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JAN 80 J MARTI, P A CUNDALL F61708-79-M-0871
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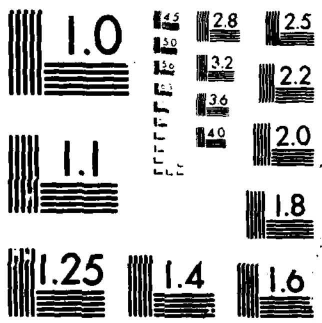

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20. Abstract

Soils are composed of a combination of solid particles, water and, often, air. The constitutive behavior of water and air are fairly well understood. Attention in this study is given only to the mechanical behavior of the soil skeleton under isothermal conditions. Characteristics which should be displayed by any soil models used under generalized loading conditions are enumerated. A summary of existing models are discussed - Elastic and Viscoelastic Models, One-Dimensional Curve Fits, Ideal Elasto-Plasticity, Elastic Non-Ideally Plastic Models, Endochronic Models, The Failure-Seeking Model as well as comparison of existing models with desired features. Recommended lines for advancement in generating a more universal model are also included.

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REPORT
CONSTITUTIVE LAWS
FOR DYNAMIC MODELLING OF SOILS

Dames & Moore Job No.: 11645-001-60

TABLE OF CONTENTS

Page No.

LIST OF FIGURES

REPORT

CONSTITUTIVE LAWS

FOR DYNAMIC MODELLING OF SOILS

1.0 INTRODUCTION

This report presents the findings of the study "Constitutive Laws for Dynamic Modelling of Soils" performed by Dr. J. Marti (principal investigator), of the Advanced Technology Group of Dames & Moore with active participation by Dr. Peter A. Cundall, Consultant, for the US Air Force. It served as a basis for discussions held between the principal investigator and Air Force experts at the Air Force Weapons Laboratory in Albuquerque, New Mexico.

The difficulties of describing the constitutive behaviour of soils are well known. The present effort is by no means unique; although each review of the state-of-the-art of this subject is affected by its particular purpose and applications, the reader should be aware of other recent efforts similar to this report. Among others, we can mention those by Nelson (1977), Christian and Desai (1977), Hardin (1978), Desai (1979) and Gudehus (1979). Not restricted to soil behaviour, but with plasticity methods in general, is that by Armen (1979).

1.1 DYNAMIC MODELLING OF SOILS

The US Air Force has a requirement to model the dynamic behaviour of soils. The possible dynamic inputs are several but of particular concern is the energy liberated by explosions, whether nuclear or conventional. Dynamic effects are of interest both at the surface and at depth.

Soils are materials composed of two or three phases. Their discrete nature and their electro-chemistry are probably the main reasons for the complex behaviour consistently observed by investigators. This complexity is demonstrated by the large number of constitutive laws which have been proposed in the past as descriptors of soil behaviour. Few of those laws have claimed to apply to all stress paths for a given soil but, even for relatively simple conditions, the options are too many for the non-specialist to choose from.

In addition to the complexity of soil behaviour, constitutive descriptions must also be attempted on the basis of very limited data; this is due to the almost insurmountable difficulties presented by physical testing of soils under complex stress paths. At present, only a few simple stress configurations and histories can be monitored with any reliability. As a consequence, results observed for a few particular cases (i.e. uniaxial compression, triaxial, etc) must be generalised to more complex stress paths, such as those induced by an explosion. This generalisation must include an untested constitutive bias. None of the generalisations so far produced has gained universal acceptance and, in view of the state-of-the-art in constitutive theories and soils testing, it is unlikely that such consensus will be reached in the near future.

1.2 OBJECTIVE

In these circumstances, the present work has one main objective; that is, in the light of what is presently known about the dynamic constitutive behaviour of soils, to compare the quality of the existing constitutive descriptions, and comment on the adequacy of their respective use.

It is hoped that this exercise will be of interest to the Air Force by providing a methodology for evaluating constitutive theories for soil. It must be remembered, however, that the list of characteristics of soil behaviour presented in this report will require updating as new experimental data become available.

1.3 SCOPE OF WORK

In the course of this study, three tasks have been undertaken:

i. From a review of the literature, preparation of a list of relevant characteristics of soil behaviour as well as any other features to be displayed by the model; this constitutes the scale against which the different theories must be measured.

ii. Again from a review of the literature, preparation of a selected list of constitutive models; this list is by no means exhaustive but tries to cover the main lines of progress in dynamic modelling of soil behaviour.

iii. Evaluation of the models (ii) in the light of the criteria described in (i) and discussion of their relative merits for dynamic modelling of soils.

1.4 LAYOUT OF REPORT

The ideal requirements of a constitutive law, based on present knowledge of soil behaviour, are given in Section 2.0. A selected number of presently used constitutive models together with their main characteristics are listed in Section 3.0. In Section 4.0, these characteristics are compared with the requirements previously described in Section 2.0 and the conclusions of this exercise are summarised in the final Section 5.0. Two appendices comprising the lists of symbols and references, respectively, complete this report.

2.0 CONDITIONS TO BE FULFILLED BY A SOIL CONSTITUTIVE MODEL

As mentioned earlier, soils are composed of a combination of solid particles, water and, often, air. The constitutive behaviour of water and air are fairly well understood. The problems of analysing the coupled equations governing the behaviour of a multiphase material (where the individual behaviour of each phase is known) are solvable and beyond the scope of this study. In the following sections, attention will be given only to the mechanical behaviour of the soil skeleton under isothermal conditions.

2.1 CHARACTERISTICS OF SOIL BEHAVIOUR TO BE DISPLAYED BY THE MODEL

A sufficient set of rules for assessing the adequacy of a given constitutive law cannot be proposed as yet. As mentioned in Section 1.1, it is believed that progress in testing has not yet allowed investigators to study soil response under more than a handful of relatively simplistic stress or strain paths. However, there are a number of characteristics of soil behaviour which are known with a fair degree of certainty; these are listed below. All of them should ideally be displayed by any soil models used under generalised loading conditions. It should be remembered that the following characteristics pertain to the law describing the behaviour of the soil skeleton alone, not the combination of skeleton plus interstitial fluids.

2.1.1 Behaviour Under Simple Stress Paths

i. One-dimensional, monotonically increasing shear

- The slope of the stress strain curve ($\Delta\tau/\Delta\gamma$) should never increase.
- A stress bound (τ_{max}) should exist.
- Non-linearity starts appearing at strains on the order of 10^{-5}.
- The model should be affected by normal stress (increased compression usually produces stiffening and a higher τ_{max}).
- Volume changes should always accompany shear strains (usually some shrinkage followed by dilation for dense soils and only shrinkage for looser materials); volume changes should be bounded.
- The law should be practically rate independent for most materials with the exception of soft clays.

ii. One-dimensional cyclic shear

- It should display elastic initial unloading.
- It should develop permanent deformations for all stress levels; these deformations should be bounded at least below a certain stress threshold.
- Cyclic shear should induce cumulative irrecoverable volume changes, which should always be bounded; these seem to be greater than those produced under monotonic shear.
- The law should generate hysteresis loops which change with the number of cycles, i.e. they usually appear progressively stiffer and narrower (both these effects should eventually stabilise); hysteresis loops should be approximately independent of rate with the possible exception of soft clays.

iii. Isotropic monotonic compression

- The pressure-volumetric strain curve displays progressively stiffer response.
- Compressive volumetric strain is bounded.

iv. Cyclic isotropic compression

- Permanent volumetric strains must be developed; they should be cumulative but bounded.
- Unloading is initially elastic.
- Energy is dissipated by essentially rate-independent hysteresis for all stress levels with the exception of soft clays.

v. Multi-dimensional stress paths

- On circular stress paths (constant mean pressure and constant octahedral shear stress) : permanent strains are accumulated faster than in one-dimensional cycling; the same applies to the associated volume changes; each cycle dissipates energy (the strain trails behind the stress).
- On multi-dimensional shear : the decrease in shear modulus with increasing strain level, associated volume changes and permanent shear strains are greater than those expected from the action of a single shear component but smaller than the sum of the independently observed effects of each component.

2.1.2. History Effects

It is known that the directional stiffness and strength characteristics of soils have a marked dependence on past history. This is shown in several ways:

i. As history-induced anisotropy

- because of orientated fabric $\begin{cases}\text{orientated contacts}\\ \text{orientated particles}\end{cases}$

- because of locked-in stresses

ii. As a certain amount of memory, e.g. the maximum τ_{oct} will be remembered and probably a number of other stress or strain peaks; on the other hand, stresses or strains which subsequently are exceeded many times are probably forgotten by the material.

2.1.3 Other Observations of Soil Behaviour

i. Energy should be dissipated even for very small amplitudes of stress changes.

ii. For small deviations from an unstressed, unstrained state, stress increments will be almost parallel to strain increments; however, for larger stresses, the strain increments will eventually tend to be parallel to the stresses rather than to their increments.

iii. The model should be able to deal with general anisotropy.

