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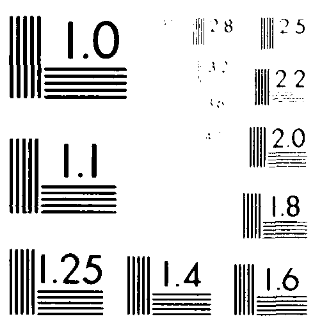

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Diffraction of optical beams with arbitrary profiles by a periodically modulated layer

R. S. Chu

Ground Systems Group, Hughes Aircraft Company, Fullerton, California 92634

J. A. Kong

Department of Electrical Engineering and Computer Science and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

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The problem of diffraction of optical beams with arbitrary profiles by a periodically modulated layer is studied for incidence at normal or at the first Bragg angle. It is shown that the far-field patterns of the *n*th diffracted order of the transmitted and reflected waves are simply the algebraic multiplications of the angular spectral amplitude of the beam profile and the transmission and reflection coefficients for the *n*th-order diffracted plane wave. Numerical results are illustrated for six different beam profiles.

INTRODUCTION

The diffraction of a plane wave by a periodically modulated medium has been treated extensively in the past.[1-5] For a layer bounded by different media on its two sides, the diffraction problem for arbitrary angle of incidence has been studied with a rigorous modal approach by Chu and Kong.[4] A simple second-order coupled-mode approach that yields closed-form solutions for the reflection and transmission coefficients has been given by Kong.[5] For a plane wave incident in the vicinity of normal incidence and in the Raman-Nath regime, Lee[6] obtained a simple closed-form formula for reflection and transmission coefficients.

In a series of previous papers,[7-9] analytical and numerical results have been given for the diffraction of optical beams with a *Gaussian profile* incident near Bragg angles on a periodically modulated half-space[7,8] or a finite layer.[9] Both near-field and far-field solutions have also been obtained by integrating over an appropriate plane-wave spectrum,[9] and the numerical results qualitatively agree very well with the experimental observations performed by Forshaw.[10]

In this paper we investigate the diffraction of optical beams with *arbitrary profiles* incident either normally or at the first Bragg angle on a finite layer of a periodically modulated medium. The far-field patterns of the nth-order reflected and transmitted waves are shown to be simply the algebraic multiplications of the angular spectral amplitude of the incident beam profile at the entrance plane and the reflection and transmission coefficients of the nth-order diffracted plane waves. Numerical results are illustrated for six different beam profiles.

I. FORMULATIONS FOR INCIDENT, REFLECTED, AND TRANSMITTED BEAMS

As shown in Fig. 1, we consider a bounded beam incident on a periodically modulated dielectric layer which is characterized by a permittivity of the form

$$\epsilon(z) = \epsilon_2(1 + M\cos 2\pi z/d), \tag{1}$$

where ϵ_2 is the relative permittivity of the slab in the absence of modulation, M is the modulation index, and d is the periodicity. The slab has a thickness L and is bounded by a dielectric medium with relative permittivity ϵ_1 for $x \le 0$ and by another dielectric medium with relative permittivity ϵ_3 for $L \le x$.

The electric field of an incident beam can be represented by[11]

$$E_{\rm inc}(x,z) = \int_{-\infty}^{\infty} G(\beta_0)\exp i(\xi_0^{(1)}x + \beta_0 z)\, d\beta_0, \tag{2}$$

where

$$\beta_0 = (2\pi/\lambda)\sqrt{\epsilon_1}\sin\theta$$

and

$$\xi_0^{(1)} = (2\pi/\lambda)\sqrt{\epsilon_1}\cos\theta = \sqrt{(2\pi/\lambda)^2\epsilon_1 - \beta_0^2}\,.$$

The function $G(\beta_0)$ is the angular spectral amplitude of the incident beam profile at the entrance plane $x = 0$, i.e.,

$$G(\beta_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} E_{\rm inc}(0,z)e^{-i\beta_0 z}\, dz. \tag{3}$$

We let

$$E_{\rm inc}(0,z) = F(z)e^{ibz}, \tag{4}$$

where

$$b = (2\pi/\lambda)\sqrt{\epsilon_1}\sin\theta_0^{(1)} = (2\pi/\lambda)\sqrt{\epsilon_3}\sin\theta_0^{(3)}$$

and $\theta_0^{(1)}$ is the angle of incidence of the beam axis, $\theta_0^{(3)}$ the angle of refracted beam, and $F(z)$ the beam-profile function at $x = 0$. Equation (3) becomes

$$G(\beta_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(z)e^{-i(\beta_0 - b)z}\, dz. \tag{5}$$

The six different beam profiles $F(z)$ and their corresponding angular spectral functions $G(\beta_0)$ to be considered later are listed in Table I.

