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**ROTOR-BEARING DYNAMICS
TECHNOLOGY DESIGN GUIDE
PART IV CYLINDRICAL ROLLER BEARINGS**

SHAKER RESEARCH CORP.
BALLSTON LAKE, NEW YORK 12019

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INTERIM REPORT FOR PERIOD APRIL 1978 - AUGUST 1979

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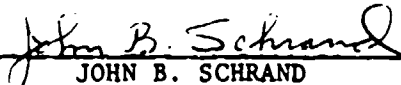
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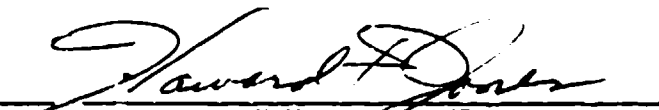
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FOREWORD

This report was prepared by Shaker Research Corporation under USAF Contract No. AF33615-76-C-2038. The contract was initiated under Project 3048, "Fuels, Lubrication, and Fire Protection," Task 304806, "Aerospace Lubrication," Work Unit 30480685, "Rotor-Bearing Dynamics Design."

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TABLE OF CONTENTS

<u>SECTION</u>	<u>PAGE</u>
I INTRODUCTION _____	1
II ANALYSIS _____	4
2.1 General Bearing Model and Coordinate System _____	4
2.2 General Bearing Support Characteristics _____	8
2.3 Cylindrical Roller Bearing Characterization _____	10
2.3.1 Stiffness _____	11
2.3.2 Damping _____	15
2.4 Cylindrical Roller Bearing Under Radial Loading _____	15
2.4.1 Bearing Applied Forces and Moments _____	16
2.4.2 Roller Geometry _____	16
III APPLICATION OF COMPUTER PROGRAM _____	35
3.1 Sample Test Case _____	35
3.2 Input Format _____	35
3.3 Output Format _____	35
APPENDIX COMPUTER PROGRAM FOR CALCULATING STIFFNESS MATRIX OF CYLINDRICAL ROLLER BEARING	43
REFERENCES	61

LIST OF ILLUSTRATIONS

<u>FIGURE</u>		<u>PAGE</u>
1a	Bearing Stiffness Model _____	5
1b	Bearing Location Coordinate System _____	5
2	Linearization of Cylindrical Roller Bearing Stiffness _____	9
3	Cylindrical Roller Bearing _____	12
4	Bearing Coordinate System _____	13
5	Cylindrical Roller Bearing Index, _____	14
6	Dimensions of Roller Profile and Crown _____	17
7	Forces and Moments Acting on Roller _____	20
8	Geometric Intersection of a Roller and Raceway _____	22
9	Sample Cylindrical Roller Bearing _____	36
10	Input Data Format _____	37
11	Sample Problem Data Input _____	38
12	Output Data for Sample Problem _____	39

NOMENCLATURE

<u>Symbol</u>		<u>Units</u>
b_x	Semi-width of contact ellipse at X	in.
B	Corner break	in.
B_{ij}	Damping component, change of force in i direction due to velocity in j direction; i = x, y, z; j = x, y, z.	$\frac{\text{lb-sec}}{\text{in}}$
$\underline{B}_N$	Damping matrix $\begin{bmatrix} (\underline{B}_N)_{\text{lineal}} & 0 & 0 \\ & 0 & 0 \\ 0 & 0 & (\underline{B}_N)_{\text{angular}} \\ 0 & 0 & \end{bmatrix}$	$\frac{\text{lb-sec}}{\text{in}}$
$(\underline{B}_N)_{\text{lineal}}$	Damping matrix due to lateral velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}}$	$\frac{\text{lb-sec}}{\text{in}}$
$(\underline{B})_{\text{angular}}$	Damping matrix due to angular velocities $\begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}}$	$\frac{\text{in-lb-sec}}{\text{radian}}$
C_i	A constant, $C_i = \begin{cases} 1 & \text{for } i = 1 \\ -1 & \text{for } i = 2 \end{cases}$	-
d	Roller diameter	in.
E	Pitch diameter	in.
E_E	Modulus of elasticity for roller body	lbs/in^2
E_R	Modulus of elasticity for race body	lbs/in^2

F_c	Roller centrifugal force	lbs.
F_i	External applied force, $i = x, y, z$	lbs.
F'_i	Reaction force, positive in direction opposite to displacements, $i = x, y, z$	lbs.
$\underline{F}$	Force Matrix = $\begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$	lbs.
G	Distance along roller element from extreme end of effective length to point where crown drop is measured	in.
H	Roller crown radius minus the rise of the arc at midpoint of effective length	in.
K	Roller-race stiffness	lbs/in
K_{ij}	Stiffness component, change of force in i direction due to displacement in j direction; $i = x, y, z$; $j = x, y, z$	lbs/in
$\underline{K}_N$	Stiffness matrix $\begin{bmatrix} (K_N)_{\text{lineal}} & 0 & 0 \\ & 0 & 0 \\ 0 & 0 & (K_N)_{\text{angular}} \\ 0 & 0 & \end{bmatrix}$	lbs/in
$(K)_{\text{lineal}}$	Stiffness matrix due to lateral displacements $\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}}$	lbs/in
$(K)_{\text{angular}}$	Stiffness matrix due to angular rotations $\begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{angular}}$	$\frac{\text{in-lb}}{\text{rad}}$

l_e	Effective length of roller load carrying surface	in.
l_F	Length of flat portion of roller measured along roller element	in.
l_T	Total length of roller measured parallel to roller axis	in.
m	Mass of roller	lbs.
M_i	External applied moment, $i = x, y, z$	lbs-in
M'_i	Reaction moment, $i = x, y, z$	lbs-in
M_1	Outer race/roller contact moment	lbs-in
M_2	Inner race/roller contact moment	lbs-in
n	Number of rollers	
N_1	Outer ring rotational speed	rpm
N_2	Inner ring rotational speed	rpm
p_x	Contact unit loading	lbs/in
p'_x	Current estimate of contact unit loading	lbs/in
P_D	Diametral clearance	in.
P_{1q}	Outer contact load on qth roller	lbs.
P_{2q}	Inner contact load on qth roller	lbs.
q	Roller position index	-
R_c	Roller crown radius	in.

$$\frac{W}{N} \quad \text{Column Matrix} = \begin{bmatrix} \delta_x \\ \delta_y \\ \theta_x \\ \theta_y \end{bmatrix}$$

x, y, z	Bearing coordinate system	in.
X, Y	Roller coordinate system	in.
x_o	Static component of displacement	in.
x'	Dynamic component of displacement	in.
x_{A_i}	Extremity of contact pattern measured parallel to roller axis to the left of the midpoint of the effective length	in.
x_{B_i}	Extremity of contact pattern measured parallel to roller axis to the right of the midpoint of the effective length	in.
x_A^*	Maximum permissible distance of contact pattern extremity from midpoint of effective length measured along race to the left	in.
x_B^*	Maximum permissible distance of contact pattern extremity from midpoint of effective length measured along race to the right	in.
$\underline{z}_N$	Impedance matrix = $\underline{K}_N + i \underline{v}_N^B$	

Other notations as defined in text.

GREEK SYMBOLS

γ	d/E	-
δ	Displacement	in.
δ_x	Linear displacement in x direction	in.
δ_y	Linear displacement in y direction	in.
δ_z	Linear displacement in z direction	in.
Δ	Approach of inner race to outer race at azimuth ϕ	in.
Δ_x	Approach of roller to race at X	in.
Δ_{1q}	Approach of roller to outer race (cup) at qth roller	in.
Δ_{2q}	Approach of roller to inner race (cup) at qth roller	in.
ϵ_i	Residues of simultaneous equations	-
η_E	Roller elastic constant = $\frac{4(1 - \frac{2}{E})}{E_E}$	$\frac{\text{in}^2}{\text{lb}}$
η_R	Race elastic constant = $\frac{4(1 - \frac{2}{R})}{E_R}$	$\frac{\text{in}^2}{\text{lb}}$
θ	Misalignment of inner race with respect to outer race	radians, ^o
θ_x	Angular rotation about x axis	radians, ^o
θ_y	Angular rotation about y axis	radians, ^o
θ_z	Angular rotation about z axis	radians, ^o
ν	Frequency of rotation	rad/sec
ν_E	Poisson's ratio for roller	
ν_R	Poisson's ratio for race	

ρ	Material density	lbs/in ³
ϕ	Circumferential roller position	radians, ^o
Ω_E	Orbital velocity of roller	rad/sec
Ω_1	Angular velocity of outer ring	rad/sec
Ω_2	Angular velocity of inner ring	rad/sec
v	Crown drop	in.
v'	Crown drop at distance G from roller extremity	in.

SUBSCRIPTS

<u>Symbol</u>	<u>Description</u>
b	Refers to bearing
E	Refers to roller
F	Refers to flat
i	Index, $i = 1, 2, 3$ or $i = x, y, z$
i,j	Refers to index of stiffness matrix; i.e., force in i direction due to displacement in j direction
p	Refers to pedestal
q	Refers to roller circumferential position
R	Refers to race
T	Refers to total
x	Refers to x direction
y	Refers to y direction
z	Refers to z direction
1	Refers to outer race
2	Refers to inner race

SECTION I

INTRODUCTION

The original Rotor-Bearing Dynamics Design Technology Series AFAPL-TR-65-45 (Parts I through X) included a volume, Part IV(1), which presented design data for typical deep-groove and angular contact ball bearings. The data was presented in graphical form and consisted of direct radial stiffness, load carrying capacity, and load levels. In addition design guidelines and limitations were discussed. The major deficiencies of this original volume were that centrifugal effects due to high speed were ignored, and axial and angular stiffness information were omitted.

Subsequent to the publication of Part IV, several extensive treatments of rolling element bearings including elastohydrodynamic, thermal, and cage effects have been published. The computer program of Mauriello, LaGasse, and Jones (2) considers both elastohydrodynamic and cage effects for ball bearings. The more recent computer based design guide prepared by Crecelius and Pirvics (3) treats elastohydrodynamic, thermal, and cage effects for a system of ball and roller bearings.

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- (1) Lewis, P. and Malanoski, S.B., "Rotor Bearing Dynamics Design Technology. Part IV: Ball Bearing Design Data Technical Report," AFAPL-TR-65-45, Part IV. Air Force Aero Propulsion Laboratory, Wright Patterson AFB, Ohio.
 - (2) Mauriello, J.A., LaGasse, and Jones, A.B., "Rolling Element Bearing Retainer Analysis," DAAJ02-69-C-0080, TR105.7.10, USAAMRDL-TR-72-45.
 - (3) Crecelius, W.T. and Pirvics, J., "Computer Program Operation Manual on "SHABERTH" a Computer Program for the Analysis of the Steady State and Transient Thermal Performance of Shaft Bearing Systems," AFAPL-TR-76-90, Air Force Aero Propulsion Laboratory, Wright Patterson AFB, Ohio, October 1976.

Thus, very sophisticated analytical tools are available for the design and application of rolling element bearings. Neither of these tools, however, provide the user with the stiffness matrix required for solution of rotor dynamics problems. In addition, both computer programs are very large and require an extensive computer facility for use.

Part II(4) of the revised series provided an update of the original Part IV (1). Those aspects of the original Part IV(1) which treated general design aspects of ball bearings, load capacity, speed limitations, etc. were deleted since their coverage is superficial compared to the more sophisticated computer tools now available (2,3). Only those parts directly connected with preparation of input for the rotordynamic response programs (Part I(5) of the revised series) were retained. The stiffness data included in the original Part IV were also updated. A later volume (Part III(6) of the revised series), enlarged the treatment to include the tapered roller bearing.

The present volume (Part IV of the revised series) extends the treatment of rolling element bearings to the cylindrical roller bearing. The complete stiffness matrix is calculated including centrifugal effects. Considerations

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- (4) Jones, A.B., and McGrew, J.M., "Rotor Bearing Dynamics Technology Design Guide--Part II: Ball Bearings," AFAPL-TR-78-6, Part II, February 1978, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.
 - (5) Pan, C.H.T., Wu, E.R., and Krauter, A.I., "Rotor Bearing Dynamics Technology Design Guide: Part I, Flexible Rotor Dynamics," AFAPL-TR-78-6, Part I, June 1978, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.
 - (6) Jones, A.B., and McGrew, J.M., "Rotor Bearing Dynamics Technology Design Guide - Part III: Tapered Roller Bearings," AFAPL-TR-78-6, Part III, February 1979, Air Force Aero Propulsion Laboratory, Wright Patterson Air Force Base, Ohio.

such as elastohydrodynamic and cage effects are not included since they have little influence on the calculation of cylindrical roller bearing stiffness. The resulting program (Appendix) is reasonably small and easy to use.

SECTION II

ANALYSIS

2.1 General Bearing Model and Coordinate System

Accurate calculation of the lateral dynamic response of a high-speed rotor depends on realistic characterization of the support bearings. In the most general case, both linear and angular motions are restrained by the support bearings at the attachment location. In the analytical model, the reaction force and the reaction moment of each bearing are felt by the rotor through a single station of the rotor axis. As schematically illustrated in Figure 1a, a coil spring restraining the lateral displacement and a torsion spring which tends to oppose an inclination are attached to the same point of the rotor axis. A complete description of the characteristics of the support bearings, however, involves much more than the specification of the two spring constants. This is because:

- . The lateral motion of the rotor axis is concerned with two displacement components and two inclination components.
- . The restraining characteristics may include cross coupling among various displacement/inclination coordinates.
- . The restraining force/moment may not be temporally in phase with the displacement/inclination.
- . The restraining characteristics of the bearing may be dependent on either the rotor speed or the frequency of vibration, or both.
- . Bearing pedestal compliance may not be negligible.

