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MATHEMATICAL THEORY OF LAMINAR COMBUSTION, X.

Effects of Shear and Strain,

9 Technical Report. No. 112 14 TR-

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The findings of this report are not to be construed as an official Department of the Army position unless so designated by other authorized documents.

Foreward

This report is Chapter X of the twelve in a forthcoming research monograph on the mathematical theory of laminar combustion. Chapter I-IV originally appeared as technical Reports Nos. 77, 80, 82 & 85; these were later extensively revised and then issued as Technical Summary Reports No's 1803, 1818, 1819 & 1888 of the Mathematics Research Center, University of Wisconsin-Madison. References to I-IV mean the MRC reports.

Contents

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Chapter X

Effects of Shear and Strain

1. Non-Uniformities.

An understanding of the response of a pre-mixed flame to non-uniformities in the flow is important in many technological situations. To sustain a flame in a high-velocity stream the turbine engineer must provide anchors, which generate strong shear. The designer of an internal combustion engine is concerned with burning rate in the swirling flow above the piston. Turbulence is all pervasive and then the flame is subject to highly unsteady shear and strain. These situations are extremely complicated, of course, and it is unlikely that mathematical analysis will ever provide detailed descriptions; those must be left to empirical studies augmented by massive numerical computations. Nevertheless, analysis of the response of a flame to a simple shear, for example, can provide useful insight into the interaction mechanism in more complex situations. Indeed the answers to such elementary questions can be used as a much-needed guide in studies of more complicated ones.

Moreover there are simple circumstances in which such analysis has direct significance. A burner flame experiences shear in the neighborhood of the rim, and its quenching depends on the local character of that shear. A flame immersed in a laminar boundary layer experiences both shear, due to velocity variations across the layer, and strain, due to streamwise variations, and its quenching will depend on their local values.

For a constant-density flow the relative motion near a point is the superposition of a simple shear and two simple strains (cf. Batchelor 1967, p. 79). These two elementary flows, simple shear and simple strain, are therefore basic to any discussion of the effect of non-uniformities on flames;

the present chapter is for the most part devoted to them. Only premixed flames are discussed and, except at the end (Sec. 5), the formulation adopted is that of near-equidiffusional flames as simplified in Sec. VIII.6.

The fundamental question is whether or not a flame can be extinguished by a non-uniform flow *per se* . In Sec. IX.5 it was shown that the proximity of a cold wall can quench an equidiffusional flame, the mechanism being heat loss from the combustion field rather than the geometrical constraint imposed on it by the wall. If flow non-uniformities can induce a similar heat flux, quenching will occur with the cold unburnt gas replacing the wall as heat sink. But even if they cannot, we may still expect unequal diffusion of heat and reactant (i.e. $\lambda \neq 0$) to provide other circumstances for which there is quenching, since that is the case for the flame tips of Sec. IX.4. When λ was sufficiently negative, the presence of a boundary quenched plane tips and flame curvature quenched axisymmetric, without the help of heat loss.

2. Response to Simple Shear.

Consider a plane parallel flow which is uniform for $y < 0$ but linearly sheared for $y < 0$, i.e.

(1) $$\mathbf{v} = (Uf, 0)$$

where

(2) $$f(y) = \begin{cases} 1 & \text{for } y > 0, \\ 1-\omega y & \text{for } y < 0. \end{cases}$$

In the limit $U \to \infty$ the equations with which we have to deal are

(3) $$f\partial(T,h)/\partial\chi = \mathbb{L}_0(T, h + \lambda T) \quad \text{for} \quad -\infty < y < F(\chi),$$

(4) $$T = T_* = H_1, \; f\,\partial h/\partial\chi = L_0(h) \quad \text{for} \quad F(\chi) < y < \infty,$$

these being the modifications of equations (IX.31,32) which take account of the flow (2). With them go the jump conditions (IX.34), together with the boundary conditions (IX.36) and

(5) $$T \to T_1, \; h \to 0 \quad \text{as} \quad y \to -\infty .$$

The uniform part of the flow can support a steady plane deflagration wave inclined at an angle such that the normal component of the gas speed equals the adiabatic flame speed, and this is assumed to be the form of the combustion field for y large and positive (Fig. 1). In other words, the initial conditions (IX.29,30) still hold, and indeed the first for $-\infty < y < F(0)$. As the flame approaches the χ-axis it is influenced by the shear as soon as its preheat zone penetrates the lower half-plane. The increased gas speed there may be expected to deflect the flame, so that its slope decreases.

