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A PREDICTION INTERVAL FOR A FIRST ORDER GAUSSIAN MARKOV PROCESS--ETC(U)
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A PREDICTION INTERVAL FOR A FIRST ORDER

GAUSSIAN MARKOV PROCESS

by

Toke Jayachandran

and

T.S. Murthy

April 1980

Prepared for:
Chief of Naval Research, Arlington, VA 22217

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Let x_t $(t = 1,2,\ldots)$ be a stationary Gaussian Markov process of order one with $E(x_t) = 0$ and $Cov(x_t, x_{t+k}) = \rho^k$. We derive a prediction interval for x_{2n+1} based on the preceding 2n observations $x_1, x_2, \ldots, x_{2n}$.

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Toke Jayachandran and T.S. Murthy

Let $x_t (t = 1,2,...)$ be a stationary Gaussian Markov process of order one with $E(x_t) = \mu$ and $Cov(x_t, x_{t+k}) = \sigma^2\rho^k$. We derive a prediction interval for x_{2n+1} based on the preceding 2n observations $x_1 x_2, ..., x_{2n}$.

1. INTRODUCTION

Consider a stationary Gaussian Markov process of order one with $E(x_t) = \mu$ and $Cov(x_t, x_{t+k}) = \sigma^2\rho^k$. For $\mu = 0$ such a process can be generated from an autoregressive model

$$x_t = \rho x_{t-1} + \epsilon_t \qquad t = 1,2,... \tag{1.1}$$

with $\{\epsilon_t\}$ a sequence of independent and identically distributed random variables with normal distributions $N(0,\sigma^2)$, $|\rho|<1$ and $x_o = 0$. The process has many applications such as in modelling certain economic and meteorological time series. From a set of sample observations $x_1, x_2, ..., x_{2k}$, in this paper, we construct a conditional prediction interval for x_{2k+1} treating one half of the observations as conditioning variables. The effect of the parameters σ^2, ρ, k and the prediction coefficient α on the prediction interval is also investigated.

2. DERIVATION OF PREDICTION INTERVAL

For the stochastic process defined above it can be shown [1] that when x_{2k-1}, $k = 1,2,...,n+1$ are fixed, x_{2k}, $k = 1,2,...,n$ are conditionally independent and are normally distributed with mean $\mu_{2k} = a + bx'_k$ and variance

σ_o^2 where

$$
\begin{aligned}
a &= m(1-\rho)^2/(1+\rho^2) \\
b &= 2\rho/(1+\rho^2) \\
x'_k &= (x_{2k-1} + x_{2k+1})/2 \\
\sigma_o^2 &= \sigma^2(1-\rho^2)/(1+\rho^2).
\end{aligned}
\tag{2.1}
$$

Conditionally, it may therefore be assumed that the x_{2k} satisfy the simple linear regression model

$$x_{2k} = a + bx'_k + e_k \qquad k = 1,2,\ldots,n$$

where $\{e_k\}$ are i·i·d $N(0,\sigma_o^2)$.

Given the sample observations $x_1,x_2,\ldots,x_{2n},x_{2n+1}$, from standard regression theory the parameters a,b and σ_o^2 in (2.1) can be estimated using the first 2n-1 observations as

$$
\begin{aligned}
\hat{b} &= s_{xy}/s_{xx} \\
\hat{a} &= \bar{x}_2 - \hat{b}\bar{x}' \\
\hat{\sigma}_o^2 &= (s_{yy} - \hat{b}^2 s_{xx})/(n-3)
\end{aligned}
\tag{2.2}
$$

where

$$\bar{x}' = \sum_{k=1}^{n-1} x'_k/(n-1) = (x_1 + 2x_3 + 2x_5 \ldots + 2x_{2n-3} + x_{2n-1})/2$$

$$\bar{x}_2 = \sum_{k=1}^{n-1} x_{2k}/(n-1)$$

$$s_{xx} = \sum_{k=1}^{n-1} x'^2_k - (n-1)\bar{x}'^2$$

$$s_{yy} = \sum_{k=1}^{n-1} x_{2k}^2 - (n-1)\bar{x}_2^{\,2}$$

$$s_{xy} = \sum_{k=1}^{n-1} x'_k x_{2k} - (n-1)\bar{x}'\bar{x}_2 \ .$$

If $\hat{x}_{2n}$ is the least squares predictor of x_{2n} i.e., $\hat{x}_{2n} = \hat{a} + \hat{b}x'_n$ then

$$\frac{x_{2n} - \hat{x}_{2n}}{\hat{\sigma}_o \left[1 + \frac{1}{n-1} + \frac{(x'_n - \bar{x}')^2}{s_{xx}}\right]^{\frac{1}{2}}}$$