2.2 OTHER CHARACTERISTICS REQUIRED OF THE MODEL

The conditions expressed so far attempt to summarise past experience of soil behaviour. Constitutive laws should be consistent with these conditions if they are to claim realistic representation of soil behaviour. However, in order to be acceptable, this is not sufficient: a law must also be useful for solving engineering problems. Even if the above conditions defined soil behaviour completely, which they obviously do not, this would require some more conditions to be fulfilled by prospective laws.

i. The constitutive law must obey the laws of thermodynamics and other principles of continuum mechanics (objectivity, etc.). Together with the field equations and the appropriate boundary and initial conditions, it should generate well posed problems in the sense of Hadamard; that is, problems for which solutions exist are unique and stable. Apart from other practical considerations, this condition must be satisfied because nature appears to satisfy it. In other words, nature is assumed to be deterministic, at least at the macroscopic level; and furthermore, infinitesimal input changes seem to produce infinitesimal changes of results. For a given model, the existence, uniqueness and stability of solutions to the boundary value problems that it generates may be extremely difficult to prove but, if disproved, the law has very limited engineering use.

ii. The constants in the law (material properties) should be few and determinable from tests. The tests should be as simple as possible and should preferably be based on existing techniques; the latter condition probably cannot be completely fulfilled but it is desirable to involve standard tests as much as possible in order to take maximum advantage of past experience.

iii. The resulting problem must be amenable to solution with existing or foreseeable computers. It is clear that laws which require remembering the complete strain history or need an inordinate number of soil descriptors at each point in a soil mass, can hardly be used in practice for solving boundary value problems.

3.0 A SUMMARY OF EXISTING MODELS

The constitutive laws considered to be of greatest interest, either because of common implementation or promising future, are summarised in the following sections.

As far as possible, a uniform notation and form of presentation has been maintained for all constitutive models since, in the opinion of the author, much of the difficulty in comparing the different models in the past has stemmed from this non-uniformity.

3.1 ELASTIC AND VISCOELASTIC MODELS

Nobody would defend that an elastic model reproduces soil behaviour under complex loading paths. The reason for mentioning elasticity here, apart from completeness, is its usefulness for producing relatively simple (sometimes even closed-form) solutions which often constitute a worthwhile qualitative guide at preliminary stages of analysis.

The equations of linear isotropic elasticity are well known:

$$\sigma_{ij} = \lambda \, \varepsilon_{kk} \, \delta_{ij} + 2 \, \mu \, \varepsilon_{ij} \tag{1}$$

where λ, μ are the Lame constants*

σ_{ij}, ε_{ij} are the stress and strain tensors, respectively.

(all stresses in this report are to be understood as effective stresses).

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* Symbols are defined in their first appearance and in Appendix B.

Anisotropic materials can be represented by giving tensor character to the material properties:

$$\sigma_{ij} = C_{ijk\ell} \, \varepsilon_{k\ell} \tag{2}$$

where $C_{ijk\ell}$ is a stiffness tensor with the obvious symmetries.

The model can be extended to non-linear elasticity by making the material constants a function of the stress level. Although such models may be useful for monotonic loading paths, their inability to handle energy dissipation, irrecoverable strains, yield, etc., make them clearly unsuitable for modelling dynamic behaviour of soils under arbitrary loading paths.

A linear viscoelastic material is represented by the following equation:

$$\sigma_{ij} = \delta_{kl} \int_o^t \lambda_{ijkl} \, (t - t') \, \frac{\partial \varepsilon_{nn}}{\partial t'} \, dt + 2 \int_o^t \mu_{ijkl} \, (t - t') \, \frac{\partial \varepsilon_{kl}}{\partial t'} \, dt' \tag{3}$$

where t is time

λ_{ijkl}, μ_{ijkl} are the relaxation moduli; if substituted by scalar functions, the material is isotropic

It can be shown that the behaviour of the most general, linear, hereditary material can be expressed in this form. If the moduli are constant, the linear elastic formulation is recovered but with complex rather than real constants.

Non-linear viscoelastic descriptions have also been used for soils (for example, Singh and Mitchell, 1968; Stevenson, 1974).

3.2 ONE-DIMENSIONAL CURVE-FITS

A number of different mathematical approaches have been proposed in the past for describing the one-dimensional, monotonic shear behaviour of soils. The most common method is to postulate a function F such that:

$$\tau = F(\gamma, \{ k \}) \tag{4}$$

$$\text{or} \quad \gamma = G(\tau, \{ k \}) \tag{5}$$

where τ, γ are one-dimensional shear stress and strain, respectively

$\{ k \}$ is a set of parameters to be determined by curve-fitting laboratory (or sometimes field) results.

Probably the most used form of F has been the hyperbola (Kondner, 1963; Kondner and Zelasko, 1963). Many other types have been used: exponentials, polynomials, logarithms, hyperbolic arctangents, power laws, etc. Again, as discrepancies are found, the parameters of the model can be made to depend on other variables, such as normal stress; this is the case, for example, of the stress dependency of the parameters of the hyperbolic model proposed by Duncan and Chang, (1970). This process can continue almost indefinitely and there is no question that test results can be approached as closely as desired by simply increasing the number of parameters in the curve.

Of more interest here is the simulation of cyclic loading histories including reversals in the direction of shear. Masing (1976) proposed a procedure for deriving cyclic behaviour from the monotonic loading curve (spline or skeletal curve): a) the tangent modulus on each reversal equals the initial one; b) the shape of unloading and reloading curves is the same as for initial loading except for a factor of two. It is basically these assumptions that have been implemented to apply the former curve fits to cyclic loading. Depending on the formulation selected, the models are called of the Davindenkov class:

$$\tau = \tau_c + \mu_o (\gamma - \gamma_c) \left[1 - H \frac{1}{n} |\gamma - \gamma_c|\right] \qquad (6)$$

or the Ramberg-Osgood class:

$$\gamma = \gamma_c + \frac{1}{\mu_o} (\tau - \tau_c) \left[1 + H \frac{1}{n} |\tau - \tau_c|\right] \qquad (7)$$

where τ_c, γ_c are the stress and strain at the last reversal point, respectively

μ_o is the initial value of the shear modulus

H is a function describing the shape of the stress-strain curve

n is 1 for initial loading and 2 thereafter.

The Ramberg-Osgood model has probably been the most popular to date (e.g. *Rosenblueth and Herrera, 1964; Constantopoulos et al., 1973;* Idriss et al., 1976); H has frequently been taken as a power function.

Although all these models are normally well behaved under regular cyclic loading, they present paradoxes under more complex loading (see for example Pyke, 1979). Problems arise particularly each time that a previous loading curve is intersected: the model must recover the continuation of previous virgin loading paths and, for this, it must be provided with a memory which remembers previous reversal. This has been done, for example, by Finn et al. (1977).

As suggested earlier, more accurate fitting of laboratory data can be obtained by increasing the number of parameters in the model. In this way, the characteristics of the law can be made to depend not only on confining pressure but on history descriptors such as strain measures (for example, Martin et al., 1974).

Furthermore, if the plastic volumetric strains can be related empirically to the shear strain history, the associated volumetric changes can be derived as the shear history progresses. This is the type of approach followed in the endochronic models used by Bazant and co-workers (Bazant and Krizeck, 1976) and Finn and co-workers, Martin et al., (1974). The resulting models can easily be used in effective stress analyses (i.e., Finn et al., 1977).

An elegant and simple model of the type considered in this section, which obeys Masing's rules except for a variation in the factor of two (second rule), has been proposed recently by Pyke (1979); at each reversal, the model starts a hyperbola with the initial slope μ_o and tending to the shear strength as an asymptote. A similar law, but based on an exponential rather than hyperbolic function, had been proposed earlier by Cundall (1976).

The models proposed by Iwan (1967) also fall within this category. They are combinations in series (or in parallel) of elements formed by a linear spring and a Coulomb slider in parallel (or in series). They obey Masing's rules and automatically display the memory required.

Finally, and in spite of not being a constitutive model but a modelling technique, this section would be incomplete without a mention of the equivalent-linear method (Schnabel et al., 1972), probably the most common in soil-structure interaction analyses to date. This is really a linear model where the shear modulus and energy dissipation characteristics at each point in a soil profile are selected so that the stress-strain curve is approached over the range of strains considered more significant to the overall response. In short, the method involves an iterative process to replace the stress-strain curve (at each point in the profile) by a straight line which intersects the real curve usually at 65% of the maximum strain developed by the earthquake (at that point in the profile); the energy dissipation is also selected to be compatible with that strain level.

3.3 IDEAL ELASTO-PLASTICITY

Soils are known to yield indefinitely if subjected to sufficient shear stress in an approximately time independent fashion and to display permanent strains; this fact prompted investigators to use plasticity for representing soil behaviour.

The existence of a failure surface, $f(\sigma_{ij})$, is postulated; all possible stress states verify $f(\sigma_{ij}) \leq 0$. For as long as the strict inequality holds, the material behaves elastically. When the stress state reaches the failure surface, plastic strains start developing. The total strain increment is:

$$d\varepsilon_{ij} = d\varepsilon^{e}_{ij} + d\varepsilon^{p}_{ij} \tag{8}$$

where e,p stand for elastic and plastic respectively.

The plastic strains are assumed to derive from a potential, g:

$$d\varepsilon^{p}_{ij} = d\lambda \frac{\partial g}{\partial \sigma_{ij}} \tag{9}$$

where $d\lambda$ is a non-negative multiplier.

When $g \equiv f$, the flow rule is said to be associated. Although the complexities of soil behaviour do not really uphold the volumetric behaviour predicted by associated flow rules, these have important mathematical advantages, such as originating well-posed problems and complying with some limit theorems.