The above formulation is due to Buckmaster (1979a), who reaches the following conclusions. For some values of the shear gradient ω and Lewis-number parameter λ, the slope actually decreases to zero. At that point the flame speed is zero and, according to the hypothesis of Sec. IX.4, quenching occurs. Invariably the quenching point lies in the uniform region although the preheat zone has, of course, penetrated the shear region. For the remaining values of ω and λ the flame is never quenched even though it may fail to penetrate the shear region. A rationalization for not being quenched in the shear region is that the non-uniformity, as measured by f'/f, is a maximum at $y = 0$ so that, if the flame manages to cross the boundary, it can survive the ever-weaker non-uniformity beyond. Certainly there is a solution of the governing equations as $y \to -\infty$ corresponding to a locally plane flame propagating with unit speed (cf. Sec. 9.5).

Clear evidence of the dichotomy is shown for $\omega = 5$ by the numerical results plotted in Fig. 2. For $\lambda = 0$ or 5 the flame penetrates the shear and shows no sign of quenching; indeed no solution showed the flame speed approaching zero in $y < 0$. But for $\lambda = 10$ the flame speed falls to zero at the point Q outside the shear region. The figure also shows that the shear has more effect on the flame as λ increases, in contrast to flame tips (Fig. IX.6) where the boundary, i.e. symmetry line, has less. Moreover, the present trend does not agree with that for slowly varying flames in Sec. 5.

The effect of increasing ω can be seen from the limit $\omega \to \infty$. There is then a fully developed combustion field, i.e. an asymptotic solution as $\chi \to \infty$, in which the flame lies at a definite level in $y > 0$ and T, h are independent of χ. In fact, in the shear region away from the boundary convection dominates for all χ, so that

(6) $$\partial T/\partial \chi = \partial h/\partial \chi = 0 \quad \text{for} \quad y < 0,$$

i.e. there is no dependence on χ anywhere. The problem is therefore reduced to one in $y > 0$ only, under the boundary conditions

(7) $$T = T_1, \ h = 0 \quad \text{on} \quad y = 0.$$

In the limit $\chi \to \infty$ the initial conditions may be ignored and the system then has the solution

(8) $$T = T_1 + Y_1 y/F(\infty), \ h = -\lambda Y_1 y/F(\infty) \quad \text{for} \quad 0 < y < F(\infty),$$

(9) $$T = T_* = H_1, \ h = -\lambda Y_1 \quad \text{for} \quad F(\infty) < y < \infty,$$

where

(10) $$F(\infty) = \exp(\lambda Y_1/2T_*^2)$$

is the asymptotic height of the flame. The expression (9b) violates the boundary condition (IX.36) for h, showing that the limits $y \to \infty$ and $\chi \to \infty$ do not commute. There is an adjustment at large values of y, which depends on the history of the flame; but that is of no importance for our purposes.

$F(\infty)$ is large even for moderate values of λ. On the other hand, inspection of Fig. 2 shows that flames are first affected by the shear when F is about 2. The implication when $\omega = \infty$ is that, for sufficiently large $\lambda > 0$, there is a minimum in F, i.e. the flame speed must have fallen to zero at some finite value of χ. Numerical solutions of the limit problem (Fig. 3) point to $\lambda = 0$ as the critical value: for $\lambda < 0$ the function F decreases monotonically to its asymptotic value (10), whereas for $\lambda > 0$ it has a minimum (which we have taken to imply quenching). We conclude that a sufficiently strong shear flow will prevent a flame from reaching the boundary $y = 0$ and actually quenches it for $\mathcal{L} > 1$.

Eventually the limit equations break down and a new coordinate χ/ω is needed, corresponding to significant diffusion in the shear region. Buckmaster (1979a) argues that F increases again when $\lambda < 0$ is sufficiently large, which implies quenching for $\mathcal{L}$ sufficiently smaller than one also. However, no numerical results exhibiting such quenching have yet been obtained.

3. Response to Simple Strain.

In contrast to simple shear, the variation of the essential velocity component in simple strain is in its own direction rather than at right angles.

The distinguished limit of Sec. VIII.6, in which the non-dimensional gas velocity and the length scale on which it varies become unbounded but their ratio remains fixed, leads to the governing equations (VIII.70). The unit of the velocity gradient represented by ϵ is that of the flame speed divided by the thickness under adiabatic conditions. As we shall see, a velocity gradient that is small on this scale has little effect on the flame, whereas a large enough gradient extinguishes it.

Equations (VIII.70) hold ahead of the flame, i.e. for $y > F(x)$. Behind the flame we have

(11) $$T = T_* = H_1,\ h = h_* \quad \text{for} \quad 0 < y < F(x)$$

if (a) the wall is insulating or (b) the flame is located in equal opposing jets (Fig. 4) so that

(12) $$\partial h/\partial y = 0 \quad \text{on} \quad y = 0.$$

(The condition ensures that wall quenching does not intrude.) For case (a) the flame will in reality be imbedded in a boundary layer (unless the Prandtl number is very small), so that the model is more directly relevant to case (b). We must add the boundary conditions

(13) $$T \to T_1,\ h \to 0 \quad \text{as} \quad y \to \infty$$

and the jump conditions (IX.34).