The integral representation (2) for the incident beam appears here as a linear superposition of plane-wave spectral components of the form

$$\exp i(\xi_0^{(1)}x + \beta_0 z)$$

with amplitude $G(\beta_0)$. For each plane-wave component incident upon the periodic layer, the reflection and the transmission coefficients for the nth-order wave are given by $R_n(\beta_0)$ and $T_n(\beta_0)$, respectively. Therefore, the nth-order transmitted beam is given by

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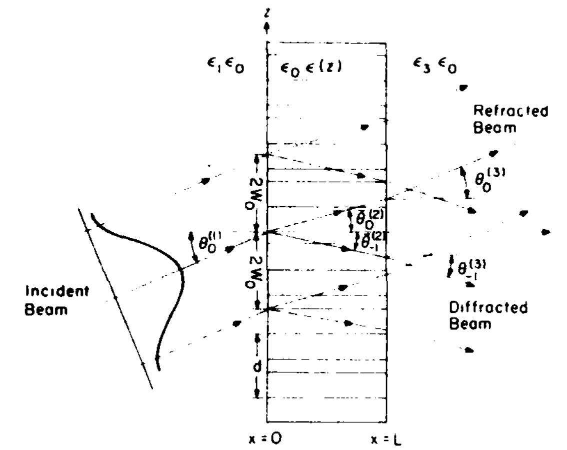

FIG. 1. Geometrical configuration of the problem.

$$E_{tn}(x,z) = \int_{-\infty}^{\infty} G(\beta_0)T_n(\beta_0)\exp i[\xi_n^{(3)}(x-L) + \beta_n z]\,d\beta_0, \qquad x \geq L \quad (6)$$

where

$$\beta_n = \beta_0 + n2\pi/d$$

and

$$\xi_n^{(3)} = \sqrt{(2\pi/\lambda)^2\epsilon_3 - \beta_n^2}\,.$$

The nth-order reflected beam is given by

$$E_{rn}(x,z) = \int_{-\infty}^{\infty} G(\beta_0)R_n(\beta_0)\exp i(-\xi_n^{(1)}x + \beta_n z)\,d\beta_0, \qquad x \leq 0 \quad (7)$$

where

$$\xi_n^{(1)} = \sqrt{(2\pi/\lambda)^2\epsilon_1 - \beta_n^2}\,. \quad (8)$$

Equations (6) and (7) are the field expressions for the transmitted and reflected fields.

As already shown,[9] the far-field pattern of a particular order is essentially the Fourier transform of the aperture field at the boundary of the layer for that particular order. The far-field pattern of the nth-order transmitted beam is given by

$$P_{tn}(\theta) = \int_{-\infty}^{\infty} E_{tn}(L,z)\exp[-i[(2\pi/\lambda)\sqrt{\epsilon_3}\sin\theta]z]\,dz. \quad (9)$$

Substituting (6) into (9), we have

$$\begin{aligned} P_{tn}(\theta) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\beta_0)T_n(\beta_0) \times \exp\left[i\left(\beta_0 + n\frac{2\pi}{d} - \frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta\right)z\right]d\beta_0 dz \\ &= \int_{-\infty}^{\infty} G(\beta_0)\,T_n\,(\beta_0) \times\left\{\int_{-\infty}^{\infty}\exp\left[i\left(\beta_0 + n\frac{2\pi}{d} - \frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta\right)z\right]dz\right\}d\beta_0 \\ &= 2\pi G\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta - n\frac{2\pi}{d}\right)T_n\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta - n\frac{2\pi}{d}\right). \end{aligned} \quad (10)$$