has a student's t-distribution with n-3 degrees of freedom; hence

$$P\left\{ |x_{2n} - \hat{x}_{2n}| < t\,\hat{\sigma}_o \left[1 + \frac{1}{n-1} + \frac{(x'_n - \bar{x}')^2}{s_{xx}}\right]^{\frac{1}{2}} \right\} = 1 - \alpha$$

where t is the $100(1 - \frac{\alpha}{2})$th percentage point of the student's t-distribution with n-3 degrees of freedom. The above probability statement can be converted into a prediction interval for x_{2n+1}, by noting that $\hat{x}_{2n}$ is a function of $x'_n = (x_{2n-1} + x_{2n+1})/2$, as shown below.

Squaring the inequality and rearranging terms the above probability statement can be expressed as

$$P\left[\left(\hat{b}^2 - \frac{t^2\sigma_o^{\,2}}{s_{xx}}\right)(x'_n - \bar{x}')^2 - 2\hat{b}(x_{2n} - \bar{x}_2)(x'_n - \bar{x}') + (x_{2n} - \bar{x}_2)^2 - \frac{nt^2\sigma_o^{\,2}}{n-1} < 0\right] = 1 - \alpha \qquad (2.3)$$

or

$$P\left[A(x'_n - \bar{x}')^2 + B(x'_n - \bar{x}') + C < 0\right] = 1 - \alpha \qquad (2.4)$$

where

$$A = \hat{b}^2 - \frac{t^2\hat{\sigma}_o^2}{s_{xx}}$$

$$B = -2\hat{b}(x_{2n} - \bar{x}_2) \tag{2.5}$$

$$C = (x_{2n} - \bar{x}_2)^2 - \frac{nt^2\hat{\sigma}_o^2}{n-1} \quad .$$

A prediction "interval" for $x'_n = (x_{2n-1} + x_{2n+1})/2$ and in turn for x_{2n+1} is now obtainable in terms of the roots of the quadratic expression in (2.4). If $B^2 - 4AC<0$ i.e., the roots are complex the prediction interval will be taken to be $(-\infty,\infty)$; in the other cases the "interval" can turn out to be a two sided interval, a one sided interval or the union of two one sided intervals. The different possibilities will now be examined in detail.

3. PROPERTIES OF THE PREDICTION INTERVAL

<u>Case 1</u>: $A > 0$

Let $$F = \frac{\hat{b}^2 s_{xx}}{t^2\hat{\sigma}_o^2} \quad .$$

Then, $A > 0 \Rightarrow F > 1$.

Also, $B^2 - 4AC>0 \Leftrightarrow$

$$F > 1 - \frac{(n-1)(x_{2n} - \bar{x}_2)^2}{nt^2\hat{\sigma}_o^2}$$

Hence, $A>0 \Rightarrow B^2 - 4AC>0$ and the prediction interval for x_{2n+1} will be of the form (D-E, D+E) where

$$D = 2\bar{x}' - x_{2n-1} + 2\hat{b}(x_{2n} - \bar{x}_2)/(\hat{b}^2 - t^2\hat{\sigma}_o^2/s_{xx})$$
$$E = 2t\hat{\sigma}_o\left[\frac{(x_{2n} - \bar{x}_2)^2}{s_{xx}} + \frac{n}{n-1}\left(\hat{b}^2 - \frac{t^2\hat{\sigma}_o^2}{s_{xx}}\right)\right]^{\frac{1}{2}} \Bigg/ \left(\hat{b}^2 - \frac{t^2\hat{\sigma}_o^2}{s_{xx}}\right) . \tag{3.1}$$

Case 2: $A < 0, \; B^2 - 4AC > 0$

$A < 0$ and $B^2 - 4AC > 0 \Leftrightarrow$

$$1 - \frac{n-1}{n} \frac{(x_{2n} - \bar{x}_2)^2}{t^2 \hat{\sigma}_o^2} < F < 1$$

and the prediction interval will be the union of two non-overlapping intervals $(-\infty, D+E)$ and $(D-E, \infty)$. Note that in this case $E < 0$.