Since x only appears in F there is a solution with the flame lying parallel to $y = 0$, the constant value of F_0 being given implicitly by

(14) $$\beta = \frac{\pi^{\frac{1}{2}}}{2} e^{d^2} \operatorname{erfc} d \exp\left[\frac{\lambda}{2H_1^2}\left(\frac{d\, e^{-d^2}}{\pi^{\frac{1}{2}} \operatorname{erfc} d} - \tfrac{1}{2} - d^2\right)\right],$$

where

$$(15) \qquad \beta = (\epsilon/2)^{\frac{1}{2}}, \ d = \beta F_0.$$

The solution ahead of the flame is

$$\left.\begin{aligned} &(16) \qquad T = T_1 + \operatorname{erfc}(\beta y)/\operatorname{erfc} d \\ &(17) \qquad h = h_* \frac{\operatorname{erfc}(\beta y)}{\operatorname{erfc} d} + \frac{\lambda}{\pi^{\frac{1}{2}}\operatorname{erfc}^2 d}\left[\beta y e^{-\beta^2 y^2}\operatorname{erfc} d - d e^{-d^2}\operatorname{erfc}(\beta y)\right] \end{aligned}\right\} \text{ for } y > F_0,$$

where

$$(18) \qquad h_* = 2H_1^2 \ln(2\beta\, e^{-d^2}/\pi^{\frac{1}{2}}\operatorname{erfc} d).$$

This determination of h_* also completes the description (11) behind the flame.

Of greatest interest is the variation in stand-off distance F_0 with the straining rate ϵ (Fig. 5). There are two kinds of response, depending on the value of λ. For $\lambda < 4H_1^2$, it is single-valued, with F_0 decreasing monotonically to zero at $\epsilon = \epsilon_c$, where

$$(19) \qquad \epsilon_c = \pi \exp(-\lambda/2H_1^2).$$

For $\epsilon > \epsilon_c$, there is no solution. For $\lambda > 4H_1^2$, the response is double-valued for some values of ϵ; and there is no solution beyond a value ϵ_e, at which F_0 ~~is~~ $= F_e$ non-zero. Corresponding values of ϵ_e and F_e for various values of λ are listed in Table 1 for the same parameter value $H_1 = 1.2$ as in Fig. 5.

Table 1

λ	5.76	6.03	6.48	7.03	7.52	8.05	9.05	10.02	11.02	12.02	14.08	16.00	18.00
F_e	0	.23	.62	1.11	1.57	2.07	3.05	4.03	5.05	6.10	8.29	10.36	12.55
ϵ_e	.212	.194	.170	.146	.130	.116	.096	.082	.072	.064	.052	.044	.038

The physical picture is now clear. Suppose the strain is increased from a small value sufficiently slowly for the combustion field to be quasi-steady. The increase causes the flame to move closer to the stagnation point. If λ is large enough, it reaches the stagnation point for the critical value ϵ_c, and is extinguished. Otherwise extinction occurs for ϵ_e, when it is still a distance F_e away. In the latter case the lower branch of the response (where F_o increases with ϵ) is unstable, as has been shown by Buckmaster (1979b). (His treatment does not rule out other parts of the response being unstable too, so that the extinction picture above may have to be modified.)

The flame speed W (Fig. 6) i.e. the ga speed normal to the flame sheet, has the value ϵF. As $\epsilon \to 0$, so that $F_o \to \infty$, it tends to the adiabatic value 1; when $\epsilon = \epsilon_c$ it vanishes, since F_o is zero. Like F_o it is single-valued for $\lambda < 4H_1^2$ and partly double-valued for $\lambda > 4H_1^2$. There is an initial increase in W for $\lambda < -2H_1^2$; otherwise it decreases monotonically until $\epsilon = \epsilon_c$ or ϵ_e. It is therefore possible for the flame to have a speed greater than its adiabatic value, even though it is being stretched at a positive rate.

In short, there is a limit (depending on the Lewis number) to the amount of strain that a premixed flame can tolerate. The conclusion is consistent with the experimental fact that a flame held at the front of a bluff body can be blown off by increasing the gas speed sufficiently. Attachment is then usually transferred to the rear.

4. Response to More General Non-Uniformity.

Reduction to a parabolic problem occurs for general non-uniform flows in which the velocity is everywhere large but changes proportionately over correspondingly large distances (Buckmaster 1979b). An example is the potential motion of a fast stream past a large cylinder, whose front stagnation point was the subject of the last section. Such flows, when plane, are characterized by velocity fields

$$\text{(20)} \qquad \underline{v} = U\underline{q}(x/U, y/U);$$

letting $U \to \infty$ gives a distinguished limit in which $\nabla \underline{v}$ remains finite. We shall see that the effective part of the limiting motion is, on the x,y-scale, made up of a fast uniform flow and a simple strain, both of which vary slowly. Shear is ineffective, so that the term general straining motion is appropriate.