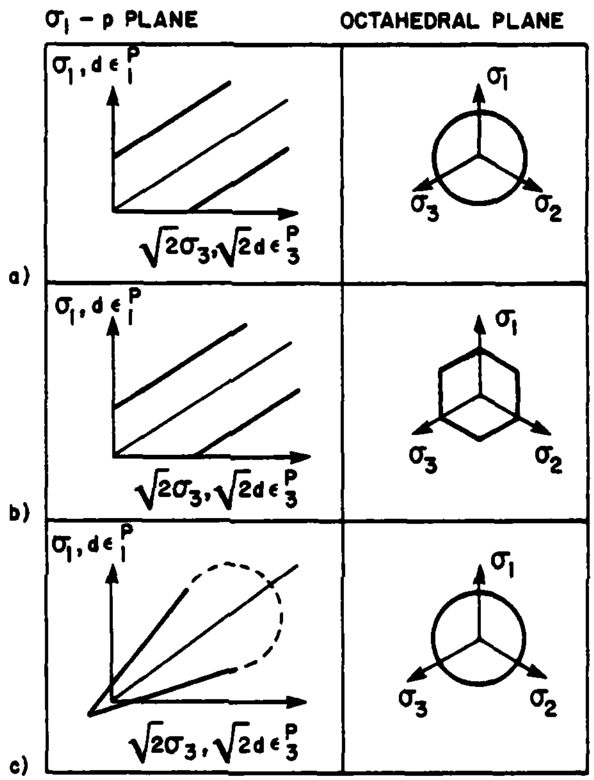

FIGURE 1 : INTERSECTION OF PLASTIC POTENTIAL SURFACES WITH THE σ_1 - p AND OCTAHEDRAL PLANES

a) Associated Von Mises
b) Associated Tresca
c) Associated Drucker-Prager

Many forms of the function $f(\sigma_{ij})$ have been proposed in the past. The classical ones are all isotropic, thus reducing the six dimensions of stress space to three (invariants or principal stresses). Among them we can mention the von Mises yield law (Figure 1a):

$$f \equiv \tau_{oct} - k \qquad (10)$$

where τ_{oct} is the octahedral shear stress

k is a material constant.

The Tresca yield law (Figure 1b):

$$f \equiv |\sigma_1 - \sigma_3| - k \qquad (11)$$

where σ_1, σ_3 are the maximum and minimum principal compressive stresses, respectively

k is a material property.

Improvements to those laws were made in order to incorporate the influence of normal stress into the degree to which a material could be sheared before yielding, an influence which cannot be neglected when dealing with the behaviour of the soil skeleton. The Drucker-Prager criterion (Drucker and Prager, 1952) (Figure 1c) is the direct generalisation of the von Mises law:

$$f \equiv \tau_{oct} - k_1 p - k_2 \qquad (12)$$

where k_1, k_2 are material properties and $p = \frac{1}{3}\sigma_{kk}$

while the Mohr-Coulomb yield criterion (Figure 2a) constitutes a similar generalisation of Tresca's:

$$f \equiv |\sigma_1 - \sigma_3| - k_1 - k_2(\sigma_1 + \sigma_3) \qquad (13)$$

where k_1, k_2 are material properties.

The latter has been by far the most popular of the above yield

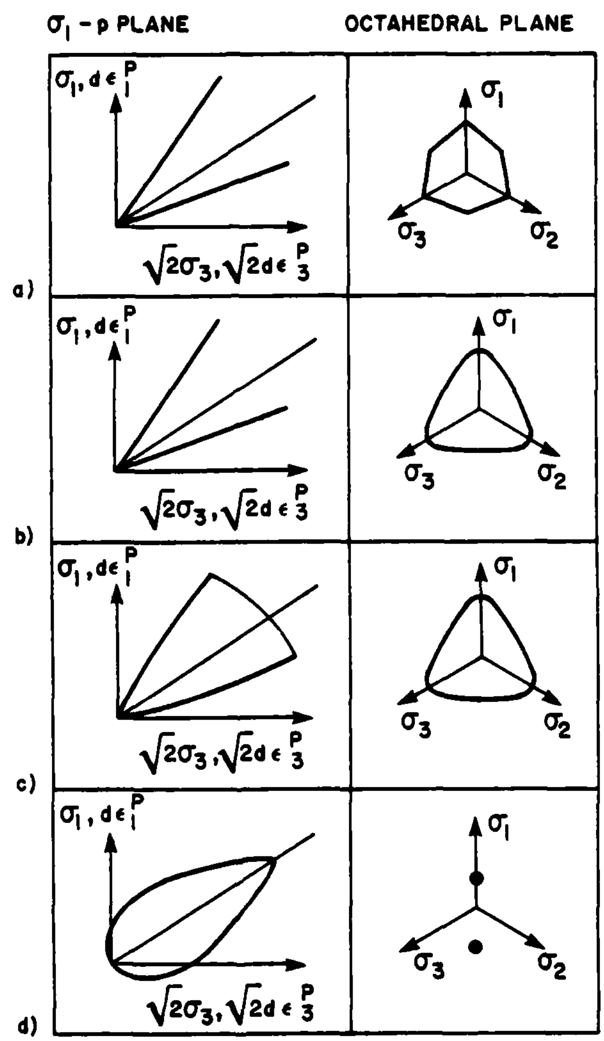

FIGURE 2 : INTERSECTION OF PLASTIC POTENTIAL SURFACES WITH THE σ_1 - p AND OCTAHEDRAL PLANES

a) Associated Mohr-Coulomb
b) Lade and Duncan
c) Lade
d) Cam-Clay

laws for describing soil failure. However, if failure is not reached, ideal elasto-plastic models are simply elastic laws. They are seldom used for describing stress-strain behaviour of soils because of this inadequacy. Also, the consequences of their associative flow rules are contradicted by experiments. Although other more complex ideal elasto-plastic models exist, the basic lines of the criticism are common for all of them. Their presentation is however, considered necessary for setting the background of elastic-plastic behaviour, which was the purpose of the present section.

3.4 ELASTIC NON-IDEALLY PLASTIC MODELS

Since elastic ideally-plastic models cannot reproduce the observed behaviour of soils (see Section 4.4), many modifications to ideal plasticity have been proposed: their goal is to maintain the ability to yield given by the originally ideal plasticity while giving better representation of other features displayed by soils.

While yield is controlled by yield surfaces, f, and the direction of the plastic strain increment is given by the plastic potential, g, the relationship between plastic strains and stress is established through the hardening function, h. The plastic strain increments can be derived, according to Hill (1950) as:

$$d\varepsilon^{p}_{ij} = h \frac{\partial g}{\partial \sigma_{ij}} df \qquad (14)$$

3.4.1 Prévost's Model

Jean Prévost, presently at Princeton University, developed a model for representing the constitutive behaviour of clays under undrained conditions (Prévost, 1977 and 1978a). He later extended the model to include volumetric behaviour (Prévost, 1978b). Models of this type had been proposed earlier in the metals literature by Mroz (1967, 1969).

Prévost's model can be described as a plastic model which incorporates isotropic and kinematic hardening. The model is characterised in stress space by a collection of nested yield surfaces, the outermost of which has the character of a failure surface. The surfaces are ellipsoids of revolution with the axis initially aligned with the hydrostatic axis.

In the absence of isotropic hardening, the material behaviour remains linear elastic inside the smallest yield surface (which can degenerate to a point). Each surface is characterised in general by a pair of plastic stiffnesses (shear and bulk) and a dilatancy property. As the stress point follows its path, it carries along all the yield surfaces that it tries to intersect. The material constants associated with each location along the stress path are those of the yield surface most recently touched. This scheme clearly provides a very natural incorporation of anisotropy, non-linearity and hysteresis.

Isotropic hardening is introduced by making the radii and other parameters of each yield surface depend on measures of the plastic volumetric and shear strains.

The flow rules are non-associative in inner yield surfaces and associative in the failure surface.

To the author's knowledge, only Prévost has published implementations of his model to date. They essentially consist of reanalysis of standard soil tests (1977, 1978a, 1978b) and pressuremeter tests (1979a), as well as applications to offshore gravity structures (1978a, 1978b, 1979b).

The general formulation can be summarised as follows:

yield surfaces:

$$f \equiv \frac{3}{2}\left[s_{ij} - \alpha_{ij}^{(m)}\right]\left[s_{ij} - \alpha_{ij}^{(m)}\right] + \frac{9}{2}\left[p - \beta^{(m)}\right]^2 - \left[k^{(m)}\right]^2 \tag{15}$$

associative flow rule (outer surface):

$$\dot{\varepsilon}_{ij}^{p} = \frac{3}{2H'_p}\left[\sigma_{ij} - \zeta_{ij}^{(p)}\right]\frac{\left[\sigma_{k\ell} - \zeta_{k\ell}^{(p)}\right]\dot{\sigma}_{k\ell}}{\left[k^{(p)}\right]^2} \tag{16}$$

non-associative flow rule (inner surfaces):

$$\dot{\varepsilon}_{ij}^{p'} = \frac{3}{2H'_m}\left[\sigma_{ij} - \zeta_{ij}^{(m)} + a_m\delta_{ij}\right]\frac{\left[\sigma_{k\ell} - \zeta_{k\ell}^{(m)}\right]\dot{\sigma}_{k\ell}}{\left[k^{(p)}\right]^2} \tag{17}$$

where $1<m<p$

$s_{ij} = \sigma_{ij} - p\delta_{ij}$ is the deviatoric stress tensor

$\zeta_{ij}^{(m)} = \alpha_{ij}^{(m)} + \beta^{(m)}\delta_{ij}$ and $k^{(m)}$ are the centre coordinates and radius of the mth surface, respectively

H'_m is the plastic modulus associated with surface m

$a_m = \frac{1}{3}\, A_m \left(\left[s_{ij} - \alpha_{ij}^{(m)}\right]\left[s_{ij} - \alpha_{ij}^{(m)}\right]\right)^{1/2}$

A_m is a material property describing dilatancy

Isotropic hardening appears by making H'_m, $k^{(m)}$ and possibly A_m functions of two measures of plastic strain, which are the following integrals along the strain path:

$$\varepsilon^p_{kk} = \int \dot{\varepsilon}^p_{kk} \tag{18}$$

$$\lambda = \int \left[\frac{2}{3}\, \dot{e}^p_{ij}\, \dot{e}^p_{ij}\right]^{\frac{1}{2}} \tag{19}$$

where $\dot{e}^p_{ij} = \dot{\varepsilon}^p_{ij} - \frac{1}{3}\, \dot{\varepsilon}^p_{kk}\delta_{ij}$ is the deviatoric strain rate tensor.

