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ANALYSIS OF A TWO-FLUID EHD POWER GENERATOR INCLUDING EFFECTS 0--ETC(U)

JAN 80 T H GAWAIN; O BIBLARZ

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ANALYSIS OF A TWO-FLUID  
EHD POWER GENERATOR INCLUDING  
EFFECTS OF COMPRESSIBILITY

by

T. H. Gawain and O. Biblarz

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A detailed analysis and method of calculation is presented for determining the complete thermodynamic cycle of either a one-fluid or a two-fluid electrohydrodynamic (EHD) power generator. The analysis takes fully into account the compressibility of the media. Parameters are included which express the thermodynamic losses in the various components of the overall system. The severe restriction on output created by the electrical breakdown limit of the medium is clearly shown. The method for computing the net-electrical work output per unit mass.		

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20. Abstract Continued

of primary fluid and the net overall thermal efficiency of the system is carefully developed.

The calculation procedure is illustrated by a completely worked out numerical example.

The techniques presented here may be used to determine the performance possibilities and limitations of various one-fluid and two-fluid EHD power generators.

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## O. Background

This report is one of a series of studies stemming from current research on electrohydrodynamic (EHD) power generation, sponsored by the Department of Energy. A certain EHD scheme proposed by the Marks Polarized Corporation is presently under development. The authors of this report have participated as consultants on various theoretical and analytical aspects of this effort. One of our main objectives has been to make an independent appraisal of the performance possibilities and limitations inherent in the general type of EHD system involved in this development. Another objective has been to create a sound analytical basis for optimizing the key design parameters of such a device, and to suggest design improvements where appropriate.

The results of our first study are reported in detail in Ref. (1) and are summarized more concisely in Ref. (2). This work indicated that the design as initially proposed would yield unacceptably poor performance and suggested a certain design revision. Further work was done to analyze such a revised design and is reported in the appendix of Ref. (1). An improved, simplified and shortened version of this work is reported in Ref. (3). Unfortunately, our results indicated that the revised design, while showing somewhat improved performance, still would not attain competitive overall efficiency.

The Marks Polarized Corporation has challenged these pessimistic conclusions. They argue that in our analysis

the compressibility of the medium was not taken into account and that if it had been, the predicted performance would have been drastically improved. We have good reasons to conclude that Marks' argument about compressibility is fallacious. In the first place our analysis did consider all of the major effects of compressibility; only certain minor effects in one part of the system were neglected in order to simplify the calculations. While a more elaborate and accurate treatment of these neglected effects is certainly possible, we maintain that this refinement will not suffice to change the generally pessimistic conclusions established in our original analysis.

Fortunately, this difference of opinion between ourselves and the Marks Polarized Corporation pertains to a question of fact about which it is entirely possible to obtain objective evidence. To this end we present in this report a revised and improved analysis which takes all compressibility effects exhaustively and minutely into account. Calculations based on this revised analysis should eventually settle the above question one way or the other. A typical trial calculation is presented later in this report. This initial calculation has been made manually, but systematic further work will be done by computer. The full results and conclusions derived from these calculations will be reported later.

It has also been conjectured by Marks and others that it might be possible to improve greatly the performance of the

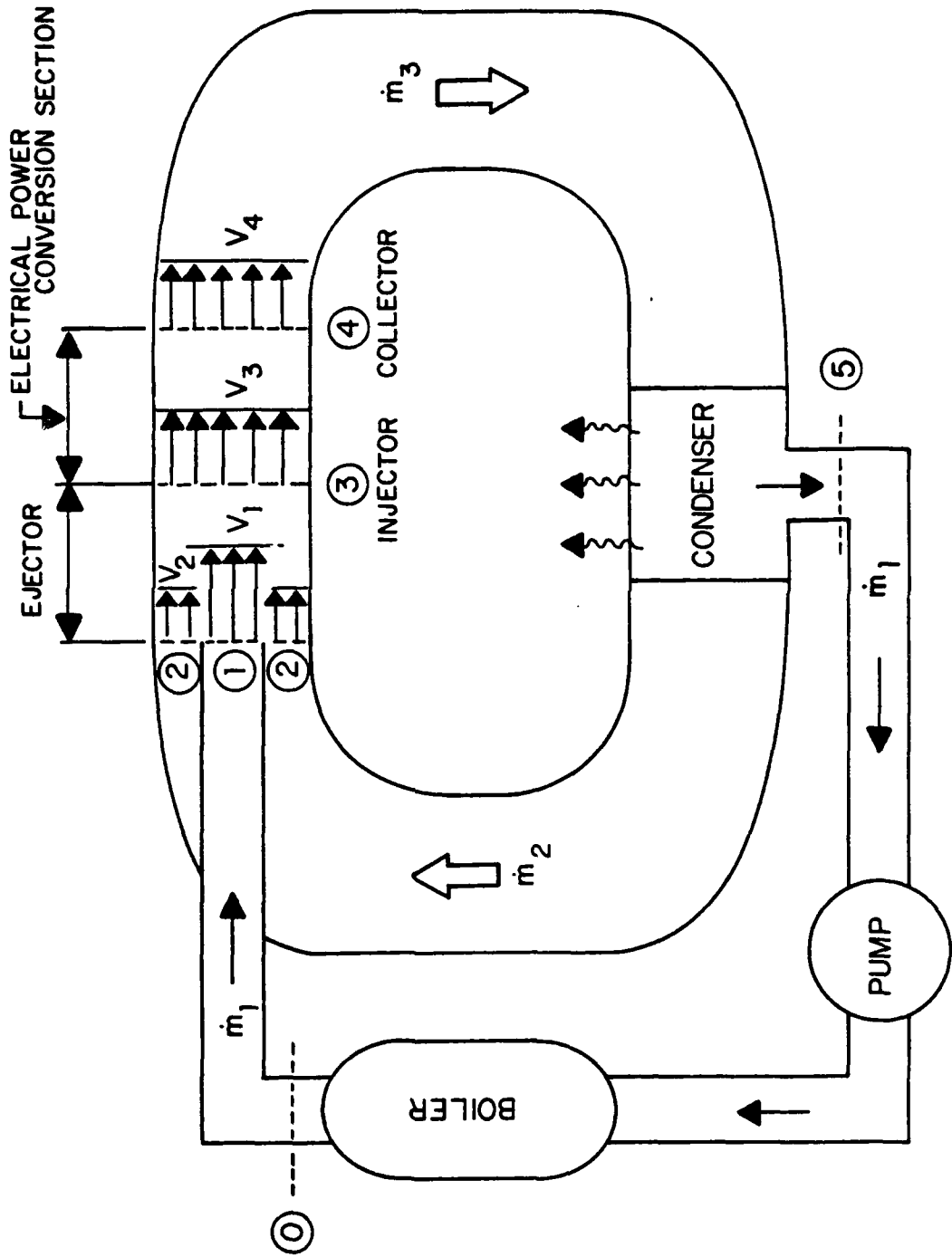
EHD generator by employing a two-fluid cycle. We therefore present in this report a detailed analysis and procedure by means of which the performance of any specified two-fluid EHD system can be calculated. Our analysis takes into account fully the compressibility of both fluids at all locations in the system.

We propose to undertake systemic computer calculations of this kind which should enable an objective assessment to be made of the performance potentialities and limitations of several proposed two-fluid EHD generators. Such results would be of great value in helping decide whether the two-fluid EHD generator shows sufficient promise to warrant further development effort and, if so, what the optimum design point values of the key parameters should be.

As far as we know, the present report is the only published analysis of the two-fluid EHD generator which includes the compressibility of both fluids and which considers the complete thermodynamic cycle. The principal earlier effort along these lines is given in Ref. (4), but the present report goes well beyond anything undertaken in Ref. (4). In particular, Ref. (4) makes no attempt to analyze the complete thermodynamic cycle. Moreover, although it contains certain useful data and is correct in some respects, Ref. (4) is flawed by a number of serious errors.

While the present report is otherwise self-contained, it does assume that the reader has a general familiarity with the type of EHD power generator described in Refs. (1), (2) and (3). It is recommended that the reader unfamiliar with this general scheme first review briefly the introductory material in the first section of any one of these references before proceeding with the more detailed treatment in this report as this earlier background information is not covered or repeated in the present report. We do, however, include here a schematic diagram, Fig. 0.1, which is taken from Ref. (3). It illustrates one particular version of an EHD power generator.

Fig.O.1 SCHEMATIC OF POWER GENERATOR



## 1. Introduction

For purposes of analysis, the overall thermodynamic process in the two-fluid EHD generator may be represented by the schematic flow diagram shown in Fig. 1.1.

Primary fluid enters the ejector at station 1, secondary fluid enters at station 2 and the mixture leaves at station 3.

The mixture then passes through a diffuser, process 3+4, the basic purpose of which is to reduce the kinetic energy of the flow and thereby decrease the associated friction losses downstream of the diffuser. On the other hand the diffuser introduces certain losses of its own so that some care is necessary to optimize the design in this respect.

The gas mixture then passes through the electrical power section, process 4+5, from which the gross electrical power is extracted.

Finally, the gas mixture enters the condenser/separator at station 5. The primary fluid is separated from the gas mixture by condensation and leaves at station 6. The fluid at this point is assumed to be compressed liquid at a known static pressure  $P_6$  and a known static temperature  $T_6$ . Kinetic energy at station 6 is regarded as negligible. The secondary fluid leaves the condenser/separator and enters the ejector at station 2. We assume, moreover, that

$$P_6 = P_2 = P_1 \quad (1.1)$$

and

$$T_6 = T_2 \quad (1.2)$$

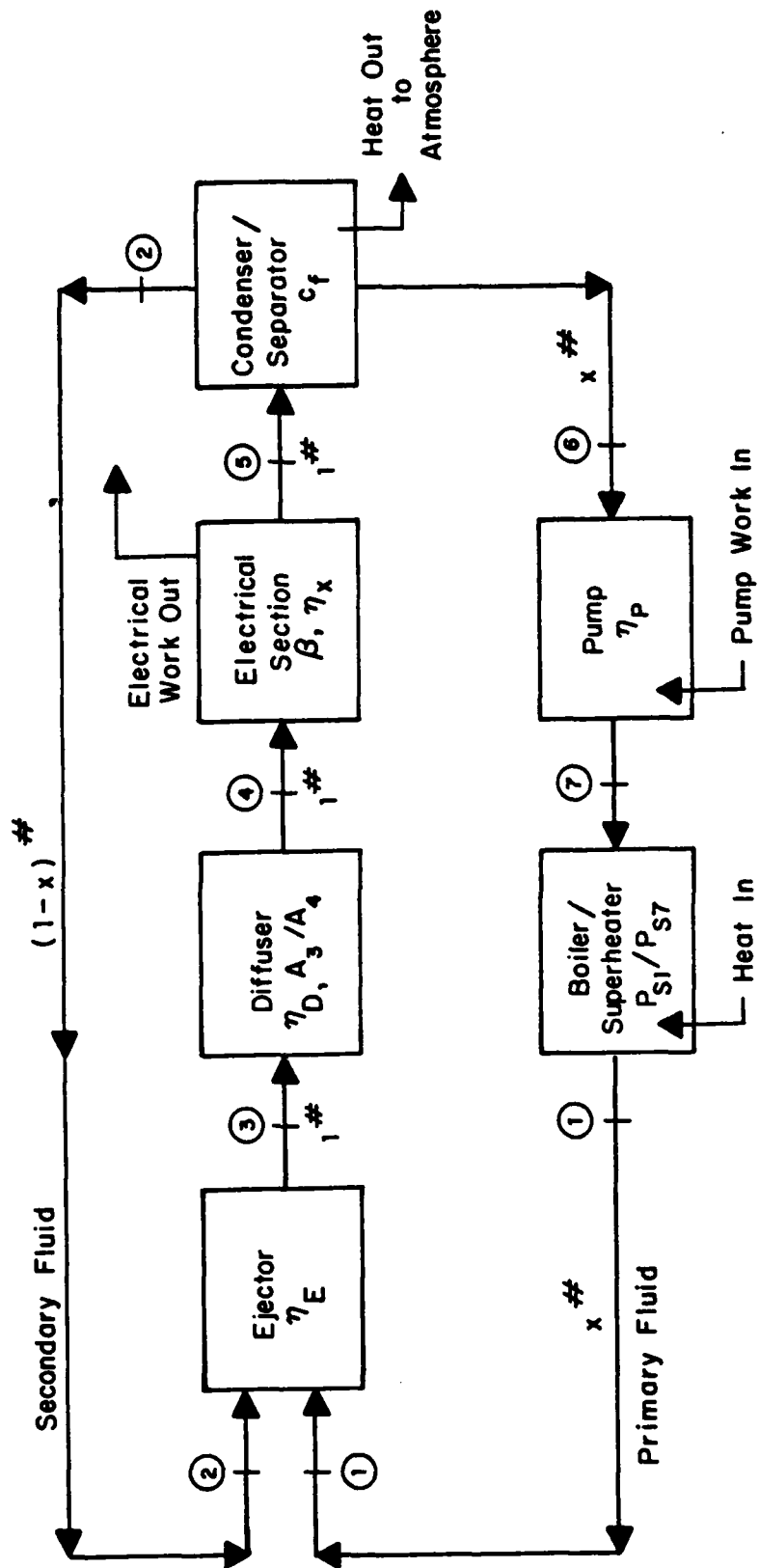


Fig. 1.1 SCHEMATIC FLOW DIAGRAM OF TWO - FLUID EHD CYCLE.

The primary fluid which leaves the condenser/separator at station 6 is circulated back through the pump and through the boiler/superheater, process 7+1.

It should also be explained that the primary fluid at station 1 is a condensible vapor of high molecular weight which contains fine liquid droplets. Unless specifically stated otherwise the quality  $z$  at this location is taken as 0.95. The important thermodynamic static properties at station 1 can be expressed in the following form.

$$h_1 = h_f + z h_{fg} \quad \text{Enthalpy} \quad (1.3)$$

$$s_1 = s_f + z s_{fg} \quad \text{Entropy} \quad (1.4)$$

Here subscript  $f$  denotes the saturated liquid and subscript  $fg$  denotes the change between saturated liquid and saturated vapor. All properties are evaluated at a specified pressure  $P_1$ ; the corresponding saturation temperature  $T_1$  is then also known.

The corresponding stagnation properties  $T_{s1}$  and  $h_{s1}$  are fixed when the stagnation pressure  $P_{s1}$  is specified. Of course the entropy has the same value  $s_1$  at the stagnation condition as at the corresponding static state. Thus specification of  $P_{s1}$  and  $s_1$  suffices to fix all other stagnation properties.

While it is convenient to start the analyses by stipulating the quality at station 1, it is also advisable later in the calculation to determine the corresponding quality at

station 4, the entrance to the electrical working section. It is essential that the fluid entering the working section contain finely dispersed liquid droplets of the proper size. These droplets carry the electrical charges that are the central feature of EHD power generation. We assume tentatively that acceptable droplet characteristics are obtained by maintaining the quality at the inlet to the working section in the range 0.92 to 0.98. These limits are only estimates.

The thermodynamic analysis would be greatly simplified if both fluids were perfect gases with constant specific heats. In fact only the secondary fluid satisfies this requirement. Nevertheless, we can define an "equivalent perfect gas" which adequately approximates the essential thermodynamic properties of the primary vapor/liquid mixture. When carried out judiciously, this procedure simplifies the analysis at the cost of only a small loss in numerical accuracy<sup>5</sup>.

The ratio of specific heats  $\gamma$  of the "equivalent perfect gas" may be deduced from the important relation

$$(P_{s1}/P_1) = (T_{s1}/T_1)^{\gamma_1/(\gamma_1 - 1)} \quad (1.5)$$

Solving this for  $\gamma_1$  gives

$$\gamma_1 = \left[ 1 - \frac{\ln(T_{s1}/T_1)}{\ln(P_{s1}/P_1)} \right]^{-1} \quad (1.6)$$

Another fundamental property of the "equivalent perfect gas" is its specific heat  $C_{p1}$  which may be evaluated from the expression

$$C_{p1} = \frac{(h_{s1} - h_1)}{(T_{s1} - T_1)} \quad (1.7)$$

where all quantities on the right are now known.

It then follows from standard perfect gas relations that the gas constant  $R_1$  and the molecular weight  $W_1$  of the "equivalent perfect gas" are, respectively,

$$R_1 = \left( \frac{\gamma_1 - 1}{\gamma_1} \right) C_{p1} \quad (1.8)$$

and

$$W_1 = \frac{\bar{R}}{R_1} \quad (1.9)$$

where  $\bar{R}$  = universal gas constant  
 = 8315 joules/kg °K

Eqs. (1.6) through (1.9) complete the definition of the "equivalent perfect gas" which adequately simulates the essential thermodynamic properties of the primary vapor/liquid mixture.

