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AN ALGORITHM FOR COMPUTERIZED ADAPTIVE DECISION ANALYSIS

James L. Raney

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PERSONNEL AND MANPOWER TECHNICAL AREA

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additive-independence model (AIM) in pair-comparisons designs with fallible data. No provisions were made for handling the systematic error problem or for accommodating more than two choice component factors. The ICM algorithm was tested in error-free data and in data with random error. The results showed that this ICM algorithm performed rather poorly.

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AN ALGORITHM FOR COMPUTERIZED ADAPTIVE DECISION ANALYSIS

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AN ALGORITHM FOR COMPUTERIZED ADAPTIVE DECISION ANALYSIS

BRIEF

Requirement:

To develop new methods for analysis of decision-making behavior--specifically, new methods for (1) modeling a decision process used to evaluate preferences for complex choice alternatives, and (2) producing measurement scales for choice component factors and composite choice alternatives based on the decision model.

Procedure:

Recent results in conjoint measurement theory research were applied to develop an algorithm for minimizing the problems of redundancy and random error in testing the additive-independence model in pair-comparisons designs with fallible data. No provisions were made for handling the systematic error problem or for accommodating more than two choice component factors.

Findings:

The algorithm was tested in error-free data and in data with random error. A reduction of approximately 44% in the number of pair-comparisons necessary to determine all model constraints was obtained in a 5 x 5 factorial design with error-free data. This reduction was decreased to 33% when a moderate amount of random error was introduced. Various deficiencies in algorithm performance were noted. In general, the results showed that this algorithm performed rather poorly.

Utilization of Findings:

This research demonstrates some limitations of a direct application of the generalized cancellation condition for testing the additive-independence model. Further research is required to develop an analytical approach which can ameliorate these limitations.

AN ALGORITHM FOR COMPUTERIZED ADAPTIVE DECISION ANALYSIS

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INTRODUCTION

General Statement of the Problem

In Army research it is often desirable to investigate the nature of a decision process used by Army personnel in making some important choice. In many instances the choice alternatives may represent composites of several component factors which may be identified and considered separately. For example, in Army career planning research it may be of interest to investigate the nature of the decision process used by officers in indicating their preferences for various assignments. In this example the choice alternatives may be described as composites of component factors such as assignment location (e.g., EUROPE, CONUS), type (e.g., COMMAND, STAFF), and duty specialty (i.e., PRIMARY, ALTERNATE).

The additive-independence model (AIM) is one particular model of the decision process which has considerable appeal because of its simplicity in comparison with other possible models and because of its general applicability in other substantive areas of psychology. In this model component factor levels are assigned specific scale values relative to one another. The scale values assigned to levels of a component factor are designated independently for each component factor. The scale value attached to a particular composite choice alternative is determined by summing the scale values assigned to levels of the component factors which are present in the composite. The relative magnitude of scale values for composite choice alternatives serves as a basis for the preference decision.

For example, EUROPE = 1 and CONUS = 2 may be the scale values assigned to assignment location; COMMAND = 2 and STAFF = 3 may be the scale values assigned to assignment type; and PRIMARY = 5 and ALTERNATE = 1 may be the scale values assigned to assignment duty specialty. If an individual uses the AIM in evaluating preferences for various assignments, the scale value attached to a PRIMARY specialty COMMAND assignment in EUROPE is $5 + 2 + 1 = 8$; the scale value attached to a PRIMARY specialty STAFF assignment in CONUS is $5 + 3 + 2 = 10$. Since the composite scale value for the latter choice alternative exceeds the composite scale value for the former choice alternative, the individual would indicate a preference for the latter choice alternative. If composite scale values for the choice alternatives are equal, an individual may indicate no preference.

In research on the nature of a decision process two specific objectives may be identified. The first objective is to develop a model of the decision process used to evaluate preferences for complex choice alternatives. If individual differences in the decision process are discovered, an individual-specific model of the decision process may be

required. The second objective is to produce measurement scales for choice component factors and composite choice alternatives based on the decision model. The general purpose of this study was to investigate an application of recent results in conjoint measurement theory research with the aim of developing a new methodology for accomplishing these two research objectives.

Specific Statement of the Problem

The conjoint measurement problem is that of obtaining measurement scales for component stimuli and composite stimuli simultaneously based on a specified composition model when only the rank order of the composite stimulus effects is known. The order constraints generate a finite system of homogeneous equations and inequalities in the composition model. If the specified composition model is valid, the resulting system should be consistent and measurement scales for component stimuli and composite stimuli may be derived by solving the system for the unknown parameters. In the case of the AIM, the derived measurement scales constitute "multidimensional ordered-metric scales" (Krantz, Luce, Suppes, and Tversky, 1971, Chapter 9).

Although conjoint measurement theory has been particularly well-developed for the additive-independence composition model, practical applications with individual subjects have been few. Three formidable problems remain to be solved for applications of conjoint measurement theory in fallible ordinal data. First, the magnitude of the pair-comparison task may exceed the capability of the individual subject when the number of composite stimuli is large. If the composite stimulus set contains N elements, $N(N-1)/2$ nontrivial pair-comparisons must be made to provide a complete ordering of the composite stimuli. Since the number of pair-comparisons usually greatly exceeds the number of parameters to be estimated for the composition model, it is clear that a large amount of redundancy of effort is inherent in the complete pair-comparisons method. Second, the presence of random experimental error may introduce ordinal inversions in the ordering of composite stimuli. The result is an inconsistent system of equations and inequalities which provides only an approximation of the latent measurement structure. Third, the possible presence of systematic error may be difficult to detect in the presence of random error. Systematic error may result when the latent composition model differs from the additive model.

The specific purpose of this study was to develop an algorithm for interactive conjoint measurement (ICM) to minimize the problems of redundancy and random error for applications of additive conjoint measurement theory in fallible ordinal data. Although the algorithm presently has no provision for handling the systematic error problem and is limited to two component factors, extensions of the algorithm to test several models simultaneously with multifactor composite stimuli may be possible. In the next section the conjoint measurement-theoretical basis for the algorithm is presented. The notion of a constraints matrix is defined

and an ICM algorithm is described. An evaluation of the performance of the algorithm is provided in the last section.

INTERACTIVE CONJOINT MEASUREMENT

Measurement-Theoretical Basis

Let us represent component stimulus variables A and P as the finite sets $\{a, b, c, \dots\}$ and $\{p, q, r, \dots\}$, respectively, where the elements of a set are the particular component stimuli included in an experiment. Each composite stimulus may be represented as an element in the Cartesian product set $A \times P$. For example, composite stimulus (a, p) represents the combination of stimulus component a in A with stimulus component p in P . Suppose all possible pairs of composite stimuli are presented to a subject whose task is to indicate the preferred composite stimulus in each pair. The experiment gives rise to a binary preference relation (denoted $\tilde{\sim}$) on $A \times P$. The two sets of component stimuli A and P together with the binary relation $\tilde{\sim}$ defined on $A \times P$ constitute an empirical relational structure (denoted $\langle A, P, \tilde{\sim} \rangle$).

The empirical relational structure $\langle A, P, \tilde{\sim} \rangle$ may be said to satisfy the additive-independence model (AIM) if there exist real-valued component stimulus-scale functions ϕ_A on A and ϕ_P on P such that the additive combination of the component stimulus-scale values preserves at least the rank ordering of the composite stimuli for all a, b in A and for all p, q in P :

$$(1) \quad (a, p) \tilde{\sim} (b, q) \text{ iff } \phi_A(a) + \phi_P(p) \geq \phi_A(b) + \phi_P(q).$$

The symbol "iff" is an abbreviation for "if and only if." Component stimulus-scale functions ϕ_A and ϕ_P map elements in sets A and P , respectively, into the set of real numbers (denoted Re). When such homomorphisms exist, the empirical relational structure $\langle A, P, \tilde{\sim} \rangle$ is said to be mapped into the numerical relational structure $\langle Re, Re, \geq \rangle$ in the sense that ϕ_A maps A into Re , ϕ_P maps P into Re , and $\tilde{\sim}$ is mapped into $\geq$ defined appropriately on $Re \times Re$. In practice, the functions ϕ_A and ϕ_P may be constructed by solving the finite system of homogeneous linear equations and inequalities generated by $\tilde{\sim}$ on $A \times P$.