Take curvilinear coordinates with x measuring distance along one particular streamline and y distance from it. Away from a stagnation point these will be slowly varying cartesian coordinates, since the curvature of streamlines is vanishingly small. The angle between the flame sheet and an intersecting streamline is also small, so that

$$\text{(21)} \qquad \chi = x/U \quad \text{and} \quad y$$

are appropriate variables for T and h. Likewise the velocity field may be approximated by

(22) $\mathbf{v} = (Uq_0, -q_0'y)$,

where $q_0(\chi)$ is the speed on $y = 0$ and continuity has been taken into account. Only leading terms have been retained, so that a simple shear due to variation of the x-component in the y-direction is ignored. Uniformity breaks down at large distances from the flame sheet, but it does not matter there. On the x,y-scale the velocity relative to that at any point is a simple strain with $\epsilon = q_0'$ (Sec. VIII.6). Together with the fast uniform flow $(Uq_0, 0)$ at the point, both of which vary with χ, it makes up the effective part of the limiting motion.

In the limit, only the approximation (22) enters into the governing equations (XIII.62, 63), which become

(23) $$\left(q_0 \frac{\partial}{\partial \chi} - q_0'y \frac{\partial}{\partial y}\right)(T,h) = L_0(T, h + \lambda T).$$

The response to an arbitrary streamwise velocity $q_0(\chi)$ may be determined from the computer as easily as that to simple shear (Sec. 2). Note that equations (VIII.67, 68), which were used in Sec. IX.4, are the special case $q_0 \equiv 1$ when the strain vanishes everywhere. The simple shear flows discussed in Sec. IX.5 and Sec. 2 are not of the type considered here since changes of the velociy field in the y-direction are large. As already noted, the analysis breaks down at a stagnation point, so that equations (VIII.70) cannot be recovered. Nevertheless, the stagnation-point solution in Sec. 3 has the required form for large x, with $q_0 = \epsilon \chi$, and hence provides initial conditions for any $q_0 \sim \epsilon \chi$ as $\chi \to 0$ when the boundary conditions on T and h are compatible.

Two examples will be considered, namely

$$q_0(\chi) = \begin{cases} \epsilon \sin \chi \text{ for } 0 < \chi < \pi, & (24) \\ \epsilon (\chi - 2\chi^3/3 + \chi^5/5) \text{ for } \chi > 0. & (25) \end{cases}$$

The first corresponds to potential flow around the cylinder and both asymptote $\epsilon\,\chi$. If therefore the same boundary conditions as in Sec. 3 are imposed, namely

(26) $$T \to T_1,\ h \to 0 \quad \text{as} \quad y \to \infty,$$

(27) $$\partial h/\partial y = 0 \quad \text{on} \quad y = 0,$$

then the results obtained there provide appropriate initial conditions here. Of course, behind the flame the appropriate solution of equations (23) has $T = T_* = H_1$.

The flows (24) and (25) are quite different in nature. For the first the stretch, represented by the Karlovitz number q_0', decreases monotonically from ϵ to $-\epsilon$ (Fig. 7a); but for the second it decreases from ϵ to a minimum of zero at $\chi = 1$ and then increases indefinitely (Fig. 7b). These characteristics are particularly relevant to an examination of the flame speed

(28) $$W = (q_0 F)',$$

which can deduced from the computations by numerical differentiation.

Fig. 7 shows that W increases monotonically for the flow (24); but that for the flow (25) it increases initially, reaches a maximum around $\chi = 1$, and then decreases. It is hard to resist the conclusion that there is an inverse correlation between the changes in flame speed and stretch, at least for moderate values of λ. The correlation is not precise (as we have already seen in Sec. 3): in Fig. 7b the flame speed achieves its maximum exactly at $\chi = 1$ for only one value of λ (which appears to be zero). But the general trend is undeniable.

The computations for the flow (25) show that the decrease in W from its maximum continues until zero is reached where, according to Sec. IX.4, quenching occurs. The results therefore suggest the general conjecture that a flame subjected to sufficiently strong and increasing strain will be extinguished. Hitherto a sound mathematical basis for such a conjecture has been lacking.

5. The Effects of Slowly Varying Shear and Strain.

The chapter ends with an examination of slowly varying flames under the influence of simple shear and simple strain. For strain the results merge, as $\mathcal{L} \to 1 \pm 0$, with those of near-equidiffusional flames, as $\lambda \to \pm\infty$; but for shear they do not. We have no explanation to offer.

