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AN INDUCTION THEOREM FOR DISCOVERING SYNTACTIC TRANSLATIONS.(U)
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(6) AN INDUCTION THEOREM FOR
DISCOVERING SYNTACTIC TRANSLATIONS.

by

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ABSTRACT

Given an input-output sequence of syntactic translations of sentences generated by a deterministic finite state grammar G into Σ^* , a method is given for discovering the function which maps productions of G into Σ^* that gives rise to the observed translation.

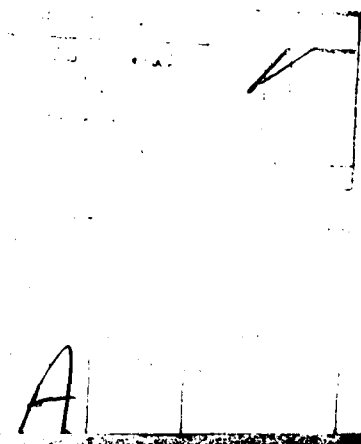
1. INTRODUCTION

Let $G = (V_N, V_T, P, S)$ be a right linear grammar [2]. Thus all productions in P are of the form

$$A \rightarrow aB \quad \text{or} \quad A \rightarrow a$$

where A and B are syntactic variables in V_N , and a is a terminal (or word) in V_T . We shall assume that G is deterministic, by which we mean that for every pair $(A, a) \in V_N \times V_T$ there is at most one production in P of the above form. We denote the set of sentences generated by G by $L(G)$.

With G we shall associate what we shall call the wiring diagram G of G .



Definition. Let G be a right linear grammar. Then the wiring diagram G of G is a directed pseudograph [3] with labelled arcs. The node set $N(G)$ is $V_N \cup \{F\}$, where F is a symbol not in $V_N \cup V_T$. The arc set $A(G)$ is determined by the productions of G : if $A \rightarrow aB$ is an element of P then $A \xrightarrow{a} B$ is a labelled arc of G ; if $A \rightarrow a$ is an element of P then $A \xrightarrow{a} F$ is a labelled arc of G .

For example, if $G = \{\{S, T, U, V\}, \{a, b, c\}, P, S\}$ where $P = \{S \rightarrow aV | bT, T \rightarrow aT | cU | b, U \rightarrow bS | a, V \rightarrow cU | bU\}$, then G is shown in Figure 1.

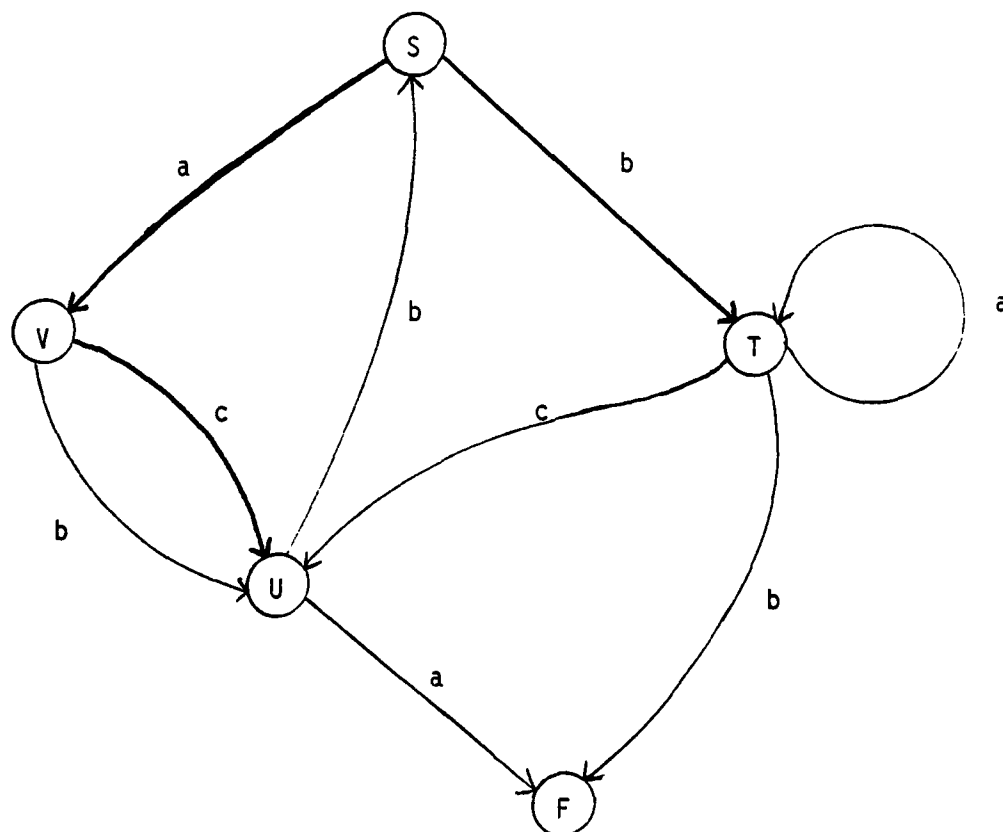


Figure 1.