It should be noticed that, in this model, the intersection of all initial yield surfaces and the failure surface with the deviatoric subspace yields loci of constant octahedral shear stress (von Mises); the shape (although not the size or position) of these surfaces remains unchanged.

A model for soils, similar to Prévost's, has recently been proposed by Mroz et al., (1978).

3.4.2 Lade's Model

The model or, rather, models proposed by Poul Lade are essentially elastoplastic models; they hinge on the interesting suggestion that, at a given mean pressure, the failure surface is curved and can be expressed as a very simple function of the first and third stress invariants. This function has the advantages of generating a shape which coincides well with experimental results (it can be seen in Figure 2b that it looks like a smoothed Mohr-Coulomb hexagon), of incorporating rather naturally the influence of the intermediate principal stress and of expanding with mean pressure in a way which is at least qualitatively consistent with experience.

Other similar surfaces have been proposed in the literature (Garidel-Thoron, 1977; Matsuoka and Nakai, 1977, among others).

The initial theory predicted elastic behaviour under proportional loading, a straight failure line in the Mohr diagram and displayed similar volumetric behaviour during yield at all confining pressures. It was used to fit sand and clay test results (Lade and Duncan, 1975; Lade and Musante, 1978).

In the more recent model (Lade, 1977 and 1978), the yield surfaces are composed of two parts: the first one, responsible for the development of "plastic expansive" strains, has a bullet shape with apex at the origin:

$$f_p \equiv \left[\frac{I_1^{\,3}}{I_3} - 27\right]\left[\frac{I_1}{P_a}\right]^m - \eta \qquad (20)$$

the failure surface has the same expression for $\eta = \eta_1$

where m, η_1 are material properties

p_a is the atmospheric pressure

The former theory is recovered for $m = o$. The second part of the yield surfaces (only existing in the new theory) also includes the incorporation of a collapse yield criterion (a spherical cap) which is responsible for the "plastic collapse" increment of strain. Its equation is:

$$f_c \equiv I_1^2 + 2I_2 \qquad (21)$$

This cap never represents a failure condition. The plastic potential function on the bullet-shaped yield surfaces is of the same form of the yield surface:

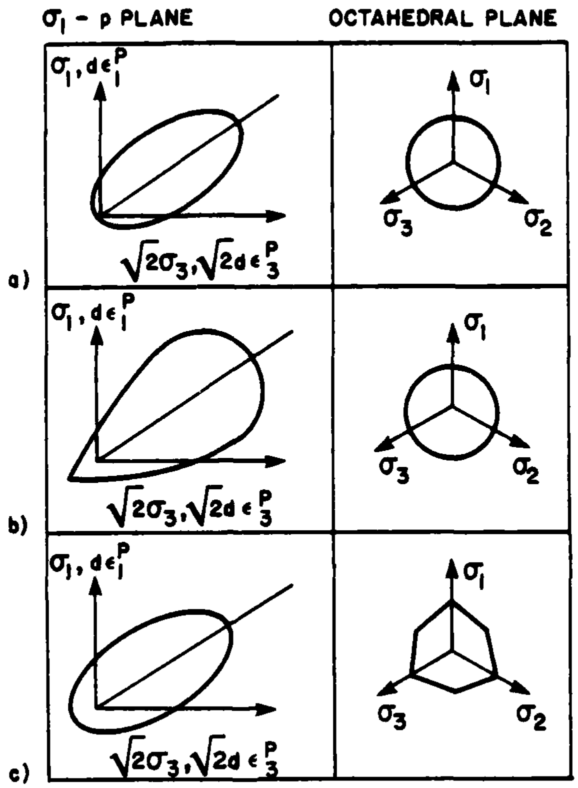

FIGURE 3 : INTERSECTION OF PLASTIC POTENTIAL SURFACES WITH THE σ_1 - p AND OCTAHEDRAL PLANES

a) Modified Cam-Clay
b) Cap Model
c) Zienkiewicz's Viscoplasticity

$$g_p = I_1^3 - \left(27 + \eta_2 \left[\frac{p_a}{I_1}\right]^m\right) I_3 \tag{22}$$

where η_2 is a constant for given values of f_p and σ_3.

The cap has an associated flow rule $g_c \equiv f_c$. The complete plastic potential surface is shown for the old and the new theories (Figures. 2b and 2c respectively).

The hardening law is obtained from empirical curve-fits of experimental data relating plastic work to the degree of hardening, f. Only isotropic hardening is contemplated. Power-law curve fits are used for both the bullet-shaped yield surfaces and the cap.

Elastic strains are obtained from a σ_3 - dependent Young's modulus and a constant Poisson's ratio.

3.4.3 Cap Models

Cap models have a yield surface composed of two parts: a fixed failure envelope of the form $f \equiv \tau_{oct} - F(p)$ and a movable cap with rotational symmetry about the p-axis (see Figure 3). The model name refers to this cap.

The idea of the cap was first introduced by Drucker et al. (1957). It was then implemented in the Cam-clay model (Schofield and Wroth, 1968) which was developed for triaxial conditions and later modified by Burland (1965) and Roscoe and Burland (1968) (see Figures 2c and 3a). It is this latter cap which has been introduced, together with a modified Drucker-Prager failure criterion, for prediction of blast effects and is now known as cap model.

Cap models are principally active at Weidlinger Associates and the US Army at WES. They are somewhat biased in that, historically, they have been mainly interested in accurate representation of blast-related effects, such as dynamic compressibility. On the other hand, investigators motivated by earthquake problems almost neglect such effects and are mainly concerned with the distortional characteristics of the soil.

In its present form, the cap model was presented by DiMaggio and Sandler (1971) and more recently by Sandler et al. (1976). The fixed failure envelope used is typically (Nelson, 1977):

$$f \equiv \tau_{oct} + k_1 \exp \{ - k_2 p \} - k_3 \qquad (23)$$

where k_1, k_2, k_3 are material constants.

Other forms of the cap have also been used; for example Baladi and Rohani (1979a) use a linear expression in τ_{oct} and p. The movable cap is somewhat arbitrarily taken as an ellipsoid:

$$f \equiv p - L^{2} + R^2 \tau_{oct}^{2} - (S - L)^2 \qquad (24)$$

where the parameters of the ellipse depend on the plastic volume change already experienced by the soil. Since the cap moves and changes shape, curve fitting of laboratory results allows reproduction of the volume changes which take place under different stress paths. The flow rule is always associative in order to guarantee a well posed problem, a central question for cap-model workers.

Behaviour is elastic inside the surface, although, in order to introduce some energy dissipation and rate effects, viscoelasticity was implemented inside the cap by Nelson (1977). Non-linear elasticity was also used inside the surface (Sandler et al., 1976).

Since the model is formulated in terms of stress invariants, anisotropic effects are unnatural and awkward to simulate. Nevertheless, Sandler and DiMaggio (1973) looked into the theoretical requirements to extend the formulation to the orthotropic case. This was implemented by Baladi (1977) simply by weighing differently the contribution of each stress component to the stress invariants.

Other recent modifications to the model are the introduction of an internal yield surface with kinematic hardening (Sandler and Baron, 1979) and the application of the model in effective stress space (Baladi and Rohani, 1977, 1978, 1979a, 1979b). A number of numerical implementations of the model as modular subroutines have been published, the most recent one being by Sandler and Rubin (1979).

3.4.4 Pender's Model

Within the general framework of critical state soil mechanics, Pender has presented a model for describing the behaviour of overconsolidated soil (1978) and extended it to normally consolidated soil (1977).

