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THESIS

STATISTICAL PROPERTIES OF OCEAN WAVE GROUPS

by

Arthur Ronald Nelson

March 1980

Thesis Advisor: W. C. Thompson

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between the parameters obtained were determined for all
wave groups, and for selected extreme wave groups. It was
found that as the relative energy of the group increases the
number of waves per group increases, and the average group
period approaches the spectral peak period. Wave steepness
was shown, however, not to depend upon the group energy.
Groups from low steepness wave records tend to contain larger
numbers of waves. It was also shown that the height of the
single highest wave in a group tends to approximate the
significant height of the wave record.

Statistical Properties of Ocean Wave Groups

by

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Submitted in partial fulfillment of the
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ABSTRACT

Individual wave groups were identified by an analysis of digitized wave records. The analysis technique is based on a comparison between short-term variance and wave record variance. Once wave groups were identified, wave parameters of interest were then derived. Analysis was performed on 338 wave records obtained from a Waverider buoy covering a wide range of height, period, and steepness. Statistical relationships between the parameters obtained were determined for all wave groups, and for selected extreme wave groups. It was found that as the relative energy of the group increases the number of waves per group increases, and the average group period approaches the spectral peak period. Wave steepness was shown, however, not to depend upon the group energy. Groups from low steepness wave records tend to contain larger numbers of waves. It was also shown that the height of the single highest wave in a group tends to approximate the significant height of the wave record.

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I.   INTRODUCTION

A visual observation of the sea surface reveals that it is a highly complex and irregular surface. The fact that most waves have no apparent relation to or dependence on each other has prompted most studies or analyses of the sea surface to be statistical in nature. This type of approach has been necessary for lack of a better one, but it has also been highly successful in enabling the mechanisms of ocean waves to be understood.

Visual observation also shows, however, that ocean waves commonly appear in packets or groups. These waves are quasi-periodic and although the wave heights are not equal they have been shown to be dependent on each other (Rye, 1974). The existence of wave groups has been known to sea-faring men for many years. Out of their experiences have come folk lore including the saying that "every seventh wave is the largest," and an old icelandic saying that "large waves rarely come alone." The fact that the phenomenon of wave grouping has been known and respected by sailors for ages makes it all the more surprising that it is only in recent years that wave groups have been systematically studied.

The economic impact of wave groups is only now being realized. It is recognized that groups of waves, not

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necessarily large, may be damaging. Some of the engineer-
ing problems that hav been found to be related to wave
groups are the stability of rubble-mound breakwaters, the
slow drift oscillations of moored floating platforms, the
stability of ships underway, and induced selching in harbors
or bays that may cause damage to moored craft and in extreme
conditions may be responsible for local flooding. Design
approaches today do not generally take wave group charac-
teristics into account.

Two approaches have been conventionally employed in the
analysis of wave groups. The first involves using a
statistical theory of runs while the second consists of
examining the statistics of wave envelopes in relation to
power spectra (Goda, 1976). Among those working on the
distribution of the lengths of runs with simulated or actual
wave data have been Goda (1970), Wilson and Baird (1972),
Rye (1974), and Siefert (1976). Ewing (1973) and Chou (1978)
have worked with the application of wave envelope statistics
to wave groups. Mollo-Christensen and Ramamonjiarisoa (1978)
recently proposed a non-linear model for wind waves in
which "envelope solitons" or groups of Stokes waves propagate
at the same speed for all frequencies rather than obeying
a linear frequency dispersion relation.

A common thread running through all of these studies
is that they employ power spectra generated by Fourier
transform techniques. Non-spectral analyses have been

relatively few in number probably owing to the success that spectral techniques and methods have enjoyed. Hamilton, Hui, and Donelan (1979) have recently proposed a simple non-spectral statistical model with decoupled wave groups that improves spectral estimates of real or laboratory wind waves. Thompson (1972), Smith (1974), and more recently Sedivy (1978) have devised a non-spectral technique for identifying wave groups and then generating wave group statistics of interest.

This study utilizes the method used by Sedivy (1978) whereby a computer program analyzes digitized wave records by means of an rms smoothing filter. This filter employs a running short-term variance computation. The resulting smoothed record is then the basis for identifying individual wave groups from which various wave-group parameters of interest are obtained. Relationships are examined between wave-group parameters, and between wave group and wave record parameters. Possible relationships between power spectra and wave-group and wave-record parameters are examined. The relation between individual waves and the power spectrum is also examined.

It is hoped that the statistics generated in this study will be of use to the engineer, and when looked at in conjunction with the results of others will help provide the basis for theories on ocean wave groups.

II. WAVE GROUP DETERMINATION

A. WAVE GROUP CONSIDERATIONS

As mentioned, the most popular approach at present for waveform or wave group analysis is to work in the frequency domain, or in other words, with power spectra. The basis of this type of analysis has its roots in the assumption that ocean waves are composed of a linear summation of an infinite number of simple sinusoidal waves of infinitesimal amplitudes and random phases. Ocean waves can then be treated as a normal or Gaussian process. Longuet-Higgins (1952) showed that ocean waves of narrow-banded spectra have a distribution of wave heights that are approximated by a Rayleigh distribution. Goda (1974), and Chakrabarti and Cooley (1977) have shown that the Rayleigh distribution holds or at least is a good approximation even if the process is not narrow banded. Other investigators, Forristall (1978), disagree and have put forward evidence to show that the Rayleigh distribution is not a good approximation for wide banded processes. They have not, however, suggested or theorized a better approximation or distribution.

The non-spectral approach operates in the time domain and generally proceeds on a wave-by-wave basis. The advantage of this method is that it includes non-linear

effects not accounted for by spectral analysis. It does,
however, have the disadvantage of using more computer time.
The time series analysis, as will be shown, has difficulty
dealing with the irregularity in height and period of ocean
waves, while spectral methods handle this problem quite
well.

Although this study uses basically a non-spectral
method of analysis to determine wave group boundaries, a
spectral analysis was performed to obtain certain wave
record parameters to assist in the time series analysis.
Thus, the irregularity and the non-linear character of
waves should both be taken into account by this combination
of the two methods. The spectral analyses also enabled
wave group parameters and individual wave parameters to
be related back to the spectral analyses themselves.

Time series analysis can be performed with either an
analog wave record, which shows all instantaneous sea
surface elevations, or with a digitized wave record which
shows sea surface elevations at time intervals equal to a
fixed sampling rate. Analog wave records must be analyzed
by hand, unless they are digitized after the fact, while
digitized wave records lend themselves easily to computer
analysis.

This study utilized wave records that were digitized
and were computer analyzed using the standard zero-upcross

technique. Here, a wave is defined between two successive upcrossings of the wave record with respect to some reference level. The reference level is taken to be the mean water level of the record. As shown in Figure 1, the wave period is defined as the time interval between successive upcrossings while the wave height is defined as the vertical separation between the highest and the lowest point of the wave record between successive up-crossings.

B. WAVE GROUP DEFINITIONS

As mentioned previously wave groups are an ocean surface phenomenon that is readily apparent to even the most casual observer simply because they represent an orderly succession of wave heights and/or periods against a random background. Even with this being the case, however, there is as yet no common agreement on how wave groups should be defined.

Many definitions of wave groups are in the literature. Most employ readily identifiable wave parameters to define wave-group boundaries. Among the many definitions in use are the largest height of a wave in a group, the height which a sequence of waves exceed, the number of successive waves, the periods of successive waves, and the time that the envelope of a wave record (with the envelope having various definitions) exceeds a certain level. The method

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of definition used by Sedivy (1978) used a new approach wherein the energy content of the wave groups was the decisive parameter.

The basic premise used by Sedivy (1978) and in this study is that the energy in a wave record is proportional to the variance of the wave record. The record variance can be calculated from a time-domain analysis or from a frequency-domain analysis. The energy of a simple sinusoidal wave can be shown to be proportional to the square of the height of the wave. Moreover, Michel (1968) states that if the wave field is thought of as the sum of an infinite number of component waves of small amplitude then the energy of the wave field may be considered to be proportional to the sums of the squares of the heights of those component waves. Thus energy, variance, and wave height are related, and if one can evaluate one of these quantities then the other two are obtainable by a pro-portionality factor.

If the concept is kept in mind that a wave group is essentially a succession of waves which are not random and in general whose heights are larger than the surrounding waves, then wave groups may be thought of as packets whose energy content is greater than that of the surrounding wave field. Over a short period of time, then, the variance of the wave groups will also be larger than the variance of the wave record as a whole.

The wave record is then analyzed by using a window of short duration to look at portions of the wave record. The window slides along the wave record at a rate equal to the digitizing rate, and at each stop the short-term variance is calculated and plotted at the midpoint of the window. The result of this process is illustrated in Figures 2 and 3, where the wave record is shown in the bottom of the figures and the corresponding short-term variance waveform is shown in the top portion of the figures. Those areas under the short-term variance curve that lie above the variance of the wave record (a straight line) denote areas of higher than wave record energy, and hence possible wave groups.

It is noticeable, however, that those areas where the short-term variance waveform crosses the record variance do not often correspond to zero upcrossings of the wave record itself. Realistically and statistically it does not make sense to think of a wave group as being anything else than a series of whole waves. Thus, the short-term variance waveform cannot be used by itself to define wave groups. The definition of a wave group was expanded so that when the boundaries are set on the wave record by the variance waveform analysis, they are then moved away from the center of the wave group to the first zero upcrossing in each direction. This is shown graphically in Figure 4.

Two additional restrictions were imposed on the placement of wave group boundaries. The first restriction is that a wave group must have at least two waves, since one wave was not considered to constitute a group against a random wave field. The second restriction is that wave groups identified as above must be separated by at least one-half the width of the window. Where this condition was not met by two successive wave groups they were treated as one wave group. This restriction was necessary to prevent the possibility of one wave being included in two separate wave groups.

Thompson (1972) and Smith (1974) have shown that the average period of the waves in a group closely approximate the period of maximum energy density in a wave record, or in other words the spectral peak period. Sedivy (1978) used twice this value for the length of the short-term sliding window after experiments on artificially generated wave records showed it to be an optimum value. Experiments on actual wave records for the purpose of this study confirmed this choice of length for the sliding window.

In this study experiments were conducted by varying the window length in fractions and multiples of the spectral peak period. It was found that large energy groups were identified by all windows and that the number of waves in a group was seldom affected by window length. In groups with

small energy, however, the number of groups in a record varied as the window length was varied, and the number of waves in these groups was quite variable. In general as the window length was increased the number of wave groups decreased. The window lengths experimented with ranged from one-half to four times the peak period and over this range the number of wave groups decreased by 50%. This was accompanied by some increase in the number of waves per group. A window length of twice the spectral peak period was therefore used in this study since it identifies all the large energy groups.

III. WAVE DATA ACQUISITION, SELECTION, AND ANALYSIS

Wave records analyzed in conjunction with this study were provided by the Coastal Engineering Data Network (CEDN) which is sponsored jointly by the California Department of Navigation and Ocean Development (CDNOD) and the U. S. Army Corps of Engineers. Additional funding support is supplied by the University of California Sea Grant Program with Scripps Institution of Oceanography providing overall direction for the actual data collection program.

The wave records received from CEDN were recorded from a Datawell Waver'der accelerometer-type buoy anchored in approximately 30 fathoms of water. The buoy was installed by the U. S. Naval Postgraduate School under contract with CDNOD, and is located, as shown in Figure 5, on the open continental shelf approximately four nautical miles south-southwest of the Santa Cruz Point light at 36°53.48' North and .2.°03.22' West. The wave record data were received from CEDN on magnetic tape digitized at a sampling rate of one second. The wave records gave an instantaneous sea level at one second intervals with respect to a mean water level. The mean water level used was the average of all sea surface elevations in the wave record and was assumed not to change significantly over the duration of the record. Each wave record contained 1024 data points (approximately

17-minute duration) with a record generally being recorded synoptically every ten hours. The wave records used in this study were selected from data recorded over a ten month period from June 1978 through March 1979.

CEDN regularly publishes a spectral analysis of the wave records obtained each month in both a graphical and tabular format as shown in Figure 6 and Figure 7, respectively. The wave records used in this study were selected from a review of the published CEDN data on the basis of displaying one main and easily distinguishable spectral peak. Only wave records with unimodal spectra were considered. Multimodal spectra present some complications in the selection of a suitable window length, and it was considered best to examine the properties of wave groups in clearly unimodal spectra first. Thus, individual wave records were initially selected if they displayed more than 30% of the energy in one, and only one, of the CEDN tabular printout spectral period bands. This limit was arbitrarily chosen to ensure that only one prominent energy peak occurred in the spectra, but yet was not so restrictive that only a few wave records would be considered in the study.

CEDN furnished 695 wave records and they were initially searched for single-peaked spectra satisfying the criteria. Approximately half, or 338 of the records, were found to possess the unimodal spectrum desired. These 338 selected

records, the characteristics of which are shown in Figure 10, were analyzed on an IBM 360/67 computer both spectrally and non-spectrally. Examples of the spectral analysis are shown in Figures 8 and 9. This type of analysis was performed in part to obtain better resolution of the spectral peak period than is possible from the two-second period bands provided in the CEDN analysis.

The non-spectral analysis of the wave records utilized the definition of a wave group, as previously defined, to identify wave groups in the wave records and to then compute the various wave record and wave group parameters desired for the statistical portion of the study. Out of 338 wave records 5598 wave groups were so identified and the pertinent statistical parameters generated.

The wave data analyzed in this study may be classified in terms of relative depth as deep water waves or intermediate water waves for the wave record spectral peak periods of interest (four to 18 seconds). Although waves with periods greater than nine seconds have begun to shoal at the location of the wave recorder (30 fathoms), the shoaling coefficient varies only between 0.91 and 1.00. Accordingly all wave records effectively contain deep water waves, and the statistics generated from these records are presumably applicable to the open ocean.

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IV.   WAVE RECORD AND WAVE GROUP PARAMETERS

The wave record and wave group parameters that were
either utilized or examined in this study are defined below.
Definitions previously used by other authors or definitions
that are in common usage are used whenever possible.

A.   WAVE RECORD PARAMETERS

1.   Spectral Peak Period: T_R

The spectral peak period of a wave record, as pre-
viously mentioned, is that period in the wave record that
represents the maximum energy concentration. Sedivy (1978)
used an interpolation technique to arrive at T_R to the
nearest second from the two-second bandwidths given in the
CEDN spectral printouts. In the present study T_R was taken
as that period corresponding to the frequency of the
spectral energy peak obtained from independent spectral
analysis of the CEDN wave data. Because of the variation
in the resolution of T_R over the range of wave record
periods dealt with, T_R was taken, for all records, to the
nearest whole second.

2.   Record Variance: V_R

As previously discussed, the variance of a wave
record is a measure of the energy in the wave record and is
related to the heights of the waves in the record.   The

variance can be determined by integration of the power
spectrum to obtain the area under the spectral density
curve, or it can be calculated directly from the digitized
sea surface elevations. The latter method was chosen for
two reasons. First, estimates of variance calculated from
the time series analysis performed for this study were
approximately the same as the variance value from the CEDN
spectral printouts and from the independent spectral analysis
of CEDN wave data. Second, the sea surface elevation
method was easily included in the wave group analysis
program at little cost of computer time, and more
importantly, is consistent with the group statistics
generated by that program.

3. Significant Height: H_R

The significant height of a wave record is defined
as the average of the highest one third waves in the wave
record, and was calculated from the record variance, V_R.

4. Wave Steepness: G_R

As defined in Table I, the steepness parameter for
spectrum waves, using H_R and T_R, may be defined in an
analogous way to the steepness parameter for monochromatic
waves. The steepness parameter can be used to generally
identify the kind of waves present in the wave record in
terms of their relative age, i.e., sea, young swell,

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moderate swell, old swell. Thompson and Reynolds (1976) state that since swell steepness diminishes with increasing travel distance from the generating area some measure of the steepness of the wave field, G_R, can be used for the purpose of estimating relative age and thus the approximate swell distance. The wave steepness parameter is used in this study to determine whether the wave records analyzed are primarily composed of sea or some type of swell, and whether this has some relationship to grouping among the waves. Figure 10 shows the distribution of wave steepness among the 338 wave records analyzed, and shows that with the exception of the old swell band, the wave records are almost evenly distributed over the range of wave age.

    5. Record Group Duration: D_R

        The record group duration is defined as the total amount of time that groups are present over the length of the wave record. It is determined simply by summing the durations of the individual groups, excluding partial groups truncated at the beginning or end of the wave record.

B. WAVE GROUP PARAMETERS

    1. Wave Group Duration: D

        Wave group duration is the interval of time that a wave group is present, to the nearest second, as determined by the wave group analysis described previously. It is measured directly from the wave record.

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2. <u>Number of Waves per Group: N_G</u>

This is the number of whole waves in a wave group
of duration D, and by definition is two or more.

3. <u>Wave Group Period: T_G</u>

The group period is defined as the average period
of the individual waves that compose a wave group. One of
the distinctive characteristics of wave groups is that the
constituent waves tend to be periodic, and T_G will thus be
an approximation of this characteristic. It is calculated
by dividing the wave group duration by the number of waves
in the group.

4. <u>Average Wave Group Variance: V_G</u>

The average variance of a wave group is a measure
of the average energy contained in the wave group and thus
is another group parameter of interest. It is calculated
by taking an average of the short-term variance values over
the length of the wave group from initial to terminal zero
upcrossing.

5. <u>Height of the Highest Individual
    Wave in a Group: H_M</u>

This parameter is defined simply as the height of
the single highest wave that occurs in a wave group. It is
calculated by the zero upcrossing method (as are all
individual wave heights in this study).

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C. WAVE GROUP TO WAVE RECORD PARAMETERS

Three wave group parameters were normalized by taking the ratio of the group parameter to the corresponding record parameter. This allows the group statistics of one wave record to be compared to the group statistics of other wave records.

1. Wave Group Variance to Wave Record Variance: V_G/V_R

This group/record parameter is the basis for the definition of the wave group, as discussed above; i.e. where V_G/V_R is greater than one a potential wave group exists. As will be seen in the data, some wave groups have a V_G/V_R value of less than one. This is explained by the fact that the wave group boundaries are moved outward to include a whole number of waves (Figure 4). This means that for groups with small V_G/V_R peak values and small numbers of waves that it is possible for more of the short-term variance curve to lie below the record variance curve than above it. As both V_G and V_R are measures of energy, their ratio is a dimensionless measure of energy in the group. Thus, the higher the value of V_G/V_R, the more energy the group contains relative to the wave record. This parameter was investigated as it seems reasonable that the more energy a group contains relative to the wave field surrounding it, the more important is that wave group.

2. **Wave Group Period to Spectral Peak Period:** T_G/T_R

Thompson (1972), Smith (1974), Thompson and Smith (1974), and Sedivy (1978) investigated the relationship of T_G and T_R. The first three of these papers report that the average group period and the spectral peak period are essentially equal in a wave group. Sedivy (1978) found a tendency for T_G/T_R to approach a value of one but not to the degree that earlier researchers have found. This parameter was included, in part in the present study, to check this relationship for essentially deep ocean waves; earlier findings were obtained from waves in shallow water recorded by bottom-mounted pressure sensors.

3. **Highest Group Wave Height to Significant Wave Height:** H_M/H_R

Sedivy (1978) found H_M/H_R to be strongly dependent on other group or record parameters, and this parameter was included to help validate those results and look for possible relationships among parameters not considered previously.

V. RESULTS AND INTERPRETATION

A. ANALYSIS SCHEDULE

All wave group and wave record parameters examined in
this study have been defined previously. Given the large
number of possible relationships between them, a systematic
method of examination was called for. To facilitate this
analysis Table II and Table III were constructed and used
as guides. As is shown in each table, a matrix was con-
structed with all relevant parameters represented (the
relationships examined are illustrated in the figures
indicated). The possible relationships that are crossed
out are either redundant or illogical. Table II represents
analyses for all of the wave group data, while Table III
illustrates the same analyses for extreme wave groups only.

The data parameters T_G/T_R, V_G/V_R, H_M/H_R, and G_R were
divided into bands of values. This alternate view of data
parameter distributions was adopted due to the wide range
of values possible for each parameter. The band divisions
were chosen either on the basis of definitions already in
use (i.e., wave record steepness) or to ensure an even
distribution of variables in the bands.

All possible relationships in this study were illus-
trated by means of frequency of occurrence graphs. As will
be seen there are two graphs for each relationship, a

percentage distribution curve or histogram, and a cumulative percentage distribution curve. Each type of curve has its own advantages and disadvantages in bringing out the details of each relationship.

The cumulative distribution curves were used to extract quantitative statistical measures for each distribution. Many of the curves approximate a normal distribution. If the distributions were normal then the cumulative 50 percentage level would represent the mean and the cumulative 16 and 84 percentage levels would approximate the first standard deviation about the mean. By analogy, the median of the actual distributions was used to represent the central tendency, and the cumulative 20 and 80 percentage levels were used as an approximation, albeit crude, for the first standard deviation distribution about the mean. In this study, the approximation for the first standard deviation will henceforth be referred to as the standard deviation for all curves examined. The median and the standard deviation (presented below in tabular format) allows rapid and convenient examination of the many distributions that are graphically presented. They also enable one graph to be compared to another graph or to another parameter band of the same graph.

B. ALL WAVE GROUPS

1. Frequency Distribution (graphs)

a. N_G

The distribution of the number of waves per group for all 338 wave records is shown in the histogram of Figure 11A and in the cumulative percentage distribution of Figure 11B. The modal number of waves per group is three, while the median falls between three and four waves per group. The standard deviation ranges between two and six waves per group. The total range of values of waves per group is from the defined minimum of two to a single group maximum of 28.

The number of waves per group as a function of T_G/T_R bands is illustrated in Figures 12A and 12B. All T_G/T_R bands closely resemble the distribution shown previously for all groups (Figure 11A). The large variability of the lowest T_G/T_R curve is due to the small sample size of the band. The cumulative percentage curve best shows the similarity between the distribution of N_G for all wave groups and for groups falling within the given T_G/T_R bands. It also shows that there is no order or gradation among the T_G/T_R bands. These two sets of curves indicate that there is no dependency of N_G on T_G/T_R. Regardless of whether the group period is less than, equal to, or greater than the spectral peak period, N_G retains the same distribution.

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The histograms and cumulative percentage curves for N_G with respect to relative energy, or V_G/V_R bands, are shown in Figures 13A and 13B respectively. The most striking feature of the histogram is that as the relative energy in a group increases (i.e., as the V_G/V_R band value increases), the peak value or the value at which the number of waves per group most frequently occurs shifts to the right toward larger values. The modal number for the lowest V_G/V_R band is two, while for the highest V_G/V_R bands it is five or six waves per group. The shape of the curve also changes as V_G/V_R increases; the curve becomes less peaked and the percentage frequency of occurrence of the peak decreases. The curve widens as it flattens, and a greater total range of values occurs for the higher V_G/V_R bands. The latter features can also be seen on the cumulative percentage curve. The standard deviation effectively doubles from lowest to highest V_G/V_R band. The number of waves per group is plainly seen to be related to the relative energy that the group possesses. The greater the relative energy that a group contains, the greater is the average number of waves in the group.

Figures 14A and 14B are the frequency of occurrence graphs for N_G as a function of H_M/H_R, the ratio of the highest wave per group to the significant height of the wave record. The curves for N_G for the H_M/H_R bands

given show similar characteristics to the curves discussed
above for N_G as a function of V_G/V_R bands. As the H_M/H_R
bands increase in value, the peak occurrence of N_G decreases
in percentage value and shifts to the right from two to six
or seven waves per group. While the curve flattens and
spreads out as H_M/H_R increases, the range of standard
deviation approximations also increases in magnitude; the
increase being from less than two to two waves per group
for the lowest H_M/H_R band to four to ten waves per group
for the highest band. The similarity between N_G for H_M/H_R
and V_G/V_R bands is evident as stated (compare Figures 13A
and 14A), and logic would dictate that this indication of
a relationship between H_M/H_R and V_G/V_R is to be expected.
H_R and V_R are statistical measures that should be proportional,
and ratios that contain these two parameters should also be
proportional. Thus, the higher the energy content of a
group the generally higher should be the waves in the group,
including the largest wave H_M, and the number of waves that
the group will possess should be larger.

The distribution of N_G with respect to wave
steepness, and hence relative wave age, shows similarities
to the distribution of N_G for all groups (Figure 11A), but
with some important differences. Figure 15A shows that the
curves for all four steepness bands are very similar to
each other. All are peaked at three waves per group,

although the maximum frequency of occurrence decreases as
wave steepness decreases. The most significant difference
between steepness bands, however, is best seen in the
orderly sequence of the cumulative curves of Figure 15B.
The median number of waves per group is seen to increase
from three to approximately five in progressing from sea
to old swell. The standard deviation varies from a range
of two to five waves per group for high steepness waves to
two to eight waves per group for low steepness waves. The
total range of N_G values also increases as steepness decreases.
Thus, the greater the wave age the greater is the probability
of wave groups having a wider range of N_G, and the
probability increases sightly of there being more waves
per group. This seems entirely reasonable when one views
seas as generally having wider frequency and direction bands
compared to swell that have propagated over long distances.

    b. T_G/T_R

      The T_G/T_R ratio for all wave groups is shown in
Figure 16A and is an approximately normal curve with a peak
value at T_G/T_R of approximately 0.90. This value is a
little less than 1.00, which would be the value of T_G/T_R
if the average group period was the same as the spectral
peak period as shown by Thompson and Smith (1974). The
overall range of T_G/T_R values is from approximately 0.30 to
3.20. The cumulative percentage curve in Figure 16B shows

that the standard deviation has a small range in value from 0.75 to 1.10 about the median of 0.90. Thus, the probability of getting a value of T_G/T_R much different from one is small.

The frequency of occurrence of T_G/T_R as a function of relative energy, expressed by V_G/V_R bands, is shown in Figure 17A. The distributions for all six V_G/V_R bands are approximately normal, but with several interesting differences between them. The peak frequency of occurrence of T_G/T_R increases as the V_G/V_R bands increase in magnitude. Also, the value of T_G/T_R for the peak shifts slightly to the right from 0.90 to 1.00 as V_G/V_R increases. Figure 17B shows that the smaller the magnitude of the V_G/V_R band the more probable is a slightly wider range of T_G/T_R values. Thus, the higher the relative energy content of a group, the more probable it is that the T_G/T_R value will be close to the peak value and also that the peak value will be close to one. Therefore, the tendency among high energy groups is for the average group period to approximately equal the spectral peak period. Since T_R is fixed for a wave record, this means that for low energy groups the average period will be more variable and will likely be less than T_R.

The T_G/T_R distribution by H_M/H_R bands is illustrated in Figures 18A and 18B. All curves with the

exception of the lowest H_M/H_R band are approximately normal, and are centered about a median T_G/T_R value of about 1.00 (Figure 18B). It may also be noted that as H_M/H_R increases, the probability increases that T_G/T_R is 1.00 (Figure 18A). The curves for the lower H_M/H_R bands have quite irregular peaks and they also possess a larger standard deviation. As was the case for N_G, the frequency of occurrence curves of T_G/T_R for both V_G/V_R and H_M/H_R bands are similar.

The effect of steepness on the T_G/T_R distribution is shown in Figure 19A. All four curves approximate a normal distribution. The peak percentage values are approximately equal, as is the total range of T_G/T_R values. It is obvious, however, that T_G/T_R is a function of steepness, as the distributions show a progressive march to the right for T_G/T_R as the steepness increases. The band corresponding to low swell ($G_R < 1/250$) peaks between T_G/T_R values of 0.60 and 0.80, while the band corresponding to seas ($1/12 < G_R < 1/40$) peaks around 1.10. The cumulative percentage curves are shown in Figure 19B. With the exception of the highest steepness band, the three remaining curves are very parallel to each other and the values for the standard deviation are approximately the same. The greater the steepness of a wave record the more probable it is that T_G will approach T_R in value for the wave groups contained in the record. However, there is also a little wider range

of possible T_G/T_R values. This is partly contrary to logic. Old swell of low steepness is expected to have a narrow frequency band. It would seem likely that the range of T_G/T_R values should thus be small and that T_G should approximate T_R. Since this is not the case then how can T_R be the period of maximum energy density for the record? This question will be looked at again later.

c. V_G/V_R

The frequency of occurrence for the ratio of average wave group variance to record variance for all groups is shown in Figure 20A. The curve is noticeably positively skewed with a very sharp increase to the left of the peak between V_G/V_R values of 0.80 to 1.00. The peak value occurs at approximately 1.10. The sharp cutoff is simply explained as due to the use of V_G/V_R equal to one for defining wave groups. If wave groups were defined solely as those parts of the wave record where V_G was greater than or equal to V_R there would be no values of V_G/V_R less than one. As explained earlier, however, other restrictions on the identification of wave group boundaries allow a small number of values less than one. The peak occurring at a value slightly larger than one means that most groups have only just enough relative energy to be included as wave groups. There was a wide range of V_G/V_R values found, extending from 0.55 to 3.60. The cumulative percentage curve

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of Figure 20B reveals that the median value of V_G/V_R is approximately 1.20 while the standard deviation about that value ranges from 1.00 to 1.50.

Previously described graphs (Figures 13A, 14A, 17A, and 18A) have shown indications of a relationship between V_G/V_R and H_M/H_R. This relationship is illustrated in a different way by Figures 21A and 21B. All curves with the exception of the curve for the highest H_M/H_R band are approximately normal. The irregularity of this curve is the result of a low number of groups in that band. As the H_M/H_R bands increase in value the percentage occurrence of the peak V_G/V_R values decreases and the peak itself shifts to the right. The curves also become less peaked, more flattened, and extend over a wider range in the progression toward higher H_M/H_R values. The cumulative percentage curves illustrate these observations nicely. The standard deviation for V_G/V_R ranges from 0.90 to 1.05 for the lowest H_M/H_R band, while for the highest it ranges from 1.75 to 2.45. The principal message obtained from these graphs is that the greater the relative energy in a group, the more likely is the chance that H_M will exceed H_R, and by a greater amount. It is also evident that the higher the value of H_M/H_R, the wider is the range of values possible for V_G/V_R.

The V_G/V_R values plotted for steepness bands are shown in Figures 22A and 22B. All four curves are very

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nearly statistically identical, and it is clear that wave

steepness does not affect V_G/V_R. In other words, V_G/V_R

will have the same distribution among wave groups regardless

of the age or distance traveled of the waves in the wave

record.

    d. H_M/H_R

The distribution of H_M/H_R for all wave groups

is shown by Figure 23A to approximate a normal curve with

a mean close to 1.00. The total range of H_M/H_R values is

approximately between 0.35 and 2.15. The standard deviation

obtained from analysis of Figure 23B varies between 0.85

and 1.21. Although a wide range of H_M/H_R values is possible,

it is surprising to note that the probability of occurrence

is greatest when H_M is approximately equal to H_R. That is,

the highest wave in a group will most frequently equal the

significant height of the record.

The relationship between H_M/H_R and steepness

is shown by Figures 24A and 24B. The peak occurrence

percentage values on the histograms are very nearly the

same with the exception of the lowest steepness band which

is slightly higher. The value of H_M/H_R at which the peak

occurs shifts to the right as the steepness of the wave

record decreases. The cumulative curves also show this

shift, however, the standard deviation about the median

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remains nearly constant. The range of values in each band
also remains almost constant. Figure 21A shows H_M/H_R and
V_G/V_R to be related,and Figure 24A shows H_M/H_R and G_R (steep-
ness) to be related. It is surprising, therefore, to find
from Figure 22A that V_G/V_R and steepness are apparently
unrelated.

     e. D_R

        The length of each wave record was 1024 seconds.
The duration or the amount of time that wave groups were
present over that time interval is shown in Figures 25A and
25B. The histogram appears quite irregular but the
irregularity is magnified in the graphical presentation
because of the small duration interval used to generate the
histogram. A best-fit curve drawn to this distribution would
yield an approximately normal distribution with a mean of
about 575 seconds. Thus, on the average, wave groups are
present over more than half of the wave record. This is in
agreement with the findings of Sedivy (1978). The standard
deviation ranges from 485 to 620 seconds.

    2. Statistical Measures (tables)

        Thus far 15 sets (two each) of frequency of occur-
rence graphs have been presented, described, and discussed
for various wave group parameters. To assist readers in
their interpretation, pertinent statistical data on the
distributions have been tabulated and are presented in
Table IV A-E. The sequence of the data is keyed to the

parameter matrix shown in Table II. The first four columns
of numerical data in Table IV A-E contain information from
the respective histograms: the parameter value at which the
peak occurs, the maximum parameter value, the minimum
parameter value, and the total range of parameter values
(the maximum value less the minimum value). The next three
columns are the 50, 20, and 80 cumulative percentage levels
from the cumulative distributions. As described previously,
these values represent the median and standard deviation
limits, respectively. The next two columns are the 50 minus
the 20 cumulative percentage value and the 80 minus the 50
cumulative percentage value. These two columns when compared
against each other give a rough idea of the skewness of the
distribution. The last column gives the approximate
standard deviation range.

C. EXTREME WAVE GROUPS

The frequency of occurrence graphs examined thus far
have utilized as a data base all 5548 wave groups identified
in the study. These groups were either examined collectively
or in bands according to wave group or wave record charac-
teristics. The engineer may, however, be particularly
interested in those groups which are the most likely to
damage a structure. These may include groups with a large
number of periodic waves that might induce resonance in the

structure, or groups containing a great deal of energy whose
waves of large amplitude are capable of exerting excessive
fluid forces.

With this in mind, a second series of frequency of
occurrence graphs were generated, with the largest wave
group in each wave record being used as the data base. Two
means of identifying the largest group per record were
employed. The largest group was first defined as the
group with the largest number of waves, and secondly as
the group with the largest V_G/V_R ratio in each record.

I. Frequency Distribution (graphs)

a. Largest N_G per Wave Record

Figure 26A shc $_3$ the distribution curve for N_G
using only that group in each record having the largest N_G
value. The peak occurrence of the 338 records analyzed was
found to be at nine to ten waves per group, with the total
range being from five to 28. The curve is approximately
normal, but is somewhat positively skewed. The cumulative
percentage curve in Figure 26B shows the standard deviation
to range between eight and 12 to 13 waves per group.

Figures 27A and 27B present T_G/T_R distributions
for the largest N_G per wave record. The most striking
features of the histogram are the sharpness of the peak
and the small range of possible values. The main peak occurs
at T_G/T_R equal to one; however, the median of the distribution

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is somewhat less than one. The standard deviation extends
from 0.74 to 1.14 and the total range is approximately from
0.50 to 2.10. The distribution shown in Figures 27A and 27B
is almost identical to that for all wave groups (Figures
16A and 16B), indicating that T_G/T_R and N_G are independent
variables. This was also shown in Figures 12A and 12B.

The distribution of V_G/V_R associated with the
largest N_G per wave record is illustrated in Figures 28A and
28B. The peak occurs at a V_G/V_R value of about 1.60,
although there are secondary maxima on either side of the
peak. The distribution is similar to that for all wave
groups (Figures 20A and 20B), but is clearly displaced toward
larger V_G/V_R values and is more narrow-banded as indicated
by a smaller standard deviation (Tables IV.C and V.A).
Thus, those groups in the records with the largest number
of waves tend to have higher relative energy values.

The frequency of occurrence of H_M/H_R with respect
to the largest N_G per wave record is shown in Figures 29A and
29B. The distribution is approximately normal, with the peak
being around 1.25 and with overall values ranging from
approximately 1.00 to 2.00. As with V_G/V_R, the distribution
of H_M/H_R among those wave groups with the largest number of
waves per wave record is similar to that for all wave groups
(Figures 23A and 23B), but is displaced toward larger H_M/H_R
values and is more narrow-banded (Tables IV.D and V.A).

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The curves suggest that for groups with large numbers of waves, the highest wave in a group will almost certainly exceed the significant height of the record. In wave records of 17 minute duration, H_M will tend to be about 30% higher than H_R.

b. Largest V_G/V_R per Wave Record

The distribution curves for N_G, as illustrated in Figure 30A and Figure 30B for the largest group V_G/V_R value per wave record are generally similar to the curves for the largest group N_G value per record (Figures 26A and 26B). However, the peak lies in the range of from five to seven waves per group, and the distribution has a slightly larger standard deviation. The long tail to the right in both distributions suggests that those groups with the most waves also have the highest relative energy since it appears that the same groups make up the tail.

Figures 31A and 31B are almost identical in every respect to the corresponding T_G/T_R graphs generated for the wave groups with largest N_G per record (Figures 27A and 27B). The description of the latter figures given above therefore describes these figures.