For simple shear, the ξ,η-coordinates of Sec. VIII.3 are now taken as shown in Fig. 8. Then the velocity field to be inserted in equation (VIII.32) is

(29) $$\underset{\sim}{v} = (Uf, 0)$$

where

(30) $$f(\eta) = \begin{cases} 1 & \text{for } \eta > 0 \\ 1 - \omega\eta & \text{for } \eta < 0. \end{cases}$$

If

(31) $$\eta = F(\xi)$$

is taken to be the locus of the flame, then for $\eta < 0$ we have

(32) $$M_n = \rho_1 U(1 - \omega F)|F'|(1 + F'^2)^{-\frac{1}{2}}, \quad v_{\perp 1} = U(1-\omega F)\operatorname{sgn} F'(1 + F'^2)^{-\frac{1}{2}}$$

if the latter is measured in the s-direction shown. Hence

(33) $$\underline{v}_{\perp 1} \cdot \underline{\nabla}_{\perp} M_n - M_n \underline{\nabla}_{\perp} \cdot \underline{v}_{\perp 1} = M_n^2 \operatorname{sgn} F'(1 + F'^2)^{-\frac{1}{2}} F''/\rho_1 F'^2$$

and the basic equation (VIII.32) becomes

(34) $$F'' = k(1-\omega F)F'^3 \ell n M_n$$

where k is the constant (IX.21).

The corresponding formulas for $\eta > 0$ are obtained on setting $\omega = 0$, and then equation (IX.20), governing the shape of a flame tip in a uniform flow, is recovered (with $\tan\psi = F'$). The appropriate solution is

(35) $$F' = -\tan \alpha$$

corresponding to a plane flame, although a curved flame (corresponding to the tip) is also available when $\mathcal{L}$ is greater than 1. We therefore take

(36) $$F(0) = 0, \quad F'(0) = -\tan \alpha$$

as initial conditions for the governing equation (34) in $\eta < 0$.

Fig. 9 shows the effect of the shear. For $\mathcal{L} < 1$ the flame is bent towards the flow direction at a rate that increases with ω, but is never actually quenched. On the other hand, the flame bends away from the flow direction for $\mathcal{L} > 1$, again at a rate that increases with ω. These results are consistent with those obtained for flame tips in a uniform flow (Sec. IX.3), where a rounded tip was possible for $\mathcal{L} > 1$ but not for $\mathcal{L} < 1$. However, they do not merge with those for a near-equidiffusional flame (Sec. 2), where the turning ability of the shear increased with λ. (The inconsistency of Secs. 2 and IX.4 has already been noted in the later section.)

We turn now to simple strain, i.e. stagnation-point flow, where the results do merge with those for near-equidiffusional flames. The comparison involves the so-called flat flames here; for $\mathcal{L} < 1$ it turns out that such flames can tolerate any amount of strain, but for $\mathcal{L} > 1$ there is a limit.

With axes as shown in Fig. 10, the velocity field with which we have to deal is

(37) $$\underset{\sim}{v} = \epsilon(\xi, -\eta).$$

For the locus (31) we find

(38) $$M_n = \rho_1 \epsilon(\xi F' + F)/(1 + F'^2)^{\frac{1}{2}}, \quad v_{\perp 1} = \epsilon(\xi - FF')/(1 + F'^2)^{\frac{1}{2}}$$

so that

$$\underset{\sim}{v}_{\perp 1} \cdot \underset{\sim}{\nabla}_{\perp} M_n - M_n \underset{\sim}{\nabla}_{\perp} \cdot \underset{\sim}{v}_{\perp 1} = \frac{M_n^2}{\rho_1 (1+F'^2)^{\frac{1}{2}}} \frac{d}{d\xi}\left(\frac{FF' - \xi}{F + \xi F'}\right)$$

and the basic equation (VIII.32) becomes

(39) $$(\xi^2 + F^2)F'' + (1 + F'^2)(\xi F' - F) = k(F + \xi F')^3 \ln M_n,$$

where

(40) $$k = -2T_*^2 \rho_1^2 \epsilon / b.$$

Solution of this equation requires two boundary conditions but only one is obvious, namely the symmetry condition

$$F'(0) = 0 . \tag{41}$$

The difficulty reflects the elliptic nature of the problem and, for determinacy, some reference must be made to the far field. If, for example, we are dealing with a fast flow past a correspondingly large cylinder (i.e. with ϵ fixed) then it is appropriate to look for a constant solution, corresponding to a flame that varies on the scale of the cylinder. Such a solution is given by

$$M_n^2 \ell n\, M_n^2 = -2\rho_1^2 \epsilon^2 / k \quad \text{with} \quad M_n = \rho_1 \epsilon\, F. \tag{42}$$

Sivashinsky (1977), who considers the axisymmetric counterpart of the present problem, proposes that such a choice is correct in all circumstances but there is no evidence for his view. Indeed, completely acceptable solutions are obtained by giving $F(0)$ a value that does not satisfy equation (42), see Fig. 11. A constant solution is just a separatrix or an asymptote in the figure.