In order for solutions ϕ_A and ϕ_P to exist, Scott (1964) proved that the following conditions were necessary and sufficient:

Connectedness. Either $(a, p) \tilde{\sim} (b, q)$ or $(b, q) \tilde{\sim} (a, p)$ for all a, b in A and for all p, q in P .

Cancellation. For all sequences $a_0, a_1, \dots, a_n$ in A , for all sequences $p_0, p_1, \dots, p_n$ in P , and for all permutations π and σ of $\{0, 1, \dots, n\}$, where $n > 0$, if for $i = 1, 2, \dots, n$ $(a_i, p_i) \tilde{\sim} (a_{\pi(i)}, p_{\sigma(i)})$, then $(a_{\pi(0)}, p_{\sigma(0)}) \tilde{\sim} (a_0, p_0)$.

The connectedness axiom simply requires that all composite stimuli must be comparable. Although the connectedness axiom may be assumed to hold trivially in many experimental applications, its validity may be questioned in others (Tversky, 1967; Krantz et al., 1971, p. 17). A more general axiomatization of the AIM which does not require connectedness is presented by Tversky (1967). For the purposes of this study, the axiom is assumed to hold in the latent psychological composition process.

Since a sequence of elements in a set may contain repetitions of elements (as opposed to a subset of elements which may not contain repetitions), the cancellation axiom actually defines a countably infinite set of cancellation axioms indexed by n . Each n th-order cancellation axiom asserts that n inequalities imply an additional inequality via the AIM provided that identical terms may be canceled from each side of the inequalities until only one term from each component stimulus set remains on each side (Krantz et al., 1971, p. 427). Thus, if the n th-order cancellation axiom fails from some n , then a fortiori the m th-order cancellation axioms must also fail for all $m > n$. Similarly, if the n th-order cancellation axiom holds for some n , then a fortiori the m th-order cancellation axioms must hold for all $m < n$.

The ICM algorithm used in this study was based on testing successive cancellation axioms (i.e., $n = 1, 2, \dots$). The algorithm minimized the redundancy problem by selecting for presentation to the subject only those composite stimulus pairs which are critical for determining AIM constraints. All other (redundant) constraints were derived via the AIM from knowledge of previously obtained ordinal constraints. The algorithm minimized the random error problem by detecting ordinal inconsistencies in real-time and attempting to rectify the discrepancies by repeating critically important pair-comparisons. An extension of the ICM algorithm based on Tversky's (1967) irreflexivity axiom may be possible for testing multi-factor polynomial composition models of specified degree.

Constraints Matrix

A central concept in the development of the ICM algorithm is the notion of a constraints matrix which indicates the preference relationships among composite choice alternatives. A constraints matrix describes the empirical binary preference relation defined over the Cartesian product-set $A \times P$. If the row composite stimulus is preferred to the column composite stimulus, enter "+" in the matrix. If the column composite stimulus is preferred to the row composite stimulus, enter "-" in the matrix. If no preference is indicated between the two composite stimuli, enter "0" in the matrix. A constraints matrix is skew-symmetric with zeroes along the main diagonal. Thus, the matrix contains $M = N(N-1)/2$ nontrivial terms, where N is the number of elements in the Cartesian product-set. A total of 3^M constraints matrices may exist for a specified Cartesian product-set. Some of these matrices may be fit with an additive model, while others may not.

In order to illustrate the concept of a constraints matrix, let us consider the simplest case of a 2 x 2 factorial design. The following two sets of levels of component factors may be considered:

$$\underline{\text{Assignment}} = \{a, b\} = \{\text{COMMAND}, \text{STAFF}\}$$

and

$$\underline{\text{Place}} = \{p, q\} = \{\text{EUROPE}, \text{CONUS}\}.$$

The corresponding Cartesian product-set is given by

$$\underline{A} \times \underline{P} = \{(a,p), (a,q), (b,p), (b,q)\},$$

where

(a,p) = ap = COMMAND assignment in EUROPE,
 (a,q) = aq = COMMAND assignment in CONUS,
 (b,p) = bp = STAFF assignment in EUROPE,
 (b,q) = bq = STAFF assignment in CONUS.

An example of a constraints matrix which may be fit with an additive model is shown in Table 1. The finite system of homogeneous linear equations and inequalities determined by this constraints matrix is:

$$\begin{aligned} a + p - a - q &< 0 \text{ or } p - q < 0 \\ a + p - b - p &> 0 \text{ or } a - b > 0 \\ a + p - b - q &> 0 \text{ or } a - b > q - p \\ a + q - b - p &> 0 \text{ or } a - b > p - q \\ a + q - b - q &> 0 \text{ or } a - b > 0 \\ b + p - b - q &< 0 \text{ or } p - q < 0. \end{aligned}$$

This system may be solved for component scale values a, b, p, and q which constitute "ordered-metric" scales (i.e., the resulting scales have properties better than mere ordinal scales but not so good as interval scales). Note the redundancy of the equations and inequalities in the system. Only the first three constraints are necessary to completely describe the system.

An example of a constraints matrix which may not be fit with an additive model is shown in Table 2. The finite system of homogeneous linear equations and inequalities determined by the constraints matrix is:

$$\begin{aligned} a + p - a - q &< 0 \text{ or } p - q < 0 \\ a + p - b - p &< 0 \text{ or } a - b < 0 \\ a + p - b - q &< 0 \text{ or } a - b < q - p \\ a + q - b - p &> 0 \text{ or } a - b > p - q \end{aligned}$$

$$a + q - b - q > 0 \text{ or } a - b > 0$$

$$b + p - b - q < 0 \text{ or } p - q < 0.$$

Note the inconsistency of the equations and inequalities in the system. This system may not be solved for component scale values which satisfy the AIM.

Table 1

Additive Constraints Matrix

	ap	aq	bp	bq
ap	0	-	+	+
aq	+	0	+	+
bp	-	-	0	-
bq	-	-	+	0

Table 2

Nonadditive Constraints Matrix

	ap	aq	bp	bq
ap	0	-	-	-
aq	+	0	+	+
bp	+	-	0	-
bq	+	-	+	0

Algorithm Logic

The objectives of the ICM algorithm are twofold: (1) to minimize the number of comparisons between choice alternatives required to generate a complete constraints matrix for the AIM; and (2) to rectify discrepancies from the AIM due to random error. A method of accomplishing these objectives will be presented below.

The ICM algorithm consists of three successive stages. In the first stage the first-order cancellation axiom is used to construct a constraints matrix that satisfies the first-order cancellation properties of the AIM. In the second stage the second-order cancellation axiom is used to construct a constraints matrix that satisfies the second-order cancellation properties of the AIM as well as the first-order cancellation properties. In the third stage the resulting constraints matrix may be passed to a linear programming subroutine which tests the constraints matrix for additivity. If a solution exists, the subroutine may generate scale values for the component factors of the composite choice alternatives based on the AIM.