In (10) we observe that the integral in the braces is in fact a δ function. Similarly, the far-field pattern of the nth-order reflected beam is

$$\begin{aligned} P_{rn}(\theta) &= \int_{-\infty}^{\infty} E_{rn}(0,z)\exp\left[-i\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_1}\sin\theta\right)z\right]dz \\ &= \int_{-\infty}^{\infty} G(\beta_0)R_n(\beta_0) \times\left\{\int_{-\infty}^{\infty}\exp\left[i\left(\beta_0 + n\frac{2\pi}{d} - \frac{2\pi}{\lambda}\sqrt{\epsilon_1}\sin\theta\right)z\right]dz\right\}d\beta_0 \\ &= 2\pi\,G\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_1}\sin\theta - n\frac{2\pi}{d}\right) \times R_n\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_1}\sin\theta - n\frac{2\pi}{d}\right). \end{aligned} \quad (11)$$

TABLE I. Beam profiles $F(z)$ and their corresponding angular spectral functions $G(\beta_0)$

	$F(z)$	$G(\beta_0)$
Gaussian beam	$e^{-(z/2W_0)^2}$	$\frac{W_0}{\sqrt{\pi}}e^{-[(\beta_0-b)W_0]^2}$
Square-wave beam	$p_{2W_0}(z) = \begin{cases}1, \lvert z\rvert \leq 2W_0 \\ 0, \lvert z\rvert > 2W_0\end{cases}$	$\frac{2W_0}{\pi}\frac{\sin[2(\beta_0-b)W_0]}{2(\beta_0-b)W_0}$
Triangle-wave beam	$q_{4W_0}(z) = \begin{cases}1 - \frac{\lvert z\rvert}{4W_0}, & \lvert z\rvert < 4W_0 \\ 0, & \lvert z\rvert > 4W_0\end{cases}$	$\frac{2W_0}{\pi}\left(\frac{\sin[2(\beta_0-b)W_0]}{2(\beta_0-b)W_0}\right)^2$
Two-side exponential beam	$e^{-\lvert z\rvert/2W_0}$	$\frac{2W_0}{\pi}\frac{1}{1+[2(\beta_0-b)W_0]^2}$
One-side exponential beam	$e^{-z/4W_0}U(z) = \begin{cases}e^{-z/4W_0}, & z > 0 \\ 0, & z < 0\end{cases}$	$\frac{2W_0}{\pi}\frac{1}{1-i4(\beta_0-b)W_0}$
Lorentzian beam	$\frac{1}{1+(4\pi z/3W_0)^2}$	$\frac{3W_0}{4}e^{-(3W_0/4)\lvert\beta_0-b\rvert}$

Hence the far-field patterns of the nth diffracted order of the transmitted and the reflected beams are simply the *algebraic multiplications* given by (10) and (11), respectively.

II. RESULTS FOR NORMAL INCIDENCE

For an optical beam incident normal to the boundary of the modulated layer, we have $\theta_0^{(1)} = \theta_0^{(3)} = 0$ and $b = 0$; thus the far field patterns for the zeroth-order reflected and transmitted beams are given by

$$P_{r0}(\theta) = 2\pi G\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_1}\sin\theta\right) R_0\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_1}\sin\theta\right), \tag{12}$$

$$P_{t0}(\theta) = 2\pi G\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta\right) T_0\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta\right). \tag{13}$$

The far-field patterns for the nth-order reflected and transmitted beams are given by

$$P_{rn}(\Delta\theta) \simeq 2\pi G\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_1}\sin\Delta\theta\cos\theta_n^{(1)}\right) \times R_n\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_1}\sin\Delta\theta\cos\theta_n^{(1)}\right), \tag{14}$$

$$P_{tn}(\Delta\theta) \simeq 2\pi G\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\Delta\theta\cos\theta_n^{(3)}\right) \times T_n\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\Delta\theta\cos\theta_n^{(3)}\right), \tag{15}$$

where

$$\theta_n^{(1)} = \sin^{-1}\left(\sin\theta_0^{(1)} + n\frac{\lambda}{d}\frac{1}{\sqrt{\epsilon_1}}\right) = \sin^{-1}\left(n\frac{\lambda}{d}\frac{1}{\sqrt{\epsilon_1}}\right), \tag{16a}$$

$$\theta_n^{(3)} = \sin^{-1}\left(\sin\theta_0^{(3)} + n\frac{\lambda}{d}\frac{1}{\sqrt{\epsilon_3}}\right) = \sin^{-1}\left(n\frac{\lambda}{d}\frac{1}{\sqrt{\epsilon_3}}\right). \tag{16b}$$

$\Delta\theta = \theta - \theta_n^{(1)}$ for reflected beams, $\Delta\theta = \theta - \theta_n^{(3)}$ for transmitted beams, and $|\Delta\theta| \ll 1$.