The distribution of V_G/V_R for the largest group V_G/V_R per wave record, shown in Figures 32A and 32B, is very similar to that for the largest group N_G (Figures 28A and 28B), but is displaced substantially toward larger V_G/V_R values. The histogram shows that the average variance of

the wave group with largest energy in a record may exceed
the record variance by a factor of more than three, although
a factor of 1.75 to 2.00 is most probable.

Figures 33A and 33B illustrate the histogram and
cumulative curves for H_M/H_R associated with the largest group
V_G/V_R per record. The distribution is very similar to that
for wave groups having the largest N_G per record (Figures
29A and 29B) although it is clearly displaced toward higher
H_M/H_R values (Tables V.A and V.B). The histogram shows that
groups having the largest relative energy in 17 minute wave
records may have H_M/H_R values as large as two, although a
factor of 1.3 to 1.6 has the greatest probability of
occurrence.

2. Statistical Measures (tables)

Statistical data on the previous eight sets of
frequency of occurrence graphs for extreme wave groups have
been tabulated and are presented in Table V.A-B. The
organization of the table is the same as previously used
(Table IV.A-E), and the same interpretational comments apply.

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VI. SUMMARY

The purpose of this study was to use a digital analysis
technique to examine the statistics of selected wave group
parameters. Wave groups were defined as packets of waves
whose energy density is greater than that of the wave
record as a whole. Wave group boundaries were mainly deter-
mined by using a sliding window to calculate the short-term
variance, which was then compared to the record variance
(variance being proportional to energy). The short-term
window length was taken to be a function of the spectral peak
period. The wave group measures of interest were calculated
for each identified wave group. One set of statistics was
then generated using all wave groups and another set using
selected extreme wave groups only. The distributions and
interrelationships of these parameters were represented for
analytical purposes by frequency of occurrence graphs.

Wave records were obtained from a Waverider buoy moored
in Monterey Bay on the outer continental shelf. The wave
records represented deep or intermediate depth waves, and
covered a wide range of significant height, spectral peak
period, and wave type (represented by wave steepness). Only
wave records that displayed a unimodal spectrum were con-
sidered for analysis, and 338 of those examined were chosen

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for analysis. The analyses were performed on wave records having a duration of 1024 seconds (about 17 minutes). The data base from these records consisted of 5598 wave groups.

The portion of the study dealing with all groups yielded results both interesting and unexpected. It was found that the number of waves per group increases as the relative energy of the wave group increases. It was also shown that the average group period approaches the spectral peak period for groups of high relative energy. The steepness of the wave record has no effect at all on the relative energy of a group, on the other hand, the lower the steepness (hence the greater the distance that the waves have traveled) the larger is the number of waves a group will tend to possess. The height of the highest wave in a group (H_M) tends to approximate the significant height of the wave record (H_R). The greater the relative energy (V_G/V_R) that a group contains, however, the greater is the probability that H_M will exceed H_R. Although relationships were shown to exist between relative energy and H_M/H_R, and between wave steepness (G_R) and H_M/H_R, no relationship could be shown to exist (or the relationship was so subtle as to not be seen) between G_R and V_G/V_R.

The extreme wave groups used for statistical analysis were divided into two sets. The first set was composed of that group from each wave record having the largest number

of waves, while the second set contained that group in each record having the largest relative energy content. The main feature of these two analyses was the similarity between the distributions of both data sets for the same parameters. Although the distributions were displaced to the right or left of each other, they were very nearly identical. The average group period of both sets of extreme groups was shown to approximate the spectral peak period ($T_G/T_R \approx 1$), and the maximum wave height was shown to substantially exceed the significant height of the wave record. It appears that those groups having the largest number of waves were also the same groups that possessed the largest relative energy.

Table 1. Definitions of wave steepness and wave age,
applicable to deep water only (Thompson and
Reynolds, 1976).

WAVE STEEPNESS, G

Monochromatic waves

$$G_M = \frac{H}{L} = \frac{H}{\frac{g}{2\pi} T^2} = \frac{H}{5.12 \, T^2}$$

H = wave height
L = wave length
T = wave period

Spectrum waves

$$G_R = \frac{H_R}{\frac{g}{2\pi} T_R^2} = \frac{H_R}{5.12 \, T_R^2}$$

H_R = significant wave height

T_R = spectral peak period

WAVE AGE (in terms of G_R)

Wave Age	G_R
Sea	1/12–1/40
Young Swell	1/40–1/100
Moderate Swell	1/100–1/250
Old Swell	1/250+

Table II. Parameter matrix for all wave groups.

Variable Bands	N_G	T_G/T_R	V_G/V_R	H_M/H_R	D_R
N_G					
T_G/T_R	Figure 12A Figure 12B				
V_G/V_R	Figure 13A Figure 13B	Figure 17A Figure 17B			
H_M/H_R	Figure 14A Figure 14B	Figure 18A Figure 18B	Figure 21A Figure 21B		
G_R	Figure 15A Figure 15B	Figure 19A Figure 19B	Figure 22A Figure 22B	Figure 24A Figure 24B	
All Groups	Figure 11A Figure 11B	Figure 16A Figure 16B	Figure 20A Figure 20B	Figure 23A Figure 23B	Figure 25A Figure 25B

TABLE III. Parameter matrix for extreme wave groups.

	N_G	T_G/T_R	V_G/V_R	H_M/H_R
Largest Group (N_G)	Figure 26A Figure 26B	Figure 27A Figure 27B	Figure 28A Figure 28B	Figure 29A Figure 29B
Largest Group (V_G/V_R)	Figure 30A Figure 30B	Figure 31A Figure 31B	Figure 32A Figure 32B	Figure 33A Figure 33B

Table IV.A. Statistical Data for N_G for all wave groups.

Group Parameter	Figure	Band	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
All Groups	11A-11B		3	28	2	26	3.5	2.1	5.9	1.4	2.4	3.8
T_G/T_R	12A-12B	0.25-0.49	2	16	2	14	3.2	2.0<	6.0	-	2.6	-
		0.50-0.74	3	22	2	20	3.7	2.3	6.2	1.4	2.5	3.9
		0.75-0.99	3	28	2	26	3.6	2.1	5.9	1.5	2.4	3.9
		1.00-1.24	3	18	2	16	3.6	2.1	6.0	1.5	2.3	3.9
		1.25-1.49	3	11	2	9	3.4	2.0	5.7	1.4	2.3	3.7
		1.50+	3	10	2	8	3.5	2.1	6.0	1.4	2.5	3.9
v_G/v_R	13A-13B	0.75-0.99	2	6	2	4	2.0<	2.0<	2.7	-	-	-
		1.00-1.24	3	13	2	11	3.0	2.1	4.5	0.9	1.5	2.4
		1.25-1.49	4	18	2	16	4.5	3.1	6.9	1.4	2.4	3.3
		1.50-1.74	5	20	2	18	5.6	3.7	8.4	1.9	2.8	2.7
		1.75-1.99	5	22	2	20	6.0	4.0	8.4	2.0	2.4	4.2
		2.00+	4	28	3	25	5.4	3.6	7.8	1.8	2.4	-
H_M/H_R	14A-14B	0.30-0.79	2	5	2	3	2.0<	2.0<	2.0<	-	-	-
		0.80-0.99	3	12	2	10	2.4	2.0<	3.4	-	1.0	3.0
		1.00-1.19	4	14	2	12	3.9	2.7	5.7	1.2	1.8	3.0
		1.20-1.39	5	16	3	14	5.6	3.7	8.2	1.9	2.6	4.5
		1.40-1.59	7	20	3	17	6.7	4.5	9.2	2.2	2.5	4.7
		1.60+	6	28	3	25	6.9	4.6	9.9	2.3	3.0	5.3
G_R	15A-15B	1/12-1/40	3	15	2	13	3.2	2.0<	5.2	-	2.0	-
		1/40-1/100	3	18	2	16	3.4	2.1	5.9	1.3	2.5	3.8
		1/100-1/250	3	22	2	20	3.8	2.3	6.5	1.3	2.7	4.2
		1/250+	3	28	2	26	4.2	2.4	7.8	1.8	3.6	5.4

Table IV.B. Statistical Data for T_G/T_R for all wave groups.

Group Parameter	Figure	Band	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	50-50%	Std.Dev. Range
All Groups	16A-16B		0.90	3.20	0.30	2.90	0.90	0.72	1.10	0.19	0.20	0.35
V_G/V_R	17A-17B	0.75-0.99	0.90	3.10	0.30	2.80	0.90	0.68	1.12	0.22	0.22	0.34
		1.00-1.24	0.90	2.80	0.40	2.40	0.88	0.70	1.06	0.18	0.18	0.36
		1.25-1.49	0.90	3.20	0.50	2.70	0.94	0.74	1.08	0.20	0.13	0.34
		1.50-1.74	0.90	2.40	0.40	2.00	0.94	0.76	1.08	0.19	0.14	0.32
		1.75-1.99	1.00	2.60	0.60	2.00	0.97	0.82	1.08	0.05	0.11	0.16
		2.00+	1.00	2.00	0.60	1.40	1.00	0.82	1.08	0.15	0.05	0.26
H_M/H_R	18A-18B	0.30-0.79	1.10	3.10	0.30	2.80	0.92	0.64	1.16	0.28	0.24	0.52
		0.80-0.99	0.90	3.20	0.40	2.80	0.94	0.73	1.13	0.21	0.19	0.40
		1.00-1.19	0.90	2.40	0.40	2.00	0.90	0.71	1.08	0.19	0.18	0.37
		1.20-1.39	0.90	2.00	0.50	1.50	0.92	0.74	1.08	0.18	0.16	0.34
		1.40-1.59	1.00	1.90	0.40	1.50	0.91	0.76	1.05	0.15	0.14	0.29
		1.60+	1.00	1.80	0.40	1.40	0.94	0.77	1.05	0.17	0.11	0.23
G_R	19A-19B	1/12-1/40	1.10	3.20	0.40	2.80	1.08	0.90	1.41	0.18	0.33	0.51
		1/40-1/100	0.90	2.40	0.30	2.10	0.92	0.74	1.08	0.18	0.16	0.34
		1/100-1/250	0.90	1.70	0.35	1.35	0.80	0.65	0.95	0.15	0.15	0.30
		1/250+	0.60	1.15	0.30	0.85	0.67	0.52	0.83	0.15	0.16	0.31

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Table IV.C. Statistical Data for V_G/V_R for all wave groups.

Group Parameter	Figure	Band	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
All Groups	20A-20B		1.10	3.55	0.50	3.05	1.20	1.00	1.52	0.20	0.32	0.52
H_M/H_R	21A-21B	0.30-0.79	1.00	1.50	0.50	1.00	0.93	0.85	1.03	0.08	0.10	0.18
		0.80-0.99	1.10	1.70	0.70	1.00	1.04	0.94	1.15	0.11	0.11	0.21
		1.00-1.19	1.30	2.10	0.90	1.20	1.24	1.10	1.40	0.14	0.16	0.30
		1.20-1.39	1.60	2.40	1.10	1.30	1.52	1.33	1.70	0.19	0.18	0.37
		1.40-1.59	1.80	2.80	1.30	1.50	1.74	1.56	2.00	0.18	0.26	0.44
		1.60+	2.40	3.55	1.40	2.15	2.12	1.74	2.42	0.38	0.30	0.63
G_R	22A-22B	1/12-1/40	1.10	5.30	0.56	2.80	1.20	1.00	1.50	0.20	0.30	0.50
		1/40-1/100	1.10	3.55	0.65	2.90	1.20	1.02	1.50	0.18	0.30	0.43
		1/100-1/250	1.10	3.25	0.75	2.50	1.22	1.02	1.55	0.20	0.51	0.51
		1/250+	1.10	2.85	0.85	2.00	1.16	1.02	1.44	0.14	0.28	0.42

Table IV.D. Statistical Data for H_M/H_R for all wave groups.

Group Parameter	Figure	Band	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
All Groups	23A-23B		1.05	2.15	0.35	1.80	1.01	0.83	1.20	0.18	0.19	0.37
G_R	24A-24B	1/12-1/40	0.95	1.75	0.35	1.40	0.95	0.79	1.14	0.16	0.19	0.35
		1/40-1/100	0.95	1.95	0.35	1.60	1.01	0.84	1.20	0.17	0.19	0.36
		1/100-1/250	1.05	2.15	0.45	1.70	1.05	0.87	1.24	0.18	0.19	0.37
		1/250+	1.05	2.05	0.55	1.50	1.05	0.90	1.27	0.15	0.22	0.37

Table IV.E. Statistical Data for D_R for all wave groups.

Group Parameter	Figure	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
All Groups	25A-25B	575.0	760.0	290.0	470.0	955.0	485.0	615.0	70.0	60.0	130.0

Table V.A. Statistical data for extreme wave groups: largest N_G per wave record.

Group Parameter	Figure	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
N_G	26A-26B	9	28	5	23	9.8	8.1	12.4	1.7	2.6	4.3
T_G/T_R	27A-27B	1.00	2.10	0.50	1.60	0.88	0.74	1.14	0.14	0.26	0.40
V_G/V_R	28A-28B	1.60	2.60	1.20	1.40	1.57	1.36	1.80	0.21	0.23	0.44
H_G/H_R	29A-29B	1.75	2.15	0.95	1.20	1.23	1.14	1.42	0.14	0.14	0.28

Table V.B. Statistical data for extreme wave groups: largest V_G/V_R per wave record.

Group Parameter	Figure	Peak Value	Maximum Value	Minimum Value	Total Range	50%	20%	80%	50-20%	80-50%	Std.Dev. Range
N_G	30A-30B	7	28	3	25	6.3	4.2	9.0	2.1	2.7	4.8
T_G/T_R	31A-31B	1.00	2.60	0.50	2.10	0.92	0.76	1.07	0.16	0.15	0.31
V_G/V_R	32A-32B	1.90	3.55	1.40	2.15	1.90	1.73	2.18	0.17	0.28	0.45
H_G/H_R	33A-33B	1.55	1.95	1.05	0.90	1.42	1.25	1.56	0.17	0.14	0.31
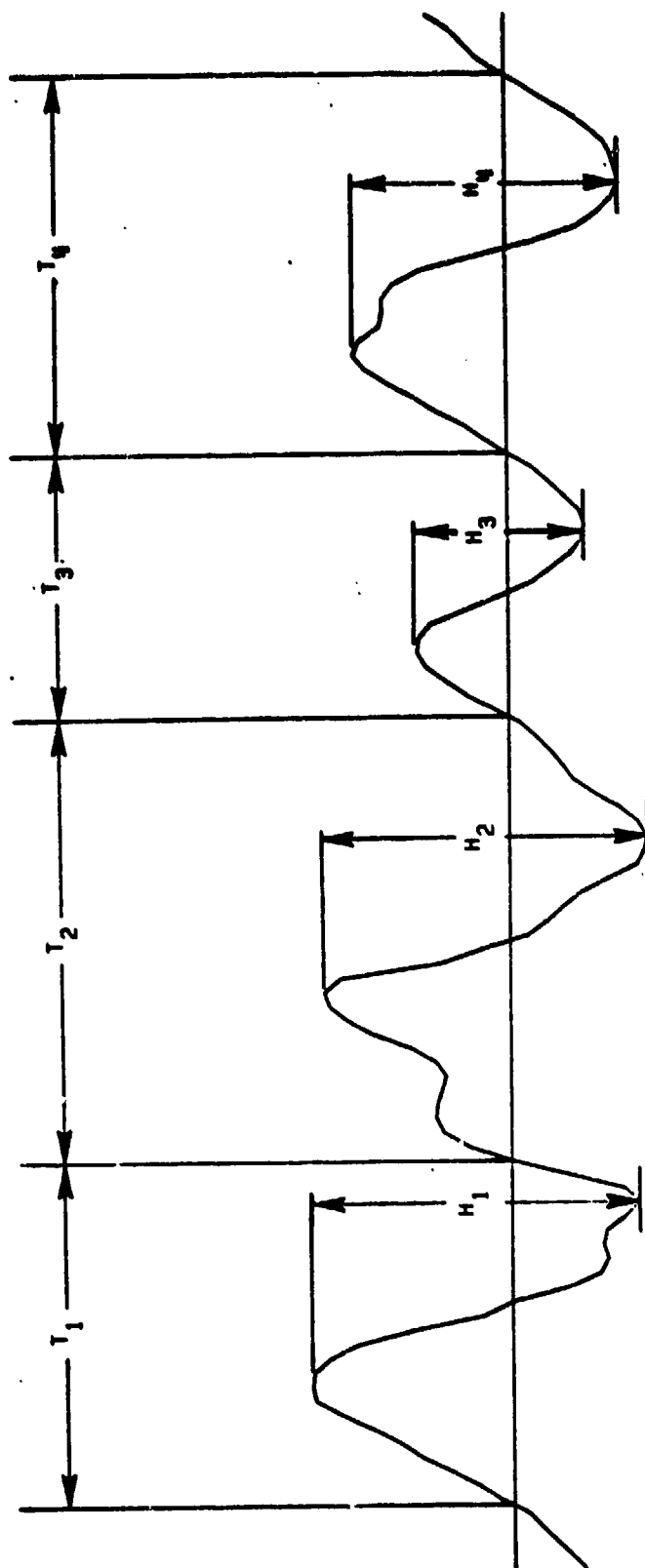
Figure 1. Illustration of zero upcrossing method definitions for wave height and period.
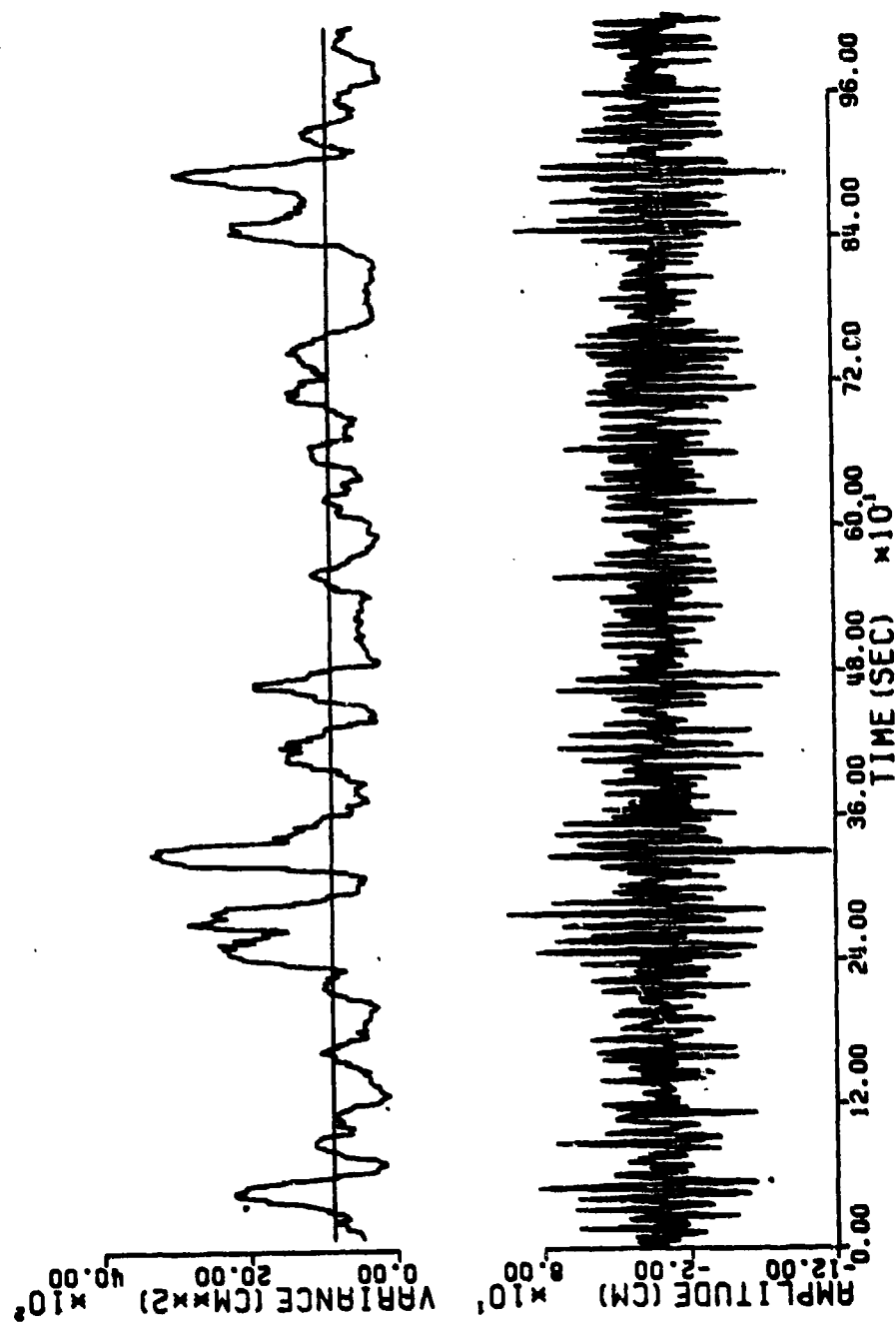
Figure 2. Wave record and group waveform analysis for wave record number 143.
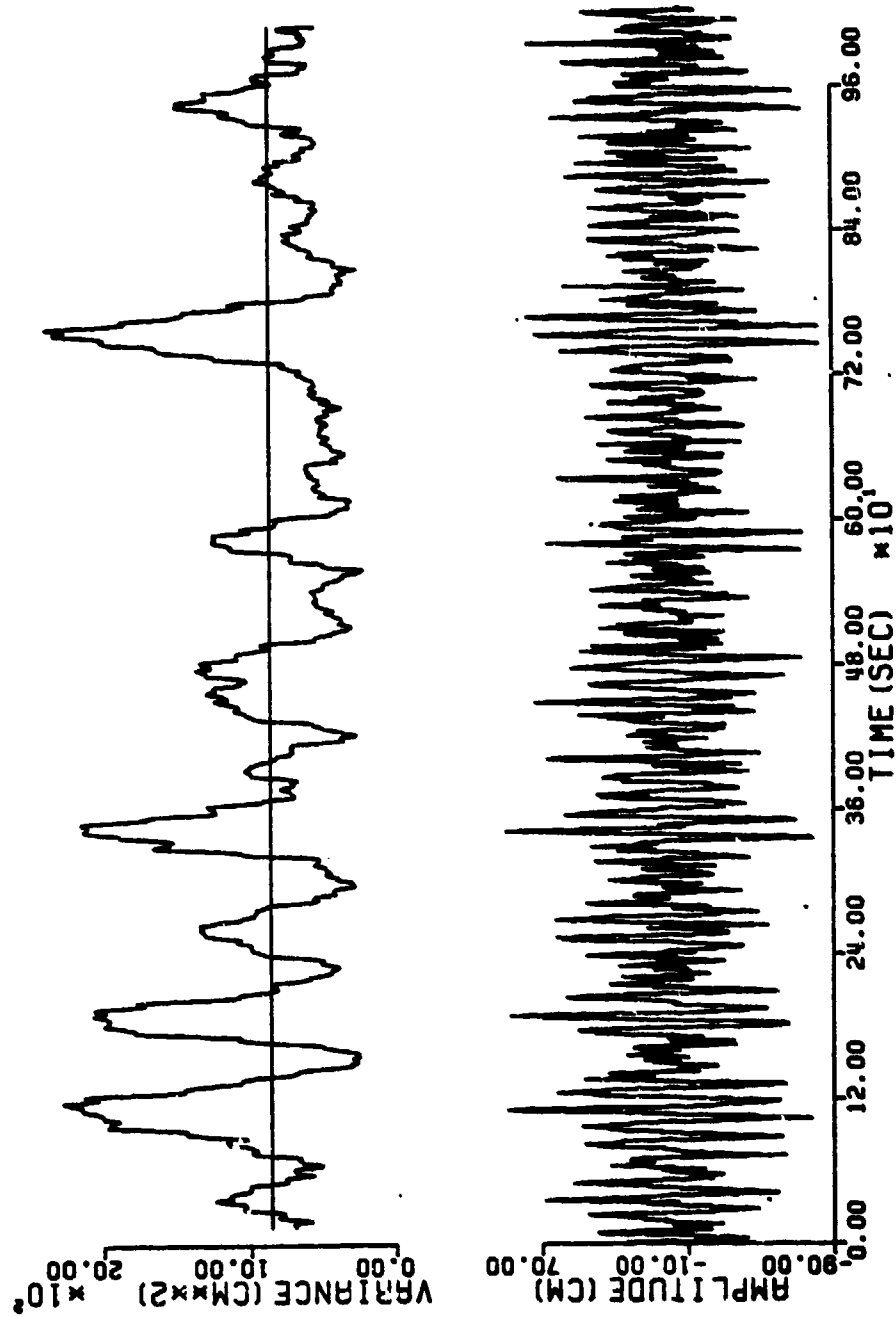
Figure 3.   Wave record and group waveform analysis for wave record number 285.

SHORT-TERM VARIANCE RECORD

VARIANCE

TIME

WAVE GROUP BOUNDARIES DETERMINED
BY VARIANCE RATIO ALONE

WAVE GROUP BOUNDARIES DETERMINED
BY ALL CRITERIA

AMPLITUDE

TIME

WAVE RECORD
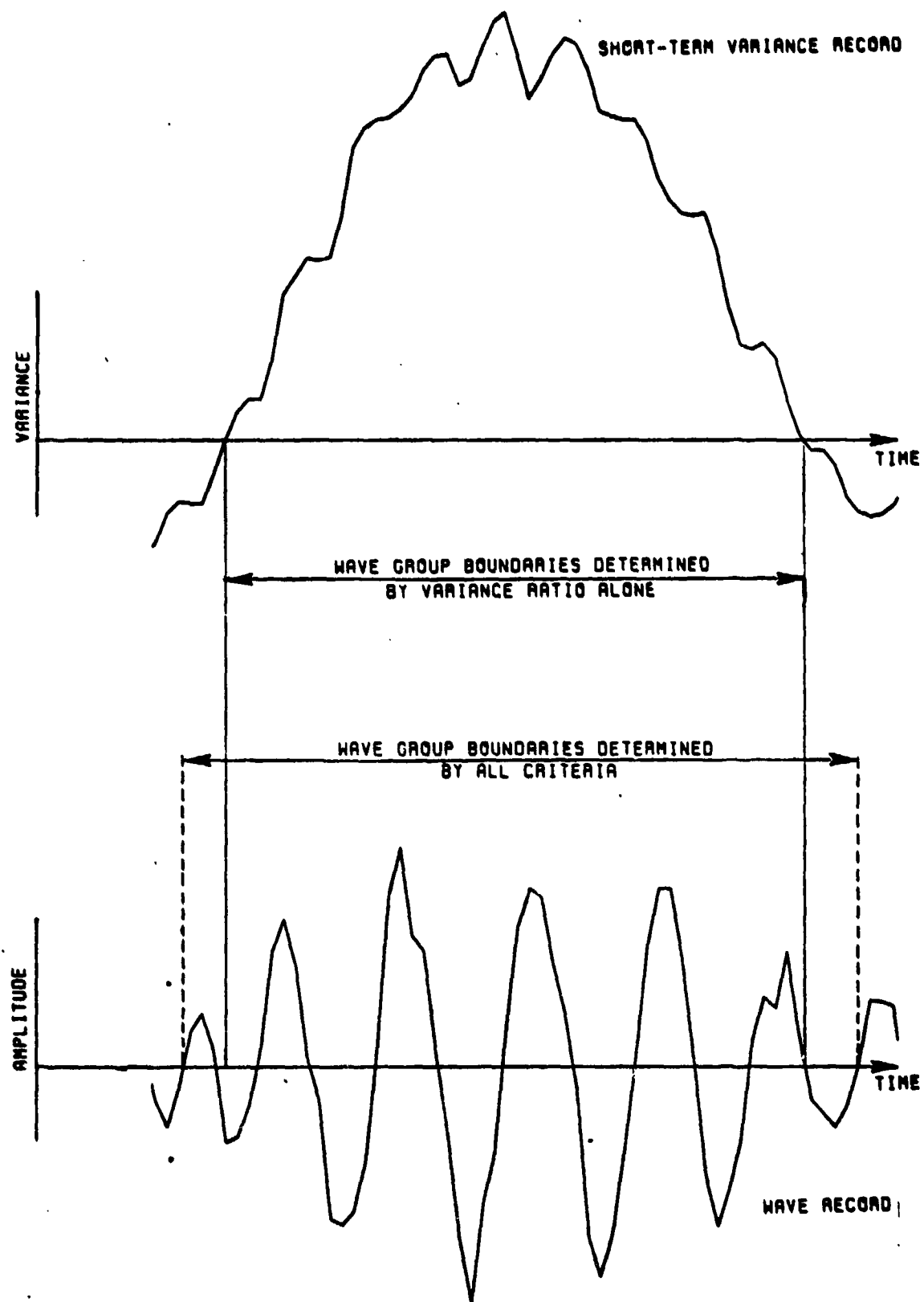
Figure 4.   Illustration of wave group boundary definition.
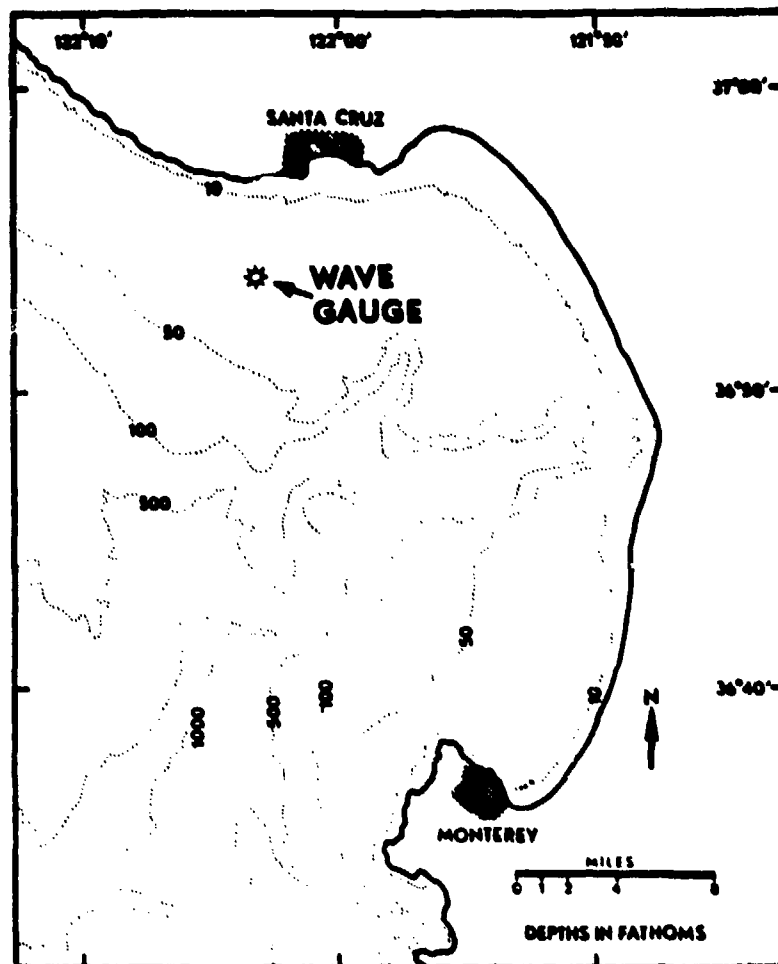
Figure 5.   Location of Monterey Bay Waverider
Buoy.

30 —

20 —
D
A
T
E

10 —

1 —

SANTA CRUZ WAVERIDER

T (sec)   20   16   12   8   4
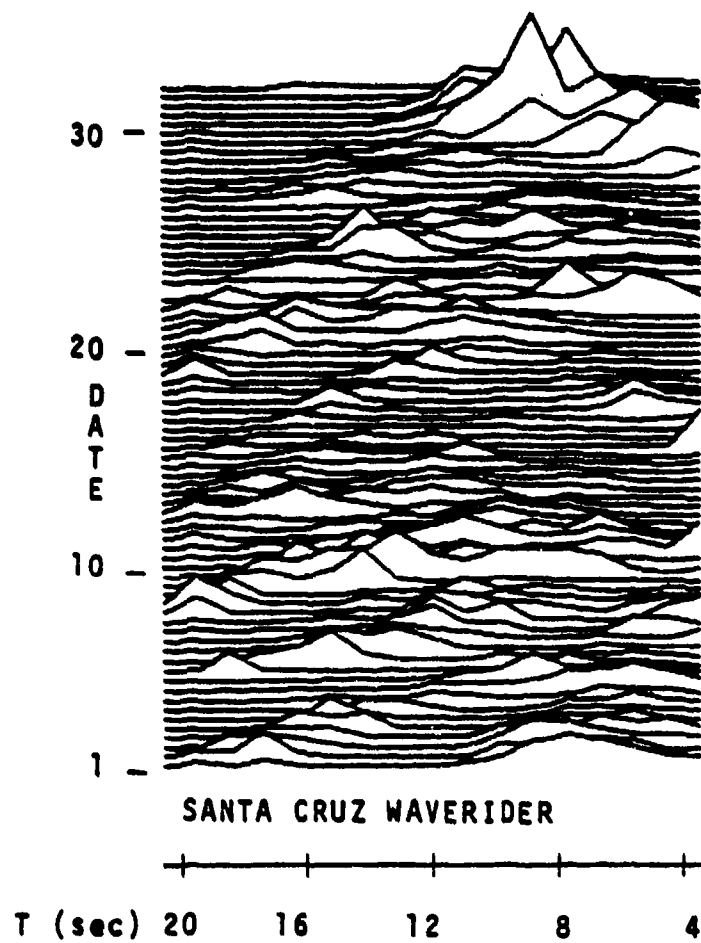
Figure 6.   Example of CEDN graphical spectra
presentation.