Common to most models within the critical state concepts is the advantage of a small number of material parameters and the disadvantages of having been developed from triaxial experience (two principal stresses equal) and not offering obvious ways of generalisation to three-dimensional conditions. In its initial version for soil with an isotropic history, apart from the assumption of constant volume unlimited distortion at the critical stress ratio, the model assumes: isotropic behaviour, no plasticity under proportional stress changes, parabolic undrained stress paths and negligible elastic distortions (see Figure 4). This results in an elasto-plastic formulation where the elastic strain (purely volumetric) is:

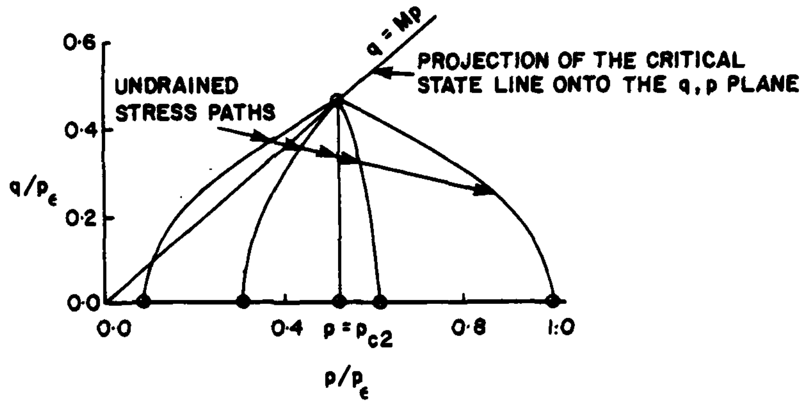

FIGURE 4 : PARABOLIC UNDRAINED STRESS PATHS IN PENDER'S MODEL

$$d\varepsilon_{kk}^{e} = \frac{\kappa \, dp}{p(1+e)} \tag{25}$$

where κ is the slope of the swelling line in the e - lnp plot

e is the void ratio

and the plastic component of strain is given by:

$$d\varepsilon_{ij}^{p} = h \frac{\partial g}{\partial \sigma_{ij}} df \tag{26}$$

where $f \equiv q - \eta p$ (27)

$$\frac{\partial g}{\partial p} = \left[\frac{p_o}{p_{cs}} - 1\right] \left[M - \eta \frac{p}{p_{cs}}\right] \tag{28}$$

$$\frac{\partial g}{\partial q} = 1 \tag{29}$$

$$h = \frac{2k\eta}{M^2 \ (1 + e) \ p_{cs}\left[\frac{2p}{p} - 1\right] \left[M - \eta \frac{p}{p_{cs}}\right]} \tag{30}$$

where η is the parameter describing the yield loci

p_o is the initial point of the loading path

p_{cs} is the end of the path,that is, p at the same void ratio on the critical state line

M is the stress ratio at the critical state.

Pender also generalised the model to a non-isotropic initial stress state with the assumptions that M is the same in extension and compression and that yield loci follow the stress point (kinematic hardening). This results in the following changes with respect to the former theory:

$$\frac{\partial g}{\partial p} = \frac{(AM)^2 \left[\frac{p_o}{p_{cs}} - 1\right] \left[(AM - \eta_o) - (\eta - \eta_o) \frac{p}{p_{cs}}\right]}{(AM - \eta_o)^2} \tag{31}$$

$$h = \frac{2k(\eta - \eta_o)}{(AM)^2\, p_{cs}\, (1 + \ell) \left[\frac{2p_o}{p} - 1\right]\left[AM - \eta_o - (\eta - \eta_o)\frac{p}{p_{cs}}\right]} \qquad (32)$$

where A is 1 in compression and -1 in extension

η_o is η at the beginning of loading.

Based on the latter model, Pender (1977a) has developed a formulation for cyclic loading, completed with an elastic shear modulus. Carter et al. (1979) have proposed a similar model.

The author is not aware of any implementations of Pender's model for solution of boundary value problems.

3.4.5 Other Elastic Non-Ideally Plastic Models

Many other elastic non-ideally plastic models have been proposed but are not included in this report. The critical state theory of soil mechanics, started with the Cam-clay (Schofield and Wroth, 1968) and the modified Cam-clay (Roscoe and Burland, 1968) has probably been one of the most fruitful in providing ideas incorporated into more sophisticated models. Although no special section has been dedicated to critical state theory because its developers did not try to implement it for irregular loading histories, the cap models (Section 3.4.3) and Pender's model (Section 3.4.4) are direct inheritors of that theory; as is, to a certain extent, Lade's latest model (Section 3.4.2). Among the interesting developments of the initial theory are the limited incorporation of anisotropy (Ohta and Wroth, 1976) and the recent extension to cyclic loading (Carter et al., 1979).

Other models of this type have been developed which try to incorporate the rate dependence displayed by some clays. This has given rise to models of the viscoplastic type, such as those developed by Zienkiewicz et al. (1975) and Adachi (1976). The former uses an ellipsoidal cap similar to the modified Cam-clay theory but producing a Mohr-Coulomb intersection with hydrostatic planes (see Figure 3). The latter is an extension of critical state theory to include rate sensitive properties.

Another interesting suggestion was made by Hardin (1978), who proposed the use of two different types of plastic potential surfaces depending on whether the loading path was quasi-hydrostatic or quasi-deviatoric.

3.5 ENDOCHRONIC MODELS

Endochronic models were first proposed by Valanis (1971a, b) for describing metal plasticity in an attempt to represent elasto-plastic behaviour as observed in nature, without the arbitrary dicotomies resulting from the introduction of yield and failure surfaces. The endochronic models are inspired on a mixture of plasticity and linear viscoelasticity concepts; from the latter they borrow their constitutive formulation:

$$\sigma_{ij} = \delta_{ij} \int_0^z \lambda(z - z') \frac{\partial \varepsilon_{kk}}{\partial z'} dz' + 2\int_0^z \mu(z - z') \frac{\partial \varepsilon_{ij}}{\partial z'} dz' \tag{33}$$

where $\lambda(z)$, $\mu(z)$ are the equivalent of relaxation moduli. Anisotropic materials can be represented by giving tensor character to those functions.

The only change from viscoelasticity is the substitution of Newtonian time by the endochronic time scale z, which is a monotonic function of a strain measure $\zeta = \int \left[P_{ijk\ell}\, d\varepsilon_{ij}\, d\varepsilon_{k\ell} \right]^{1/2}$ where the integration is carried out along the strain path and $p_{ijk\ell}$ is a metric tensor. The function relating z and ζ is responsible for the hardening or softening characteristics of the model. As can be seen, the model is practically identical to the "thermorheologically simple" material, where temperature effects are introduced as a change of the time scale in the convolution integrals.

The initial theory was rather attractive. It reproduced strain hardening and softening, cross-hardening and other features of metal plasticity well (Valanis, 1972 and 1974); it could also be formulated in elegant thermodynamic terms (Valanis, 1975) and implemented in dynamic problems (Wu et al., 1975) as well as creep and relaxation of metals (Valanis and Wu, 1975). However, it presented the problem of being unable to close hysteresis loops in the first quadrant and showed paradoxes in the patterns of energy dissipation.

Because of the latter problems, Valanis (1978) introduced a new endochronic time, where the strain measure is based on something similar to plastic rather than total strains. For example, in simple shear:

$$d\zeta^2 = p_{ijk\ell}\, d\Theta_{ij}\, d\Theta_{k\ell} \tag{34}$$

where $d\Theta_{ij} = d\varepsilon_{ij} - \dfrac{k}{2\mu_o}\, d\sigma_{ij}$

μ_o is the initial shear modulus (for $z = 0$)

k is a material property close to 1; when $k = 1$, Θ_{ij} is the plastic strain.

Valanis has applied this improved model to soils (1978). Meanwhile, Bazant and his co-workers have continued using the older model for describing concrete (Bazant and Bhat, 1976; Bazant and Kim, 1979) and sands

(Bazant and Krizek, 1976: Cuellar et al., 1977; Cuellar, 1977; Krizek et al., 1978). Reed and Hegemier (1976) were among the early proposers of this theory for soil modelling.

Unfortunately, most of the applications to date have been one-dimensional, in particular all of those for soils. Also, $p_{ijk\ell}$ has usually been taken as $\delta_{ij}\delta_{k\ell}$. Furthermore, since convolution integrals with non-linear terms are very awkward to handle and the relaxation moduli are not easy to measure and implement, they have degenerated into combinations of a few springs and "endochronic sliders". This is similar to representing viscoelastic models by combinations of springs and dashpots, rather than using general moduli or compliances determined from tests. But it must be realised that, when taken in this simplified context, the "theory" becomes a differential equation representing an arbitrary combination of springs and endochronic sliders (with history dependent characteristics) where all the parameters are determined by curve-fitting.

3.6 THE FAILURE-SEEKING MODEL

The Failure-Seeking model (Cundall, 1979) extends the one-dimensional scheme of Pyke (1979) and Cundall (1976) to two-dimensions. It is not a true constitutive model but a rule to generalise a one-dimensional model for monotoric shear to arbitrary paths in more dimensions of deviatoric stress. In a way, it is comparable to Masing's rule for generating cyclic from monotonic behaviour. Although Cundall (1979) has only presented it for two dimensions, its generalisation to the complete deviatoric subspace is relatively straightforward. In one dimension, a given stress-strain curve in shear is used as a basis for all excursions in the stress-strain space. The curve is scaled at each stress strain reversal, with the scaling factor depending on the ratio of current stress to failure stress; i.e. the size of the stress-strain curve is adjusted starting from the current reversal point, so that the calculated stress will be asymptotic to the failure stress at large strain. The "magnification factor" (by which the given curve is multiplied) is:

$$m = \frac{\tau_{max} - \tau_c}{\tau_{max}} \qquad (35)$$

In two-dimensions the given curve is used to compute the magnitude of the stress increment tensor from the magnitude of the strain increment tensor. The curve is scaled according to the distance of the current stress point from the failure surface in stress space; hence the name 'Failure-Seeking'. The direction of strain increment may be related to the direction of stress and the direction of stress increment, but the relation is unspecified as yet.