To accommodate the above considerations, the support bearing characteristics are described in Reference 5 by a four-degrees-of-freedom impedance matrix as defined in Equation (1):

$$\underline{R}_N = - \underline{Z}_N \cdot \underline{W}_N \quad (1)$$

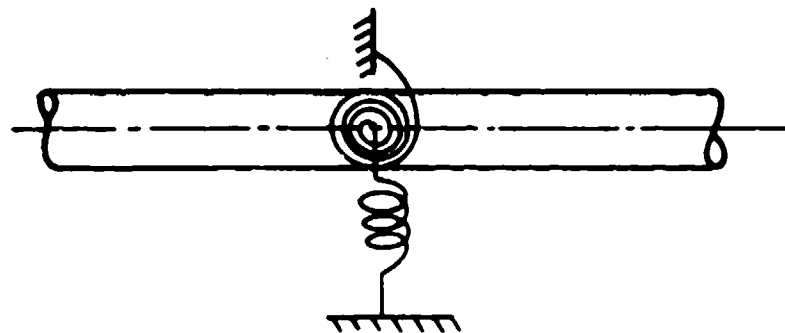


Figure 1a Bearing Stiffness Model

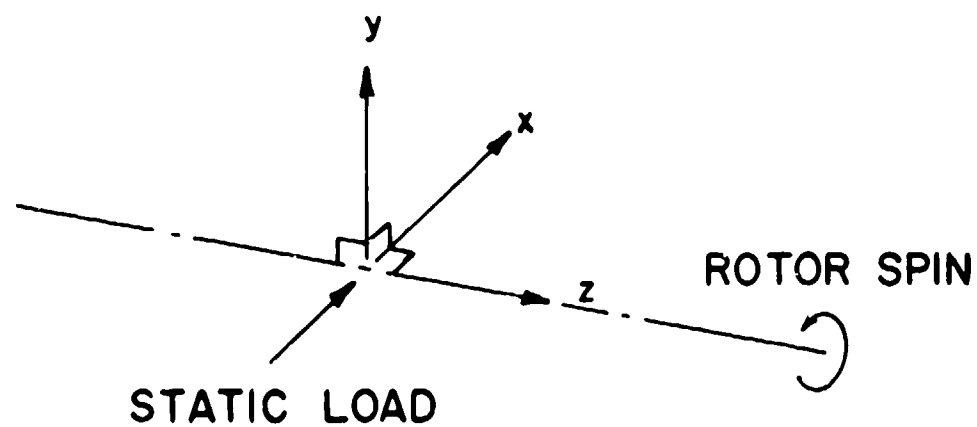


Figure 1b Bearing Location Coordinate System

where $\underline{W}_N$ is a column vector containing elements which are the two lateral displacements (δ_x, δ_y) and the two lateral inclinations (θ_x, θ_y) of the rotor axis at the bearing station N.

Employing a right-handed Cartesian representation in a lateral plane as depicted in Figure 1b, the z-axis is coincident with the spin vector of the rotor. The x-axis is oriented in the direction of the external static load, and the y-axis is perpendicular to both z and x axes forming the right-handed triad (x, y, z). (δ_x, δ_y) are respectively lateral lineal displacement components of the rotor axis along the (x, y) directions. (θ_x, θ_y) are lateral inclination components respectively in the (z-x, z-y) planes. Note that θ_y is a rotation about the y-axis, while θ_x is a rotation about the negative x-axis.

$\underline{Z}_N$ is a complex (4 x 4 matrix), and in accordance with the common notation for stiffness and damping coefficients, may be expressed as

$$\underline{Z}_N = \underline{K}_N + i\nu\underline{B}_N \quad (2)$$

where $\underline{K}_N$ is the stiffness matrix and $\underline{B}_N$ is the damping matrix. ν is the frequency of vibration. Most commonly, lateral lineal and angular displacements do not interact with each other so that the non-vanishing portions of $\underline{K}_N$ and $\underline{B}_N$ are separate 2 x 2 matrices. That is

$$\underline{K}_N = \begin{bmatrix} (\underline{K}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{K}_N)_{\text{angular}} \end{bmatrix} \quad (3)$$

$$\underline{B}_N = \begin{bmatrix} (\underline{B}_N)_{\text{lineal}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\underline{B}_N)_{\text{angular}} \end{bmatrix} \quad (4)$$

Accordingly, a total characterization of a support bearing would include sixteen coefficients which make up the 4 (2 x 2) matrices:

$$(\underline{K})_{\text{lineal}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{lineal}} \quad (5)$$

$$(\underline{B})_{\text{lineal}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{lineal}} \quad (6)$$

$$(\underline{K})_{\text{angular}} = \begin{bmatrix} K_{xx} & K_{xy} \\ K_{yx} & K_{yy} \end{bmatrix}_{\text{angular}} \quad (7)$$

$$(\underline{B})_{\text{angular}} = \begin{bmatrix} B_{xx} & B_{xy} \\ B_{yx} & B_{yy} \end{bmatrix}_{\text{angular}} \quad (8)$$

In the event that the pedestal compliance is significant, then the effective support impedance can be calculated from

$$\underline{Z}_N = (\underline{Z}_b^{-1} + \underline{Z}_p^{-1}) \quad (9)$$

where subscripts "p" and "b" refer to the pedestal and bearing respectively. Note that both pedestal inertia and damping may be included in $\underline{Z}_p$.

2.2 General Bearing Support Characteristics

The function of a bearing is to restrict the rotor axis to a nominal axis under realistic static and dynamic load environments. Deviation of any particular point of the rotor axis from the nominal line can be characterized by three lineal and two angular displacements. These may be designated as $(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)$ in accordance with a right-handed Cartesian reference system. The z-coordinate is coincident with the reference axis and is directed toward the spin vector. (θ_x, θ_y) are rotor axis inclinations respectively in the z-x and z-y planes. The x-coordinate is directed toward the predominant static load; e.g., earth gravity. Ideally, the bearing would resist the occurrence of any displacement so that the reaction force system imparted by the bearing to the rotor is generally expressed in matrix notation as

$$\underline{F} = \underline{Z} \cdot \underline{x} \quad (10)$$

$\underline{F}$ is a column vector comprising the five reaction components $(F_x, F_y, F_z, M_x, M_y)$, while $\underline{x}$ is the displacement vector $(\delta_x, \delta_y, \delta_z, \theta_x, \theta_y)$. $\underline{Z}$ is a (5×5) matrix containing the elements Z_{ij} with both indices (i, j) ranging from 1 to 5. The values of Z_{ij} characterize how rotor displacements are being resisted by the bearing.

From the standpoint of dynamic perturbation, distinction is made between a static equilibrium component and a dynamic perturbation component for both the displacements and the reactions. Thus,

$$\underline{x} = \underline{x}_0 + \underline{x}'; \quad \underline{F} = \underline{F}_0 + \underline{F}' \quad (11)$$

$(\underline{x}', \underline{F}')$ are respectively presumed to be infinitesimal in comparison with $(\underline{x}_0, \underline{F}_0)$. Accordingly, Z_{ij} are regarded as dependent on $\underline{x}_0$ but not on $\underline{x}'$. To illustrate the idea of perturbation linearization, one may examine the one-dimensional load-displacement curve shown in Figure 2.

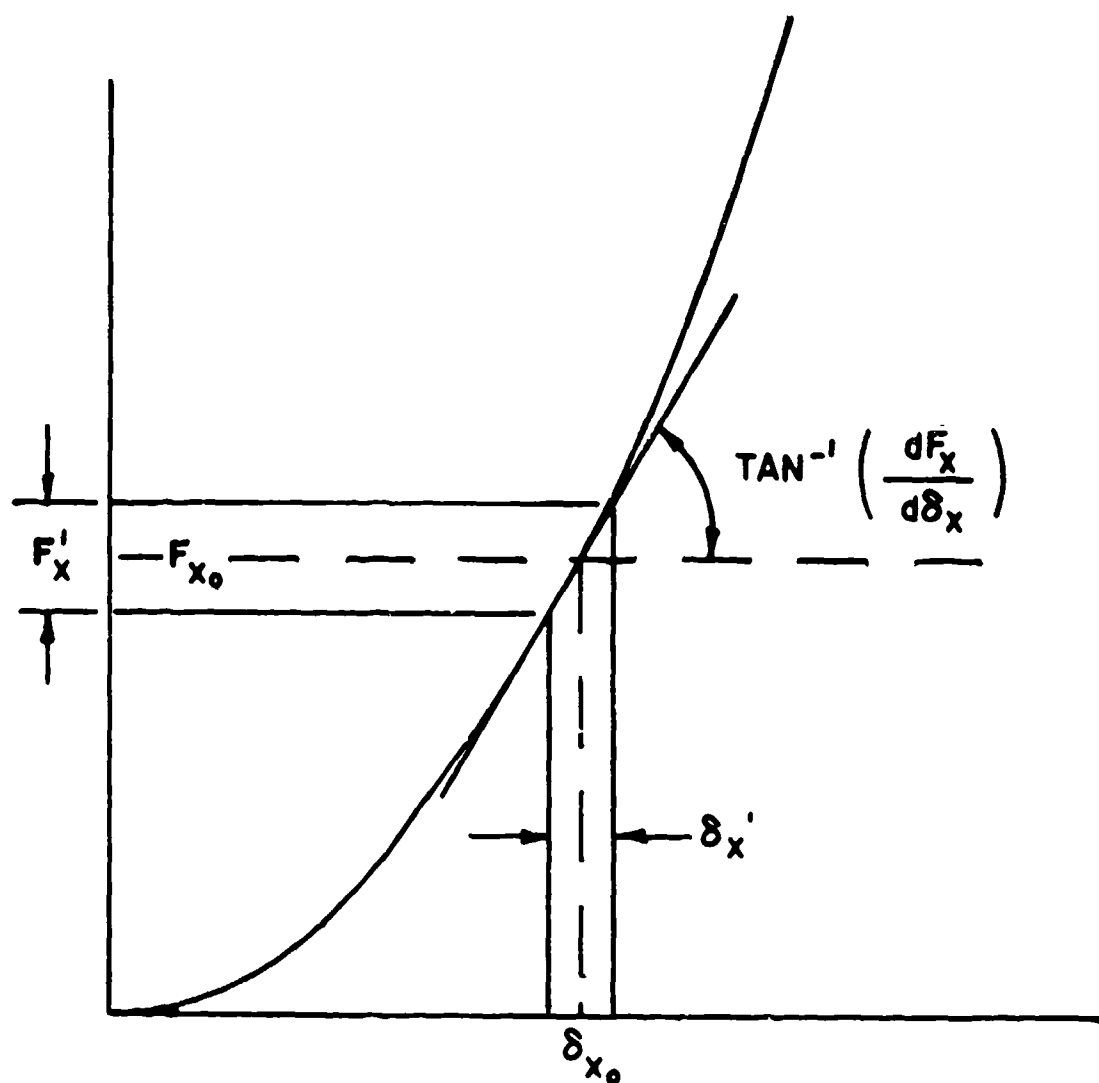


Figure 2 Linearization of Cylindrical Roller Bearing Stiffness

As illustrated, the load-displacement relationship is a 10/9 power law in accordance with the Hertzian point contact formula. It is not possible to describe the entire range by a linear approximation. However, if a small dynamic perturbation is taken around a static equilibrium point, $\delta'_x < \delta_{x_0}$, the small segment of the load-displacement curve can be approximated by a local tangent line. The corresponding force increment is

$$F'_x = \frac{\partial F_x}{\partial \delta_x} \delta'_x \quad (12)$$

where δ'_x is the incremental displacement. $\partial F_x / \partial \delta_x$ will depend on the amplitude of δ_{x_0} .

The question of history dependence is resolved by regarding x' as periodic motions at any frequency ν of interest, and Z_{ij} accordingly would have both real and imaginary parts and may also be dependent on both the rotor speed ω and the vibration frequency ν .

To avoid notational clumsiness, the primes will be dropped from (F', x') which are understood to be dynamic perturbation quantities unless the subscript "o" is used to designate the static equilibrium condition.

2.3 Cylindrical Roller Bearing Characterization

In many ways the cylindrical roller bearing is much simpler to model from a rotor dynamic point of view than a fluid film bearing. In general, the following two simplifications can be made:

- . The restraining characteristics do not include cross coupling among the various displacement/inclination coordinates.
- . The restraining force/moment is normally temporally in phase with the displacement/inclination.

Figure 3 illustrates a typical radial roller bearing having cylindrical rollers. In the bearing illustrated the rollers are guided by closely fitting flanges on the outer ring. Other constructions are common and all carry load in the manner to be described.

The bearing is referred to an orthogonal xyz coordinate system with the bearing axis directed along z. The bearing reacts to the linear displacements δ_x and δ_y and to the rotations or misalignments θ_x and θ_y . All are shown in their positive senses in Figure 4.

Figure 5 illustrates the roller index q.