There is obviously a natural correspondence between the elements of $L(G)$ and the set of walks from S to F in G ; i.e.,
 $L(G) = \{x_1 \dots x_n \mid S \xrightarrow{x_1} X_1, X_1 \xrightarrow{x_2} X_2, \dots, X_{n-1} \xrightarrow{x_n} F \text{ are labelled arcs of } G, \text{ for some } X_1, \dots, X_{n-1} \in V_N\}$. We shall assume throughout this paper that for each $A \in V_N$ in G there is a path from S to F that passes through A .

Definition. Given a deterministic right linear grammar G and a finite abstract set of symbols $\Phi = \{\phi_1, \dots, \phi_s\}$, a syntactic translation is a map f from $A(G)$ to Φ^* .

If $A \xrightarrow{a} B$ is a labelled arc of G and if the image of this arc under f is ϕ where $\phi \in \Phi^*$, then graphically we write

$$A \xrightarrow{a \mid \phi} B$$

(Φ^* is the set of finite length sequences from Φ , including Λ , the empty string).

This definition is basically equivalent to the definition of a generalized sequential machine (gsm) [1], where f is called an output function.

By extending the definition of f in the natural way we have

$$f^{ex}: L(G) \rightarrow \Phi^* ;$$

i.e., if we have under f

$$S \xrightarrow{a_1 \mid \phi^{(1)}} A_1, \dots, A_{n-1} \xrightarrow{a_n \mid \phi^{(n)}} F$$

with $\phi^{(1)}, \dots, \phi^{(n)} \in \Phi^*$, then the sentence

$$a_1 a_2 \dots a_n \xrightarrow{f^{ex}} \phi^{(1)} \phi^{(2)} \dots \phi^{(n)} .$$

In the syntactic translation as shown in Figure 2,

$$ba^2b \rightarrow \phi_5\phi_4\phi_5\phi_5\phi_4\phi_1$$

$$acbaba \rightarrow \phi_3\phi_1\phi_3\phi_1\phi_2\phi_3\phi_1\phi_3\phi_1\phi_1\phi_2,$$

etc.