In two-dimensions, it is not clear what constitutes a reversal since the strain increment vector in strain space can change direction by anything from 0 to π. An empirical relation is used to adjust the scaling factor, m, as a function of $\Delta\theta$, the change in strain-increment direction.

The Failure-Seeking model has only been proposed in skeleton form to date; it does not include volumetric behaviour or its coupling to shear behaviour. The Failure-Seeking model was put forward as an attempt to obtain realistic cyclic behaviour from a simple scheme that did not need to store large numbers of variables; it is also as a means for generalising the applicability of the data obtainable with existing testing equipment.

4.0 COMPARISON OF EXISTING MODELS WITH DESIRED FEATURES

In this chapter, the models presented in Chapter 3 are discussed and assessed on the basis of the characteristics that they should ideally display and which were listed in Chapter 2. For clarity, the models are discussed in the same order as in Chapter 3.

4.1 ELASTIC AND VISCOELASTIC MODELS

There is no point in listing the desired characteristics of soil that elastic models fail to display since this includes practically all of those mentioned in Chapter 2. However, it is worth mentioning that, as the deformation of soil is restricted to smaller strains, progressively better approximations can be obtained with linear elastic models. Because of this, they can be used for analysis of low-energy wave propagation with some success. Absolute adequacy cannot be assured even for those cases since a) soils appear to dissipate energy internally even at very low deformation levels, and b) in some high explosive tests, field measurements did not show elastic behaviour even at large distances and small energy levels (Trulio, 1978).

On the other hand, the material properties required are few (e.g. 2 in the isotropic case, 5 in an orthotropic material) and can be determined easily from standard tests. Analysis becomes very inexpensive compared with other methods. Linear elasticity fulfills all continuum mechanics requirements and there are theorems available which demonstrate existence, uniqueness and stability of solutions to elastic problems.

The same comments can be applied to linear viscoelasticity, irrespective of the complexity or the anisotropy of the compliances. Biot's correspondence principle guarantees that viscoelastic solutions can be extracted from elastic ones in most engineering problems by means of Laplace transforms, a requirement easily handled by modern FFT (Fast Fourier Transforms) routines. Unfortunately, proportionality, superposition and viscous energy dissipation are phenomena which are seldom observed in soils, the exception probably being small-strain deformation of soft clays. For these reasons, linear viscoelasticity is not used very frequently for dynamic analysis in geotechnical problems.

As for non-linear viscoelasticity, it raises very difficult questions of determination of properties and solution of boundary value problems if considered with some generality. In practice, it is usually applied as rather arbitrary curve fits which reflect more the curve-fitting technique than the actual soil behaviour (i.e. straight line fits in log-log plots originate power laws of the type $\dot{\gamma} = k\tau^m$, etc). In this respect they suffer from limitations similar to those pointed out in the next section.

4.2 ONE-DIMENSIONAL CURVE FITS

Although Cundall (1979) has recently presented a two-dimensional generalisation of Pyke's model (see Section 3.6), and Prévost's (see Section 3.4.1) can be thought of as a six-dimensional Iwan model, the basic problem presented by all models mentioned in this section is that they allow infinite generalisations to more dimensions. This occurs basically because such models are not formulations of theories of soil deformation but curve-fits of experimental data under specific conditions. The basic reason for their extended use in dynamic modelling is the common assumption among earthquake engineers that seismic effects may be sufficiently understood by modelling the earthquake as a train of vertically-propagating, horizontally-polarised shear waves.

Let us take, for example, the model used by Finn and his co-workers. It has been derived by curve-fitting the results from cyclic, simple-shear tests. It includes all the desired features sought from models under one-dimensional shear, including the coupling to the volumetric behaviour and the influence of effective normal stress. Since the parameters of the model are determined by fitting simple shear tests, it is to be expected that, as long as the samples tested are representative of the field material and the test configuration is adequate, the model will correctly predict soil behaviour in the field when that soil is deformed in simple shear. But, what happens if the conditions in the field are different from simple shear? The question cannot be answered; there is no justification for any generalisation of Finn's hyperbolic functions or his quadratic shear-volumetric coupling to cases other than one-directional simple shear. Professor Finn is perfectly aware of this limitation and has restricted the application of his model to such problems.

Finn's model has been selected for comment in this section because it is probably one of the best for reproducing soil behaviour under the conditions for which it is derived. But identical criticisms can be tabled for all the other models presented in Section 3.2. Notice that the natural generalisations through the invariants (which assume isotropy and some form of coaxiality) are certain to fail due to history induced anisotropy and the non-coaxiality of stress and strain or stress and strain rate. Gross errors can be expected from neglecting such effects.

The conclusion hence must be that, for the time being, none of these models can be used in general dynamic calculations, particularly if applied to analysis of blast effects. This does not mean that their development should not be closely monitored. They do provide sometimes useful information for restricted stress paths; besides, it is possible that plausible rules (similar to Masing's for generalising monotonic to cyclic behaviour or to Cundall's in Section 3.6) may be formulated for generalising cyclic to multidimensional behaviour, in which case the usefulness of those models could be enhanced. In the meantime, their utilisation should be restricted to the same conditions for which they are derived.

4.3 *IDEAL ELASTO-PLASTICITY*

As already suggested in Section 3.3, elastic ideally-plastic models cannot be applied with generality to describe soil behaviour. This is not to say that they are useless for all purposes; on the contrary, they are sufficient for many applications. For example, bearing capacitites can usually be estimated fairly accurately with simple elasto-plastic models, the reason being that the failure mechanism is often essentially independent of the behaviour of the non-failing material. But for the rather arbitrary stress paths generated by blasting, particularly as the distance from the blast increases, these laws are clearly insufficient.

The reasons are obvious from an examination of Chapter 2. Von Mises and Tresca lack any normal stress dependence. Apart from that, and common to all elastic ideally-plastic models, no non-linearity or plastic volume changes occur prior to failure. At failure, non-associated flow rules must be used since associated Mohr-Coulomb and Drucker-Prager plastic potentials only predict constant dilation (usually too large) and Von Mises and Tresca require no volume change. No permanent deformations or energy dissipation appear prior to failure. Isotropic compression always produces elastic volume changes (linear and recoverable) and no memory or history dependence.is displayed by the models.

There is a point left over for comment related to the behaviour under circular stress paths (p and τ_{oct} constant). The different intersections (circular or hexagonal) with the octahedral plane presented by the models suggest different behaviours on those paths. The difference relates basically to the influence of the intermediate principal stress, σ_2 on the failure criterion. Experiments tend to show non-circular shapes for that intersection, curves between Mohr-Coulomb and that displayed by Lade's model. However, it is not clear whether such differences in the effect of σ_2 generate significant departures in the behaviour of the soil masses under analysis; the answer is probably no.

Finally, it is worth noting that the existence of corners in plastic potential surfaces (Tresca and Mohr-Coulomb) create discontinuities in the strain-rate direction, the physical meaning and numerical implementation of which are not free of problems.

The considerations contained in the last two paragraphs should be kept in mind while assessing other more complex models.

4.4 ELASTIC NON-IDEALLY PLASTIC MODELS

As will be seen throughout the present section, elastic non-ideally plastic models afford much greater flexibility than their ideal counter-linearities, permanent straining and hysteretic damping observed in soils prior to failure. Using non-associative plastic potentials, the direction of the plastic strain increment can be adequately controlled (assuming that the required experimental data exist).

Because of their greater flexibility, elastic non-ideally plastic models are more affected by lack of reliable data on soil behaviour. An example is the transition from isotropic to deviatoric behaviour. It would be reasonable to expect that small deviations in the stress path would produce small deviations in the strain path; hence, plastic potentials should have a continuous normal in the region where they intersect the p-axis and the transition from isotropic to other loading paths should be smooth. But observed behaviour under purely deviatoric paths is so different from that under purely isotropic paths, that it is not always simple to reconcile the two, a difficulty which led Hardin (1978) to propose two different types of plastic potentials depending on the loading path.

4.4.1 Prévost's Model

From the viewpoint of representing soil behaviour, Prévost's model is probably one of the best which have been proposed. It appears to have the capability to fulfill most of the conditions set out in Chapter 2 for representing soil behaviour. As can be seen, the stress-strain law has been "discretised"; this results in errors which, in principle, need not be worse than those following from the space or time discretisations used for solving boundary value problems by most numerical procedures. However, it must be noted that small amplitude excursions will not dissipate energy, which may result in unrealistic enhancements of high frequencies, if the spacing between yield surfaces is too coarse. Non-uniform, such as logarithmic, spacing can be used to minimise this problem.

Of particular attraction is the handling of stress-induced anisotropy through kinematic hardening; although it is not obvious that all history-induced anisotropy can be accounted for by this procedure, it is at least qualitatively correct. The isotropic hardening function may however prove difficult to determine in practice. Besides, the two invariant measures in which Prevost bases the isotropic hardening have identical weight for all components of the strain increment. It is not evident on theoretical grounds why those two measures are sufficient. Third order measures (of the type $\left[d\dot{e}^p_{ij}\ d\dot{e}^p_{jk}\ d\dot{e}^p_{k\ell}\right]^{1/3}$) and/or weighted measures (for example, $\left[p_{ijk}\ \ d\dot{e}_{ij}\ d\dot{e}_{k\ell}\right]^{1/2}$) may be required. Although, with respect to the latter, based on the curve fitting experience of the endochronic school, $p_{ijk\ell} = \delta_{ij}\ \delta_{k\ell}$ may well prove sufficient.

