1,BFSK0,BFSK1,SFSK0,SFSK1)
      IB=1+IFIX((KS-.5)/NSPB)
      IF(MOD(KS,NSPB).NE.0) GO TO 1000
      CALL DDCOM(KS,SUM0,SUM1,XEST0,XEST1,BB,ERROR,ERRATE)
      CALL STDCOM(KS,SUMS,SFSK0,SFSK1,BB,ERRORS,ERRATS)
      PHEEOD=360.*PHEEO/(2.*4.*ATAN(1.))
      IF(MOD(IB,NPRNT).EQ.0) WRITE(6,100) IB,ERROR,ERRATE,ERRORS,ERRATS
     1,PHEEOD,MEAN,VARANS
  100 FORMAT(2X,I5,5E18.6,2E13.6)
 1000 CONTINUE
      CALL GTIM(JTIME)
      CALL TIMASC(JTIME,STRNG)
      WRITE(6,7273) (STRNG(II),II=1,8)
 7273 FORMAT(1X,'TIME IS ',8A1)
      REWIND 6
 2000 CONTINUE
      STOP
      END
C
      BLOCK DATA
      COMMON/SEED/IXS,JXS,IXJ1,JXJ1,IXJ2,JXJ2,IXN1,JXN1,IXN2,JXN2
      COMMON/SAMPLE/NSPB,TB
      COMMON/OPTION/NOS
      COMMON/DELAY/DELPHI,DELMEG
      COMMON/SIGMA/SIGMAJ,SIGMAN
      COMMON/PHASE/PHEES,PHEEO
      COMMON/COLORD/PHIDJ,PHIOJ,GAMDJ,GAMOJ
      COMMON/QDB/ENODB,SJRDB
      COMMON/PLLFLT/BNP,ESP,DELF
      COMMON/FREQJ/FJ
      COMMON/PHASIN/HO,P,Z,KI,KQ,PHASP,PHASQ
      REAL KI,KQ
      COMMON/TRACK/GAMMA(4,4),PHEE(4,4)
      INTEGER*2 IX1(2),JX1(2),IX2(2),JX2(2),IX3(2),JX3(2),IX4(2),JX4(2)
     1,IX5(2),JX5(2)
      INTEGER*4 IXS,JXS,IXJ1,JXJ1,IXJ2,JXJ2,IXN1,JXN1,IXN2,JXN2
      EQUIVALENCE (IXS,IX1),(JXS,JX1),(IXJ1,IX2),(JXJ1,JX2),(IXJ2,IX3)
     1,(JXJ2,JX3),(IXN1,IX4),(JXN1,JX4),(IXN2,IX5),(JXN2,JX5)
      DATA IX1,JX1/"136303,"053354,"041256,"141560/
      DATA IX2,JX2,IX3,JX3/"176303,"037702,"141236,"056407,
     1                     "125537,"103453,"055052,"032461/
      DATA IX4,JX4,IX5,JX5/"034313,"103400,"021165,"104262,
     1                     "072063,"122076,"016415,"041540/
      END
C
      SUBROUTINE INPUT
      COMMON/SAMPLE/NSPB,TB
      COMMON/OPTION/NOS
      COMMON/DELAY/DELPHI,DELMEG
      COMMON/PHASE/PHEES,PHEEO
      COMMON/FREQJ/FJ
      COMMON/QDB/ENODB,SJRDB
      COMMON/PLLFLT/BNP,ESP,DELF
      COMMON/PHASIN/HO,P,Z,KI,KQ,PHASP,PHASQ
      READ(5,1) NSPB,TB
      READ(5,1) NOS,DELPHI
      READ(5,2) ENODB,SJRDB
      PHEES=0.
C     INITIALIZE PHEES AS PHEEO
      PHEEO=0.