2.3.1 Stiffness

The total characterization of a cylindrical roller bearing's stiffness can be expressed by the matrix.

$$[K] = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} & \frac{\partial F_x}{\partial z} & \frac{\partial F_x}{\partial \theta_x} & \frac{\partial F_x}{\partial \theta_y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} & \frac{\partial F_y}{\partial z} & \frac{\partial F_y}{\partial \theta_x} & \frac{\partial F_y}{\partial \theta_y} \\ \frac{\partial F_z}{\partial x} & \frac{\partial F_z}{\partial y} & \frac{\partial F_z}{\partial z} & \frac{\partial F_z}{\partial \theta_x} & \frac{\partial F_z}{\partial \theta_y} \\ \frac{\partial M_x}{\partial x} & \frac{\partial M_x}{\partial y} & \frac{\partial M_x}{\partial z} & \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial x} & \frac{\partial M_y}{\partial y} & \frac{\partial M_y}{\partial z} & \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{bmatrix} \quad (13)$$

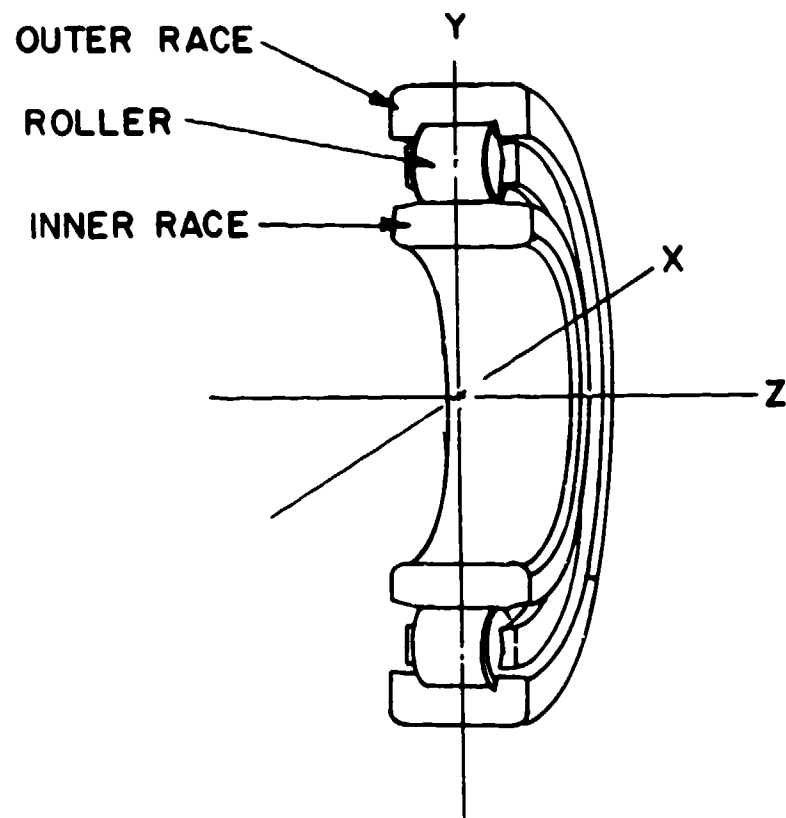


Figure 3. Cylindrical Roller Bearing

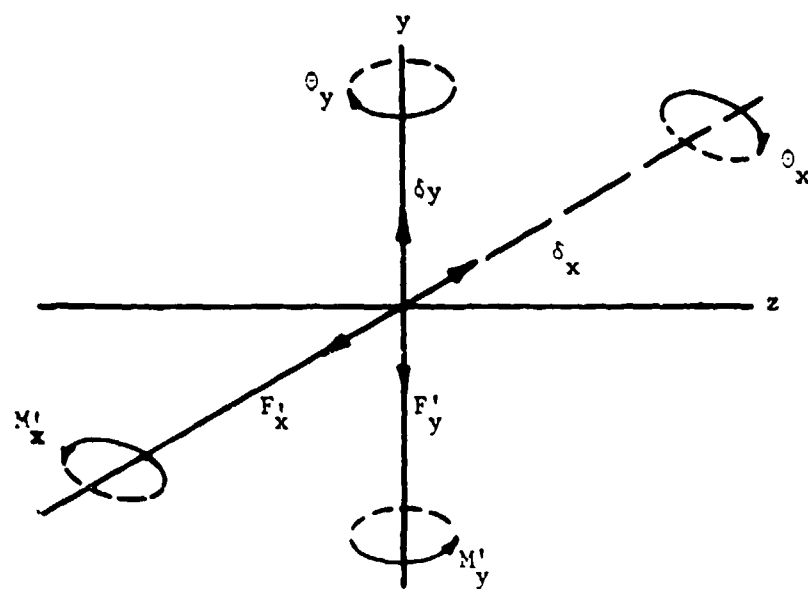


Figure 4. Bearing Coordinate System

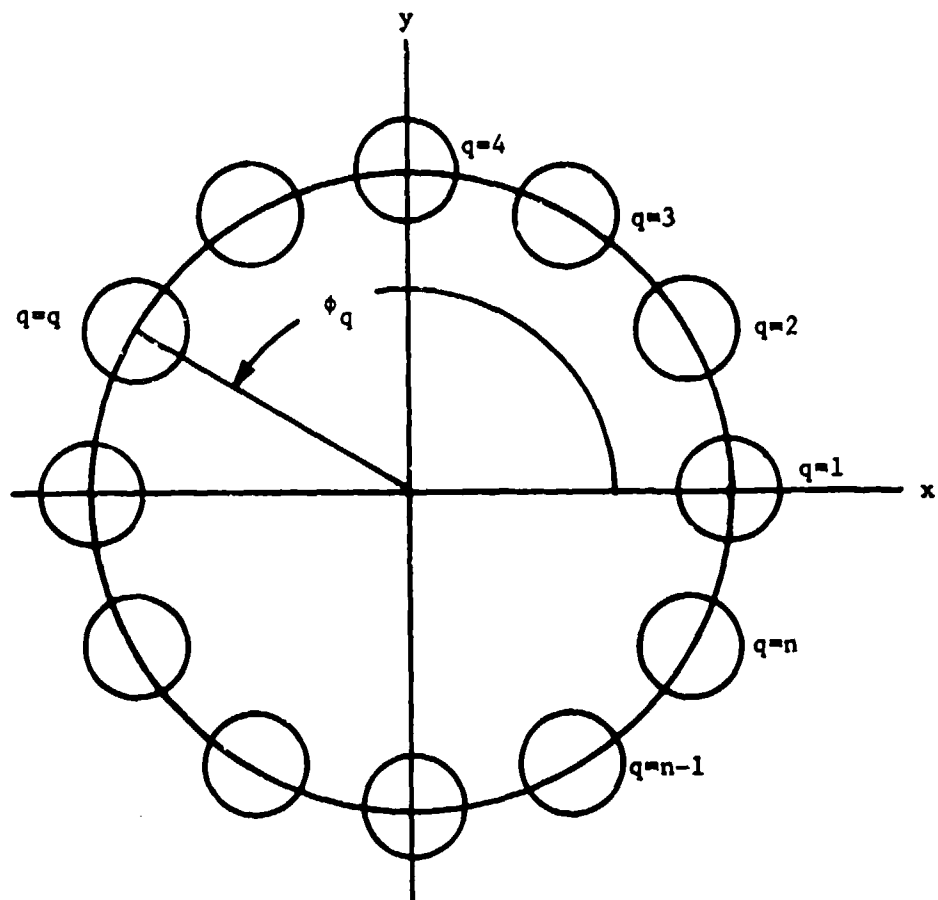


Figure 5. Cylindrical Roller Bearing Index, q

The lineal and angular stiffness matrices (Equations 5 and 7) can be derived from Equation (13). For example:

$$\begin{matrix} (K) \\ \text{lineal} \end{matrix} = \begin{bmatrix} \frac{\partial F_x}{\partial x} & \frac{\partial F_x}{\partial y} \\ \frac{\partial F_y}{\partial x} & \frac{\partial F_y}{\partial y} \end{bmatrix} \quad (14)$$

$$\begin{matrix} (K) \\ \text{angular} \end{matrix} = \begin{bmatrix} \frac{\partial M_x}{\partial \theta_x} & \frac{\partial M_x}{\partial \theta_y} \\ \frac{\partial M_y}{\partial \theta_x} & \frac{\partial M_y}{\partial \theta_y} \end{bmatrix} \quad (15)$$

Note that the axial components of stiffness are zero for a cylindrical roller bearing. However, these terms have been retained on the general cylindrical roller bearing stiffness matrix for consistency with the ball and tapered roller bearing analyses included in earlier volumes of this series (4,6).

2.3.2 Damping

An extensive search of the literature revealed no experimental damping data for cylindrical roller bearings. As the current state-of-the-art does not permit accurate calculation of cylindrical roller bearing damping, no damping data is included in this report.

2.4 Cylindrical Roller Bearing Under Radial Loading

Solution for the stiffness matrix of a cylindrical roller bearing under radial loading is a tedious problem and requires the use of a digital computer. In this section, the derivation of the solution is described. A computer program for obtaining the solution is included in the Appendix.

2.4.1 Bearing Applied Forces and Moments

In the present problem an external force is applied to the inner ring along x, only, and the inner ring is constrained to movement along x. There may, however, be initial displacements in other modes which are fixed and which result in reactions in other modes as will be shown.

As the result of the four displacements described previously in Figures 3 and 4, there are the reactions F'_x , F'_y , M'_x , and M'_y . F'_x and F'_y are forces. M'_x and M'_y are moments. All are shown in their positive sense in Figure 4. External force F_x is applied at the inner ring center. The senses of the signs are the same as for the reaction F'_x .

2.4.2 Roller Geometry

Figure 6 shows the boundary dimensions of a typical cylindrical roller. Roller mass, moment of inertia, and location of the center of gravity are calculated assuming the roller is a flat-ended, cylinder bounded by d and ℓ_T .

ℓ_e is the effective length of the roller and represents the maximum working length of the roller. The contact pattern between roller and race must lie within the effective length.

ℓ_T is the total length of the roller.

ℓ_F is the length of the flat portion of the roller profile and may be zero for a fully-crowned roller.

R_c is the roller crown radius and is struck from the central plane of the roller. ∇ is the drop of the crown measured at the extremities of the effective length. Thus we can write:

$$\nabla = H - \sqrt{R_c^2 - \left(\frac{\ell_e}{2}\right)^2} \quad (16)$$

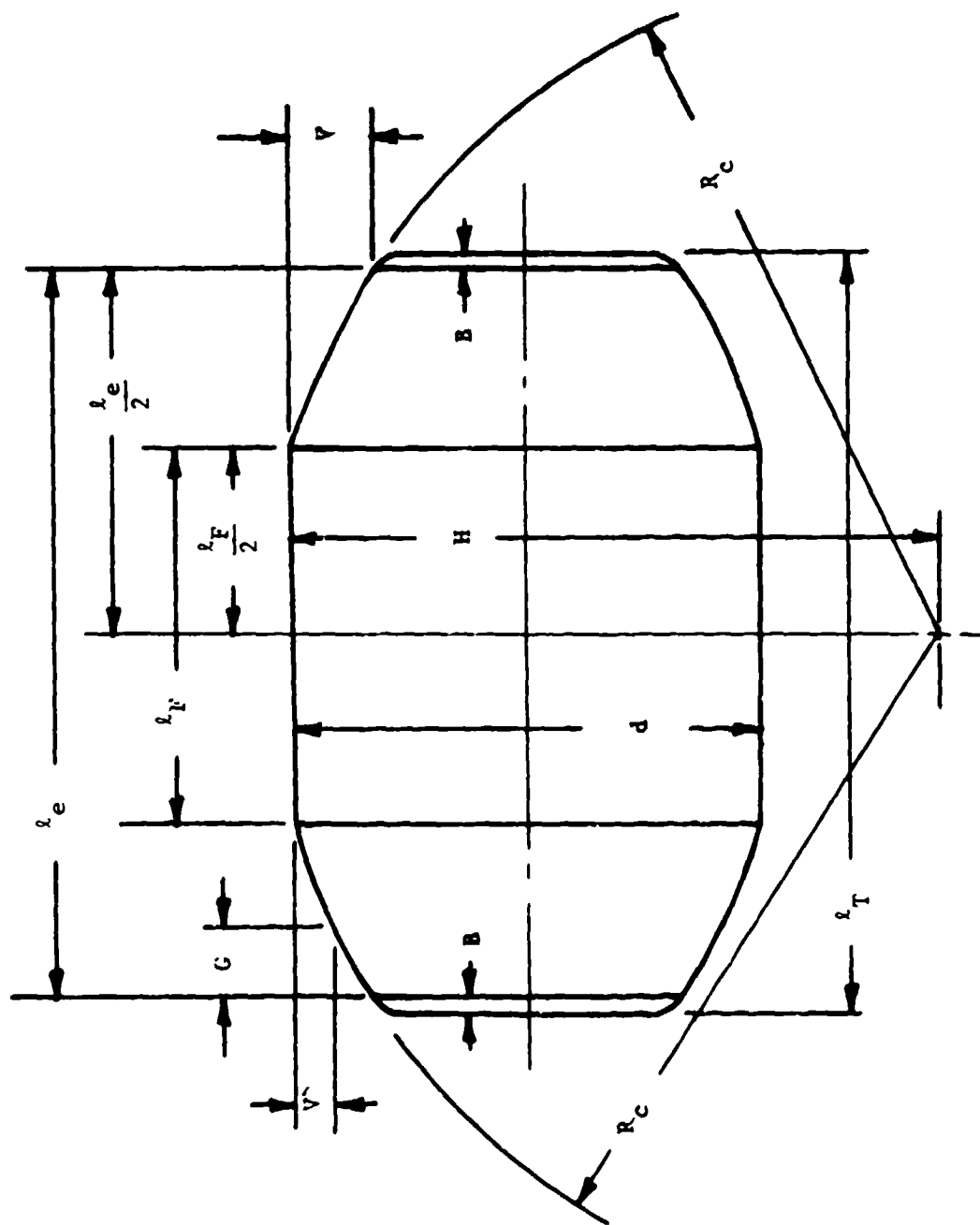


Figure 6. Dimensions of Roller Profile and Crown

where

$$H = \sqrt{R_c^2 - \left(\frac{l_F}{2}\right)^2} \quad (17)$$

The crown drop, ∇ , is used in checking the roller profile. If the crown radius is known, the drop at a distance G from the extremity of the effective length is

$$\nabla' = H - \sqrt{R_c^2 - \left(\frac{l_e}{2} - G\right)^2} \quad (18)$$

If the drop ∇' at G is known the crown radius is

$$R_c = \sqrt{\left[\frac{\left(\frac{l_e}{2} - G\right)^2 - \left(\frac{l_F}{2}\right)^2 - \nabla'^2}{2\nabla'} \right]^2 + \left(\frac{l_e}{2} - G\right)^2} \quad (19)$$

B is the corner break which is the same at both ends of the roll. The exact form of the corner break is unimportant as long as the corner blends smoothly into the roller crown. The effects of the corner breaks are neglected in calculating roller mass. The effect of the crown is also neglected.