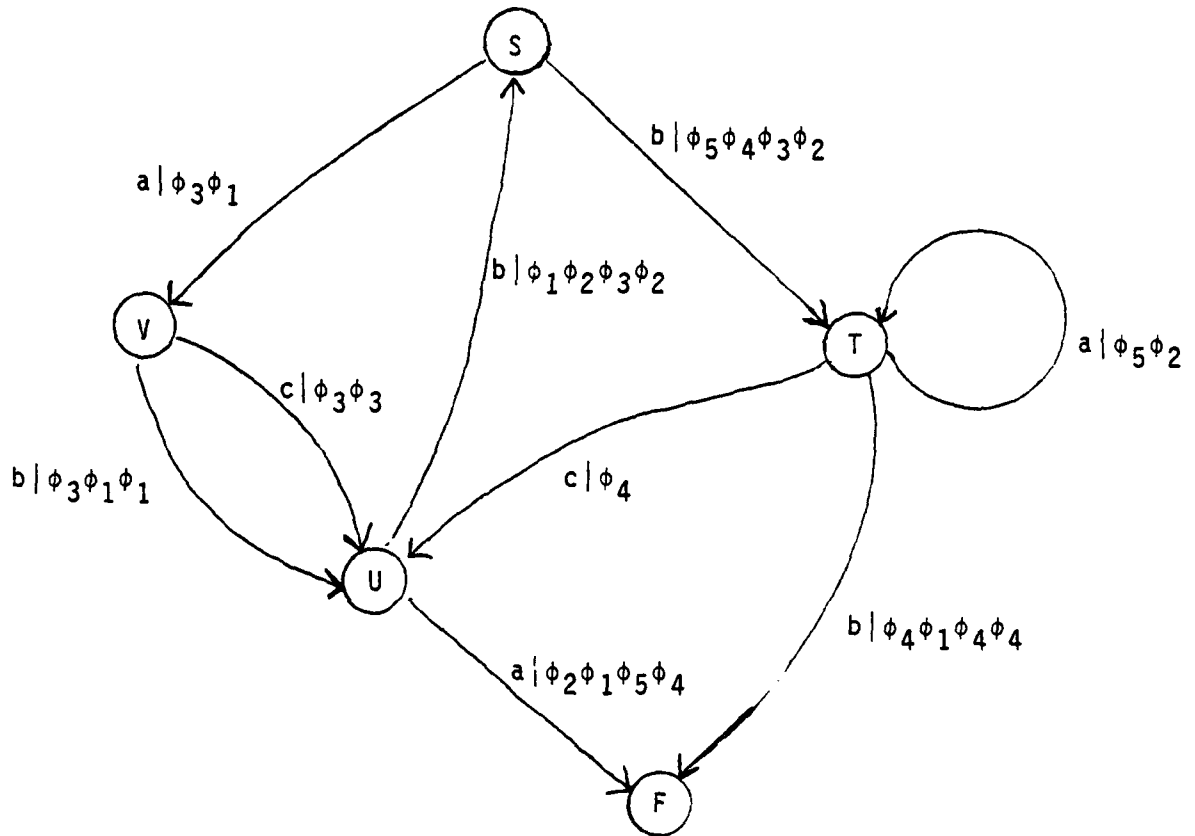


Figure 2.

Let $A(G, \phi^*)$ be the set of syntactic translations of G , and let $A^{ex}(L(G), \phi^*)$ be the extension of A to $(\phi^*)^{L(G)}$. We shall refer to elements of $A^{ex}(L(G), \phi^*)$ as syntactic maps.

2. TREE COMPOSITIONS

Definition. Let Σ be a finite alphabet, and $x \in \Sigma^*$. A k-composition of x is defined to be an ordered k -tuple $c \in (\Sigma^*)^k$, $c = (c_1, \dots, c_k)$ having the property that $c_1 c_2 \dots c_k = x$. The set of k -compositions of x is denoted $C_k(x)$.

For example, if $\Sigma = \{a, b, c\}$, then $C_3(ab^2c)$ is the set $\{(\Lambda, \Lambda, ab^2c), (\Lambda, a, b^2c), (\Lambda, ab, bc) \dots\}$ where Λ denotes the empty word. In general, $|C_k(x)| = \binom{n+k-1}{k-1} = \binom{n+k-1}{n}$ if $|x| = n$.

The notion of composition is extended to trees.

Definition. Let Σ be a finite alphabet, T a rooted directed tree $T = (N(T), A(T))$. Thus T is a directed tree with a distinguished node $R \in N(T)$, and for each node $N \in N(T)$ there is a unique directed path from R to N . The leaves of T , denoted $L(T) \subset N(T) - R$ are the nodes of T with degree 1. Assume the elements of $L(T)$ are ordered $L_1, \dots, L_\ell$ where $\ell = |L(T)|$. For a given element $x = (x_1, \dots, x_\ell) \in (\Sigma^*)^\ell$ a T-composition of x is defined by a function

$$A(T) \xrightarrow{t^c} \Sigma^*$$

having the property that for each leaf L_j of T , and unique path $a_1, \dots, a_k \in A(T)$ from R to L_j ,

$$t^c(a_1) t^c(a_2) \dots t^c(a_k) = x.$$

Thus a tree composition reduces to a k -composition when the tree is a rooted path consisting of k connected arcs. An example of a tree composition of (ab, ab, b, ba) is shown in Figure 3, for the complete binary tree with 7 nodes. Given T , along with an ordering for the leaves, and $x \in (\Sigma^*)^{L(T)}$ we denote the set of all tree compositions of x by $TC(T, x)$.