It should also be explained that the secondary fluid at station 2 is predominantly a noncondensing gas of low molecular weight primary vapor.

It can be shown that the mass ratio  $\omega$  of condensible primary vapor to dry secondary gas at station 2 is given by the expression

$$\omega = \frac{W_V}{W_G} \frac{P_V(T_6)}{[P_2 - P_V(T_6)]} \quad (1.10)$$

where

$W_v$  = true molecular weight of condensible vapor (not to be confused with the "equivalent molecular weight  $W_1$ " considered earlier)

$W_G$  = molecular weight of noncondensing gas

$P_v(T_G)$  = vapor pressure of condensible primary fluid at known temperature  $T_G$

$P_2$  = known static pressure at station 2

The present analysis is restricted to conditions under which  $\omega$  is very small compared with unity and may be neglected. Thus the fluid at station 2 may be treated as dry gas. This greatly simplifies the analysis. Eq. (1.10) is useful for verifying that this assumption is indeed satisfied in any particular instance.

It should be added that if it later becomes of interest to make calculations for circumstances under which parameter  $\omega$  is not negligible, the present analysis can in fact be generalized to include this effect. For the present, however, we prefer to deal only with the simpler situation in which  $\omega$  may be neglected.

## 2. Ejector

The ejector receives a primary stream of high molecular weight gas at station 1, a secondary stream of low molecular weight gas at station 2 and discharges the resulting mixture at station 3. Static conditions at these three stations are designated by subscripts 1, 2, 3. Stagnation conditions at the corresponding stations are designated by subscripts  $s_1$ ,  $s_2$  and  $s_3$ .

The following quantities are arbitrarily specified or known at stations 1 and/or 2, namely,

Molecular weights:  $W_1$  and  $W_2$

Ratios of specific heats:  $\gamma_1$  and  $\gamma_2$

Stagnation pressure of primary jet:  $P_{s1}$

Stagnation temperature of primary jet:  $T_{s1}$

Static pressures:  $P_1 = P_2 (= P_6)$

Static temperature of primary jet:  $T_1$  (= saturation temperature of primary fluid at pressure  $P_1$ )

Static temperature of secondary jet:  $T_2 (= T_6)$

Mass flow ratio:  $\dot{m}_1/\dot{m}_3 = \dot{m}_1/(\dot{m}_1 + \dot{m}_2) = x$

Velocity ratio:  $V_1/V_2 = y$

We treat the fluids at stations 1, 2, 3, 4 and 5 as perfect gases with constant specific heats.

The velocity ratio of the ejector may be developed as follows:

$$\frac{V_1}{V_2} = y = \frac{a_1}{a_2} \frac{M_1}{M_2} = \sqrt{\frac{\gamma_1 R_1 T_{s1}}{\gamma_2 R_2 T_2}} \sqrt{\frac{M_1}{T_{s1}/T_1}} \cdot \frac{1}{M_2} \quad (2.1)$$

We can now use this result and other standard perfect gas relations to complete the calculation of key thermodynamic properties at stations 1 and 2. It is convenient to arrange these calculations sequentially as follows:

$$M_1 = \sqrt{\frac{2}{(\gamma_1 - 1)} \left( \frac{T_{s1}}{T_1} - 1 \right)} \quad (2.2)$$

$$M_2 = \left( \frac{V_2}{V_1} \right) \sqrt{\frac{\gamma_1 R_1 T_{s1}}{\gamma_2 R_2 T_2}} \frac{M_1}{\sqrt{T_{s1}/T_1}} \quad (2.3)$$

$$\frac{T_{s2}}{T_2} = 1 + \frac{\gamma_2 - 1}{2} M_2^2 \quad (2.4)$$

$$T_{s2} = T_2 (T_{s2}/T_2) \quad (2.5)$$

$$\left( \frac{P_{s2}}{P_2} \right) = \left( \frac{T_{s2}}{T_2} \right)^{\frac{\gamma_2}{\gamma_2 - 1}} \quad (2.6)$$

$$P_{s2} = P_2 \left( \frac{P_{s2}}{P_2} \right) \quad (2.7)$$

Before we can determine the actual conditions at station 3, the outlet of the ejector, it is first necessary to analyze two other hypothetical cases as indicated schematically in Fig. 2.1. In all three of these cases the flow is taken as steady and adiabatic.

The first case, that shown in Fig. 2.1(a), involves a hypothetical device which receives two gas streams at stations 1 and 2 and discharges the resulting mixture at station x. Inlet conditions at 1 and 2 are identical to those of the actual ejector.

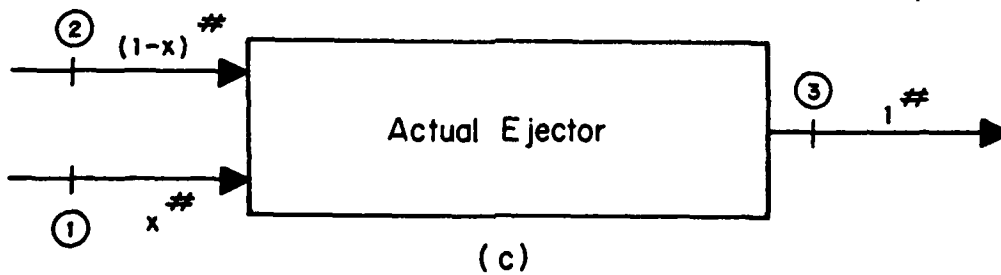
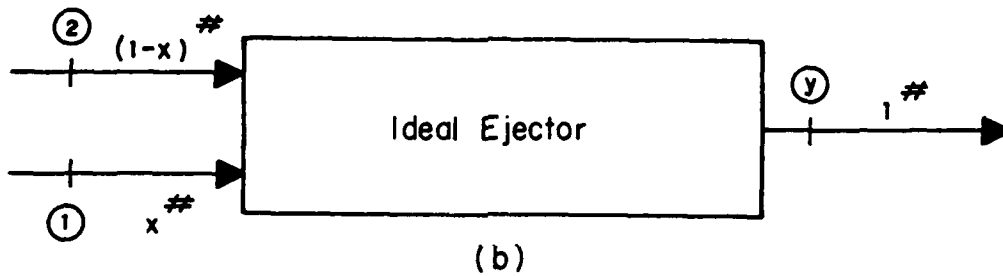
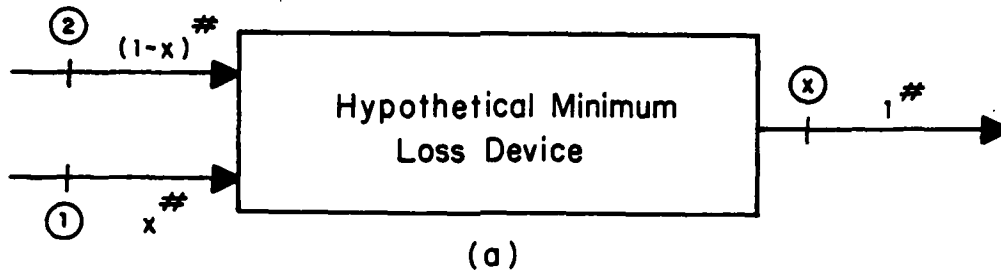


Fig. 2.1 COMPARISON OF ACTUAL EJECTOR WITH IDEAL EJECTOR AND WITH HYPOTHETICAL MINIMUM LOSS DEVICE.

Recall that the adiabatic mixing of two different gases is an inherently irreversible process that always involves a corresponding entropy increase. We stipulate that the first case is such that the only irreversibility which occurs is that associated with this mixing. We term this mass mixing to distinguish it from another type of mixing considered below which we term momentum mixing. ' .

The second case, that shown in Fig. 2.1(b), involves an ideal ejector which receives two gas streams at stations 1 and 2 and discharges the resulting mixture at station y. Again, the inlet conditions at 1 and 2 are identical to those of the actual ejector. This case satisfies, among other relations, the idealized one-dimensional momentum equation for frictionless, constant area flow. It can be shown that in this case there is an overall entropy increase which includes not only the previously mentioned effect of mass mixing but also a further increase associated with momentum mixing.

The third case, that shown in Fig. 2.1(c), represents the actual ejector itself which receives input streams at stations 1 and 2 which discharges the resulting mixture at station 3.

All three of the above cases refer to constant area processes in the sense that

$$A_x = A_y = A_3 = (A_1 + A_2) \quad (2.8)$$

Once the hypothetical operating conditions at stations x and y have been found, it then becomes possible to express the

corresponding actual conditions at station 3. This is accomplished through the use of an ejector effectiveness  $\eta_E$ . This parameter is defined more explicitly in the later analysis. Values of  $\eta_E$  must be estimated by reference to test data on ejectors (i.e., Ref. 6, 7, 8).

### 3. Adiabatic Mixing of Two Streams

The process in the ejector is treated as the adiabatic mixing of two streams. The following relations can be shown to apply.

$$\bar{R} = 8315 \text{ Joule/kg}^\circ\text{K} \quad (3.1)$$

$$R_1 = \frac{\bar{R}}{W_1} \quad (3.2)$$

$$R_2 = \frac{\bar{R}}{W_2} \quad (3.3)$$

$$R_x = R_y = R_3 = x R_1 + (1 - x) R_2 \quad (3.4)$$

$$C_{p1} = \frac{\gamma_1 R_1}{(\gamma_1 - 1)} \quad (3.5)$$

$$C_{p2} = \frac{\gamma_2 R_2}{(\gamma_2 - 1)} \quad (3.6)$$

$$C_{px} = C_{py} = C_{p3} = x C_{p1} + (1 - x) C_{p2} \quad (3.7)$$

$$\gamma_x = \gamma_y = \gamma_3 = \frac{C_{p3}}{(C_{p3} - R_3)} \quad (3.8)$$

$$T_{sx} = T_{sy} = T_{s3} = \frac{1}{C_{p3}} [x C_{p1} T_{s1} + (1 - x) C_{p2} T_{s2}] \quad (3.9)$$

$$A_x = A_y = A_3 = (A_1 + A_2) \quad (3.10)$$

Notice that the quantities  $R$ ,  $C_p$ ,  $\gamma$ ,  $T_s$  and  $A$  all have the same values at stations  $x$ ,  $y$  and  $3$ .

This fact may be used to simplify many of the subsequent expressions. It is always permissible to substitute subscript  $3$  for subscripts  $x$  or  $y$  on any of these quantities.

#### 4. Mass Flow

It is convenient temporarily to omit station subscripts 1, 2, 3, x, y and thereby develop certain needed mass flow relations in generalized form. Thus the mass flow across an arbitrary station may be written and developed as follows.

$$\begin{aligned} \dot{m} &= \rho AV = \left( \frac{P}{RT} \right) A (\sqrt{\gamma RT} M) = PAM \sqrt{\frac{\gamma}{RT}} \\ &= \frac{P_s A}{\sqrt{RT_s/\gamma}} \sqrt{\frac{T_s/T}{(P_s/P)}} M \end{aligned} \quad (4.1)$$

This may be rewritten as

$$\dot{m} = \frac{P_s A}{\sqrt{RT_s/\gamma}} f(M) \quad (4.2)$$

where the auxiliary function  $f(M)$  is defined as follows.

$$f(M) = \sqrt{\frac{T_s/T}{(P_s/P)}} M = M \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M^2 \right]^{-\frac{(\gamma + 1)}{2(\gamma - 1)}} \quad (4.3)$$

Eq. (4.2) can be applied specifically to stations 1, 2, x, y, 3 as follows

$$\dot{m}_1 = \frac{P_{s1} A_1}{\sqrt{R_1 T_{s1}/\gamma_1}} f(M_1) \quad (4.4)$$

$$\dot{m}_2 = \frac{P_{s2} A_2}{\sqrt{R_2 T_{s2}/\gamma_2}} f(M_2) \quad (4.5)$$

$$\dot{m}_x = \frac{P_{sx} A_3}{\sqrt{R_3 T_{s3}/\gamma_3}} f(M_x) \quad (4.6)$$

$$\dot{m}_y = \frac{P_{sy} A_3}{\sqrt{R_3 T_{s3}/\gamma_3}} f(M_y) \quad (4.7)$$

$$\dot{m}_3 = \frac{P_{s3} A_3}{\sqrt{R_3 T_{s3}/\gamma_3}} f(M_3) \quad (4.8)$$

also

$$\left(\frac{\dot{m}_1}{\dot{m}_3}\right) = x \quad (4.9)$$

$$\dot{m}_x = \dot{m}_y = \dot{m}_3 = \dot{m}_1 + \dot{m}_2 \quad (4.10)$$

From Eqs. (4.4), (4.5), (4.9) and (4.10) we readily find that

$$\left(\frac{A_1}{A_2}\right) = \frac{x}{(1-x)} \left(\frac{P_{s2}}{P_{s1}}\right) \sqrt{\frac{R_1 T_{s1} \gamma_2}{R_2 T_{s2} \gamma_1}} \frac{f(M_2)}{f(M_1)} \quad (4.11)$$

Then from Eq. (2.10) we infer also that

$$\left(\frac{A_1}{A_3}\right) = \frac{(A_1/A_2)}{(1 + A_1/A_2)} \quad (4.12)$$

Since all quantities on the right side of Eq. (4.11) are now known, Eqs. (4.11) and (4.12) fix the area ratios  $A_1/A_2$  and  $A_1/A_3$ .

From Eqs. (4.5), (4.6), (4.9), (4.10) and (2.8) we deduce further that

$$f(M_x) = \frac{(1 - A_1/A_3)}{(1 - x)} \left( \frac{P_{s2}}{P_{sx}} \right) \sqrt{\frac{R_3 T_{s3} \gamma_2}{R_2 T_{s2} \gamma_3}} f(M_2) \quad (4.13)$$

All quantities on the right side of Eq. (4.13) except  $P_{sx}$  are now known. The method of determining  $P_{sx}$  is explained in a later section; refer to Eq. (5.15). Once  $P_{sx}$  has been found, Eq. (4.13) fixes  $f(M_x)$  whereupon Eq. (4.3) fixes  $M_x$  itself. The solution of Eq. (4.3) for  $M$  when  $f(M)$  is known involves an iterative procedure which is explained later in this section.

From Eqs. (4.6), (4.7), (4.8) and (4.10) we also find that

$$P_{sx} f(M_x) = P_{sy} f(M_y) = P_{s3} f(M_3) \quad (4.14)$$

Assuming  $P_{sx}$  and  $M_x$  known, this relation along with Eq. (4.3) fixes  $P_{sy}$  when  $M_y$  is specified; it also fixes  $f(M_3)$  and  $M_3$  when  $P_{s3}$  is specified. The methods of calculating  $M_y$  and  $P_{s3}$  are explained later; refer to Eqs. (6.8) and (7.14).

Fig. 4.1 is a rough sketch of the function  $f(M)$  as defined by Eq. (4.3). This function vanishes at  $M = 0$  and at  $M = \infty$ . The maximum value occurs at  $M = 1$  and equals

$$f_{\max} = \left[ \frac{2}{\gamma + 1} \right]^{2 \frac{(\gamma + 1)}{(\gamma - 1)}} \quad (4.15)$$

It is clear from the figure that in the range  $0 < f < f_{\max}$ , Eq. (4.3) has two positive real roots, one subsonic and the other supersonic. Usually the supersonic root must be discarded because it is not consistent with a further constraint imposed by the second law of thermodynamics.