The mass of the roller is

$$m = \frac{(l_e + 2B)d^2\rho}{491.98} \quad (20)$$

where ρ is the material density of the roller in lb/in³.

The centrifugal force acting on the roller is

$$F_c = \frac{mE\Omega_E^2}{2} \quad (21)$$

E is the pitch diameter of the bearing in inches. Ω_E is the orbital velocity of the roller and is

$$\Omega_E = \frac{\Omega_1(1+\gamma) + \Omega_2(1-\gamma)}{2} \quad (22)$$

where

$$\gamma = \frac{d}{E} \quad (23)$$

Ω_1 and Ω_2 are the angular velocities of outer and inner rings, respectively, in radians/sec.

The subscripts 1 and 2, used in Equation 7 and subsequently, refer to outer and inner contacts respectively.

Figure 7 shows the forces and moments acting on a roller which is in contact with both outer and inner races.

P_1 and P_2 are the contact loads. M_1 and M_2 are the contact moments resulting from non-uniform loading along the roller's length. F_c is the centrifugal force acting at the mass center of the roller.

In the present problem the radial roller bearing is loaded by a single force applied along x and working displacements are limited to a single deflection along x.

Assume, for the moment, that working displacements are possible along x and y and about x and y. Then the approach of inner and outer race for a roller at azimuth ϕ is

$$\Delta = (\delta_x + \delta_x'') \cos\phi + (\delta_y + \delta_y'') \sin\phi - \frac{P_D}{2} \quad (24)$$

where P_D is the diametral clearance.

The relative misalignment of the inner race with respect to the outer is

$$\theta = (\theta_x + \theta_x'') \sin\phi + (\theta_y + \theta_y'') \cos\phi \quad (25)$$

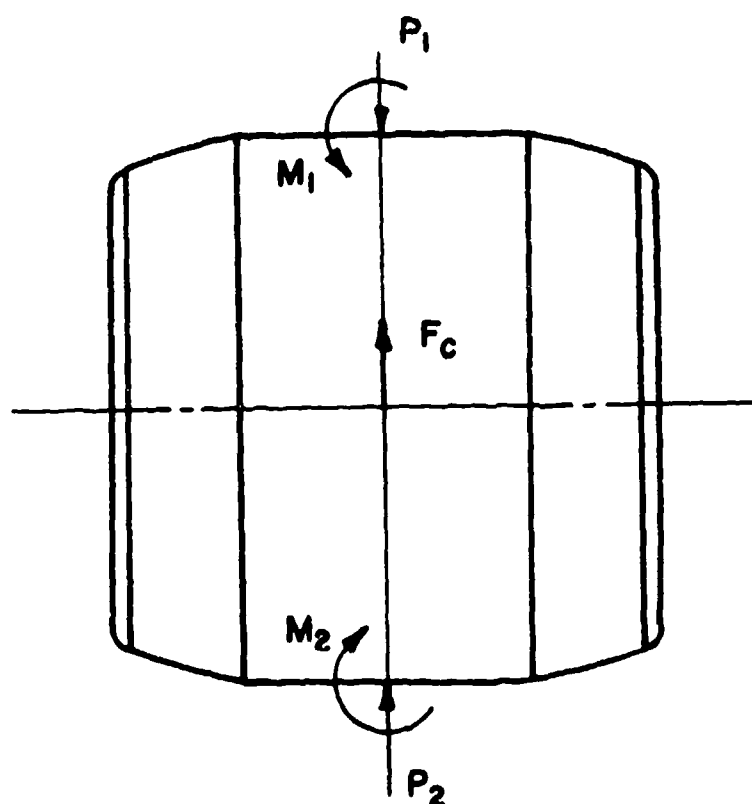


Figure 7. Forces and Moments Acting on Roller

The double primed items in Equations 24 and 25 are initial displacements. If initial displacements exist for a particular mode, working displacements in that mode are prevented.

For the present problem the linear displacement δ_y and the rotations θ_x and θ_y are non-existent and the approach and misalignment of the races are, for this case,

$$\Delta = (\delta_x + \delta_x'') \cos\phi + \delta_y'' \sin\phi - \frac{P_D}{2} \quad (26)$$

and

$$\theta = \theta_x'' \sin\phi + \theta_y'' \cos\phi \quad (27)$$

When θ is positive the left end of the roller, viewed with the axis horizontal, tends to be pinched.

If Δ_1 is the approach of the roller to the outer race, the approach of the roller to the inner race, at the midpoint of the roller is

$$\Delta_2 = \Delta - \Delta_1 \quad (28)$$

and the misalignment at the inner contact is

$$\theta_2 = \theta - \theta_1 \quad (29)$$

Figure 8 illustrates the geometric intersection of a roller and raceway.

The profiles of race and roller bodies are referred to an XY coordinate system. Note that the X axis is positive to the left of the origin.

The equation of the race surface is

$$Y = 0 \quad (30)$$

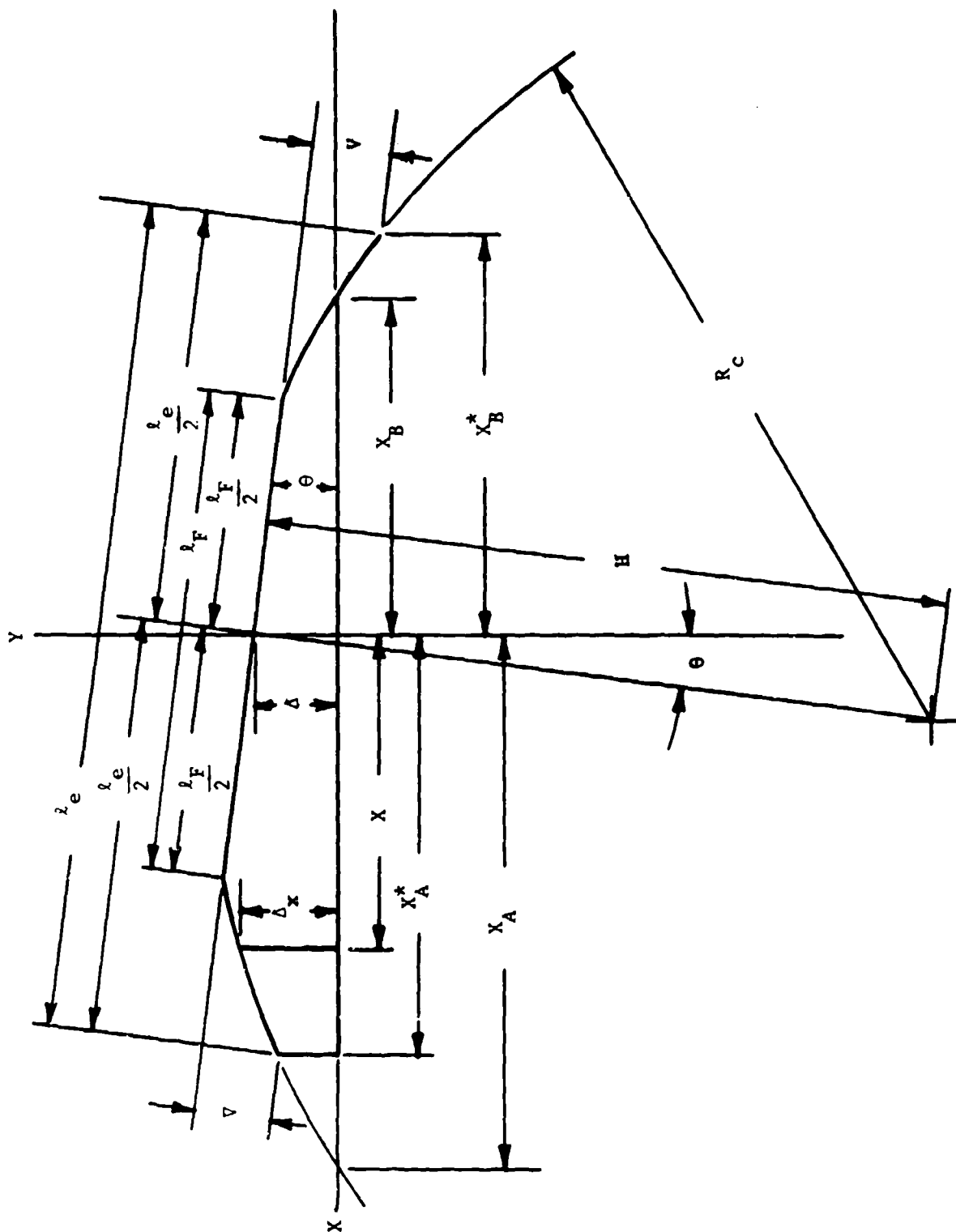


Figure 8. Geometric Intersection of a Roller and Raceway

The equation of the flat portion of the roller is

$$Y = \Delta_i + X \tan \theta_i \quad (31)$$

The equation of the crowned portion of the roller is

$$(X - H \sin \theta_i)^2 + (Y + H \cos \theta_i - \Delta_i)^2 = R_c^2 \quad (32)$$

In the above the subscript i is 1 for an outer contact and 2 for an inner contact.

The intersections of the race and the crowned surface of the roller occur at X_{A_i} and X_{B_i} .

$$x_{A_i} = \sqrt{R_c^2 - (H \cos \theta_i - \Delta_i)^2} + H \sin \theta_i \quad (33)$$

$$x_{B_i} = -\sqrt{R_c^2 - (H \cos \theta_i - \Delta_i)^2} + H \sin \theta_i \quad (34)$$

X_{A_i} and X_{B_i} must be within the projected extremities of the roller crown.

That is

$$X_{A_i} \leq X_{A_i}^* \quad (35)$$

$$X_{B_i} \geq X_{B_i}^* \quad (36)$$

where

$$X_{A_1}^* = \frac{l_e}{2} \cos \theta_1 + \sqrt{v \sin \theta_1} \quad (37)$$

$$X_{B_1}^* = \frac{l_e}{2} \cos \theta_1 + \sqrt{v \sin \theta_1} \quad (38)$$

If the quantity under the radical in Equations 33 and 34 is zero or negative there is no contact between roll and race.

If $\frac{l_e}{2} \cos \theta_1 \geq X_{A_1}$ there is also no contact.

If $X_{A_1} > X_{A_1}^*$, X_{A_1} is set equal to $X_{A_1}^*$.

If $X_{B_1} < X_{B_1}^*$, X_{B_1} is set equal to $X_{B_1}^*$.

If $\frac{l_F}{2} \cos \theta_1 > X_{B_1} > -\frac{l_F}{2} \cos \theta_1$ and

$X_{A_1} > \frac{l_F}{2} \cos \theta_1$ the value of X_{B_1} is

$$X_{B_1} = -\frac{\Delta_1}{\tan \theta_1} \quad (39)$$

From Figure 7 the conditions for roller equilibrium are

$$-P_1 + P_2 + F_c = 0 \quad (40)$$

$$M_1 - M_2 = 0 \quad (41)$$

Equations 40 and 41 are a set of non-linear simultaneous equations in which the variables are Δ_1 and θ_1 at the outer contact of the particular roller under consideration.

From Figure 8, the intrusion of the roller into the raceway is

$$\Delta_x = \Delta_1 + X \tan \theta_1 \quad |X| \leq \frac{l_F}{2} \cos \theta_1 \quad (42)$$

$$\Delta_x = -\sqrt{R_c^2 - (X - H \sin \theta)^2} - H \cos \theta_1 + \Delta_1 \quad |X| > \frac{l_F}{2} \cos \theta_1 \quad (43)$$

The derivatives of Δ_x with respect to θ_1 will be required later. They are

$$\frac{d\Delta_x}{d\theta_1} = \frac{X}{\cos^2 \theta_1} \quad |X| \leq \frac{l_F}{2} \cos \theta_1 \quad (44)$$

$$\frac{d\Delta_x}{d\theta_1} = \frac{(X - H \sin \theta_1) H \cos \theta_1}{\sqrt{R_c^2 - (X - H \sin \theta_1)^2}} + H \sin \theta_1 \quad |X| > \frac{l_F}{2} \cos \theta_1 \quad (45)$$

Lundberg (7) gives the approach Δ_x of two cylindrical bodies pressed together with the uniform loading p_x as

$$\Delta_x = \frac{(\eta_R + \eta_E)}{2\pi} p_x \left[1.8864 + \ln \left(\frac{X_A - X_B}{2b_x} \right) \right] \quad (46)$$

η_R and η_E are elastic constants for race and roller, or respectively, having the form

$$\eta_R = \frac{(1 - \nu_R^2)}{E_R} \quad \text{or} \quad \eta_E = \frac{(1 - \nu_E^2)}{E_E} \quad (47)$$

where ν is Poisson's ratio and $E_{R,E}$ is the modulus of elasticity.

b_x is the semi-width of the pressure area in the rolling direction

7. G. Lundberg: Elastische Berührung zweier Halbraume, VDI Forschung, Sept./Oct., 1939.

$$b_x = \left[\frac{(\eta_R + \eta_E)}{2\pi} p_x d (1 + C_1 \gamma_1) \right]^{1/2} \quad (48)$$

C_1 is 1 for an outer contact and -1 for an inner contact. γ is given by Equation 23.