One of the aspects of greatest concern is whether the volumetric coupling introduced through the A_m properties (affected by isotropic hardening) is sufficient to generate the observations gathered, for example, under multi-dimensional shear. This would be a most effective way of assessing whether the elaborate six-dimensional approach followed is really adequate for drained soil conditions. The problem is really that of the smooth transition between isotropic and deviatoric loading which was discussed ealier in the section.

Unfortunately, the model has been applied very sparingly to boundary value problems. Also, since the questions on the validity of the model cannot easily be resolved by tests with existing equipment, a comparison with field measurements could have been helpful. Although some practical work seems to have been done with the model (Prévost, 1979c), the results are not in the public domain.

The author is not aware of a proof that Prévost's model generates well posed problems. With the exception of the isotropic hardening rules, all parameters are easily determinable from standard tests: for example, one triaxial extension test and one triaxial compression test are sufficient for their determination.

A problem of this model is the large number of variables which require storage during computer calculations: apart from those common to all models (stresses, etc), at least the six coordinates of the centre of each ellipsoid require storage for each point in the soil mass. This is believed to greatly restrict the general applicability of the model.

4.4.2 Lade's Model

Lade's model is extremely convincing as long as isotropy is preserved. With that exception it complies with all the requirements listed in Chapter 2 for describing soil behaviour. Its representation of monotonic loading and the shear-volumetric coupling is indeed excellent. The model does require over ten parameters, some of which are purely curve-fitting constants but all of which can be determined from isotropic compression and triaxial compression tests.

Unfortunately, it is not yet known how well the model can reproduce cyclic behaviour. The author suspects that, due to its formulation in terms of invariants and lack of kinematic hardening, Lade's model is unable to deal effectively with either natural or stress-induced anisotropy. Although Lade appears to be experimenting with ways of introducing kinematic hardening in his model (Lade, 1979) no results in that direction are known to the author. The stress-strain response is a function of the position of the stress point in stress space but is otherwise independent of past deformation.

Those are very serious limitations for applicability of the model to blast problems. However, it is hoped that further work will be carried out to correct such deficiencies in this otherwise promising model.

The model can be readily incorporated into a computer program in its present form without special penalties of time or storage requirements. However, the author is not aware of any applications of the model for solution of boundary value problems.

4.4.3 Cap Models

It has already been mentioned that the developers of the cap models (in the form presented in Section 3.4.3.) have historically been interested primarily in reproduction of blast effects. Close-in, such effects are known to consist essentially of a single high-frequency, uniaxial compression cycle, subject to whatever spatial symmetry constraints are presented by the configuration of the blast. Because of this, cap workers worried primarily about achieving acceptable behaviour on excursions along the hydrostatic axis and only later tried to extend the reliability of the model to paths more complex and further away from the p-axis, the ones which take place in the lower energy, outrunning region. The result is that cap models represent soil much better under hydrostatic than deviatoric loading.

Even in more advanced versions, cap models have a single yield surface (before the failure surface) inside which shear behaviour is elastic with the natural consequences on energy dissipation, linearity and permanent straining. For a model of the complexity of the cap model, this is rather limited. The fact that the yield surface moves to generate kinematic hardening improves somewhat the former inadequacy. Notice that viscoelastic behaviour inside this surface cannot be considered a solution (see Section 4.4.5.). The circular intersection of the failure surface compares unfavourably with others such as Mohr-Coulomb, Lade, etc, but this is probably not too important.

The handling of anisotropy in the cap models is also limited. Because of its formulation in terms of stress invariants, cap models should have been restricted to isotropic behaviour. Differential weighting of stress components in the expression of the invariants is a clever though cumbersome procedure to extend the applicability to orthotropic (even fully anisotropic) materials, but only as long as their principal axes of anisotropy and stress coincide. This condition cannot be guaranteed under arbitrary stress paths.

The number of constants to be determined from tests is rather large, usually more than ten even in the simpler versions and many more in the more complex ones. Most of the constants are non-standard and lack physical significance. The model is amenable to implementation in present computers and has been used frequently for calculations in the past.

A good point of the model is the guarantee of generating well-posed problems which is afforded to it by Drucker's postulates under the conditions of associated flow rule and convex yield law. But this point is usually over emphasised by cap model defenders. In the first place, there is a fair amount of evidence against associated flow rules for soils; this is why the cap model is one of the very few models still using them. Hence, it appears that the associated flow rule is kept on the grounds of mathematical safety, rather than soil mechanics. Second, Drucker's postulates are sufficient but not necessary conditions. Proofs of existence, uniqueness and stability of solutions may take decades or not come at all for certain complex problems, even if those properties hold.

Very often it is simpler to find a solution than to prove the existence of solutions to the general problem; and not being able to do the latter should not preclude trying the former. Engineering judgement should in any case be exercised when considering computer solutions; the chances of finding more than one "credible" solution are obviously smaller than those of finding two solutions. Also, parametric runs are always a very commendable exercise which, among other things, has the virtue of showing the sensitivity of the solution to the input data. In summary, if a model generates problems which can be shown to be well posed, this is a happy circumstance; but this should never be an overriding factor in the selection of a model if doing so requires disregarding experimental evidence.

4.4.4. Pender's Model

More attention should be dedicated to this model than has been given in the past. The model has two basic limitations : it is restricted to triaxial paths and it is formulated in terms of invariants with its well known restrictions on anisotropy. But, within those constraints, the reproduction of soil behaviour is indeed very good, particularly when account is taken of the fact that only five well established parameters have to be determined for the soil (only four if the problem tolerates a material with unbounded shear stiffness for small strains).

The parameters in Pender's model have clear physical meanings and are easily measureable in standard tests. The model should be easy to implement in a computer program although this does not appear to have been done to date. Minimum storage will be required for the model free of kinematic hardening; however, with the latter, the model may require large storage under unfavourable stress paths since it will need to remember all stress reversals of progressively decreasing amplitude. This effect can be minimised by dynamic allocation of storage.

The worst limitation of the model is its restriction to triaxial paths. It is hoped that generalisations to other paths, although not immediately obvious, might be possible so that the encouraging prospects of the model might be more fully realised.

4.4.5. Other Elastic Non-Ideally Plastic Models

Little can be said in this section since the models in Section 3.4.5 were mentioned rather than described; a few words, however, on viscous effects in soils. Viscous effects certainly arise as a consequence of movements of the interstitial water. But if, as in the present report, the problem is posed in effective stresses (with the behaviour of the fluid and its interaction with the particles being handled by appropriate independent equations) rate effects practically disappear. This is particularly true for sands and lean clays where a wealth of information points out this rate-insensitivity (see for example Youd, 1972; Hardin and Drnevich, 1972a & b; Seed, 1976). For soft clays there seems to be some rate sensitivity which usually does not result in changes of more than 10% in the properties over a decade of strain rates. As a consequence, although consolidation effects may be approximately simulated by viscous effects in total stress analyses, neglecting them appears to be reasonable in effective stress models. This statement would obviously need re-examination in problems where long term creep of soft clays is of primary concern, particularly at higher temperatures.

Finally, although Hardin's suggestion of double plastic potentials may in some cases be used to advantage, it cannot be considered more than a temporary solution since a true model must somehow reconcile the quasi-deviatoric with the quasi-hydrostatic behaviour at their meeting point.

4.5 ENDOCHRONIC MODELS

The endochronic models could have constituted a very encouraging step forward if their initial formulation had provided a good representation of soil behaviour. The idea of the material keeping track of the flow of events by measuring the amount of deformation undergone during those events is very attractive. However, the complexity of the formulation, the need to solve convolution integrals with non-linear factors and the number of functions which had to be determined precluded the application of that formulation for cases other than those sufficiently simple to admit close-form solutions. Further, the need to correct empirically the strain measure and to introduce more than one endochronic time have forced the utilisation of more simplified formulations for soils.

In the form in which they have been applied, they are really curve-fits of one-dimensional shear behaviour and they belong in Sections 3.2 and 4.2 rather than here. Some of them have also partly lost their rigorous mathematical formulation and have been shown to be thermodynamically inconsistent (Sandler, 1977). The criticisms in Section 4.2 should therefore be considered to apply to the endochronic models so far used for soils. The only reason why they have been given a special section is because of their applications in other materials and the discussion that they have generated in the soil mechanics community.

4.6 THE FAILURE-SEEKING MODEL

The Failure Seeking model is treated here to show that, for crude analyses, it is possible to construct models which are very simple, involve very few constants, require minimium, storage and still behave, at least qualitatively, in the manner observed in soil experiments. Admittedly, many of the features displayed (take for example the energy dissipated during small stress changes from an overall loading path) arise directly from the different curvatures at different points in the curve; therefore they cannot be tuned to match experimental data. But, at least, they are qualitatively correct, which is already better than can be said of other, more complex models.

No volumetric coupling yet exists, although those used by Finn, Krizek, etc., could be borrowed directly. In any case, the work done on this model is still insufficient to admit complete assessment. But there are certainly applications for the idea of using simple models which naturally display most of the known characteristics of soil behaviour, even if only a few of those can be independently controlled; this must be compared with the idea of taking a simple mode (say linear elastic-ideally plastic) and complicate it progressively in attempts to include more and more features of soil deformation in a well controlled fashion.

