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**Steady State Magnetic Diffusion from
Resistive Interchange Modes**

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STEADY STATE MAGNETIC DIFFUSION FROM RESISTIVE INTERCHANGE MODES

It is now generally assumed that reverse field pinches (RFPs) can exist neither in states which are MHD unstable nor in states which are tearing mode unstable. Indeed, diffuse pinch profiles which are stable to all of these have been calculated.^{1,2,3} It seems reasonable that this conclusion also applies to spheromaks. The remaining problem is the effect of resistive interchange modes. The simplest theory, which mocks up pressure gradient with an effective gravity, indicates that any pressure gradient drives these modes unstable.^{4,5,6} Apparently the plasma must either exist in such an unstable state, or else be driven to the Taylor configuration with zero pressure gradient.⁷ This seems to be a crucial issue and one can examine it in two ways. First one can do the linear theory more accurately, modeling geometry and/or kinetic effects more realistically, and hope that stable regimes emerge. Second, one can examine whether the plasma can exist in the presence of these modes. This report follows the second approach. It finds that the plasma can indeed exist (but has anomalous transport), with a spectrum of nonlinearly saturated resistive interchange modes. The anomalous magnetic diffusion is calculated in terms of the fluid velocity fluctuation and an estimate is then given of an upperbound for this. Also, it is worth pointing out that since mode rational surfaces for resistive g modes occur

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everywhere in an RFP, a theory which predicts stabilization by flattening the pressure profile over for instance a few widths of the interaction region is not a viable theory, because these interaction regions will almost certainly overlap.

This work is motivated to a large extent by recent studies in Eta-Beta II.⁸ There it was found that a reverse field pinch plasma with $n \approx 2 \times 10^{14}$, $T \approx 100$ ev and $I = 200$ KA could exist in a quiescent state for as long as 500 μ sec before disrupting. The growth time for the resistive g mode is less than 10 μ sec, while the resistive diffusion time is about 10-20 milisec. Thus something allows this plasma to exist for many growth times, but to be lost rapidly compared to a classical diffusion time. Also, while magnetic probes show that low frequency fluctuations virtually disappear during the quiescent phase, the high frequency fluctuations are reduced, but are still present. During this quiescent period, the electron temperature increases by 50-100 ev. It is simple to show that classical Ohmic heating will give rise to a much larger temperature increase, and also to show that ion thermal conduction does not significantly cool the plasma during a relevant time scale. Thus there appears to be an anomalous heat loss also.

There are several reasons to examine the resistive interchange mode as well as other micro-instabilities. First, the high frequency probes pick up magnetic signals so the mode cannot be

purely electrostatic. Secondly, an RFP has strong shear and this usually has a great stabilizing effect on micro-instabilities,^{9,10} whereas a resistive interchange mode cannot be shear stabilized. Finally a resistive interchange mode is simpler than a micro-instability since it can be described within a fluid framework, so a nonlinear theory of it is interesting in its own right.

We begin by briefly reviewing (following Ref. 6) the linear theory of the resistive interchange mode. The idea is that resistivity allows the fluid to slip through the field lines, and thereby defeat the shear stabilization which would occur in ideal MHD. That is, the $\mathbf{V} \times \mathbf{B}$ force in the plasma does not generate an electric field, but is balanced by resistive heating, so that instead of $\tilde{\mathbf{E}} + \frac{\tilde{\mathbf{V}}}{c} \times \mathbf{B} = 0$, Ohm's law becomes

$$\eta \tilde{\mathbf{J}} = \frac{\tilde{\mathbf{V}}}{c} \times \mathbf{B} \quad (1)$$

where a top squiggle indicates a fluctuating quantity, and η is assumed to be a constant. In slab geometry, with $\mathbf{B} = B_0 (\hat{\mathbf{i}}_z + \frac{x}{L_s} \hat{\mathbf{i}}_y)$, incompressible perturbed motion and with the fluctuating quantities varying as $f(x) \exp(iky + \gamma t) + \text{c.c.}$, Ref. 6 calculates the mode structure and growth rate. The result is

$$\tilde{v}_x = \frac{ik\ell^2}{x} \tilde{v}_y = - \frac{\eta c^2}{4\pi \frac{\partial B_y}{\partial x} \ell^2} \tilde{B}_y = \tilde{v}_0 \exp - 1/2 \left(\frac{x}{\ell}\right)^2 \quad (a)$$

$$\tilde{B}_x = - ik \frac{4\pi \tilde{v}_0}{\eta c^2} \frac{\partial B_y}{\partial x} \ell^3 \sqrt{\frac{\pi}{2}} \operatorname{erf} \left(\frac{x}{\sqrt{2} \ell} \right) \quad (b) \quad (2)$$

$$\ell = (\eta c^2)^{1/3} k^{-1/3} \left(\frac{\partial B_y}{\partial x} \right)^{-2/3} \left(-g \frac{\partial \rho}{\partial x} \rho \right)^{1/6} \quad (c)$$

(2)

$$\gamma = \left\{ -g \frac{\partial \rho}{\partial x} k \frac{\partial B_y}{\partial x} \right\}^{2/3} \left(\frac{\eta c^2}{4\pi} \right)^{1/3} \quad (d)$$

where g is the equivalent gravity $g \sim \frac{T}{MR}$, and R is the radius of curvature of the field line. The plasma is unstable only if $\frac{\partial \rho}{\partial x}$ and R have opposite sign. For a reversed field pinch, R is roughly the minor radius r . Also, it is important to note that for a resistive interchange mode, $\tilde{E} = 0$ according to Eq. (1). Notice that all quantities except \tilde{B}_x are localized to within a distance ℓ of the rational surface. Far from the rational surface, \tilde{B}_x is actually determined by the full MHD equations⁴ and Eq. (2d) is not accurate, however, this is not important in what follows. In a future publication, a more detailed discussion of the linear theory (and nonlinear theory) will be given in both slab and more realistic geometry.