The value of p_x corresponding to Δ_x is required. It cannot be obtained from Equation 46 in closed form. It can be obtained numerically as follows.

Let p'_x be an estimate of p_x . A good starting value is

$$p'_x = \frac{5 \times 10^{7.10/9} \Delta_x}{(X_A - X_B)^{1/9}} \quad (49)$$

An improved value of p'_x is

$$p_x = p'_x - \frac{(\Delta'_x - \Delta_x)}{\frac{d\Delta'_x}{dp'_x}} \quad (50)$$

Δ'_x is the approach of race and roller bodies calculated for the current estimate p'_x using Equation 46.

$\frac{d\Delta'_x}{dp'_x}$ is obtained from Equations 46 and 48 using the current estimate p'_x and is

$$\frac{d\Delta'_x}{dp'_x} = \frac{(\eta_R + \eta_E)}{2\pi} \left[1.3864 + \ln \left(\frac{X_A - X_B}{2b_x} \right) \right] \quad (51)$$

Iteration of Equation 50 yields p_x to any desired accuracy.

The contact force P and the moment M are

$$P_1 = \int_{x_{B_1}}^{x_{A_1}} p_x dx \quad (52)$$

$$M_1 = \int_{x_{B_1}}^{x_{A_1}} p_x dx \quad (53)$$

Equations 40 and 41 may now be solved numerically for Δ_1 and θ_1 .

If estimates are made of the variables Δ_1 and θ_1 Equations 40 and 41 may not be satisfied and there will be the residues ϵ_1 and ϵ_2 for Equations 40 and 41 respectively. Differentiating Equations 40 and 41 gives:

$$\frac{d\epsilon_1}{d\Delta_1} = -\frac{dP_1}{d\Delta_1} + \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} \quad (54)$$

$$\frac{d\epsilon_1}{d\theta_1} = -\frac{dP_1}{d\theta_1} + \frac{dP_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} \quad (55)$$

$$\frac{d\epsilon_2}{d\Delta_1} = \frac{dM_1}{d\Delta_1} - \frac{dM_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta_1} \quad (56)$$

$$\frac{d\epsilon_2}{d\theta_1} = \frac{dM_1}{d\theta_1} - \frac{dM_2}{d\theta_2} \frac{d\theta_2}{d\theta_1} \quad (57)$$

where

$$\frac{d\Delta_2}{d\Delta_1} = -1 \quad (58)$$

$$\frac{d\theta_2}{d\theta_1} = -1 \quad (59)$$

If Δ'_1 and θ'_1 are current estimates of the variables, improved values are Δ_1 and θ_1

$$\Delta_1 = \Delta'_1 - \frac{\begin{vmatrix} \epsilon_1 & \frac{d\epsilon_1}{d\theta_1} \\ \epsilon_2 & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_2}{d\theta_1} \end{vmatrix}} \quad (60)$$

$$\theta_1 = \theta'_1 - \frac{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \epsilon_1 \\ \frac{d\epsilon_2}{d\Delta_1} & \epsilon_2 \end{vmatrix}}{\begin{vmatrix} \frac{d\epsilon_1}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \\ \frac{d\epsilon_2}{d\Delta_1} & \frac{d\epsilon_1}{d\theta_1} \end{vmatrix}} \quad (61)$$

The right members of Equations 60 and 61 are evaluated at current estimates.

Iteration of Equations 60 and 61 yields Δ_1 and θ_1 to any desired accuracy.

The derivatives of P_1 and M_1 with respect to Δ_1 and θ_1 are

$$\frac{dP_1}{d\Delta_1} = \int_{X_{B_1}}^{X_{A_1}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Delta_1} dx \quad (62)$$

$$\frac{dP_1}{d\theta_1} = \int_{X_{B_1}}^{X_{A_1}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\theta_1} dx \quad (63)$$

$$\frac{dM_1}{d\Delta_1} = \int_{X_{B_1}}^{X_{A_1}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\Delta_1} dx \quad (64)$$

$$\frac{dM_1}{d\theta_1} = \int_{X_{B_1}}^{X_{A_1}} \frac{dp_x}{d\Delta_x} \frac{d\Delta_x}{d\theta_1} dx \quad (65)$$

The value of $\frac{dp_x}{d\Delta_x}$ is obtained from Equation 51 and the value of $\frac{d\Delta_x}{d\Delta_1}$ is unity.

If Equations 27, 28, 40 and 41 are differentiated with respect to Δ , there results four simultaneous equations which are linear in

$\frac{d\Delta_1}{d\Delta}$, $\frac{d\Delta_2}{d\Delta}$, $\frac{d\theta_1}{d\Delta}$, and $\frac{d\theta_2}{d\Delta}$ and from which the latter can be obtained.

Only $\frac{d\Delta_1}{d\Delta}$ and $\frac{d\theta_1}{d\Delta}$ are of interest here.

$$-\frac{dP_1}{d\Delta_1} \frac{d\Delta_1}{d\Delta} + \frac{dP_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta} - \frac{dP_1}{d\Theta_1} \frac{d\Theta_1}{d\Delta} + \frac{dP_2}{d\Theta_2} \frac{d\Theta_2}{d\Delta} = 0 \quad (66)$$

$$\frac{dM_1}{d\Delta_1} \frac{d\Delta_1}{d\Delta} - \frac{dM_2}{d\Delta_2} \frac{d\Delta_2}{d\Delta} + \frac{dM_1}{d\Theta_1} \frac{d\Theta_1}{d\Delta} - \frac{dM_2}{d\Theta_2} \frac{d\Theta_2}{d\Delta} = 0 \quad (67)$$

$$\frac{d\Delta_1}{d\Delta} + \frac{d\Delta_2}{d\Delta} = 1 \quad (68)$$

$$\frac{d\Theta_1}{d\Delta} + \frac{d\Theta_2}{d\Delta} = 0 \quad (69)$$

Equations 66 through 69 are easily solved for $\frac{d\Delta_1}{d\Delta}$, $\frac{d\Theta_1}{d\Delta}$, $\frac{d\Delta_2}{d\Delta}$ and $\frac{d\Theta_2}{d\Delta}$ in a similar manner.

The reactions of the bearing on the shaft are

$$F'_x = \sum_{q=1}^n P_{1q} \cos \phi_q \quad (70)$$

$$F'_y = \sum_{q=1}^n P_{1q} \sin \phi_q \quad (71)$$

$$F'_z = 0 \quad (72)$$

$$M'_x = \sum_{q=1}^n M_{1q} \sin \phi_q \quad (73)$$

$$M'_y = \sum_{q=1}^n M_{1q} \cos \phi_q \quad (74)$$

The conditions for equilibrium under the single radial load F_x is

$$F'_x + F_x = 0 \quad (75)$$

The only working response of the bearing is the linear displacement δ_x .

If δ'_x is an estimate of δ_x an improved value of δ_x is

$$\delta_x = \delta'_x - \frac{(F'_x + F_x)}{\frac{dF'_x}{d\delta_x}} \quad (76)$$

The right member of Equation 76 is calculated at current values of δ_x .

Iteration of Equation 76 yields δ_x to any desired accuracy.

Although only $\frac{dF'_x}{d\delta_x}$ is required in the foregoing, all the derivatives are required to complete the matrix needed for stiffness calculations. The elements of the complete matrix are

$$\frac{dF'_x}{d(\delta_x, \delta_y, \theta_x, \theta_y)} = \sum_{q=1}^n \cos \phi_q \frac{dP_{1q}}{d(\delta_x, \delta_y, \theta_x, \theta_y)} \quad (77)$$

$$\frac{dF'_y}{d(\delta_x, \delta_y, \theta_x, \theta_y)} = \sum_{q=1}^n \sin \phi_q \frac{dP_{1q}}{d(\delta_x, \delta_y, \theta_x, \theta_y)} \quad (78)$$

$$\frac{dF'_z}{d(\delta_x, \delta_y, \theta_x, \theta_y)} = 0 \quad (79)$$

$$\frac{dM'_x}{d(\delta_x, \delta_y, \theta_x, \theta_y)} = \sum_{q=1}^n \sin \phi_q \frac{dM_{1q}}{d(\delta_x, \delta_y, \theta_x, \theta_y)} \quad (80)$$

$$\frac{dN'_y}{d(\delta_x, \delta_y, \theta_x, \theta_y)} = \sum_{q=1}^n \cos \phi_q \frac{dM_{1q}}{d(\delta_x, \delta_y, \theta_x, \theta_y)} \quad (81)$$

$$\frac{d(F'_x, F'_y, F'_z, M'_x, M'_y)}{d\delta_z} = 0 \quad (82)$$

where

$$\begin{aligned} \frac{dP_{1q}}{d(\delta_x, \delta_y, \theta_x, \theta_y)} &= \left[\frac{dP_{1q}}{d\Delta_{1q}} \frac{d\Delta_{1q}}{d\Delta_q} + \frac{dP_{1q}}{d\theta_{1q}} \frac{d\theta_{1q}}{d\Delta_q} \right] \frac{d\Delta_q}{d(\delta_x, \delta_y, \theta_x, \theta_y)} \\ &+ \left[\frac{dP_{1q}}{d\Delta_{1q}} \frac{d\Delta_{1q}}{d\theta_q} + \frac{dP_{1q}}{d\theta_{1q}} \frac{d\theta_{1q}}{d\theta_q} \right] \frac{d\theta_q}{d(\delta_x, \delta_y, \theta_x, \theta_y)} \end{aligned} \quad (83)$$

$$\begin{aligned} \frac{dM_{1q}}{d(\delta_x, \delta_y, \theta_x, \theta_y)} &= \left[\frac{dM_{1q}}{d\Delta_{1q}} \frac{d\Delta_{1q}}{d\Delta_q} + \frac{dM_{1q}}{d\theta_{1q}} \frac{d\theta_{1q}}{d\Delta_q} \right] \frac{d\Delta_q}{d(\delta_x, \delta_y, \theta_x, \theta_y)} \\ &+ \left[\frac{dM_{1q}}{d\Delta_{1q}} \frac{d\Delta_{1q}}{d\theta_q} + \frac{dM_{1q}}{d\theta_{1q}} \frac{d\theta_{1q}}{d\theta_q} \right] \frac{d\theta_q}{d(\delta_x, \delta_y, \theta_x, \theta_y)} \end{aligned} \quad (84)$$

The derivatives of Δ_q and θ_q with respect to the inner-ring displacements are, from Equations 24 and 25.

$$\frac{d\Delta_q}{d\delta_x} = \cos\phi_q \quad (85)$$

$$\frac{d\Delta_q}{d\delta_y} = \sin\phi_q \quad (86)$$

$$\frac{d\Delta_q}{d(\theta_x, \theta_y)} = 0 \quad (87)$$

$$\frac{d\theta_q}{d(\delta_x, \delta_y)} = 0 \quad (88)$$

$$\frac{d\theta_q}{d\theta_x} = \sin\phi_q \quad (89)$$

$$\frac{d\theta_q}{d\theta_y} = \cos\phi_q \quad (90)$$

Some rollers may be out-of-contact with the inner race and are loaded against the outer race by centrifugal force. Such rollers contribute to the bearing's reactions but have no effect on the derivatives of P_{1q} and M_{1q} .

The complete matrix for stiffness calculations is then:

$$\begin{bmatrix} \frac{dF'_x}{d\delta_x} & \frac{dF'_x}{d\delta_y} & 0 & \frac{dF'_x}{d\theta_x} & \frac{dF'_x}{d\theta_y} \\ \frac{dF'_y}{d\delta_x} & \frac{dF'_y}{d\delta_y} & 0 & \frac{dF'_y}{d\theta_x} & \frac{dF'_y}{d\theta_y} \\ 0 & 0 & 0 & 0 & 0 \\ \frac{dM'_x}{d\delta_x} & \frac{dM'_x}{d\delta_y} & 0 & \frac{dM'_x}{d\theta_x} & \frac{dM'_x}{d\theta_y} \\ \frac{dM'_y}{d\delta_x} & \frac{dM'_y}{d\delta_y} & 0 & \frac{dM'_y}{d\theta_x} & \frac{dM'_y}{d\theta_y} \end{bmatrix} \quad (91)$$

As discussed earlier, the axial components of stiffness are zero for a cylindrical roller bearing.

SECTION III

APPLICATION OF COMPUTER PROGRAM

The analysis of Section II has been programmed in Fortran IV for a digital computer and is suitable for use on the CDC 6600. A program listing is presented in the Appendix.

3.1 Sample Test Case

To illustrate a typical case consider the cylindrical roller bearing in Figure 9. This is a cylindrical bearing typical of a design employed in high speed applications. The geometry of this sample bearing is summarized below.