      READ(5,2) FJ,HO
      READ(5,3) BNP
    1 FORMAT(I5,E15.6)
    2 FORMAT(2E15.6)
    3 FORMAT(E15.6)
      RETURN
      END
C
      SUBROUTINE INIT(XJI,XJQ,XESTO,XEST1,VESTO,VEST1,XPI,XPQ,VCO
     1,ERROR,ERRORS,MEAN,VARANS)
      COMMON/SAMPLE/NSPB,TB
      COMMON/OPTION/NOS
      COMMON/DELAY/DELPHI,DELMEG
      COMMON/SIGMA/SIGMAJ,SIGMAN
      COMMON/QDB/ENODB,SJRDB
      COMMON/PLLFLT/BNP,ESP,DELF
      COMMON/FREQJ/FJ
      COMMON/COLORD/PHIDJ,PHIOJ,GAMDJ,GAMOJ
      COMMON/PHASE/PHEES,PHEEO
      COMMON/PHASIN/HO,P,Z,KI,KQ,PHASP,PHASQ
      REAL KI,KQ,MEAN
      COMMON/TRACK/GAMMA(4,4),PHEE(4,4)
      DIMENSION XESTO(4),XEST1(4),VESTO(4,4),VEST1(4,4)
      PI=4. *ATAN(1. )
      DELMEQ=DELPHI*2. *PI/TB
      IF(NOS.EQ.1) GO TO 10
      SUMF=0.
      DO 15 K=1,NSPB
   15 SUMF=SUMF+(SIN((K-.5)*TB*DELMEQ/NSPB))**2
      SIGMAN=SQRT(SUMF)*10. **(-ENODB/20. )
      GO TO 20
   10 CONSTP=SQRT(NSPB/2. )*ABS(SIN(DELPHI))
      SIGMAN=CONSTP*10. **(-ENODB/20. )
   20 SIGMAJ=10. **(-SJRDB/20. )/SQRT(2. )
C     GENERATE THE COLOURED NOISE PARAMETERS AND ITS BANDWIDTH
C          "    THE RHO-FILTER AND ITS BANDWIDTH
      T=TB/NSPB
      POLEJ=-2. *PI*FJ
      PHIDJ=EXP(POLEJ*T)
      GAM=(PHIDJ-1. )/POLEJ
      GAINK=1. /SQRT(GAM**2/(1. -PHIDJ**2))
      GAMDJ=GAINK*GAM
      PHIOJ=0.
      GAMOJ=0.
      BNJ=-POLEJ/4.
C
      BNR=BNP
      POLER=-4. *BNR
      PHIDR=EXP(POLER*T)
      GAM=(PHIDR-1. )/POLER
      GAINK=1. /SQRT(GAM**2/(1. -PHIDR**2))
      GAMDR=GAINK*GAM
      PHIOR=0.
      GAMOR=0.
      DO 50 I=1,4
      DO 50 J=1,4
      GAMMA(I,J)=0.
   50 PHEE(I,J)=0.
      GAMMA(1,1)=GAMDR
      GAMMA(2,2)=GAMDR
      GAMMA(3,3)=GAMDJ
      GAMMA(4,4)=GAMDJ
      PHEE(1,1)=PHIDR
      PHEE(2,2)=PHIDR
      PHEE(3,3)=PHIDJ
      PHEE(4,4)=PHIDJ
C     GENERATE THE PHASE ESTIMATOR PARAMETERS
C      A=2. *PI*ESP/360.
C      TANHO=SIN(A)/COS(A)
C      HO=(2. *PI*DELF)/TANHO
      KG=(8. /3. )*BNP
      Z=-(4. /3. )*BNP
      P=KG*Z/HO
      KI=P/Z
      PHASP=EXP(P*T)
      PHASG=(PHASP-1. )/P
C
C     INITIALIZATION
C
      XJI=0.
      XJG=0.
      XPG=0.
      XPI=1. /(KI*(P-Z))
      VCO=0.
      DO 60 I=1,4
      XESTO(I)=0.
   60 XEST1(I)=0.
      DO 65 I=1,4
      DO 65 J=1,4
      VESTO(I,J)=0.
      IF(I.EQ.J) VESTO(I,J)=1.
   65 CONTINUE
      DO 70 I=1,4
      DO 70 J=1,4
   70 VEST1(I,J)=VESTO(I,J)
      ERROR=0.
      ERRORS=0.
      PHEEO=0.
      MEAN=0.
      VARANS=0.