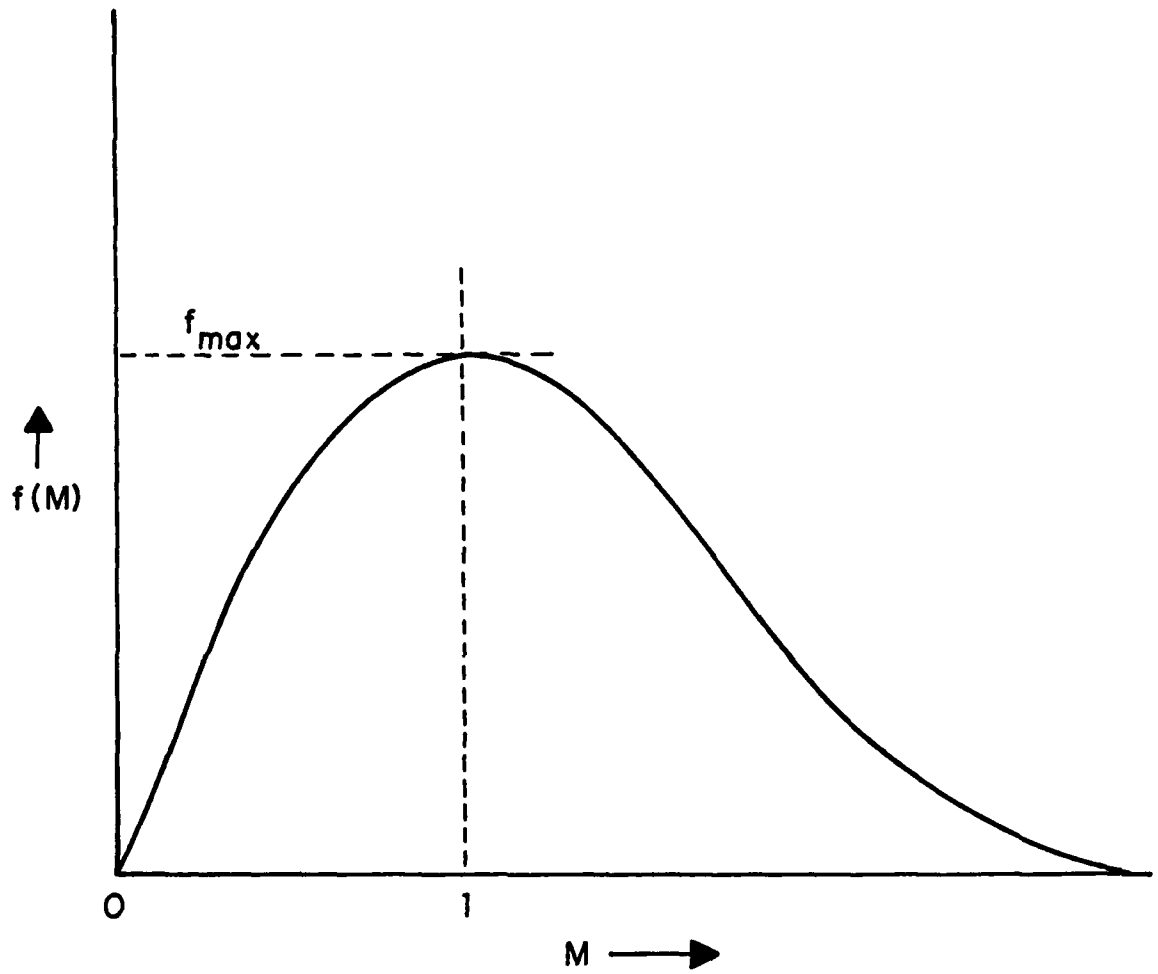


Fig. 4.1 APPROXIMATE SKETCH OF FUNCTION  $f(M)$

In order to solve Eq. (4.3) for  $M$  when values of  $f$  and  $\gamma$  are specified, we utilize an iterative procedure based on Newton's method. Let  $M_n$  denote the  $n$ th trial value of  $M$ . We then define a function  $F(M_n)$  and its derivative  $F'(M_n)$  as follows.

$$F(M_n) = f \left[ 1 + \frac{\gamma - 1}{2} M_n^2 \right]^{\frac{(\gamma + 1)}{2(\gamma - 1)}} - M_n \quad (4.16)$$

$$F'(M_n) = f \left( \frac{\gamma + 1}{2} \right) M_n \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M_n^2 \right]^{\frac{(3 - \gamma)}{2(\gamma - 1)}} - 1 \quad (4.17)$$

The  $(n + 1)$ st trial value of  $M$  may now be taken as

$$M_{(n + 1)} = M_n - \frac{F(M_n)}{F'(M_n)} \quad (4.18)$$

The cycle of calculations defined by Eqs. (4.16), (4.17) and (4.18) is repeated until the result for  $M_n$  converges to a stable value at the desired level of accuracy.

Thus

$$M = \lim_{n \rightarrow \infty} M_n \quad (4.19)$$

It is readily apparent that the value of  $M$  so obtained satisfies Eq. (4.3) for the prescribed values of  $f$  and  $\gamma$ .

## 5. Entropy

Consider the case shown in Fig. 2.1(a). The specific entropy  $s_x$  of the discharged gas may be expressed in the form

$$s_x = x \left[ C_{p1} \ln \left( \frac{T_x}{T_o} \right) - R_1 \ln \left( \frac{P_{x1}}{P_o} \right) \right] + (1 - x) \left[ C_{p2} \ln \left( \frac{T_x}{T_o} \right) - R_2 \ln \left( \frac{P_{x2}}{P_o} \right) \right] \quad (5.1)$$

where  $P_{x1}$  and  $P_{x2}$  are the partial pressures of the two components of the gas mixture. Symbols  $P_o$  and  $T_o$  denote the pressure and temperature of the ambient atmosphere. The entropy of each component is assigned the value zero at this reference state  $P_o, T_o$ .

Incidentally, the primary fluid, being condensible, might well exist only in the liquid state at the reference condition  $P_o, T_o$ . Nevertheless, so long as it is in the gaseous state at station  $x$ , Eq. (5.1) may still be used.

The partial pressures of the two components are proportional to the respective mol fractions. Hence

$$\left( \frac{P_{x1}}{P_o} \right) = \frac{x R_1}{x R_1 + (1 - x) R_2} \left( \frac{P_x}{P_o} \right) = \frac{x R_1}{R_3} \left( \frac{P_x}{P_o} \right) \quad (5.2)$$

$$\left( \frac{P_{x2}}{P_o} \right) = \frac{(1 - x) R_2}{x R_1 + (1 - x) R_2} \left( \frac{P_x}{P_o} \right) = \frac{(1 - x) R_2}{R_3} \left( \frac{P_x}{P_o} \right) \quad (5.3)$$

Upon substituting Eqs. (5.2) and (5.3) into (5.1) and simplifying we may obtain the result in the form

$$s_x = s_m + C_{p3} \ln \left( \frac{T_x}{T_0} \right) - R_3 \ln \left( \frac{P_x}{P_0} \right) \quad (5.4)$$

where

$$s_m = R_3 \ln R_3 - x R_1 \ln (x R_1) - (1 - x) R_2 \ln [(1 - x) R_2] \quad (5.5)$$

It can be shown that this quantity  $s_m$  represents the entropy increase associated with the mass mixing of the two different gases.

Moreover, since the entropy of the stagnation state  $s_x$  is by definition identical to that of the corresponding static state  $x$ , we may replace  $T_x$  and  $P_x$  in Eq. (5.4) by  $T_{sx}$  and  $P_{sx}$ , respectively. Thus we obtain

$$s_x = s_m + C_{p3} \ln \left( \frac{T_{sx}}{T_0} \right) - R_3 \ln \left( \frac{P_{sx}}{P_0} \right) \quad (5.6)$$

In previous studies of this series, we have considered only the special case in which the same fluid is employed for both the primary and secondary streams. In that case we have

$$R_1 = R_2 = R_3 = R \quad (5.7)$$

$$C_{p1} = C_{p2} = C_{p3} = C_p \quad (5.8)$$

$$\gamma_1 = \gamma_2 = \gamma_3 = \gamma \quad (5.9)$$

Moreover, in this special case there is no mass mixing, so that we must set

$$s_m = 0 \quad (5.10)$$

Consequently, Eq. (5.6) now simplifies to

$$s_x = 0 + C_p \ln \left( \frac{T_{sx}}{T_o} \right) - R \ln \left( \frac{P_{sx}}{P_o} \right) \quad (5.11)$$

Next reverting to the case of two different gases, let  $\Delta t$  denote a time interval during which unit mass crosses station  $x$ . During this same interval  $x$  units of mass cross station 1 and  $(1 - x)$  units of mass cross station 2. The entropy which leaves across station  $x$  must equal the entropy which enters across stations 1 and 2 plus the entropy increase caused by the mass mixing. Thus

$$s_x = s_m + x s_1 + (1 - x)s_2 \quad (5.12)$$

where

$$s_1 = C_{p1} \ln \left( \frac{T_{s1}}{T_o} \right) - R_1 \ln \left( \frac{P_{s1}}{P_o} \right) \quad (5.13)$$

$$s_2 = C_{p2} \ln \left( \frac{T_{s2}}{T_o} \right) - R_2 \ln \left( \frac{P_{s2}}{P_o} \right) \quad (5.14)$$

Upon eliminating  $s_x$  between Eqs. (5.6) and (5.12), we find that  $s_m$  also cancels from the result. It is then a simple matter to solve for the exit stagnation pressure in the form

$$\ln \left( \frac{P_{sx}}{P_o} \right) = \frac{1}{R_3} \left\{ C_{p3} \ln \left( \frac{T_{s3}}{T_o} \right) - x s_1 - (1 - x)s_2 \right\} \quad (5.15)$$

Thus the sequence defined by Eqs. (5.13), (5.14) and (5.15) now fixes  $P_{sx}$ . Next referring back to Eq. (4.13), we can calculate the value of  $f(M_x)$ ; finally, from Eqs. (4.16), (4.17), (4.18) we can calculate  $M_x$  itself. Once  $P_{sx}$ ,  $T_{sx} = T_{s3}$  and  $M_x$  are known, it is a simple matter to calculate the corresponding conditions  $P_x$ ,  $T_x$  from the usual perfect gas relations.

## 6. Momentum

Consider the idealized one dimensional constant area ejector shown in Fig. 2.1(b). The momentum equation for this system may be written

$$(P_1 + \rho_1 V_1^2)A_1 + (P_2 + \rho_2 V_2^2)A_2 = (P_y + \rho_y V_y^2)A_y \quad (6.1)$$

The mass conservation relation for this system can be expressed in the form

$$\frac{\rho_1 A_1 V_1}{x} = \frac{\rho_2 A_2 V_2}{(1-x)} = \rho_y A_y V_y \quad (6.2)$$

Let us divide the three terms of Eq. (6.1) by the corresponding three terms of Eq. (6.2). Notice that the areas cancel out of the result. In this way we obtain

$$x \frac{(P_1 + \rho_1 V_1^2)}{\rho_1 V_1} + (1-x) \frac{(P_2 + \rho_2 V_2^2)}{\rho_2 V_2} = \frac{(P_y + \rho_y V_y^2)}{\rho_y V_y} \quad (6.3)$$

Temporarily dropping the station subscript, we next develop the typical term of Eq. (6.3) in generalized format as follows.

$$\begin{aligned} \frac{(P + \rho V^2)}{\rho V} &= \frac{\left[ P + \frac{P}{RT} (\gamma RT) M^2 \right]}{\frac{P}{RT} \sqrt{\gamma RT} M} = \sqrt{\frac{RT_s}{\gamma}} \frac{(1 + \gamma M^2)}{M \sqrt{T_s/T}} \\ &= \sqrt{\frac{RT_s}{\gamma}} \frac{(1 + \gamma M^2)}{M \sqrt{1 + (\frac{\gamma-1}{2}) M^2}} \end{aligned} \quad (6.4)$$

Observe that the pressure  $P$  cancels from the result.

It is convenient to define the auxiliary function

$$g(M) = \frac{M}{(1 + \gamma M^2)} \sqrt{1 + \left(\frac{\gamma - 1}{2}\right) M^2} \quad (6.5)$$

With this notation Eq. (6.3) may be rewritten in the form

$$\frac{x}{g(M_1)} \sqrt{\frac{R_1 T_{s1}}{\gamma_1}} + \frac{(1-x)}{g(M_2)} \sqrt{\frac{R_2 T_{s2}}{\gamma_2}} = \frac{1}{g(M_y)} \sqrt{\frac{R_3 T_{s3}}{\gamma_3}} \quad (6.6)$$

Rearranging gives

$$g(M_y) = \sqrt{\frac{R_3 T_{s3}}{\gamma_3}} \left\{ \frac{x}{g(M_1)} \sqrt{\frac{R_1 T_{s1}}{\gamma_1}} + \frac{(1-x)}{g(M_2)} \sqrt{\frac{R_2 T_{s2}}{\gamma_2}} \right\}^{-1} \quad (6.7)$$

Since all quantities on the right are known, Eq. (6.7) fixes  $g(M_y)$ . Then  $M_y$  itself can be found by inverting Eq. (6.5). Fortunately, an explicit solution is possible in this case as Eq. (6.5) can be reduced to a quadratic in  $M^2$ . The result is

$$M^2 = \frac{(1 - 2\gamma g^2) \pm \sqrt{1 - 2(\gamma + 1)g^2}}{1 - \gamma(1 - 2\gamma g^2)} \quad (6.8)$$

The general character of the function  $g(M)$  is sketched in Fig. 6.1. Notice that  $g(M)$  vanishes at  $M = 0$ , reaches its peak value  $g_{\max}$  at  $M = 1$ , and decreases toward the limit  $g_{\infty}$  as  $M \rightarrow \infty$ . It can be shown that

$$g_{\max} = \frac{1}{\sqrt{2(\gamma + 1)}} \quad (6.9)$$

$$g_{\infty} = \frac{1}{\gamma} \sqrt{\frac{\gamma - 1}{2}} \quad (6.10)$$

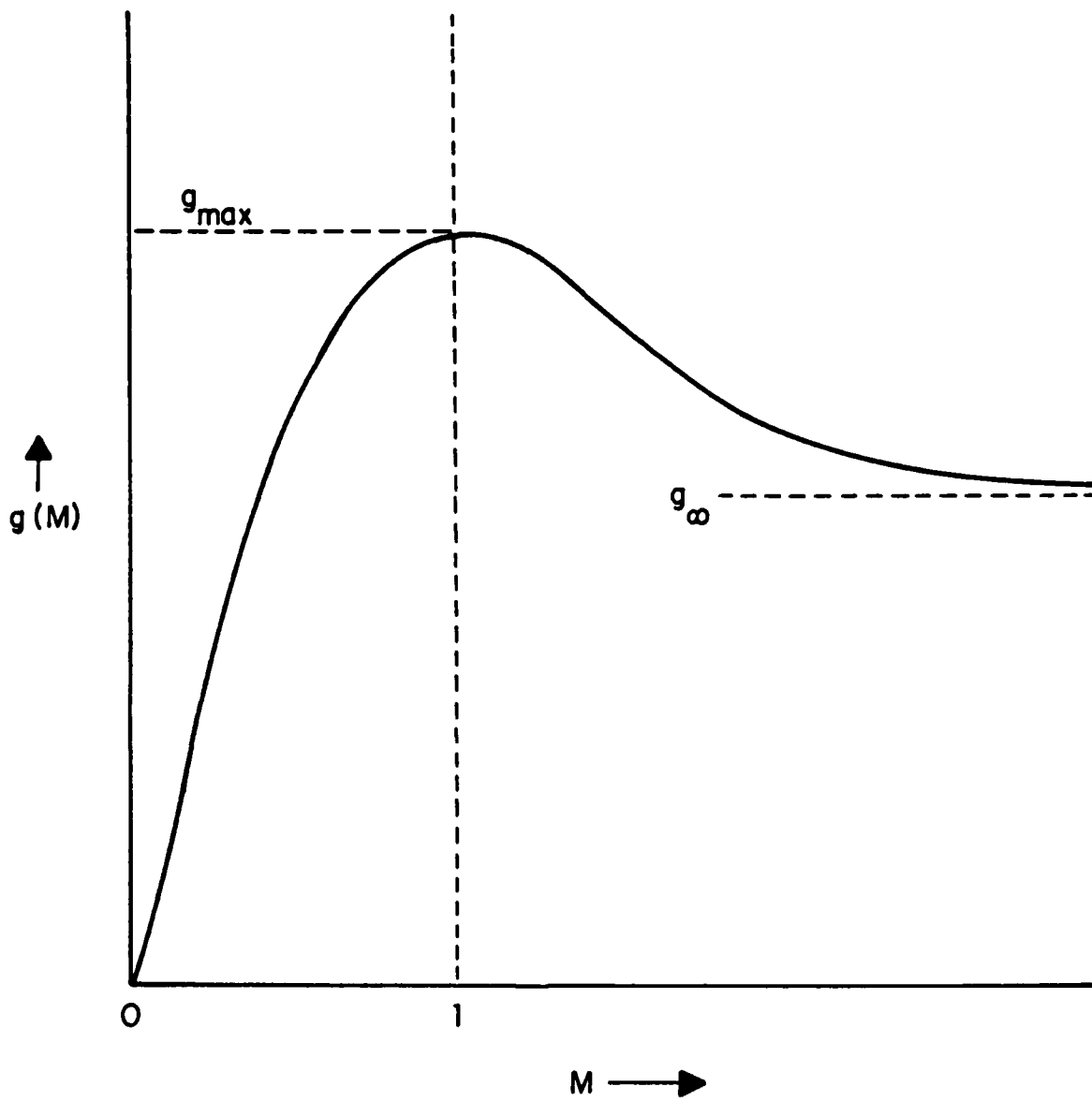


Fig. 6.1 APPROXIMATE SKETCH OF FUNCTION  $g(M)$

Fig. 6.1 discloses the existence of a supersonic root over the range  $g_{\infty} < g < g_{\max}$ . This root should not be accepted until a check calculation is made to determine whether the result is consistent with the second law of thermodynamics. The subsonic root is found by retaining only the negative sign before the radical in Eq. (6.8).