Number of rollers, n	32
Roller diameter, d	.6299 in.
Pitch diameter, E	7.75 in.
Total length of roller, ℓ_T	.6299 in.
Length of flat portion of roller ℓ_F	.2 in.
Roller crown radius	17 in.
Roller corner break	.05 in
Crown drop gage point	0

The operating conditions for the sample case are:

Diametrical clearance, $P_D = .0058$ in.

Rotational speed, $N_2 = 13,230$ rpm

Radial load, $F_x = 1,465$ lbs.

3.2 Input Format

Figure 10 presents the input data format and Figure 11 shows the actual input data for the sample case operating conditions.

3.3 Output Format

Figure 12 presents the output data for the sample case. The input data

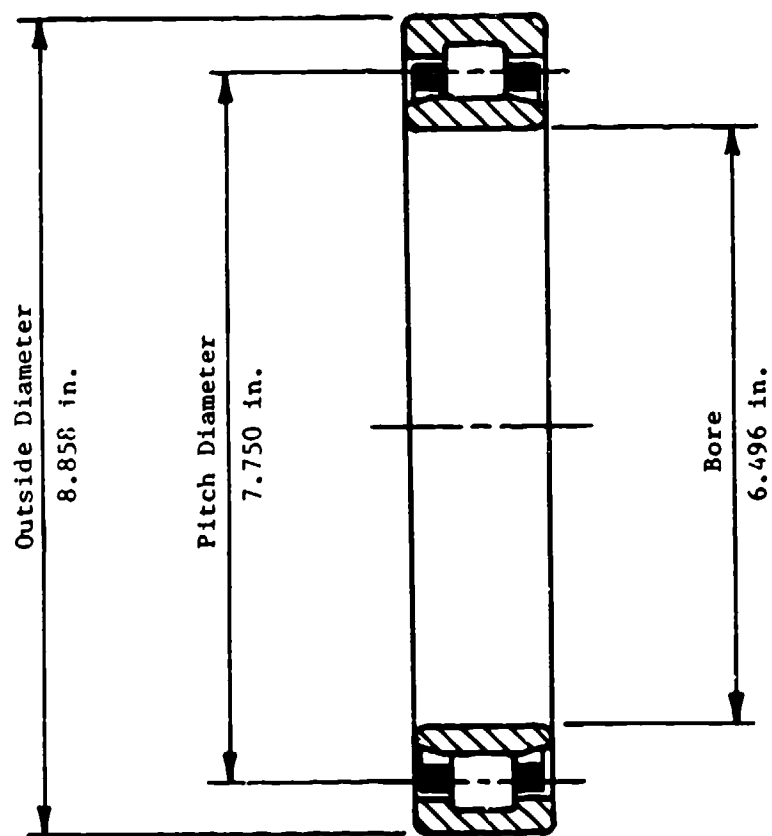


Figure 9 Sample Cylindrical Roller Bearing

Number of Rollers (60 max/min)	Roller Diameter (inches)	Pitch Diameter (inches)	Total Length of Roller (inches)	Effective Length of Roller (inches)	Length of Flat Portion of Roller (inches)	Crown Radius (inches)	Crown Drop (inches)
(C) TITLE							
TITLE							
TITLE							
Cage Point (inches)	Corner Break (inches)	Modulus of Elasticity of Outer Race (lb/in ²) (D)	Modulus of Elasticity of Inner Race (lb/in ²) (D)	Modulus of Elasticity of Rollers (lb/in ²) (D)	Poisson's Ratio for Outer Race (if blank - assume .25)	Poisson's Ratio for Inner Race (if blank - assume .25)	Poisson's Ratio for Rollers (if blank - assume .25)
Roller Material Density (lb/in ³) If blank assume 2.83	Tolerance for Elastic Approach of Rollers If blank assume 1.8-7	Tolerance for Angular Deflection of Roller If blank assume 1.8-7	Tolerance for Bearing Deflection Along x If blank assume 5.8-7				
RPM of Outer Ring	RPM of Inner Ring	Load Along x (lb)	Initial Displacement Along x (inches)	Initial Displacement Along y (inches)	Initial Displacement About x (Radians)	Initial Displacement About y (Radians)	Diametral Clearance (inches)

Z 10.0 Format

(A) If total length is given omit effective length.
If effective length is given omit total length.

(B) If crown radius is given omit crown drop.
If crown drop is given omit crown radius.

(C) Punch 1 to start printout at top of a new page.

(D) If blank program assumes 29.86.
To run additional load cases with same bearing repeat card 5 as required.
To run new bearing place one blank after last card 6 and repeat card 1 et seq.
To stop place two blanks after last card 6.

Figure 10 Input Data Format

PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISPLACEMENTS

DEX/DX LB/IN 3.4980+06	DEX/DY LB/IN 1.9336-01	DEX/DZ LB/IN 0.0000	DEX/DALX LB/RAD 2.1193-04	DEX/DALY LB/RAD 1.3479-02
DEY/DX LB/IN 1.9531-01	DEY/DY LB/IN 2.6504+05	DEY/DZ LB/IN 0.0000	DEY/DALX LB/RAD 9.5776-04	DEY/DALY LB/RAD 2.1193-04
DEZ/DX LB/IN 0.0000	DEZ/DY LB/IN 0.0000	DEZ/DZ LB/IN 0.0000	DEZ/DALX LB/RAD 0.0000	DEZ/DALY LB/RAD 0.0000
DMX/OX LB/IN 3.0981-04	DMX/DY LB/IN 9.5084-04	DMX/DZ LB/IN 0.0000	DMX/DALX LB/RAD 1.6056+03	DMX/DALY LB/IN/RAD 9.7046-03
DMY/DX LB/IN 1.3566-02	DMY/DY LB/IN 3.0981-04	DMY/DZ LB/IN 0.0000	DMY/DALX LB/IN/RAD 9.7046-03	DMY/DALY LB/IN/RAD 2.3117+04

Figure 12 Output Data for Sample Problem

105 MM CYLINDRICAL ROLLER BEARING
CROWNED ROLLERS

DESIGN DATA FOR BEARING NO. 1

NO. OF ROLLS	ROLL DIAMETER IN	PITCH DIAMETER IN	TOTAL LENGTH IN	FLAT LENGTH IN	EFFECTIVE LENGTH IN	CORNER BREAK IN	CROWN RADIUS IN	CROWN DROP IN	GAGE POINT IN	ROLL WEIGHT LB
3.2000-01	6.2990-01	7.7500-00	6.2990-01	2.0000-01	5.2990-01	5.0000-02	1.7000-01	1.7707-03	0.0000	5.5551-02

MODULUS OF ELASTICITY ROLLS
INNER 1.6710E+02
OUTER 2.9000E+07

POISSON'S RATIO ROLL
INNER 0.0000
OUTER 0.0000

INITIAL DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

DIAMETRAL CLEARANCE 5.0000-03

INPUT DATA FOR LOAD NO. 1, BEARING NO. 1

RPM OF OUTER 1.2235-04

LOAD ALONG X 1.0650-03

LOAD ALONG Y 0.0000

REARINGS OF BEARING ON SHAFT
ALONG X 0.0000

REARINGS OF BEARING ON SHAFT
ALONG Y 0.0000

CONTACT LENGTH IN

CONTACT WIDTH IN

CONTACT AREA IN²

CONTACT STRESS IN

CONTACT DEFLECTION IN

CONTACT MOMENT LB-IN

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

CONTACT DISPLACEMENTS
ALONG X 0.0000
ALONG Y 0.0000

Figure 12 Output Data for Sample Problem (Continued)

are summarized in Figure 12, followed by the output data including the internal load distribution as well as various other stress and displacement parameters. The stiffness matrix is given on the last page of Figure 12.

APPENDIX

COMPUTER PROGRAM
FOR
CALCULATING STIFFNESS MATRIX
OF
CYLINDRICAL ROLLER BEARING

PRECEDING PAGE BLANK-NOT FILMED

COMMON RB(2,60),C(2),CROWN,CORNER,CPH,COR2,D,DKOP,DENS,DFL11(5),
 10FLD1(5),DFL2,DTV(5,5),DLX(2,60),E,EL(2),FLT,FLT2,FC,GAGE,GAM,H,
 2HERTZ(2,60),IRR,ILOAD,IQUIT,ITER,JPASS,N,NOLOAD,OME,PR(2),PRR,PD,
 3P(2,60),RPM(2),SPH,TOL1,TOL2,TOL3,THD1(5),THSAV(2,60),WEIGHT,XN,
 4XLT,XLC,XMASS,XLE2,XF2,XF1(5),XNM(2,60),XX(2,2,60),XMSAV(2,60),
 5Y(2),YR

COMMON ABIG,DELLX,HRZ,X9IG,XX0(2)

COMMON CRN,FLT2D,HD,XLE2D

DOUBLE PRECISION CRN,FLT2D,XLE2D,HD

CARD	COLS.	ITEM
1	1-10	NUMBER OF ROLLS - 60 MAXIMUM
1	11-20	ROLL DIAMETER - IN
1	21-30	PITCH DIAMETER - IN
1	31-40	TOTAL LENGTH OF ROLL - IN. MEASURED BETWEEN ROLL ENDS
1	41-50	EFFECTIVE LENGTH OF ROLL - IN. MAXIMUM WORKING LENGTH OF ROLL MEASURED BETWEEN OUTER EXTREMITIES OF CROWNED SURFACE

NOTE IF TOTAL LENGTH IS GIVEN OMIT EFFECTIVE LENGTH

IF EFFECTIVE LENGTH IS GIVEN OMIT TOTAL LENGTH

51-60 LENGTH OF FLAT PORTION OF ROLL - IN. MAY BE ZERO FOR A FULLY-CROWNED ROLL

61-70 CROWN RADIUS - IN

71-80 CROWN DROP MEASURED AT GAGE POINT - IN

NOTE IF CROWN RADIUS IS GIVEN OMIT CROWN DROP

IF CROWN DROP IS GIVEN OMIT CROWN RADIUS

2 1 PUNCH 1 TO START PRINTOUT AT THE TOP OF A NEW PAGE

2 2-80 TITLE - PUNCH ANYTHING

3 1 LEAVE BLANK

3 2-80 TITLE - PUNCH ANYTHING

4 1-10 GAGE POINT - IN. THE AXIAL DISTANCE IN FROM THE END OF THE CROWNED SURFACE OF THE ROLL TO WHERE THE CROWN DROP IS MEASURED

11-20 CORNER BREAK - IN. MEASURED PARALLEL TO ROLL AXIS

21-30 MODULUS OF ELASTICITY OF OUTER RACE - LB/IN**2. IF BLANK PROGRAM ASSUMES 29.E6

31-40 SAME FOR INNER RACE

41-50 SAME FOR ROLLS

51-60 POISSON'S RATIO FOR OUTER RACE. IF BLANK PROGRAM ASSUMES .25

61-70 SAME FOR INNER RACE
 71-80 SAME FOR ROLLS
 1-10 ROLL MATERIAL DENSITY - LB/IN**3. IF BLANK PROGRAM
 ASSUMES .283
 11-20 TOLERANCE FOR ELASTIC APPROACH OF ROLL AND RACE. IF BLANK
 PROGRAM ASSUMES 1.E-7
 21-30 TOLERANCE FOR ANGULAR DEFLECTION OF ROLL RELATIVE TO RACE
 IF BLANK PROGRAM ASSUMES 1.E-7
 31-40 TOLERANCE FOR BEARING DEFLECTION ALONG X. IF BLANK
 PROGRAM ASSUMES 5.E-7

5

1-10 RPM OF OUTER RING
 11-20 RPM OF INNER RING
 21-30 LOAD ALONG X - LB
 31-40 INITIAL DISPLACEMENT ALONG X - IN
 41-50 INITIAL DISPLACEMENT ALONG Y - IN
 51-60 INITIAL DISPLACEMENT ABOUT X - RADIANS
 61-70 INITIAL DISPLACEMENT ABOUT Y - RADIANS
 71-80 DIAMETRAL CLEARANCE - IN. A POSITIVE VALUE INDICATES
 LOOSENESS

6

TO RUN ADDITIONAL LOAD CASES WITH SAME BEARING REPEAT CARD 6 AS
REQUIRED

TO RUN NEW BEARING PLACE ONE BLANK AFTER LAST CARD 6 AND REPEAT
CARD 1 ET SEQ

TO STOP PLACE TWO BLANKS AFTER LAST CARD 6

```

IPR=0
C(1)=1.
C(2)=-1.
READ(5,10)AN,OE,XLT,XLF,FLT,CROWN,DROP
IF(XN.EQ.O.)STOP
IPR=IPR+1
FORMAT(REF10.0)
READ(5,20)
FORMAT(R04H
1
2
WRITE(6,20)
WRITE(6,30)IPR
FORMAT(30H) DESIGN DATA FOR BEARING NO.,T3)
READ(5,10)GAGE,CORNER,YM(1),YM(2),YMR,PR(1),PR(2),PRR
1,DENS,TOL1,TOL2,TOL3
IF(XLE.EQ.O.)XLE=XLT-2.*CORNER
IF(XLT.EQ.O.)XLT=XLE+2.*CORNER
IF(YMR.EQ.O.)YMR=29.E6
IF(PRR.EQ.O.)PRR=.25
IF(TOL1.EQ.O.)TOL1=1.F-7
IF(TOL2.EQ.O.)TOL2=1.F-7
IF(TOL3.EQ.O.)TOL3=5.F-7
DO 40 N=1,2
IF(YM(K).EQ.O.)YM(K)=29.E6
IF(PR(K).EQ.O.)PR(K)=.25
EL(K)=.6366198*((1.-PR(K)**2)/YM(K)+(1.-PRR**2)/YMR)
CONTINUE
IF(DENS.EQ.O.)DENS=.283
X'ASSE=J**2*XLT*DENS/491.9798
N=KN
ILOAD=U
XLE2=.5*XLE
FLT2=.5*FLT
XLE2D=XLE2
FLT2D=FLT2
CRNE=CROWN
CALL RCALC(CROWN,DROP,FLT2,GAGE,H2,XLF2)
H=HD