Fig. 12 shows that there is always a unique solution of equation (42) for $b > 0$ but that the condition

$$\epsilon \leq -T_*^2/eb \tag{43}$$

must be met for $b < 0$, and then there are two solutions. We conclude that there is always a flat flame for $\mathcal{L} < 1$ but that there is a limit to the straining rate for $\mathcal{L} > 1$. To reconcile these results with those for near equidiffusional flames note that here ϵ is measured on the scale θ^{-1} so that the limit (43) tending to infinity as $\mathcal{L} \to 1 + 0$ is $(b \to -0)$ is compatible with ϵ_e remaining finite there as $\lambda \to \infty$ (suggested by Table 1). On the other hand, $\epsilon_c \to \infty$ as $\lambda \to -\infty$ agrees with the absence of a limit here for any $\mathcal{L} > 1$ (in particular $\mathcal{L} = 1 + 0$).

Fig. 12 could have been introduced after the general relation (VIII.43) to suggest that extinction will occur whenever

$$\frac{1}{\Delta}\frac{d\Delta}{d\tau} \lessgtr -\frac{T_*^2}{be} \quad \text{for} \quad b \gtrless 0. \tag{44}$$

However, without a guarantee that such voluminal stretch is possible, the speculation would have been idle. Stagnation-point flow provides the first example, and so far the only one. If we restrict attention to flat flames, then the voluminal stretch is constant and equal to ϵ. The condition (44) for $b > 0$ can never be realized but that for $b < 0$ can: if, when $\mathcal{L}$ is greater than one, ϵ is increased sufficiently slowly for the combustion field to be quasi-steady, then extinction occurs when it reachers $-T_*^2/be$.

These remarks apply only to the constant solutions in Fig. 12. The non-constant solutions are available whatever the value of ϵ or the sign of b. No evidence was found of quenching in the computations (cf. Sec. 2) i.e. the curves never became tangent to streamlines.

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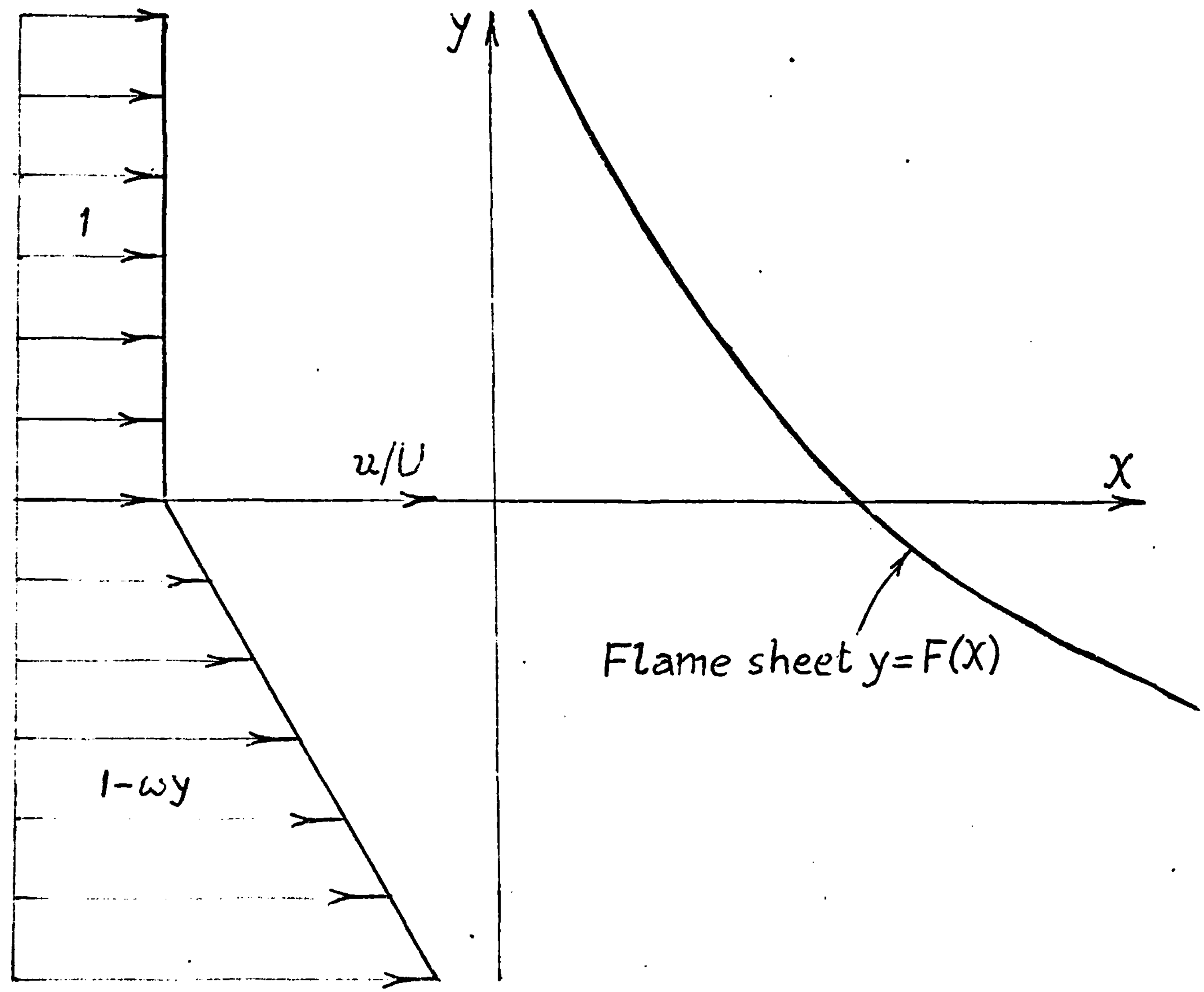