Once  $M_y$  has been found from Eq. (6.8), the corresponding value of  $P_{sy}$  may be found from Eqs. (4.3) and (4.14). This suffices to fix all properties at states  $y$  and  $sy$ .

## 7. Availability

The entropies  $s_1$  and  $s_2$  of the two input streams were defined earlier in Eqs. (5.13) and (5.14). The corresponding steady flow availability functions with respect to an ambient atmosphere at pressure  $P_o$  and temperature  $T_o$  may be written

$$\psi_1 = C_{p1} (T_{s1} - T_o) - T_o s_1 \quad (7.1)$$

$$\psi_2 = C_{p2} (T_{s2} - T_o) - T_o s_1 \quad (7.2)$$

Consequently the total available energy entering the system becomes

$$\psi_m = x \psi_1 + (1 - x) \psi_2 \quad (7.3)$$

The respective entropies of the streams leaving at stations  $x$ ,  $y$  and  $3$  are

$$s_x = s_m + C_{p3} \ln \left( \frac{T_{s3}}{T_o} \right) - R_3 \ln \left( \frac{P_{sx}}{P_o} \right) \quad (7.4)$$

$$s_y = s_m + C_{p3} \ln \left( \frac{T_{s3}}{T_o} \right) - R_3 \ln \left( \frac{P_{sy}}{P_o} \right) \quad (7.5)$$

$$s_3 = s_m + C_{p3} \ln \left( \frac{T_{s3}}{T_o} \right) - R_3 \ln \left( \frac{P_{s3}}{P_o} \right) \quad (7.6)$$

where  $s_m$  has been previously defined in Eq. (5.5).

The corresponding availabilities of the streams leaving at stations  $x$ ,  $y$  and  $3$  may therefore be written

$$\psi_x = C_{p3} (T_{s3} - T_o) - T_o s_x \quad (7.7)$$

$$\psi_y = C_{p3} (T_{s3} - T_o) - T_o s_y \quad (7.8)$$

$$\psi_3 = C_{p3} (T_{s3} - T_o) - T_o s_3 \quad (7.9)$$

Now consider the losses of availability listed below. By substituting the above expressions for the quantities on the left and simplifying we readily obtain the expressions shown on the right. Thus

$$(\psi_m - \psi_x) = T_o s_m \quad (7.10)$$

$$(\psi_x - \psi_y) = R_3 T_o \ln \left( \frac{P_{sx}}{P_{sy}} \right) \quad (7.11)$$

$$(\psi_x - \psi_3) = R_3 T_o \ln \left( \frac{P_{sx}}{P_{s3}} \right) \quad (7.12)$$

These results are very significant. Eq. (7.10) defines the loss of available energy caused by the mass mixing of the two different gases. Eq. (7.11) defines the further loss of available energy caused by the momentum mixing in an idealized ejector. Eq. (7.12) defines the augmented loss of available energy caused by the momentum mixing in the real ejector.

It is useful to postulate an empirical relation between these last two losses which is shown below on the left. The expression on the right then follows from Eqs. (7.11) and (7.12). Thus

$$\frac{(\psi_x - \psi_y)}{(\psi_x - \psi_3)} = \eta_E = \frac{\ln (P_{sy}/P_{sx})}{\ln (P_{s3}/P_{sx})} \quad (7.13)$$

We term  $\eta_E$  the ejector effectiveness. Its value, which is always less than unity, must be estimated from appropriate test data on ejectors.

If we treat  $\eta_E$  as known, Eq. (7.13) fixes  $P_{s3}$ . The solution is simply

$$P_{s3} = P_{sx} \left( \frac{P_{sy}}{P_{sx}} \right)^{\frac{1}{\eta_E}} \quad (7.14)$$

We can next find  $f(M_3)$  from Eq. (4.14) and  $M_3$  from Eqs. (4.16), (4.17) and (4.18). This suffices to fix all properties at states 3 and s3 thereby completing the analysis of the ejector.

## 8. Diffuser

It is convenient to analyze the diffuser in terms of the concept of availability in steady flow. However, we wish to deal with the characteristics of the diffuser itself and these cannot depend on the arbitrary values  $P_0$ ,  $T_0$  which happen to characterize the condition of the ambient atmosphere. We can achieve our objective by defining the availability with respect to a reference state  $P_3$ ,  $T_3$  which characterizes the diffuser rather than with respect to the ambient state  $P_0$ ,  $T_0$ .

Under these circumstances we may write the appropriate availabilities at diffuser inlet and outlet as follows

$$\psi_{s3} = C_{p3} (T_{s3} - T_3) - T_3 \left\{ C_{p3} \ln \left( \frac{T_{s3}}{T_3} \right) - R_3 \ln \left( \frac{P_{s3}}{P_3} \right) \right\} \quad (8.1)$$

$$\psi_{s4} = C_{p3} (T_{s3} - T_3) - T_3 \left\{ C_{p3} \ln \left( \frac{T_{s3}}{T_3} \right) - R_3 \ln \left( \frac{P_{s4}}{P_3} \right) \right\} \quad (8.2)$$

These equations make use of the fact that  $T_{s4} = T_{s3}$ . Also, states  $s3$  and  $3$  are at the same entropy so that

$$(s_{s3} - s_3) = 0 = C_p \ln \left( \frac{T_{s3}}{T_3} \right) - R_3 \ln \left( \frac{P_{s3}}{P_3} \right) \quad (8.3)$$

From Eqs. (8.1) and (8.3) we conclude that

$$\psi_{s3} = C_{p3} (T_{s3} - T_3) = \left( \frac{V_3^2}{2} \right) \quad (8.4)$$

This result shows that the available energy at the diffuser inlet is simply the inlet kinetic energy itself.

The loss of availability through the diffuser may be found by subtracting Eq. (8.2) from (8.1). The result is

$$(\psi_{s4} - \psi_{s3}) = \Delta\psi = + R_3 T_3 \ln \left( \frac{P_{s3}}{P_{s4}} \right) \quad (8.5)$$

We now define the diffuser effectiveness as

$$\left( \frac{\psi_{s4}}{\psi_{s3}} \right) = \eta_D \quad (8.6)$$

so that

$$\left( \frac{\psi_{s3} - \psi_{s4}}{\psi_{s3}} \right) = \frac{\Delta\psi}{\psi_{s3}} = (1 - \eta_D) = \frac{R_3 T_3 \ln \left( \frac{P_{s3}}{P_{s4}} \right)}{C_{p3} (T_{s3} - T_3)} \quad (8.7)$$

This reduces readily to

$$(1 - \eta_D) = \frac{\ln \left( \frac{P_{s3}}{P_{s4}} \right)}{\left( \frac{\gamma_3 M_3^2}{2} \right)} \quad (8.8)$$

Solving for the pressure ratio gives

$$\frac{P_{s4}}{P_{s3}} = e^{- (1 - \eta_D) \frac{\gamma_3 M_3^2}{2}} \quad (8.9)$$

This is the result required. It fixes  $P_{s4}$  when  $P_{s3}$ ,  $M_3$  and  $\eta_D$  are specified.

Assuming the area ratio  $A_3/A_4$  to be arbitrarily specified, we can find  $M_4$  from the continuity relation. Referring to Eq. (4.8) we may write

$$\dot{m}_3 = \frac{P_{s3} A_3}{\sqrt{R_3 T_{s3} / \gamma_3}} f(M_3) = \frac{P_{s4} A_4}{\sqrt{R_3 T_{s3} / \gamma_3}} f(M_4) = \dot{m}_4 \quad (8.10)$$

Consequently

$$f(M_4) = \left( \frac{P_{s3}}{P_{s4}} \right) \left( \frac{A_3}{A_4} \right) f(M_3) \quad (8.11)$$

Eq. (8.11) fixes  $f(M_4)$ . Then  $M_4$  follows in the usual way from Eqs. (4.16), (4.17) and (4.18). The result suffices to fix all properties at states 4 and  $s_4$ . Thus

$$\left( \frac{T_{s3}}{T_4} \right) = \left( \frac{T_{s4}}{T_4} \right) = 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_4^2 \quad (8.12)$$

$$T_4 = T_{s3} / (T_{s3} / T_4) \quad (8.13)$$

$$\left( \frac{P_{s4}}{P_4} \right) = \left( \frac{T_{s3}}{T_4} \right)^{\frac{\gamma_3}{(\gamma_3 - 1)}} \quad (8.14)$$

$$P_4 = P_{s4} / (P_{s4} / P_4) \quad (8.15)$$

The configurations studied in earlier work did not incorporate a diffuser. For the purpose of comparing the results of the present analysis with corresponding earlier work, it is desirable to be able to eliminate the effects of the diffuser in certain cases. This can be accomplished in the present analysis simply by setting

$$\eta_D = 1 \quad (8.16)$$

and

$$\left(\frac{A_3}{A_4}\right) = 1 \quad (8.17)$$

It is easy to confirm that under these circumstances

$$T_{s4} = T_{s3} \quad (8.18)$$

$$P_{s4} = P_{s3} \quad (8.19)$$

$$M_4 = M_3 \quad (8.20)$$

$$A_4 = A_3 \quad \text{etc.} \quad (8.21)$$

and all diffuser effects disappear.

## 9. Electrical Power Section

The power conversion process in an EHD duct is usually treated on the basis either of constant area or of constant static state. Inasmuch as electrical power output per unit mass is small, the numerical differences between the results computed by these two methods is negligible. For definiteness in this analysis, however, we assume constant area.

The electrical power that can be obtained from an EHD duct of constant area, negligible change of density and optimum length can be estimated from the one dimensional version of Poisson's equation which governs the electrical field. The solution is well known and will not be derived here; a detailed derivation may be found in Ref. (3). The essential result may be written in the form

$$P_e = \frac{1}{2} \epsilon E_b^2 A_4 V_4 \quad (9.1)$$

where

$P_e$  = gross electric power output, watts

$\epsilon$  = permittivity of medium

=  $8.854 \times 10^{-12}$  farad/m (for any gas)

$E_b$  = dielectric strength of medium, volts/m

$A_4$  = area of duct (constant)

$V_4$  = velocity

According to the test data analyzed in Ref. 4, the dielectric strength is well approximated by the expression

$$E_b = C_{B3} R_3 \rho_4 \quad (9.2)$$

where  $C_{B3}$  is a characteristic constant of the medium. The data show that Eq. (9.2) applies to air or steam up to about 10 atmospheres pressure with

$$C_{B3} \doteq 9.49 \times 10^3 \text{ m}^2 \text{ }^\circ\text{K/cmb} \quad (9.3)$$

In this study we assume that Eq. (9.2) can be extrapolated up to about 100 atmospheres pressure. We also assume that an expression of the same form applies to other media besides air or steam but that each medium has its own characteristic value of the breakdown constant  $C_B$ .

In the two-fluid system, the separate values of the breakdown constants  $C_{B1}$  and  $C_{B2}$  are usually known, but the breakdown constant  $C_{B3}$  of the resulting mixture is seldom known. In the absence of adequate test data bearing on this point, we tentatively assume that  $C_{B3}$  can be estimated from the hypothetical relation

$$C_{B3} = \frac{1}{R_3} [xR_1 C_{B1} + (1-x)R_2 C_{B2}] \quad (9.4)$$

Upon substituting Eq. (9.2) into (9.1) and dividing through by the mass flow rate, we find the electrical work output per unit mass of fluid in the form:

$$\frac{P_e}{\dot{m}_4} = \left( \frac{\epsilon C_{B3}^2 R_3^2}{2} \right) \rho_4 = C_{p3} (T_{s4} - T_{s5}) \quad (9.5)$$

By use of standard perfect gas relations, Eq. (9.5) can be changed to a more useful form. In this connection set

$$(T_{s4} - T_{s5}) = \Delta T_s \quad (9.6)$$

and

$$\beta = \frac{\epsilon C_{B3}^2 P_o}{T_o^2} \quad (9.7)$$

where

$$P_o = \text{ambient pressure, N/m}^2$$

$$T_o = \text{ambient temperature, } ^\circ\text{K}$$

Notice that  $\beta$  represents a dimensionless version of the electrical breakdown constant.

Eq. (9.5) now yields the important result

$$\frac{\Delta T_s}{T_{s4}} = \beta \left( \frac{\gamma_3 - 1}{2\gamma_3} \right) \left( \frac{P_{s4}}{P_o} \right) \left( \frac{T_o}{T_{s4}} \right)^2 \left( 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_4^2 \right)^{-\frac{1}{\gamma_3 - 1}} \quad (9.8)$$

The corresponding gross electrical work output per unit mass of fluid is then simply

$$W_e^* = C_{p3} \Delta T_s \quad (9.9)$$

Unfortunately, the known value of  $\epsilon$  and the typical experimentally measured values of  $C_B$  are very small. Consequently, the dimensionless breakdown constant  $\beta$  is extremely small compared to unity. Hence the drop in stagnation temperature  $\Delta T_s$  and the gross electrical work per unit mass  $W_e^*$  are also typically very small. This is a very basic and serious limitation on the performance that can be obtained from an EHD generator.

Treating the process 4→5 as isentropic, we may write

$$P_{s5} = P_{s4} \left( \frac{T_{s5}}{T_{s4}} \right)^{\frac{\gamma_3}{(\gamma_3 - 1)}} \quad (9.10)$$

By analogy with Eq. (4.8) we write the continuity relation as

$$\dot{m}_4 = \frac{P_{s4} A_4}{\sqrt{R_3 T_{s4}/\gamma_3}} f(M_4) = \frac{P_{s5} A_5}{\sqrt{R_3 T_{s5}/\gamma_3}} f(M_5) = \dot{m}_5 \quad (9.11)$$

It then follows that

$$f(M_5) = \sqrt{\frac{T_{s5}}{T_{s4}}} \left( \frac{P_{s4}}{P_{s5}} \right) f(M_4) \quad (9.12)$$

This fixes  $f(M_5)$  whereupon  $M_5$  may be found in the usual way from Eqs. (4.16), (4.17) and (4.18). However, convergence should now be very rapid since we may take as a first approximation

$$M_5 \approx M_4 \quad (9.13)$$

This solution now suffices to fix all properties at states 4 and s4 .

## 10. Condenser/Separator

The primary fluid which leaves the condenser/separator at station 6 is assumed to be compressed liquid at known static pressure  $P_6 = P_2 = P_1$  and at known static temperature  $T_6 = T_2$ . The kinetic energy of the liquid at station 6 is assumed to be negligible.

Temperature  $T_6$  must be equal to or greater than the ambient temperature  $T_0$  in order to satisfy the requirements of a heat balance on the condenser/separator. No attempt is made in this analysis to formulate this actual heat balance; instead  $T_6$  is simply treated as a given or known quantity. However, it may be of interest later to investigate the effects of varying  $T_6$  on the overall performance of the EHD system.

The secondary fluid which leaves the condenser/separator and enters the ejector at station 2 consists primarily of a low molecular weight noncondensing gas. It also contains some secondary vapor, but the mass fraction of this vapor is treated as negligible in this analysis. Thus the secondary fluid at station 2 is treated as dry gas. The kinetic energy at station 2 is not negligible and must be taken into account in the analysis.

In analyzing the thermodynamic process in the condenser/separator, it is again useful to employ the concept of availability in steady flow. The availability function is always defined with respect to some suitable reference state. Nor-

mally this is fixed by the conditions  $P_0$ ,  $T_0$  of the ambient atmosphere. In some cases, however, we seek to characterize the performance of some component of the system in a manner which is independent of the ambient conditions  $P_0$ ,  $T_0$ . In such cases it is useful to choose as a reference state some condition that actually occurs in the component in question. We used such a procedure to good advantage in analyzing the diffuser and in defining the diffuser effectiveness  $\eta_D$ . We now use it again in analyzing the condenser/separator and in defining a friction loss coefficient  $c_f$  which characterizes the operation of this component. For this purpose we choose as the appropriate reference state of zero availability the static conditions  $P_6$ ,  $T_6$  in the condenser/separator.

It also clarifies matters further if we choose the condition  $P_6$ ,  $T_6$  as the datum state of zero enthalpy and zero entropy. In this way we ensure that our analysis in this section makes no reference whatever to the state  $P_0$ ,  $T_0$ . Remember also that  $P_6 = P_2 = P_1$  and  $T_6 = T_2$ .