```

```

WEIGHT=XMASS*386.4
WRITE(0,50)XN,D,E,XLT,FLT,XLE,CORNER,CROWN,DROP,GAGE,WEIGHT
FORMAT(129H) NO. OF ROLL PITCH TOTAL GAGE FL
1AT EFFECTIVE CORNER CROWN CROWN LENGTH L
2 ROLL/130H ROLL DIAMETER DIAMETER DROP POINT
3 LENGTH LENGTH BREAK RADIUS
4 WEIGHT/18X,9(12HIN),2HLB,/1P11E12.4)
WRITE(0,60)YM(1),YM(2),YMR,PR(1),PR(2),PRR
FORMAT(62H) MODULUS OF ELASTICITY POISSON'S
1 RATIO/69H OUTER INNER ROLLS OUTER INNE
2R ROLLS/3(12H L9/IN**2)/1P6E12.4)
GAMED/E
DFL11(1)=0.
DELD1(4)=0.
DELD1(5)=0.
THD1(2)=0.
THD1(3)=0.
65 READ(5,10)RPM(1),RPM(2),XF2,DFL11(2),NFL11(3),DFL11(4),DFL11(5),PD
IF(ABS(RPM(1))+ABS(RPM(2))).EQ.0.)GO TO 5
OME=.5*(RPM(1)*(1.+GAM)+RPM(2)*(1.-GAM))
ILOAD=ILOAD+1
FC=XMASS*(.1)47198*OME)**2*.5*E
IQUIT=0
WRITE(0,70)ILOAD,IBR
FORMAT(26H) INPUT DATA FOR LOAD NO.,13,13H, BEARING NO.,13)
WRITE(0,80)RPM(1),RPM(2),XF2,(DFL11(1),I=2,5),PD
80 FORMAT(95H) RPM OF LOAD INITI
1AL DISPLACEMENTS DIAMETRAL/95H OUTER INNER
2 ALONG X ALONG Y ABOUT X ABOUT Y CLE
3 ARANCE/30X,6:HLB IN IN RADIANS RADIANS
4 IN/1P8E12.4)
DFL2=.005*D
IF(PD.GT.0.)DFL2=DFL2+.5*PD
DFL2=-SIGN(DFL2,XF2)
DO 140 ITER=1,20
NLOAD=0
DO 100 K=1,5
XF1(K)=0.
DO 100 L=1,5

```

```

100 OTV(L,K)=0.
    DO 130 J=1,N
      JPASS=J
      XJ=J
      PHI=6.283185*(XJ-1.)/XM
      SPH=SI.(PHI)
      CPH=CO.(PHI)
      CALL CYLROL
      IF(IQUAT)150,130,110
110 WRITE(6,120)IRR,ILOAD,ITER,J,DFL2
120 FORMAT(17H0MAIN PROGRAM 120,4I6,1P1E12.4)
      GO TO 65
130 CONTINUE
      IF(DFL11(2))160,135,160
135 COR2=(XF1(2)+XF2)/OTV(2,2)
      DFL2=DFL2-COR2
      IF(ABS(COR2)-TOL3)160,140,140
140 CONTINUE
      WRITE(6,150)DFL2,COR2
150 FORMAT(17H0MAIN PROGRAM 150,1P2F12.4)
      GO TO 65
160 CALL OUTPT1
      GO TO 65
    END
  SUBROUTINE CYLROL
    COMMON RB(2,60),C(2),CROWN,CORNER,CPH,COR2,0,DROP,DENS,DFL11(5),
    1DEL01(5),DFL2,OTV(5,5),DLX(2,60),E,EL(2),FLT,FLT2,FC,GAGE,GAM,H,
    2HERTZ(2,60),IRR,ILOAD,IQUIT,ITER,JPASS,N,NLOAD,OME,PR(2),PRR,PD,
    3P(2,60),RPM(2),SPH,TOL1,TOL2,TOL3,THD1(5),THSAV(2,60),WEIGHT,XN,
    4XLT,XLC,XMASS,XLE2,XF1(5),XMM(2,60),YXX(2,2,60),XHS(2,60),
    5YM(2),YMR
    COMMON ABIG,DELLX,HRZ,XRIG,XX0(2)
    COMMON CRN,FLT2D,HD,XLE2D
    DOUBLE PRECISION CRN,FLT2D,XLE2D,HD
    DOUBLE PRECISION XTH,XDEL,X1D,X2D
    DIMENSION A(4,4),COR(4),DEL(2),DPDEL(2),DMDEL(2),DPH(2),MTH(2),
    1ER(4),PX(2),TH(2),XM(2)
    J=JPASS
    DELTA=(DFL2+DFL11(2))*CPH+DFL11(3)*SPH-.5*PD

```

```

THETA=JFL11(4)*SPH+DFL11(5)*CPH
IF(ABS(THETA).LE.1.E-9)THETA=0.
DEL(1)=.55*DELTA
DEL(2)=DELTA-DEL(1)
TH(1)=.5*THETA
TH(2)=THETA-TH(1)
DELD1(2)=CPH
DELD1(3)=SPH
THD1(4)=SPH
THD1(5)=CPH
DO 140 IT=1,20
DO 120 K=1,2
P(K,J)=0.
XMM(K,J)=0.
PX(K)=0.
XM(K)=0.
DPDEL(K)=0.
DPH(K)=0.
DMDEL(K)=0.
DMTH(K)=0.
XTH=TH(K)
CTH=COS(XTH)
STH=SIN(XTH)
TANTH=STH/CTH
XDEL=DEL(K)
ZDEL=DEL(K)
CALL XTREME(CRN,FLT2D,HJ,KM,XTH,X1D,X2D,XDEL,XLE2D)
IF(KM)GO TO 10
P(1,J)=FC
P(2,J)=0.
NOLOAD=NOLOAD+1
GO TO 240
X1=X1D
X2=X2D
EFL=X1-X2
IF(EFL.LT..01*XLE2)GO TO 10
XINC=EFL/18.
XX(1,K,J)=X1
XX(2,K,J)=X2

```

10

20

```

XHEX1+AINC
SMINC=1.
RGTMPEU.
DO 110 L=1,19
SM=3.-SMINC
IF((L-20.1).OP.(L-20.19))SM=1.
SMINC=-SMINC
XH=XH-XINC
DELX=ZDEL+AH*TANTH
DO TH=XH/CTH**2
IF(ABS(XH).LE.FLT2*CTH)GO TO 50
TEMP=XH-H*STH
TEMP1=CROWN**2-TEMP**2
IF(TEMP1)30,30,50
IQUIT=1
WRITE(3,40)IBR,ILOAD,ITER,J,K,L,DFL2,THETA,ZDEL,DELX
FORMAT(10HUCYLR0L 40,6I6,1P4E12.4)
RETURN
30
TEMP1=SQRT(TEMP1)
DELX=TEMP1-H*CTH+ZDEL
DO TH=TEMP*H*CTH/TEMP1+H*STH
IF(DELX.LT.1.E-8)GO TO 110
TEMP=5.E7*DELX**1.11111/EFL**1.11111
DO 70 LLL=1,20
A1=SQRT(EL(K)*TEMP*D*(1.+C(K)*GAM))
A2=1.8064+ALOG(EFL*.5/A1)
A3=EL(K)*TEMP*A2
A4=(A3-DELX)/((A2-.5)*EL(K))
TEMP=TEMP-A4
IF(TEMP.LE.0.)GO TO 110
IF(ABS(A3-DELX)-TOL1)90,70,70
CONTINUE
WRITE(3,80)IBR,ILOAD,ITER,IT,J,K,L,TEMP,DELX,DFL2
FORMAT(10HUCYLR0L 80,7I6,1P3E12.4)
IQUIT=1
RETURN
90
PX(K)=PX(K)+TEMP*SM
XM(K)=XM(K)+TEMP*XH*SM
PDEL=SM/((A2-.5)*EL(K))
70
80

```

```

      DPDEL(K)=DPDEL(K)+PDEL
      DPTH(K)=DPTH(K)+DNTH*PDEL
      DMDEL(K)=DMDEL(K)+XH*PDEL
      DMTH(K)=DMTH(K)+XH*DNTH*PDEL
      IF(TEMP-RGIMP)110,110,100
100  RGIMP=TEMP
      DLX(K,J)=DELX
      HERTZ(K,J)=.6366198*TEMP/AL
      BB(K,J)=2.*A1
      XHSAV(K,J)=XH
110  CONTINUE
      IF(PX(K).EQ.0.)GO TO 10
      TEMP=XINC/3.
      PX(K)=PX(K)*TEMP
      XM(K)=XM(K)*TEMP
      DPDEL(K)=DPDEL(K)*TEMP
      DPTH(K)=DPTH(K)*TEMP
      DMDEL(K)=DMDEL(K)*TEMP
      DMTH(K)=DMTH(K)*TEMP
120  CONTINUE
      PS1=-PA(1)+PX(2)+FC
      PS2=XM(1)-XM(2)
      PS1DEL=-DPDEL(1)-DPDEL(2)
      PS1TH=-DPTH(1)-DPTH(2)
      PS2DEL=DMDEL(1)+DMDEL(2)
      PS2TH=DMTH(1)+DMTH(2)
      DET=PS1DEL*PS2TH-PS2DEL*PS1TH
      CORDEL=(PS1*PS2TH-PS2*PS1TH)/DET
      CORTH=(PS1DEL*PS2-PS2DEL*PS1)/DET
      DEL(1)=DEL(1)-CORDEL
      DEL(2)=DEL(1)-CORDEL
      TH(1)=TH(1)-CORTH
      TH(2)=TH(1)-CORTH
      IF(ABS(CORDEL)-TOL1)130,140,140
130  IF(ABS(CORTH)-TOL2)160,140,140
140  CONTINUE
      WRITE(6,150)IRR,ILOAD,ITER,J,DELTA,THETA,CORDEL,CORTH
150  FORMAT(11HUCYLRQ 150,4I6,1P4E12.4)
      IQUIT=1

```

```

160      RETURN
      GO TO 170 K=1,2
      P(K,J)=PX(K)
      THSAV(K,J)=TH(K)
170      XMM(K,J)=XM(K)
      MM=1
      ER(1)=0.
      ER(2)=0.
      ER(3)=1.
      ER(4)=0.
180      A(1,1)=-DPDEL(1)
      A(1,2)=DPDEL(2)
      A(1,3)=-DPTH(1)
      A(1,4)=DPTH(2)
      A(2,1)=DMDEL(1)
      A(2,2)=-DMDEL(2)
      A(2,3)=DMTH(1)
      A(2,4)=-DMTH(2)
      A(3,1)=1.
      A(3,2)=1.
      A(3,3)=0.
      A(3,4)=0.
      A(4,1)=0.
      A(4,2)=0.
      A(4,3)=1.
      A(4,4)=1.
      NZ=4
      CALL SIMULT(A,NZ,ER,COR,IQUIT)
      IF(IQUIT)210,210,190
190      WRITE(0,200) IGR,ILOAD,ITER,J,MM,DELTA,THETA
200      FORMAT(11HUCYLR 200,5I6,1P2E12.4)
      RETURN
210      GO TO(220,230),MM
220      DELDEL=COR(1)
      THDEL=COR(3)
      MM=2
      ER(3)=0.
      ER(4)=1.
      GO TO 180