First-order cancellation conditions consist of two forms of single-component-wise cancellation. The A-component-wise cancellation form is:

$$\begin{array}{ll} \text{If} & (a,p) \succsim (a,q), \\ \text{then} & (b,p) \succsim (b,q) \end{array}$$

for all a, b in A and for all p, q in P . The P-component-wise cancellation form is:

$$\begin{array}{ll} \text{If} & (a,p) \succsim (b,p), \\ \text{then} & (a,q) \succsim (b,q) \end{array}$$

for all a, b in A and for all p, q in P . Since the antecedent inequality implies the conclusion inequality, only the antecedent constraint must be ascertained by querying the subject. The conclusion constraint may be derived since first-order cancellation is a necessary condition for the AIM.

Second-order cancellation conditions consist of two forms of double-component-wise cancellation. In the simple transitivity form, the A-component and P-component of one composite stimulus in each of the two antecedents are identical:

$$\begin{array}{ll} \text{If} & (a,p) \succsim (b,q) \\ \text{and} & (b,q) \succsim (c,r), \\ \text{then} & (a,p) \succsim (c,r) \end{array}$$

for all a, b, c in A and for all p, q, r in P . In the generalized transitivity form, the A-component and P-component of each composite stimulus in the antecedents are different:

If $(a,p) \succsim (b,q)$
 and $(b,r) \succsim (c,p)$,
 then $(a,r) \succsim (c,q)$

for all a, b, c in A and for all p, q, r in P . Since the antecedent inequalities imply the conclusion inequality, only the antecedent constraints must be ascertained by querying the subject. The conclusion constraint may be derived since second-order cancellation is a necessary condition for the AIM.

In the general case first- and second-order cancellation properties are necessary but not sufficient for a constraints matrix to satisfy the AIM since higher-order cancellation properties which are not tested may fail to hold. However, some recent results by Arbuckle and Larimer (1976) indicate that the double cancellation requirement becomes stronger as the size of the factorial design increases. For large factorial designs (e.g., 5×5 , 6×6), the study showed that the chance probability of observing a constraints matrix which satisfies both first- and second-order cancellation is very small. Since the ICM algorithm may be most useful with large factorial designs, and since the computational requirements in testing third- and fourth-order cancellation were found to require large amounts of computer time, only the first- and second-order cancellation tests were implemented in the present version of the algorithm. The test for sufficiency of the constraints matrix for the AIM may be performed by a linear programming subroutine.

The antecedent constraints ascertained by querying the subject may be incorrect when random error is present in an experiment. Any conclusion constraints derived from cancellation conditions based on an incorrect antecedent constraint will also be incorrect. The ICM algorithm must include a mechanism to detect ordinal inconsistencies in the constraints matrix and to rectify those discrepancies from the AIM when they are discovered.

The mechanism for detecting ordinal inconsistencies involves simply checking the conclusion constraint for each possible set of antecedent constraints generated on the basis of Scott's (1964) cancellation axiom. When a particular set of antecedent constraints implies a conclusion constraint, the following logic is used:

1. If no conclusion constraint has been entered in the constraints matrix, the implied constraint is entered and the algorithm proceeds to test the next set of antecedent constraints.
2. If the correct conclusion constraint has been entered in the constraints matrix, the constraints are consistent and the algorithm proceeds to test the next set of antecedent constraints.

3. If an incorrect conclusion constraint has been entered in the constraints matrix, an ordinal inconsistency has been detected. An error-correction strategy must be adopted to rectify the discrepancy from the AIM. An ordinal inconsistency detected in this manner may stem from one of two sources: (1) one or more of the antecedent constraints may be incorrect; (2) the conclusion constraint may be incorrect. For the purposes of this study, a simple error-correction strategy was adopted. Each antecedent constraint was queried in turn until a constraint different from the table entry was obtained. If all the antecedent constraints were found to correspond to the table entries, the table value for the conclusion constraint was changed to the derived value. Since any constraints which were altered may have been used in previously tested conditions, it was necessary to restart the cancellation condition testing process from the beginning each time an ordinal inconsistency was corrected.

The ICM algorithm logic described above was implemented on a CDC 3300 computer. A listing of the FORTRAN program is provided in the appendix. Some internal program documentation is provided by means of comment statements. Program MAIN is the driver program which initializes the algorithm and successively calls up the first- and second-order cancellation test subroutines CAN1 and CAN2, respectively. Subprograms ASK, OUTPUT, INDEX, and XNORM perform a variety of utility functions as indicated by comments in the listing. Since the linear programming subroutine LPSUB was taken directly from Davisson (1972), the listing is not included in the appendix. The results of testing the first- and second-stages of the algorithm produced discouraging results, so the third-stage of the algorithm represented by LPSUB was not fully implemented.

ALGORITHM EVALUATION

Error-Free Data

The purpose of testing the ICM algorithm in error-free data was to determine the degree of success in minimizing the redundancy problem. In a complete pair-comparison experimental design with N composite stimuli, $M = N(N-1)/2$ nontrivial pair-comparisons are made to generate a complete constraints matrix. The degree of success in minimizing the redundancy problem may be measured by contrasting the number of queries of the subject required by the ICM algorithm to generate a complete constraints matrix in error-free data with the theoretical maximum number M.

A 5 x 5 factorial design was chosen for the test. The 25 composite stimuli yield a constraints matrix with $25(25-1)/2 = 300$ constraints. The five values of each component stimulus were generated in accordance with a random-effects model by sampling randomly from independent normal distributions with means equal to zero and variances equal to unity. The 25 values of composite stimuli were generated in accordance with a

strict additive-independence model by summing appropriate component stimulus scale values. The resulting 5 x 5 matrix of composite stimuli is the PSI matrix in the ICM algorithm computer program listing. The PSI matrix generated for the error-free data test is shown in Figure 1.

PSI MATRIX

	P	U	K	S	T
A	1.3290	.8141	.5348	.3234	.6083
B	-0.5655	-1.0804	-1.3597	-1.5707	-1.2862
C	1.9932	1.4783	1.1990	.9880	1.2725
D	1.0116	.4967	.2174	.0064	.2909
E	.0796	-0.4353	-0.7146	-0.9256	-0.6411

Figure 1. PSI matrix of composite scale values.

In the first stage of the ICM algorithm, the test of first-order cancellation required only 20 queries in order to fill in 100 constraints in the matrix. These 100 constraints are shown as 1 or -1 above the main diagonal in the CONSTRAINTS matrix in Figure 2. The table value of 9 indicates that a constraint was neither queried nor derived in the first-order cancellation stage. The 20 constraints which were queried in the first-order cancellation stage are shown as 1 in the TIMES matrix in Figure 3. The table value of 0 indicates that a constraint was not queried in the first-order cancellation stage.

In the second stage of the ICM algorithm, the test of second-order cancellation required 149 queries in order to fill in the remaining 200 constraints. The complete CONSTRAINTS matrix is shown in Figure 4. The 149 queries required by second-order cancellation plus the 20 queries required by first-order cancellation are shown as 1 in the TIMES matrix in Figure 5. The table value of 0 indicates that a constraint was not queried in either cancellation stage.