SANTA CRUZ PIER WAVERIDER
OCT. 1978

PERCENT ENERGY IN BAND
(TOTAL ENERGY INCLUDES RANGE 2048-4 SECS)

LOCAL DAY/TIME	SIG.HT (CM)	TOT.EN (CM.SQ)	22+	22-18	18-16	16-14	14-12	12-10	10-8	8-6	6-4
1 0640	138.9	1078.2	0.2	3.7	4.7	0.8	1.4	10.0	30.8	28.0	12.9
1 1640	123.3	990.1	0.2	9.3	21.3	0.4	0.9	11.7	19.2	25.6	11.5
2 0240	130.7	1281.3	0.4	0.6	13.6	2.3	1.5	10.7	30.8	17.6	14.4
2 1240	130.5	1195.9	0.4	2.3	13.8	7.9	2.1	9.7	20.5	24.0	19.3
2 2240	126.9	1006.9	0.2	0.6	14.2	20.0	2.5	2.0	20.4	24.0	14.7
3 0042	113.6	806.6	0.2	1.3	0.0	25.9	4.3	2.0	10.3	25.1	17.1
3 1040	110.0	757.0	0.4	1.9	2.7	12.9	14.6	2.5	16.1	30.6	18.3
4 0440	110.0	767.0	0.9	0.9	4.9	10.2	13.7	9.7	15.6	26.1	18.4
4 1736	133.8	1104.9	0.1	1.2	2.9	2.8	7.6	11.9	21.3	33.2	19.3
5 0333	123.1	947.3	0.1	0.7	2.7	5.2	5.4	10.3	31.1	28.1	16.4
5 1347	112.8	784.5	0.2	21.7	2.2	3.6	5.3	7.4	25.9	21.2	12.4
6 0002	122.0	938.5	0.1	1.8	20.2	29.6	3.6	0.7	11.7	15.4	0.0
6 1207	112.9	796.1	0.2	1.4	7.9	33.6	29.9	4.9	9.0	9.6	6.7
6 2207	114.3	816.7	1.1	0.5	5.5	26.4	24.3	11.5	6.3	9.7	14.3
7 0007	110.7	801.3	1.0	1.2	4.0	6.9	35.0	14.6	2.8	15.0	16.2
7 1007	126.9	1086.9	0.6	1.3	3.2	7.4	23.1	21.9	5.0	5.3	31.0
8 0407	125.0	977.2	0.2	11.4	1.5	6.2	10.5	21.2	12.6	10.3	10.1
8 1407	122.0	943.0	0.3	20.9	0.9	0.1	9.5	17.0	11.3	9.7	14.3
9 0007	111.3	773.0	0.3	20.6	1.3	6.3	11.5	22.1	16.2	9.1	12.6
9 1157	129.9	1054.3	0.4	9.6	15.3	36.0	6.0	10.3	7.9	6.9	6.9
9 2150	105.6	2153.1	0.2	1.6	11.7	10.6	22.1	14.3	10.2	9.2	4.0
10 0756	131.0	1425.3	0.3	1.5	14.7	15.3	19.0	20.2	12.7	0.0	0.3
10 1640	152.3	1450.0	0.3	1.3	3.0	15.2	14.2	11.7	13.3	17.5	22.6
11 0240	161.2	1624.6	0.7	0.9	4.6	13.3	17.9	23.3	13.3	16.0	9.2
11 1240	135.5	1146.7	1.3	3.3	3.1	5.7	15.3	17.3	19.4	24.1	10.0
11 2240	130.6	1066.5	1.0	4.0	2.0	12.0	9.5	27.4	10.4	14.7	9.5
12 0040	123.6	954.0	1.3	6.3	5.0	7.0	10.4	20.0	22.7	10.0	0.6
12 1040	139.2	1210.0	0.5	11.1	24.0	12.6	0.3	13.6	14.0	0.2	6.0
13 0440	122.3	935.3	0.5	0.0	7.1	19.7	21.5	10.1	9.3	0.3	7.4
13 1514	122.3	935.2	0.4	9.2	12.1	0.1	23.9	19.9	12.7	0.2	5.6
14 0113	121.5	922.6	0.6	6.3	15.4	7.2	24.1	15.6	13.3	10.0	7.4
14 1113	97.5	594.6	0.4	5.0	19.0	7.5	10.7	16.6	20.9	9.6	10.3

Figure 7. Example of CEDN tabular spectra presentation.
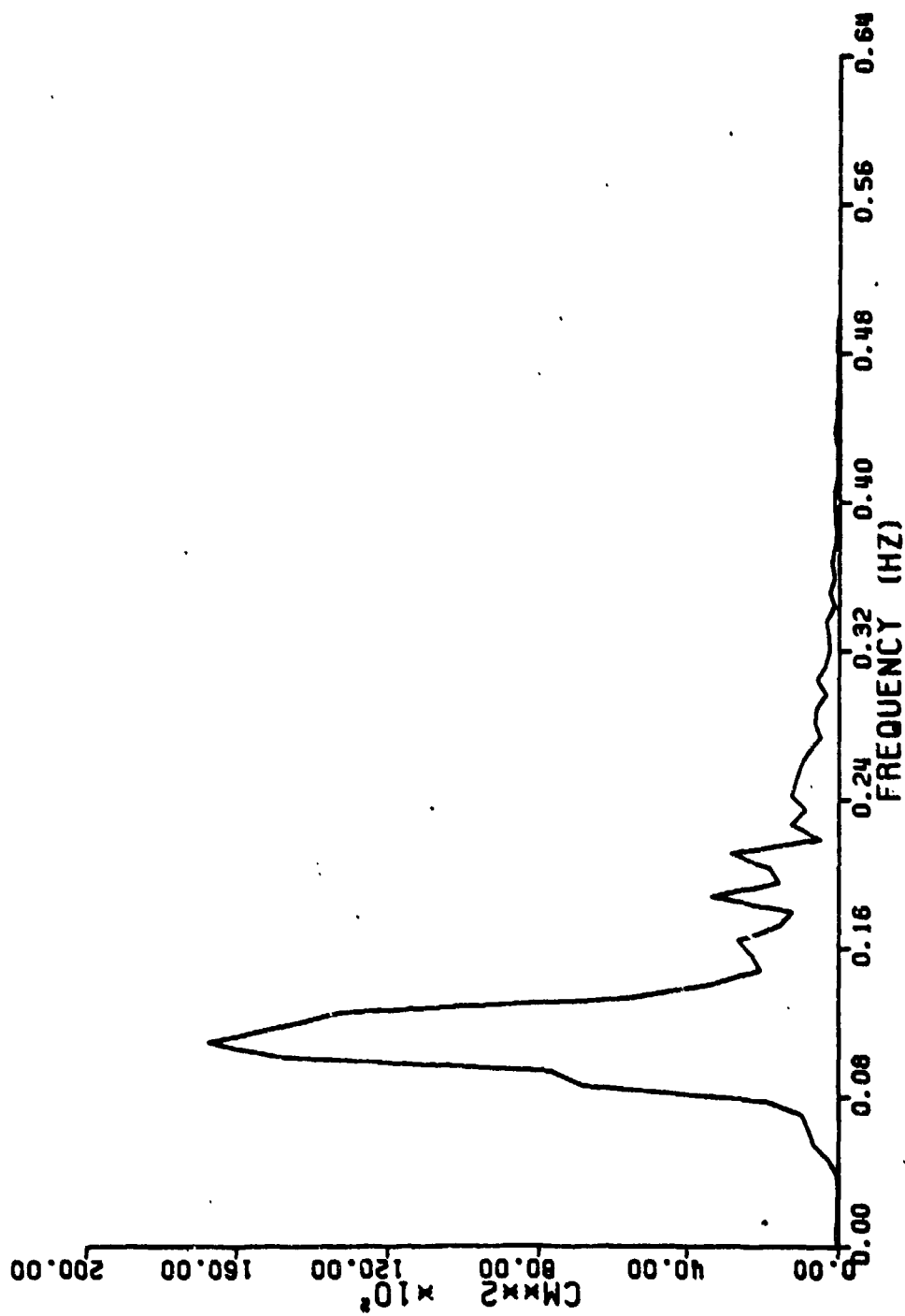
Figure 8. Spectral analysis of wave record number 143.
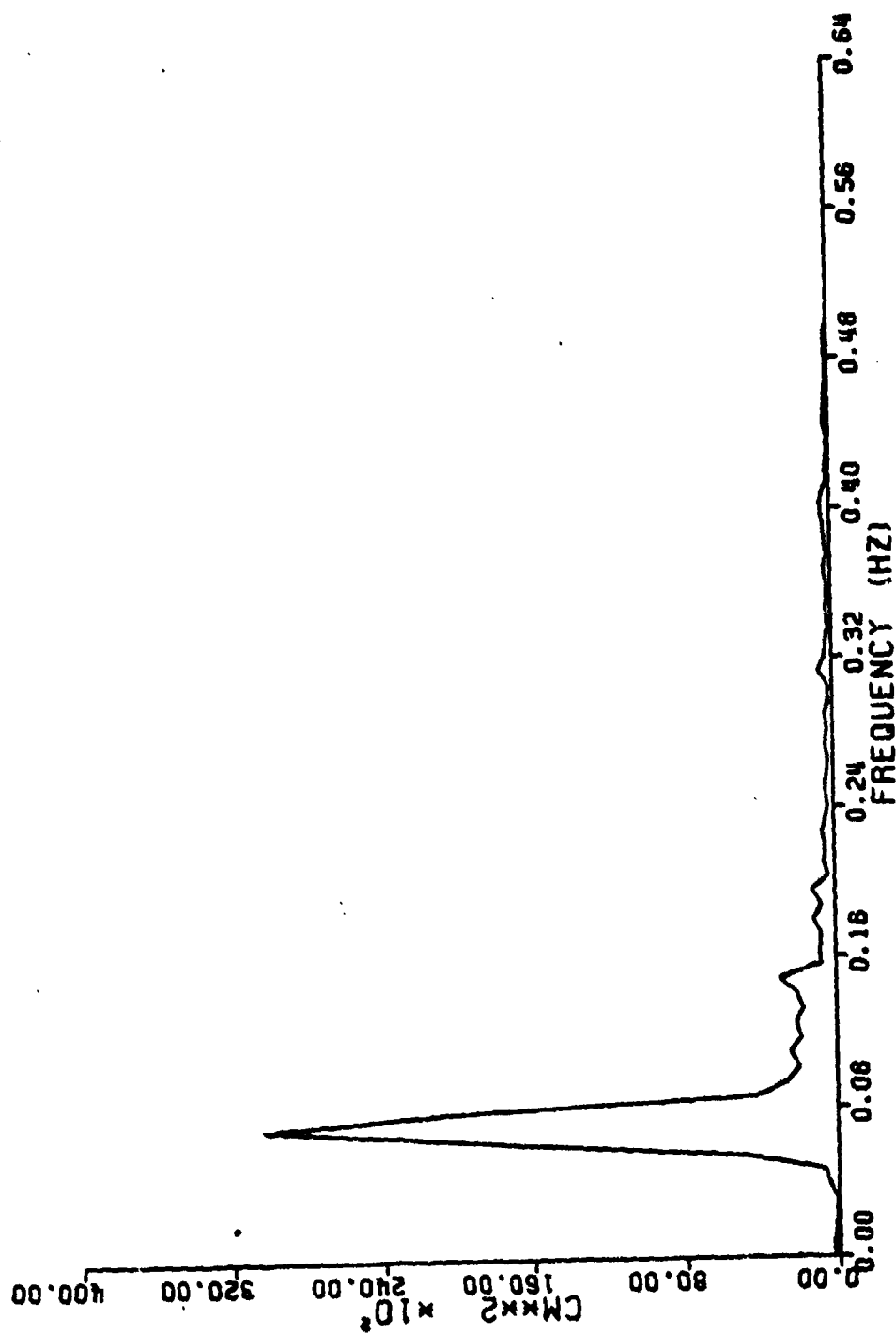
Figure 9. Spectral analysis of wave record number 285.