We now turn to the quasi-linear theory. In ideal MHD, quasi-linear theory shows that the background plasma responds to the unstable mode only while that mode is growing.¹¹ This corresponds to nonresonant quasi-linear theory in an infinite homogeneous plasma.¹² However, for a resistive interchange mode, dissipation is present. As we will see, this means that the background plasma can respond even if the fluctuations are at steady state. This corresponds to resonant quasi-linear theory in an infinite homogeneous plasma.¹³

Now imagine that some nonlinear effect stops the growth of the fluctuation (i.e., so $\gamma = 0$) at some saturated value which we will denote \tilde{V}_0 . (In addition to specifying a \tilde{V}_0 , there are other subtle requirements on such a nonlinear effect which will be discussed more fully elsewhere). This paper does not speculate on what this nonlinear effect is; it only assumes a saturated value for \tilde{V}_0 and proceeds.

The idea then is that the plasma is not quiescent, but has a fluctuating velocity. This fluctuating velocity is balanced by Ohmic dissipation in steady state according to Eq. (1). However, this fluctuating velocity gives rise to a steady state electric field in the z direction according to Ohms law:

$$E_z = -\frac{1}{c} \left(\tilde{V}_x \tilde{B}_y^* - \tilde{V}_y \tilde{B}_x^* \right) + c.c. + \eta J. \quad (3)$$

This field gives rise to a diffusion in the magnetic field

$$\begin{aligned} \frac{\partial B_y}{\partial t} - \frac{\partial}{\partial x} \frac{\eta c^2}{4\pi} \frac{\partial B_y}{\partial x} &= -\frac{\partial}{\partial x} \left(V_x B_y^* - V_y B_x^* \right) + c.c. \\ &= 2 \frac{\partial}{\partial x} \sum_i \frac{4\pi}{\eta c^2} |\tilde{V}_{0i}|^2 \left(\ell_i^2 \exp\left(-\frac{(x-x_i)^2}{\ell_i^2}\right) \right. \\ &\quad \left. + \sqrt{\frac{\pi}{2}} \ell_i (x-x_i) \operatorname{erf} \frac{x-x_i}{\sqrt{2} \ell_i} \exp\left(-\frac{1}{2} \left(\frac{x-x_i}{\ell_i}\right)^2\right) \right) \frac{\partial B_y}{\partial x} \end{aligned} \quad (4)$$

where the term on the right hand side is obtained from Eq. (2) and the factor of 2 in front comes from adding the complex conjugate. Also, we have changed the notation slightly and now the index i denotes the i^{th} rational surface. Thus the stabilized resistive interchange mode gives rise to magnetic diffusion on each rational surface. If the

different mode widths overlap, there will be magnetic diffusion over the entire plasma.

In addition to magnetic diffusion, the fluctuations also give rise to energy transport. The total energy equation for the plasma is

$$\frac{\partial}{\partial t} \left(\frac{1}{2} \rho v^2 + \frac{3}{2} p \right) + \frac{\partial}{\partial x} \left(\frac{5}{2} p v_x + Q + \frac{5}{2} \tilde{p}^* \tilde{v}_x \right) = J_z E_z \quad (5)$$

where p is the pressure and E_z is given by Eq. (3). Note that because $\tilde{E} = 0$, there is no $\tilde{E} \cdot \tilde{J}^*$ term on the right hand side of Eq. (5). As is apparent from Eq. (5), the presence of the fluctuating velocity has two effects. First it gives rise to an anomalous energy flux from the $\frac{\partial}{\partial x} \frac{5}{2} \tilde{p}^* \tilde{v}_x$ term, and second it gives rise to an anomalous energy exchange with the magnetic field arising from the $(-J_z/c)(\tilde{v}_x \tilde{B}_y^* - \tilde{v}_y \tilde{B}_x^*)$ contribution to the right hand side of Eq. (5). A more detailed discussion of the energy flux will be published elsewhere.

To get a rough estimate for the anomalous magnetic diffusion, let us assume that nonlinear effects limit the y fluctuating velocity to some fraction α of the sound speed so that $\tilde{v}_0 = 0.5 \alpha k l \frac{T}{M}$. In this case, the value of the magnetic diffusion coefficient at $x = x_i$ is given roughly by

$$D \approx \frac{1}{2} \left(\frac{4\pi}{\eta c^2} \right) \alpha^2 k^2 l^4 \left(\frac{T}{M} \right). \quad (6)$$

It is interesting to note that in this case $D \sim \eta^{1/3}$ as one might expect for resistive interchange modes. Also \tilde{B}_x is given by $1/2 \beta \frac{\tilde{v}_0}{\sqrt{1/M}} B_y \approx 1/2 \beta k l \alpha B_0$ so that the basic phenomena is fluid convection and vortex motion; the radial fluctuating magnetic field

being, by contrast, small. For Eta-Beta II, we find that if $kr \sim 10$, Eq.(6) with $\alpha \ll 1$ gives roughly the correct confinement time. However, we emphasize that since $\ell \sim \rho_1$ (the ion larmor radius), the theory developed here does not directly apply. Nevertheless it would undoubtedly be interesting to try to measure small scale velocity fluctuations during the quiescent phase of a reverse field pinch.

To conclude, we have shown that if resistive interchange modes exist in a nonlinearly stabilized state, they can give rise to anomalous magnetic diffusion which is localized about each rational surface. The diffusion coefficient and energy transport can be calculated easily in terms of the velocity fluctuation. Since simple theory of this mode shows that it is always unstable if there is a pressure gradient opposite in sign to R , it seems that it will be an important effect in both RFP's and spheromaks.

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