In total, 169 queries were required by the ICM algorithm to fill in all 300 constraints for a reduction of approximately 44%. Although the savings were substantial, more than three minutes of CDC 3300 computer time were required to compile, load, and execute the program to implement the first two stages of the algorithm. If the third stage of the algorithm (i.e., linear programming for scaling solutions) had been implemented for this test, the computer time and core memory requirements would have been much more substantial. Thus, the computational requirements of the ICM algorithm based on a direct application of Scott's (1964) cancellation condition may be considered rather excessive. A more sophisticated

CONSTRAINTS MATRIX

	AP	AQ	AR	AS	AT	BP	BQ	BR	BS	BT	CP	CQ	CR	CS	CT	DP	DQ	DR	DS	DT	EP	EQ	ER	ES	ET
AP	0	1	1	1	1	1	9	9	9	9	-1	9	9	9	9	1	9	9	9	9	1	9	9	9	9
AQ	-1	0	1	1	1	9	1	9	9	9	9	-1	9	9	9	9	1	9	9	9	9	1	9	9	9
AR	-1	-1	0	1	-1	9	9	1	9	9	9	9	-1	9	9	9	1	9	9	9	9	1	9	9	9
AS	-1	-1	-1	0	-1	9	9	9	1	9	9	9	9	-1	9	9	9	1	9	9	9	9	1	9	9
AT	-1	-1	1	1	0	9	9	9	9	1	9	9	9	9	-1	9	9	9	9	1	9	9	9	9	1
BP	-1	9	9	9	9	0	1	1	1	1	-1	9	9	9	9	-1	9	9	9	9	-1	9	9	9	9
BQ	9	-1	9	9	9	-1	0	1	1	1	9	-1	9	9	9	9	-1	9	9	9	9	-1	9	9	9
BR	9	9	-1	9	9	-1	-1	0	1	-1	9	9	-1	9	9	9	-1	9	9	9	9	-1	9	9	9
BS	9	9	9	-1	9	-1	-1	-1	0	-1	9	9	9	-1	9	9	9	-1	9	9	9	9	-1	9	9
BT	9	9	9	9	-1	-1	-1	1	1	0	9	9	9	9	-1	9	9	9	9	-1	9	9	9	9	-1
CP	1	9	9	9	9	1	9	9	9	9	0	1	1	1	1	1	9	9	9	9	1	9	9	9	9
CQ	9	1	9	9	9	9	1	9	9	9	-1	0	1	1	1	9	1	9	9	9	9	1	9	9	9
CR	9	9	1	9	9	9	9	1	9	9	-1	-1	0	1	-1	9	9	1	9	9	9	9	1	9	9
CS	9	9	9	1	9	9	9	9	1	9	-1	-1	-1	0	-1	9	9	9	1	9	9	9	9	1	9
CT	9	9	9	9	1	9	9	9	9	1	-1	-1	1	1	0	9	9	9	9	1	9	9	9	9	1
DP	-1	9	9	9	9	1	9	9	9	9	-1	9	9	9	9	0	1	1	1	1	1	9	9	9	9
DQ	9	-1	9	9	9	9	1	9	9	9	9	-1	9	9	9	-1	0	1	1	1	9	1	9	9	9
DR	9	9	-1	9	9	9	9	1	9	9	9	9	-1	9	9	-1	-1	0	1	-1	9	9	1	9	9
DS	9	9	9	-1	9	9	9	9	1	9	9	9	9	-1	9	-1	-1	-1	0	-1	9	9	9	1	9
DT	9	9	9	9	-1	9	9	9	9	1	9	9	9	9	-1	-1	-1	1	1	0	9	9	9	9	1
EP	-1	9	9	9	9	1	9	9	9	9	-1	9	9	9	9	-1	9	9	9	9	0	1	1	1	1
EQ	9	-1	9	9	9	9	1	9	9	9	9	-1	9	9	9	9	-1	9	9	9	-1	0	1	1	1
ER	9	9	-1	9	9	9	9	1	9	9	9	9	-1	9	9	9	-1	9	9	-1	-1	0	1	-1	9
ES	9	9	9	-1	9	9	9	1	9	9	9	9	-1	9	9	9	-1	9	9	-1	-1	-1	0	-1	9
ET	9	9	9	9	-1	9	9	9	9	1	9	9	9	9	-1	9	9	9	9	-1	-1	-1	1	1	0

Figure 2. CONSTRAINTS matrix after first-order cancellation in error-free data.

TIMES MATRIX

	AP	AQ	AR	AS	AT	BP	BQ	BR	BS	BT	CP	CQ	CR	CS	CT	DP	DQ	DR	DS	DT	EP	EQ	ER	ES	ET
AP	0	1	1	1	1	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
AQ	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AR	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AS	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
AT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BP	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	0
BQ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
CQ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
CT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
DQ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
DT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EQ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ES	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ET	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 3. TIMES matrix after first order cancellation in error-free data.

CONSTRAINTS MATRIX

	AP	AQ	AR	AS	AT	BP	BQ	BR	BS	BT	CP	CQ	CR	CS	CT	DP	DQ	DR	DS	DT	EP	EQ	ER	ES	ET
AP	0	1	1	1	1	1	1	1	1	1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1
AQ	-1	0	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
AR	-1	-1	0	1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
AS	-1	-1	-1	0	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
AT	-1	-1	1	1	0	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
BP	-1	-1	-1	-1	-1	0	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1
BQ	-1	-1	-1	-1	-1	-1	0	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
BR	-1	-1	-1	-1	-1	-1	-1	0	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
BS	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
BT	-1	-1	-1	-1	-1	-1	-1	1	1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
CP	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CQ	-1	1	1	1	1	1	1	1	1	1	-1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
CR	-1	1	1	1	1	1	1	1	1	1	-1	-1	0	1	-1	1	1	1	1	1	1	1	1	1	1
CS	-1	1	1	1	1	1	1	1	1	1	-1	-1	-1	0	-1	-1	1	1	1	1	1	1	1	1	1
CT	-1	1	1	1	1	1	1	1	1	1	-1	-1	1	1	0	1	1	1	1	1	1	1	1	1	1
DP	-1	1	1	1	1	1	1	1	1	1	-1	-1	-1	1	-1	0	1	1	1	1	1	1	1	1	1
DQ	-1	-1	-1	1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	0	1	1	1	1	1	1	1	1
DR	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	0	1	-1	1	1	1	1	1
DS	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	1	1	1	1
DT	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	1	0	1	1	1	1	1
EP	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	0	1	1	1	1
EQ	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1	1	1
ER	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1	-1
ES	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1
ET	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	0

Figure 4. CONSTRAINTS matrix after second-order cancellation in error-free data.

[illegible][illegible]

mathematical approach seems required to provide a more efficient solution to the redundancy problem. The principal disadvantage of the present approach seems to be the inherent rigidity in constructing the second-order cancellation tests based on specific sequences for testing all a, b, c , in A and all p, q, r in P . In order to further reduce the number of queries required in the second-order cancellation stage of the algorithm, it seems necessary to abandon this rigid procedure in favor of an adaptive procedure based on the nature of all the known constraints. Just how to accomplish this goal in the present context of Scott's (1964) cancellation condition is not clear. Perhaps an adaptation of the general algorithm of McClelland and Coombs (1975) may be feasible for this purpose of interactive conjoint measurement.

Data With Random Error

The purpose of testing the ICM algorithm in data with random error was to determine the degree of success in minimizing the problem of random error. This degree of success may be assessed in terms of (1) the number of additional queries required by the ICM algorithm to resolve detected discrepancies from the AIM, and (2) the extent to which the estimated constraints matrix corresponds to the true constraints matrix (i.e., the extent to which the obtained measurement structure approximates the true latent measurement structure).

The 5×5 PSI matrix of composite choice alternative scale values generated for the test of the ICM algorithm in error-free data was also used for the test of the algorithm in data with random error. Whenever the algorithm queried the simulated "subject," the true absolute difference in composite choice alternative scale values was perturbed by adding an error component prior to ascertaining the preference relation. The error components were sampled independently from a normal distribution with mean equal to zero and variance equal to σ_E^2 . The error variance was set at a moderate value of .25.

The results of this test showed that a total of 200 queries were required to generate a complete constraints matrix which satisfied both first- and second-order cancellation conditions. Thus, the 44% reduction obtained in error-free data was decreased to 33% reduction in data with a moderate amount of random error. The number of queries made for each pair of composite choice alternatives is shown in the TIMES matrix in Figure 6. Note that comparisons (a,r) versus (a,s) and (a,q) versus (a,t) were made nine times and eight times, respectively. The large numbers of queries required for these comparisons may be attributed to the small absolute differences in scale values (approximately .21) which make these comparisons especially sensitive to random error (standard deviation .50). The fact that the algorithm made repeated queries of these comparisons rather than queries of equivalent comparisons [e.g., (b,r) versus (b,s) ; (b,q) versus (b,t)] indicates another deficiency of this algorithm which is due to the inherent rigidity of constructing cancellation conditions based on specific sequences of elements in the component stimulus sets.