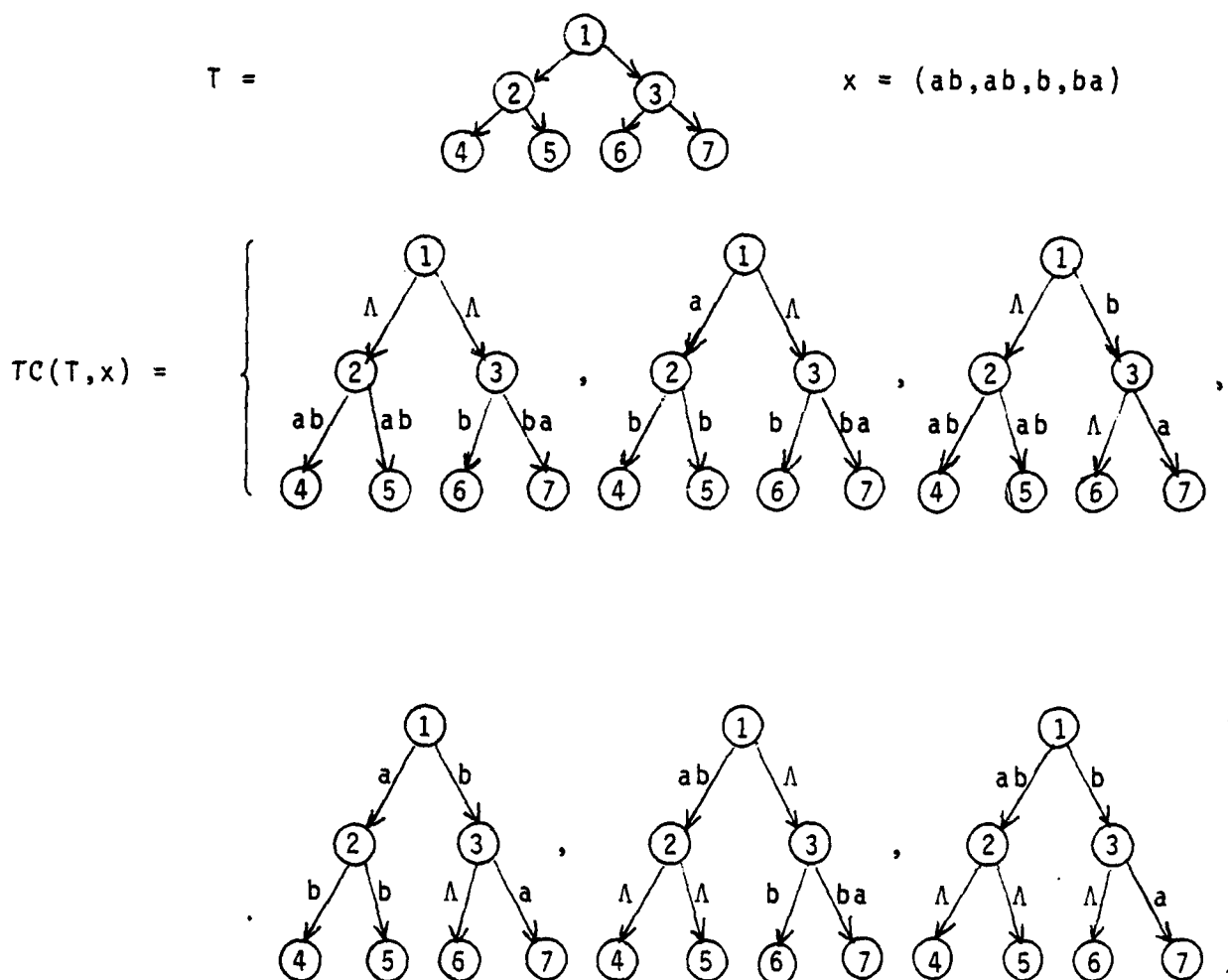


Figure 3.

An element of $TC(T, x)$ can be represented as a non-negative integer lattice point in a natural way:

If $a_1, \dots, a_{|A(T)|}$ is some ordering of the arcs, then

$$t^C(a) \longrightarrow |t^C(a)| \quad a \in A(T)$$

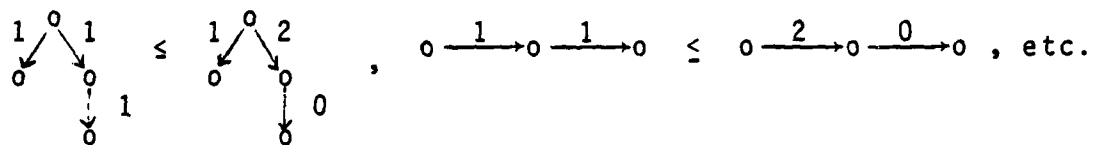
specifies a lattice point in $L = \mathbb{N}^{|A(T)|}$, $\mathbb{N}$ = non-negative integers.

We denote by $S[TC(T,x)]$ the set of lattice points defined above. A partial order $\leq_T$ is defined in L : for $s, t \in L$

$$s \leq_T t \text{ iff } t \text{ is obtained from } s$$

by moving objects up the tree.

For example,



We define, for $S \subseteq L$, $\max S$ = the elements of S having the property that for no $t \in S$: $s \leq t$, $t \neq s$.

3. THE INDUCTION PROBLEM

It is possible for two distinct syntactic translations to be extended to the same syntactic map. Thus we define an equivalence relation, $\sim$, on $S(G, \phi^*)$ by defining $f_1 \sim f_2$ iff f_1 and f_2 are extended to the same element of $S^{ex}(L(G), \phi^*)$.

The induction problem for syntactic translations is this: an observer O , who we assume knows the internal structure of the wiring diagram G except for the syntactic translation, can observe sentences from $L(G)$ along with their image in ϕ^* under the unknown syntactic translation. Thus he can observe the syntactic map for a few sentences in $L(G)$. O wishes to discover an element $f \in S(G, \phi^*)$ (up to equivalence) such that f^{ex} holds. We assume O can pick the sentences he wishes to observe. The theorem that follows shows, essentially, that O can pick a finite

number of sentences from $L(G)$ from which syntactic translation discovery is possible.