The reflection coefficient $R_n(\beta_0)$ and transmission coefficient $T_n(\beta_0)$ for a plane wave incident at an angle θ such that $|\theta| \ll 1°$ can be obtained by solving the matrix equation (45) of Ref. 4 with proper truncation. For the case of $\epsilon_1 = \epsilon_3$ when a plane wave is incident at an angle close to normal incidence and is in the Raman-Nath regime, a simple closed form three-mode formula for the reflection coefficient R_n and the transmission coefficient T_n has been obtained by Lee[4] as follows:

$$\delta_{n0} + R_n = \sum_{\alpha=1}^{3} [(m_\alpha^0)^+ D_\alpha^n(1 + e^{i\tilde{k}_\alpha L}) + (m_\alpha^0)^- D_\alpha^n(1 - e^{i\tilde{k}_\alpha L})], \tag{17}$$

$$T_n = e^{-ik_{tn}L}\sum_{\alpha=1}^{3} [(m_\alpha^0)^+ D_\alpha^n(1 + e^{i\tilde{k}_\alpha L}) - (m_\alpha^0)^- D_\alpha^n(1 - e^{i\tilde{k}_\alpha L})],$$

$$n = 0, \pm 1 \tag{18}$$

where

$$D_\alpha^n = 2(\tilde{k}_\alpha^2 - \tilde{k}_{\lambda n}^2)/k_1^2, \tag{19a}$$

$$k_1 = (2\pi/\lambda)\sqrt{\epsilon_1} = k_0\sqrt{\epsilon_1}, \quad k_0 = 2\pi/\lambda, \tag{19b}$$

$$k_{2n} = \sqrt{\xi_n^2 + k_1^2(\epsilon_2 - \epsilon_1)/\epsilon_1}, \tag{19c}$$

$$k_{tn} = \sqrt{\xi_n^2 + k_1^2(\epsilon_3 - \epsilon_1)/\epsilon_1}, \tag{19d}$$

$$\xi_n = \sqrt{k_0^2\epsilon_1 - \beta_n^2}, \tag{19e}$$

$$\beta_n = \beta_0 + n2\pi/d, \quad \beta_0 = (2\pi/\lambda)\sqrt{\epsilon_1}\sin\theta, \tag{19f}$$

and $\tilde{k}_\alpha$ is obtained by solving the following equation:

$$(\tilde{k}_\alpha)^3 - (k_{1n} + k_{2n} + k_{3n})(\tilde{k}_\alpha)^2 + [k_{1n}k_{2n} + k_{2n}k_{3n} + k_{1n}k_{3n} - 2(k_0\epsilon_2 M/2)^2](\tilde{k}_\alpha) - k_{1n}k_{2n}k_{3n} + (1/2)k_0\epsilon_2 M(k_{1n} + k_{3n}) = 0 \tag{20}$$

The functions $(m_\alpha^0)^\pm$ in (17)–(18) are given below:

$$(m_1^0)^\pm = (-\xi_0/\Delta^\pm)[A^\pm(-1,2)A^\pm(1,3)D_2^{-1}D_3^1 - A^\pm(1,2)A^\pm(-1,3)D_2^1D_3^{-1}], \tag{21}$$

$$(m_2^0)^\pm = (\xi_0/\Delta^\pm)[A^\pm(-1,1)A^\pm(1,3)D_1^{-1}D_3^1 - A^\pm(1,1)A^\pm(-1,3)D_1^1D_3^{-1}], \tag{22}$$

$$(m_3^0)^\pm = (-\xi_0/\Delta^\pm)[A^\pm(-1,1)A^\pm(1,2)D_1^{-1}D_2^1 - A^\pm(1,1)A^\pm(-1,2)D_1^1D_2^{-1}], \tag{23}$$

$$A^\pm(n,\alpha) = (\xi_n + \tilde{k}_\alpha) \pm (\xi_n - \tilde{k}_\alpha)e^{i\tilde{k}_\alpha L}, \tag{24}$$

$$\Delta^\pm = A^\pm(-1,1)D_1^{-1}[A^\pm(0,2)A^\pm(1,3)D_3^1 - A^\pm(0,3)A^\pm(1,2)D_2^1] - A^\pm(-1,2)D_2^{-1}[A^\pm(0,1)A^\pm(1,3)D_3^1 - A^\pm(0,3)A^\pm(1,1)D_1^1] + A^\pm(-1,3)D_3^{-1}[A^\pm(0,1)A^\pm(1,2)D_2^1 - A^\pm(0,2)A^\pm(1,1)D_1^1]. \tag{25}$$