Case 3: $A \neq 0, \; B^2 - 4AC < 0$

As indicated earlier, the roots will be complex and the prediction interval is defined to be $(-\infty, \infty)$. We will call the prediction intervals resulting from the above three cases a type 1, type 2 and a type 3 interval, respectively. There are two other cases viz., $A = 0$ and $B^2 - 4AC = 0$ and we have ignored these possibilities since their probability of occurrence is zero. It should be clear that the following identity holds for the prediction coefficient $1 - \alpha$.

$$\sum_{i=1}^{3} P[\text{an interval of type } i \text{ is obtained}].\; P[\text{the interval will contain } x_{2n+1}] = 1 - \alpha.$$

To study the effect of the parameters n, ρ, σ^2 and α on the probability of occurrence of the different types of intervals we conducted a simulation. For each choice of the parameter values, $2n+1$ samples are generated from the autoregressive process (1.1) and the prediction interval for x_{2n+1} is calculated using the first $2n$ values. We then calculated the empirical frequencies of the three types of intervals, in 1000 replications, and also for each type of interval the frequency of inclusion of x_{2n+1}. In Table 1 we present the results for $\alpha = .05$, $\sigma = 1.0$, $n = 14,22,30,38$ as ρ takes on the values .1, .3, .5, .7, .9. Table 2 shows the effect of increasing n as the other parameters are held fixed. In Table 3 the standard deviation σ is varied from 1 to 5 while the other parameter values are fixed. Some of the results are also presented in graphical form in figures 1-5.

The following general conclusions can be drawn from the results of the simulation. The probability of obtaining a type 1 interval increases with ρ, n and α. For $n \geq 15$ (30 or more samples) and $\rho \geq .5$ the probability of a type 1 interval is of the order of .85. The standard deviation σ does not appear to have any effect on this probability.

4. AN EXAMPLE

The following data represents the monthly Dow-Jones industrial averages for the years 1966-67.

1966		1967	
Jan 31	983.51	Jan 31	879.87
Feb 28	951.89	Feb 28	839.37
Mar 31	924.77	Mar 31	865.98
Apr 30	933.68	Apr 28	897.05
May 31	884.07	May 31	852.56
Jun 30	870.10	Jun 30	860.26
Jul 29	847.38	Jul 31	904.24
Aug 31	788.41	Aug 31	901.29
Sep 30	774.22	Sep 29	926.66
Oct 31	807.07	Oct 31	879.74
Nov 30	791.59	Nov 30	875.81
Dec 30	785.69	Dec 29	905.11

Assuming that the data is generated by a Gaussian Markov process of order one (a calculation of lagged correlations supports the assumption with $\rho = .8$) we computed prediction intervals for March 1967, May 1967, July 1967, September

1967 and November 1967 based on all the preceding data and the results are presented below.

Month	n	Lower Prediction Limit	Upper Prediction Limit	Length of Interval	True Value
Mar 67	7	727.11	1080.63	353.52	865.98
May 67	8	598.39	900.56	302.16	852.56
Jul 67	9	708.70	1009.10	300.40	904.24
Sep 67	10	573.59	864.87	291.28	926.66
Nov 67	11	633.12	899.73	266.61	875.81

All the intervals except for September 1967 contain the true value. As is to be expected the length of the interval decreases with an increase in sample size.

REFERENCES

[1] Ogawara, Masami (1951), "A Note on the Test of Serial Correlation Coefficients", Annals of Mathematical Statistics, 22, 115-118.

PROBABILITIES OF OCCURRENCE OF PREDICTION INTERVALS
OF TYPES 1,2,3 IN 1000 REPLICATIONS

TABLE I

$\sigma = 1$ $\alpha = .05$

n	ρ	P(type 1)	P(type 2)	P(type 3)	Empirical Prediction Coefficient
7	.1	.193 (.917)	.066 (.576)	.741	.956
	.3	.342 (.915)	.080 (.775)	.578	.953
	.5	.269 (.888)	.098 (.786)	.633	.949
	.7	.369 (.902)	.119 (.874)	.512	.949
	.9	.539 (.939)	.128 (.930)	.333	.958
11	.1	.492 (.935)	.048 (.438)	.460	.941
	.3	.516 (.924)	.077 (.714)	.407	.939
	.5	.483 (.909)	.110 (.818)	.407	.936
	.7	.791 (.934)	.071 (.873)	.138	.939
	.9	.902 (.947)	.050 (.920)	.048	.948

Table I (Continued)

n	ρ	P(type 1)	P(type 2)	P(type 3)	Empirical Prediction Coefficient
15	.1	.554 (.960)	.036 (.722)	.410	.968
	.3	.861 (.940)	.033 (.758)	.106	.940
	.5	.874 (.952)	.044 (.841)	.082	.951
	.7	.914 (.951)	.028 (.929)	.058	.953
	.9	.979 (.952)	.008 (1.000)	.013	.953
21	.1	.573 (.932)	.047 (.723)	.380	.948
	.3	.719 (.947)	.044 (.773)	.237	.952
	.5	.853 (.943)	.047 (.894)	.100	.946
	.7	.980 (.948)	.007 (.857)	.013	.948
	.9	.998 (.947)	.001 (1.000)	.001	.947

The numbers in parentheses are the probabilities that x_{2n+1} is contained in the interval; for a type 3 interval this probability is always 1.