Fig. 1 Notation for flame approaching shear flow.

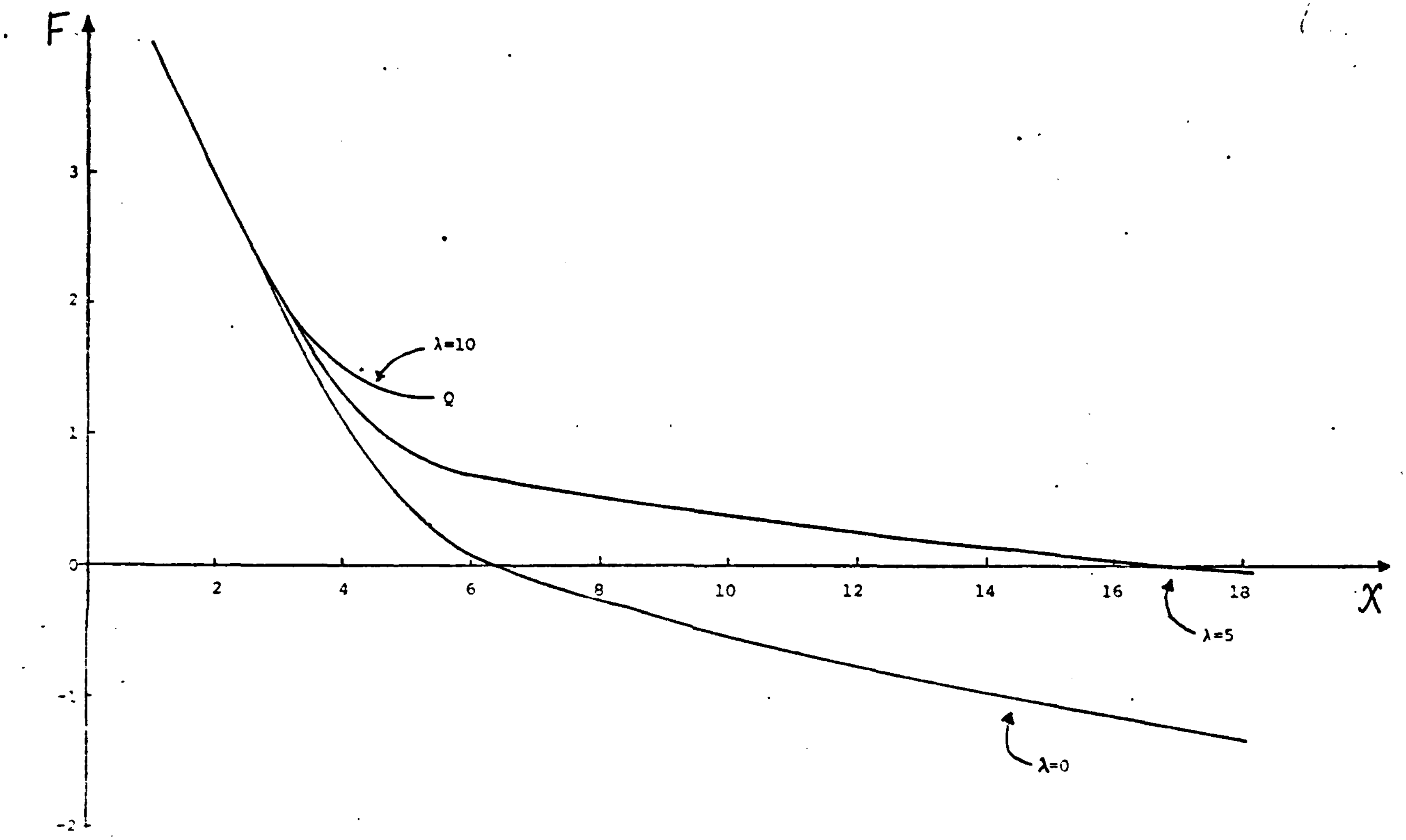

Fig. 2 Flame configurations for various λ. Drawn for $\omega=5$, $T_1=0.2$ & $Y_1=1$.