C
      WRITE(6,99) ENODB,SJRDB
   99 FORMAT(2X,6HENODB=,E13.6,5X,6HSJRDB=,E13.6,/)
      WRITE(6,100)  NOS,NSPB,TB,DELPHI,PHEES
  100 FORMAT(2X,4HNOS=,I2,5X,5HNSPB=,I5,5X,3HTB=,E13.6,5X
     1,7HDELPHI=,E13.6,5X,6HPHEES=,E13.6,/)
      WRITE(6,101) GAMDJ,PHIDJ,BNJ
  101 FORMAT(5X,6HGAMDJ=,E13.6,5X,6HPHIDJ=,E13.6,5X,4HBNJ=,E13.6)
      WRITE(6,102) GAMDR,PHIDR,BNR
  102 FORMAT(5X,6HGAMDR=,E13.6,5X,6HPHIDR=,E13.6,5X,4HBNR=,E13.6)
      WRITE(6,103) PHASG,PHASP,BNP
  103 FORMAT(5X,6HPHASG=,E13.6,5X,6HPHASP=,E13.6,5X,4HBNP=,E13.6)
      WRITE(6,105) HO,P,Z,KI,KQ
  105 FORMAT(2X,18HPARAMETERS IN VCO=,/,5X,5HH(0)=,E13.6,5X,2HP=,E13.6
     1,5X,2HZ=,E13.6,5X,3HKI=,E13.6,5X,3HKQ=,E13.6,///)
      REWIND 6
      RETURN
      END
C
      SUBROUTINE SIGNAL(KS,BB,SI,SQ)
      COMMON/SEED/IXS,JXS,IXJ1,JXJ1,IXJ2,JXJ2,IXN1,JXN1,IXN2,JXN2
      INTEGER*4 IXS,JXS,IXJ1,JXJ1,IXJ2,JXJ2,IXN1,JXN1,IXN2,JXN2
      COMMON/SAMPLE/NSPB,TB
      COMMON/OPTION/NOS
      COMMON/PHASE/PHEES,PHEEO
      COMMON/DELAY/DELPHI,DELMEG
      IF(MOD(KS-1,NSPB).NE.0) GO TO 10
      CALL RANC(IXS,JXS,QB)
      BB=AINT(QB+.5)
   10 C=1.-2*BB
      TK=(KS-.5)/NSPB
      TKMOD=(TK-IFIX(TK))*TB
      A=1.
      GO TO (1,2),NOS
    1 PHEEM=DELPHI*C
      GO TO 20
    2 PHEEM=DELMEG*C*TKMOD
   20 SI=A*COS(PHEEM+PHEES)
      SQ=A*SIN(PHEEM+PHEES)
      RETURN
      END
C
      SUBROUTINE RFI(KS,XJI,XJQ,YI,YQ)
      COMMON/SEED/IXS,JXS,IXJ1,JXJ1,IXJ2,JXJ2,IXN1,JXN1,IXN2,JXN2
      INTEGER*4 IXS,JXS,IXJ1,JXJ1,IXJ2,JXJ2,IXN1,JXN1,IXN2,JXN2
      COMMON/COLORD/PHIDJ,PHIOJ,GAMDJ,GAMOJ
      COMMON/SIGMA/SIGMAJ,SIGMAN
      REAL NI,NQ
      CALL MARSA(IXJ1,JXJ1,WI)
      CALL MARSA(IXJ2,JXJ2,WQ)
      CALL MARSA(IXN1,JXN1,NI)
      CALL MARSA(IXN2,JXN2,NQ)
      XJI1=PHIDJ*XJI+PHIOJ*XJQ+GAMDJ*WI+GAMOJ*WQ
      XJQ1=-PHIOJ*XJI+PHIDJ*XJQ-GAMOJ*WI+GAMDJ*WQ
      YI=SIGMAJ*XJI+SIGMAN*NI
      YQ=SIGMAJ*XJQ+SIGMAN*NQ
      XJI=XJI1
      XJQ=XJQ1
      RETURN
      END
C
      SUBROUTINE DATA(SI,SQ,YI,YQ,ZI,ZQ)
      COMMON/PHASE/PHEES,PHEEO
      ZI=SI+YI
      ZQ=SQ+YQ
      ZI=ZI*COS(PHEEO)+ZQ*SIN(PHEEO)
      ZQ=-ZI*SIN(PHEEO)+ZQ*COS(PHEEO)
      RETURN
      END
C
      SUBROUTINE VCOUT(KS,ZI,ZQ,XPI,XPQ,VCO,MEAN,VARANS)
      COMMON/SAMPLE/NSPB,TB
      COMMON/PHASE/PHEES,PHEEO
      COMMON/PHASIN/HO,P,Z,KI,KQ,PHASP,PHASQ
      REAL KI,KQ,MEAN
      XPQ1=PHASP*XPQ+PHASQ*ZQ
      ZQ1=KQ*((P-Z)*XPQ+ZQ)
      XPQ=XPQ1
      XPI1=PHASP*XPI+PHASQ*ZI
      ZI1=KI*((P-Z)*XPI+ZI)
      XPI=XPI1
      VCOP1=ZQ1/ZI1
      T=TB/NSPB
      PHEEO=PHEEO+(VCOP1+VCO)*T/2.