As an example, consider a plane wave at normal incidence upon a periodically modulated layer with the following parameters: $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.0$, $\lambda = 0.6328\ \mu m$, $d = 6.328\ \mu m$, $kL = 700$ (or $L = 70.4992\ \mu m$), and

$$Q = 2\pi\lambda L/\sqrt{\epsilon_2}d^2 = 7.$$

(Q is a structure constant defined by Klein and Cook[2] such that in the Raman-Nath regime Q is much greater than unity.) The zeroth- and the first-order transmitted intensities $I_0 = |T_0|^2$ and $I_1 = |T_{+1}|^2$ as functions of M or $v = \pi LM\sqrt{\epsilon_2}/\lambda$ computed from (18) are quite accurate up to $v \sim 2.0$, compared with the results obtained by modal solution [Eq. (45) of Ref. 4].

The transmitted beams and the reflected beams for the Gaussian beam profile are shown in Figs. 2 and 3, respectively. Two cases of modulation index, namely, $M = 2 \times 10^{-3}$ and $M = 5 \times 10^{-3}$, have been given. Note that $\Delta\theta = \theta$ for the zeroth order beam and $\Delta\theta = (\theta - \theta_{-1})$ for the first-order beam where $\theta_{-1} = \sin^{-1}(-\lambda/d) = -5.7392°$ and that the scale for reflected beams is magnified by a factor of 1000. Figure 4 shows the amplitudes of the first-order transmitted beams for six different beam profiles. The modulation index $M = 5 \times 10^{-3}$ and $\overline{W}_0 = \pi W_0/d = 500$. We observe that a square-wave beam profile gives high side-lobe ripples while a Lorentzian beam profile gives very wide beam width.

III. RESULTS FOR BRAGG INCIDENCE

For an optical beam incident at the first Bragg angle, we have $\theta_0^{(1)} = \theta_B^{(1)}$, $\theta_0^{(3)} = \theta_B^{(3)}$, and

$$b = (2\pi/\lambda)\sqrt{\epsilon_1}\sin\theta_B^{(1)} = (2\pi/\lambda)\sqrt{\epsilon_3}\sin\theta_B^{(3)} = \pi/d.$$

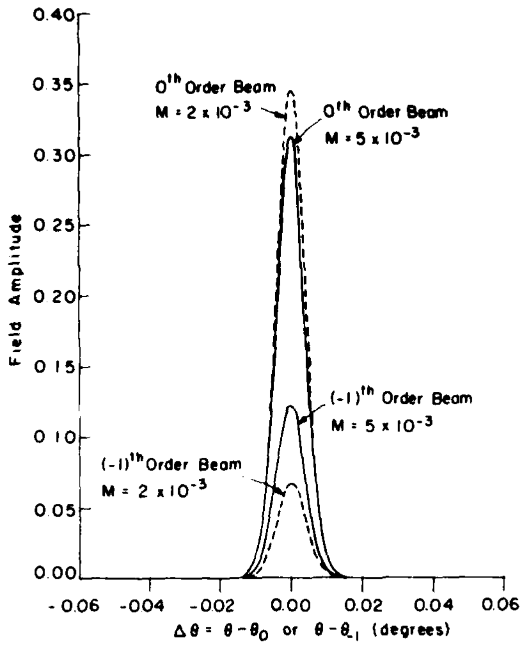

FIG. 2. Transmitted field amplitudes for a normally incident Gaussian beam. $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.0$, $\lambda = 0.6328\ \mu m$, $d = 6.328\ \mu m$, $k_0L = 700$, $\overline{W}_0 = \pi W_0/d = 500$, $Q = 7$.