Figure 10. Height, period, and steepness distribution of wave records sampled.
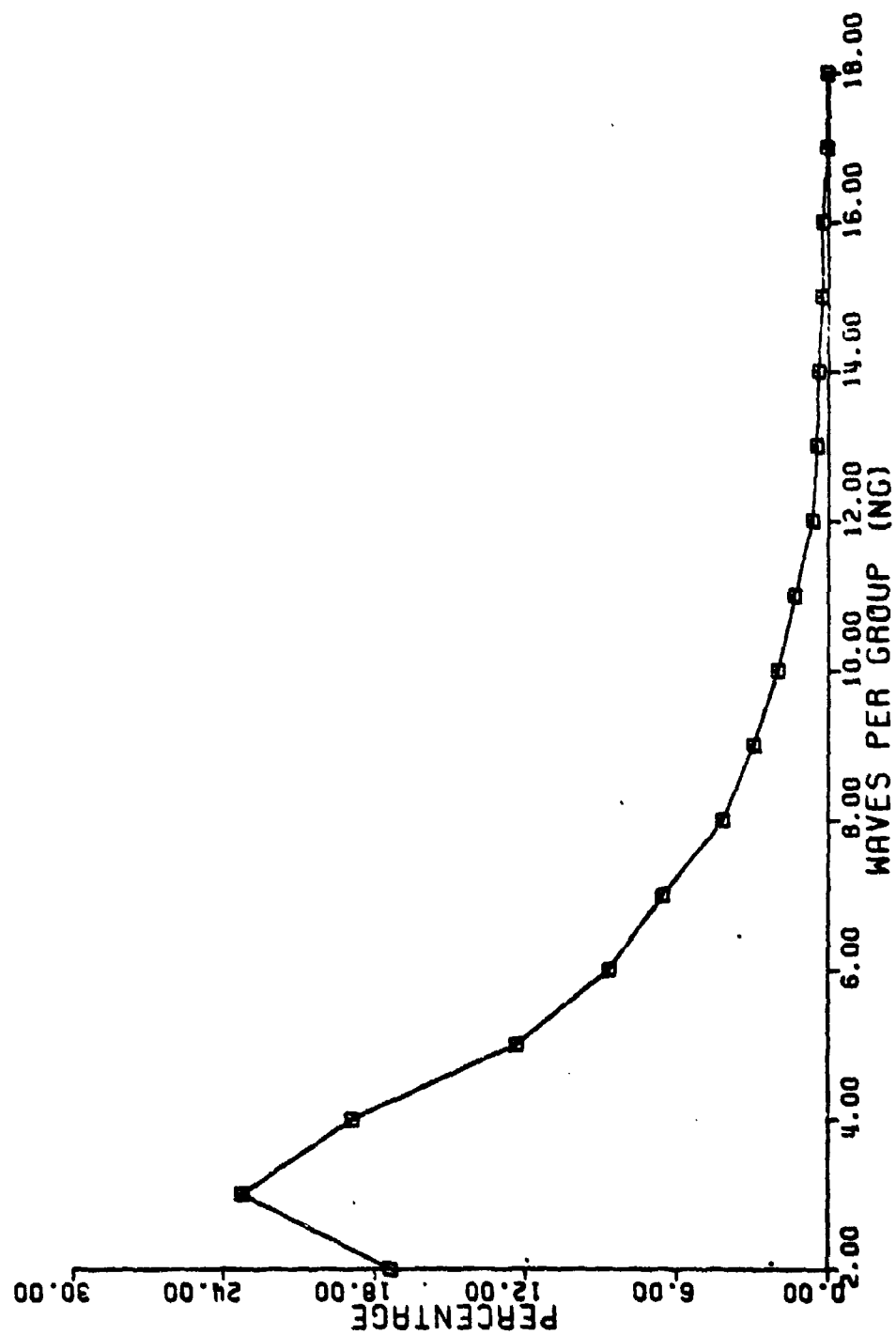
Figure IIA. Histogram of N_G for all wave groups.
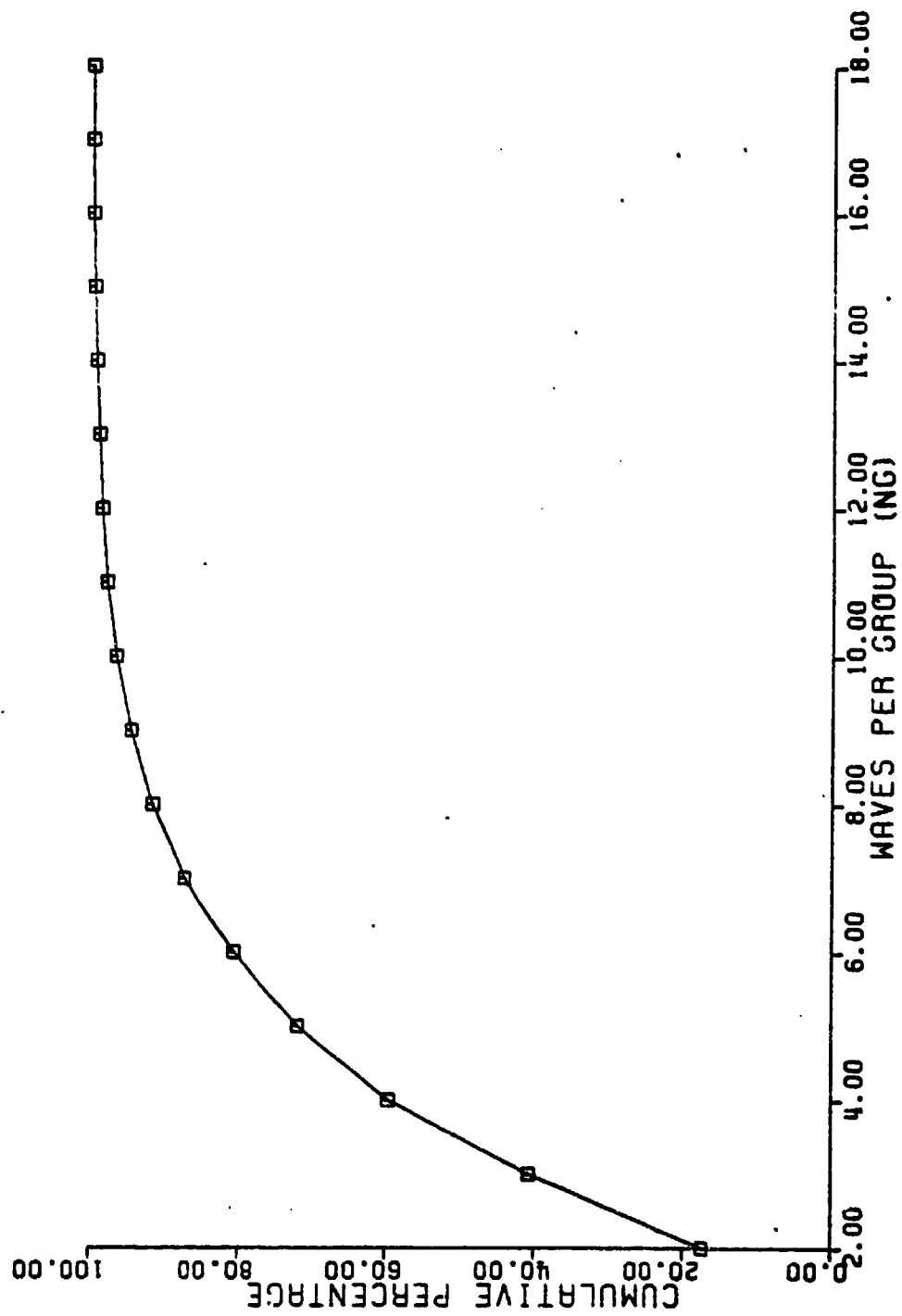
Figure IIB. Cumulative percentage distribution of N_G for all wave groups.
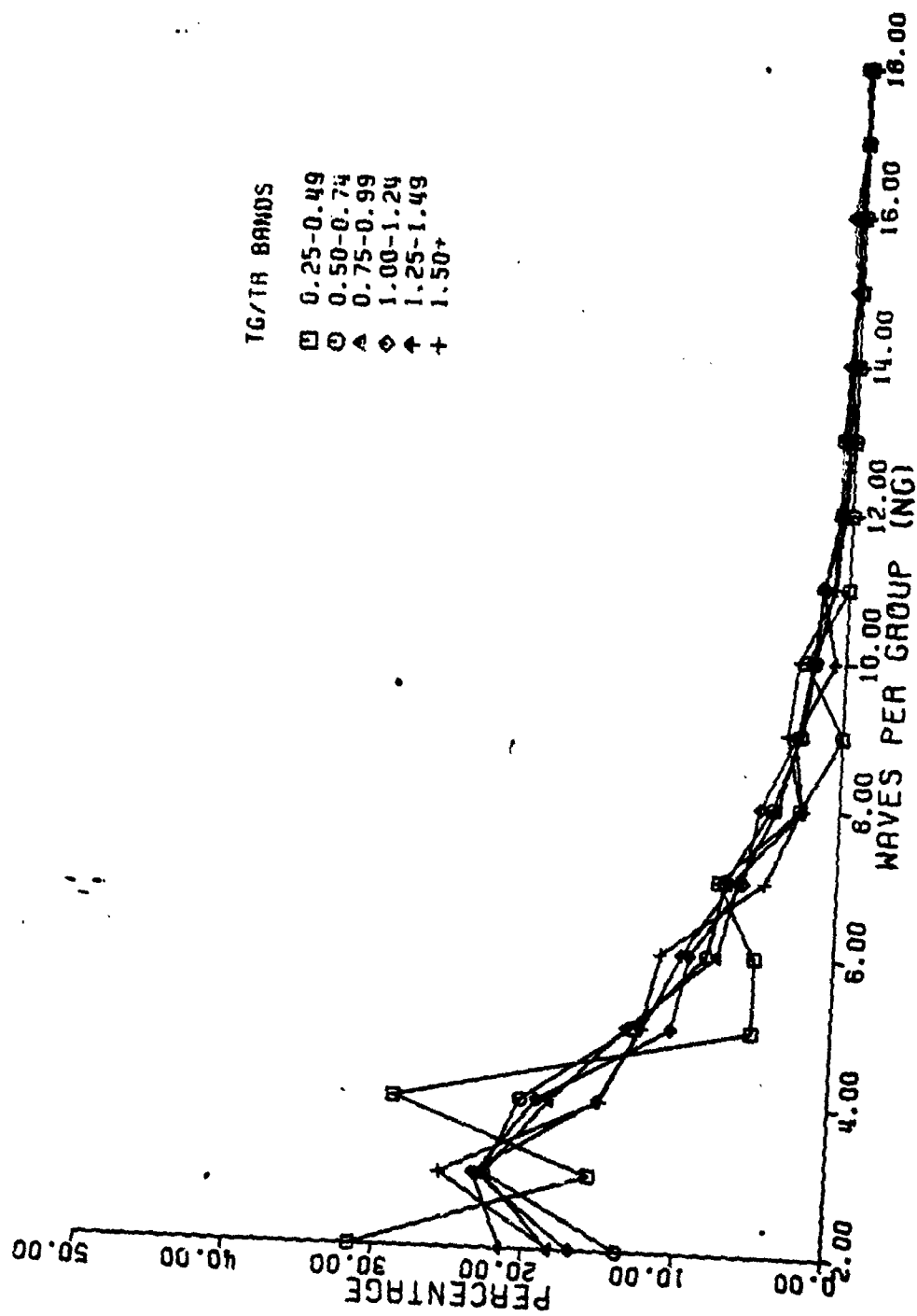
Figure 12A. Histogram of N_G for T_G/T_R wave group bands.
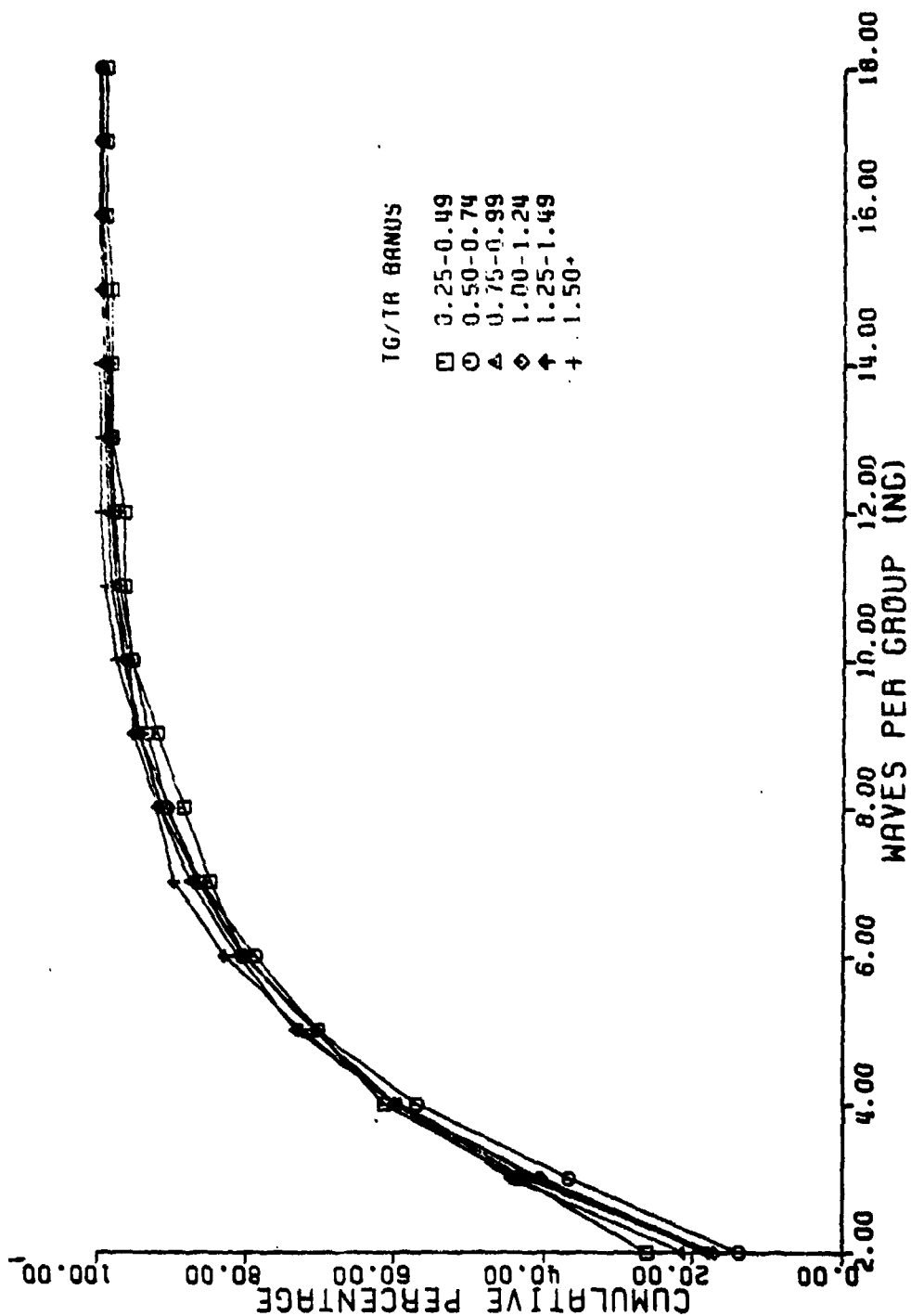
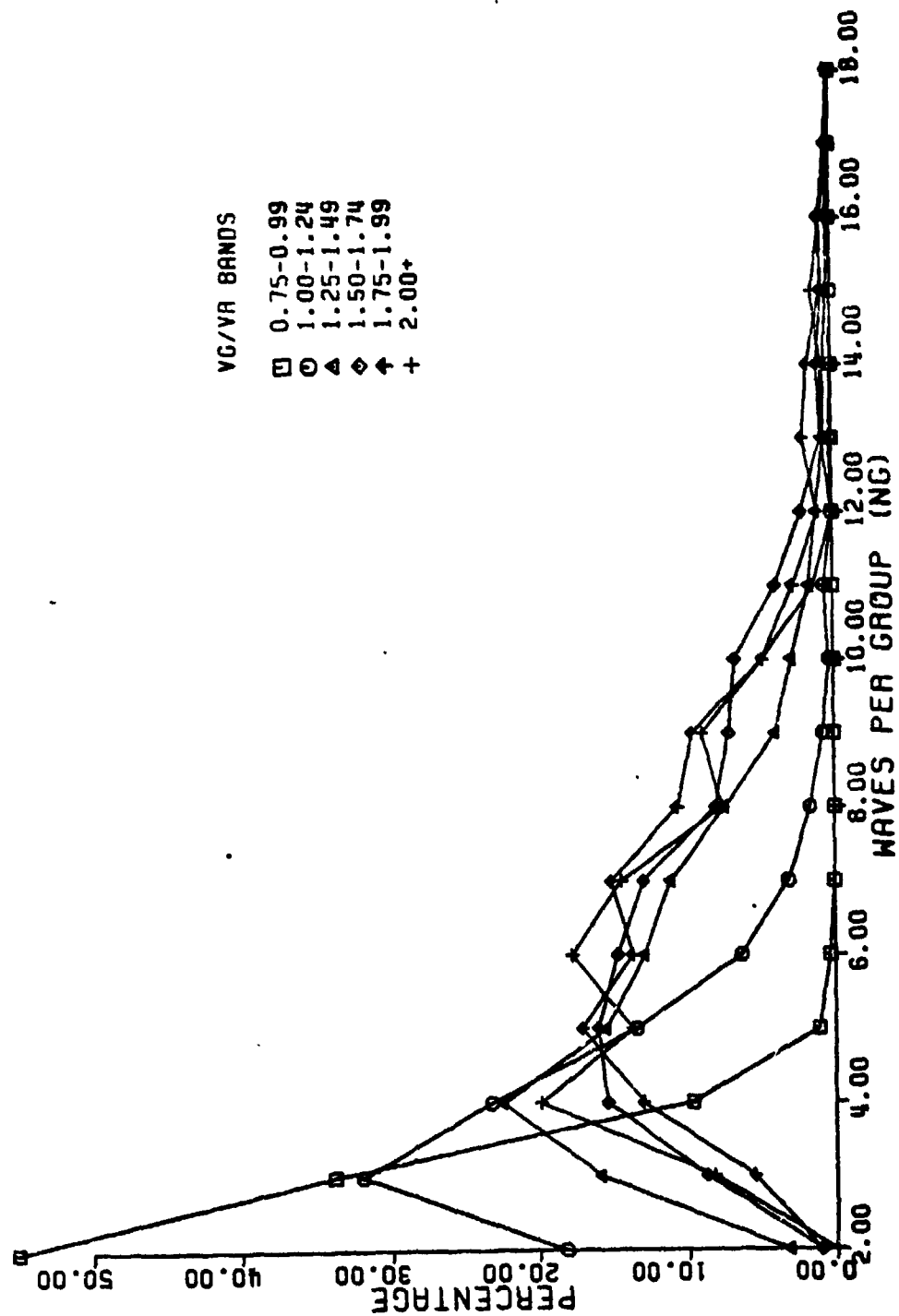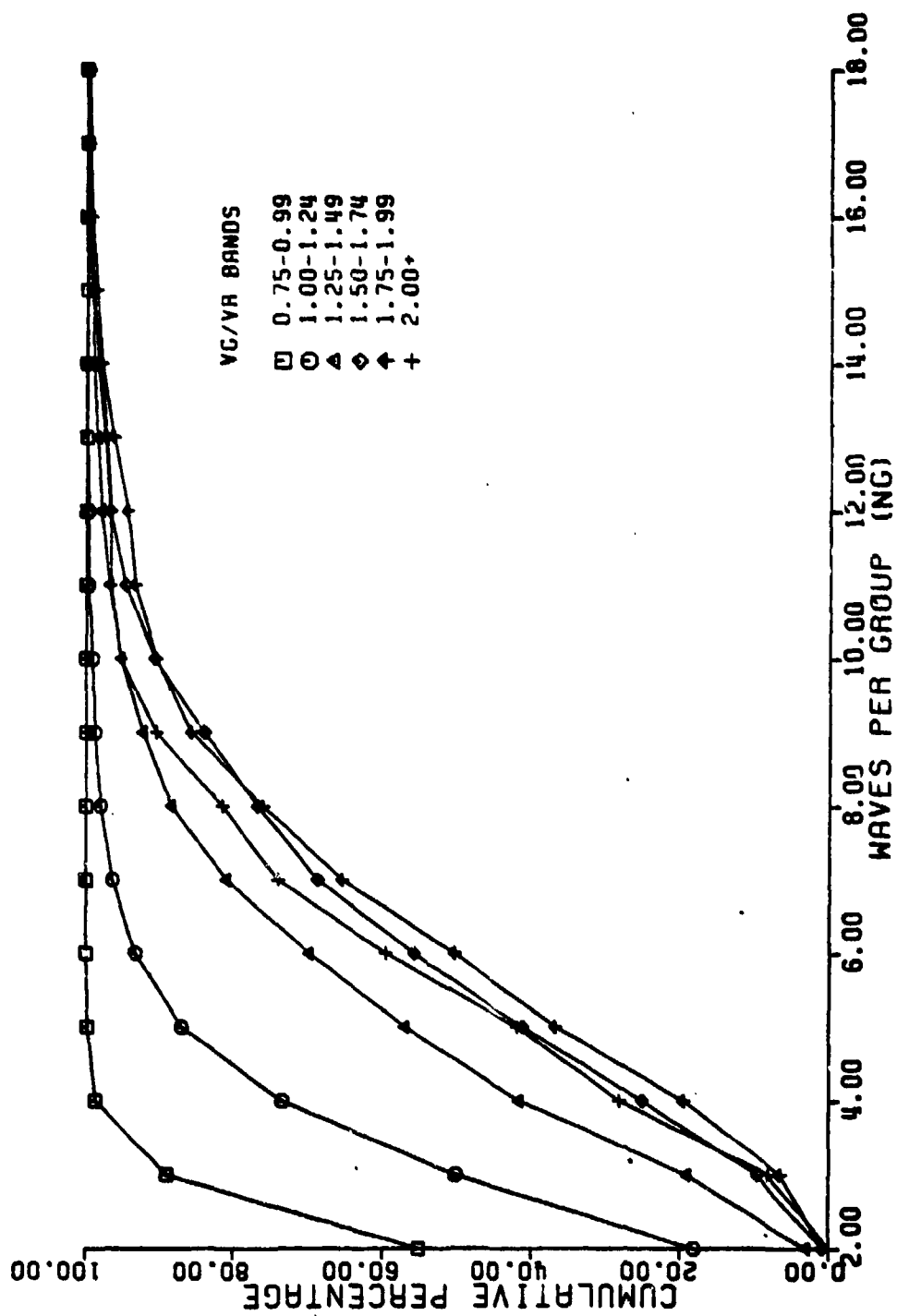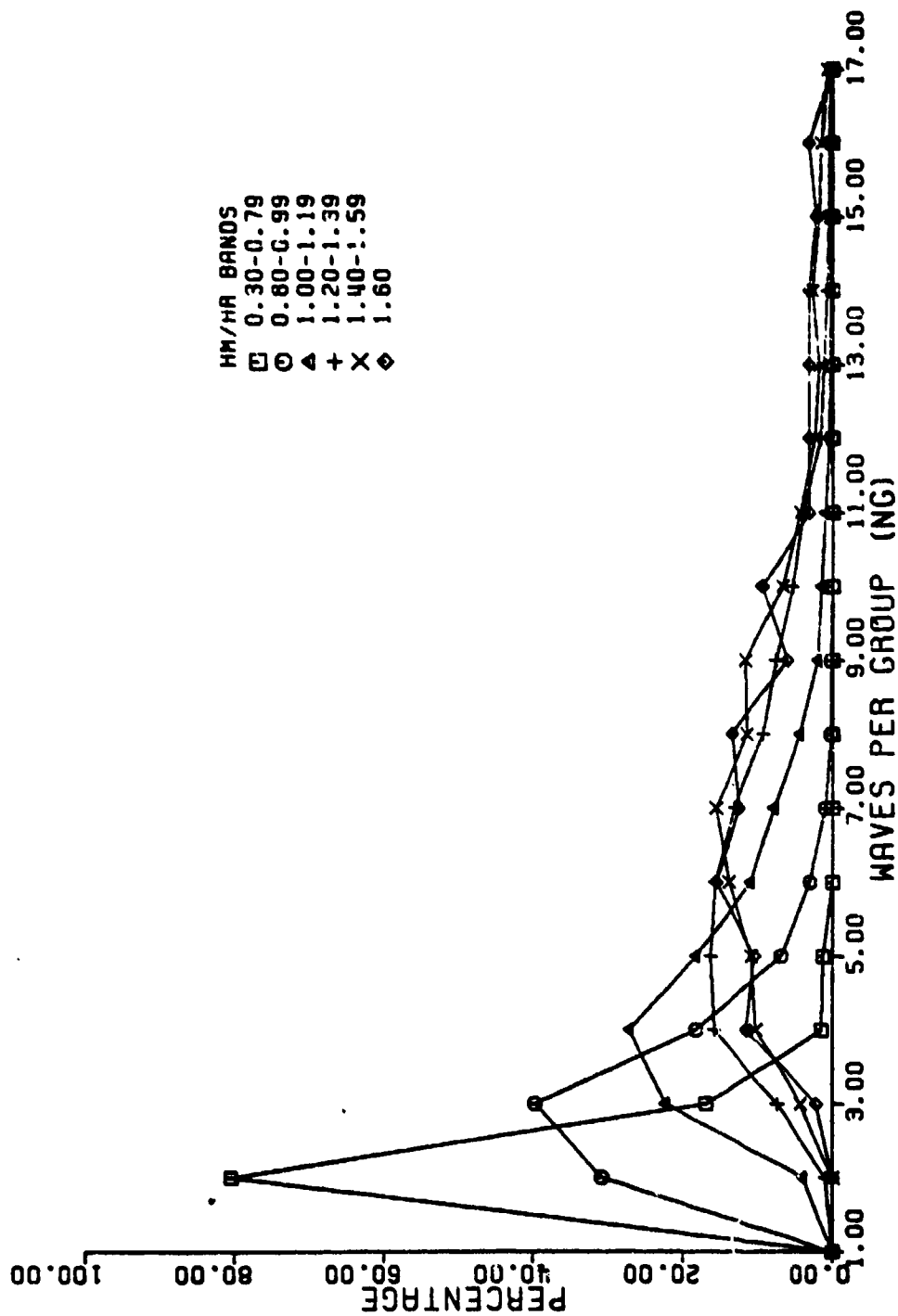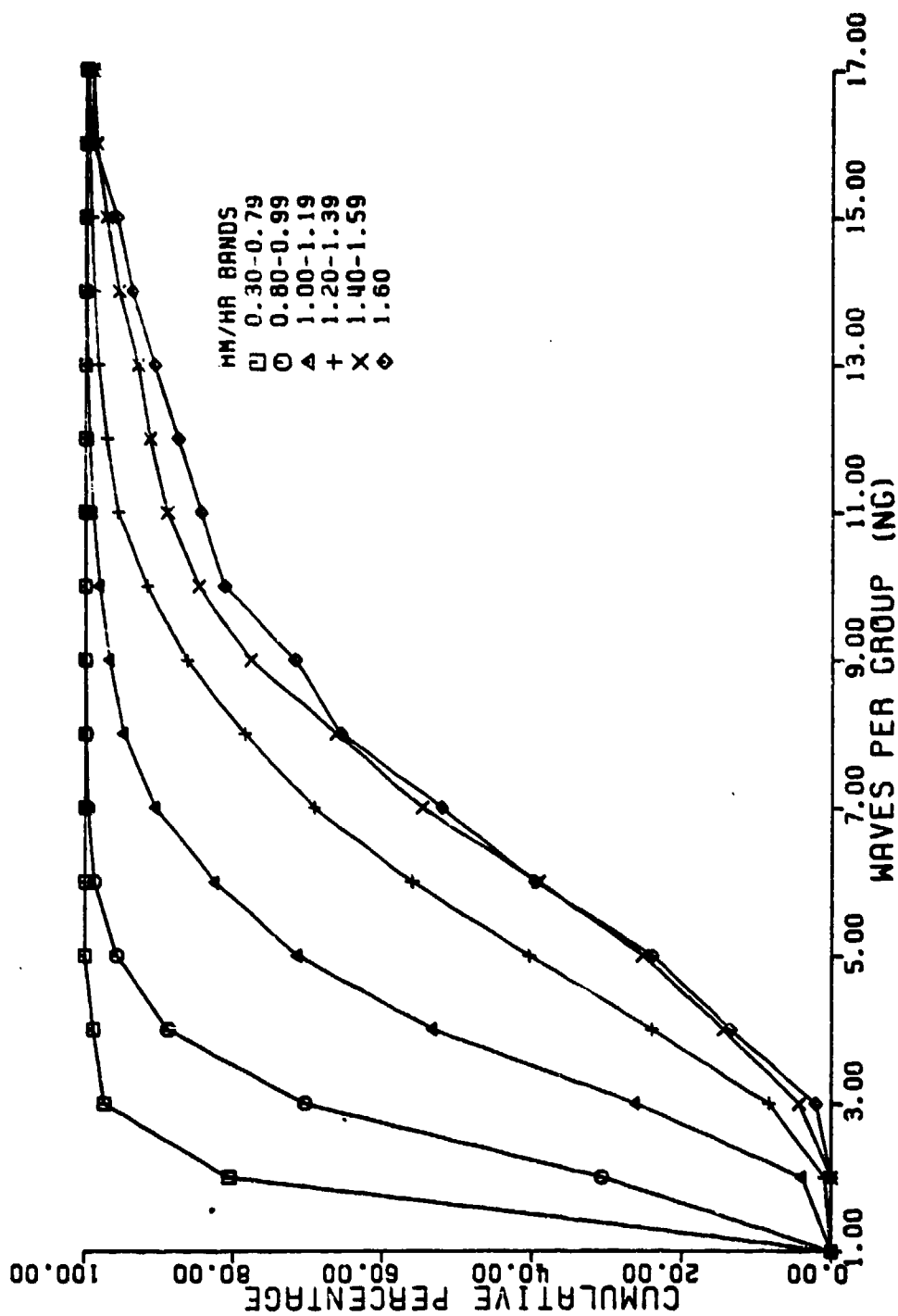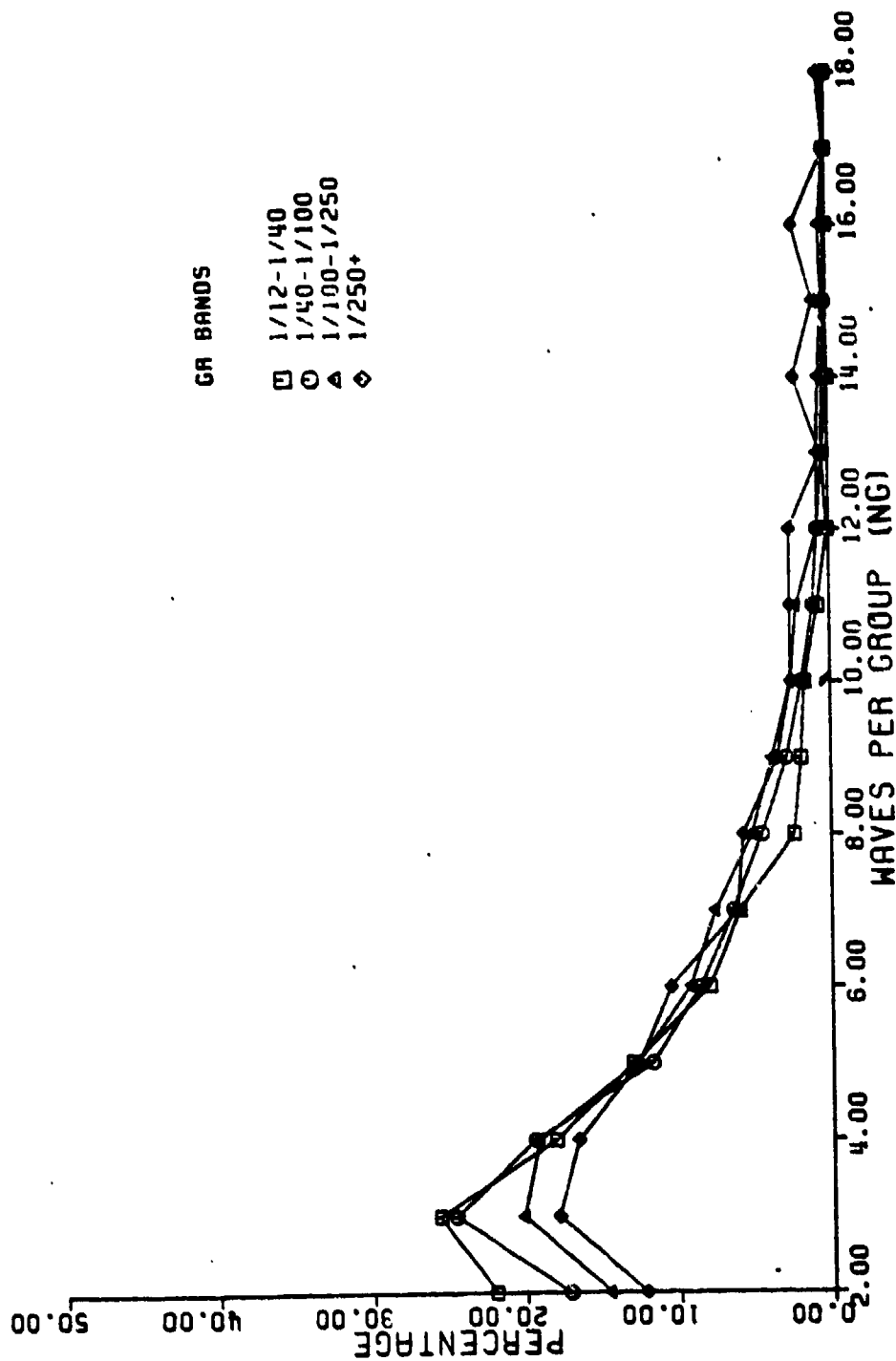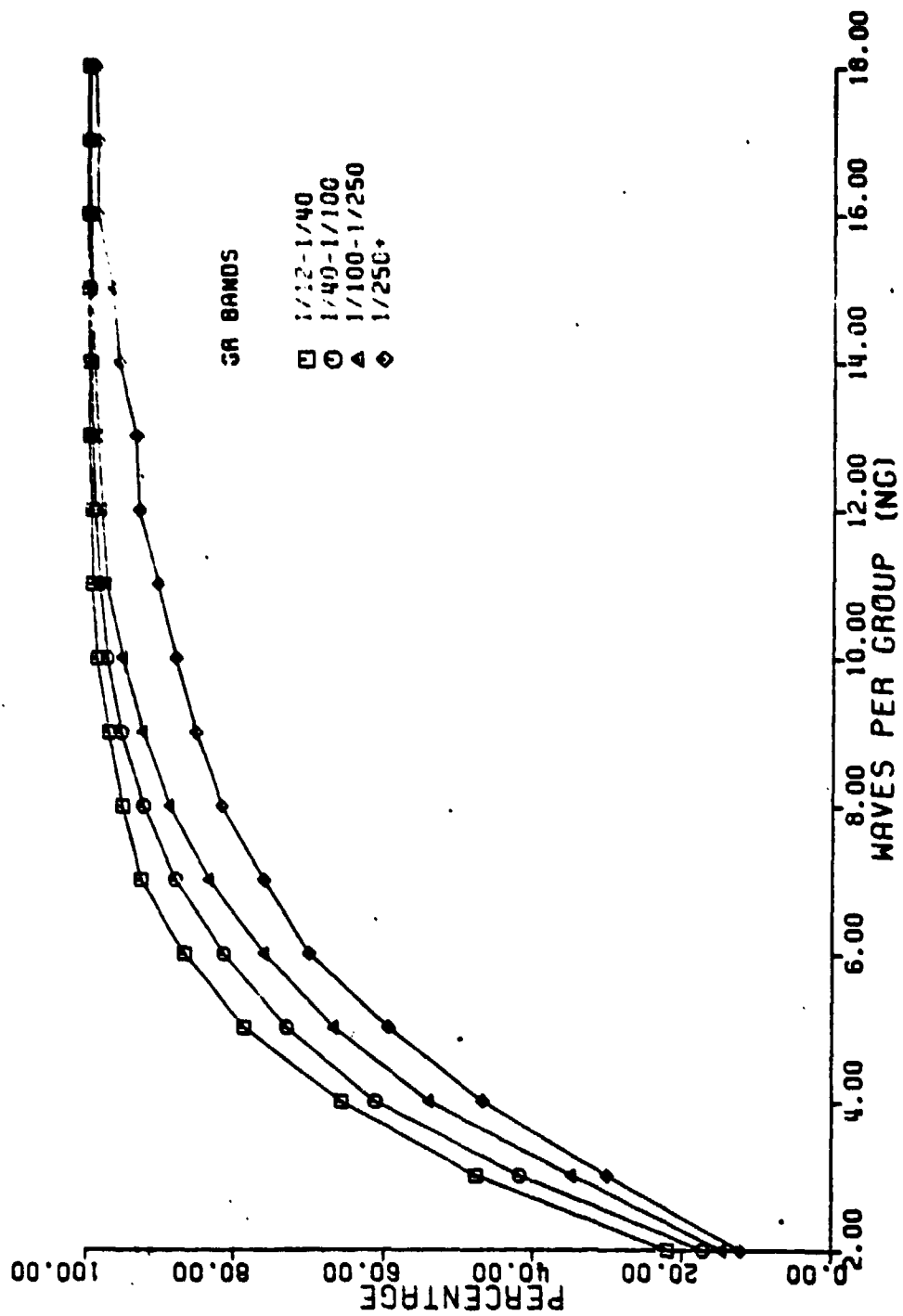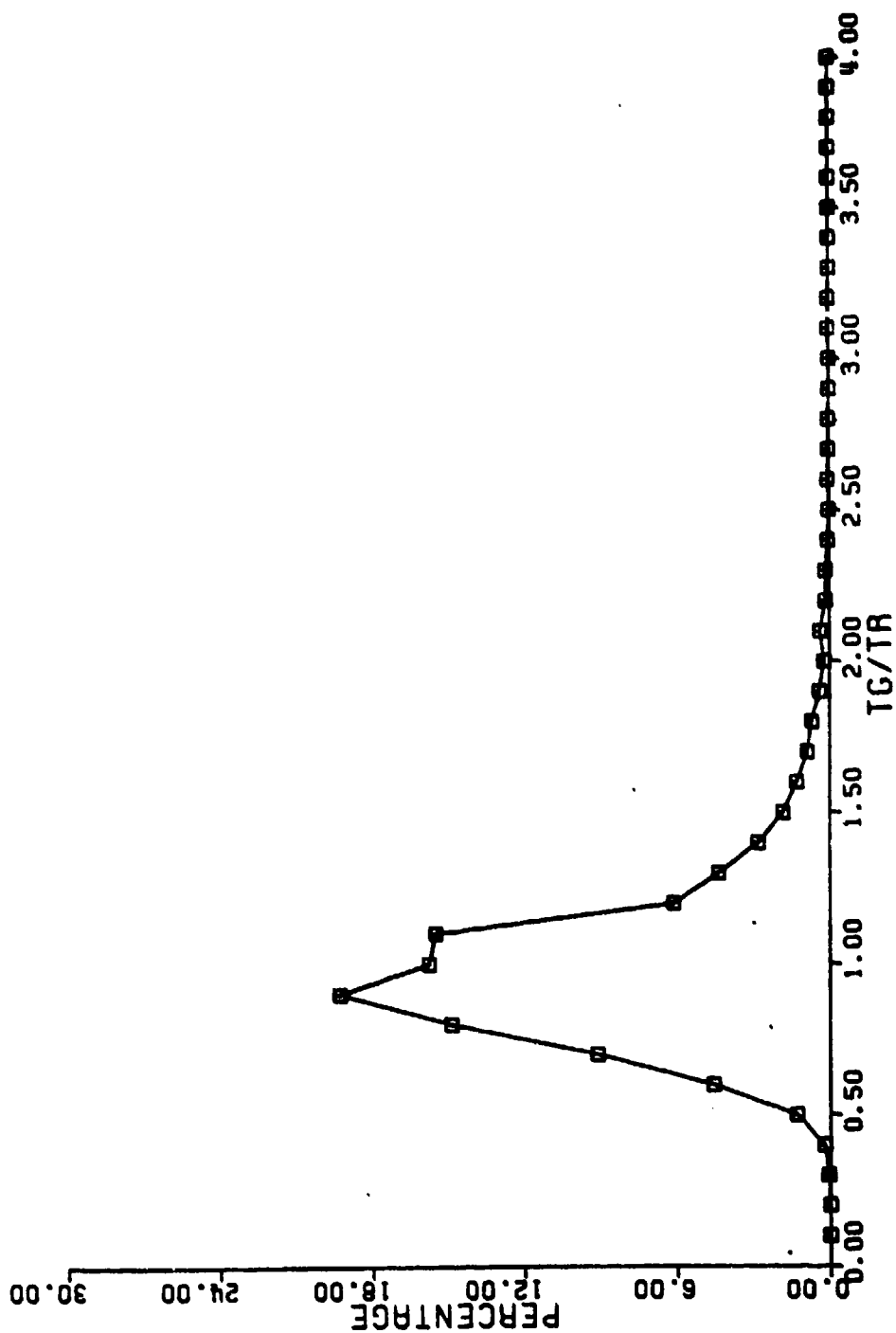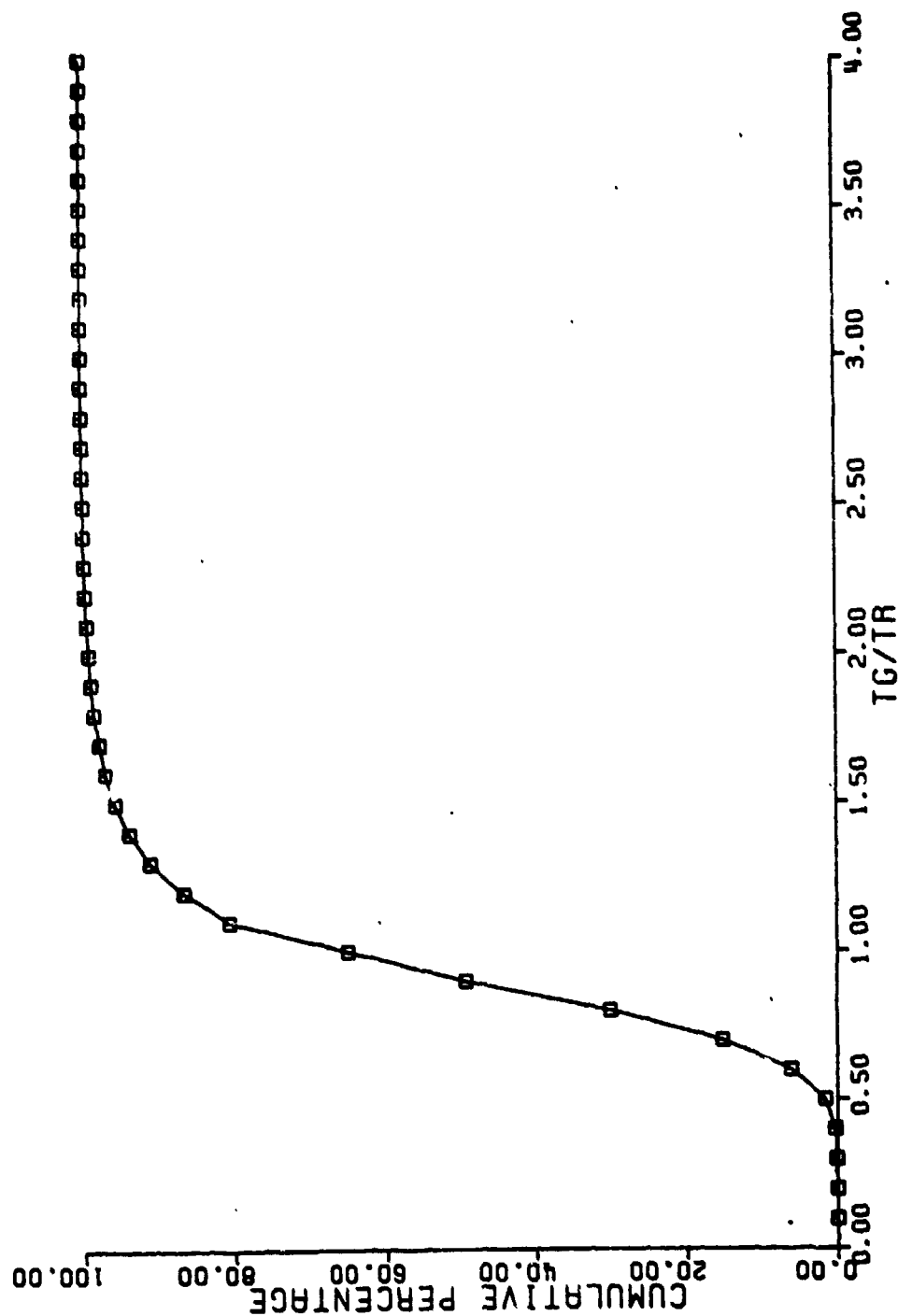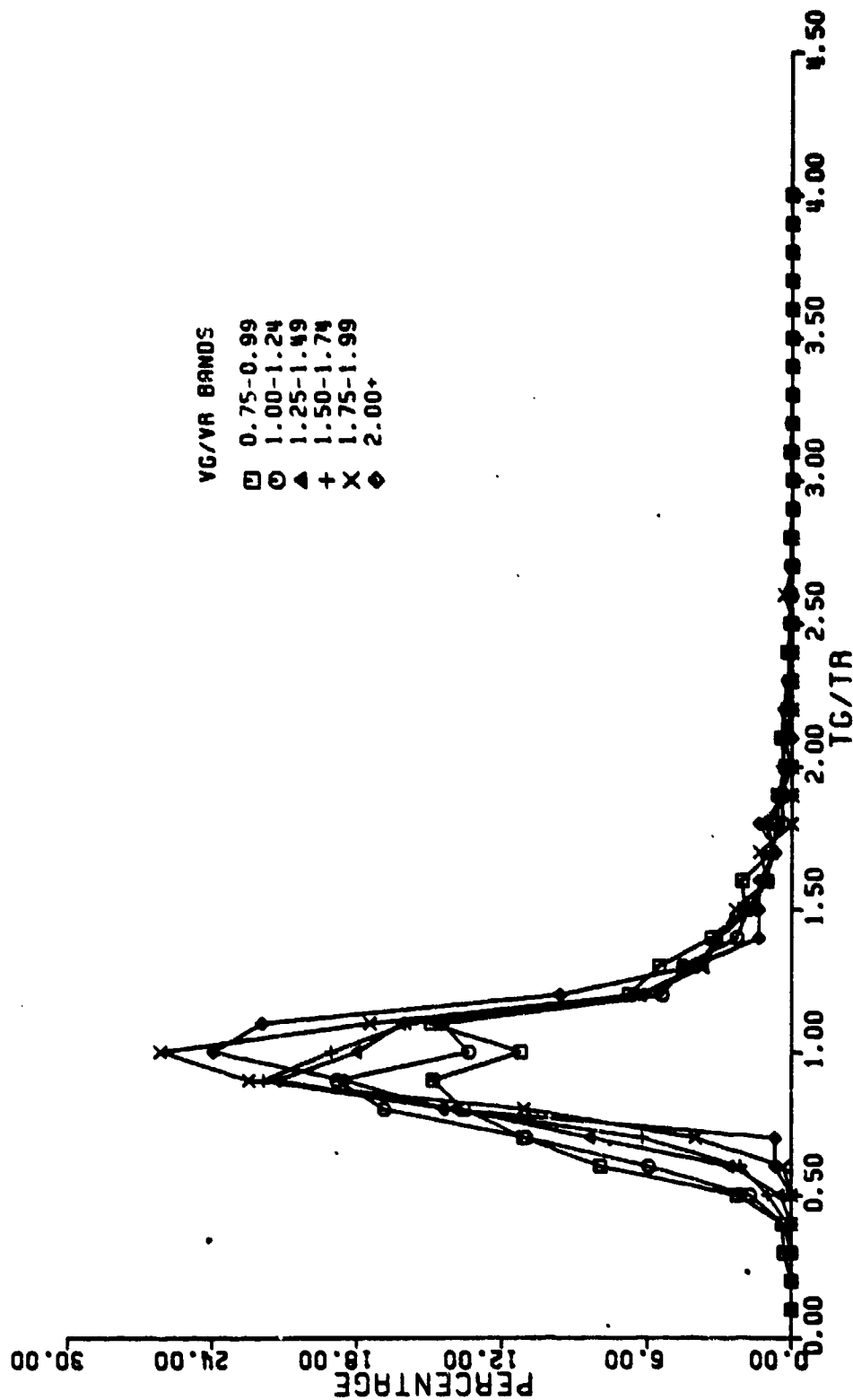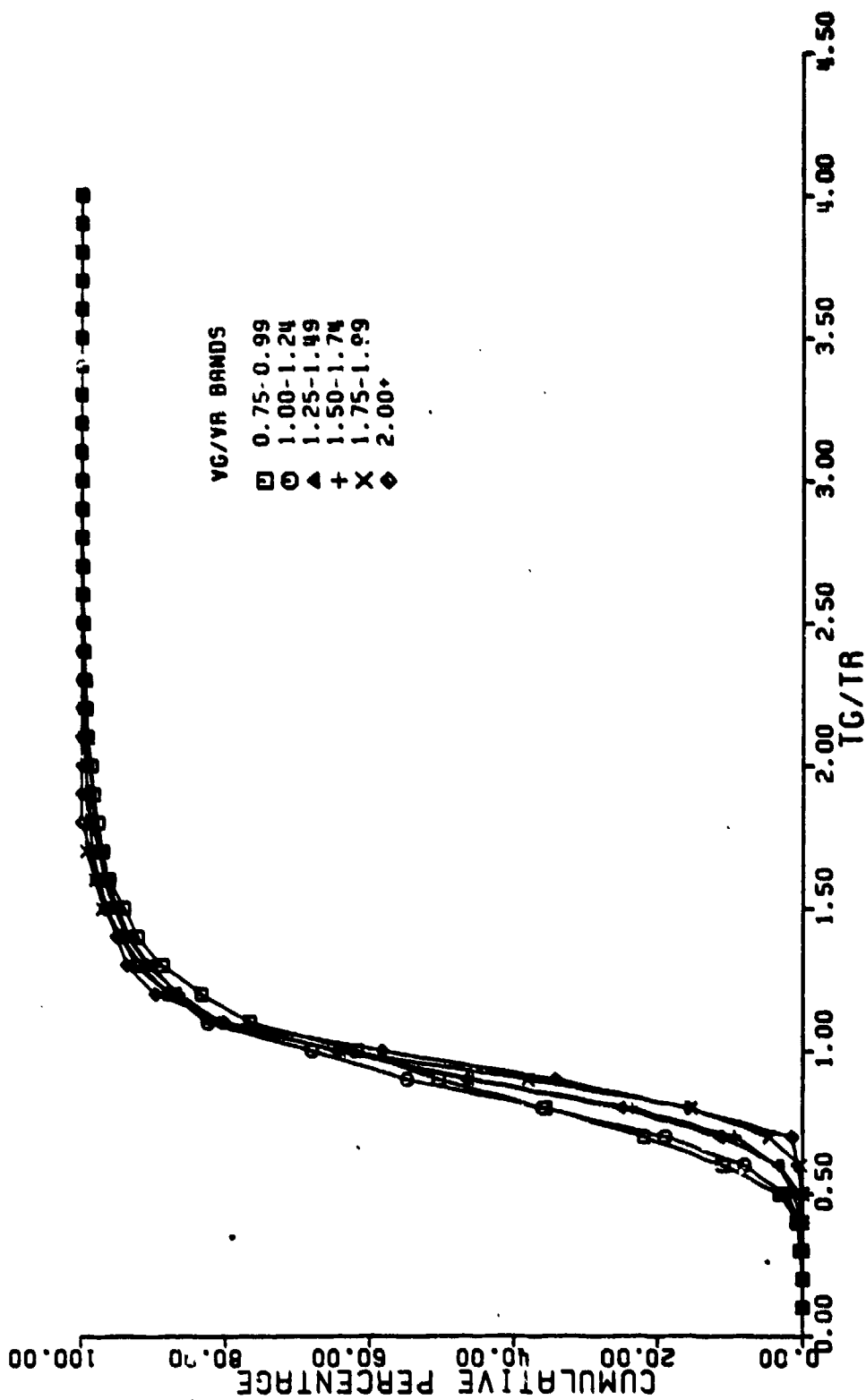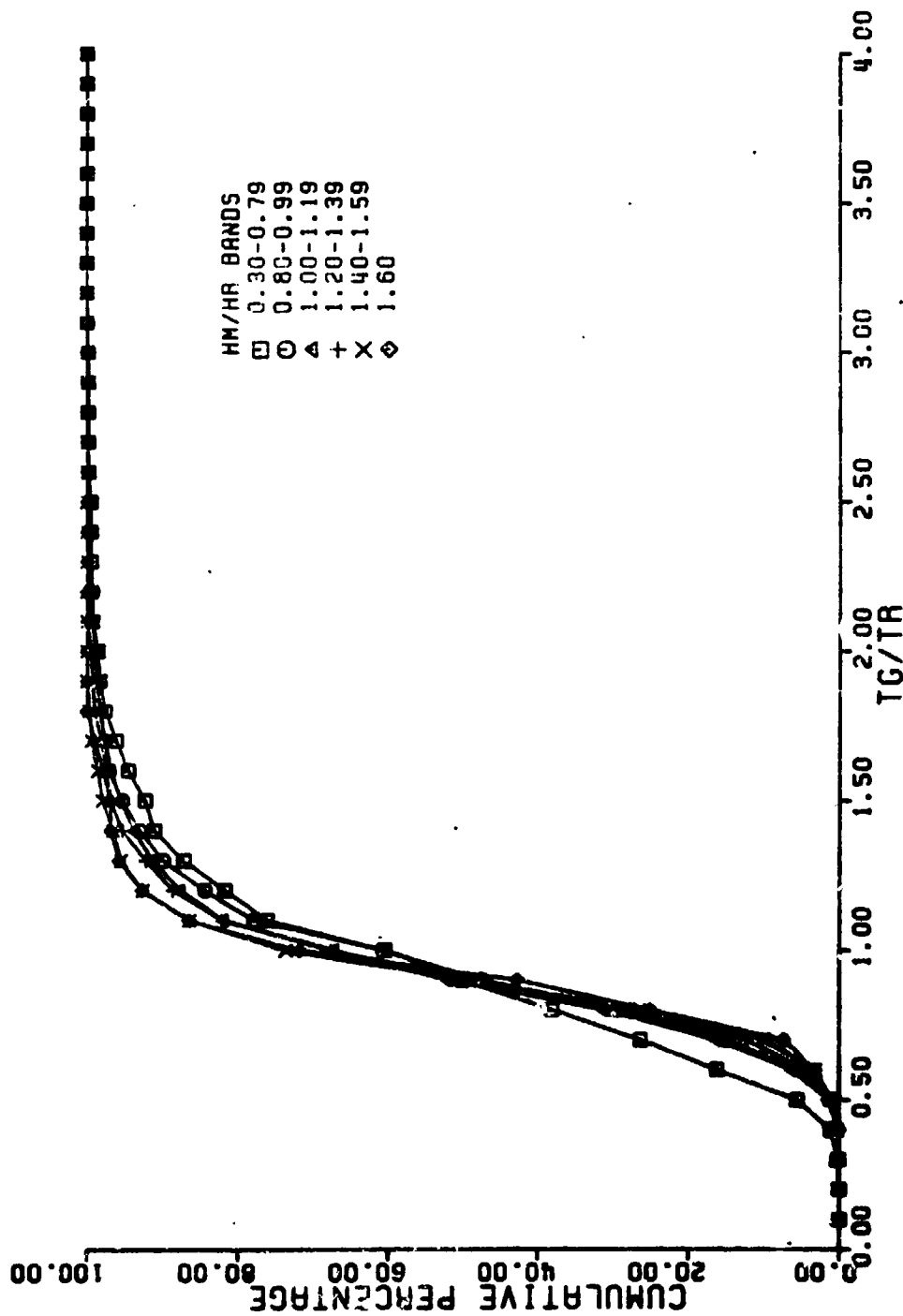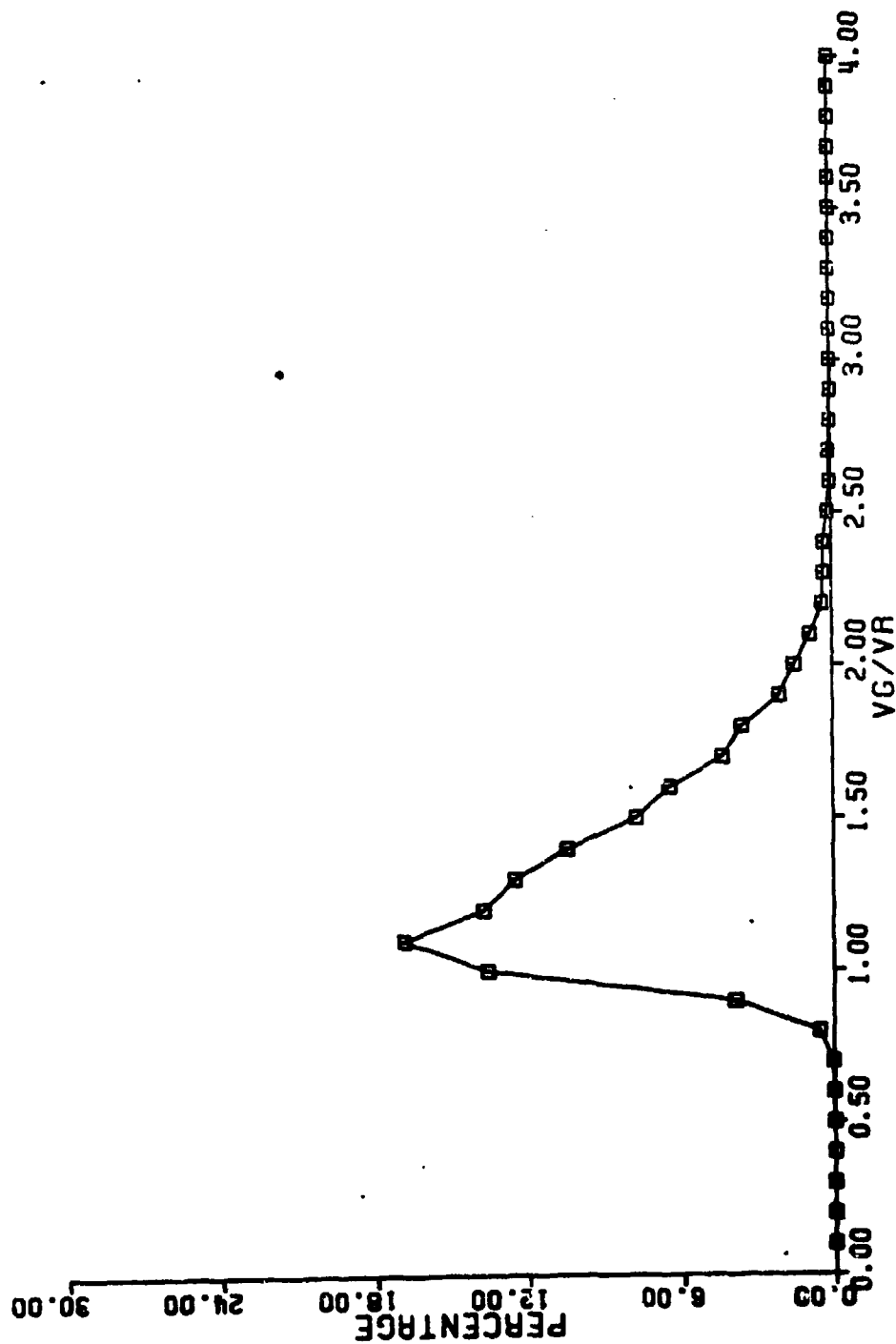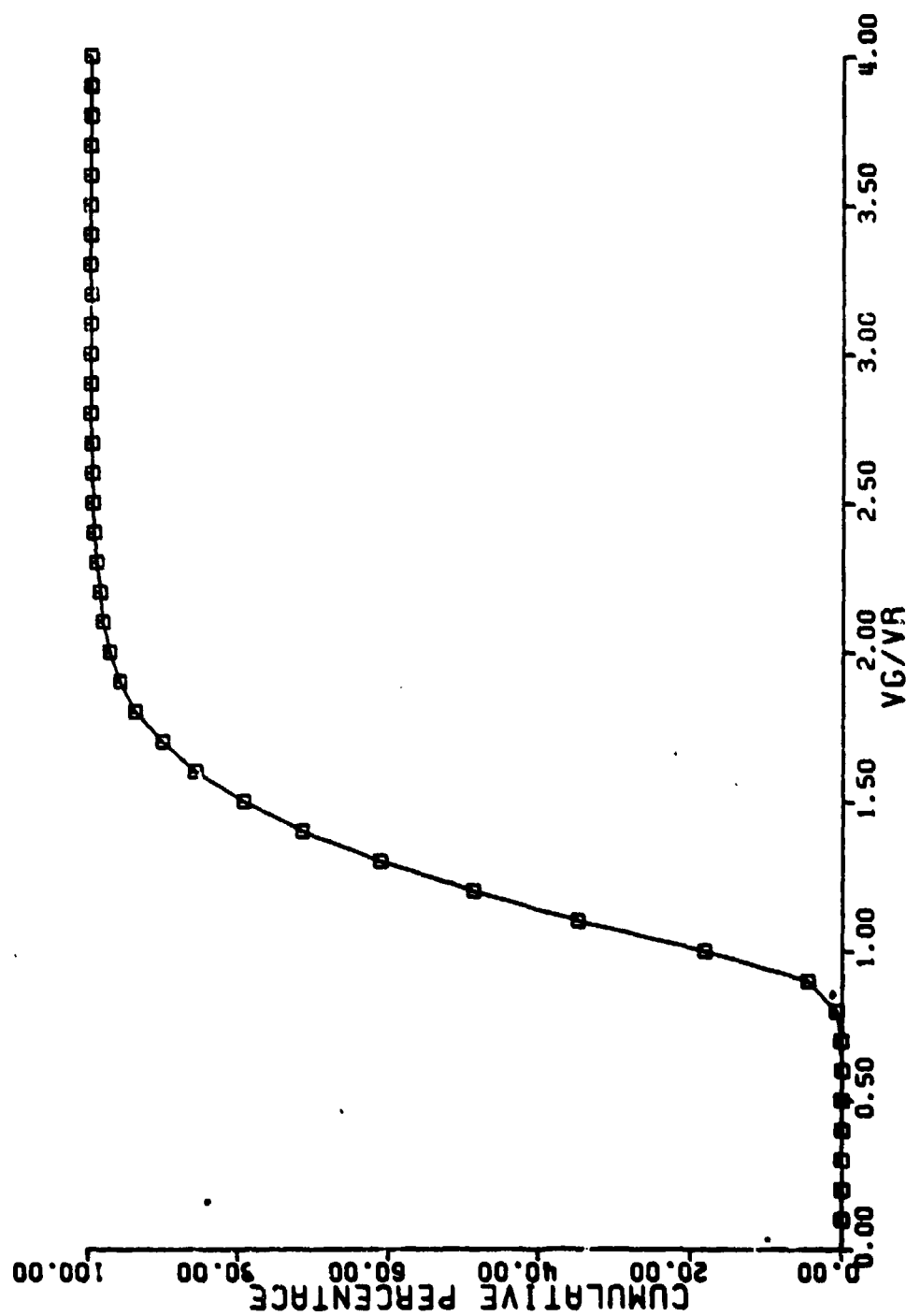Figure 12B. Cumulative percentage distribution of N_G to T_G/T_R wave group bands.

Figure 13A. Histogram of N_G for V_G/V_R wave group bands.