PROBABILITIES OF OCCURRENCE OF PREDICTION INTERVALS
OF TYPES 1,2,3 IN 1000 REPLICATIONS

TABLE II

$\sigma = 1$ $\alpha = .05$

ρ	n	P(type 1)	P(type 2)	P(type 3)	Empirical Prediction Coefficient
.3	3	.300 (.867)	.040 (.600)	.660	.944
	4	.342 (.915)	.080 (.775)	.578	.953
	5	.587 (.942)	.057 (.737)	.356	.951
	6	.516 (.924)	.077 (.714)	.407	.939
	7	.259 (.946)	.106 (.698)	.635	.954
	8	.861 (.940)	.033 (.758)	.106	.940
.5	3	.232 (.853)	.049 (.694)	.719	.951
	4	.269 (.888)	.098 (.786)	.633	.949
	5	.540 (.935)	.083 (.819)	.377	.950
	6	.483 (.909)	.110 (.818)	.407	.936
	7	.561 (.941)	.109 (.853)	.330	.951
	8	.874 (.952)	.044 (.841)	.082	.951

Table II (Continued)

ρ	n	P(type 1)	P(type 2)	P(type 3)	Empirical Prediction Coefficient
	3	.308 (.896)	.072 (.861)	.620	.958
	4	.369 (.902)	.119 (.874)	.512	.949
	5	.586 (.920)	.095 (.884)	.319	.942
.7	6	.791 (.934)	.071 (.873)	.138	.939
	7	.840 (.940)	.065 (.908)	.095	.944
	8	.914 (.951)	.028 (.929)	.058	.953

PROBABILITIES OF OCCURRENCE OF PREDICTION INTERVALS OF TYPES 1,2,3 IN 1000 REPLICATIONS

TABLE III

$\alpha = .05$

	σ	P(type 1)	P(type 2)	P(type 3)	Empirical Prediction Coefficient
n=8, ρ=.7	1	.369 (.902)	.119 (.874)	.512	.949
	2	.352 (.901)	.123 (.886)	.525	.951
	3	.354 (.898)	.122 (.885)	.524	.950
	4	.354 (.898)	.121 (.884)	.525	.950
	5	.353 (.898)	.122 (.885)	.525	.950
n=12, ρ=.5	1	.483 (.909)	.110 (.818)	.407	.936
	2	.446 (.901)	.115 (.826)	.439	.936
	3	.445 (.899)	.117 (.828)	.438	.935
	4	.446 (.899)	.115 (.826)	.439	.935
	5	.444 (.901)	.115 (.817)	.441	.935

Fig. 1. n versus P(Intervals of types 1,2,3)