TIMES MATRIX

	AP	AQ	AR	AS	AT	BP	BQ	BR	BS	BT	CP	CQ	CR	CS	CT	DP	DQ	DR	DS	DT	EP	EQ	ER	ES	ET
AP	0	5	2	1	1	1	1	1	1	1	2	3	2	1	1	1	1	1	1	1	1	1	1	0	1
AQ	0	0	1	1	8	1	0	0	0	0	1	0	2	1	1	1	0	0	0	0	1	0	0	0	0
AR	0	0	0	9	1	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0
AS	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
AT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
BP	0	0	1	1	1	0	0	0	0	0	1	0	0	0	0	1	1	1	1	1	1	1	0	0	1
BQ	0	0	1	1	1	1	0	0	0	0	1	0	1	1	1	1	0	1	1	1	1	0	2	1	1
BR	0	1	0	1	1	0	0	0	0	1	1	1	0	1	1	1	1	0	1	1	1	1	0	1	1
BS	0	1	1	0	1	0	0	0	0	0	1	1	1	0	1	1	1	1	0	1	1	1	1	0	1
BT	0	1	1	1	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1	0	1	1	2	1	0
CP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0
CQ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1	1	0	1	1	1
CR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	0	1	1	1	1	0	1	1
CS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	1	2	0	1	1	1	1	0	1
CT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	1	1	1	1	0
DP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
DQ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1	1
DR	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1	1
DS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1
DT	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0
EP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
EQ	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ER	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ES	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
ET	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 6. TIMES matrix after second-order cancellation in data with random error.

The estimated CONSTRAINTS matrix obtained in this test is shown in Figure 7. Note that this constraints matrix corresponds to the true constraints matrix shown in Figure 4 with the exception of three constraints: (a,r) versus (d,q), (b,p) versus (e,q), and (c,r) versus (d,p). Since perfect correspondence was not obtained, these results imply a need to test higher-order cancellation conditions prior to passing the constraints matrix to the linear programming third stage of the algorithm. More than six minutes of CDC 3300 computer time were required to compile, load, and execute the program for this test with random error. Again, the computational requirements of the ICM algorithm based on a direct application of Scott's (1964) cancellation condition seem excessive. A more computationally efficient algorithm seems required to serve as a basis for computerized adaptive model testing.

CONCLUSIONS

This study investigated an application of recent results in conjoint measurement theory research with the aim of developing a new methodology for (1) modeling a decision process used to evaluate preferences for complex choice alternatives and (2) producing measurement scales for choice component factors and composite choice alternatives based on the decision model. An algorithm for interactive conjoint measurement (ICM) was developed to minimize the problems of redundancy and random error in testing the additive-independence model (AIM) in pair-comparisons designs with fallible data. No provisions were made for handling the systematic error problem or for accommodating more than two choice component factors.

The results showed that a reduction of approximately 44% in the number of pair-comparisons necessary to determine all model constraints was possible in a 5 x 5 factorial design with error-free data. This reduction was decreased to 33% when a moderate amount of random error was introduced. Although only first- and second-order cancellation conditions were built into the algorithm for this test, the test in error-free data required more than three minutes of computer time and the test in data with a moderate amount of random error required more than 6 minutes of computer time. The main conclusion from the study was that the ICM algorithm based on a direct application of Scott's (1964) cancellation axiom performs rather poorly, and that a more computationally efficient algorithm seems required to serve as a basis for computerized adaptive model testing.

CONSTRAINTS MATRIX

	AP	AQ	AR	AS	AT	BP	BQ	BR	BS	BT	CP	CQ	CR	CS	CT	DP	DQ	DR	DS	DT	EP	EQ	ER	ES	ET
AP	0	1	1	1	1	1	1	1	1	1	-1	-1	1	1	1	1	1	1	1	1	1	1	1	1	1
AQ	-1	0	1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
AR	-1	-1	0	1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
AS	-1	-1	-1	0	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
AT	-1	-1	1	1	0	1	1	1	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	1	1	1
BP	-1	-1	-1	-1	-1	0	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	1	1	1	1
BQ	-1	-1	-1	-1	-1	-1	0	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
BR	-1	-1	-1	-1	-1	-1	-1	0	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
BS	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
BT	-1	-1	-1	-1	-1	-1	-1	1	1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
CP	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1
CQ	1	1	1	1	1	1	1	1	1	1	-1	0	1	1	1	1	1	1	1	1	1	1	1	1	1
CR	-1	1	1	1	1	1	1	1	1	1	-1	-1	0	1	-1	-1	1	1	1	1	1	1	1	1	1
CS	-1	1	1	1	1	1	1	1	1	1	-1	-1	-1	0	-1	-1	1	1	1	1	1	1	1	1	1
CT	-1	1	1	1	1	1	1	1	1	1	-1	-1	1	1	0	1	1	1	1	1	1	1	1	1	1
DP	-1	1	1	1	1	1	1	1	1	1	-1	-1	1	1	-1	0	1	1	1	1	1	1	1	1	1
DQ	-1	-1	1	1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	0	1	1	1	1	1	1	1	1
DR	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	1	1	1	1	1
DS	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1	-1	1	1	1	1
DT	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	1	1	0	1	1	1	1	1
EP	-1	-1	-1	-1	-1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	1	-1	0	1	1	1	1
EQ	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1	1	1
ER	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	1	-1
ES	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0	-1
ET	-1	-1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	0

Figure 7. CONSTRAINTS matrix after second-order cancellation
in data with random error.

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APPENDIX

MSFORTRAN (4.3) / MSUS 5.1

07/25/79

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```

PROGRAM MAIN
DIMENSION ROWA(5),COLP(5),ITER(2)
INTEGER CONMAT,TIMES,SUMIT
INTEGER A,P,FLAG
CHARACTER AA,PP,AST
COMMON /DATA/ NA,NAM1,NAM2,NAM3,NAM4,NP,NPM1,NPM2,NPM3,NPM4,
1A(5),P(5),FLAG,AA(5),PP(5),AST,PSI(5,5),CONMAT(625),TIMES(625),SE
2,IX

```

C
C
C

INITIALIZATIONS

```

AA(1) = 218
AA(2) = 228
AA(3) = 238
AA(4) = 248
AA(5) = 258
PP(1) = 478
PP(2) = 508
PP(3) = 518
PP(4) = 628
PP(5) = 638
AST = 543
IX = 4321567
SA = 1.
SP = 1.
SE = 0.
NA = 5
NP = 5
NAM1 = NA - 1
NAM2 = NA - 2
NAM3 = NA - 3
NAM4 = NA - 4
NPM1 = NP - 1
NPM2 = NP - 2
NPM3 = NP - 3
NPM4 = NP - 4
MAXIT = 150
SUMIT = 0
FLAG = 0
DO 101 I=1,2
ITER(I) = 0
101 CONTINUE
I = (NA*NP)**2
DO 102 J=1,1
CONMAT(J) = 9
TIMES(J) = 0
102 CONTINUE
DO 103 I=1,NA
DO 103 J=1,NP
IND = INDEX(I,J,I,J)
CONMAT(IND) = 0
103 CONTINUE
DO 104 I=1,NA
ROWA(I) = XNORM(IX,0.,SA)
104 CONTINUE
DO 105 J=1,NP

```