THEOREM: The syntactic translation (up to equivalence) can be discovered by observing a finite number of sentences W .

Remark: What the theorem says is that on observing a finite set W (to be constructed below), θ is presented with a finite number of word equations:

$$(E) \quad \begin{array}{l} a_{11}a_{12} \cdots a_{1i_1} = \phi(1) \\ \vdots \\ a_{k1}a_{k2} \cdots a_{ki_k} = \phi(k) \end{array}$$

where $|W| = k$, $a_{mn} \in A(G)$ (the arc set of G) and $\phi(j)$ the observed image in ϕ^* corresponding to the sentence determined by the walk $a_{j1} \cdots a_{ji_j}$ in G . A solution of E (that is, an assignment of values in ϕ^* to the arcs $A(G)$ so that E is satisfied) will solve the induction problem.

Proof: The proof follows the construction of the implicit functions in [4].

We construct a spanning tree T in G , rooted at S and connecting all nodes in V_N . F is not connected to the spanning tree. For the example of Figure 1, a spanning tree T is indicated by darkened lines.

Label the arc set $A(G)$ in such a way that $A(T)$, the set of arcs in the spanning tree are $a_1, \dots, a_t$.

From ϕ and $A(T) = \{a_1, \dots, a_t\}$ we create a new set of symbols. In general let X be a finite alphabet $\{x_1, \dots, x_n\}$. Then define X^0 to be the group freely generated by the symbols of X , with Λ the identity element. Form $(\phi \cup A(T))^0$.

Begin at F and consider all arcs a entering F . Call this set $A(F)$, $A(F) \neq \phi$. Take an element a in $A(F)$. In what follows if a is the arc $A \xrightarrow{x} F$ then $\alpha(a) = A$, $\omega(a) = F$. Thus $\alpha(a) \in V_N$ and thus there is some walk $w = a_{i_1}, \dots, a_{i_j}, a$ from S to F with $a_{i_1}, \dots, a_{i_j} \in A(T)$. The sentence determined by the walk w , call it s , is mapped to $\phi(s)$, which 0 observes and writes

$$a = a_{i_j}^{-1} \dots a_{i_1}^{-1} \phi \in (\phi \cup A(T))^0.$$

This is done for each element of $A(F)$.

0 now considers the arcs of $A(G) - (A(T) \cup A(F))$. Let $A^{(j)} =$ the set of arcs a of G not in $A(T)$ such that the number of arcs in the shortest path (a walk with no repeated nodes) from $\omega(a)$ to F is j (i.e., $A^{(0)} = A(F)$). Suppose 0 has computed the equations for the arcs in $A^{(0)}, \dots, A^{(j-1)}$. Let $a \in A^{(j)}$ and let $a, b_1, \dots, b_j$ be a shortest path from $\omega(a)$ to F . Now $\alpha(a) \in V_N$ hence

$$a_{i_1} \dots a_{i_j} a b_1 \dots b_j,$$

a walk from S to F , $a_{i_1}, \dots, a_{i_j} \in A(T)$. If this corresponds

to sentence s then 0 observes $\phi(s)$, so that

$$a = a_{i_j}^{-1} \dots a_{i_1}^{-1} \phi b_j^{-1} \dots b_1^{-1} \in (\phi \cup A(T))^0 \text{ by using the equations}$$

for $b_1, \dots, b_j$ from previous computations. This process terminates with a list of equations

$$(I) \begin{cases} a_{k+1} = g_1 \\ \vdots \\ a_q = g_{q-k} \end{cases}$$

where $g_1, \dots, g_{q-k}$ are elements of $(\Phi \cup A(T))^0$.

For example, from Figure 3 if we define the arcs

$$\left. \begin{array}{lcl} a_1 & S \xrightarrow{a} & V \\ a_2 & S \xrightarrow{b} & T \\ a_3 & V \xrightarrow{c} & U \\ a_4 & U \xrightarrow{a} & F \\ a_5 & T \xrightarrow{b} & F \\ a_6 & T \xrightarrow{a} & T \\ a_7 & T \xrightarrow{c} & U \\ a_8 & U \xrightarrow{b} & S \\ a_9 & V \xrightarrow{b} & U \end{array} \right\} \text{spanning tree .}$$