$\rho = .5 \quad \sigma = 3.0 \quad \alpha = .05$

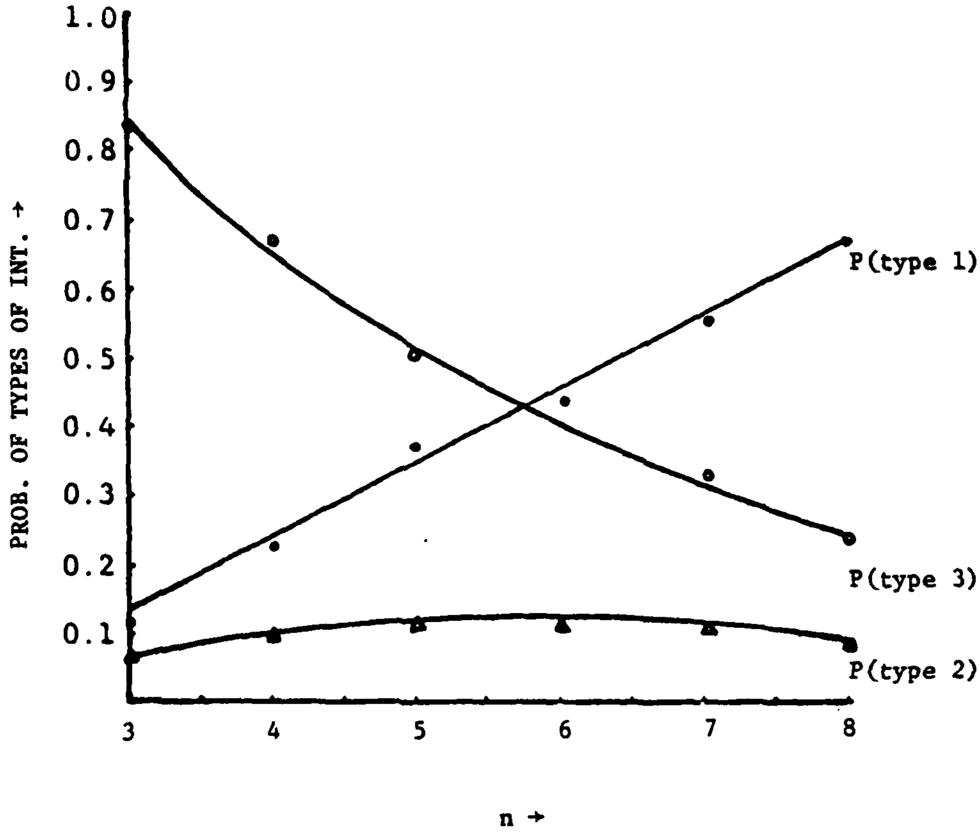


Fig. 2. n versus P(Intervals of types 1,2,3)

$\rho = .7 \quad \sigma = 1.0 \quad \alpha = .05$

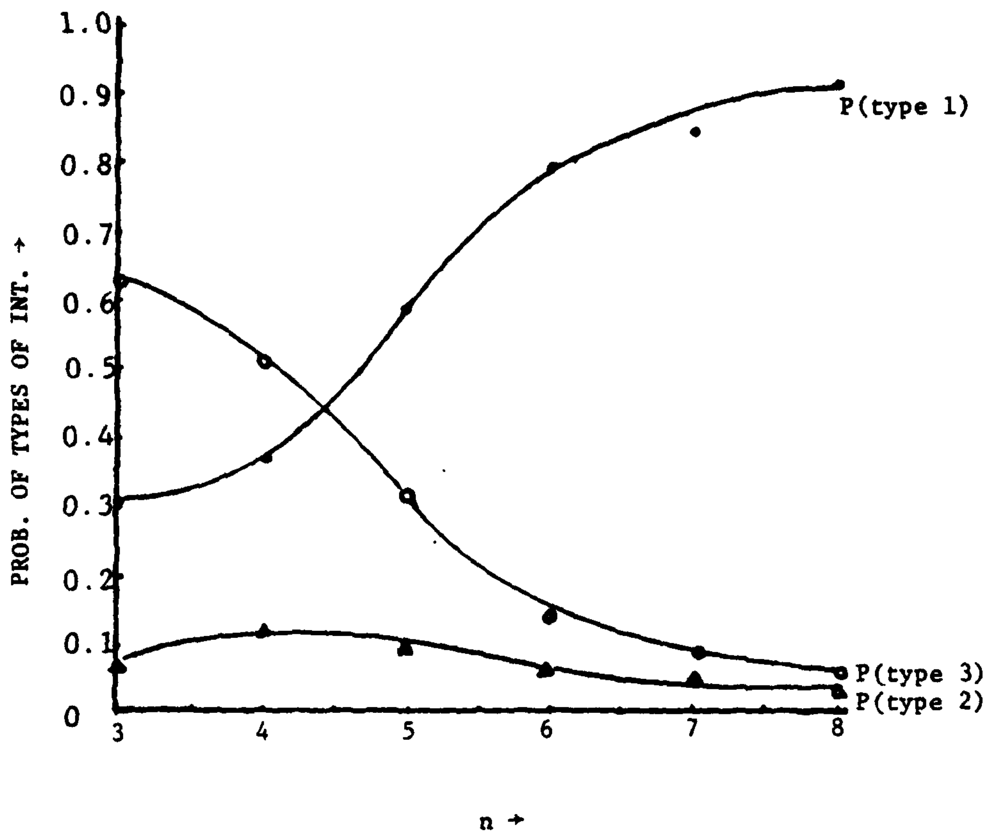


Fig. 3. ρ versus P(Intervals of types 1,2,3)