The far-field pattern for the zeroth-order transmitted beam is given by

$$P_{t0}(\theta) = 2\pi G\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta\right) T_0\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta\right), \quad (26)$$

and the far-field pattern for the Bragg-scattered beam by

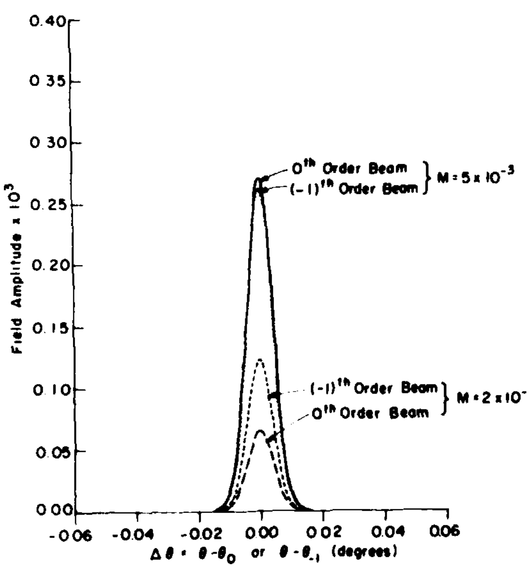

FIG. 3. Reflected field amplitudes for a normally incident Gaussian beam. $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.0$, $\lambda = 0.6328\ \mu m$, $d = 6.328\ \mu m$, $k_0L = 700$, $\overline{W}_0 = 500$, $Q = 7$.

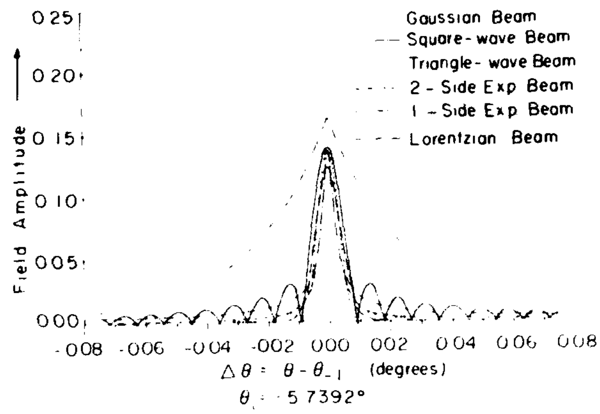

FIG. 4. Amplitudes of the first-order transmitted beams for six different beam profiles. $\epsilon_1 = \epsilon_2 = \epsilon_3 = 1.0$, $\lambda = 0.6328\ \mu m$, $d = 6.328\ \mu m$, $k_0L = 700$, $M = 5 \times 10^{-3}$, $\overline{W}_0 = 500$, $Q = 7$.

$$P_{t-1}(\theta) = 2\pi G\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta + \frac{2\pi}{d}\right) T_{-1}\left(\frac{2\pi}{\lambda}\sqrt{\epsilon_3}\sin\theta + \frac{2\pi}{d}\right), \quad (27)$$

where the transmission coefficients T_0 and T_{-1} have been given in Ref. 5 as

$$T_0 = 4k_{1x}^a(\alpha_2 - \alpha_1)(\alpha_1 A_{bb} - \alpha_2 B_{bb})\exp(-ik_{3x}^a L)/\mathrm{Det}, \quad (28)$$

and

$$T_{-1} = 4k_{1x}^a(\alpha_2 - \alpha_1)(A_{ba} - B_{ba})\exp(-ik_{3x}^b L)/\mathrm{Det}, \quad (29)$$

where

$$\mathrm{Det} = (\alpha_2 A_{aa} - \alpha_1 B_{aa})(\alpha_1 A_{bb} - \alpha_2 B_{bb}) - \alpha_1\alpha_2(A_{ab} - B_{ab})(A_{ba} - B_{ba}) \quad (30)$$

$$\alpha_1 = \frac{1}{Mk^2\epsilon_2}\{\beta_{-1}^2 - \beta_0^2 + [(\beta_{-1}^2 - \beta_0^2)^2 + (Mk_0^2\epsilon_2)^2]^{1/2}\}, \quad (31)$$

$$\alpha_2 = \frac{1}{Mk^2\epsilon_2}\{\beta_{-1}^2 - \beta_0^2 - [(\beta_{-1}^2 - \beta_0^2)^2 + (Mk_0^2\epsilon_2)^2]^{1/2}\}, \quad (32)$$