Figure 13B. Cumulative percentage distribution of N_G for V_G/V_R wave group bands.
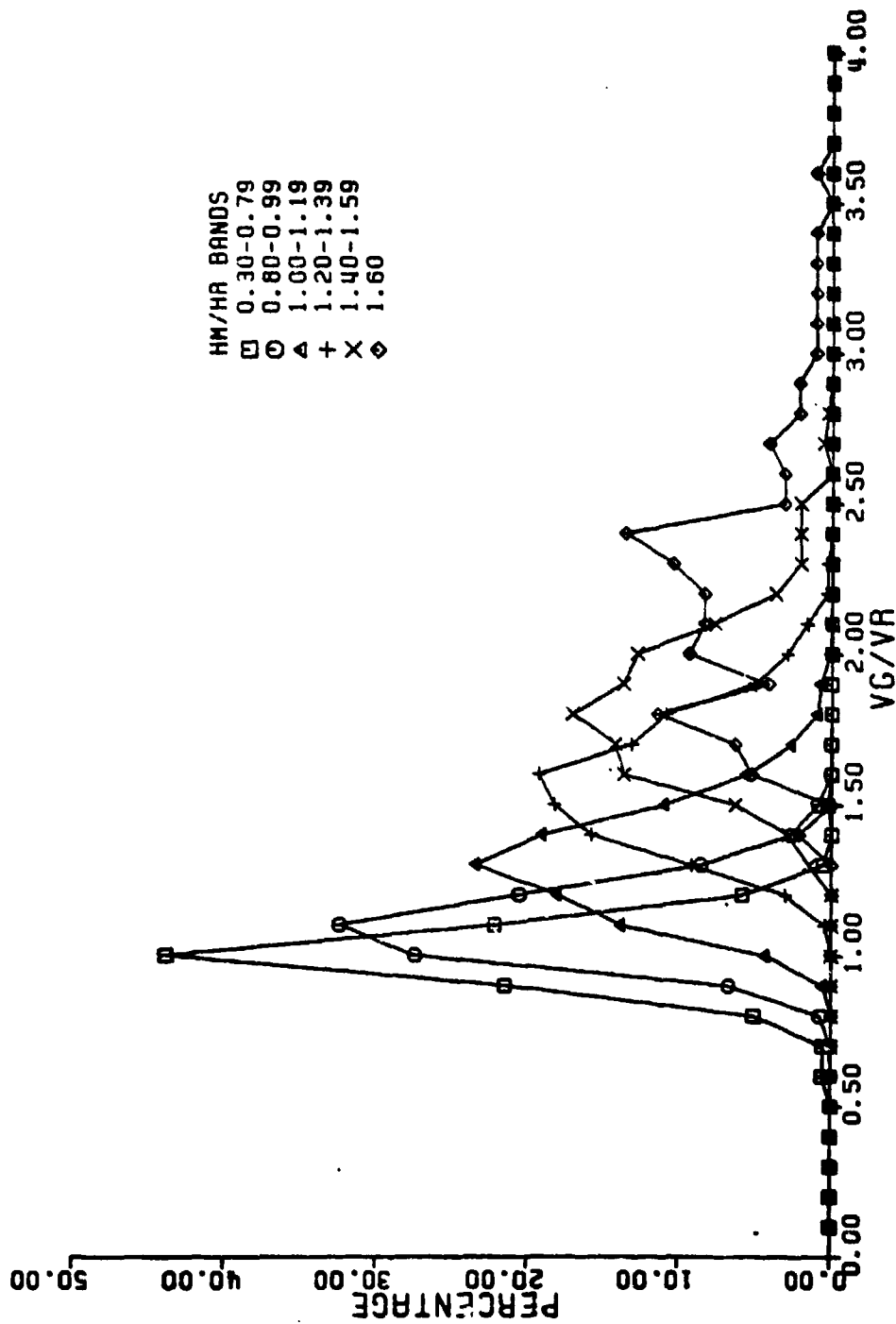
Figure 14A. Histogram of N_G for H_M/H_R wave group bands.

Figure 14B. Cumulative percentage distribution of N_G for H_M/H_R wave group bands.

Figure 15A. Histogram of $\bar{n}_G$ for G_R wave group bands.

Figure 15B. Cumulative percentage distribution of N_G for G_R wave group bands.

Figure 16A. Histogram of T_G/T_R for all wave groups.

Figure 16B. Cumulative percentage distribution of T_G/T_R for all wave groups.

Figure 17A. Histogram of T_G/T_R for V_G/V_R wave group bands.

Figure 17B. Cumulative percentage distribution of T_G/T_R for V_G/V_R wave group bands.
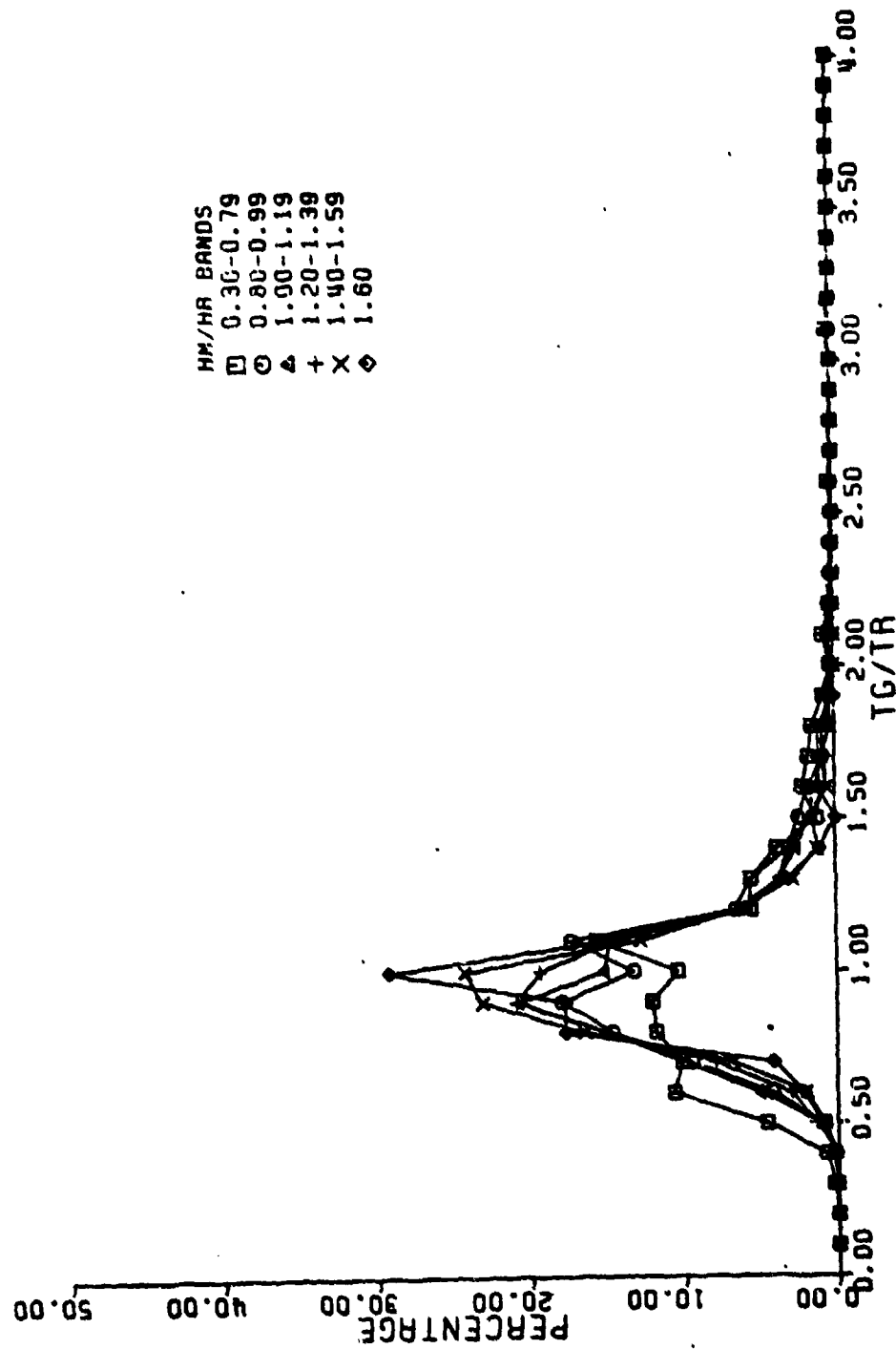
Figure 18A. Histogram of $\tilde{T}_G/T_R$ for H_M/H_R wave group bands.

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Figure 18B. Cumulative percentage distribution of T_G/T_R for H_M/H_R wave group bands.
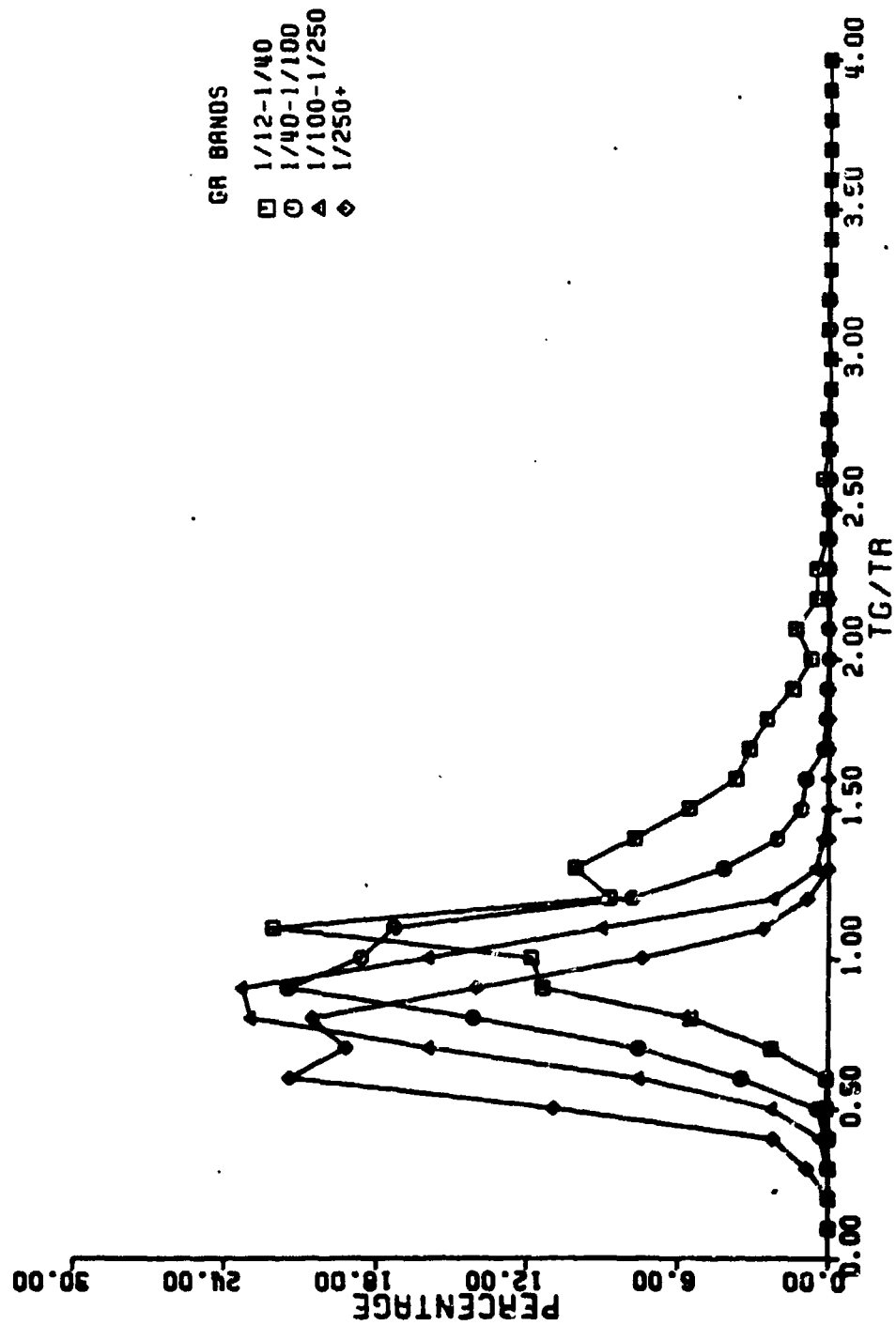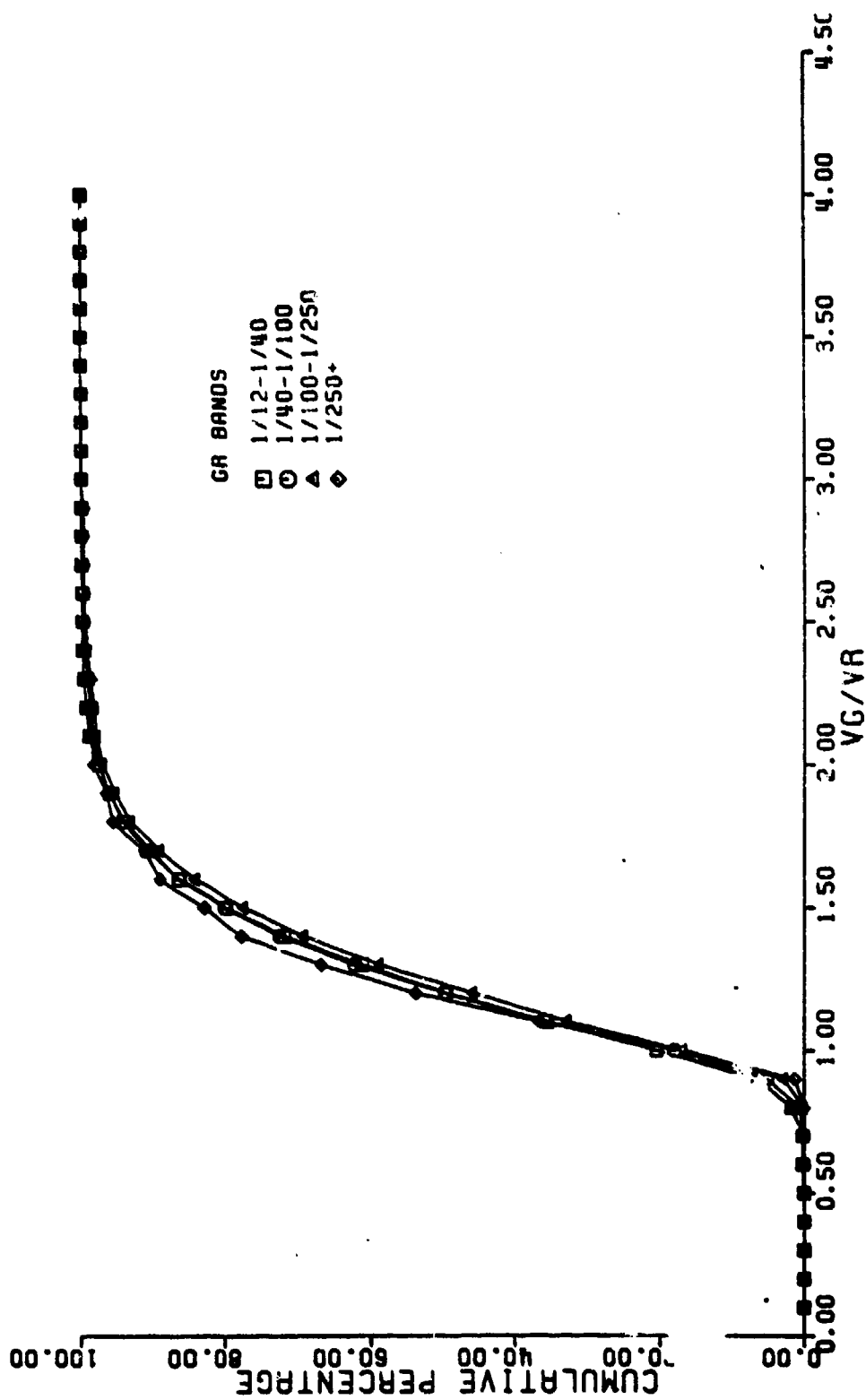
Figure 19A. Histogram of T_G/T_R for G_R wave group bands.

GR BANDS

☐ 1/12–1/40
⊙ 1/40–1/100
△ 1/100–1/250
◇ 1/250+
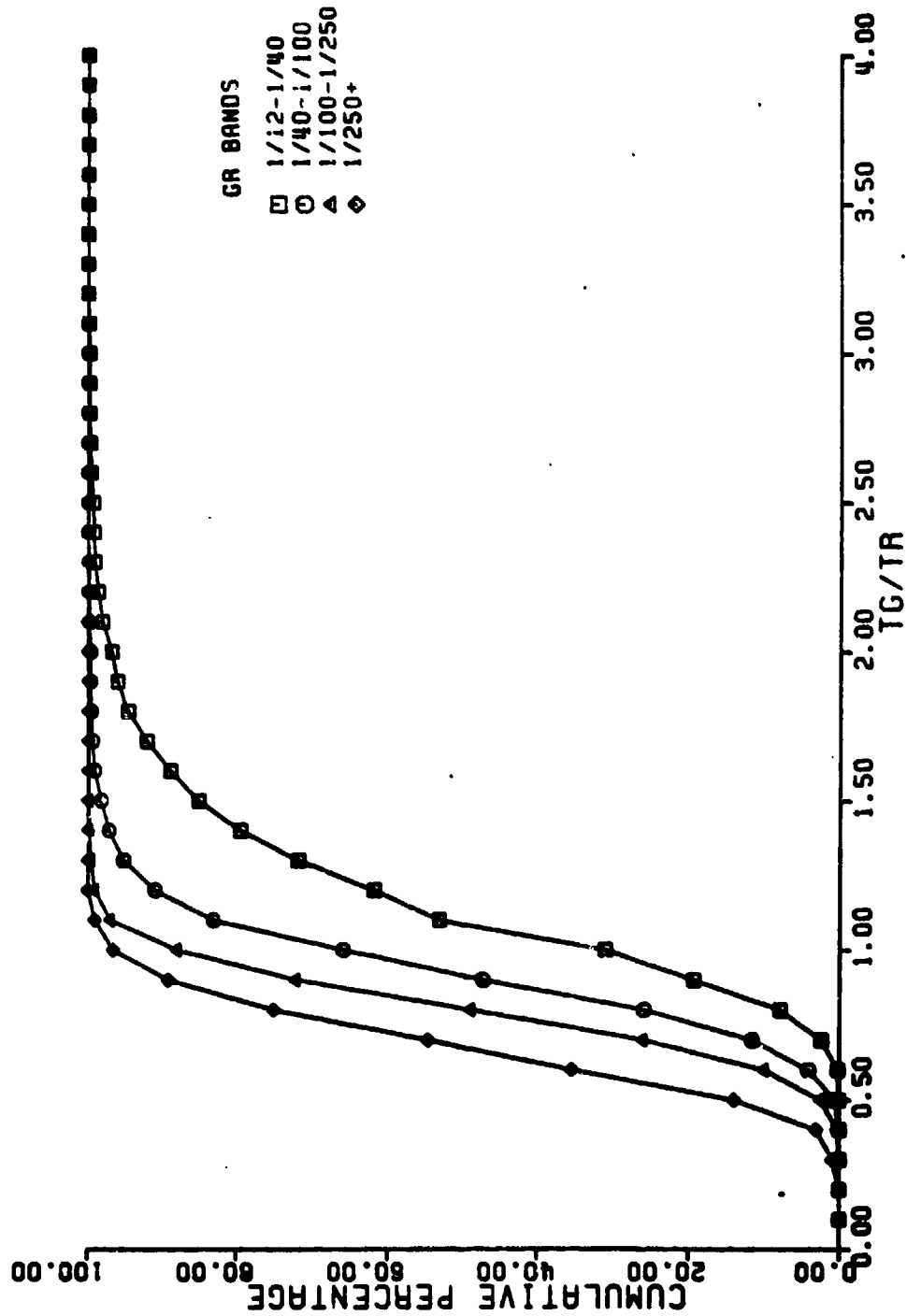
Figure 19B. Cumulative percentage distribution of T_G/T_R for G_R wave group bands.
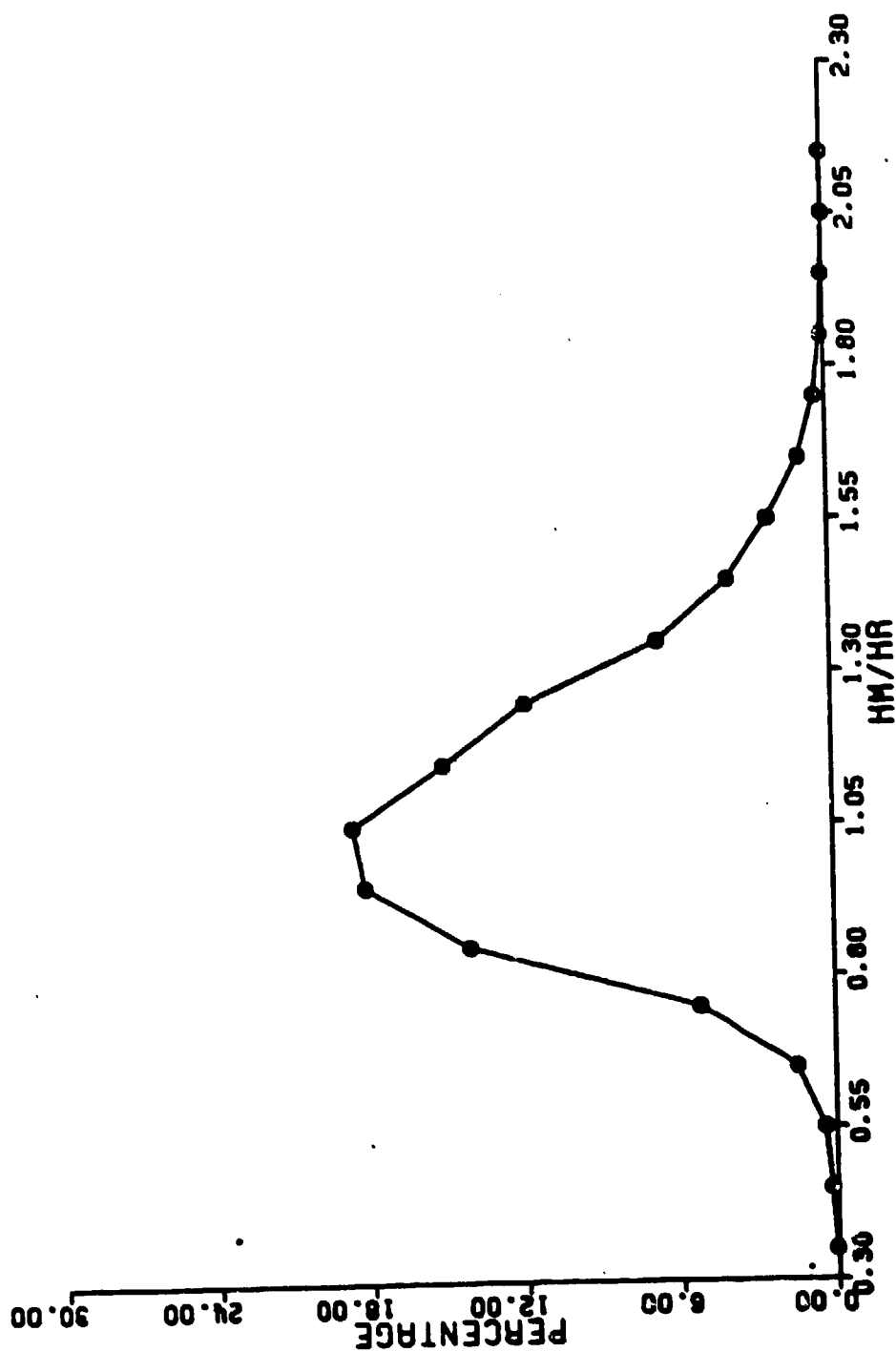
84

Figure 20A. Histogram of V_G/V_R for all wave groups.

Figure 20B. Cumulative percentage distribution of V_G/V_R for all wave groups.

Figure 21A. Histogram of V_G/V_R for H_M/H_R wave group bands.

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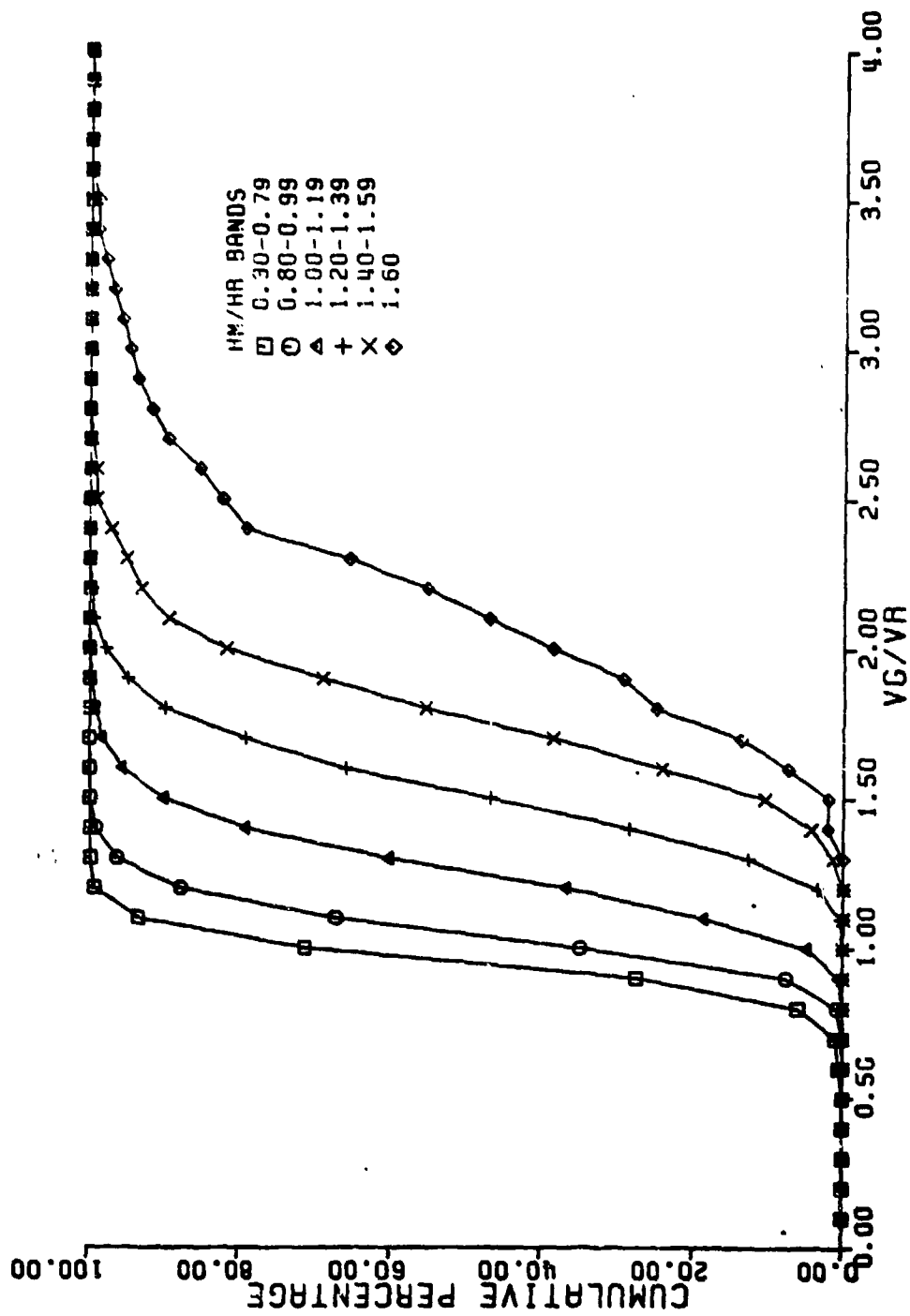
Figure 21B. Cumulative percentage distribution of V_G/V_R for H_M/H_R wave group bands.
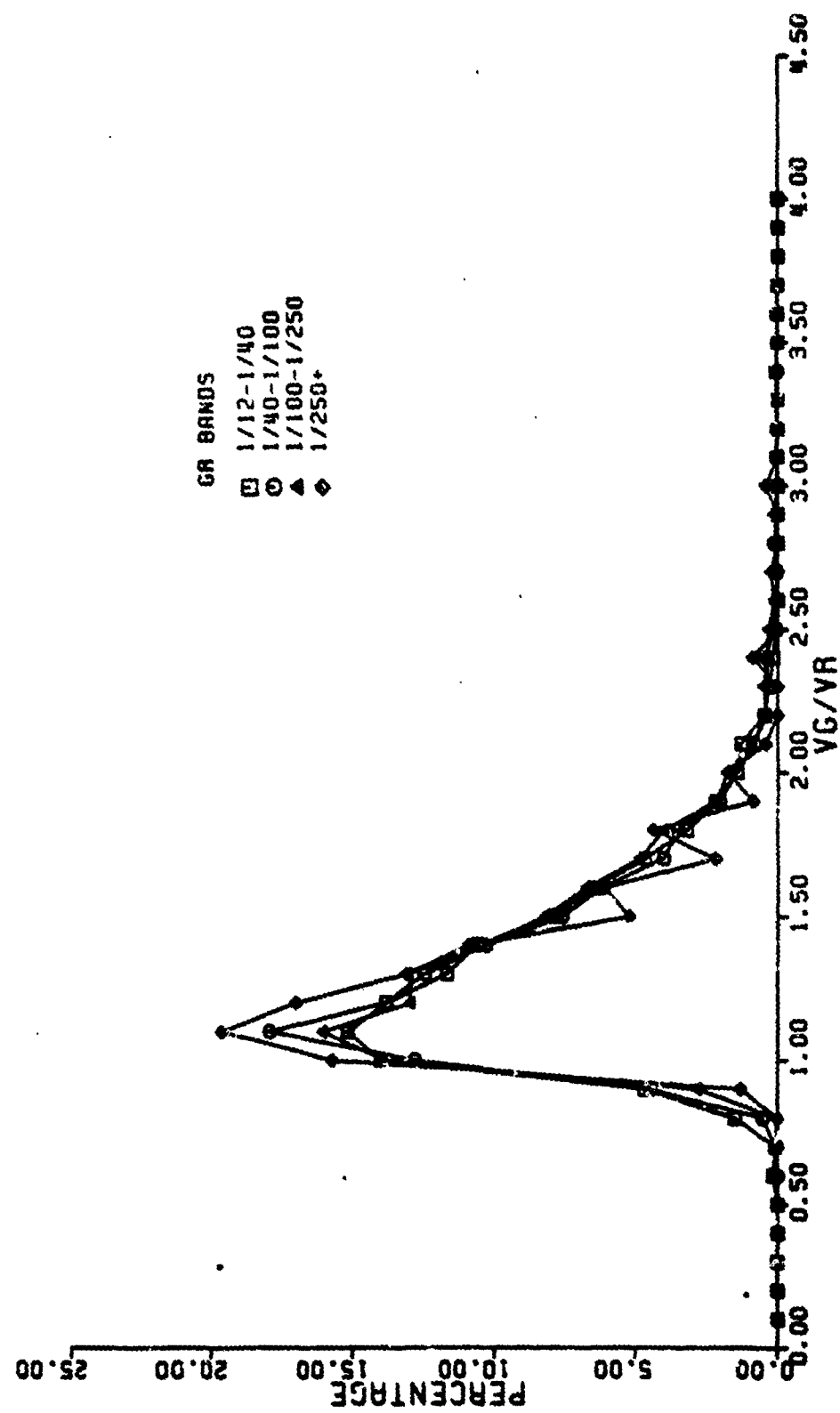
Figure 22A. Histogram of V_G/V_R for G_R wave group bands.

GR BANDS

- 1/12-1/40
- 1/40-1/100
- 1/100-1/250
- 1/250+

PERCENTAGE

VG/VR

Figure 22B. Cumulative percentage distribution of V_G/V_R for G_R wave group bands.
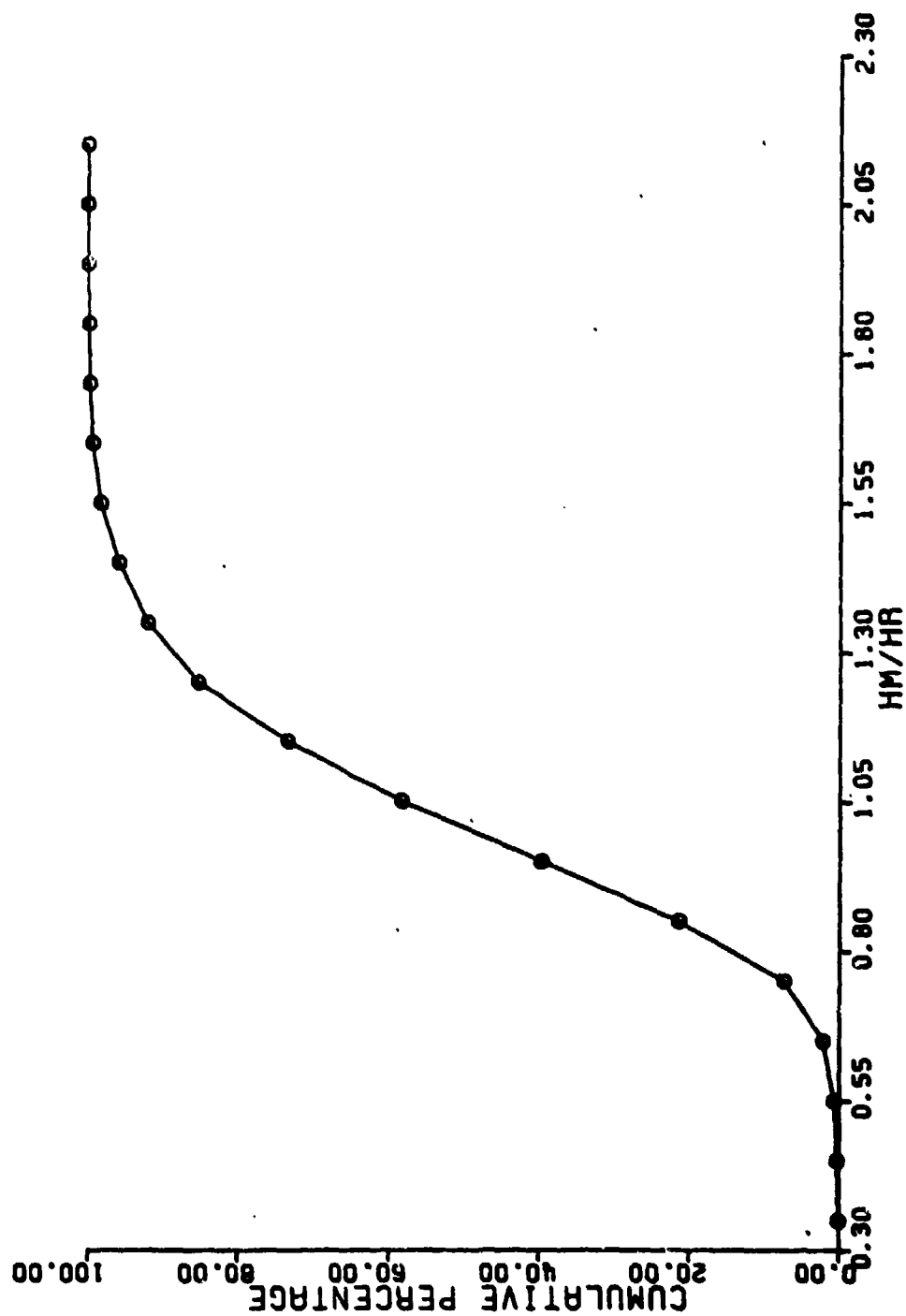
Figure 23A. Histogram of H_M/H_R for all wave groups.