n = 5 σ = 3 α = .05

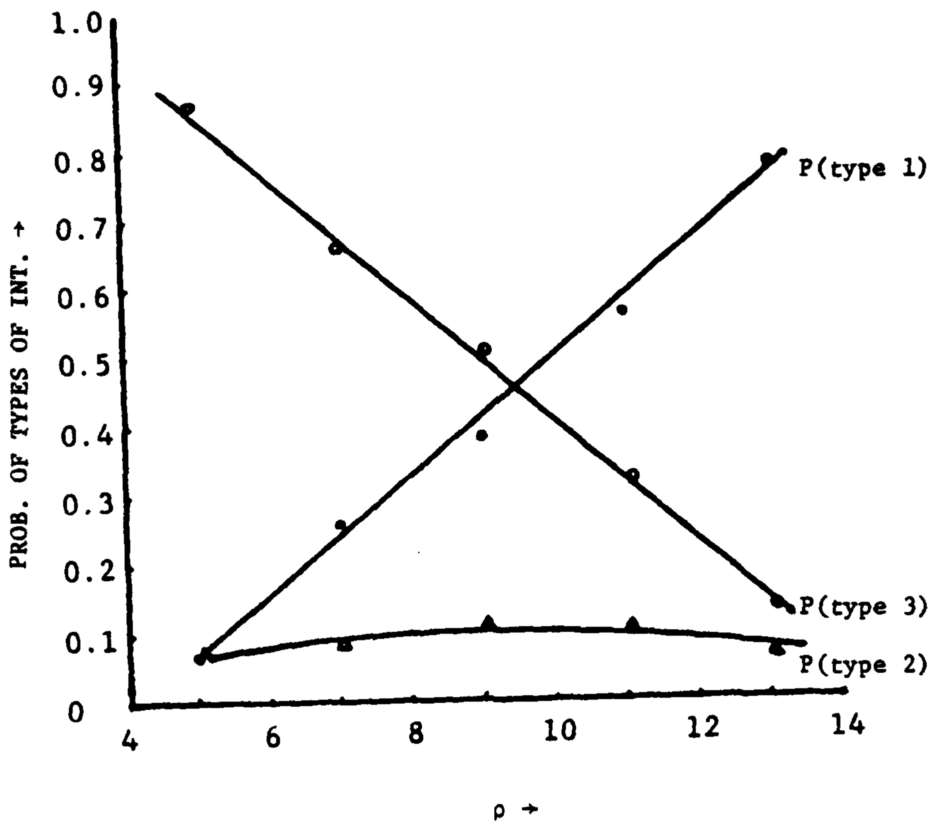


Fig. 4. ρ versus P(Intervals of types 1,2,3)