Then

$$\begin{aligned} a_1 a_3 a_4 &= \phi_3 \phi_1 \phi_3^2 \phi_2 \phi_1 \phi_5 \phi_4 \\ a_2 a_5 &= \phi_5 \phi_4 \phi_3 \phi_2 \phi_4 \phi_1 \phi_4^2 \\ a_2 a_6 a_5 &= \phi_5 \phi_4 \phi_3 \phi_2 \phi_5 \phi_2 \phi_4 \phi_1 \phi_4^2 \\ a_2 a_7 a_4 &= \phi_5 \phi_4 \phi_3 \phi_2 \phi_4 \phi_2 \phi_1 \phi_5 \phi_4 \\ a_1 a_9 a_4 &= \phi_3 \phi_1 \phi_3 \phi_1^2 \phi_2 \phi_1 \phi_5 \phi_4 \\ a_1 a_3 a_8 a_2 a_5 &= \phi_3 \phi_1 \phi_3^2 \phi_1 \phi_2 \phi_3 \phi_2 \phi_5 \phi_4 \phi_3 \phi_2 \phi_4 \phi_1 \phi_4^2 . \end{aligned}$$

These equations can be solved in the group $(\Phi \cup A(T))^0$ by the method indicated.

It follows from [4] that, given (I), the syntactic map is the same for all assignments of $a_1, \dots, a_k$ to elements of Φ^0 , and

hence ϕ^* . What this means is that, given the finite equations (I), an assignment of values in ϕ^* to the arcs of the spanning tree $a_1, \dots, a_k$ so that $a_{k+1}, \dots, a_q$ as defined by (I) are in ϕ^* will solve the induction problem. $\square$.

A sequence $a_1, \dots, a_k \in \phi^*$ such that $a_{k+1}, \dots, a_q$ are in ϕ^* is called a feasible point.

4. THE INDUCTION SOLUTION

The structure of equations (I) will help in solving the word equations. Instead of the equation $a_{k+r} = g_r$ in (I) let us consider its associated equation $r = 1, \dots, q-k$

$$\phi(r) = a_{i_1} \dots a_{i_j} \underline{a_{k+r}} b_1 \dots b_j$$

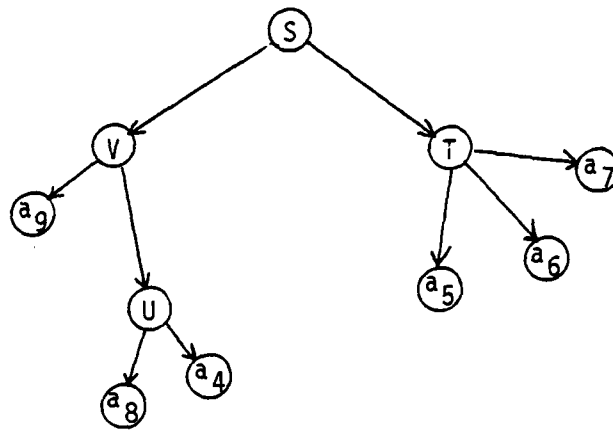
as determined in the proof of Theorem 1. Thus $a_{i_1} \dots a_{i_j}$ denotes a descent down the spanning tree T , a_{k+r} the unknown in (I), $b_1 \dots b_j$ a shortest path from $w(a_{k+r})$ to F .

From T we shall construct a new tree T' by adding leaves to T as follows. The new leaves will be labelled $a_{j+1}, \dots, a_q$ and will be directed respectively to the nodes

$$\alpha(a_{j+1}), \dots, \alpha(a_q).$$

Thus the spanning tree T of Figure 1 becomes T' in Figure 4.

If we consider $TC(T', x)$ where $x \in (\phi^*)^{q-k}$ $x = (\phi(1), \dots, \phi(q-k))$ is the vector of observed sentences from ϕ^* , then obviously the set of feasible points $a_1, \dots, a_k$ are in $TC(T', x)|_{a_1, \dots, a_k}$; that is, $TC(T', x)$ restricted to the arcs $a_1, \dots, a_k$. In some examples it turns out that a feasible point can be discovered by computing $\max [TC(T', x)]$, but this is not always the case. Consider Figures 5 and 6.



T'

Figure 4.