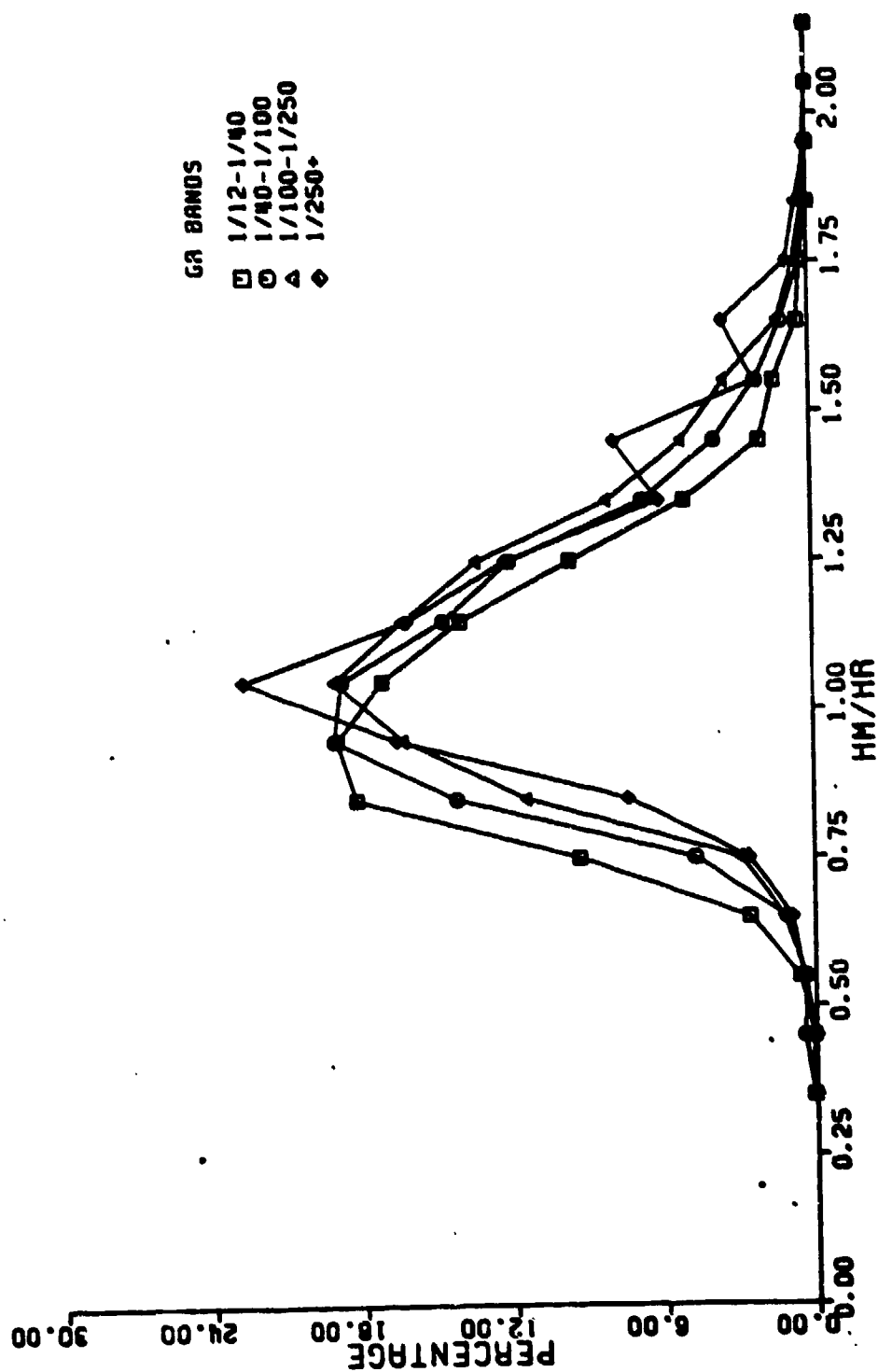Figure 23B. Cumulative percentage distribution of H_M/H_R for all wave groups.

Figure 24A. Histogram of H_M/H_R for G_R wave group bands.
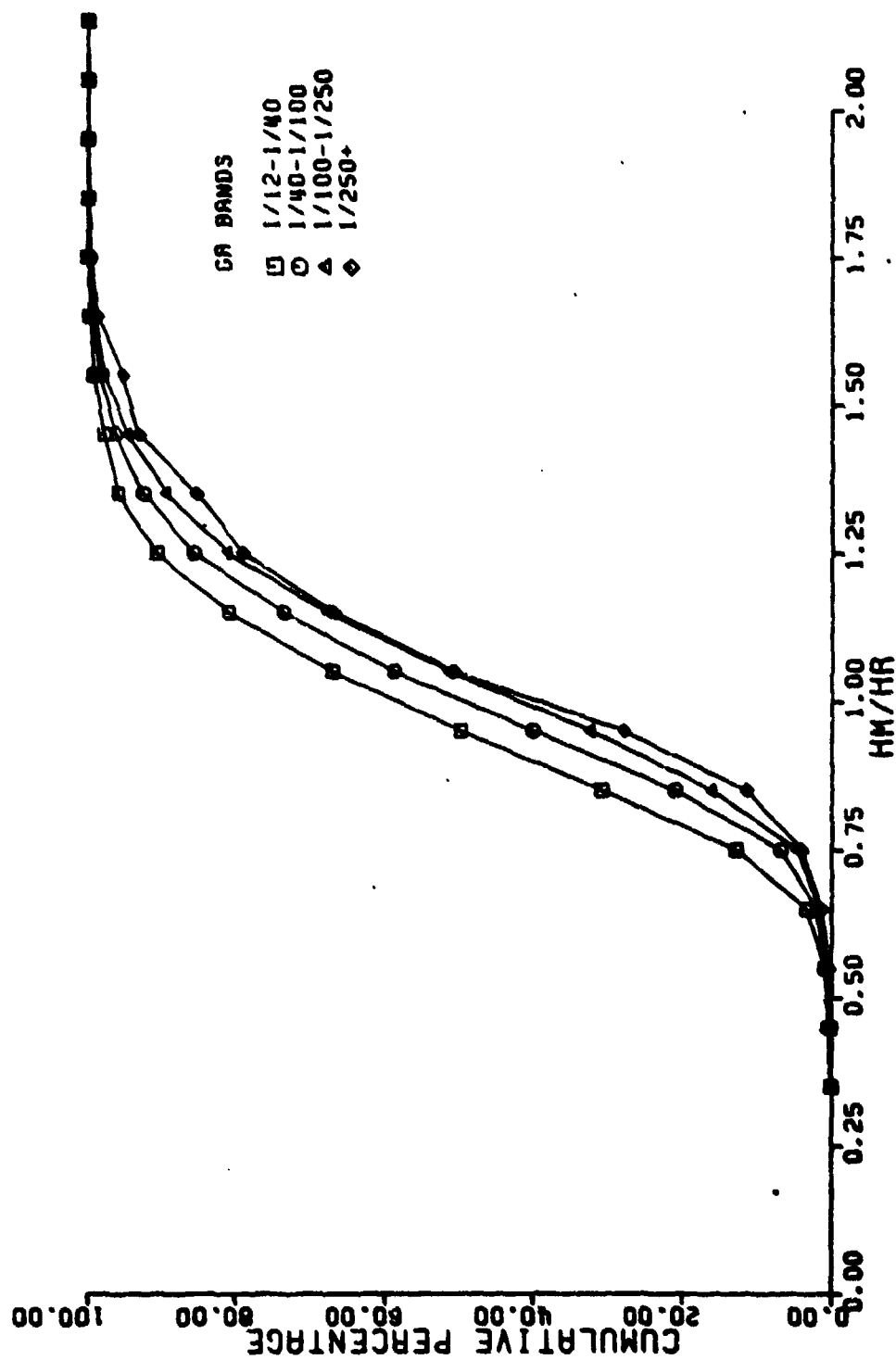
Figure 24B. Cumulative percentage distribution of H_M/H_R for G_R wave group bands.
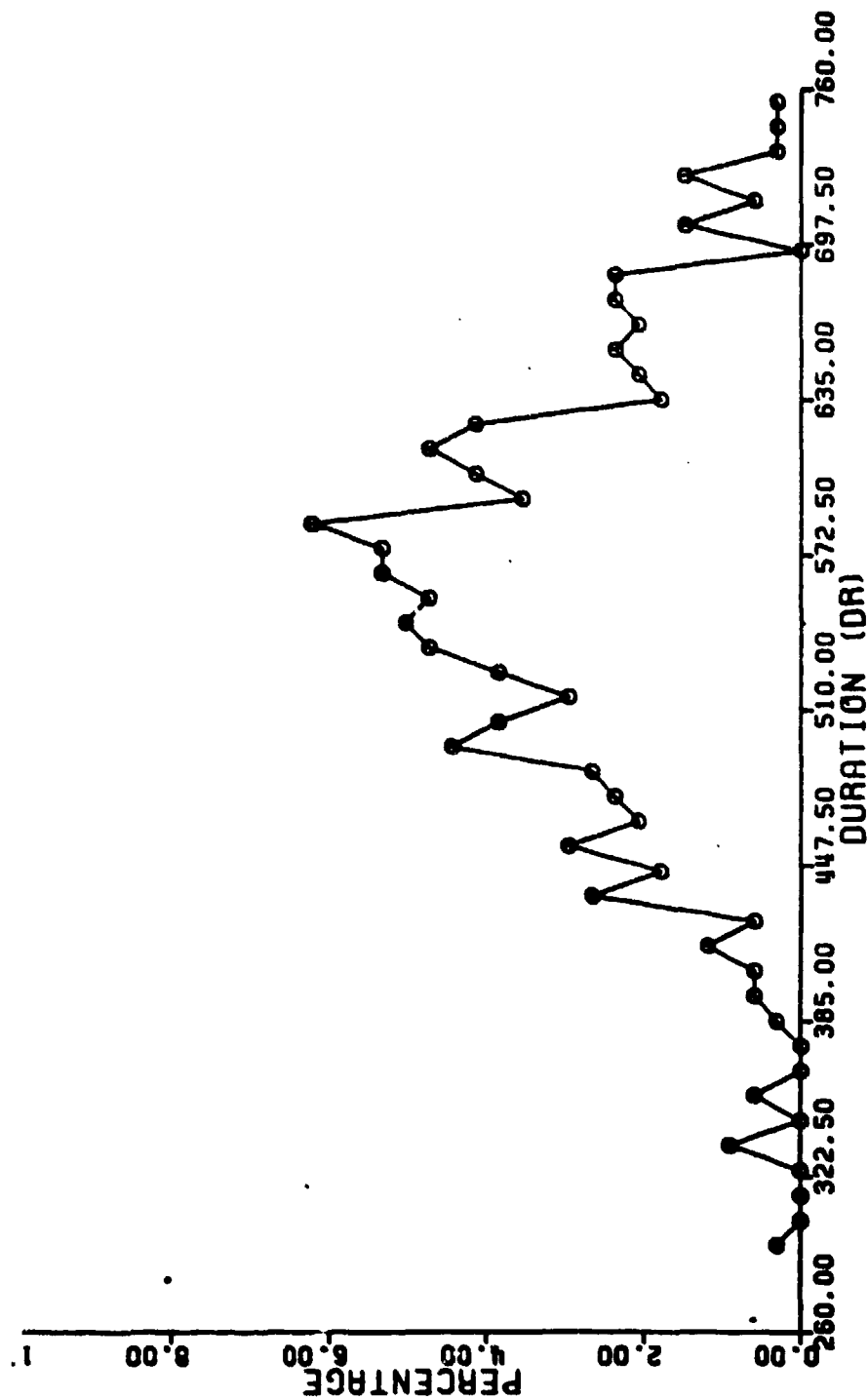
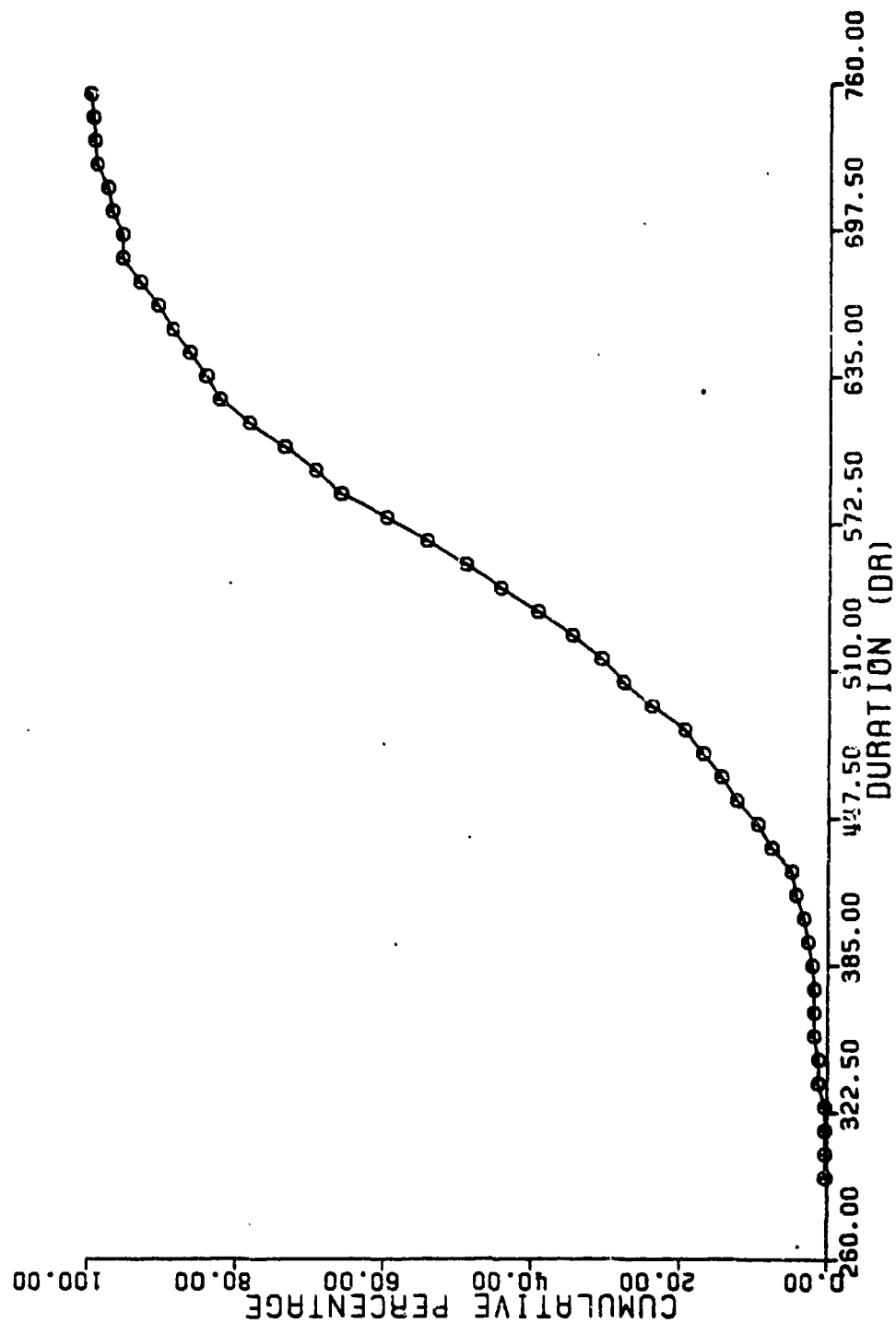Figure 25A. Histogram of D_R for all wave groups.

Figure 25B. Cumulative percentage distribution of D_R for all wave groups.
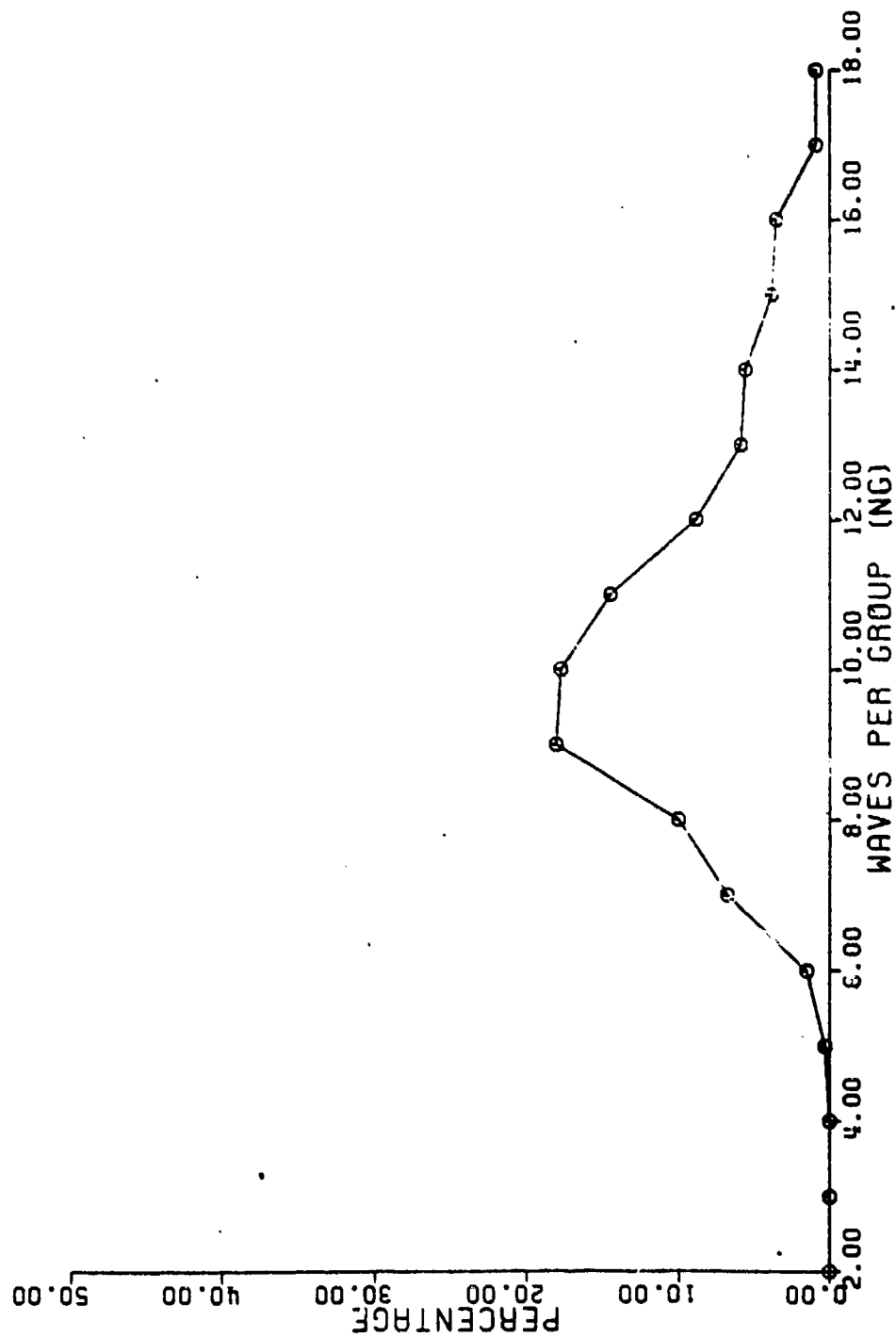
Figure 26A. Histogram of N_G for extreme wave groups: largest group N_G per wave record.
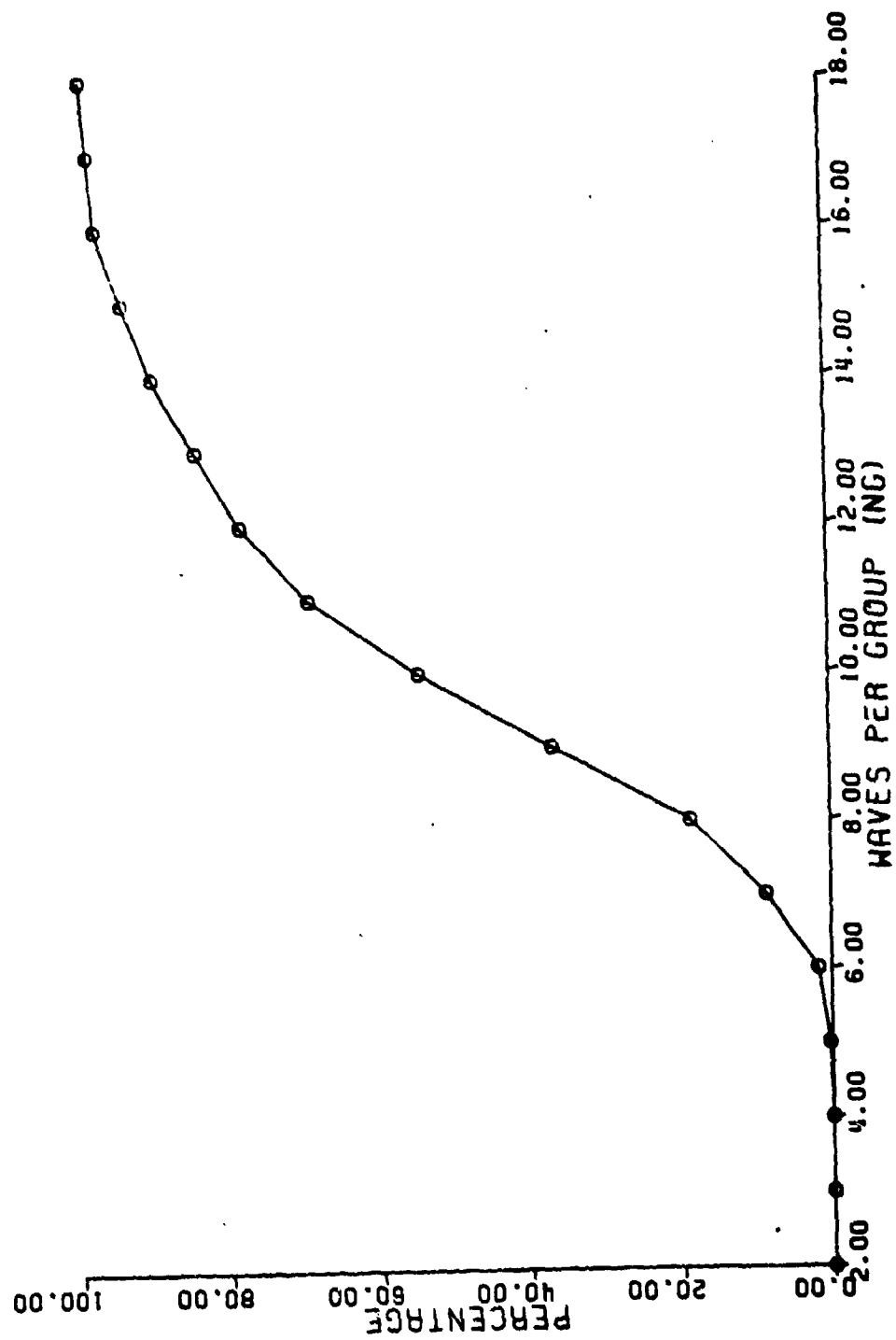
Figure 26B. Cumulative percentage distribution of N_G for extreme wave groups: largest group N_G per wave record.

Figure 27A. Histogram of T_G/T_R for extreme wave groups: largest group N_G per wave record.

Figure 27B. Cumulative percentage distribution of T_G/T_R for extreme wave groups: largest group N_G per wave record.

Figure 28A. Histogram of V_G/V_R for extreme wave groups: largest group N_G per wave record.

Figure 28B. Cumulative percentage distribution of V_G/V_R for extreme wave groups: largest group N_G per wave record.
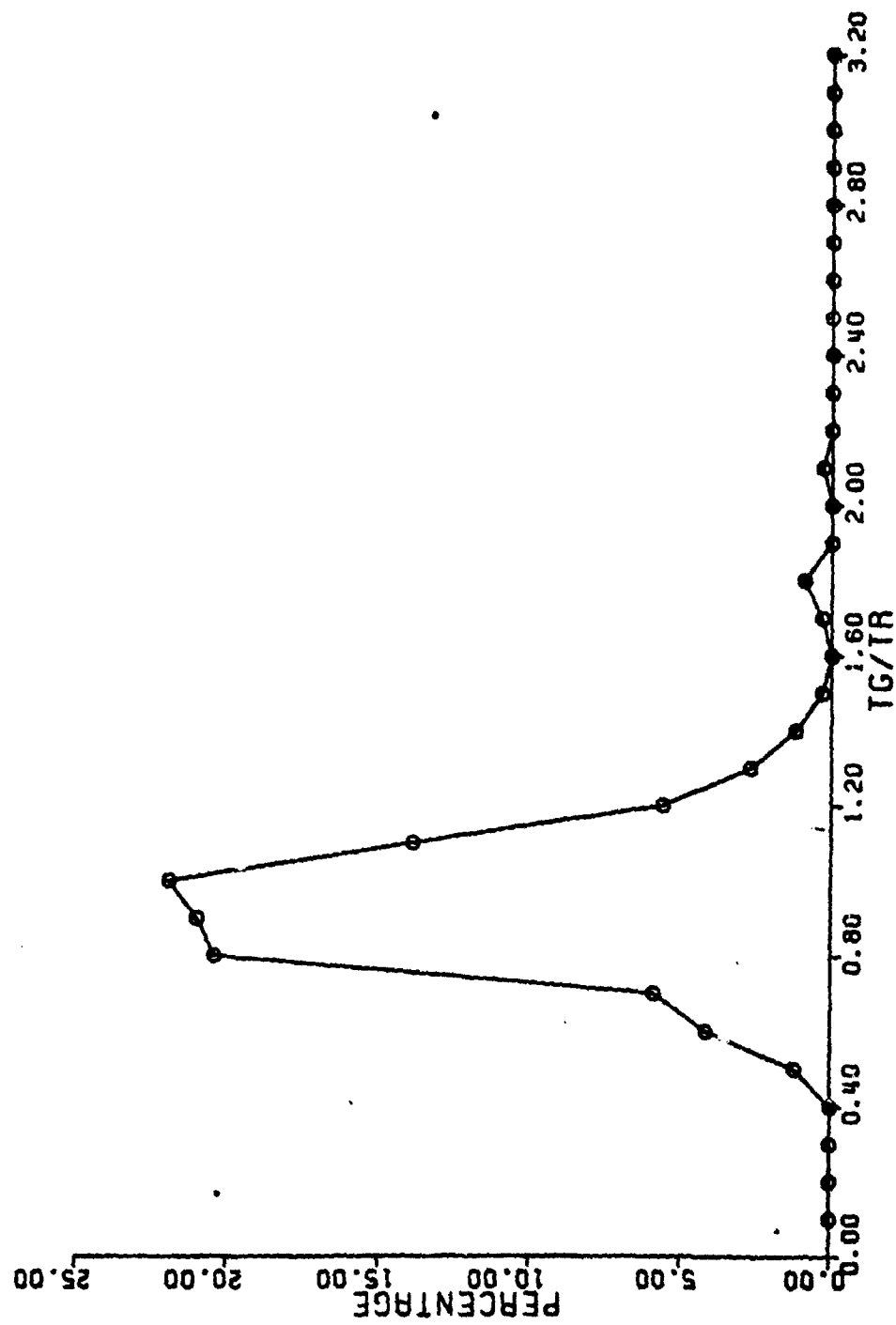
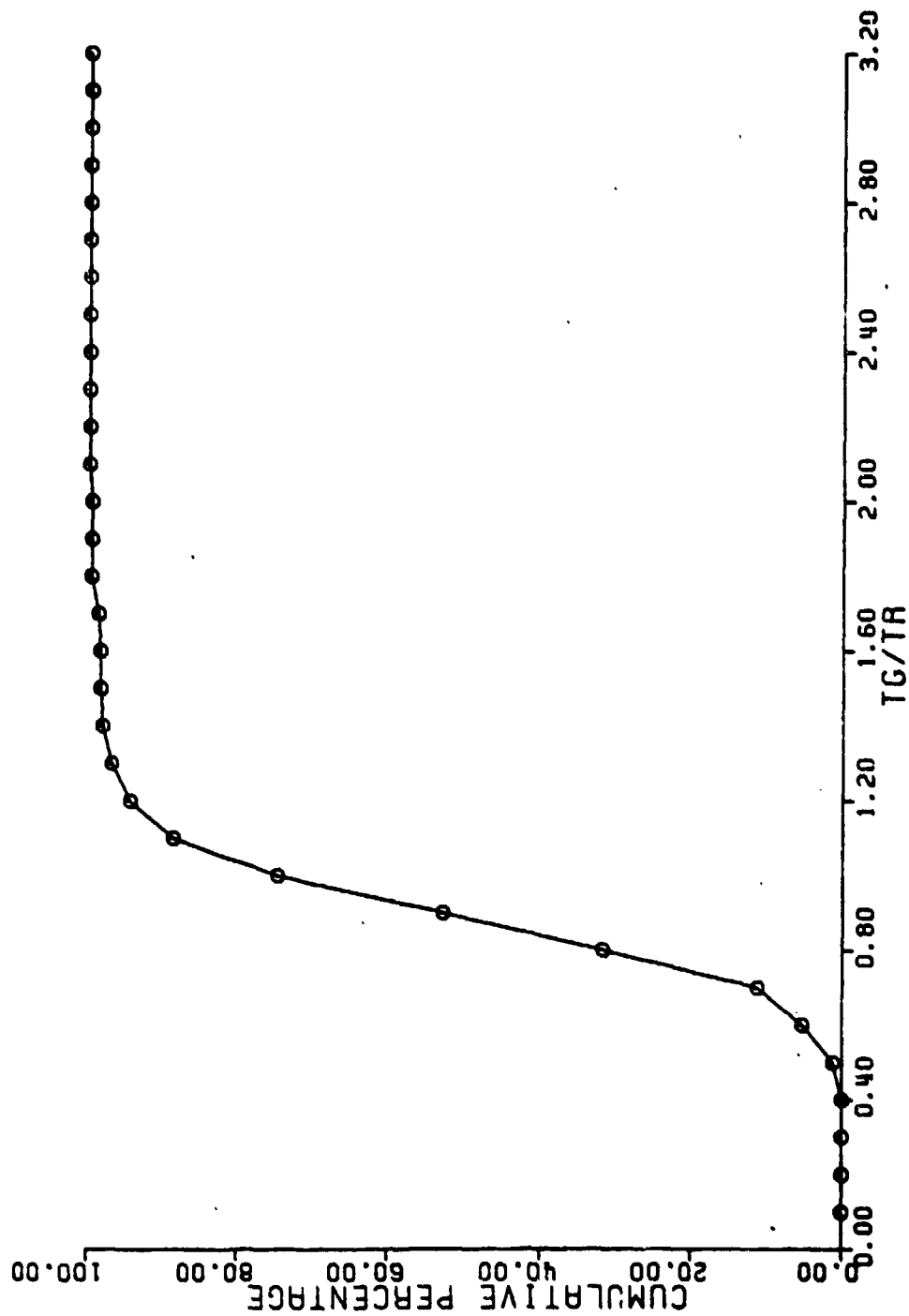Figure 29A. Histogram of H_M/H_R for extreme wave groups: largest group N_G per wave record.

Figure 29B. Cumulative percentage distribution of H_M/H_R for extreme wave groups: largest group N_G per wave record.

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Figure 30A. Histogram of N_G for extreme wave groups: largest group V_G/V_R per wave record.

Figure 30B. Cumulative percentage distribution of N_G for extreme wave groups: largest group V_G/V_R per wave record.

Figure 31A. Histogram of T_G/T_R for extreme wave groups: largest group V_G/V_R per wave record.