$$A_{\rho\sigma} = k_{2x}^a\left(1 + \frac{k_{1x}^\rho}{k_{2x}^a}\right)\left(1 + \frac{k_{3x}^\sigma}{k_{2x}^a}\right) \times [\exp(-ik_{2x}^a L) - R_{21}^{a\rho}R_{23}^{a\sigma}\exp(ik_{2x}^a L)], \quad (33)$$

$$B_{\rho\sigma} = k_{2x}^b\left(1 + \frac{k_{1x}^\rho}{k_{2x}^b}\right)\left(1 + \frac{k_{3x}^\sigma}{k_{2x}^b}\right) \times [\exp(-ik_{2x}^b L) - R_{21}^{b\rho}R_{23}^{b\sigma}\exp(ik_{2x}^b L)], \quad (34)$$

$$R_{ij}^{\rho\sigma} = \frac{k_{ix}^\rho - k_{jx}^\sigma}{k_{ix}^\rho + k_{jx}^\sigma} \quad \rho,\sigma = a,b, \quad i,j = 1,2,3, \quad (35)$$

$$k_{1x}^a = k_0\sqrt{\epsilon_1}\cos\theta = \sqrt{(2\pi/\lambda)^2\epsilon_1 - \beta_0^2} = \xi_0^{(1)}, \quad (36)$$

$$k_{1x}^b = (k_0^2\epsilon_1 - \beta_{-1}^2)^{1/2} = \xi_{-1}^{(1)}, \quad (37)$$

$$k_{2x}^a = \{[1 - (1/2)M\alpha_1]k_0^2\epsilon_2 - \beta_0^2\}^{1/2}, \quad (38)$$

$$k_{2x}^b = \{[1 - (1/2)M\alpha_2)k_0^2\epsilon_2 - \beta_0^2\}^{1/2}, \quad (39)$$

$$k_{3x}^a = (k_0^2\epsilon_3 - \beta_0^2)^{1/2} = \xi_0^{(3)}, \quad (40)$$

$$k_{3x}^b = (k_0^2\epsilon_3 - \beta_{-1}^2)^{1/2} = \xi_{-1}^{(3)}. \quad (41)$$

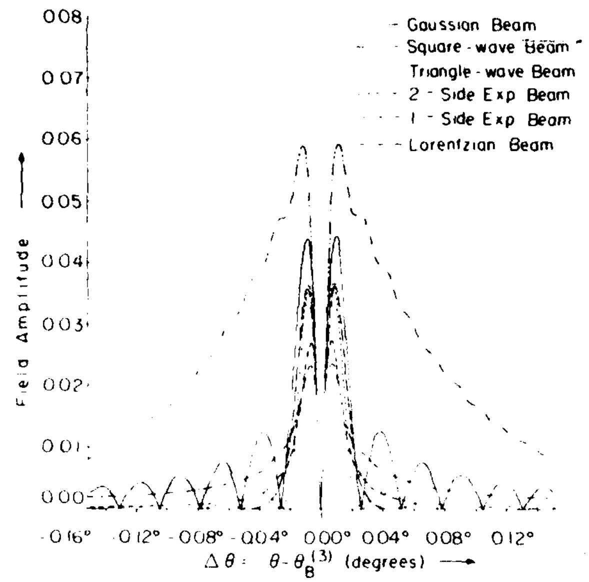

FIG. 5. Zeroth-order transmitted beams for $L = D_1/2$, $M = 1 \times 10^{-4}$, $\epsilon_1 = 1.0$, $\epsilon_2 = 2.25$, $\epsilon_3 = 3.0$, $\lambda = 0.6328\ \mu m$, $d = 1.2656\ \mu m$, $D_1 = 8.3193$ mm, $W_0 = 500$, $\theta_0^{(1)} = 14.4775°$, $\theta_0^{(3)} = 8.2989°$.

and $k_0 = 2\pi/\lambda$.