```
      COLP(J) = XNORM(IX,0.,SP)
105  CONTINUE
      DO 106 I=1,NA
      DO 106 J=1,NP
      PSI(I,J) = ROWA(I) + COLP(J)
106  CONTINUE
C
C      TEST CANCELLATION AXIOMS
C
201  CALL CAN1
      IF (FLAG .NE. 0) GO TO 401
      CALL OUTPUT (NA,NP,SUMIT,ITER,PSI,CONMAT,TIMES,AA,PP)
      CALL CAN2
      IF (FLAG .NE. 0) GO TO 401
301  CALL OUTPUT (NA,NP,SUMIT,ITER,PSI,CONMAT,TIMES,AA,PP)
      GO TO 501
401  SUMIT = SUMIT + 1
      ITER(FLAG) = ITER(FLAG) + 1
      IF (SUMIT .GT. MAXIT) GO TO 301
      FLAG = 0
      GO TO 201
501  CONTINUE
      STOP
      END
```

```

SUBROUTINE CAN1
  INTEGER CONMAT,TIMES
  INTEGER A1,A2,A3,A4,P1,P2,P3,P4
  INTEGER A,P,FLAG
  CHARACTER AA,PP,AST
  COMMON /DATA/ NA,NAM1,NAM2,NAM3,NAM4,NP,NPM1,NPM2,NPM3,NPM4,
1A(5),P(5),FLAG,AA(5),PP(5),AST,PSI(5,5),CONMAT(625),TIMES(625),SE
2,IX
  EQUIVALENCE (A(1),IA),(A(2),JA),(A(3),KA)
  EQUIVALENCE (P(1),IP),(P(2),JP),(P(3),KP)

C
C  CAN1 TESTS FIRST ORDER CANCELLATION
C
  IF (NA .LT. 2 .OR. NP .LT. 2) GO TO 9999

C
C  IN FIRST ORDER CANCELLATION, THE SEQUENCE LOGIC IN SCOTT'S THEOREM
C  MAY BE REPLACED BY SUBSET LOGIC WITH NO LOSS OF GENERALITY AND WITH
C  SUBSTANTIAL SAVINGS IN COMPUTER TIME.
C
C  TEST EACH SUBSET OF 2 ELEMENTS IN A
C
  IA = 0
1  IA = IA + 1
  IF (IA .EQ. NA) GO TO 9999
  JA = IA
2  JA = JA + 1
  IF (JA .GT. NA) GO TO 1
  IF (IA .EQ. JA) GO TO 2

C
C  TEST EACH SUBSET OF 2 ELEMENTS IN P
C
  IP = 0
3  IP = IP + 1
  IF (IP .EQ. NP) GO TO 2
  JP = IP
4  JP = JP + 1
  IF (JP .GT. NP) GO TO 3
  IF (IP .EQ. JP) GO TO 4

C
C  TEST EACH PERMUTATION OF ELEMENTS IN SUBSET OF A
C
  DO 102 I=1,2
  DO 101 J=1,2
  IF (I .EQ. J) GO TO 101

C
C  TEST EACH (NONTRIVIAL) PERMUTATION OF ELEMENTS IN SUBSET OF P
C
  II = J
  JJ = I

C
C  TEST FIRST ORDER CANCELLATION
C
  A1 = IA
  A2 = A(I)
  A3 = A(J)
  A4 = JA

```

```
P1 = IP
P2 = P(II)
P3 = P(JJ)
P4 = JP
IND1 = INDEX(A1,P1,A2,P2)
IOR1 = CONMAT(IND1)
IF (IOR1 .EQ. 9)
1  CALL ASK (IOR1,A1,P1,A2,P2)
  IND2 = INDEX(A3,P3,A4,P4)
  IOR2 = CONMAT(IND2)
  IF (IOR2 .NE. 9) GO TO 51
  CONMAT(IND2) = IOR1
  IND = INDEX(A4,P4,A3,P3)
  CONMAT(IND) = -IOR1
  GO TO 101
51  IF (IOR2 .EQ. IOR1) GO TO 101
    FLAG = 1
    WRITE (61,1001) AA(A1),PP(P1),AST,AA(A2),PP(P2),
1      AA(A3),PP(P3),AST,AA(A4),PP(P4)
1001 FORMAT (//(1X,2A1,1X,A1,1X,2A1))
    WRITE (61,1002) IOR1,IOR2
1002 FORMAT (/1X,2I5)
    CALL ASK (IOR,A1,P1,A2,P2)
    IF (IOR .NE. IOR1) GO TO 9999
    CONMAT(IND2) = IOR1
    IND = INDEX(A4,P4,A3,P3)
    CONMAT(IND) = -IOR1
    GO TO 9999
101  CONTINUE
102  CONTINUE
    GO TO 4
9999 RETURN
END
```

```

SUBROUTINE CAN2
INTEGER CONMAT,TIMES
INTEGER A1,A2,A3,A4,A5,A6,P1,P2,P3,P4,P5,P6
INTEGER A,P,FLAG
CHARACTER AA,PP,AST
COMMON /DATA/ NA,NAM1,NAM2,NAM3,NAM4,NP,NPM1,NPM2,NPM3,NPM4,
1A(5),P(5),FLAG,AA(5),PP(5),AST,PSI(5,5),CONMAT(625),TIMES(625),SE
2,IX
EQUIVALENCE (A(1),IA),(A(2),JA),(A(3),KA)
EQUIVALENCE (P(1),IP),(P(2),JP),(P(3),KP)

```

```

C
C CAN2 TESTS SECOND ORDER CANCELLATION
C
C IF (NA .LT. 3 .OR. NP .LT. 3) GO TO 9999
C
C TEST EACH (NONREPEATING) SEQUENCE OF 3 ELEMENTS IN A
C
C   IA = 0
1  IA = IA + 1
   IF (IA .GT. NA) GO TO 9999
   JA = 0
2  JA = JA + 1
   IF (JA .GT. NA) GO TO 1
   IF (IA .EQ. JA) GO TO 2
   KA = 0
3  KA = KA + 1
   IF (KA .GT. NA) GO TO 2
   IF (IA .EQ. KA .OR. JA .EQ. KA) GO TO 3
C
C TEST EACH (NONREPEATING) SEQUENCE OF 3 ELEMENTS IN P
C
C   IP = 0
4  IP = IP + 1
   IF (IP .GT. NP) GO TO 3
   JP = 0
5  JP = JP + 1
   IF (JP .GT. NP) GO TO 4
   IF (IP .EQ. JP) GO TO 5
   KP = 0
6  KP = KP + 1
   IF (KP .GT. NP) GO TO 5
   IF (IP .EQ. KP .OR. JP .EQ. KP) GO TO 6
C
C TEST EACH PERMUTATION OF ELEMENTS IN SEQUENCE OF A
C
C   DO 106 I=1,3
C   DO 105 J=1,3
C   IF (I .EQ. J) GO TO 105
C   DO 104 K=1,3
C   IF (I .EQ. K .OR. J .EQ. K) GO TO 104
C
C TEST EACH (NONTRIVIAL) PERMUTATION OF ELEMENTS IN SEQUENCE OF P
C
C   DO 103 II=1,3
C   IF (I .EQ. 1 .AND. II .EQ. 1) GO TO 103
C   DO 102 JJ=1,3

```