n = 15 σ = 1.0 α = .05

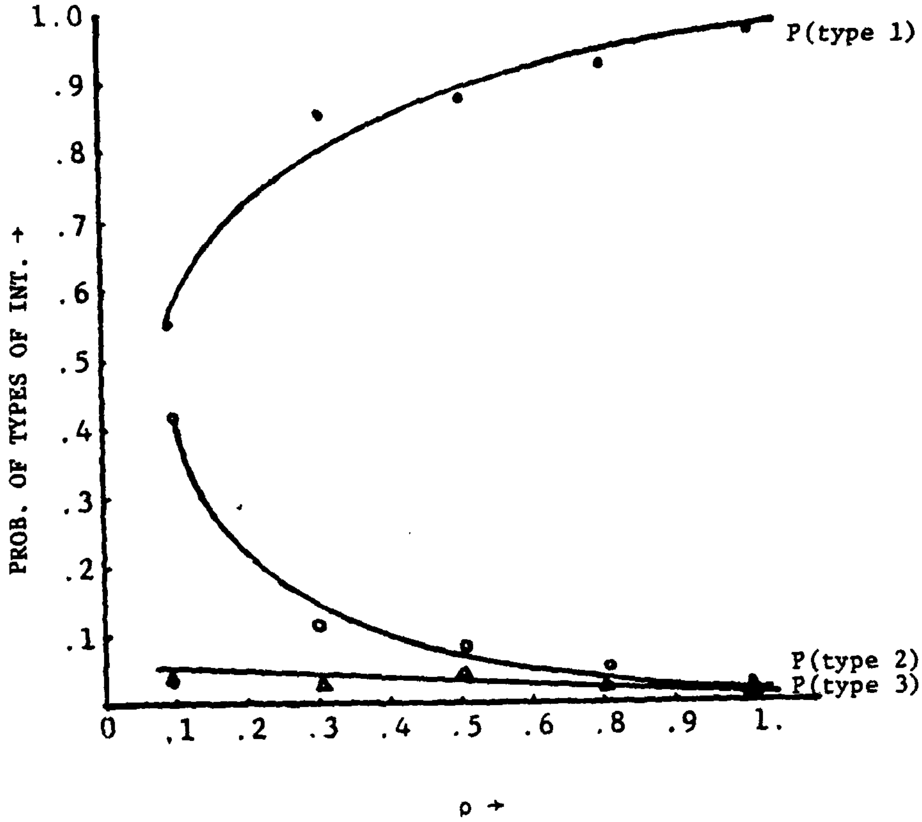
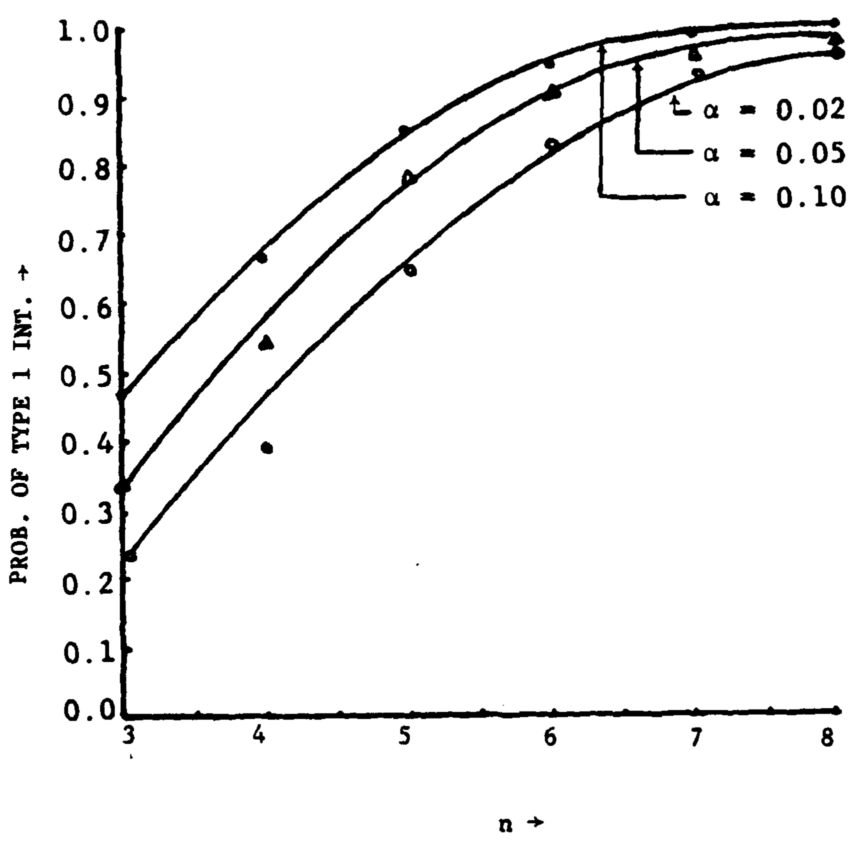


Fig. 5. n versus P(type 1 interval)