5.0 CONCLUSIONS AND RECOMMENDATIONS

5.1 SUMMARY AND CONCLUSIONS

Based on past experience, a number of features observed in soil deformation have been listed, all of which should ideally be displayed by soil constitutive models. Other desirable characteristics which affect the applicability and usefulness of models, even though not directly related to soil behaviour, have been proposed to complete the model requirements. Obviously, both lists can and will be built upon as more modelling experience is gathered.

A survey of the literature on constitutive laws of soil has been conducted. It is by no means comprehensive, but the author hopes to have included representatives of the main past and present trends in constitutive modelling of soils. A succinct presentation has been made of the models considered to have special interest or future possibilities.

The models presented have been discussed mainly on the basis of the criteria gathered previously. This discussion is summarised in Table 1. To the obvious question of which model is the best, no unique answer exists. Lade's model seems best for quasi-monotonic loading paths; Prévost's, for mainly deviatoric irregular loading; cap models for mainly isotropic irregular loading; Pender's is ideal for analysing triaxial paths economically. Overall, taking account of the requirements of modelling of blast effects, Prévost's is probably superior although, for close-in calculations, it will need time to accumulate the experience already gathered by cap models in stress excursions along (or close to) the hydrostatic axis. The requirements

Constitutive models \ Requirements of the model (Sec. 2)	characteristics of soil (Sec. 2.1): behaviour under simple stress paths (Sec. 2.1.1): 1-D monotonic shear	1-D cyclic shear	isotropic monotonic compression	cyclic isotropic compression	multi dimensional paths	history effects (Sec.2.1.2)	other observations (Sec.2.1.3)	other desirable features (Sec.2.2): - satisfy principals of mechanics - well posed problems	- few constants - easy to measure	implementable in present technology
Elasticity (E) and Viscoelasticity (V/E)	linear, unbounded, uncoupled from normal stress, no induced volume changes, V/E is rate dependent	no cumulative strains or volume changes; no hysteresis	linear and unbounded	no permanent strains or hysteresis (E); rate dependent (V/E)	bad	not considered	bad	- yes - yes	- yes - yes	yes
One-dimensional Curve fits (ODCF)	potentially good	good if provided with memory; usually adequate volume treatment	potentially good	potentially good	not considered	potentially good but only for paths fitted	potentially good	- usually yes - usually no	- no, when good - not very easy	yes but may require unlimited memory
Elastic Ideally Plastic (EIP)	as E until failure; then, volume changes are not realistic	as E if not failed; unaffected by no. of cycles	as E	as E	bad	not considered	in general, no energy dissipated by small changes	- yes - yes	- yes - yes	yes
Elastic Non-Ideally Plastic (ENIP): Prévost (Pr)	good	good	good in the version including it	good	circular paths are neutral	probably good	good if yield loci are carefully spaced	- probably - unknown	- no - yes, except for hardening function	yes but require large storage and expense
Elastic Non-Ideally Plastic (ENIP): Lade (Ld)	good	does not include provisions for unloading	good	good	not considered	no kinematic hardening	limited by not treating reversals; no anisotropy	- yes - unknown	- no - not very easy	probably yes but not done
Elastic Non-Ideally Plastic (ENIP): Cap (Cp)	good if variable moduli are used; otherwise, as EIP	good capabilities of fit with variable moduli; otherwise, as EIP	good	good	circular paths are neutral	very restricted kinematic hardening	as EIP; also limited anisotropy	- yes - yes	- no - not very easy	yes
Elastic Non-Ideally Plastic (ENIP): Fowler (Fn)	good in the version including finite initial shear modulus	unaffected by number of cycles	good	good	only considers triaxial conditions	no kinematic hardening	very limited anisotropy possible	- yes - unknown	- yes - yes	yes
Endochronic (End)	good	good	as ODCF	as ODCF	not considered	no kinematic hardening	potentially good	- some doubts - unknown	- no - not very easy	yes
Failure Seeking (FS)	no volumetric coupling	no volume changes	not considered	not considered	potentially good	potentially good	potentially good	- probably - unknown	- no - probably not	yes

TABLE 1 : SUMMARY OF THE DISCUSSION OF PROPOSED MODELS

of a computer memory for implementation of Prévost's model are admittedly large; but the feature of general kinematic hardening is considered to be a very important requirement for more accurate calculations by future models.

5.2 LIMITATIONS OF THE PRESENT STUDY AND RECOMMENDATIONS FOR THE FUTURE

The limitations of the present study arise from two main sources:

- o The physical impossibility of covering all constitutive models proposed to date. This problem is not of major concern since, in spite of the large number of existing models, a fair cross-section has been selected for review, including those models which are thought or expected to be of greater interest to the US Air Force.
- o The scarcity of data on which to base comparison of the different models. This is not to say that too few laboratory experiments have been performed but, rather, that the experiments performed include too few different test configurations and stress paths. Also, problems of sampling disturbance and stress relief are far from solved, with obvious consequences on the interpretation of test results. In summary, the jump between the laboratory experience and the field predictions is still too large and, at the present pace of progress, it is likely to remain so for at least a decade.

Several paths are open to the US Air Force for improving this situation. Pursuing all those in parallel and, if possible, in cooperation with other interested organisations, is probably the best alternative. The recommended lines for advancement are:

i. Laboratory equipment needs to be specifically designed to generate stress paths on which little, or no information exists. In particular, there is urgent need for devices which can rotate the principal directions of stress or strain in a well controlled manner. Professors R. Arthur (University College, London) and C. Ladd (MIT, US) have first generation equipment for tests of this type but much more work and further development are required.

ii. Since hardware cannot be expected to progress very fast, particularly into the six-dimensional stage, it seems advisable to obtain as much information as possible on deformation mechanisms and constitutive behaviour by numerical modelling at the fundamental level. This produces fast, repeatable results free of experimental problems. Dr. P.A. Cundall (Consultant, England) and Professor O. Strack (University of Minnesota, US) are among the initiators of such techniques (Cundall and Strack, 1979a & b) but this type of work is only starting compared with its possibilities.

iii. The same set of test data should be represented by several constitutive models, bearing in mind the recommendations of the present report. The different models could then be used for predicting ground motion and the results compared with actual measurements. This has been done occasionally in the past; but it needs to be carried out much more frequently in order to determine the true reliability of each model on a safer basis than the rather theoretical considerations used in the present report. This procedure can be followed for both past and future tests.

iv. There is also a need for more in situ testing to define the behaviour of soils without the problems related to sampling and laboratory testing. However, this is a much longer term effort due to the comparatively large investment required (both in development and applications) since in situ testing traditionally lags behind laboratory techniques in capabilities. A good example of this type is the Borehole Shear Device sponsored by the US Air Force (Sidey, et al., 1979; Marti and Rodriguez-Ovejero, 1980; Sidey and Bassett, 1980).

APPENDIX A - SYMBOLS

a_m	a variable in Prevost's model related to yield surface m
A	1 or -1 (Pender)
A_m	a material property related to yield surface m
$C_{ijk\ell}$	anisotropic elastic property tensor
$d\lambda$	multiplier in flow rule
e	voids ratio
f	yield function
f_c	cap-like f in Lade's model
f_p	bullet-like f in Lade's model
F,G,H	arbitrary functions
H'_p	plastic modulus
g	plastic potential
g_c	cap-like g in Lade's model
g_p	bullet-like g in Lade's model
h	hardening function
i,j,k,ℓ	co-ordinate indices
I_1, I_2, I_3	stress invariants
$\{k\} = \{k_1, k_2 \ldots\}$	material constants (different meanings in different models)
$k^{(m)}$	radius of yield surface m
L	parameter of cap model
m	exponent (Lade) magnification factor (Failure-Seeking Model)
M	slope of critical state line
n	an integer

p	$\left\{\begin{array}{l}\text{mean pressure}\\ \text{number of yield surfaces}\end{array}\right.$
p_a	atmospheric pressure
p_{cs}	p at critical state
$P_{ijk\ell}$	metric tensor
p_o	initial p
q	stress deviator
R	parameter of the cap model
s_{ij}	deviatoric stress tensor
S	parameter of the cap model
t	time
t'	integration variable
z	endochronic time scale
z'	integration variable
$\alpha_{ij}^{(m)}$	deviatoric co-ordinates of centre of yield surface m
$\rho_o^{(m)}$	isotropic co-ordinates of centre of yeild surface m
γ	shear strain
γ_c	shear strain at last reversal
δ_{ij}	Kronecker delta
Δ	increment
ε_{ij}	strain tensor
ε_{ij}^e	elastic strain tensor
ε_{ij}^p	plastic strain tensor
ηo	initial η
η_1, η_2, η_3	identification of yield or potential surfaces (Lade, Pender)
Θ_{ij}	reduced strain tensor (endochronic)

ζ	strain measure (endochronic)
$\zeta_{ij}^{(m)}$	co-ordinates of centre of yeld surface m (including deviatoric and isotropic co-ordinates)
κ	slope of compression line
λ	$\left\{\begin{array}{l}\text{1st Laine constant}\\ \text{deviatoric strain measure}\end{array}\right.$
$\lambda_{ijk\ell}$	anisotropic 1st Laine constant or 1st Laine relaxation modulus
μ	shear modulus
μ_o	initial shear modulus
$\mu_{ijk\ell}$	anisotropic shear modulus or shear relaxation modulus
σ_{ij}	stress tensor
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
τ	shear stress
τ_c	shear stress at last reversal
τ_{max}	limiting shear stress
τ_{oct}	octahedral shear stress

APPENDIX B

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