      VCO=VCOP1
C     ESTIMATE THE MEAN AND VARIANCE OF THE PHASE ERROR, RECURSIVELY
      PHEEOD=360.*PHEEO/(2.*4.*ATAN(1.))
      MEAN=((KS-1.)*MEAN+PHEEOD)/KS
      EVAR=(PHEEOD-MEAN)**2
      VARANS=((KS-1.)*VARANS+EVAR)/KS
      RETURN
      END
C
      SUBROUTINE REFGEN(KS,M,FTR,QTR,HM)
      COMMON/SAMPLE/NSPB,TB
      COMMON/DELAY/DELPHI,DELMEG
      COMMON/OPTION/NOS
      DIMENSION HM(2,2)
      TK=(KS-.5)/NSPB
      TKMOD=(TK-IFIX(TK))*TB
      AR=1.
      IF(NOS.NE.1) GO TO 1
      IF(M.EQ.0) PHEEMR=DELPHI
      IF(M.EQ.1) PHEEMR=-DELPHI
      GO TO 2
    1 IF(M.EQ.0) PHEEMR=DELMEG*TKMOD
      IF(M.EQ.1) PHEEMR=-DELMEG*TKMOD
    2 FTR=AR*COS(PHEEMR)
      QTR=AR*SIN(PHEEMR)
      HM(1,1)=COS(PHEEMR)
      HM(1,2)=SIN(PHEEMR)
      HM(2,1)=SIN(PHEEMR)
      HM(2,2)=-COS(PHEEMR)
      RETURN
      END
C
      SUBROUTINE KALMAN(KS,ZI,ZQ,HM,VEST,XEST,GAIN,VARINV,DET,V)
      COMMON/TRACK/GAMMA(4,4),PHEE(4,4)
      COMMON/SIGMA/SIGMAJ,SIGMAN
      DIMENSION VEST(4,4),PVP(4,4),QTQ(4,4),VPRED(4,4),VHT(4,2)
     1,HVHT(2,2),VAR(2,2),VARINV(2,2),GAIN(4,2),GH(4,4),HM(2,2)
```

```
      DIMENSION VNN(2,2)
      REAL IMGH(4,4)
      DIMENSION XEST(4),XPRED(4),HXPRED(2),V(2),GV(4),HX(2,4)
      DO 1 I=1,2
      DO 1 J=1,2
    1 HX(I,J)=HM(I,J)
      HX(1,3)=SIGMAJ
      HX(1,4)=0.
      HX(2,3)=0.
      HX(2,4)=SIGMAJ
      VNN(1,1)=SIGMAN**2
      VNN(1,2)=0.
      VNN(2,1)=0.
      VNN(2,2)=SIGMAN**2
C     CALCULATE THE STEADY-STATE KALMAN GAIN
      CALL MABCT(PHEE,4,4,VEST,4,PHEE,4,PVP,4,4,4,4,4,4,4,4)
      CALL MATMUL(2,GAMMA,4,4,GAMMA,4,GTG,4,4,4,4,4,4)
      CALL MATAS(1,PVP,4,4,GTG,VPRED,4,4)
      CALL MABCT(HX,2,4,VPRED,4,HX,2,HVHT,2,4,4,4,2,4,2,2)
      CALL MATAS(1,HVHT,2,2,VNN,VAR,2,2)
      DET=VAR(1,1)*VAR(2,2)-VAR(1,2)*VAR(2,1)
      VARINV(1,1)=VAR(2,2)/DET
      VARINV(1,2)=-VAR(1,2)/DET
      VARINV(2,1)=-VAR(2,1)/DET
      VARINV(2,2)=VAR(1,1)/DET
      CALL MATMUL(2,VPRED,4,4,HX,2,VHT,4,4,2,4,4,2)
      CALL MATMUL(1,VHT,4,2,VARINV,2,GAIN,4,2,2,2,4,2)
      CALL MATMUL(1,GAIN,4,2,HX,4,GH,4,2,2,4,4,4)
      DO 10 I=1,4
      DO 10 J=1,4
      IMGH(I,J)=-GH(I,J)
      IF(I.EQ.J) IMGH(I,J)=1.-GH(I,J)
   10 CONTINUE
      CALL MATMUL(1,IMGH,4,4,VPRED,4,VEST,4,4,4,4,4,4)
C
      CALL MATVEC(PHEE,4,4,XEST,XPRED,4,4)
      CALL MATVEC(HX,2,4,XPRED,HXPRED,2,4)
      V(1)=ZI-HXPRED(1)
      V(2)=ZQ-HXPRED(2)
      CALL MATVEC(GAIN,4,2,V,GV,4,2)
      CALL VECAS(1,XPRED,GV,XEST,4)
      RETURN
      END
C
      SUBROUTINE COST(KS,V,VARINV,DET,SUM)
      COMMON/SAMPLE/NSPB,TB
      DIMENSION V(2),VARINV(2,2)
      IF(MOD(KS-1,NSPB).EQ.0) SUM=0.