α = .02, .05, .10 ρ = .9 σ = 1.0



APPENDIX

```
C      PROGRAM TO CALCULATE PROBABILITY OF TYPE 1,TYPE 2,TYPE
C      3,TYPE 4 INTERVALS FOR SIMULATED SAMPLES.
C      PROGRAMMER T S MURTHY   SEP 1979.
C      ********************************************************
       DIMENSION Z(55),X(55),V(100,55),XI(50),YI(50),S(55),
      1IC(5),IR(5),IPC(5),STAT(10,7),VS(10,10,7),IVS(10)
       CALL OVFLOW
       INDEX=1
       SIGMA=1.0
1      READ(5,2) K,T
       WRITE (6,2)K,T
2      FORMAT( 1X, I2, 2X F5.3)
       IF(K .EQ. 0 ) GO TO 460
       ROW=0.10
       DO 300 IB=1,5
       DO 10 J=1,5
       B=(1-ROW**2 )**.5
C      SIMULATION OF SAMPLES
       ISEED=12345
10     IPC(J)=0
       DO 250 IA=1,10
       DO 50 M=1,100
       CALL SNORM(ISEED,Z,K)
       X(1)=SIGMA*Z(1)
       DO 30 J=2,K
30     X(J)=ROW*X(J-1)+B*SIGMA*Z(J)
       DO 40 L=1,K
40     V(M,L)=X(L)
50     CONTINUE
       DO 60 II=1,5
60     IR(II)=0
       DO 200 I=1,100
       DO 70 J=1,K
70     S(J)=V(I,J)
       K=K-5
       KK=K/2
```

```
      DO 80 L=1,KK
80    YI(L)=S(2*L)
      N=KK-1
      DO 90 LL=1,N
90    XI(LL)=(S(2*LL-1)+S(2*LL+1))/2.0
      XSUM=0.0
      YSUM=0.0
      SXX=0.0
      SXY=0.0
      SYY=0.0
      DO 100 KL=1,5
100   IC(KL)=0
      DO 110 M=1,N
      YSUM=YSUM+YI(M)
      XSUM=XSUM+XI(M)
110    CONTINUE
      XB=XSUM/N
      YB=YSUM/N
      DO 120 M=1,N
      SXX=SXX+(XI(M)-XB)**2
      SYY=SYY+(YI(M)-YB)**2
      SXY=SXY+(XI(M)-XB)*(YI(M)-YB)
120    CONTINUE
      VRES=(SYY-((SXY**2 )/SXX))/(N-2)
      BH=SXY/SXX
      AH=YB-BH*XB
      SS=(T**2 )*VRES
      A=(BH**2 )-SS/SXX
      P=S(2*KK)-YB
      B=-(2*BH*P)
      C=(P**2)-(SS*(N+1))/N
      F=(BH**2 )*SXX/SS
      SSQ=1.0-N*(P**2)/(SS*(N+1))
      IF (F .LT. SSQ) GO TO 150
      IF(F .EQ.1.0) GO TO 500
      D=(2.*XB)-S(K-1)+2.*BH*P/A
```

```
      E=(2./A)*(((SS*(P**2)/SXX)+(N+1)*A*SS/N)**.5)
      PIL=D-E
      PIR=D+E
      PVAL=S(K+1)
      IF(F .GT. 1.0) GO TO 130
      IC(2)=IC(2)+1
      IF(PVAL.LE.PIR .OR. PVAL.GE.PIL) IC(5)=IC(5)+1
      GO TO 160
130   IC(1)=IC(1)+1
      IF(PIL.LE.PVAL.AND.PIR.GE.PVAL) IC(4)=IC(4)+1
      GO TO 160
150   IC(3)=IC(3)+1
160   DO 170 J=1,5
170   IR(J)=IR(J)+IC(J)
      K=K+5
200   CONTINUE
      DO 220 J=1,5
220   IPC(J)=IPC(J)+IR(J)
250   CONTINUE
C     PRINT STATISTICS
      STAT(IB,1)=ROW
      STAT(IB,2)=IPC(1)/1000.0
      STAT(IB,3)=IPC(2)/1000.0
      STAT(IB,4)=IPC(3)/1000.0
      IF(STAT(IB,2).EQ.0.0) GO TO 275
      STAT(IB,5)=IPC(4)/(STAT(IB,2)*1000.0)
      GO TO 280
275    STAT(IB,5)=IPC(4)
280   IF(STAT(IB,3) .EQ. 0.0) GO TO 285
      STAT(IB,6)=IPC(5)/(STAT(IB,3)*1000.0)
      GO TO 290
285   STAT(IB,6)=IPC(5)
290   SIG=STAT(IB,2)*STAT(IB,5)+STAT(IB,3)*STAT(IB,6)
     1 +STAT(IB,4)
      STAT(IB,7)=SIG
      ROW=ROW+0.2
```

```
300   CONTINUE
      DO 305 IQ=1,5
      DO 305 JQ=1,7
      VS(INDEX,IQ,JQ)=STAT(IQ,JQ)
305   CONTINUE
      IVS(INDEX)=N
      INDEX=INDEX+1
      ISZ=K-5
      WRITE(6,310)ISZ,SIGMA,N
      WRITE (6,325)
      WRITE(6,350)((STAT(K,L),L=1,7),K=1,5)
310   FORMAT(1X,'   SAMPLE SIZE = ',I5,' SIGMA = ',F5.0,   '
     1N = ',I5,/)
325   FORMAT(1X,'    ROW      TYPE 1     TYPE 2     TYPE 3
     1PROB.1    PROB.2    CON.REG    ',/,70('-'))
350   FORMAT(   7(F8.3,2X),/        )
      WRITE(6,485)
      GO TO 1
460   CORR=0.1
      INDEX=INDEX-1
      DO 470 J=1,5
      WRITE (6,472) CORR,SIGMA
      WRITE(6,475)
       DO 471 I=1,INDEX
      WRITE(6,480)(IVS(I),(VS(I,J,K),K=2,7))
471   CONTINUE
      CORR=CORR+0.2
470   CONTINUE
472   FORMAT(  ' CORR.COEFF. = ',F5.3,' SIGMA = ',F5.0,//)
475   FORMAT(' SAMPLE SIZE TYPE 1     TYPE 2     TYPE 3     PRO
     1B.1    PROB.2    CON.REG ',/,70('-'))
480   FORMAT(I5,5X,6(F8.3,2X),/)
500   STOP
      END
```

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