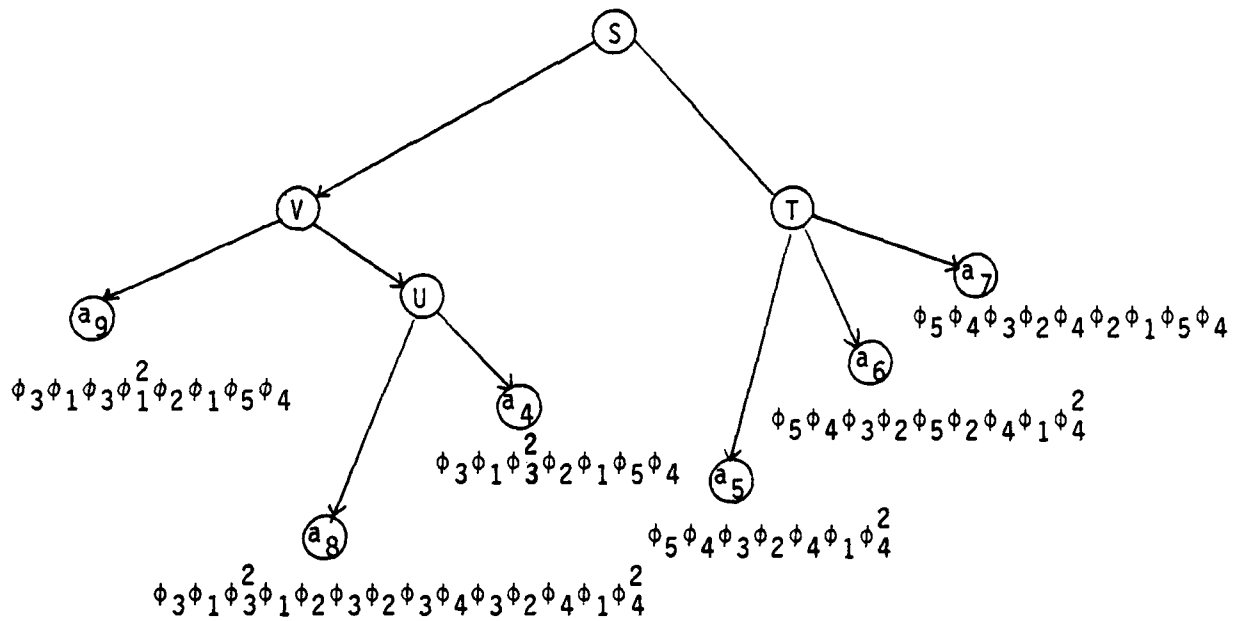
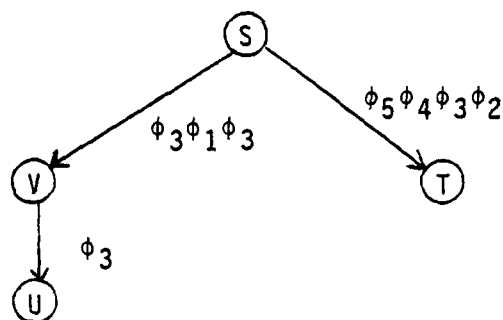


Figure 5.



$$\max TC(T', x) \Big|_{a_1, a_2, a_3}$$

Figure 6.

Figure 6 gives $\max TC(T', x) \Big|_{a_1, a_2, a_3}$, a feasible point (which is easily verified).

Figure 7 gives an example of a case where $\max TC(T', x) \Big|_{a_i \in T}$ is not a feasible point.

An obvious necessary condition, in addition to the feasible points being in $TC(T', x) \Big|_{a_i \in T}$, is

$$|\phi(r)| = |a_{i_1}| + \dots + |a_{i_j}| + |a_{k+r}| + |b_1| + \dots + |b_j|.$$

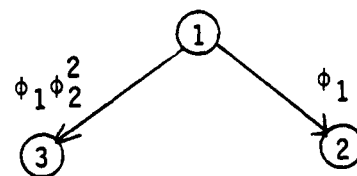
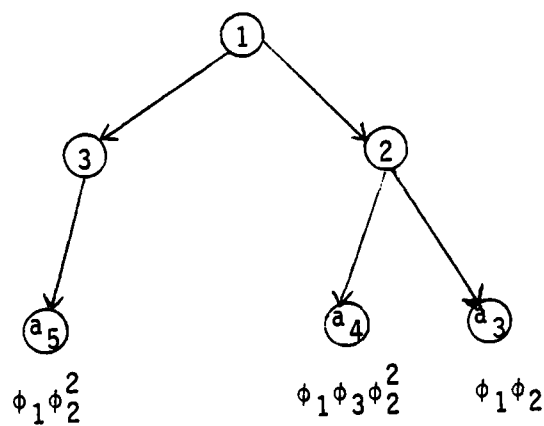
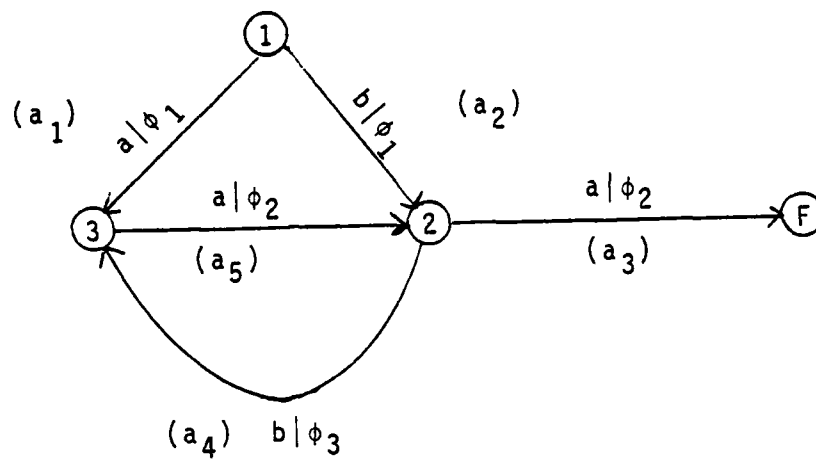
Note for the example in Figure 7, if we let $|a_i| = x_i$ then