```

      IF (II .EQ. JJ) GO TO 102
      IF (J .EQ. 2 .AND. JJ .EQ. 2) GO TO 102
      DO 101 KK=1,3
      IF (II .EQ. KK .OR. JJ .EQ. KK) GO TO 101
      IF (K .EQ. 3 .AND. KK .EQ. 3) GO TO 101
      IF (K .EQ. 1 .AND. KK .EQ. 1 .AND. I .EQ. 3 .AND. II .EQ. 3)
1    GO TO 101
      IF (K .EQ. 2 .AND. KK .EQ. 2 .AND. J .EQ. 3 .AND. JJ .EQ. 3)
1    GO TO 101
C
C      TEST SECOND ORDER CANCELLATION
C
      A1 = IA
      A2 = A(I)
      A3 = JA
      A4 = A(J)
      A5 = A(K)
      A6 = KA
      P1 = IP
      P2 = P(II)
      P3 = JP
      P4 = P(JJ)
      P5 = P(KK)
      P6 = KP
      IND1 = INDEX(A1,P1,A2,P2)
      IOR1 = CONMAT(IND1)
      IF (IOR1 .EQ. 9)
1    CALL ASK (IOR1,A1,P1,A2,P2)
      IND2 = INDEX(A3,P3,A4,P4)
      IOR2 = CONMAT(IND2)
      IF (IOR2 .EQ. 9)
1    CALL ASK (IOR2,A3,P3,A4,P4)
      IF (IOR2 .NE. IOR1) GO TO 101
      IND3 = INDEX(A5,P5,A6,P6)
      IOR3 = CONMAT(IND3)
      IF (IOR3 .NE. 9) GO TO 51
      CONMAT(IND3) = IOR2
      IND = INDEX(A6,P6,A5,P5)
      CONMAT(IND) = -IOR2
      GO TO 101
51    IF (IOR3 .EQ. IOR2) GO TO 101
      FLAG = 2
      WRITE (61,1001) AA(A1),PP(P1),AST,AA(A2),PP(P2),
1          AA(A3),PP(P3),AST,AA(A4),PP(P4),
2          AA(A5),PP(P5),AST,AA(A6),PP(P6)
1001  FORMAT (//(1X,2A1,1X,A1,1X,2A1))
      WRITE (61,1002) IOR1,IOR2,IOR3
1002  FORMAT (/1X,3I5)
      CALL ASK (IOR,A1,P1,A2,P2)
      IF (IOR .NE. IOR1) GO TO 9999
      CALL ASK (IOR,A3,P3,A4,P4)
      IF (IOR .NE. IOR2) GO TO 9999
      CONMAT(IND3) = IOR2
      IND = INDEX(A6,P6,A5,P5)
      CONMAT(IND) = -IOR2
      GO TO 9999

```

101 CONTINUE
102 CONTINUE
103 CONTINUE
104 CONTINUE
105 CONTINUE
106 CONTINUE
GO TO 6
9999 RETURN
END

```
      SUBROUTINE ASK (IOR,I,J,K,L)
      INTEGER A,P,FLAG
      INTEGER CONMAT,TIMES
      CHARACTER AA,PP,AST
      COMMON /DATA/ NA,NAM1,NAM2,NAM3,NAM4,NP,NPM1,NPM2,NPM3,NPM4,
1A(5),P(5),FLAG,AA(5),PP(5),AST,PSI(5,5),CONMAT(625),TIMES(625),SE
2,IX
C
C      ASK QUERIES THE SUBJECT CONCERNING THE PREFERENCE RELATION
C      FOR STIMULUS (I,J) RELATIVE TO STIMULUS (K,L).
C
      PREF = PSI(I,J) - PSI(K,L)
      PREF = PREF + XNORM(IX,0.0,SE)
      IF (PREF) 1,2,3
1      IOR = -1
      GO TO 4
2      IOR = 0
      GO TO 4
3      IOR = 1
4      CONTINUE
      IND = INDEX(I,J,K,L)
      CONMAT(IND) = IOR
      TIMES(IND) = TIMES(IND) + 1
      IND = INDEX(K,L,I,J)
      CONMAT(IND) = -IOR
      RETURN
      END
```

```

      SUBROUTINE OUTPUT (NA,NP,SUMIT,ITER,PSI,CONMAT,TIMES,AA,PP)
      DIMENSION ITER(2),PSI(5,5)
      INTEGER SUMIT,CONMAT(625),TIMES(625),TEMP(5,5)
      CHARACTER AA(5),PP(5)
C
C      OUTPUT ITERATIONS
C
      WRITE (61,1001) SUMIT, (ITER(I),I=1,2)
1001  FORMAT (1H1//11H ITERATIONS
           1          //6H SUMIT,2X,I3
           2          //6H ITER1,2X,I3
           3          //6H ITER2,2X,I3)
C
C      OUTPUT CONSTRAINTS MATRIX
C
      WRITE (61,1002) ((AA(I),PP(J),J=1,NP),I=1,NA)
1002  FORMAT (1H1//19H CONSTRAINTS MATRIX//6X,25(2A1,2X))
      DO 101 I=1,NA
      DO 101 J=1,NP
      DO 1 K=1,NA
      DO 1 L=1,NP
      IND = INDEX(I,J,K,L)
      TEMP(K,L) = CONMAT(IND)
1      CONTINUE
      WRITE (61,1003) AA(I),PP(J),((TEMP(K,L),L=1,NP),K=1,NA)
1003  FORMAT (/2X,2A1,2X,25(12,2X))
101   CONTINUE
C
C      OUTPUT TIMES MATRIX
C
      WRITE (61,1004) ((AA(I),PP(J),J=1,NP),I=1,NA)
1004  FORMAT (1H1//13H TIMES MATRIX//6X,25(2A1,2X))
      DO 102 I=1,NA
      DO 102 J=1,NP
      DO 2 K=1,NA
      DO 2 L=1,NP
      IND = INDEX(I,J,K,L)
      TEMP(K,L) = TIMES(IND)
2      CONTINUE
      WRITE (61,1003) AA(I),PP(J),((TEMP(K,L),L=1,NP),K=1,NA)
102   CONTINUE
C
C      OUTPUT PSI MATRIX
C
      WRITE (61,1005) (PP(J),J=1,NP)
1005  FORMAT (1H1//11H PSI MATRIX//6X,5(5X,A1,4X))
      DO 103 I=1,NA
      WRITE (61,1006) AA(I), (PSI(I,J),J=1,NP)
1006  FORMAT (/3X,A1,2X,5F10.4)
103   CONTINUE
      RETURN
      END

```

```
      FUNCTION INDEX(I,J,K,L)
      COMMON /DATA/ M,LOUM(4),N
C
C      INDEX COMPUTES THE LOCATION OF ELEMENT (I,J,K,L) IN A LINEAR
C      FOUR-DIMENSIONAL ARRAY OF SIZE (M,N,M,N).
C
      INDEX = I + (J-1)*M + (K-1)*M*N + (L-1)*M*N*M
      RETURN
      END
```