By substituting (28) and (29) into (26) and (27), the far-field patterns for the zeroth-order transmitted beam and the Bragg-scattered beam are obtained for a Gaussian beam profile and are shown in Figs. 6 and 7. The slab width has been chosen for two cases, namely, $L = D_1/4$ and $D_1/2$, and D_1 is defined as

$$D_1 = \frac{2d}{q}\cot\bar{\theta}_B^{(2)} = \frac{2d}{q}\frac{\bar{\xi}_0 d}{\pi}, \tag{42}$$

where

$$q = 2(d/\lambda)^2 M\epsilon_2,$$

$$\bar{\theta}_B^{(2)} = \sin^{-1}\left(\frac{\lambda}{2d}\frac{1}{\sqrt{\epsilon_2}}\right),$$

and

$$\bar{\xi}_0 = \sqrt{(2\pi/\lambda)^2\epsilon_2 - \beta_0^2}.$$

The meaning of D_1 is that, for a slab width $L = D_1/2$ and for a plane wave incident at exactly the first Bragg angle, complete conversion of energy occurs from the zeroth-order wave into the Bragg-scattered wave. The zeroth-order beams for $L = D_1/2$ for six different beam profiles are shown in Fig. 5 and the Bragg-scattered beams for $L = D_1/2$ for six different beam profiles are shown in Fig. 6. It is seen from Fig. 5 that all the beams have a deep null at the beam center and that the square-wave beam profile gives high side-lobe ripples while the Lorentzian beam profile gives wide beam width for the zeroth-order beam. The physical reason for these deep nulls is that, at $L = D_1/2$, the central portions of the beam spectra of the zero-order waves have completely converted their energies into the Bragg-scattered waves, which results in a depletion of energies from their beam-center portions, as shown. For the Bragg-scattered beams, it is seen from Fig.

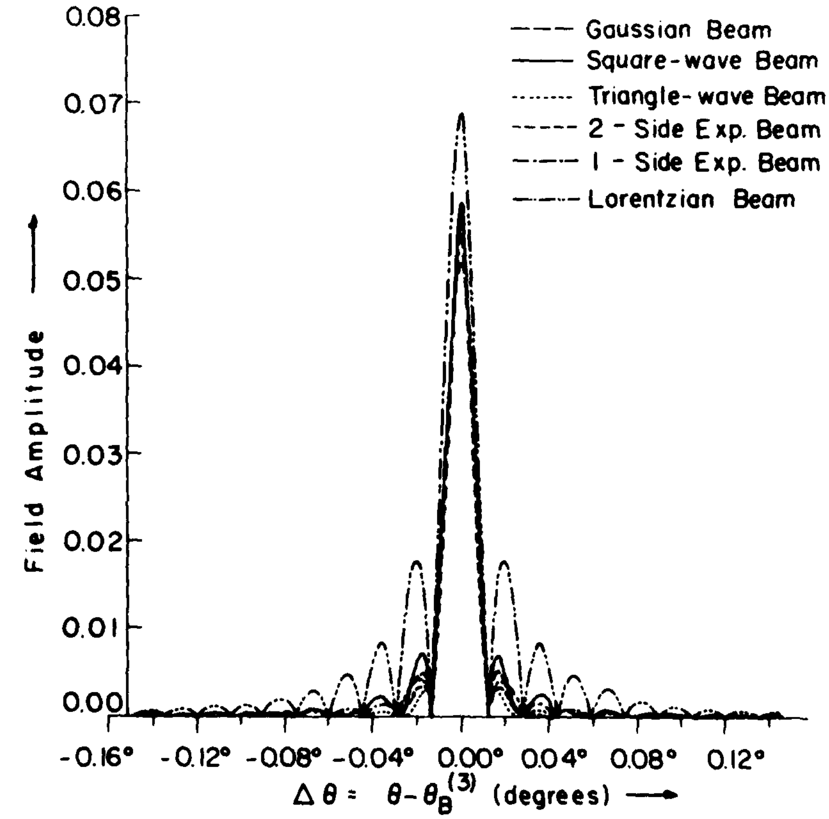

FIG. 6. Bragg-centered beams for $L = D_1/2$, $M = 1 \times 10^{-4}$, $\epsilon_1 = 1.0$, $\epsilon_2 = 2.25$, $\epsilon_3 = 3.0$ $\lambda = 0.6328\ \mu m$, $d = 1.2656\ \mu m$, $D_1 = 8.3193$ mm, $W_0 = 500$, $\theta_0^{(1)} = 14.4775°$ and $\theta_0^{(3)} = 8.2989°$.

6 that the beam widths are about the same for all beam profiles.

ACKNOWLEDGMENT

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