Thus we may now write

$$s_5 = s_m + C_{p3} \ln \left( \frac{T_{s5}}{T_6} \right) - R_3 \ln \left( \frac{P_{s5}}{P_1} \right) \quad (10.1)$$

$$s_2 = C_{p2} \ln \left( \frac{T_{s2}}{T_2} \right) - R_2 \ln \left( \frac{P_{s2}}{P_2} \right) = 0 \quad (10.2)$$

$$s_6 = 0 \quad (10.3)$$

The quantity  $s_m$  in Eq. (10.1) is the same as previously defined in Eq. (5.5). It expresses the entropy increase associated with mass mixing.

The availabilities at stations 5, 2 and 6 are therefore

$$\psi_5 = C_{p3} (T_{s5} - T_2) - T_6 s_5 \quad (10.4)$$

$$\psi_2 = C_{p2} (T_{s5} - T_2) - 0 \quad (10.5)$$

$$\psi_6 = h_6 - T_6 s_6 = 0 \quad (10.6)$$

The irreversibility  $I$  of the process which occurs in the condenser/separator with respect to the specified reference state  $P_6, T_6$  may be expressed in terms of the above availabilities as follows

$$I = \psi_5 - (1 - x) \psi_2 - x \psi_6^{\circ} \quad (10.7)$$

We postulate that this irreversibility can also be correlated with the inlet kinetic energy in terms of an empirical friction loss coefficient  $c_f$  as follows

$$I = c_f \left( \frac{V_5^2}{2} \right) = \left( \frac{c_f \gamma_3 R_3 T_{s5}}{2} \right) \frac{M_5^2}{(T_{s5}/T_5)} \quad (10.8)$$

where as usual

$$\left( \frac{T_{s5}}{T_5} \right) = \left[ 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_5^2 \right] \quad (10.9)$$

Of course the above friction loss coefficient  $c_f$  must be estimated from suitable experimental data.

Assuming that  $c_f$  is a specified constant, the trial solution obtained for an arbitrary trial value of  $x$  and arbitrary fixed values of the various other input parameters

will not necessarily satisfy Eq. (10.7). It is therefore convenient to represent the residual error in this equation in the form below. The factor  $R_3 T_6$  is inserted on the left to make  $E(x)$  dimensionless. Thus

$$R_3 T_6 E(x) = \psi_5 - (1 - x) \psi_2 - 0 - I \quad (10.10)$$

The foregoing relations can be assembled and rearranged as follows. Let

$$K_0 = \ln \left( \frac{P_{s5}}{P_6} \right) + \frac{(C_{p3} - C_{p2})}{R_3} \left( \frac{T_{s5}}{T_6} - 1 \right) - \frac{\gamma_3}{(\gamma_3 - 1)} \ln \left( \frac{T_{s5}}{T_6} \right) - \frac{s_m}{R} - \frac{c_f \gamma_3}{2} \left( \frac{T_5}{T_6} \right) M_5^2 \quad (10.11)$$

$$K_1 = \frac{C_{p2}}{R_3} \left( \frac{T_{s5}}{T_6} - 1 \right) \quad (10.12)$$

Then the final energy balance equation reduces to

$$E(x) = K_0 + K_1 x \quad (10.13)$$

By holding all other input parameters fixed and allowing  $x$  to take on a range of possible values, it will normally be possible to find by trial a value of  $x$  such that

$$E(x) = 0 \quad (10.14)$$

This then is the value of  $x$  which is consistent with the other specified input parameters and which defines a possible EHD thermodynamic cycle.

Of course, if the various fixed input parameters are not properly chosen, Eq. (10.14) cannot be satisfied for any trial value of  $x$  which lies in the useful range  $0 < x < 1$  and no solution exists under these conditions.

If a solution exists it can be found by some suitable iteration technique. One useful method is to set  $E(x) = 0$  in Eq. (10.13) and then solve this equation for  $x$  but to relabel this as  $x_{\text{new}}$ . Then

$$x_{\text{new}} = - \frac{K_0}{K_1} \quad (10.15)$$

Assuming that conditions have been so specified that a solution for  $x$  actually exists, and assuming that an initial trial value of  $x$  has been chosen that is sufficiently close to the true root, repeated application of Eq. (10.15) should finally yield convergence to the true root itself.

Incidentally, any trial solution for which  $E(x) < 0$  represents a physical impossibility because it violates the second law of thermodynamics. Any trial solution for which  $E(x) > 0$  satisfies the second law but represents a condition in which the losses in the condenser/separator exceed the value prescribed by Eq. (10.8). Only the case for which  $E(x) = 0$  satisfies both Eq. (10.8) and the second law.

## 11. Cycle Efficiency

Once a solution has been found such that  $E(x) = 0$ , it becomes a straightforward matter to calculate the corresponding overall cycle efficiency.

Because of the pressure drop through the boiler/superheater, the pressure  $P_7 = P_{s7}$  at boiler inlet is slightly higher than the stagnation pressure  $P_{s1}$  of the primary fluid at the ejector inlet. We assume that the ratio  $P_{s1}/P_{s7}$  is a specified constant. Hence with  $P_{s1}$  specified,  $P_{s7}$  is also known. Also the pressures  $P_6 = P_1$  are known.

Consider the ideal pump work  $w_p^*$  done per unit mass of primary fluid. For the present consider the hypothetical case of a reversible pump. Also note that density changes across the pump are negligible. Hence we may write

$$w_p^* = \frac{(P_{s7} - P_6)}{\rho_6} \quad (11.1)$$

The ideal gross electrical work output per unit mass of mixed fluid has earlier been established in Eq. (9.5). It is denoted by symbol  $w_e^*$ .

Let us now denote the net useful electrical work output per unit mass of primary fluid by symbol  $w_{net}$ . It may be related to  $w_e^*$  and  $w_p^*$  in the following way

$$w_{net} = \left( \frac{\eta_x w_e^*}{x} - \frac{w_p^*}{\eta_p} \right) \text{ Joule/kg} \quad (11.2)$$

where  $\eta_x$ , the excitation efficiency, allows for the small electrical power expended to excite the system and where  $\eta_p$ ,

the pump efficiency, allows for irreversibilities in the pump.

The heat input in the boiler/superheater per unit mass of primary fluid may be written

$$q_{in} = (h_{s1} - h_6) - \frac{w_p^*}{\eta_p} \quad (11.3)$$

where  $h_6$  is the enthalpy of the primary liquid at conditions  $P_6$ ,  $T_6$  as listed in suitable tables of properties.

The overall thermal efficiency of the cycle can now be calculated from the simple formula

$$\eta_c = \frac{w_{net}}{q_{in}} \quad (11.4)$$

Both  $\eta_c$  and  $w_{net}$  are useful parameters which characterize the overall thermodynamic performance of the EHD system.

Eq. (11.2) shows clearly how the ejector serves as a kind of amplifier which increases the electrical work output per unit mass of primary fluid. This effect is shown by the presence of the parameter  $x$  in the denominator of the first term. Notice the beneficial effect of a low value of  $x$  on parameters  $w_{net}$  and  $\eta_c$ . Unfortunately, the value of  $x$  cannot be stipulated independently in advance; it is fixed by the other specified input parameters as explained in the previous section.

Notice that owing to the fact that  $w_e^*$  is normally very small, it is possible in some circumstances for  $w_{net}$

to take on values which are actually negative. When this happens it means that electrical power output is insufficient to drive the pump.

The relationships developed in this and the preceding sections make it possible to carry out systematic parametric studies of various one-fluid and two-fluid EHD cycles, with realistic allowances for the various losses that occur. Such studies can establish optimum design parameters and performance limits under various circumstances. These results in turn can finally permit informed conclusions to be drawn concerning the ultimate feasibility of this general type of EHD power generator.

## 12. Calculation Sequence for Two-Fluid System

In this section we summarize the various equations previously derived in the approximate order in which they would be used in the calculation of system performance. Also listed are the initial input parameters whose values must be specified in order to start the calculation and various further input parameters whose values must be specified in order to proceed with various successive stages of the calculation.

Initial Input Data:  $P_1 = P_2 = P_6$  ,  $T_1$  ,  $P_{s1}$  ,  $T_{s1}$  ,  $(h_{s1} - h_1)$  ,  
 $\bar{R} = 8315$  Joule/kg°K

$$\gamma_1 = \left( 1 - \frac{\ln(T_{s1}/T_1)}{\ln(P_{s1}/P_1)} \right)^{-1} \quad (12.1)$$

$$C_{p1} = \frac{(h_{s1} - h_1)}{(T_{s1} - T_1)} \quad (12.2)$$

$$R_1 = \left( \frac{\gamma_1 - 1}{\gamma_1} \right) C_{p1} \quad (12.3)$$

$$W_1 = \frac{\bar{R}}{R_1} \quad (12.4)$$

Further Input Data:  $W_v$  ,  $W_G = W_2$  ,  $T_2 = T_6$  ,  $P_v(T_6)$

$$\omega = \frac{W_v}{W_G} \frac{P_v(T_6)}{[P_2 - P_v(T_6)]} \quad (12.5)$$

Verify that  $\omega \ll 1$

Further Input Data:  $\gamma_2 \cdot \left(\frac{V_1}{V_2}\right) = y$

$$M_1 = \sqrt{\frac{2}{(\gamma_1 - 1)} \left[ \left(\frac{T_{s1}}{T_1}\right) - 1 \right]} \quad (12.6)$$

$$M_2 = \left(\frac{V_2}{V_1}\right) \sqrt{\frac{\gamma_1 R_1 T_{s1}}{\gamma_2 R_2 T_2}} \frac{M_1}{\sqrt{T_{s1}/T_1}} \quad (12.7)$$

Verify that  $M_2 < 1$  (12.8)

$$\left(\frac{T_{s2}}{T_2}\right) = 1 + \left(\frac{\gamma_2 - 1}{2}\right) M_2^2 \quad (12.9)$$

$$T_{s2} = T_2 (T_{s2}/T_2) \quad (12.10)$$

$$\left(\frac{P_{s2}}{P_2}\right) = \left(\frac{T_{s2}}{T_2}\right)^{\frac{\gamma_2}{(\gamma_2 - 1)}} \quad (12.11)$$

$$P_{s2} = P_2 \left(\frac{P_{s2}}{P_2}\right) \quad (12.12)$$

Further Input Data:  $x$  (trial value)

$$R_2 = \frac{\bar{R}}{W_2} \quad (12.13)$$

$$R_3 = xR_1 + (1 - x) R_2 \quad (12.14)$$

$$C_{p2} = \frac{\gamma_2 R_2}{(\gamma_2 - 1)} \quad (12.15)$$

$$C_{p3} = x C_{p1} + (1 - x) C_{p2} \quad (12.16)$$

$$\gamma_3 = \frac{C_{p3}}{(C_{p3} - R_3)} \quad (12.17)$$

$$T_{s3} = \frac{1}{C_{p3}} [x C_{p1} T_{s1} + (1 - x) C_{p2} T_{s2}] \quad (12.18)$$

$$f(M_1) = M_1 \left[ 1 + \frac{\gamma_1 - 1}{2} M_1^2 \right]^{-\frac{(\gamma_1 + 1)}{2(\gamma_1 - 1)}} \quad (12.19)$$

$$f(M_2) = M_2 \left[ 1 + \frac{\gamma_2 - 1}{2} M_2^2 \right]^{-\frac{(\gamma_2 + 1)}{2(\gamma_2 - 1)}} \quad (12.20)$$

$$\frac{A_1}{A_2} = \frac{x}{(1 - x)} \left( \frac{P_{s2}}{P_{s1}} \right) \sqrt{\frac{R_1 T_{s1} \gamma_2}{R_2 T_{s2} \gamma_1}} \frac{f(M_2)}{f(M_1)} \quad (12.21)$$

$$\frac{A_1}{A_3} = \frac{(A_1/A_2)}{(1 + A_1/A_2)} \quad (12.22)$$

Further Input Data:  $P_o, T_o$

$$s_1 = C_{p1} \ln \left( \frac{T_{s1}}{T_o} \right) - R_1 \ln \left( \frac{P_{s1}}{P_o} \right) \quad (12.23)$$

$$s_2 = C_{p2} \ln \left( \frac{T_{s2}}{T_o} \right) - R_2 \ln \left( \frac{P_{s2}}{P_o} \right) \quad (12.24)$$

$$\ln \left( \frac{P_{sx}}{P_o} \right) = \frac{1}{R_3} \left\{ C_{p3} \ln \left( \frac{T_{s3}}{T_o} \right) - x s_1 - (1 - x) s_2 \right\} \quad (12.25)$$

$$P_{sx} = P_o e^{\ln (P_{sx}/P_o)} \quad (12.26)$$

$$f(M_x) = f \frac{(1 - A_1/A_3)}{(1 - x)} \left( \frac{P_{s2}}{P_{sx}} \right) \sqrt{\frac{R_3 T_{s3} \gamma_1}{R_2 T_{s2} \gamma_3}} f(M_2) \quad (12.27)$$

$$f_{\max} = \left( \frac{2}{\gamma_3 + 1} \right)^{\frac{(\gamma_3 + 1)}{2(\gamma_3 - 1)}} \quad (12.28)$$

Verify that  $f(M_x) = f < f_{\max}$  (12.29)

For first approximation set  $M_x = M_2$  (12.30)

Iterate using Eqs. (12.31), (12.32), (12.33).

$$F(M_n) = f \left( 1 + \frac{(\gamma_3 - 1)}{2} M_n^2 \right)^{\frac{(\gamma_3 + 1)}{2(\gamma_3 - 1)}} - M_n \quad (12.31)$$

$$F'(M_n) = f \frac{(\gamma_3 + 1)}{2} M_n \left( 1 + \frac{(\gamma_3 - 1)}{2} M_n^2 \right)^{\frac{(3 - \gamma_3)}{2(\gamma_3 - 1)}} \quad (12.32)$$

$$M_{(n+1)} = M_n - \frac{F(M_n)}{F'(M_n)} \quad (12.33)$$

Iteration converges to fix  $M_x$

$$g(M_1) = \frac{M_1}{(1 + \gamma_1 M_1^2)} \sqrt{1 + \left( \frac{\gamma_1 - 1}{2} \right) M_1^2} \quad (12.34)$$

$$g(M_2) = \frac{M_2}{(1 + \gamma_2 M_2^2)} \sqrt{1 + \left( \frac{\gamma_2 - 1}{2} \right) M_2^2} \quad (12.35)$$

$$g(M_Y) = g = \sqrt{\frac{R_3 T_{s3}}{\gamma_3}} \left( \frac{x}{g(M_1)} \sqrt{\frac{R_1 T_{s1}}{\gamma_1}} + \frac{(1-x)}{g(M_2)} \sqrt{\frac{R T_{s2}}{\gamma_2}} \right)^{-1} \quad (12.36)$$

$$M_Y^2 = \frac{(1 - 2\gamma_3 g^2) - \sqrt{1 - 2(\gamma_3 + 1)g^2}}{1 - \gamma_3 (1 - 2\gamma_3 g^2)} \quad (12.37)$$

Eq. (12.37) applies provided that  $g \neq g_\infty = \frac{1}{\gamma_3} \sqrt{\frac{(\gamma_3 - 1)}{2}}$  (12.38)

If  $g = g_\infty$  the solution reduces to

$$M_Y = \sqrt{\frac{(\gamma_3 - 1)}{2\gamma_3}} \quad (12.39)$$

Once  $M_Y$  is known from Eq. (12.37) or (12.39)

$$f(M_Y) = M_Y \left( 1 + \frac{(\gamma_3 - 1)}{2} M_Y^2 \right)^{\frac{(\gamma_3 + 1)}{2(\gamma_3 - 1)}} \quad (12.40)$$

$$P_{sy} = P_{sx} \frac{f(M_x)}{f(M_Y)} \quad (12.41)$$

Further Input Data:  $\eta_E$

$$P_{s3} = P_{sx} \left( \frac{P_{sy}}{P_{sx}} \right)^{1/\eta_E} \quad (12.42)$$

$$f(M_3) = \left( \frac{P_{sx}}{P_{s3}} \right) f(M_x) \quad (12.43)$$

Verify that  $f(M_3) = f < f_{Max}$  (12.44)

For a first approximation take  $M_3 = M_y$  (12.45)

Iterate using Eqs. (12.31); (12.32), (12.33).

Iteration converges to fix  $M_3$ .

$$\left(\frac{T_{s3}}{T_3}\right) = \left[1 + \frac{(\gamma_3 - 1)}{2} M_3^2\right] \quad (12.46)$$

$$T_3 = T_{s3} / (T_{s3} / T_3) \quad (12.47)$$

$$\left(\frac{P_{s3}}{P_3}\right) = \left(\frac{T_{s3}}{T_3}\right)^{\frac{\gamma_3}{(\gamma_3 - 1)}} \quad (12.48)$$

$$P_3 = P_{s3} / (P_{s3} / P_3) \quad (12.49)$$

Further Input Data:  $\eta_D$ ,  $\left(\frac{A_3}{A_4}\right)$

$$P_{s4} = P_{s3} e^{-\left(1 - \eta_D\right)\left(\frac{\gamma_3 M_3^2}{2}\right)} \quad (12.50)$$

$$f(M_4) = f = \left(\frac{P_{s3}}{P_{s4}}\right)\left(\frac{A_3}{A_4}\right) f(M_3) \quad (12.51)$$

Verify that  $f(M_4) = f < f_{\max}$  (12.52)

For a first approximation take  $M_4 = \left(\frac{A_3}{A_4}\right) M_3$  (12.53)

Iterate using Eqs. (12.31), (12.32), (12.33).