      ARG=-ALOG(DET)-(V(1)**2*VARINV(1,1)+V(2)**2*VARINV(2,2)
     1+V(1)*V(2)*(VARINV(1,2)+VARINV(2,1)))
      SUM=SUM+ARG
      RETURN
      END
C
      SUBROUTINE STAND(KS,ZI,ZQ,SUM,FTR0,GTR0,FTR1,GTR1
     1,AFSK0,AFSK1,BFSK0,BFSK1,SFSK0,SFSK1)
      COMMON/SAMPLE/NSPB,TB
      COMMON/OPTION/NOS
      GO TO (1,2),NOS
    1 IF(MOD(KS-1,NSPB).EQ.0) SUM=0.
      SUM=SUM+ZQ
      RETURN
    2 IF(MOD(KS-1,NSPB).NE.0) GO TO 20
      AFSK0=0.
      AFSK1=0.
      BFSK0=0.
      BFSK1=0.
   20 AFSK0=AFSK0+ZI*FTR0+ZQ*QTR0
      AFSK1=AFSK1+ZI*FTR1+ZQ*QTR1
      BFSK0=BFSK0+ZI*QTR0-ZQ*FTR0
      BFSK1=BFSK1+ZI*QTR1-ZQ*FTR1
      IF(MOD(KS,NSPB).NE.0) RETURN
      SFSK0=AFSK0**2+BFSK0**2
      SFSK1=AFSK1**2+BFSK1**2
      RETURN
      END
C
      SUBROUTINE DDCOM(KS,SUM0,SUM1,XEST0,XEST1,BB,ERROR,ERRATE)
      COMMON/SAMPLE/NSPB,TB
      DIMENSION XEST0(4),XEST1(4)
      IF(SUM0.GT.SUM1) GO TO 10
      BBHAT=1.
      DO 1 I=1,4
    1 XEST0(I)=XEST1(I)
      GO TO 20
   10 BBHAT=0.
      DO 2 I=1,4
    2 XEST1(I)=XEST0(I)
   20 IF(BB.EQ.BBHAT) ERR=0.
      IF(BB.NE.BBHAT) ERR=1.
      ERROR=ERROR+ERR
      IB=1+IFIX((KS-.5)/NSPB)
      ERRATE=ERROR/IB
      RETURN
      END
C
      SUBROUTINE STDCOM(KS,SUM,SFSK0,SFSK1,BB,ERROR,ERRATE)
      COMMON/SAMPLE/NSPB,TB
      COMMON/OPTION/NOS
      GO TO (1,2),NOS
    1 IF(SUM.GE.0.) BBHAT=0.
      IF(SUM.LT.0.) BBHAT=1.
      GO TO 10
    2 IF(SFSK0.GT.SFSK1) BBHAT=0.
      IF(SFSK1.GT.SFSK0) BBHAT=1.
   10 IF(BB.EQ.BBHAT) ERR=0.
      IF(BB.NE.BBHAT) ERR=1.
      IB=1+IFIX((KS-.5)/NSPB)
      ERROR=ERROR+ERR
      ERRATE=ERROR/IB
      RETURN
      END
C
      SUBROUTINE MARSA(IXA,JXA,V)
      INTEGER*4 IXA,JXA
      CALL RANC(IXA,JXA,X1)
      CALL RANC(IXA,JXA,X2)
      X1=(X1-.5)*2.
      X2=(X2-.5)*2.
    5 W=X1**2+X2**2
      IF(W.LE.1.) GO TO 10
      CALL RANC(IXA,JXA,X1)
      CALL RANC(IXA,JXA,X2)
      X1=(X1-.5)*2.
      X2=(X2-.5)*2.
      GO TO 5
   10 XX=X1*SQRT(-2.*ALOG(W)/W)
      V=X2*XX/X1
      RETURN
      END
```

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