Figure 31B. Cumulative percentage distribution of T_G/T_R for extreme wave groups: largest group V_G/V_R per wave Record.
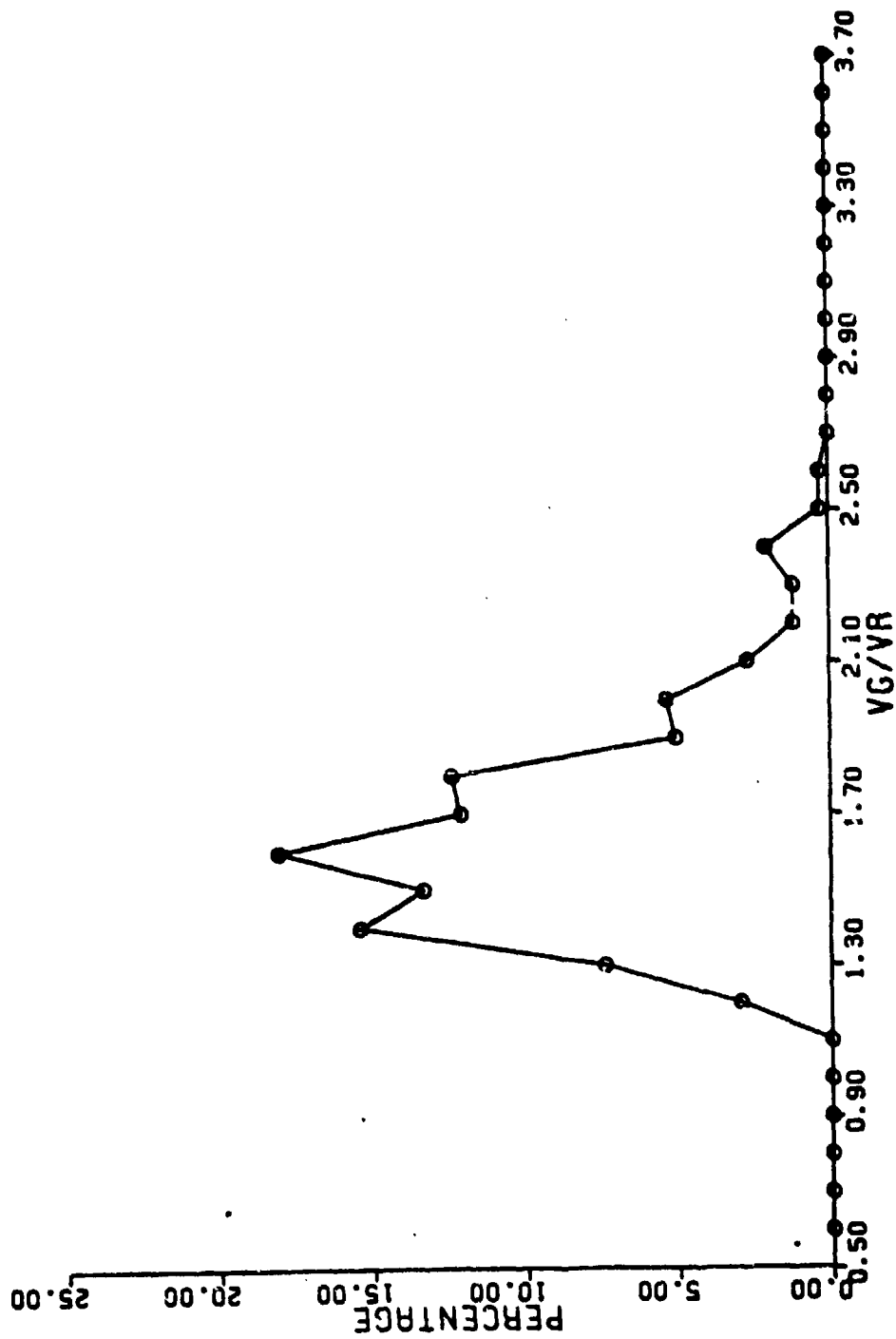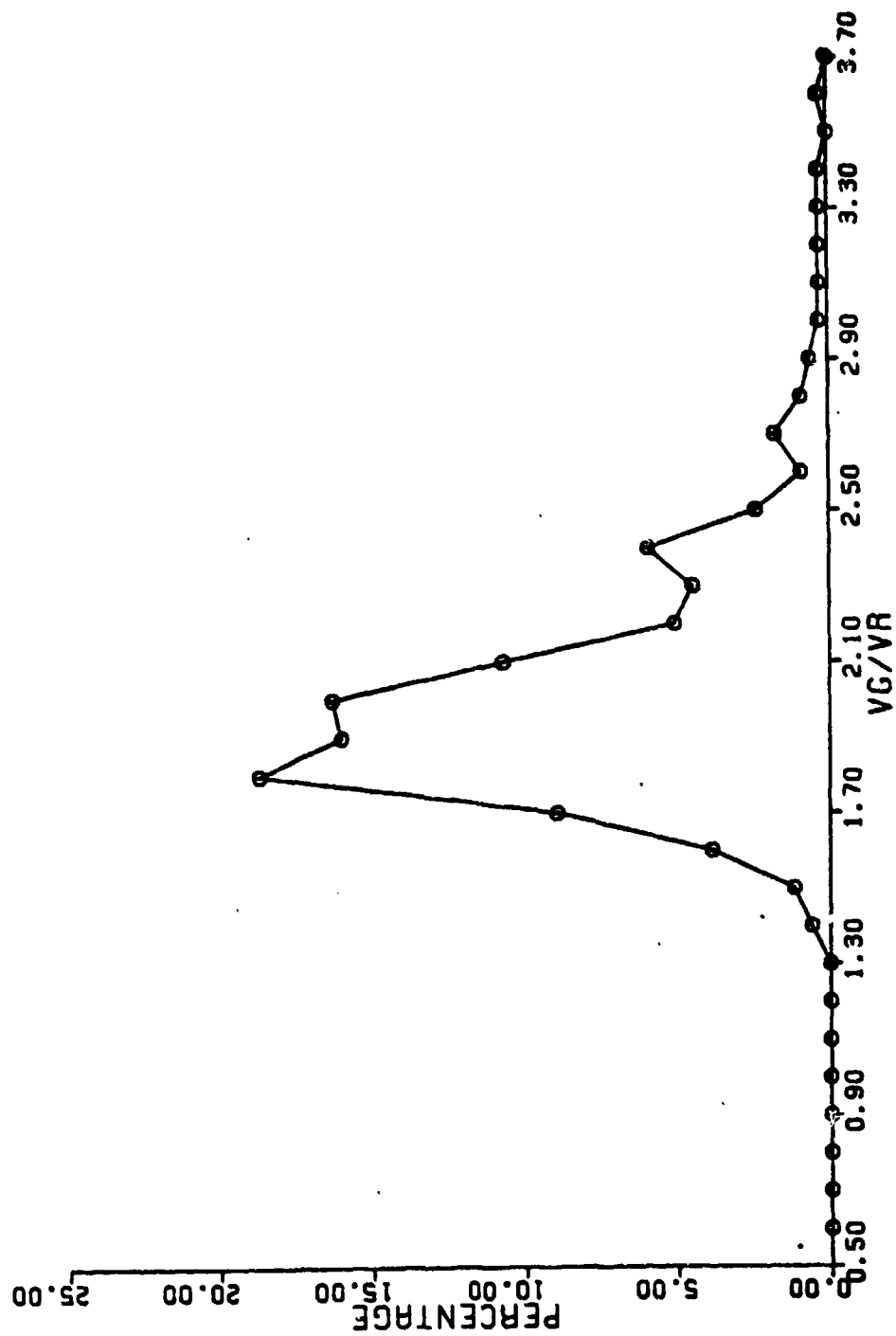
Figure 32A.   Histogram of V_G/V_R for extreme wave groups: largest group V_G/V_R per wave record.
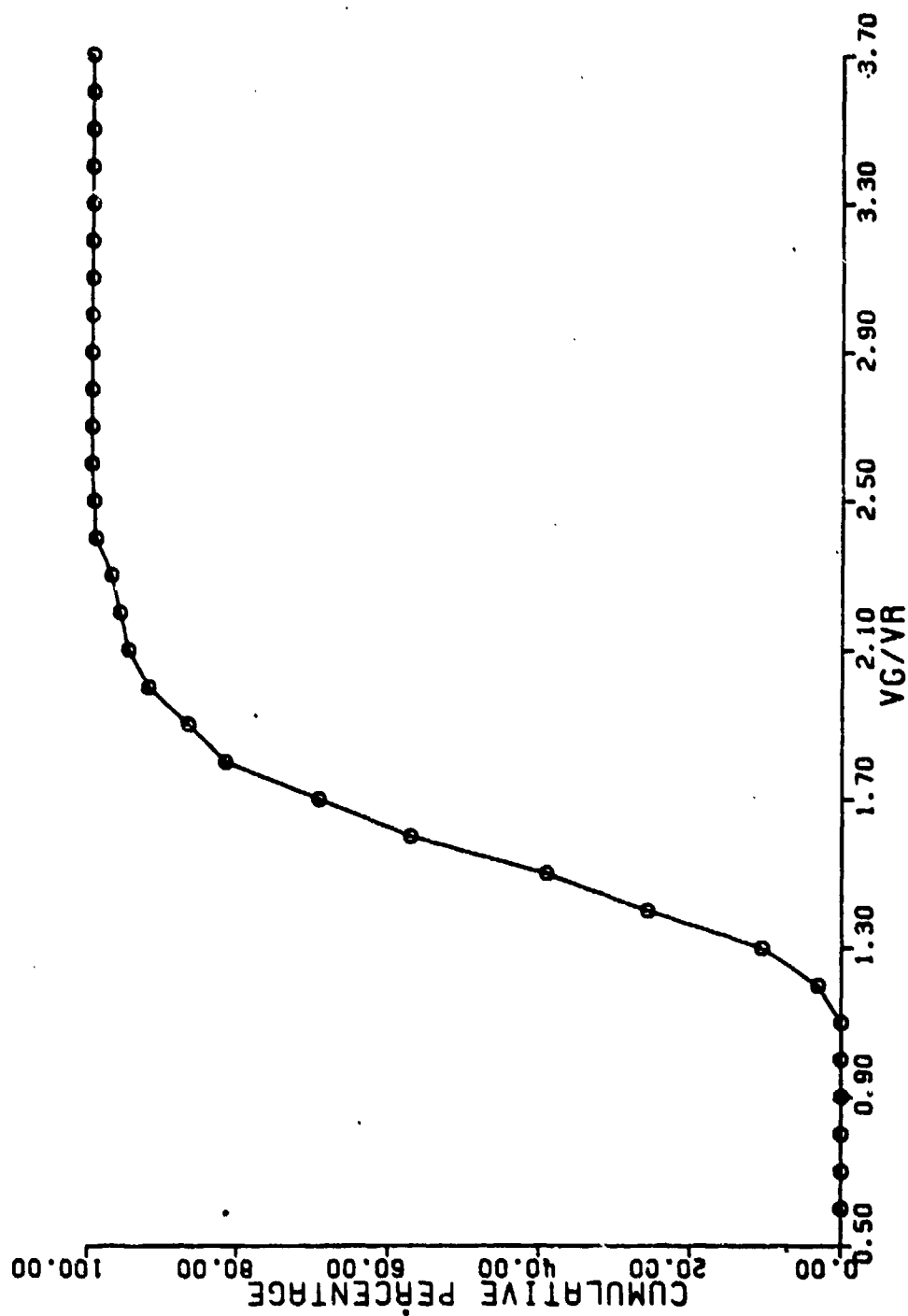
109
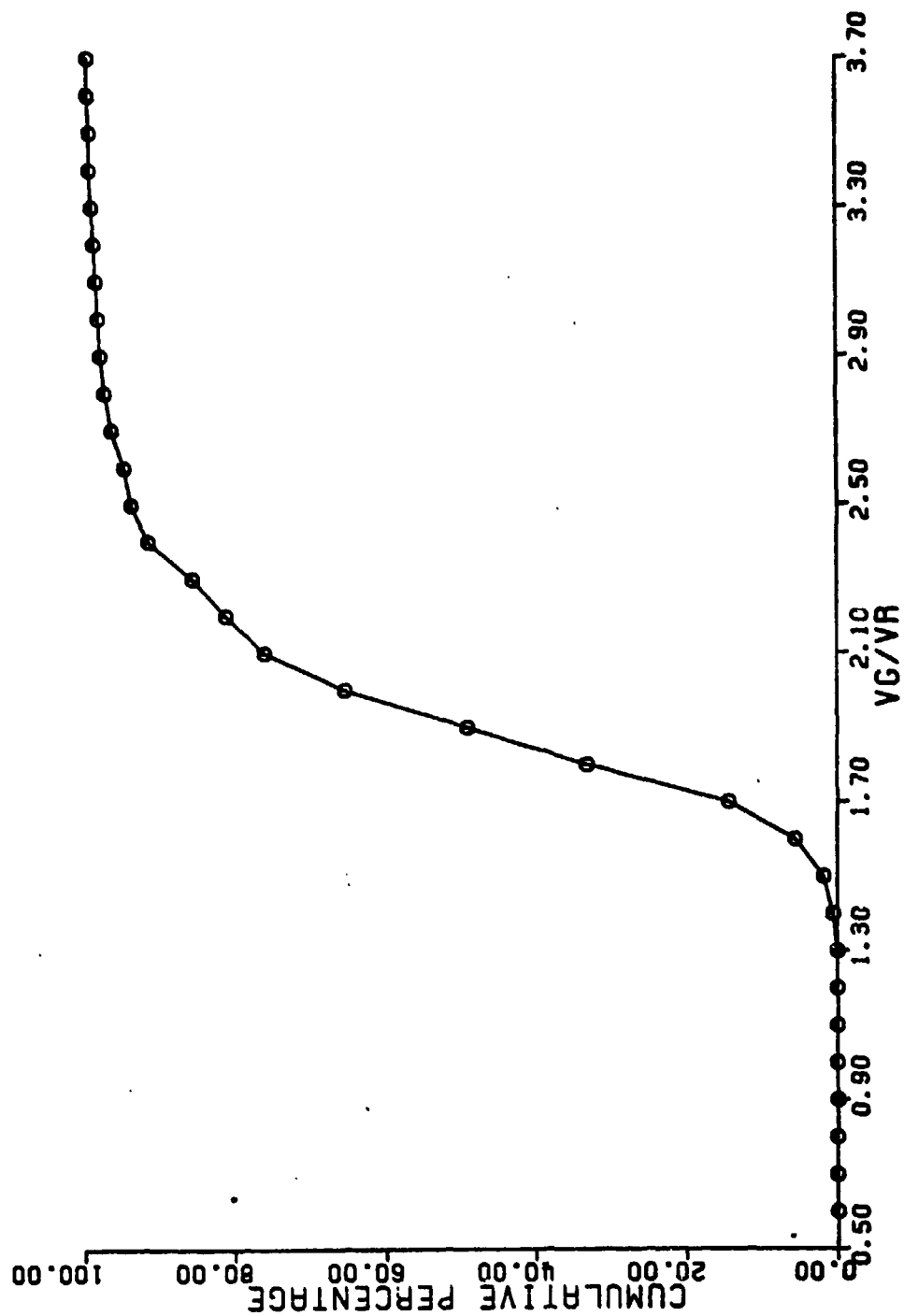
Figure 32B. Cumulative percentage distribution of V_G/V_R for extreme wave groups: largest group V_G/V_R per wave record.
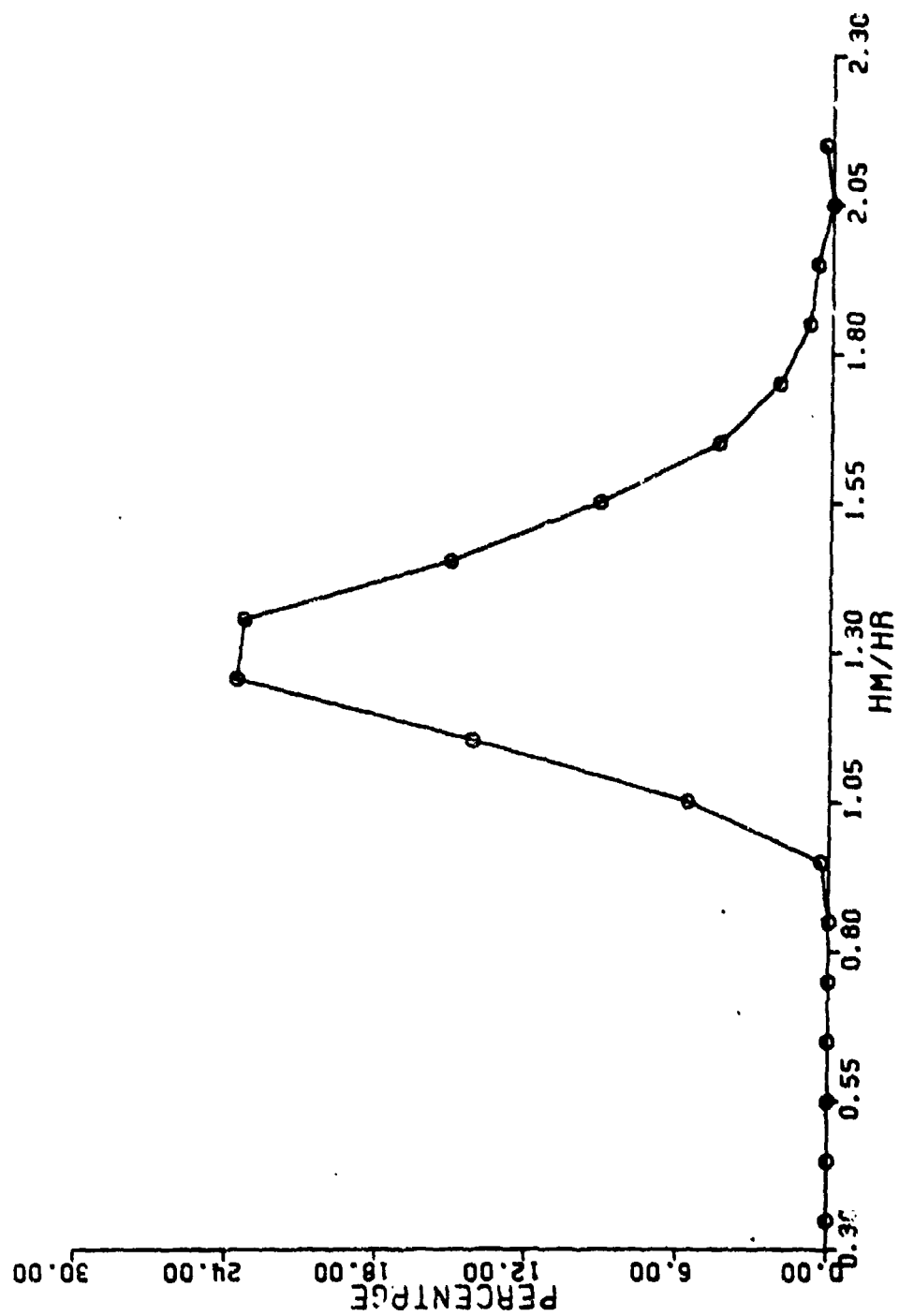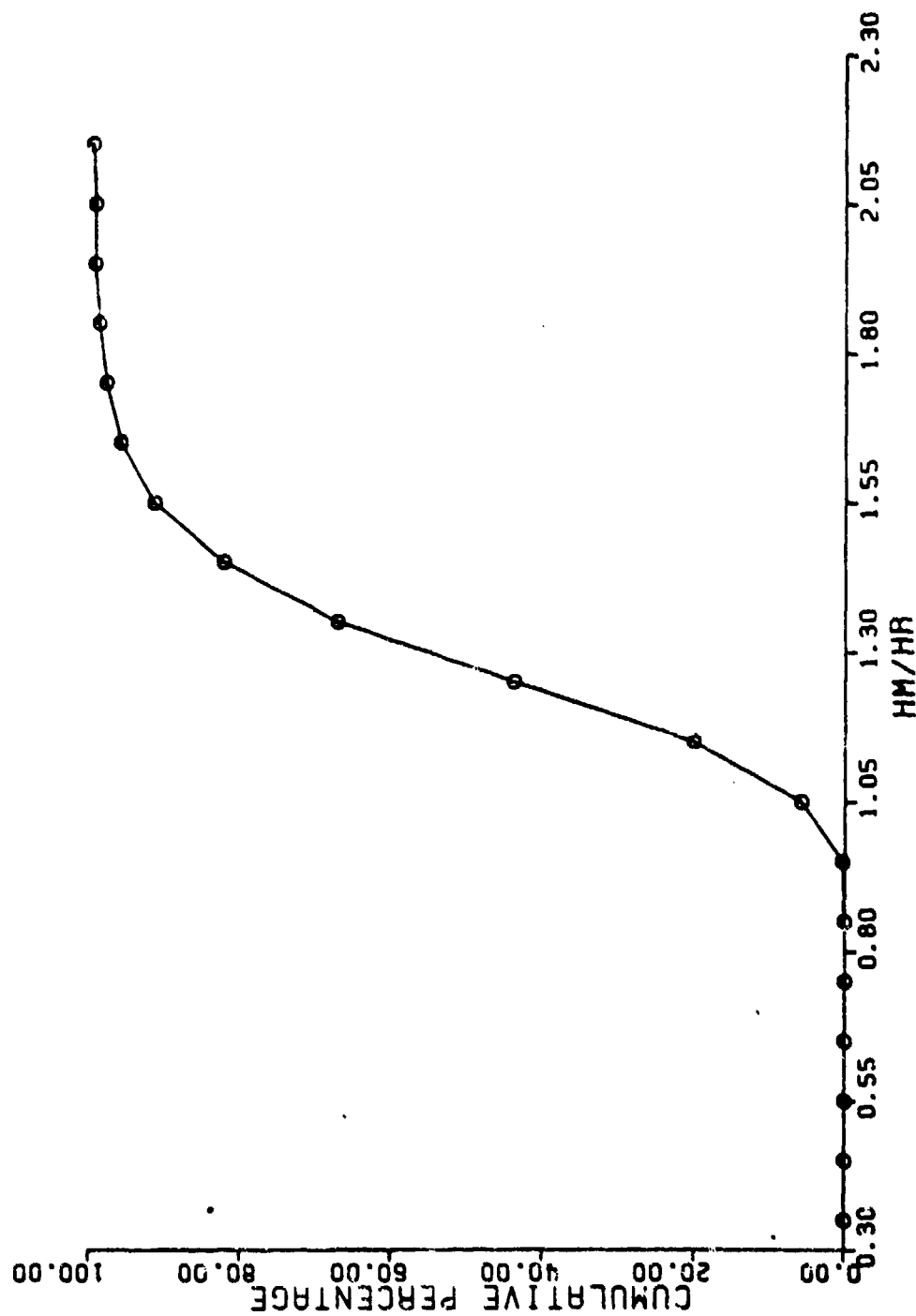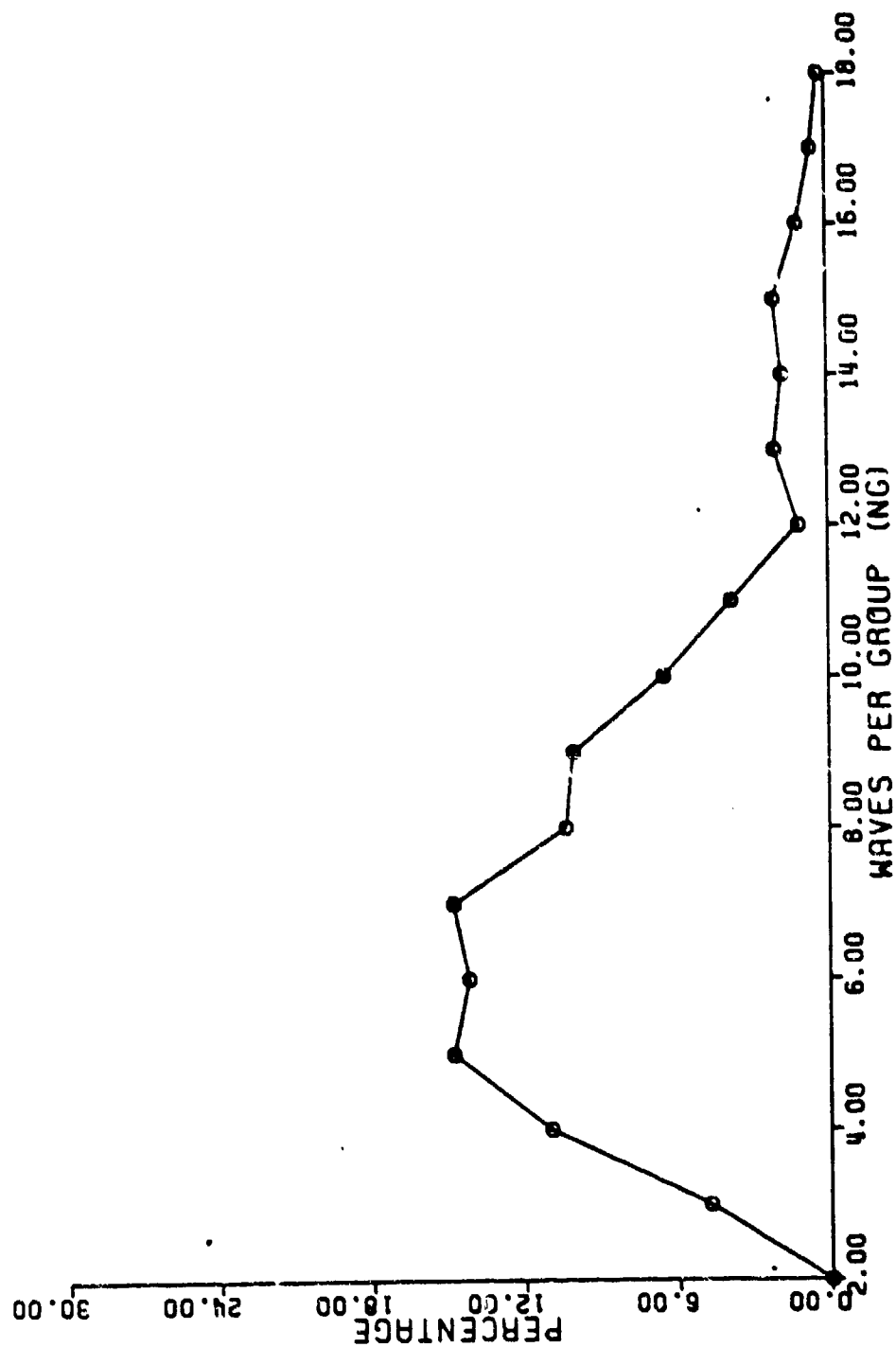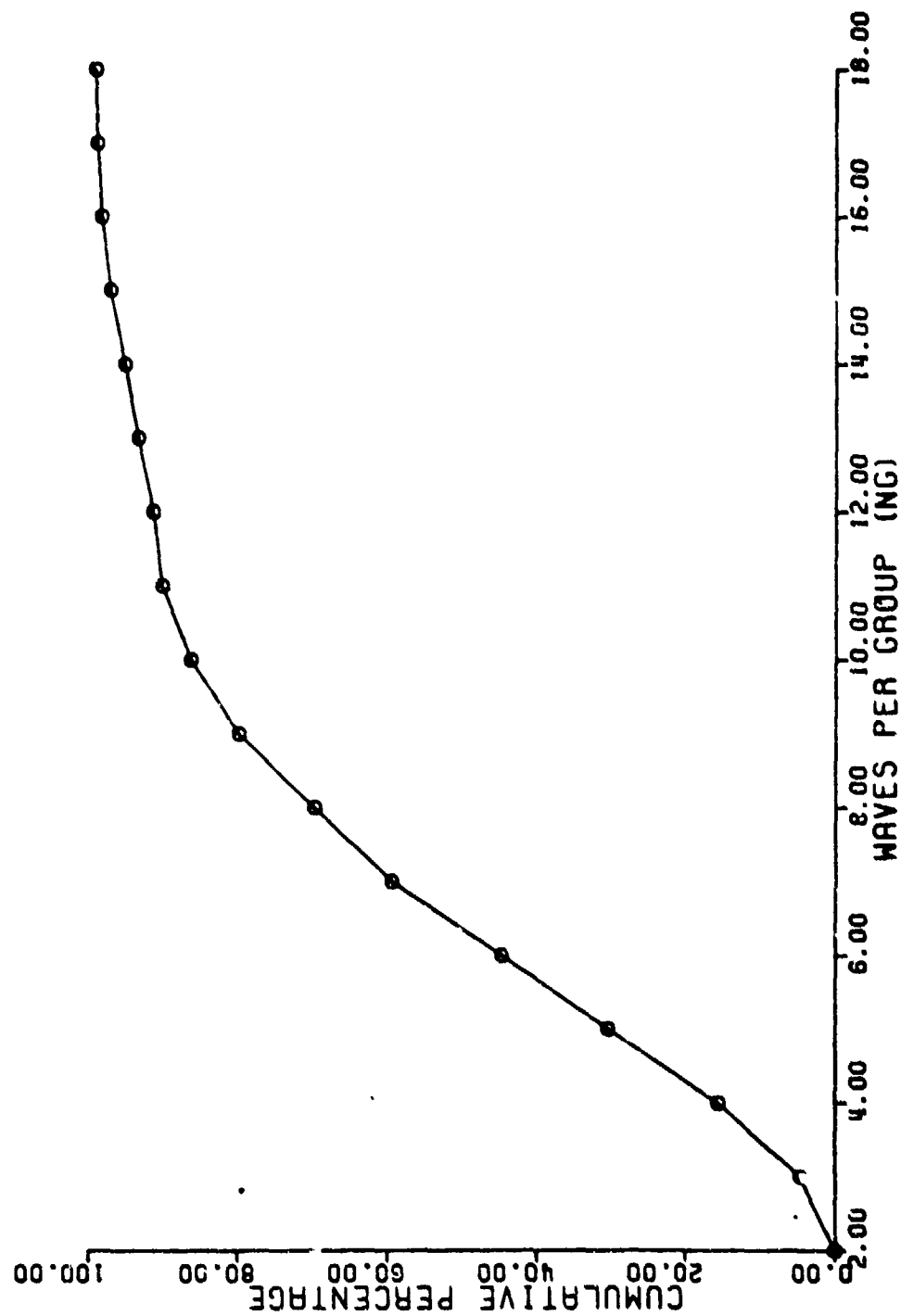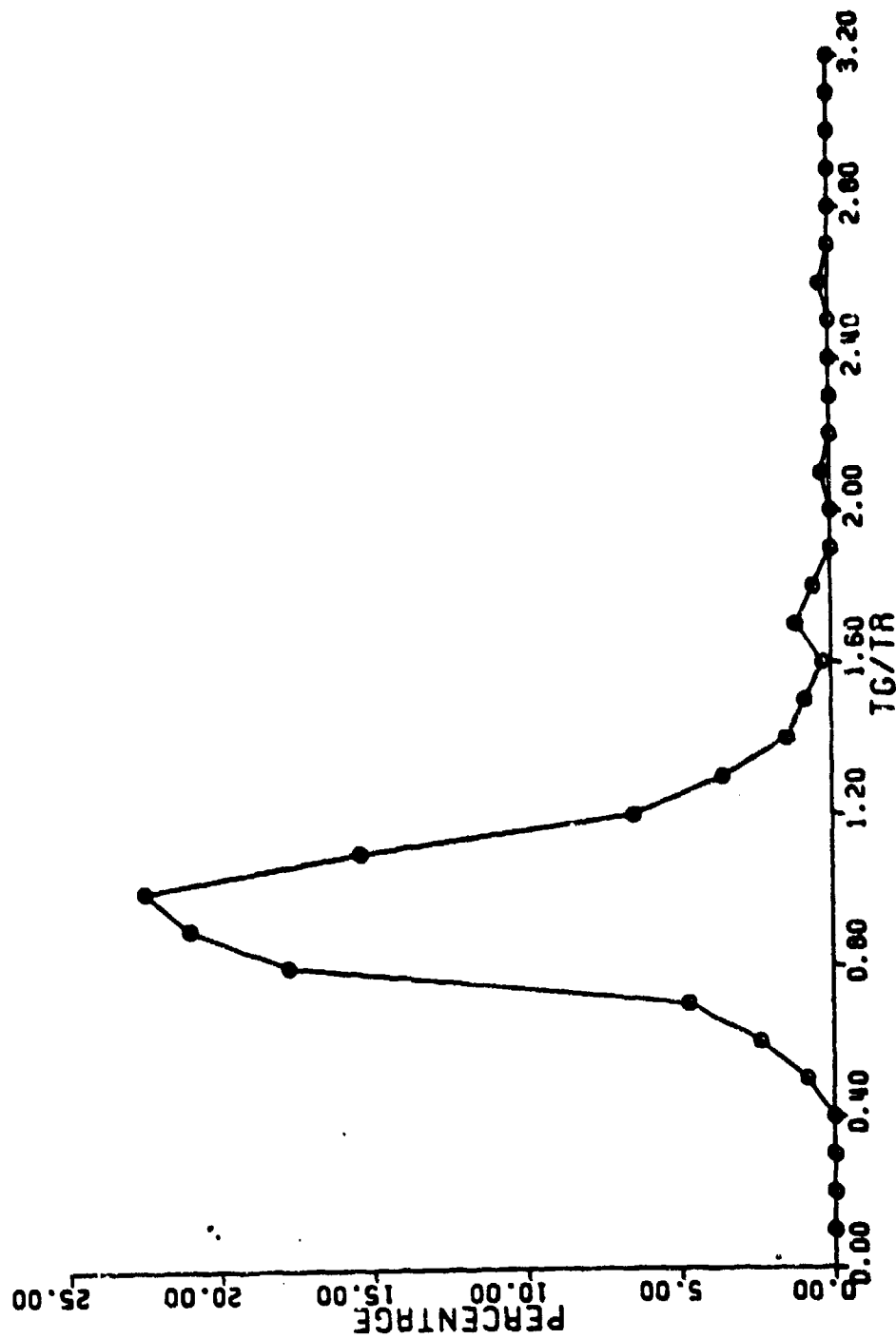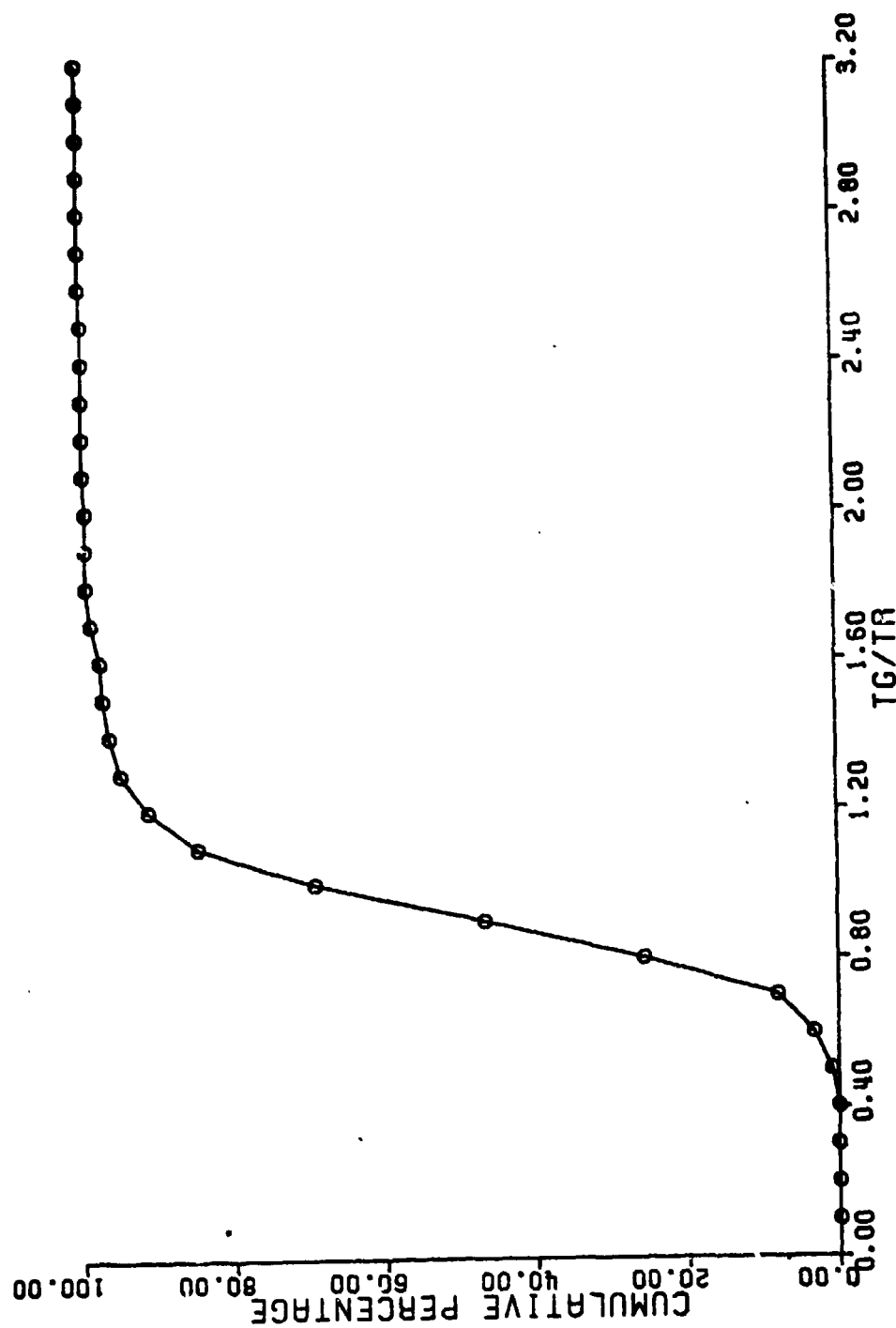
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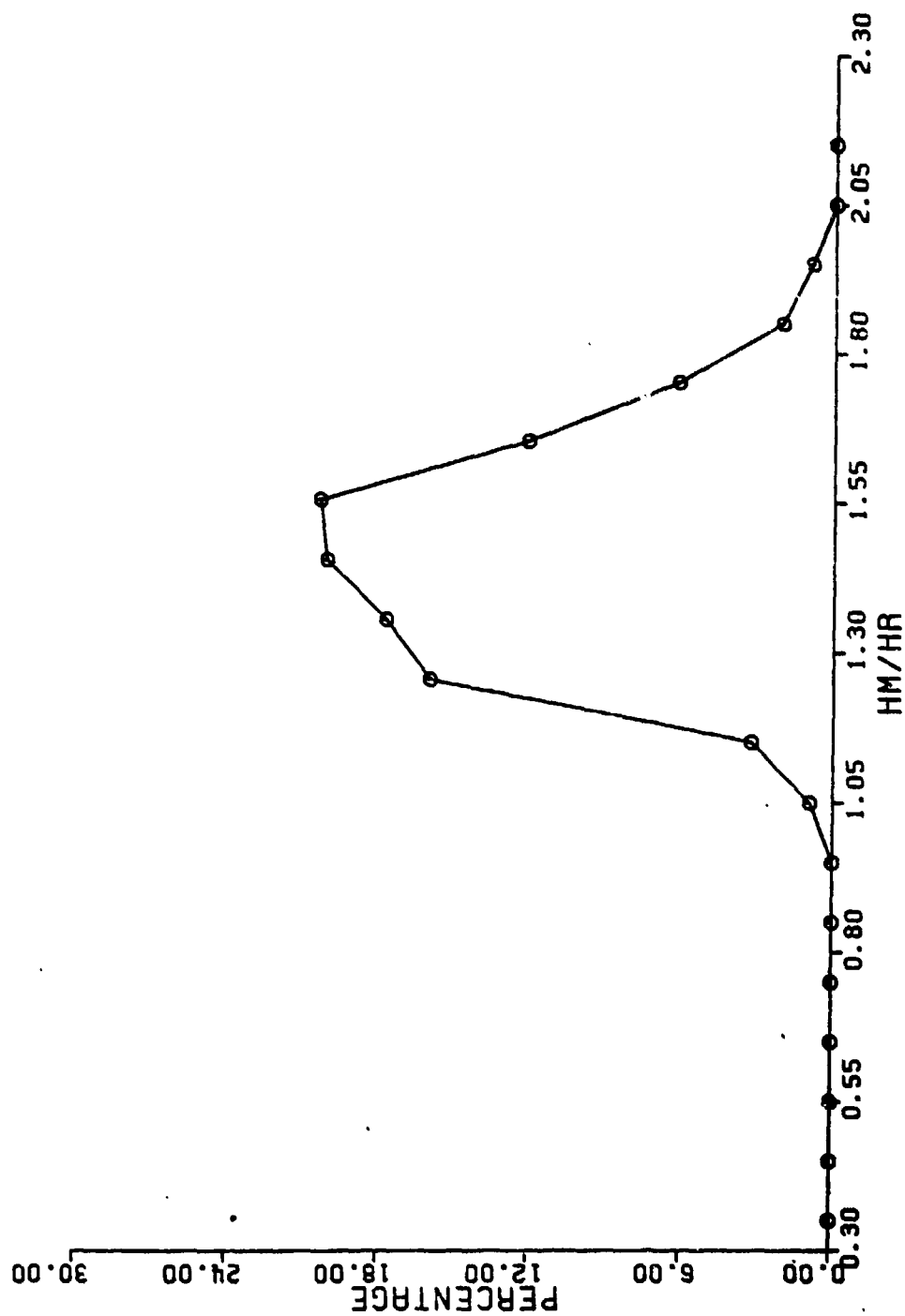
Figure 33A. Histogram of H_M/H_R for extreme wave groups: largest group V_G/V_R per wave record.
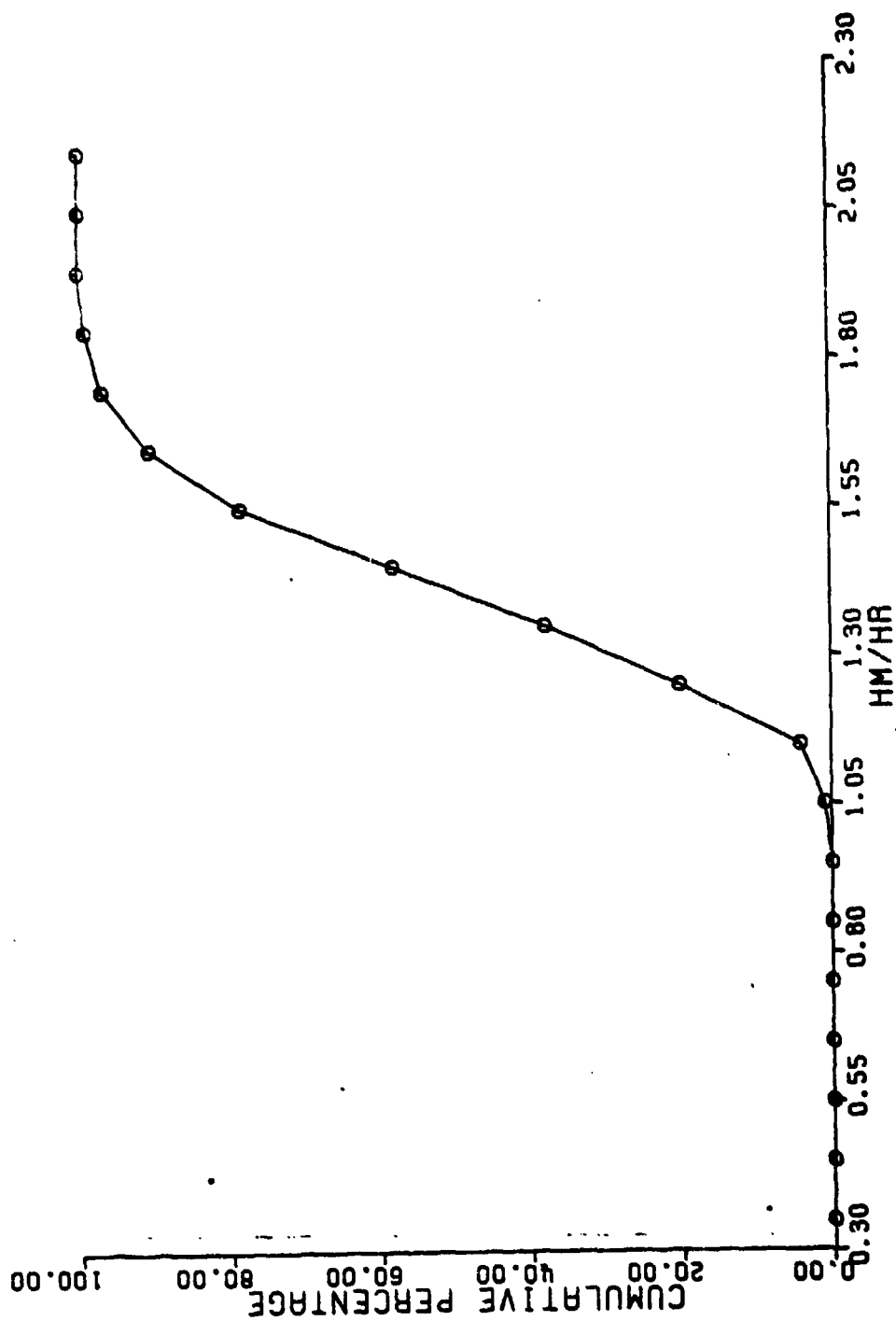
Figure 33B. Cumulative percentage distribution of H_M/H_R for extreme wave groups: largest group V_G/V_R per wave record.