Iteration converges to fix  $M_4$ .

$$T_{s4} = T_{s3} \quad (12.54)$$

$$\frac{T_{s4}}{T_4} = 1 + \frac{(\gamma_3 - 1)}{2} M_4^2 \quad (12.55)$$

$$T_4 = T_{s4} / (T_{s4}/T_4) \quad (12.56)$$

$$\frac{P_{s4}}{P_4} = \frac{T_{s4}}{T_4} \frac{\gamma_3}{(\gamma_3 - 1)} \quad (12.57)$$

$$P_4 = P_{s4} / (P_{s4}/P_4) \quad (12.58)$$

Further Input Data:  $\epsilon = 8.854 \times 10^{-12}$  farad/m,  $C_{B1}$ ,  $C_{B2}$ ,

( $C_B = 9.5 \times 10^3 \text{ m}^2 \text{ }^\circ\text{K/cmb}$  for air or steam)

$$C_{B3} = \frac{1}{R_3} [x R_1 C_{B1} + (1 - x) R_2 C_{B2}] \quad (12.59)$$

$$\beta = \frac{\epsilon C_{B3}^2 P_0}{T_0^2} \quad (12.60)$$

$$\frac{\Delta T_s}{T_{s4}} = \beta \left( \frac{\gamma_3 - 1}{2\gamma_3} \right) \left( \frac{P_{s4}}{P_0} \right) \left( \frac{T_0}{T_{s4}} \right)^2 \left( 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_4^2 \right)^{-1/(\gamma_3 - 1)} \quad (12.61)$$

$$\frac{T_{s5}}{T_{s4}} = \left( 1 - \frac{\Delta T_s}{T_{s4}} \right) \quad (12.62)$$

$$P_{s5} = P_{s4} \left( \frac{T_{s5}}{T_{s4}} \right)^{\gamma_3/(\gamma_3 - 1)} \quad (12.63)$$

$$f(M_5) = f = \sqrt{\frac{T_{s5}}{T_{s4}}} \left( \frac{P_{s4}}{P_{s5}} \right) f(M_4) \quad (12.64)$$

Verify that  $f(M_5) = f < f_{\max}$  (12.65)

For a first approximation take  $M_5 = M_4$  (12.66)

Iterate using Eqs. (12.31), (12.32), (12.33)

Iteration converges to fix  $M_5$ .

$$\left( \frac{T_{s5}}{T_5} \right) = 1 + \left( \frac{\gamma_3 - 1}{2} \right) M_5^2 \quad (12.67)$$

$$T_5 = T_{s5} / (T_{s5} / T_5) \quad (12.68)$$

$$\left( \frac{P_{s5}}{P_5} \right) = \left( \frac{T_{s5}}{T_5} \right)^{\frac{\gamma_3}{(\gamma_3 - 1)}} \quad (12.69)$$

$$P_5 = P_{s5} / (P_{s5} / P_5) \quad (12.70)$$

Further Input Data:  $c_f$

$$s_m = R_3 \ln R_3 - x R_1 \ln (x R_1) \quad (12.71)$$

$$- (1 - x) R_2 \ln [(1 - x) R_2]$$

$$K_0 = \ln \left( \frac{P_{s5}}{P_6} \right) + \frac{(C_{p3} - C_{p2})}{R_3} \left( \frac{T_{s5}}{T_6} - 1 \right) - \frac{\gamma_3}{(\gamma_3 - 1)} \ln \left( \frac{T_{s5}}{T_6} \right) - \frac{s_m}{R} - \frac{c_f \gamma_3}{2} \left( \frac{T_5}{T_6} \right) M_5^2 \quad (12.72)$$

$$K_1 = \frac{C_{p2}}{R_3} \left( \frac{T_{s5}}{T_6} - 1 \right) \quad (12.73)$$

Verify that  $K_0 < 0$

(12.74)

$$x_{\text{new}} = - \frac{K_0}{K_1}$$

(12.75)

Iterate from Eq. (12.14) until  $x$  converges.

Further Input Data:  $(P_{s1}/P_{s7})$ ,  $\rho_6$ ,  $(h_{s1} - h_6)$ ,  $\eta_x$ ,  $\eta_p$

$$P_{s7} = P_{s1} / (P_{s1}/P_{s7})$$

(12.76)

$$w_e^* = C_{p3} \Delta T_s$$

(12.77)

$$w_p^* = \frac{(P_{s7} - P_6)}{\rho_6}$$

(12.78)

$$w_{\text{net}} = \left( \frac{\eta_x w_e^*}{x} - \frac{w_p^*}{\eta_p} \right)$$

RESULT

(12.79)

$$q_{\text{in}} = (h_{s1} - h_6) - \frac{w_p^*}{\eta_p}$$

(12.80)

$$\eta_c = \frac{w_{\text{net}}}{q_{\text{in}}}$$

RESULT

(12.81)

### 13. OPTIMUM VELOCITY RATIO

The calculation sequence summarized in the preceding section reveals that a complete cycle calculation requires us to specify numerical values for about thirty input parameters. One of these is the velocity ratio  $(V_1/V_2) = y$ . The calculation procedure outlined above then permits us to find the corresponding value of the mass flow ratio  $x$ , assuming that there exists a value of  $x$  which satisfies the governing thermodynamic relations for the specified values of the other input parameters.

If we now change the value of  $y$  but hold all other input parameters fixed, we can repeat the above procedure and find a corresponding new value of  $x$ , if such a value exists. It is evident, therefore, that  $x$  becomes some definite function of  $y$  as long as we remain in the domain where a real solution exists. Of course all other dependent variables of the cycle are also functions of  $y$ . In particular, the overall cycle efficiency  $\eta_c$  is some definite function of  $y$ . Moreover, for prescribed values of the other input parameters, there will be some definite value of  $y$ , let us designate it as the optimum value  $y_{opt}$ , which yields the greatest value of  $\eta_c$  that is possible under the specified input conditions. Our basic purpose therefore is to determine for any prescribed values of the other input parameters, the value of  $y_{opt}$  and the corresponding value of  $\eta_c)_{max}$ . Of course the values of  $x_{opt}$  and other dependent variables are also of interest.

Once a working computer program is available to establish values of  $y_{opt}$ ,  $\eta_c)_{max}$ ,  $x_{cpt}$  and so on for prescribed values of the other input parameters, we can then proceed to study the effects of changes in these other parameters and to search systematically for such values of the most important input parameters as will yield the best overall performance of the system.

#### 14. CALCULATION SEQUENCE FOR ONE-FLUID SYSTEM

The calculation sequence summarized in Section 12 simplifies for the special case where the same condensible fluid is used for both the primary and secondary streams. This special case is summarized below.

Initial Input Data:  $P_1 = P_2 = P_6$  ,  $z_1$  ,  $P_{s1}$  ,  $T_2 = T_6$

Assuming pressure  $P_1$  and quality  $z_1$  are specified, we may find from tables of properties of the saturated fluid

$$h_1 = h_g - (1 - z_1)h_{fg} \quad (14.1)$$

$$s_1 = s_g - (1 - z_1)s_{fg} \quad (14.2)$$

Then with  $s_1$  fixed and  $P_{s1}$  specified, we may also find  $T_{s1}$  and  $h_{s1}$  from tables of properties of the superheated vapor.

In addition with  $T_6$  specified somewhere in the range  $T_0 \leq T_6 \leq T_1$  , and with  $P_6 = P_1$  specified, we may find  $h_6$  , the enthalpy of the liquid at state 6 from the tables of properties.

The primary fluid can now be approximated by an "equivalent perfect gas" whose properties are as follows

$$\gamma = \left( 1 - \frac{\ln(T_{s1}/T_1)}{\ln(P_{s1}/P_1)} \right)^{-1} \quad (14.3)$$

$$C_p = \frac{(h_{s1} - h_1)}{(T_{s1} - T_1)} \quad (14.4)$$

$$R = \left( \frac{\gamma - 1}{\gamma} \right) C_p \quad (14.5)$$

The secondary fluid is represented by the same "equivalent perfect gas" and is therefore assigned the same values of  $\gamma$ ,  $C_p$  and  $R$ .

Further Input Data:  $(V_1/V_2) = \gamma$

$$M_1 = \sqrt{\frac{2}{(\gamma - 1)}} \left[ \left( \frac{T_{s1}}{T_1} \right) - 1 \right] \quad (14.6)$$

$$M_2 = \left( \frac{V_1}{V_2} \right) \sqrt{\frac{T_1}{T_2}} M_1 \quad (14.7)$$

$$\text{Verify that } M_2 < 1 \quad (14.8)$$

$$\left( \frac{T_{s2}}{T} \right) = 1 + \left( \frac{\gamma - 1}{2} \right) M_2^2 \quad (14.9)$$

$$T_{s2} = T_2 (T_{s2}/T_2) \quad (14.10)$$

$$\left( \frac{P_{s2}}{P_2} \right) = (T_{s2}/T_2)^{\gamma/(\gamma - 1)} \quad (14.11)$$

$$P_{s2} = P_2 (P_{s2}/P_2) \quad (14.12)$$

Further Input Data:  $x$  (trial value)

$$T_{s3} = x T_{s1} + (1 - x) T_{s2} \quad (14.13)$$

$$f(M_1) = M_1 \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M_1^2 \right]^{-(\gamma + 1)/2(\gamma - 1)} \quad (14.14)$$

$$f(M_2) = M_2 \left[ 1 + \left( \frac{\gamma - 1}{2} \right) M_2^2 \right]^{-(\gamma + 1)/2(\gamma - 1)} \quad (14.15)$$

$$\left( \frac{A_1}{A_2} \right) = \left( \frac{P_{s2}}{P_{s1}} \right) \sqrt{\frac{T_{s1}}{T_{s2}}} \frac{f(M_2)}{f(M_1)} \frac{x}{(1 - x)} \quad (14.16)$$

$$\left(\frac{A_1}{A_3}\right) = \frac{A_1/A_2}{(1 + A_1/A_2)} \quad (14.17)$$

Further Input Data:  $P_o$ ,  $T_o$  (ambient conditions)

$$s_1 = c_p \ln \left( \frac{T_1}{T_o} \right) - R \ln \left( \frac{P_1}{P_o} \right) \quad (14.18)$$

$$s_2 = c_p \ln \left( \frac{T_2}{T_o} \right) - R \ln \left( \frac{P_2}{P_o} \right) \quad (14.19)$$

$$\ln \left( \frac{P_{sx}}{P_o} \right) = \frac{1}{R} \left\{ c_p \ln \left( \frac{T_{s3}}{T_o} \right) - x s_1 - (1 - x) s_2 \right\} \quad (14.20)$$

$$P_{sx} = P_o e^{\ln(P_{sx}/P_o)} \quad (14.21)$$

$$f(M_x) = \frac{(1 - A_1/A_3)}{(1 - x)} \left( \frac{P_{s2}}{P_{sx}} \right) \sqrt{\frac{T_{s3}}{T_{s2}}} f(M_2) \quad (14.22)$$

$$f_{\max} = \left( \frac{2}{\gamma + 1} \right)^{(\gamma + 1)/2(\gamma - 1)} \quad (14.23)$$

$$\text{Verify that } f(M_x) = f < f_{\max} \quad (14.24)$$

$$\text{For a first approximation set } M_x \approx M_2 \text{ and iterate} \quad (14.25)$$

Iterate using Eqs. (14.26), (14.27), (14.28).

$$F(M_n) = f \left( 1 + \left( \frac{\gamma - 1}{2} \right) M_n^2 \right)^{(\gamma + 1)/2(\gamma - 1)} - M_n \quad (14.26)$$

$$F'(M_n) = f \left( \frac{\gamma + 1}{2} M_n \right) \left( 1 + \left( \frac{\gamma - 1}{2} \right) M_n^2 \right)^{(3 - \gamma)/2(\gamma - 1)} - 1 \quad (14.27)$$

$$M_{(n+1)} = M_n - \frac{F(M_n)}{F'(M_n)} \quad (14.28)$$

Iteration converges to fix  $M_x$  .

$$g(M_1) = \frac{M_1}{(1 + \gamma M_1^2)} \sqrt{1 + \left(\frac{\gamma - 1}{2}\right) M_1^2} \quad (14.29)$$

$$g(M_2) = \frac{M_2}{(1 + \gamma M_2^2)} \sqrt{1 + \left(\frac{\gamma - 1}{2}\right) M_2^2} \quad (14.30)$$

$$g(M_Y) = g = \left\{ \frac{x}{g(M_1)} \sqrt{\frac{T_{s1}}{T_{s3}}} + \frac{(1-x)}{g(M_2)} \sqrt{\frac{T_{s2}}{T_{s3}}} \right\}^{-1} \quad (14.31)$$

$$M_Y^2 = \frac{(1 - 2\gamma g^2) - \sqrt{1 - 2(\gamma + 1)g^2}}{1 - \gamma(1 - 2\gamma g^2)} \quad (14.32)$$

It can also be shown that in the special case where

$g = g_\infty = \frac{1}{\gamma} \sqrt{\frac{\gamma - 1}{2}}$  , the above solution degenerates to

$$M_Y = \sqrt{\frac{(\gamma - 1)}{2\gamma}} . \quad (14.33)$$

Once  $M_Y$  is found from Eq. (14.32) or (14.33)

$$f(M_Y) = M_Y \left[ 1 + \left(\frac{\gamma - 1}{2}\right) M_Y^2 \right]^{- (\gamma + 1)/2(\gamma - 1)} \quad (14.34)$$

$$P_{sY} = P_{sX} \frac{f(M_X)}{f(M_Y)} \quad (14.35)$$

Further Input Data:  $\eta_E$

$$P_{s3} = P_{sX} \left( \frac{P_{sY}}{P_{sX}} \right)^{1/\eta_E} \quad (14.36)$$

$$f(M_3) = \left( \frac{P_{sX}}{P_{s3}} \right) f(M_X) \quad (14.37)$$

$$\text{Verify that } f(M_3) = f < f_{\max} \quad (14.38)$$

$$\text{For a first approximation, take } M_3 \approx M_y \quad (14.39)$$

Iterate using Eqs. (14.26) , (14.27) , (14.28) .

Iteration converges to fix  $M_3$  .

$$\frac{T_{s3}}{T_3} = 1 + \frac{(\gamma - 1)}{2} M_3^2 \quad (14.40)$$

$$T_3 = T_{s3} / (T_{s3}/T_3) \quad (14.41)$$

$$\frac{P_{s3}}{P_3} = (T_{s3}/T_3)^{\gamma/(\gamma - 1)} \quad (14.42)$$

$$P_3 = P_{s3} / (P_{s3}/P_3) \quad (14.43)$$

Further Input Data:  $\eta_D$  ,  $\frac{A_3}{A_4}$

$$P_{s4} = P_{s3} e^{-(1 - \eta_D) (\gamma M_3^2/2)} \quad (14.44)$$

$$f(M_4) = f = \left( \frac{P_{s3}}{P_{s4}} \right) \left( \frac{A_3}{A_4} \right) f(M_3) \quad (14.45)$$

$$\text{Verify that } f(M_4) = f < f_{\max} \quad (14.46)$$

$$\text{For a first approximation take } M_4 \approx \left( \frac{A_3}{A_4} \right) M_3 . \quad (14.47)$$

Iterate using Eqs. (14.26) , (14.27) , (14.28) .

Iteration covers to fix  $M_4$  .

$$T_{s4} = T_{s3} \quad (14.48)$$

$$\frac{T_{s4}}{T_4} = 1 + \frac{(\gamma - 1)}{2} M_4^2 \quad (14.49)$$

$$T_4 = T_{s4} / (T_{s4} / T_4) \quad (14.50)$$

$$\frac{P_{s4}}{P_4} = (T_{s4} / T_4)^{\gamma / (\gamma - 1)} \quad (14.51)$$

$$P_4 = P_{s4} / (P_{s4} / P_4) \quad (14.52)$$

Note: If it be desired to omit the diffuser, merely set  $\eta_D = 1$  and  $A_3/A_4 = 1$ . The result is

$$T_{s4} = T_{s3} \quad (14.53)$$

$$T_4 = T_3 \quad (14.54)$$

$$P_{s4} = P_{s3} \quad (14.55)$$

$$P_4 = P_3 \quad (14.56)$$

$$M_4 = M_3 \text{ etc} \quad (14.57)$$

and all effects of a diffuser disappear from the result.