```
      FUNCTION XNORM(IX,XM,XS)
C
C      XNORM COMPUTES A NORMALLY DISTRIBUTED RANDOM VARIABLE WITH
C      MEAN XM AND STANDARD DEVIATION XS.
C
      XNORM = 0.
      DO 101 I=1,12
      IY = IX * 4093
      IF (IY) 1,2,2
1     IY = IY + 8388607 * I
2     YFL = IY
      YFL = YFL * 0.1192093E-06
      IX = IY
      XNORM = XNORM + YFL
101  CONTINUE
      XNORM = (XNORM - 6.) * XS + XM
      RETURN
      END
```

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 1 USA Combined Arms Cmbt Dev Act, Ft Leavenworth, ATTN: ATCACO-E
 1 USA Combined Arms Cmbt Dev Act, Ft Leavenworth, ATTN: ATCACO-CI
 1 USAECOM, Night Vision Lab, Ft Belvoir, ATTN: AMSEL-NV-SD
 3 USA Computer Sys Cmd, Ft Belvoir, ATTN: Tech Library
 1 USAMERDC, Ft Belvoir, ATTN: STSFB-DQ
 1 USA Eng Sch, Ft Belvoir, ATTN: Library
 1 USA Topographic Lab, Ft Belvoir, ATTN: ETL TD-S
 1 USA Topographic Lab, Ft Belvoir, ATTN: STINFO Center
 1 USA Topographic Lab, Ft Belvoir, ATTN: ETL GSL
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: CTD MS
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: ATS-CTD-MS
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: ATSI-TE
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: ATSI-TEX-GS
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: ATSI-CTS-OR
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: ATSI-CTD-DT
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: ATSI-CTD-CS
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: DAS/SRD
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: ATSI-TEM
 1 USA Intelligence Ctr & Sch, Ft Huachuca, ATTN: Library
 1 CDR, HQ Ft Huachuca, ATTN: Tech Ref Div
 2 CDR, USA Electronic Prg Grd, ATTN: STEEP-MT-S
 1 HQ, TCATA, ATTN: Tech Library
 1 HQ, TCATA, ATTN: ATCAT-OP-Q, Ft Hood
 1 USA Recruiting Cmd, Ft Sheridan, ATTN: USARCPM-P
 1 Senior Army Adv., USAFAGOD/TAC, Elgin AF Aux Fld No. 9
 1 HQ, USARPAC, DCSPER, APO SF 96558, ATTN: GPPE-SE
 1 Stimson Lib, Academy of Health Sciences, Ft Sam Houston
 1 Marine Corps Inst., ATTN: Dean-MCI
 1 HQ, USMC, Commandant, ATTN: Code MTMT
 1 HQ, USMC, Commandant, ATTN: Code MPI-20-28
 2 USCG Academy, New London, ATTN: Admission
 2 USCG Academy, New London, ATTN: Library
 1 USCG Training Ctr, NY, ATTN: CO
 1 USCG Training Ctr, NY, ATTN: Educ Svc Ofc
 1 USCG, Psychol Res Br, DC, ATTN: GP 1/62
 1 HQ Mid-Range Br, MC Det, Quantico, ATTN: P&S Div

1 US Marine Corps Liaison Ofc, AMC, Alexandria, ATTN: AMCGS-F
 1 USATRADO, Ft Monroe, ATTN: ATRO-ED
 6 USATRADO, Ft Monroe, ATTN: ATPR AD
 1 USATRADO, Ft Monroe, ATTN: ATTS EA
 1 USA Forces Cmd, Ft McPherson, ATTN: Library
 2 USA Aviation Test Bd, Ft Rucker, ATTN: STEBG-PO
 1 USA Agcy for Aviation Safety, Ft Rucker, ATTN: Library
 1 USA Agcy for Aviation Safety, Ft Rucker, ATTN: Educ Advisor
 1 USA Aviation Sch, Ft Rucker, ATTN: PO Drawer O
 1 HQUSA Aviation Sys Cmd, St Louis, ATTN: AMSAV-ZDR
 2 USA Aviation Sys Test Act., Edwards AFB, ATTN: SAVTE--T
 1 USA Air Def Sch, Ft Bliss, ATTN: ATSA TEM
 1 USA Air Mobility Rsch & Dev Lab, Moffett Fld, ATTN: SAVDL-AS
 1 USA Aviation Sch, Res Tng Mgt, Ft Rucker, ATTN: ATST-T-RTM
 1 USA Aviation Sch, CO, Ft Rucker, ATTN: ATST-D-A
 1 HQ, DARCOM, Alexandria, ATTN: AMXCD-TL
 1 HQ, DARCOM, Alexandria, ATTN: CDR
 1 US Military Academy, West Point, ATTN: Serials Unit
 1 US Military Academy, West Point, ATTN: Ofc of Milt Ldrshp
 1 US Military Academy, West Point, ATTN: MAOR
 1 USA Standardization Gp, UK, FPO NY, ATTN: MASE-GC
 1 Ofc of Naval Rsch, Arlington, ATTN: Code 452
 3 Ofc of Naval Rsch, Arlington, ATTN: Code 458
 1 Ofc of Naval Rsch, Arlington, ATTN: Code 450
 1 Ofc of Naval Rsch, Arlington, ATTN: Code 441
 1 Naval Aerosp Med Res Lab, Pensacola, ATTN: Acous Sch Div
 1 Naval Aerosp Med Res Lab, Pensacola, ATTN: Code L51
 1 Naval Aerosp Med Res Lab, Pensacola, ATTN: Code L5
 1 Chief of NavPers, ATTN: Pers-OR
 1 NAVAIRSTA, Norfolk, ATTN: Safety Ctr
 1 Nav Oceanographic, DC, ATTN: Code 6251, Charts & Tech
 1 Center of Naval Anal, ATTN: Doc Ctr
 1 NavAirSysCom, ATTN: AIR-5313C
 1 Nav BuMed, ATTN: 713
 1 NavHelicopterSubSqua 2, FPO SF 96601
 1 AFHRL (FT) Williams AFB
 1 AFHRL (TT) Lowry AFB
 1 AFHRL (AS) WPAFB, OH
 2 AFHRL (DOJZ) Brooks AFB
 1 AFHRL (DOJN) Lackland AFB
 1 HQUSAF (INYSO)
 1 HQUSAF (DPXXA)
 1 AFVTG (RD) Randolph AFB
 3 AMRL (HE) WPAFB, OH
 2 AF Inst of Tech, WPAFB, OH, ATTN: ENE/SL
 1 ATC (XPTD) Randolph AFB
 1 USAF AeroMed Lib, Brooks AFB (SUL-4), ATTN: DOC SEC
 1 AFOSR (NL), Arlington
 1 AF Log Cmd, McClellan AFB, ATTN: ALC/DPCRB
 1 Air Force Academy, CO, ATTN: Dept of Bel Scn
 5 NavPers & Dev Ctr, San Diego
 2 Navy Med Neuropsychiatric Rsch Unit, San Diego
 1 Nav Electronic Lab, San Diego, ATTN: Res Lab
 1 Nav TrngCen, San Diego, ATTN: Code 9000-Lib
 1 NavPostGraSch, Monterey, ATTN: Code 55Aa
 1 NavPostGraSch, Monterey, ATTN: Code 2124
 1 NavTrngEquipCtr, Orlando, ATTN: Tech Lib
 1 US Dept of Labor, DC, ATTN: Manpower Admin
 1 US Dept of Justice, DC, ATTN: Drug Enforce Admin
 1 Nat Bur of Standards, DC, ATTN: Computer Info Section
 1 Nat Clearing House for MH-Info, Rockville
 1 Denver Federal Ctr, Lakewood, ATTN: BLM
 12 Defense Documentation Center
 4 Dir Psych, Army Hq, Russell Ofcs, Canberra
 1 Scientific Advsr, Mil Bd, Army Hq, Russell Ofcs, Canberra
 1 Mil and Air Attache, Austrian Embassy
 1 Centre de Recherche Des Facteurs Humaine de la Defense Nationale, Brussels
 2 Canadian Joint Staff Washington
 1 C/Air Staff, Royal Canadian AF, ATTN: Pers Std Anal Br
 3 Chief, Canadian Def Rsch Staff, ATTN: C/CRDS(W)
 4 British Def Staff, British Embassy, Washington
 1 Def & Civil Inst of Enviro Medicine, Canada
 1 AIR CRESS, Kensington, ATTN: Info Sys Br
 1 Militaerpsychologisk Tjeneste, Copenhagen
 1 Military Attache, French Embassy, ATTN: Doc Sec
 1 Medecin Chef, C.E.R.P.A.-Arsenal, Toulon/Naval France
 1 Prin Scientific Off, Appl Hum Engr Rsch Div, Ministry of Defense, New Delhi
 1 Pers Rsch Ofc Library, AKA, Israel Defense Forces
 1 Ministeris van Defensie, DOOP/KL Afd Sociaal Psychologische Zaken, The Hague, Netherlands