APPENDIX:

WAVE RECORD SPECTRAL AND WAVE
GROUP STATISTICAL RELATIONS

A. BACKGROUND

The initial objective of this study was to examine sta-
tistical relationships among wave group and wave record
parameters, and the results have been presented in the main
text. As the study progressed, however, it became evident
that it might be possible to relate group statistics to the
frequency spectra of the wave records analyzed. Time was a
critical factor and the analytical methods employed were
simple, but the results were very promising and have been
included in this appendix. The need for a more in-depth
study is evident.

It is generally thought that spectra that are unimodal
or bimodal (one or two main spectral peaks) indicate wave
trains that have traveled from one or two sources, respec-
tively. In the study of unimodal wave record spectra,
Thompson (1972), Smith (1974), and Sedivy (1978) have shown
that the spectral peak occurs at a certain frequency or
period in a spectrum because the wave groups contain quasi-
periodic waves that approximate this period of maximum
energy density. This seems logical since wave groups may
be thought of as packets of energy concentration. In other
words, wave groups with the most energy compared to the
energy contained in the wave record as a whole $(V_G/V_R$

highest) should be responsible for that period marking the
spectral peak. It also seems reasonable to expect that
aside from the main peak of the spectrum, wave groups might
possibly be responsible for subsidiary maxima, including
bumps, peaks, and other irregularities, that commonly are
observed in frequency spectra.

Limited time permitted only a preliminary examination
of these suppositions, and the results obtained are pre-
sented below. The results are promising but far from con-
clusive. More research is needed in this area for
definitive conclusions to be drawn.

B. RELATIONSHIP BETWEEN GROUP STATISTICS AND THE FREQUENCY SPECTRUM

The first area examined was the relationship of average
wave group periods and their respective V_G/V_R ratios in a
given record to the frequency spectrum of the record. Ten
wave records were examined for this relationship. The fre-
quency spectra were printed out and the T_G and V_G/V_R value
for each group plotted thereon. Figure A-I shows an example.
As expected, most of the wave groups are seen to fall in
the area of the spectral peak although not necessarily on
the peak itself. An interesting feature of the figure is
that not a single group falls to the left of the peak. Also
noteworthy is the fact that those groups with high values of
relative energy do not necessarily fall closest to the
spectral peak period, nor do groups of low relative energy

fall in the tails of the curve. The other feature that deserves attention is that some of the more prominent secondary peaks show no wave group falling near their peak periods. It is notable that without exception every wave record so examined showed similar results.

In an attempt to determine why the plotted wave groups do not bracket the frequency peak as logic would dictate, four of the digitized wave records were printed out in analog form to large scale and hand analyzed. The first important observation that was immediately apparent concerned the accuracy to which the average wave group period can be calculated. The computer, which was constrained by the one-second digitizing rate of the data collection system, was accordingly limited to calculating wave periods to the nearest second. The hand analysis method could use interpolation and determine wave periods to a tenth of a second. Since the average group period is an average of the periods of the individual waves in the group, it was possible to determine T_G to an accuracy an order of magnitude better than the computer analysis. With the accuracy of T_G thus increased the spectra and group statistics were replotted as shown in Figure A-2. The same problems as before were still apparent. The positions of the wave groups had all shifted slightly to the left but their relationship to the spectral peak remained unchanged.

A problem area that became readily apparent was that most wave groups contain one or more waves of smaller amplitude,

and almost without exception also of shorter period, relative to the other waves in the group. Figures A-3 and A-4 show groups with waves radically different from the other waves in the group. As wave groups can be thought of in terms of not only energy content but also in terms of regularity of both the height and period of the successive waves that comprise the groups, the question arose as to whether anomalous waves should be included in a group for statistical purposes.

This particular problem arose from using the zero upcrossing method that identifies waves solely on the basis of the waveform rising (however slightly) above the mean water level. Relatively small or insignificant waves would not be counted if wave groups were being observed visually. In fact, at times, they might not even be seen against the random wave field background. It was therefore apparent that another restriction needed to be added to the wave group definition. A disqualification factor based on either the height or the period of the subject wave would be logical.

Using wave period as the basis for disqualification, a wave deletion parameter was applied under the following conditions. If a group contains only two waves neither of the two waves would be disqualified. If the group has three or four waves the shortest period wave, if any, would be disqualified. If the group contains five or more waves the two shortest period waves would be disqualified. Using the

more accurate hand-analyzed data, this method of wave dis-
qualification was applied and T_G was calculated and was
plotted as before on the power spectrum. The results are
illustrated in Figure A-5 for the same wave record as shown
in Figures A-1 and A-2. The results are startlingly differ-
ent. All wave groups now fall close to and on either side
of the main spectral peak. The results were similar for
the other wave records so analyzed. The V_G/V_R values,
however, again displayed no tendency toward being larger
as T_G/T_R approaches one. No wave groups were found to fall
on or near secondary maxima; however, later analysis showed
that those waves occurring in the intervals between wave
groups when systematically analyzed fall in the tails of
the spectrum. These results show that a relationship
evidently exists between the quasi-periodic waves constitut-
ing a group and the peak period of the system, but that the
methods and definitions used in the machine analysis were
too coarse to spell out this relationship.

It should be noted that the wave disqualification scheme
described above and used in the subsequent analyses repre-
sented a first attempt in the interest of time. This is a
very crude method for wave deletion and more sophisticated
methods (such as filtering) or definitions would probably
yield better results.

C. RELATIONSHIP BETWEEN INDIVIDUAL WAVES AND THE FREQUENCY SPECTRUM

As mentioned previously, the energy represented in a wave record may be thought of as proportional to the squares of the heights of the frequency components that are summed linearly to give the observable waves on the sea surface. However, if the wave record is considered to be composed of a finite number of components and if each wave in the record can be assumed to be representative of one of these components, then the energy of the wave record may be expected to be proportional to the squares of the heights of the waves in the wave record. Data from the hand analysis of wave records accomplished previously were coupled with this assumption and the results used to construct a form of an energy density curve. If these pseudo-energy curves thus constructed are similar to the frequency spectra for the same wave records, then a statement could be made relating individual waves in a wave record to the frequency spectrum.

The first analysis approach was made by identifying each successive wave in a record as belonging to a wave group or to the interval between wave groups. The wave groups were also identified by the standard V_G/V_R bands specified earlier. The heights of the individual waves were squared and plotted against the respective frequency. Figure A-6 illustrates this analysis for the same wave record shown in Figure A-5 (and several earlier figures). This figure is remarkably

similar in appearance to the actual spectral density curve
for the same wave record. This is true with regard to both
the distribution and magnitude of the energy. Waves belong-
ing to wave groups fall within a relatively narrow frequency
band while waves belonging to intervals between groups,
termed interval waves, are uniformly distributed over the
full range of frequencies found in the spectrum. As expected,
the interval waves are of low energy content while most of
the group waves are high in energy. This is shown by the
gradation from interval to group waves from the bottom to
the top of the graph. The other notable feature of this
figure is that the high energy waves that comprise the
higher values around the spectral peak are waves from groups
with a high V_G/V_R ratio.

Because this type of analysis is clearly suggestive of
a spectral energy distribution, another method was employed
to generate a pseudo-spectrum. This involved summing the
squares of the height values and cumulating them over the
full frequency range. Differential values, corresponding
to the width of the frequency band desired, were read from
the cumulative curve and represented the amount of energy
in these bands. Values were then plotted at the midpoint of
each band and a continuous pseudo-spectral curve generated.
The results of this method are shown in Figure A-7. The
plotted values were obtained from the cumulative curve using

a frequency bandwidth of 0.01 Hz. Curves for bandwidths

other than 0.01 Hz were generated, including a bandwidth of

0.0078 Hz which was used in the computer program to generate

the actual frequency spectrum. The bandwidth of 0.01 Hz

was used for several reasons. Shorter bandwidths tended

to give too much irregularity in the pseudo-spectral curve

while longer bandwidths tended to smooth secondary features

out entirely. Also, this choice of bandwidth was extremely

simple to work with, especially in the initial hand analysis.

A comparison between the actual spectrum and the pseudo-

spectrum shows that the locations of the main and secondary

peaks are in close agreement with those of the frequency

distribution. The relative heights between peaks do not

closely agree, however.

D. SUMMARY

A preliminary analysis of the relationships of wave groups

and individual waves (both within and without wave groups)

with the energy density spectrum was performed as a natural

outgrowth of the wave group statistical analysis. Digital

determination of the group period from the wave records

contained accuracy limitations related to the digital sampling

rate, and anomalous waves within groups (both in period and

height) were shown to be a possible source of deviation of

the average group period from the spectral peak period.

This may explain why T_G/T_R values in this study tend to be

less than unity and why the T_G/T_R distributions found are not in close agreement with the results of manual analyses of wave groups from previous studies. Indications that groups of high relative energy are responsible for the period of the spectral peak were shown, and it was also demonstrated that individual waves of large amplitude (and energy) contained in these high energy groups appear to be the factor controlling the spectral peak period. Also shown is the generation of a pseudo-spectrum from a simple statistical analysis of individual wave heights that is very similar in most respects to the actual energy spectrum.
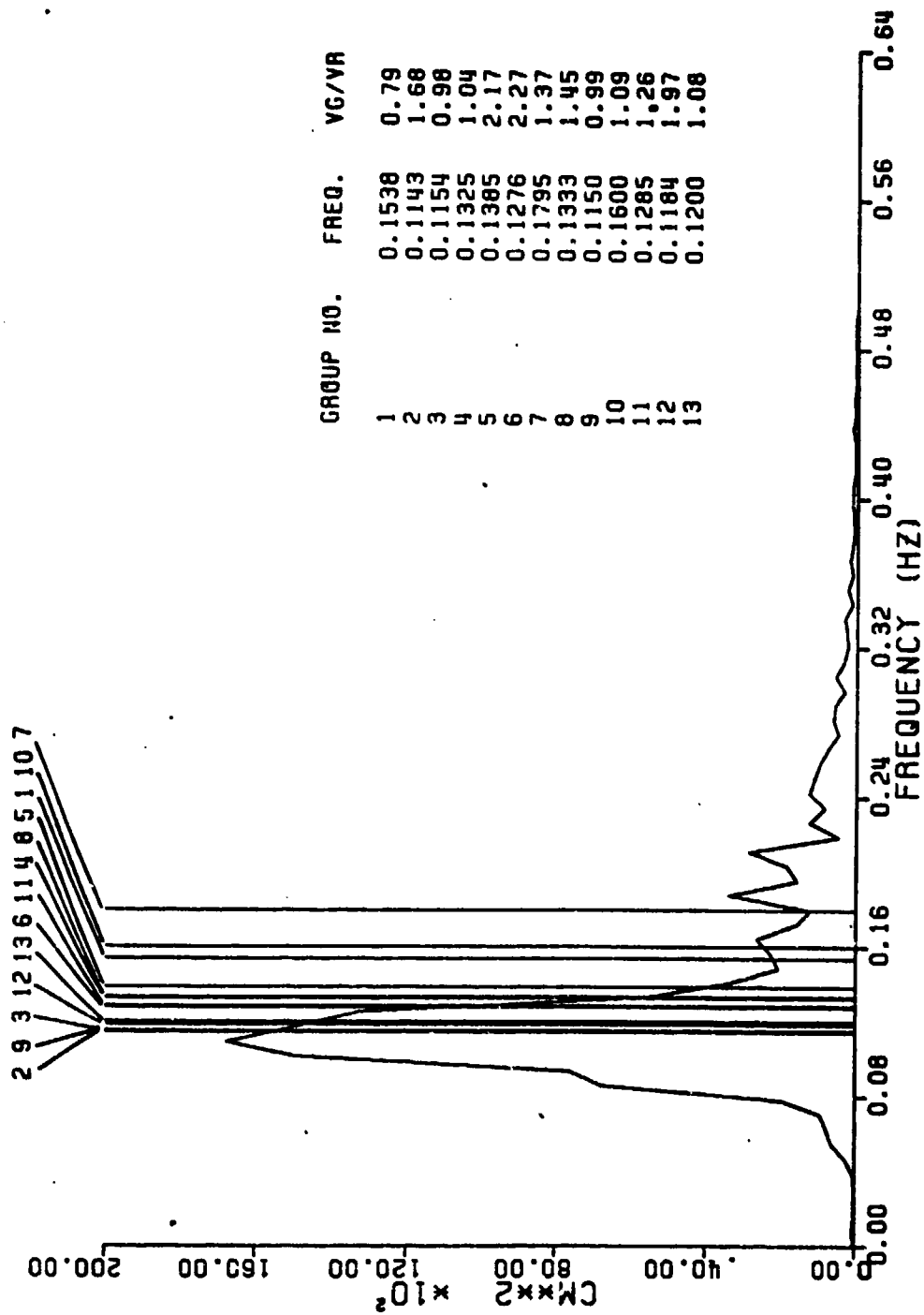
Figure A-1. Machine analysis of group T_G and y_G/y_R for wave record number 143 plotted on corresponding frequency spectrum.

GROUP NO.	FREQ.	YG/YR
1	0.1538	0.79
2	0.1143	1.68
3	0.1154	0.98
4	0.1325	1.04
5	0.1385	2.17
6	0.1276	2.27
7	0.1795	1.37
8	0.1333	1.45
9	0.1150	0.99
10	0.1600	1.09
11	0.1285	1.26
12	0.1184	1.97
13	0.1200	1.08

GROUP NO.	FREQ.	VG/VR
1	0.1379	0.79
2	0.1118	1.68
3	0.1100	0.98
4	0.1230	1.04
5	0.1358	2.17
6	0.1256	2.27
7	0.1739	1.37
8	0.1260	1.45
9	0.1143	0.99
10	0.1568	1.09
11	0.1245	1.26
12	0.1180	1.97
13	0.1153	1.08
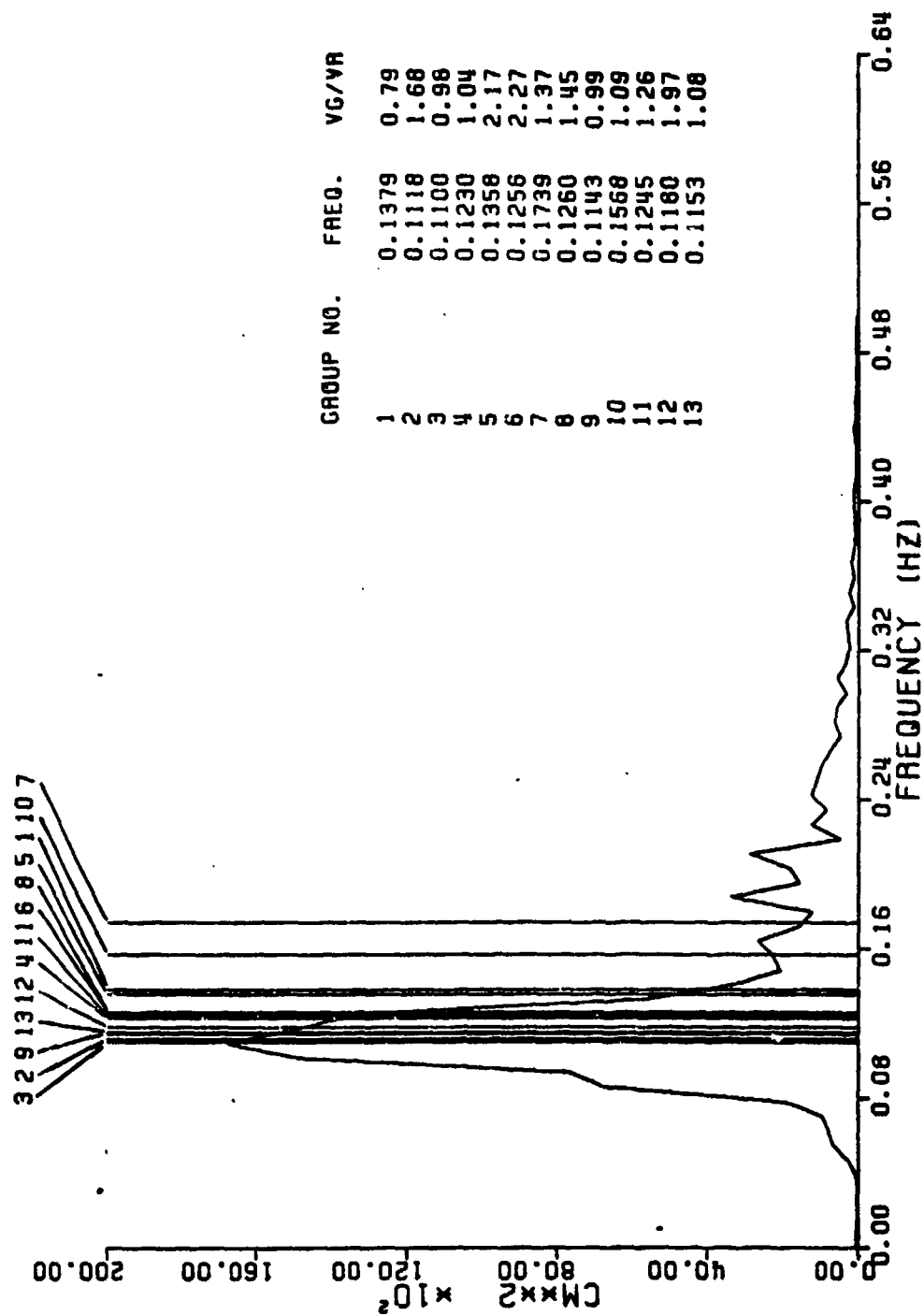
Figure A-2. Hand analysis of group T_G and V_G/V_R for wave record number 143 plotted on corresponding frequency spectrum.

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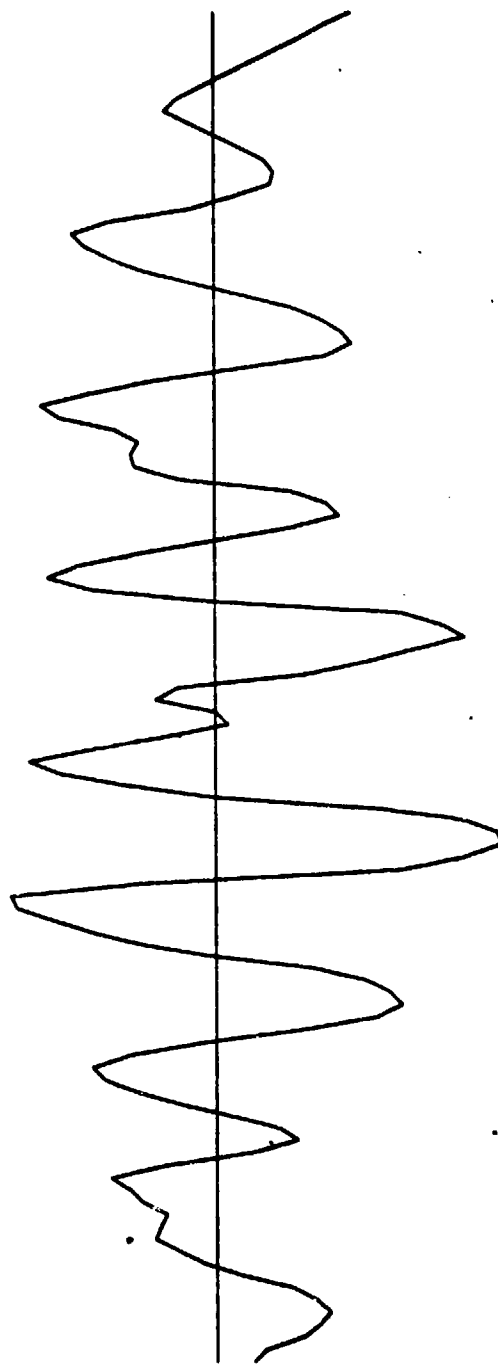
Figure A-3. Example of wave group containing non-periodic anomalous wave in middle of wave group.
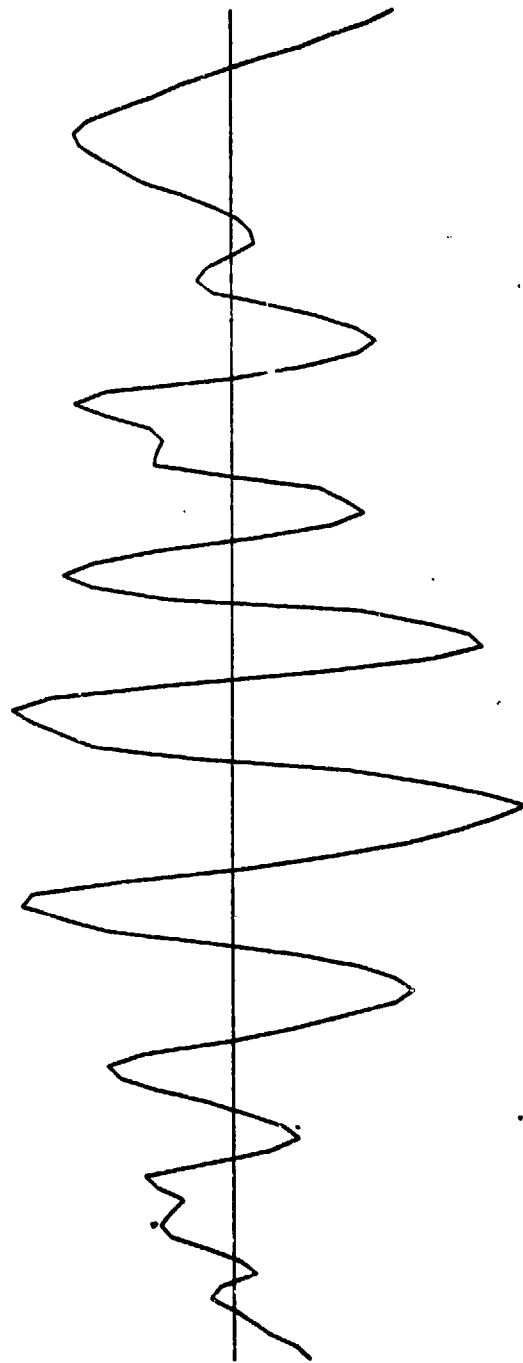
Figure A-4. Example of wave group containing non-periodic anomalous waves at beginning and end of wave group.
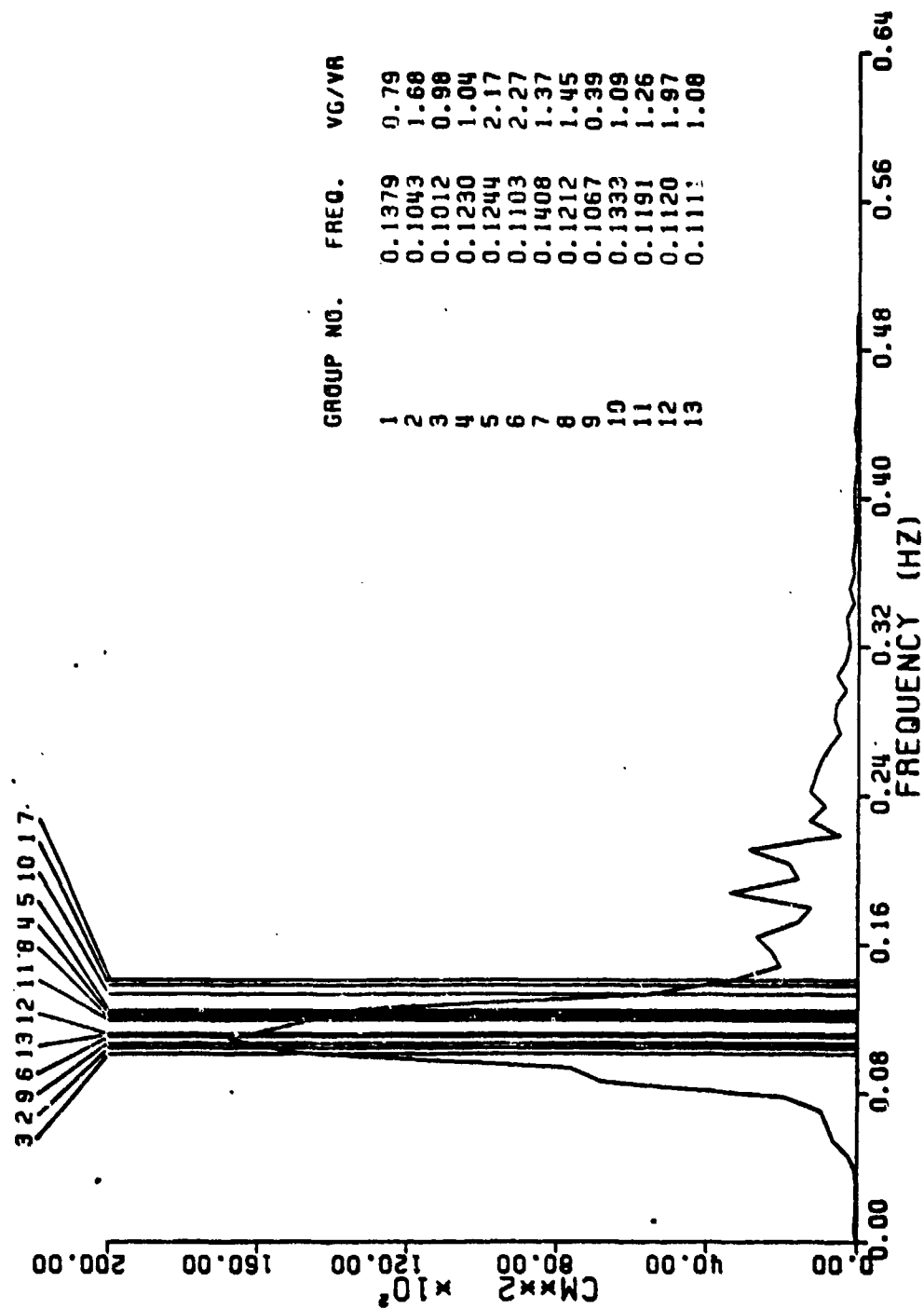
Figure A-5. Improved (anomalous wave deletion) hand analysis of group T_G and V_G/V_R for wave record number 143 plotted on corresponding frequency spectrum.

GROUP NO.	FREQ.	VG/VR
1	0.1379	0.79
2	0.1043	1.68
3	0.1012	0.98
4	0.1230	1.04
5	0.1244	2.17
6	0.1103	2.27
7	0.1408	1.37
8	0.1212	1.45
9	0.1067	0.39
10	0.1333	1.09
11	0.1191	1.26
12	0.1120	1.97
13	0.1111	1.08

Figure A-6. Plot of squares of individual wave heights versus frequency for wave record number 143.

Figure A-7. Plot of pseudo-spectrum for wave record number 143 generated by cumulating squared wave heights.

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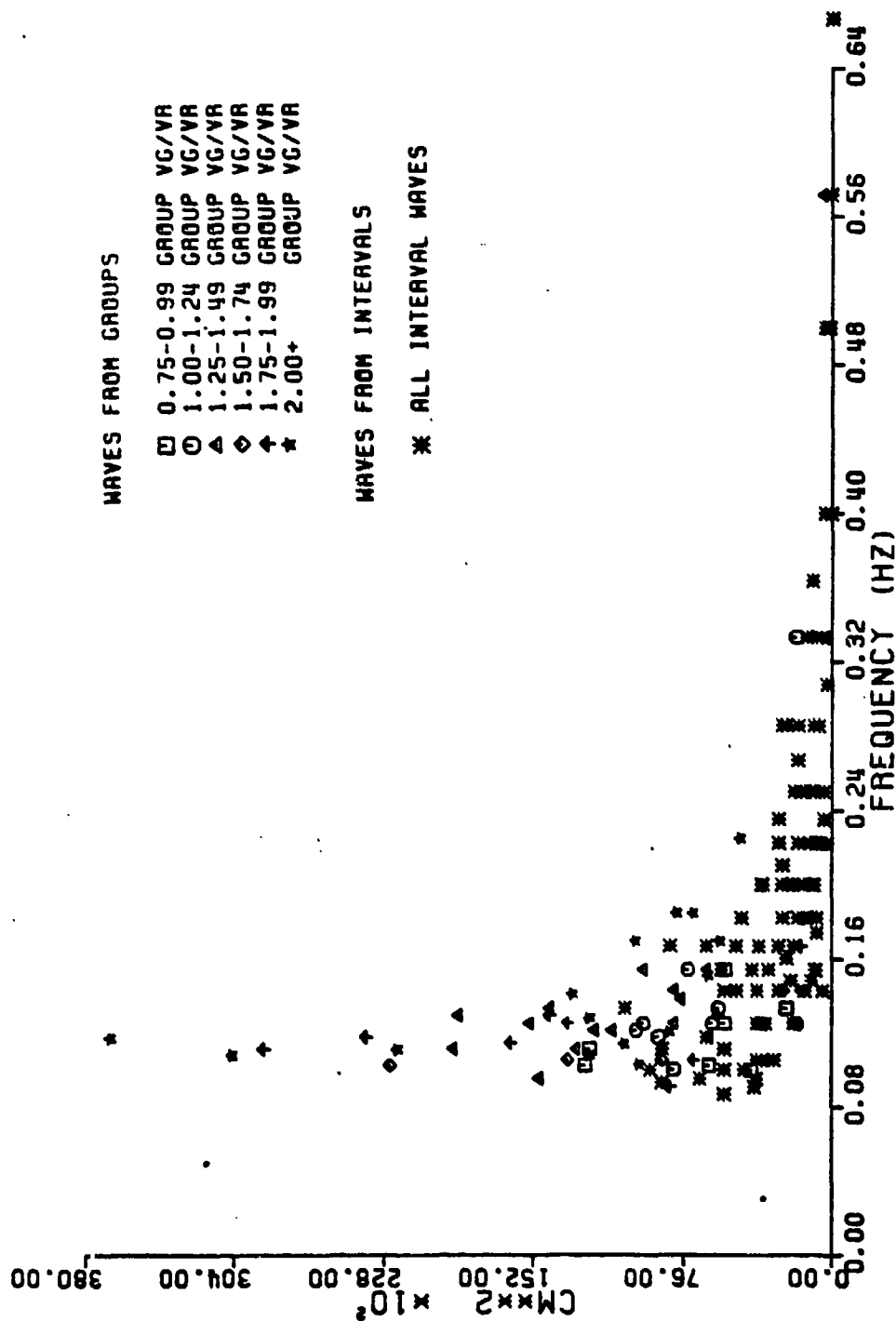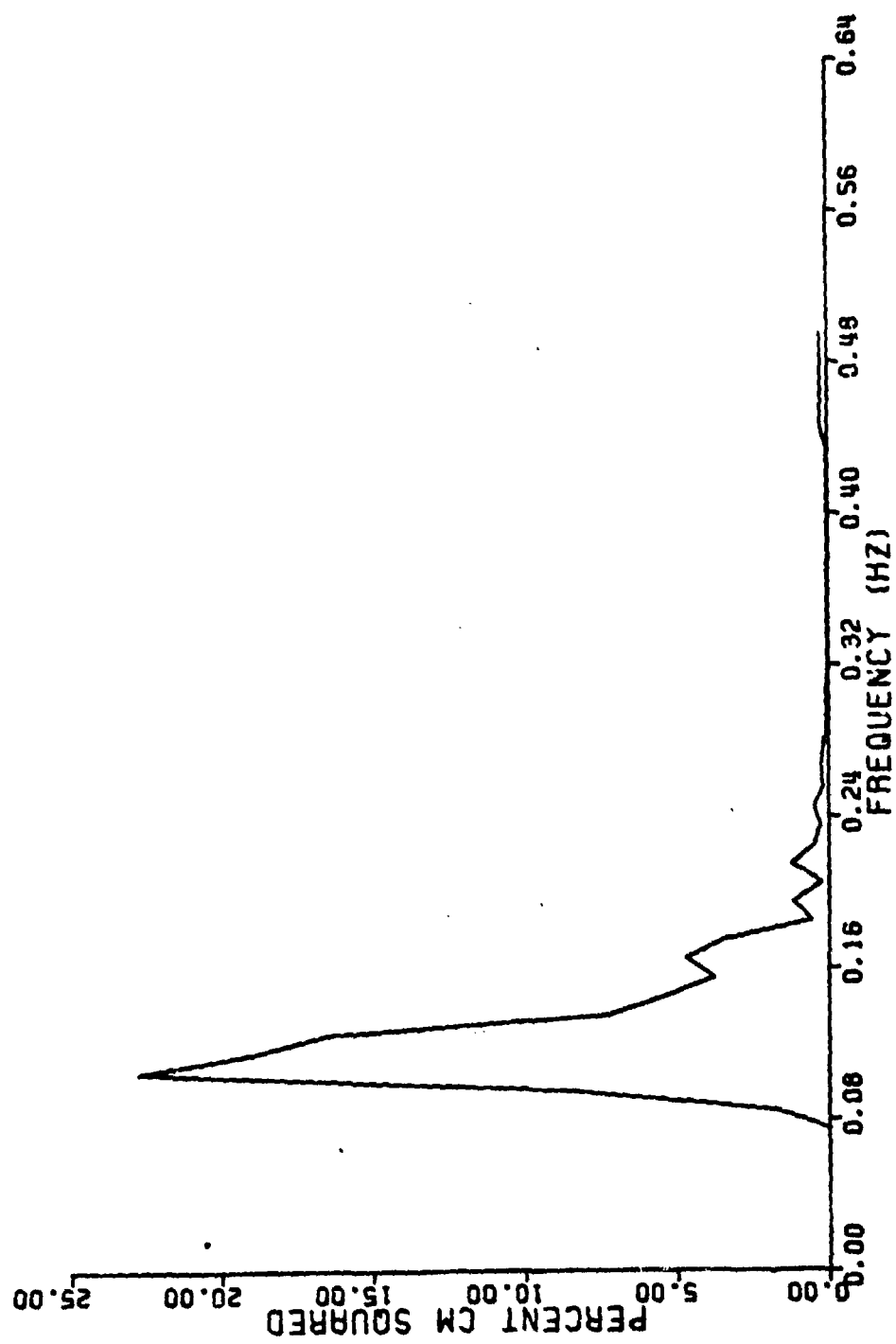4. Forristall, G. Z., 1978, "On the Statistical Distribution of Wave Heights in a Storm", Journal of Geophysical Research, v. 83 no. C5, p. 2353-2358.

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