Further Input Data:  $\epsilon = 8.854 \times 10^{-12}$  farad/m for all gases  
 $C_B = 9.5 \times 10^3 \text{ m}^2 \text{ }^\circ\text{K/cmb}$  for air or steam

$$\beta = \frac{\epsilon C_B^2 P_o}{T_o^2} \quad (14.58)$$

$$\frac{\Delta T_s}{T_{s4}} = \beta \left( \frac{\gamma - 1}{2\gamma} \right) \left( \frac{P_{s4}}{P_o} \right) \left( \frac{T_o}{T_{s4}} \right)^2 \left( 1 + \frac{\gamma - 1}{2} M_4^2 \right)^{-1/(\gamma - 1)} \quad (14.59)$$

$$T_{s5} = T_{s4} - \Delta T_s \quad (14.60)$$

$$P_{s5} = P_{s4} (T_{s5} / T_{s4})^{\gamma / (\gamma - 1)} \quad (14.61)$$

$$f(M_5) = \left( \frac{T_{s5}}{T_{s4}} \right) \left( \frac{P_{s4}}{P_{s5}} \right) f(M_4) \quad (14.62)$$

$$\text{Verify that } f(M_5) = f < f_{\max} \quad (14.63)$$

$$\text{For a first approximation take } M_5 \approx M_4 . \quad (14.64)$$

Iterate using Eqs. (14.26), (14.27), (14.28) .

Iteration converges to fix  $M_5$  .

$$\left(\frac{T_{s5}}{T_5}\right) = 1 + \frac{(\gamma - 1)}{2} M_5^2 \quad (14.65)$$

$$T_5 = T_{s5} / (T_{s5} / T_5) \quad (14.66)$$

$$\left(\frac{P_{s5}}{P_5}\right) = (T_{s5} / T_5)^{\gamma / (\gamma - 1)} \quad (14.67)$$

$$P_5 = P_{s5} / (P_{s5} / P_5) \quad (14.68)$$

Further Input Data:  $c_f$

Note: For a single fluid cycle  $s_m = 0$

$$K_0 = \ln \left(\frac{P_{s5}}{P_6}\right) - \frac{\gamma}{(\gamma - 1)} \ln \left(\frac{T_{s5}}{T_6}\right) - \frac{c_f \gamma}{2} \left(\frac{T_5}{T_6}\right) M_5^2 \quad (14.69)$$

$$K_1 = \frac{\gamma}{(\gamma - 1)} \left(\frac{T_{s5}}{T_6} - 1\right) \quad (14.70)$$

$$E(x) = K_0 + K_1 x \quad (14.71)$$

$$x_{\text{new}} = - \frac{K_0}{K_1} \quad (14.72)$$

Return to Eq. (14.13) and iterate until  $x$  converges and

$$E(x) = 0 .$$

Further Input Data:  $(P_{s1}/P_{s7})$  ,  $\rho_6$  ,  $(h_{s1} - h_6)$  ,  $\eta_x$  ,  $\eta_p$

$$P_{s7} = P_{s1}/(P_{s1}/P_{s7}) \quad (14.73)$$

$$w_e^* = C_p \Delta T_s \quad (14.74)$$

$$w_p^* = \frac{(P_{s7} - P_6)}{\rho_6} \quad (14.75)$$

$$w_{net} = \frac{\eta_x w_e^*}{x} - \frac{w_p^*}{\eta_p} \quad \underline{\underline{RESULT}} \text{ (Joule/kg)} \quad (14.76)$$

$$q_{in} = (h_{s1} - h_6) - \frac{w_p^*}{\eta_p} \quad (14.77)$$

$$\eta_c = \frac{w_{net}}{q_{in}} \quad \underline{\underline{RESULT}} \quad (14.78)$$

## 15. SAMPLE CALCULATION FOR A ONE-FLUID SYSTEM

In this section we illustrate numerically how the calculations for a one-fluid system may be carried out for one arbitrary but typical operating point. To find an optimum operating condition would require the repetition of similar calculations for a large number of such trial operating points. No attempt is made in this section to find such an optimum.

We arbitrarily choose to calculate a case which is in some respects similar to that identified as case B in the appendix of Ref. (1). Thus, we have for both of these cases -

Medium		Steam
Maximum pressure	$P_{s1}$	1500 psia
Quality at state 1	$z_1$	0.95
Ejector effectiveness	$\eta_E$	0.9
Friction factor	$c_f$	0.1
Dielectric strength	$C_B$	9,490 m <sup>2</sup> °K/cmb
Diffuser characteristics		No diffuser
Excitation efficiency	$\eta_x$	1
Pump efficiency	$\eta_p$	0.9
Pressure ratio across boiler	$P_{s1}/P_{s7}$	1

Nevertheless, despite these similarities, these two cases are not identical. Case B treats the densities at states 1, 2, x, y, 3, 4, 5 as equal. The present analysis takes the true

density changes at these states fully into account. Also it is expedient in the present compressible analysis to define the parameters  $\eta_E$  and  $c_f$  somewhat differently than they are defined in Ref (1).

Ref.(1) also shows that a comparatively good performance is obtained by choosing

$$P_1 = P_2 = P_6 = 600 \text{ psia} \quad (15.1)$$

and

$$\frac{V_1}{V_2} = \gamma = 4.25 \quad (15.2)$$

Therefore we arbitrarily choose the same values for the present numerical example, despite the fact that it is not entirely equivalent to case B of Ref (1). We also choose to set

$$T_6 = T_1 \quad (15.3)$$

The above conditions now suffice to fix the following properties. These are obtained from Keenan and Kayes steam tables and are listed on page 10A of Ref. (1).

$$\begin{aligned} T_{s1} &= 1115.9 \text{ }^\circ\text{R} \\ T_1 &= T_2 = T_6 = 946.2 \text{ }^\circ\text{R} \end{aligned} \quad (15.3)$$

$$(h_{s1} - h_1) = 78.9 \text{ Btu/lbm}$$

$$(h_{s1} - h_6) = 673.9 \text{ Btu/lbm}$$

We designate ambient conditions as follows

$$\begin{aligned} P_0 &= 14.696 \text{ psia} = 1.013 \times 10^5 \text{ N/m}^2 \\ T_0 &= 520 \text{ }^\circ\text{R} = 288.9 \text{ }^\circ\text{K} \end{aligned} \quad (15.4)$$

The properties of the "equivalent perfect gas" may now be computed as follows

$$\begin{aligned}\gamma &= \left(1 - \frac{\ln(T_{s1}/T_1)}{\ln(P_{s1}/P_1)}\right)^{-1} \\ &= \left(1 - \frac{\ln(1115.9/946.2)}{\ln(1500/600)}\right)^{-1} = 1.2196\end{aligned}\quad (15.5)$$

$$\begin{aligned}C_p &= \frac{(h_{s1} - h_1)}{(T_{s1} - T_1)} = \frac{78.9}{(1115.9 - 946.2)} \\ &= 0.4649 \text{ Btu/lbm}\end{aligned}\quad (15.6)$$

$$\begin{aligned}R &= \frac{(\gamma - 1)}{\gamma} C_p = \frac{0.2196}{1.2196} (0.4649) \\ &= 0.08371 \text{ Btu/lbm } ^\circ\text{R} = 65.15 \text{ ft lbf/lbm } ^\circ\text{R}\end{aligned}\quad (15.7)$$

The following constants will prove useful for later application

$$\begin{aligned}\frac{\gamma - 1}{2} &= 0.1098 & \frac{1}{(\gamma - 1)} &= 4.5537 \\ \frac{\gamma + 1}{2} &= 1.1098 & \frac{(\gamma + 1)}{2(\gamma - 1)} &= 5.0537 \\ \frac{\gamma}{(\gamma - 1)} &= 5.5537 & \frac{(3 - \gamma)}{2(\gamma - 1)} &= 4.0537\end{aligned}\quad (15.8)$$

Also the dielectric breakdown constant can be expressed in dimensionless form as

$$\begin{aligned}\beta &= \frac{\epsilon C^2 B^P O}{T_O^2} = \frac{(8.854 \times 10^{-12}) (9.49 \times 10^3)^2 (1.013 \times 10^5)}{(288.9)^2} \\ &= 0.9678(10)^{-3}\end{aligned}\quad (15.9)$$

The Mach numbers at the ejector inlet are

$$M_1 = \sqrt{\frac{2}{(\gamma - 1)} \left( \frac{T_{s1}}{T_1} - 1 \right)} = \sqrt{\frac{2}{0.2196} \left( \frac{1115.9}{94.62} - 1 \right)}$$

$$= 1.2781 \quad (15.10)$$

$$M_2 = \left( \frac{V_2}{V_1} \right) \sqrt{\frac{T_2}{T_1}} M_1 = \frac{1}{4.25} (1) (1.2781)$$

$$= 0.3007 \quad (15.11)$$

The following functions are calculated for later use

$$f(M_1) = M_1 \left( 1 + \frac{\gamma - 1}{2} M_1^2 \right)^{-(\gamma + 1)/2(\gamma - 1)}$$

$$= 1.2781 \left( 1 + 0.1098 (1.2781)^2 \right)^{-5.50537}$$

$$= 0.5552 \quad (15.12)$$

$$f(M_2) = M_2 \left( 1 + \frac{\gamma - 1}{2} M_2^2 \right)^{-(\gamma + 1)/2(\gamma - 1)}$$

$$= 0.3007 \left( 1 + 0.1098 (0.3007)^2 \right)^{-5.0537}$$

$$= 0.2861 \quad (15.13)$$

$$f_{\max} = \left( \frac{2}{\gamma + 1} \right)^{(\gamma + 1)/2(\gamma - 1)} = \left( \frac{2}{2.2196} \right)^{5.0537}$$

$$= 0.5907 \quad (15.14)$$

$$g(M_1) = \frac{M_1 \sqrt{1 + \frac{\gamma - 1}{2} M_1^2}}{(1 + \gamma M_1^2)}$$

$$= \frac{1.2781 \sqrt{1 + 0.1098 (1.2781)^2}}{(1 + 1.2196 (1.2781)^2)}$$

$$= 0.4639 \quad (15.15)$$

$$g(M_2) = \frac{M_2 \sqrt{1 + \frac{\gamma - 1}{2} M_2^2}}{(1 + \gamma M_2^2)}$$

$$\begin{aligned}
&= \frac{0.3007 \sqrt{1 + 0.1098(0.3007)^2}}{(1 + 1.2196(0.3007)^2)} \\
&= 0.2722 \qquad (15.16)
\end{aligned}$$

Stagnation conditions at station 2 are calculated next.

$$\begin{aligned}
(T_{s2}/T_1) &= 1 + \left(\frac{\gamma - 1}{2}\right) M_2^2 = 1 + 0.1098(0.3007)^2 \\
&= 1.0099 \qquad (15.17)
\end{aligned}$$

$$\begin{aligned}
T_2 &= T_2 (T_{s2}/T_2) = 946.2(1.0099) \\
&= 955.6 \text{ }^\circ\text{R} \qquad (15.18)
\end{aligned}$$

$$\begin{aligned}
(P_{s2}/P_2) &= (T_{s2}/T_2)^{\gamma/(\gamma - 1)} = (1.0099)^{5.5537} \\
&= 1.0562 \qquad (15.19)
\end{aligned}$$

$$\begin{aligned}
P_{s2} &= P_s (P_{s2}/P_2) = 600.0 (1.0562) \\
&= 633.7 \text{ psia} \qquad (15.20)
\end{aligned}$$

In this particular case the entropies at stations 1 and 2 happen to be equal. They can be calculated either from the stagnation temperatures and pressures or from the static temperatures and pressures. The latter choice gives:

$$\begin{aligned}
s_1 &= s_2 = c_p \ln(T_1/T_0) - R \ln(P_1/P_0) \\
&= 0.4649 \ln(946.2/520.0) - 0.08371 \ln(600.0/14.696) \\
&= -0.0322 \text{ Btu/lbm }^\circ\text{R} \qquad (15.21)
\end{aligned}$$

Further Data Input:  $x = 0.472$  (trial value)

The above value of  $x$  was found by previous trial calculations not shown here. The calculations which follow verify that this value of  $x$  does indeed satisfy the energy balance equation of the system to within the round-off error.

$$T_{s3} = x T_{s1} + (1 - x) T_{s2} = 0.472(1115.9) + 0.528(955.6)$$

$$= 1031.3 \text{ } ^\circ\text{R}$$

$$\frac{A_1}{A_2} = \frac{x}{(1-x)} \left( \frac{P_{s2}}{P_{s1}} \right) \sqrt{\frac{T_{s1}}{T_{s2}}} \frac{f(M_2)}{f(M_1)} \quad (15.21)$$

$$= \frac{0.472}{0.528} \left( \frac{633.7}{1500.0} \right) \sqrt{\frac{1115.9}{955.6}} \left( \frac{0.2861}{0.5552} \right) = 0.2103 \quad (15.22)$$

$$\frac{A_1}{A_3} = \frac{A_1/A_2}{(1 + A_1/A_2)} = \frac{0.2103}{1.2103} = 0.1738 \quad (15.23)$$

Since  $s_1 = s_2$  in this case, Eq. (14.20) simplifies to

$$\ln\left(\frac{P_{sx}}{P_o}\right) = \frac{1}{R} [c_p \ln\left(\frac{T_{s3}}{T_o}\right) - s_1]$$

$$= \frac{1}{0.08371} [0.4649 \ln\left(\frac{1031.3}{520.0}\right) + 0.0322] = 4.1876 \quad (15.24)$$

$$P_{sx} = P_{oe}^{4.1876} = 14.696e^{4.1876} = 967.9 \text{ psia} \quad (15.25)$$

$$f(M_x) = \frac{(1 - A_1/A_3)}{(1-x)} \left( \frac{P_{s2}}{P_{sx}} \right) \sqrt{\frac{T_{s3}}{T_{s2}}} f(M_2)$$

$$= \frac{(1 - 0.1726)}{0.528} \left( \frac{633.7}{967.9} \right) \sqrt{\frac{1031.3}{955.6}} (0.2861) = 0.3045 \quad (15.26)$$

Solving for  $M_x$  from Eqs. (4.16), (4.17), (4.18) gives

$$M_x = 0.3225 \quad (15.27)$$

For an ideal ejector

$$g(M_y) = \left( \frac{x}{g(M_1)} \sqrt{\frac{T_{s1}}{T_{s3}}} + \frac{(1-x)}{g(M_x)} \sqrt{\frac{T_{s2}}{T_{s3}}} \right)^{-1}$$

$$= \left( \frac{0.472}{0.4639} \sqrt{\frac{1115.9}{1031.3}} + \frac{0.528}{0.2722} \sqrt{\frac{955.6}{1031.3}} \right)^{-1} = 0.3418 \quad (15.28)$$

$$M_y^2 = \frac{(1 - 2\gamma g^2) - \sqrt{1 - 2(\gamma + 1)g^2}}{1 - \gamma(1 - 2\gamma g^2)}$$

$$\begin{aligned}
&= \frac{[1 - 2.4392(0.3418)^2] - \sqrt{1 - 4.4392(0.3418)^2}}{1 - 1.2196 [1 - 2.4392(0.3418)^2]} \\
&= 0.1658 \qquad \qquad \qquad M_y = 0.4071 \qquad \qquad \qquad (15.29)
\end{aligned}$$

$$\begin{aligned}
f(M_y) &= \frac{M_y^{(\gamma+1)/2(\gamma-1)}}{[1 + \frac{\gamma-1}{2} M_y^2]} \\
&= \frac{0.4071}{[1 + 0.1098(0.1658)]^{5.0537}} = 0.3716 \qquad \qquad \qquad (15.30)
\end{aligned}$$

$$P_{sY} = P_{sX} \frac{f(M_x)}{f(M_y)} = 967.9 \left( \frac{0.3045}{0.3716} \right) = 793.1 \text{ psia} \qquad \qquad \qquad (15.31)$$

$$P_{s3} = P_{sX} \left( \frac{P_{sY}}{P_{s3}} \right)^{1/\eta_E} = 967.9 \left( \frac{793.1}{967.7} \right)^{1/0.9} = 775.7 \text{ psia} \qquad \qquad \qquad (15.32)$$

$$f(M_3) = \left( \frac{P_{sX}}{P_{s3}} \right) f(M_x) = \left( \frac{967.9}{775.7} \right) (0.3045) = 0.3799 \qquad \qquad \qquad (15.33)$$