$$x_2 + \underline{x_3} = 2$$

$$x_1 + \underline{x_5} + x_3 = 3$$

$$x_2 + \underline{x_4} + x_5 + x_3 = 4.$$

If $x_1 = 3$ and $x_2 = 1$, as we have in the $\max TC(T', x) \Big|_{a \in T}$ solution, then there is no (x_3, x_4, x_5) non-negative solution.



$$\max TC(T', x) \mid a_i \in T$$

Figure 7.

As before, we denote the word equation for the variable a_{k+r} by $(a_{k+r} \in A^{(j)})$

$$a_{i_1} \dots a_{i_\ell} \underline{a_{k+r}} b_1 \dots b_j = \phi(r) .$$

Let us now assume that $b_1 \dots b_j$ (a shortest path from $w(a_{k+r})$ to F) is chosen so that it is a suffix of a previously defined walk.

THEOREM: A sufficient condition for an assignment of arcs $a \in T$ to values in ϕ^* to be feasible is that it satisfies

$$\begin{aligned} & \max TC(T', \phi) \\ \text{subject to} & \quad (*) |w(j)| = \phi(j) \end{aligned}$$

where $w(j)$ is the walk from S to F corresponding to the variable a_{k+j} .

Proof: Let $\hat{\phi}(a)$, $a \in A(G)$, be the "true" unknown syntactic translation, so for $r = 1, \dots, g-k$

$$\hat{\phi}(a_{i_1}) \dots \hat{\phi}(a_{i_\ell}) \hat{\phi}(a_{k+r}) \hat{\phi}(b_1^{(r)}) \dots \hat{\phi}(b_j^{(r)}) .$$

Let $\phi(a) \big|_{a \in T}$ be the assignment determined by the criteria stated in the theorem.

We claim that for each $s = 1, \dots, \ell$

$\phi(a_{i_s}) \dots \phi(a_{i_\ell}) \phi(a_{k+r}) \dots \phi(b_j)$
is a suffix of

$$\hat{\phi}(a_{i_s}) \dots \hat{\phi}(a_{i_\ell}) \hat{\phi}(a_{k+r}) \dots \hat{\phi}(b_j) .$$

If this were not true, then we would have, for some s ,

$$\phi(a_{i_1}) \dots \phi(a_{i_{s-1}})$$

being a proper prefix of

$$\hat{\phi}(a_{i_1}) \dots \hat{\phi}(a_{i_{s-1}})$$

and this contradicts maximality.

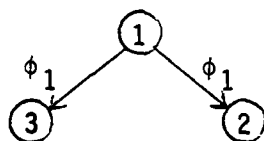
Consequently, $\phi(b_1) \dots \phi(b_j)$ is a suffix of $\phi(r)$ (by induction, $b_1 \dots b_j$ is of the form $a'_{i_s} \dots a'_{i_\ell} a'_{k+r} b'_1 \dots b'_j$ for a previously computed walk) $\phi(a_{i_1}) \dots \phi(a_{i_\ell})$ is a prefix of $\phi(r)$, so by (*) we have a solution in ϕ^* of $\phi(a_{k+r})$. $\square$

The example of Figure 7 shows that

$$\begin{cases} x_2 + x_3 & = 2 \\ x_1 + x_5 + x_3 & = 3 \\ x_2 + x_4 + x_5 + x_3 & = 4 \end{cases}$$

$$\Rightarrow (x_1, x_2) \in \{(0,0), (0,1), (1,0), (1,1), (1,2)\}.$$

$(x_1, x_2) = (1,1)$ corresponds to



$$\max TC(T', x) \Big|_{a \in T}$$

subject to (*)

which is indeed feasible.

It is evident that we may replace $TC(T', \phi)$ with a set of inequalities, i.e., for the example in Figure 7 we must have

$$\begin{aligned} x_1 &\leq 3 \\ x_2 &\leq 1, \end{aligned}$$

for the example in Figure 5

$$\begin{aligned} x_1 &\leq 3 \\ x_2 &\leq 4 \\ x_1 + x_3 &\leq 5. \end{aligned}$$

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