```

```

230 DELTH=COR(1)
    THTH=CVR(3)
    DPDEL=DPDEL(1)*DELDEL+DPTH(1)*THDEL
    DMDEL=DMDEL(1)*DELDEL+DMTH(1)*THDEL
    NPITH=NPDEL(1)*DELTH+DPTH(1)*THTH
    DM1TH=DMDEL(1)*DELTH+DMTH(1)*THTH
240 XF1(2)=XF1(2)+P(1,J)*CPH
    XF1(3)=XF1(3)+P(1,J)*SPH
    XF1(4)=XF1(4)+XMM(1,J)*SPH
    XF1(5)=XF1(5)+XMM(1,J)*CPH
    IF(P(2,J).EQ.0.)RETURN
    DO 250 L=2,5
    Z7=DP1JEL*DEL(1(L))+DP1TH*THD1(L)
    DTV(2,L)=DTV(2,L)+ZZ*CPH
    DTV(3,L)=DTV(3,L)+ZZ*SPH
    Z7=DM1JEL*DEL(1(L))+DM1TH*THD1(L)
    DTV(4,L)=DTV(4,L)+ZZ*SPH
    DTV(5,L)=DTV(5,L)+ZZ*CPH
250 RETURN
    END
    SUBROUTINE CONOUT
    COMMON RB(2,60),C(2),CROWN,CORNER,CPH,COR2,D,DROP,DENS,DFL11(5),
    1DEL(5),DFL2,DTV(5,5),DLX(2,60),E,EL(2),FLT,FLT2,FC,GAGE,GAM,H,
    2HRTZ(2,60),IPR,ILOAD,IQUIT,ITER,JPASS,N,NLOAD,OME,PR(2),PRR,PD,
    3P(2,60),RPM(2),SPH,TOL1,TOL2,TOL3,THD1(5),THSAV(2,60),WEIGHT,XN,
    4XLT,XLE,XMASS,XLE2,XF2,XF1(5),XMM(2,60),XX(2,2,60),XHS(2,60),
    5Y(2),YR
    COMMON ARIG,DELLX,HRZ,X9IG,XX0(2)
    COMMON CRV,FLT2,HD,XLE2D
    DOUBLE PRECISION CRN,FLT2D,XLE2D,HD
    EL1=EL(1)
    DELL=.005*D
    DO 50 ITR=1,20
    PI=0.
    P1DEL=0.
    S'C=1.
    XI=SQRT(CROWN**2-(H-DELL)**2)
    IF(X1.GT.XLE2)XI=XLE2
    EFL=2.*XI

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X'IC=EFL/10.
X'G=X1+X'G
X'G(1)=Y1
X'G(2)=X1
DO 40 L=1,11
X'G=X'G-X'G
S'W=3.-S'W
S'G=-S'G
IF((L.EQ.1).OR.(L.EQ.11))S'W=1.
DLTX=SQRT(CROWN**2-XS**2)- H*DELL
IF(DLTX.LT.1.E-R)GO TO 40
TEMP=5.57*DLTX**1.11111/EFL**1.11111
DO 10 IK=1,20
IF(TEMP.LT.0.)GO TO 40
A1=SQRT(EL1*D*TEMP*(1.+GAM))
A2=1.8364+ALOG(EFL*.5/A1)
A3=EL1+A2*TEMP
A4=(A3-DLTX)/((A2-.5)*EL1)
TEMP=TEMP-A4
IF(ABS(A3-DLTX)-1.E-7)GO,10,10
CONTINUE
WRITE(6,20)ISD,ILOAD,DLTX,TEMP,FL2
FORMAT(10HOUTCNT 20,2I6,1P3E12.4)
IQUIT=1
RETURN
P1=P1+TEMP*S'W
PDL=S'W/((A2-.5)*EL1)
P1DEL=P1DEL+PDL
CONTINUE
P1=.6666667*P1*XNC+TEMP*FL1
P1DEL=.6666667*P1DEL*XNC+PDL*FL1
CRR=(P1-FC)/P1DEL
DELL=DELL-CRR
IF(ABS(CRR)-1.E-7)70,50,50
CONTINUE
WRITE(6,60)DELL,CRR
FORMAT(10HOUTCNT 60,1P2E12.4)
IQUIT=1
RETURN

```



```

WRITE(3,40)J,PHI,P(1,J),P(2,J),VVM(1,J),VVM(2,J),XXX(1,1,J),XXX(2,
11,J),XXX(1,2,J),XXX(2,2,J)
FORMAT(17,5X,1P10F12.4)
CONTINUE
IF(NOLoad.EQ.0)GO TO 100
CALL CONOUT
IF(IQUIT)80,80,60
WRITE(6,70)
FORMAT(9HOUTPT1 70)
RETURN
TEMP=0.
WRITE(6,90)FC,TEMP,XX0(1),XX0(2)
FORMAT(104 OTHERS,14X,1P1E12.4,12X,1P1F12.4,12X,1P2E12.4)
WRITE(6,110)
FORMAT(132H0 ROLL CONTACT LENGTH MAXIMUM CONTACT W
110TH MAX. CONTACT DEFLECTION MAXIMUM HERTZ STRESS LOCATION OF
2MAX. VALUES/129H NUMBER OUTER INNER OUTER
3 INNER OUTER INNER OUTER INNER OUTER
4R INNER/14X,2HIN,10X,2HIN,10X,2HIN,10X,2HIN,10X,2HIN,10X,2HI
5N,7X,8H1B/IN**2,4X,8H1B/IN**2,7X,2HIN,10X,2HIN)
DO 120 J=1,N
IF(P(2,J).EQ.0.)GO TO 120
TEMP=XXX(1,1,J)-XXX(2,1,J)
TEMP1=XXX(1,2,J)-XXX(2,2,J)
WRITE(6,40)J,TEMP,TEMP1,RR(1,J),RR(2,J),DLX(1,J),DLX(2,J),HERTZ(1,
1J),HERTZ(2,J),XHSAB(1,J),XHSAB(2,J)
CONTINUE
IF(NOLoad.EQ.0)GO TO 140
TEMP=XX0(1)-XX0(2)
WRITE(6,130)TEMP,ABIG,DELLX,HRZ,XBIG
FORMAT(104 OTHERS,1P1E14.4,12X,1P1F12.4,12X,1P1E12.4,12X,1P1E12
1.4,12X,1P1E12.4)
WRITE(6,150)
FORMAT(34H0 ROLL CONTACT MISALIGNMENT/33H NUMBER OU
150TER INNER/15X,19HRADIANS RADIANS)
DO 160 J=1,N
IF(P(2,J).EQ.0.)GO TO 160
WRITE(6,40)J,THSAV(1,J),THSAV(2,J)
CONTINUE

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```

IF(NOLJAD.EQ.0)GO TO 180
TEMP=0.
WRITE(0,17J)TEMP
170 FORMAT(10H OTHERS,1P1E14.4)
180 WRITE(0,195)FC
185 FORMAT(32H0 CENTRIFUGAL FORCE OF ROLLER =,1P1E11.4,3H LB)
190 WRITE(0,190)
195 FORMAT(65H0 PARTIAL DERIVATIVES OF REACTIONS WITH RESPECT TO DISP
1 PLACEMENTS)
200 WRITE(0,200)
205 FORMAT(59H0 DFX/DX DFX/DY DFX/DZ DFX/DALX DFX/D
LB/RAD)
1ALY/58H LB/IN LB/IN LB/IN LB/IN LB/RAD
WRITE(6,20)DTV(2,2),DTV(2,3),DTV(2,1),DTV(2,4),DTV(2,5)
210 WRITE(0,210)
215 FORMAT(59H0 DFY/DX DFY/DY DFY/DZ DFY/DALX DFY/D
LB/RAD)
1ALY/58H LB/IN LB/IN LB/IN LB/IN LB/RAD
WRITE(6,20)DTV(3,2),DTV(3,3),DTV(3,1),DTV(3,4),DTV(3,5)
220 WRITE(0,220)
225 FORMAT(59H0 DFZ/DX DFZ/DY DFZ/DZ DFZ/DALX DFZ/D
LB/RAD)
1ALY/58H LB/IN LB/IN LB/IN LB/IN LB/RAD
WRITE(6,20)DTV(1,2),DTV(1,3),DTV(1,1),DTV(1,4),DTV(1,5)
230 WRITE(0,230)
235 FORMAT(59H0 DMX/DX DMX/DY DMX/DZ DMX/DALX DMX/D
LBIN/RAD)
1ALY/59H LBIN/IN LBIN/IN LBIN/IN LBIN/RAD
2)
WRITE(6,20)DTV(4,2),DTV(4,3),DTV(4,1),DTV(4,4),DTV(4,5)
240 WRITE(0,240)
245 FORMAT(59H0 DMY/DX DMY/DY DMY/DZ DMY/DALX DMY/D
LBIN/RAD)
1ALY/50H LBIN/IN LBIN/IN LBIN/IN LBIN/RAD
2)
WRITE(6,20)DTV(5,2),DTV(5,3),DTV(5,1),DTV(5,4),DTV(5,5)
RETURN
END
SUBROUTINE RCALC(CROWN,DROP,FLT02,GAGF,HD,XLEO2)
DOUBLE PRECISION CRD,DRP,DSORT,FLT2D,GAG,HD,XLEO2D
FLT2D=FLT02
GAG=GAGE
XLEO2D=XLEO2
IF(CROWN)10,10,20

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10  DD=DDDD
   CD=DSORT((XLEO2D-GA9)**2-FLT2D**2+CRD**2)/(2.0D*DRP)**2+
   FLT2D**2)
   HD=DSORT(CRD**2-FLT2D**2)
   DD=DDDD
   RETURN
20  CD=CD*2
   HD=DSORT(CRD**2-FLT2D**2)
   DD=DDDD-DSORT(CRD**2-(ALEO2D-GA9)**2)
   RETURN
   END
   SUBROUTINE SIMULT(AA,N,RR,XX,KX)
   DIMENSION AA(4,4),RR(4),XX(4),KOL(4)
   DOUBLE PRECISION A(4,4),R(4),X(4),ROW(4),AMPY,TEMP
   DO 10 I=1,N
   DO 5 J=1,N
   A(J,I)=AA(J,I)
   R(I)=RR(I)
   DO 50 J=1,N
   TEMP=0.00
   DO 30 K=1,N
   IF(DABS(A(J,K))-TEMP)30,30,20
   TEMP=DABS(A(J,K))
30  CONTINUE
   DO 40 K=1,N
   A(J,K)=A(J,K)/TEMP
   R(J)=R(J)/TEMP
50  CONTINUE
5000 KOL(1)=1
5001 DO 5002 IROW=2,N
5002 KOL(IROW)=KOL(IROW-1)+1
5004 DO 5025 KUUNT=1,N
   LARGSTEN=KUUNT+1
   IERASE=KOL(1)
   JCOLD=1
5005 IF (N-KUUNT) 5035,5014,5006
5006 AMPY=DABS ( A(1,1) )
5007 DO 5010 IROW=2,LARGST
5008 IF ( AMPY -DABS ( A(IROW,1) ) ) 5009, 5010, 5010

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SIMU0190
SIMU0200
SIMU0210
SIMU0220
SIMU0230
SIMU0240
SIMU0250
SIMU0260
SIMU0270
SIMU0280
SIMU0290

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5009 JCOL=IROW
      AMPY =DABS ( A(IROW,1) )
      IERASE=KOL(IROW)
5010 CONTINUE
5011 IF (KOL(1)-IERASE) 5012,5014,5012
5012 KOL(JCOL)=KOL(1)
      KOL(1)=IERASE
5014 IF(A(JCOL,1))5015,5035,5015
5015 AMPY=A(JCOL,1)
5017 DO 5018 IROW=2,N
      ROW(IROW-1)=A(JCOL,IROW)/AMPY
5018 A(JCOL,IROW-1)=A(1,IROW-1)
      ROW(N)=1.00/AMPY
      A(JCOL,N)=A(1,N)
5019 DO 5022 IROW=2,N
      AMPY=A(IROW,1)
5020 DO 5021 JCOL=2,N
5021 A(IROW-1,JCOL-1)=A(IROW,JCOL)-AMPY*ROW(JCOL-1)
5022 A(IROW-1,N)=-AMPY*ROW(N)
5023 DO 5024 JCOL=1,N
      KOL(JCOL)=KOL(JCOL+1)
5024 A(N,JCOL)=ROW(JCOL)
5025 KOL(N)=IERASE
5026 DO 5034 KOUNT=1,N
5027 IF (KOL(KOUNT)-KOUNT) 5035,5034,5028
5028 DO 5032 IROW=KOUNT,N
5029 IF (KOL(IROW)-KOUNT) 5035,5030,5032
5030 DO 5031 JCOL=1,N
      ROW(1)=A(JCOL,IROW)
5031 A(JCOL,IROW)=A(JCOL,KOUNT)
      IERASE=KOL(KOUNT)
      KOL(KOUNT)=ROW(1)
      KOL(IROW)=IERASE
      GO TO 5034
5032 CONTINUE
      GO TO 5035
5034 CONTINUE
997 IF(KX-3)908,9000,998

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SIMU0300
SIMU0310
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SI MU0690
SI MU0700
SI MU0710
SI MU0720

SI MU0750
SI MU0780
SI MU0790
SI MU0800
SI MU0810

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9000 KY=0
      RETURN
99A  50 5042 IROW=1,N
5040 X(IROW)=0.00
5041 50 5043 JKOL=1,N
5043 X(IROW)=X(IROW)+A(IROW,JKOL)*3(JKOL)
5042 XY(IROW)=X(IROW)
      KY=0
      RETURN
5035 KY=1
      RETURN
      END
SI9ROUINE XTREVE(CRN,FLT2D,HD,KM,XTH,X1D,X2D,XDEL,XLE2D)
DOUBLE PRECISION CRN,CTH,DSIN,DCOS,DSORT,DNABLA,FLT2D,HD,STH,TEMP,
IXTH,X1D,X2D,XDEL,XLE2D,XTHAB,XSTAR1,XSTAR2
XTHAB=JABS(XTH)
KY=0
STH=DSIN(XTHAB)
CTH=DCOS(XTHAB)
TEMP=CRN**2-(HD*CTH-XDEL)**2
IF(TEMP)10,10,20
10  KY=1
      RETURN
20  TEMP=DSORT(TEMP)
      X1D=TEMP+HD*STH
      X2D=-TEMP+HD*STH
      DNABLA=HD-DSORT(CRN**2-XLE2D**2)
      XSTAR1=XLE2D*CTH+DNABLA*STH
      XSTAR2=-XLE2D*CTH+DNABLA*STH
      IF(X2D.GT.XSTAR1)GO TO 10
      IF(X1D.LE.FLT2D*CTH)GO TO 10
      IF(X1D.GT.XSTAR1)X1D=XSTAR1
      IF(X2D.LT.XSTAR2)X2D=XSTAR2
      IF(XTH.EQ.0.00)RETURN
      IF((X2D.GT.-FLT2D*CTH).AND.(X2D.LT.FLT2D*CTH))X2D=-XDEL*CTH/STH
      IF(XTH.GE.0.00)RETURN
      TEMP=-X1D
      X1D=-X2D
      X2D=TEMP

```

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