Solving for  $M_3$  from Eqs. (4.16), (4.17), (4.18) gives

$$M_3 = 0.4183 \qquad \qquad \qquad (15.34)$$

In the absence of a diffuser we may write

$$\begin{aligned}
T_{s4} &= T_{s3} = 1031.3 \text{ }^\circ\text{R} \\
P_{s4} &= P_{s3} = 775.7 \text{ psia} \\
M_4 &= M_3 = 0.4183
\end{aligned} \qquad \qquad \qquad (15.35)$$

$$f(M_4) = f(M_3) = 0.3799$$

The drop in stagnation temperature through the working section is given by the following expressions.

$$\left(\frac{\Delta T_s}{T_{s4}}\right) = \frac{\beta \left(\frac{\gamma - 1}{2\gamma}\right) \left(\frac{P_{s4}}{P_o}\right) \left(\frac{T_o}{T_{s4}}\right)^2}{\left[1 + \left(\frac{\gamma - 1}{2}\right) M_4^2\right]^{1/(\gamma - 1)}} \quad (15.36)$$

$$= \frac{0.9678(10)^{-3} \left(\frac{0.1098}{1.2196}\right) \left(\frac{775.7}{14.696}\right) \left(\frac{520.0}{1031.3}\right)^2}{\left[1 + 0.1098(0.4183)^2\right]^{4.5537}} = 1.0722(10)^{-3}$$

$$\Delta T_s = T_{s4} (\Delta T_s / T_{s4}) = 1031.3(1.0722)(10)^{-3} = 1.1057 \text{ }^\circ\text{R} \quad (15.37)$$

Notice that this temperature drop is only about 1.1 °R or about one tenth of one percent of the absolute stagnation temperature!

$$T_{s5} = T_{s4} - \Delta T_s = 1031.3 - 1.1 = 1030.2 \text{ }^\circ\text{R} \quad (15.38)$$

$$P_{s5} = P_{s4} (T_{s5}/T_{s4})^{\gamma/(\gamma - 1)}$$

$$= 775.7 \left(\frac{1030.2}{1031.3}\right)^{5.5537} = 771.1 \text{ psia} \quad (15.39)$$

$$f(M_5) = \sqrt{\frac{T_{s5}}{T_{s4}}} \left(\frac{P_{s4}}{P_{s5}}\right) f(M_4)$$

$$= \sqrt{\frac{1030.2}{1031.3}} \left(\frac{775.7}{771.1}\right) (0.3799) = 0.3820 \quad (15.40)$$

Solving for  $M_5$  from Eqs. (4.16), (4.17), and (4.18)

$$M_5 = 0.4210$$

$$(T_{s5}/T_5) = 1 + \left(\frac{\gamma - 1}{2}\right) M_5^2 = 1 + 0.$$

$$T_5 = T_{s5} / (T_{s5}/T_5) = \frac{1030.2}{1.0195} = 1010.5 \text{ }^\circ\text{R} \quad (15.43)$$

To verify the energy balance in the condenser/separator we calculate

$$\begin{aligned}
 K_0 &= \ln\left(\frac{P_{s5}}{P_6}\right) - \frac{\gamma}{(\gamma - 1)} \ln\left(\frac{T_{s5}}{T_6}\right) - \frac{c_p \gamma}{2} \left(\frac{T_5}{T_6}\right) M_5^2 \\
 &= \ln\left(\frac{771.1}{600.0}\right) - 5.5537 \ln\left(\frac{1030.2}{946.2}\right) - \frac{0.12196}{2} \left(\frac{1010.5}{946.2}\right) (0.4210)^2 \\
 &= -0.2330 \qquad (15.44)
 \end{aligned}$$

$$K_1 = \frac{\gamma}{(\gamma - 1)} \left(\frac{T_{s5}}{T_6} - 1\right) = 5.5537 \left(\frac{1030.2}{946.2} - 1\right) = 0.4930 \qquad (15.45)$$

The final energy balance now gives

$$\begin{aligned}
 E(x) &= K_0 + K_1 x = -0.2330 + 0.4930 (0.472) \\
 &= -0.0003 \qquad (15.46)
 \end{aligned}$$

This verifies that the energy balance is satisfied to within round-off error. It also confirms that the value  $x = 0.472$  is the mass flow ratio that corresponds to the various independent input conditions stipulated in this sample calculation.

Now that the value of  $x$  has been confirmed, we may complete the thermodynamic cycle.

Since the pressure drop across the boiler is taken as negligible in this example

$$P_{s7} = P_{s1} = 1500.0 \text{ psia} \qquad (15.47)$$

The gross electrical work output per unit mass is

$$w_e^* = C_p \Delta T_s = 0.4649(1.1057) = 0.5140 \text{ Btu/lbm} \qquad (15.48)$$

The ideal pump work is

$$\begin{aligned}
 w_p^* &= \frac{(P_{s7} - P_6)}{\rho_6} = \frac{144}{778.26} \frac{(1500.0 - 600.0)}{49.75} \\
 &= 3.3472 \text{ Btu/lbm} \qquad (15.49)
 \end{aligned}$$

The net useful work of the cycle is therefore

$$\begin{aligned}
 w_{\text{net}} &= \frac{\eta_x w_e^*}{x} - \frac{w_p^*}{\eta_p} = \frac{(1)(0.5140)}{0.472} - \frac{3.3472}{0.9} \\
 &= -2.6301 \text{ Btu/lbm} \qquad (15.50)
 \end{aligned}$$

Notice that the net work of the cycle happens to be negative at this particular operating point. In other words the electrical power output is insufficient to drive the pump under these conditions.

The net heat input to the cycle is

$$\begin{aligned}
 q_{\text{in}} &= (h_{s1} - h_6) - \frac{w_p^*}{\eta_p} = 673.9 - \frac{3.3472}{0.9} \\
 &= 670.2 \text{ Btu/lbm} \qquad (15.51)
 \end{aligned}$$

Finally, the overall thermal efficiency of the system at this operating point is

$$\eta_c = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{-2.6301}{670.2} = -0.003924 \qquad (15.52)$$

Although the performance at this arbitrary operating conditions is very poor, the fact remains that there exists some (as yet unknown) value of the parameter  $V_1/V_2 = y$  for which the efficiency  $\eta_c$  reaches its maximum possible value.

This optimum value of  $\gamma$  can be determined by further systematic trial calculations similar to the above sample calculation.

The foregoing calculations have established the values of  $T_s$ ,  $P_s$  and  $M$  at all stations. The corresponding values of  $T$ ,  $P$ ,  $\rho$  and  $s$  at these locations can then be determined from the following relations.

$$\frac{T_s}{T} = 1 + \left(\frac{\gamma - 1}{2}\right)M^2 = 1 + 0.1098M^2 \quad (15.53)$$

$$T = T_s / (T_s/T) \text{ } ^\circ\text{R} \quad (15.54)$$

$$\frac{P_s}{P} = \left(\frac{T_s}{T}\right)^{\gamma/(\gamma - 1)} = \left(\frac{T_s}{T}\right)^{5.5537} \quad (15.55)$$

$$P = P_s / (P_s/P) \text{ psia} \quad (15.56)$$

$$\rho = \frac{P}{RT} = \left(\frac{144}{65.15}\right) \frac{P}{T} \text{ lbm/ft}^3 \quad (15.57)$$

$$s = C_p \ln \left(\frac{T_s}{T_o}\right) - R \ln \left(\frac{P_s}{P_o}\right) \quad (15.58)$$

$$= 0.4649 \ln \left(\frac{T_s}{520.0}\right) - 0.08371 \ln \left(\frac{P_s}{14.696}\right) \text{ Btu/lbm } ^\circ\text{R} \quad (15.59)$$

The calculation of the above properties has been completed in this way and the results are summarized in Table 15.1. These same states are shown on a temperature entropy diagram in Fig. 15.1.

Notice from the tabulation that in passing through the ejector, the fluid density  $\rho$  changes from  $1.402 \text{ lbm/ft}^3$  to  $1.524 \text{ lbm/ft}^3$ , an increase of about 8.5%. In passing through the working section, the density changes from  $1.524 \text{ lbm/ft}^3$  to

TABLE 15.1 THERMODYNAMIC PROPERTIES

STATE	1	2	x	Y	3	4	5	6
T <sub>s</sub> °R	1115.9	955.6	1031.3	1031.3	1031.3	1031.3	1030.2	946.2
P <sub>s</sub> psia	1500.0	633.7	967.9	793.1	775.7	775.7	771.1	600.0
M	1.2781	0.3007	0.3225	0.4071	0.4183	0.4183	0.4210	0
T °R	946.2	946.2	1019.7	1012.9	1011.9	1011.9	1010.5	946.2
P psia	600.0	600.0	908.8	717.5	697.9	697.9	692.8	600.0
ρ lbm/ft <sup>3</sup>	1.402	1.402	1.970	1.566	1.524	1.524	1.515	49.75
s Btu/lbm °R	-0.0322	-0.0322	-0.0322	-0.0155	-0.0137	-0.0137	-0.0137	-

$$Y = 4.25$$

$$x = 0.472$$

$$Y = 1.2196$$

$$R = 0.08371 \text{ Btu/lbm } ^\circ\text{R}$$

$$C_p = 0.4649 \text{ Btu/lbm } ^\circ\text{R}$$

$$\beta = 9.678 \times 10^{-4}$$

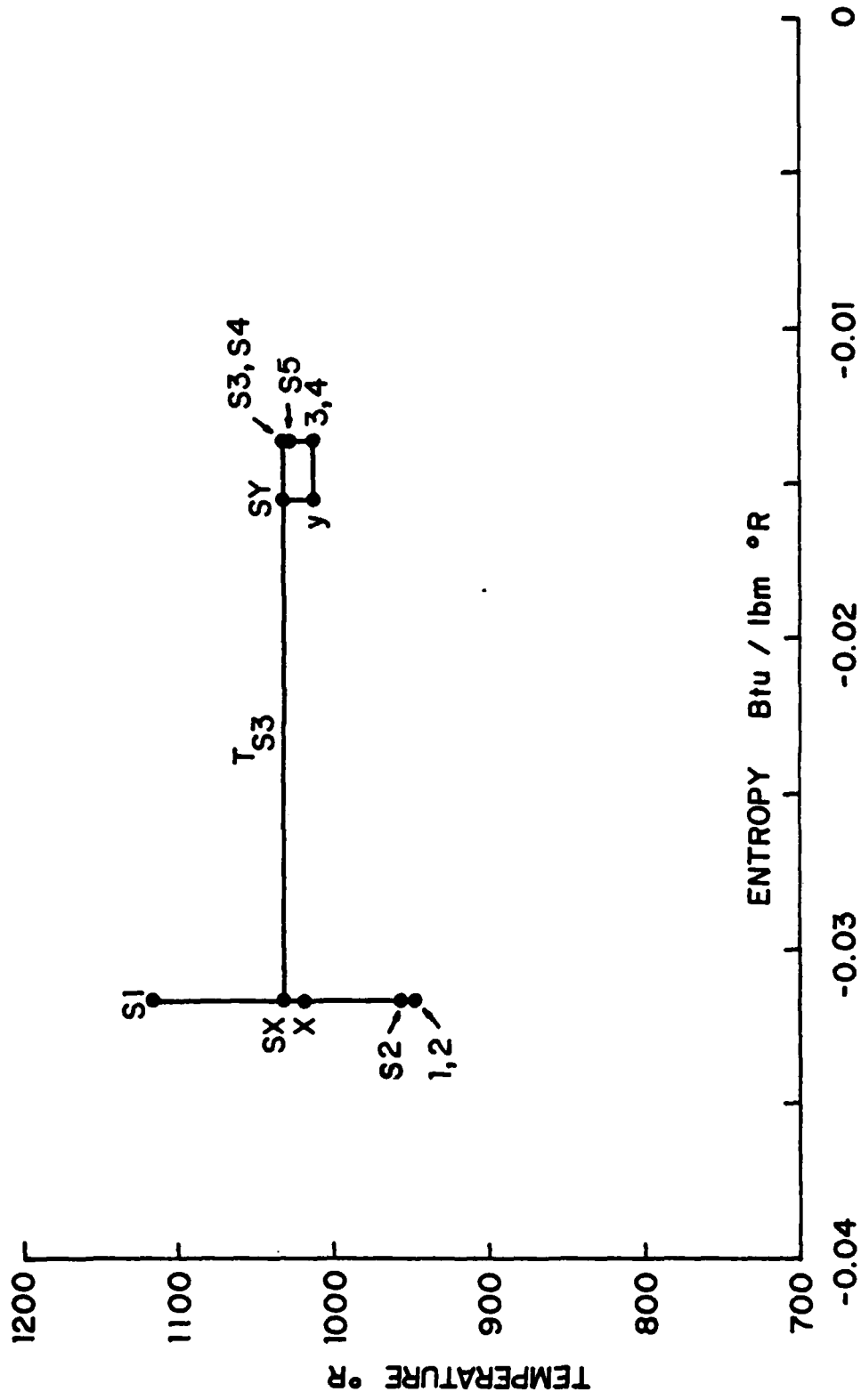


Fig. 15.1 TEMPERATURE-ENTROPY DIAGRAM OF THERMODYNAMIC STATES

1.515 lbm/ft<sup>3</sup> , a decrease of well under 1% . These data support the principle that it is permissible to neglect the density changes across these components for purposes of simplifying the analysis. This is particularly justified because at present the uncertainty in the value of the empirical ejector effectiveness  $\eta_E$  far exceeds the small error introduced by neglecting these minor density variations. Moreover, the present sample calculation applies to a velocity ratio  $V_1/V_2$  of 4.25 which is an extreme case. For efficient ejector operation the ratio  $V_1/V_2$  should be only slightly greater than unity and under these circumstances the density change across the ejector is expected to be far less than 8% . Thus there is little substance in the argument that an analysis which neglects these minor compressibility effects must be seriously in error. Such an argument runs counter to the fundamental principles of fluid mechanics. A more correct appraisal of the situation is that the inclusion of these detailed compressibility effects can be expected to improve the accuracy of the analysis to some degree, but that they should not be expected to change the overall results in any radical way.

## 16. Nomenclature

A	-	area
a	-	speed of sound
$C_B$	-	characteristic breakdown constant (Eqn 9.3)
$C_p$	-	specific heat at constant pressure
$c_f$	-	friction loss coefficient - condenser
$E_b$	-	breakdown electric field strength
$E(x)$	-	Eqn. 10.13
$F(M_n)$	-	Eqn. 4.16
$f(M)$	-	Eqn. 4.3
$f_{max}$	-	Eqn. 4.15
$g(M)$	-	Eqn. 6.5
g	-	Eqn. 6.7
$g_{max}$	-	Eqn. 6.9
$g_\infty$	-	Eqn. 6.10
h	-	enthalpy
I	-	irreversibility
$K_0$	-	Eqn. 10.11
$K_1$	-	Eqn. 10.12
M	-	Mach number
$\dot{m}$	-	mass flow rate
$P_e$	-	gross electrical power
p	-	pressure
$q_{in}$	-	heat input
R	-	gas constant
$\bar{R}$	-	universal gas constant

$s$	-	entropy
$s_m$	-	entropy of mixing
$T$	-	temperature
$V$	-	velocity
$W$	-	molecular weight
$w_e^*$	-	gross work out per unit mass
$w_p^*$	-	ideal pump work per unit mass
$w_{net}$	-	useful electrical output per unit mass
$x$	-	$\dot{m}_1/\dot{m}_3$
$x_{new}$	-	Eqn. 10.15
$\bar{y}$	-	$v_1/v_3$
$z$	-	quality of vapor
$\beta$	-	Eqn. 9.7, dimensionless breakdown constant
$\gamma$	-	ratio of specific heats
$\Delta( )$	-	change of
$\epsilon$	-	permittivity of the medium
$\eta_c$	-	overall cycle efficiency
$\eta_E$	-	Eqn. 7.13, ejector effectiveness
$\eta_D$	-	Eqn. 8.6, diffuser effectiveness
$\eta_p$	-	pump efficiency
$\eta_x$	-	excitation efficiency
$\rho$	-	gas density
$\psi$	-	steady flow availability
$\omega$	-	mass ratio of condensible primary to dry secondary

### Subscripts

- 0 - ambient conditions
- 1 - primary, entrance to ejector (exit from boiler/superheater)
- 2 - secondary, entrance to ejector (exit from separator)
- 3 - mixture, exit from ejector entrance to diffuser
- 4 - mixture, exit from diffuser entrance to generator
- 5 - mixture, exit from generator entrance to condenser
- 6 - primary, exit from condenser entrance to pump
- 7 - primary, exit from pump entrance to boiler
- f - saturated liquid
- G - gas or secondary
- g - saturated vapor
- n - next value
- s - stagnation
- v - vapor
- x - exit station of hypothetical minimum loss device
- y - exit station of ideal ejector

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