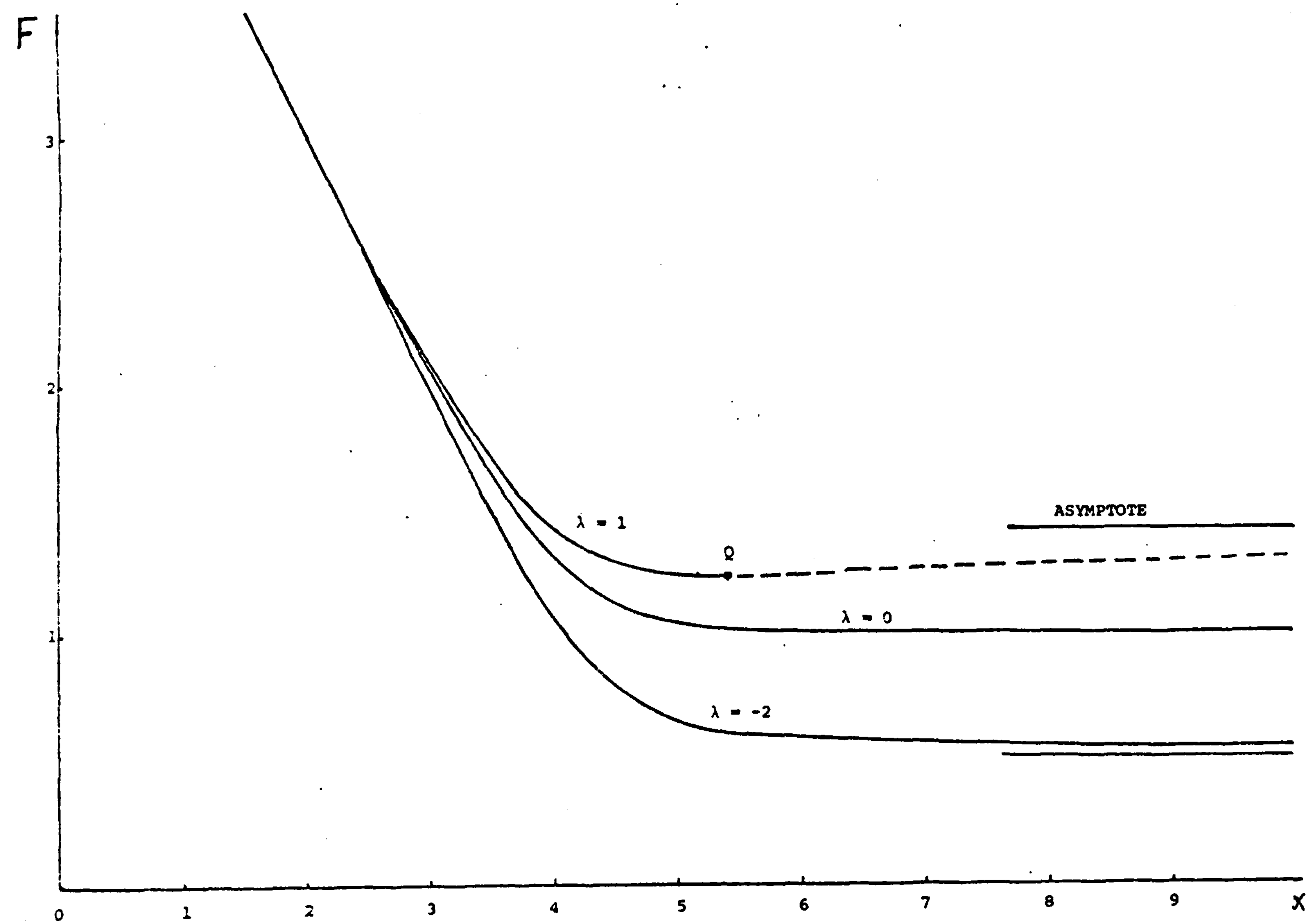

Fig. 3 Flame configurations in limit $\omega \to \infty$. Drawn for $T_1 = 0.2$ & $Y_1 = 1$.

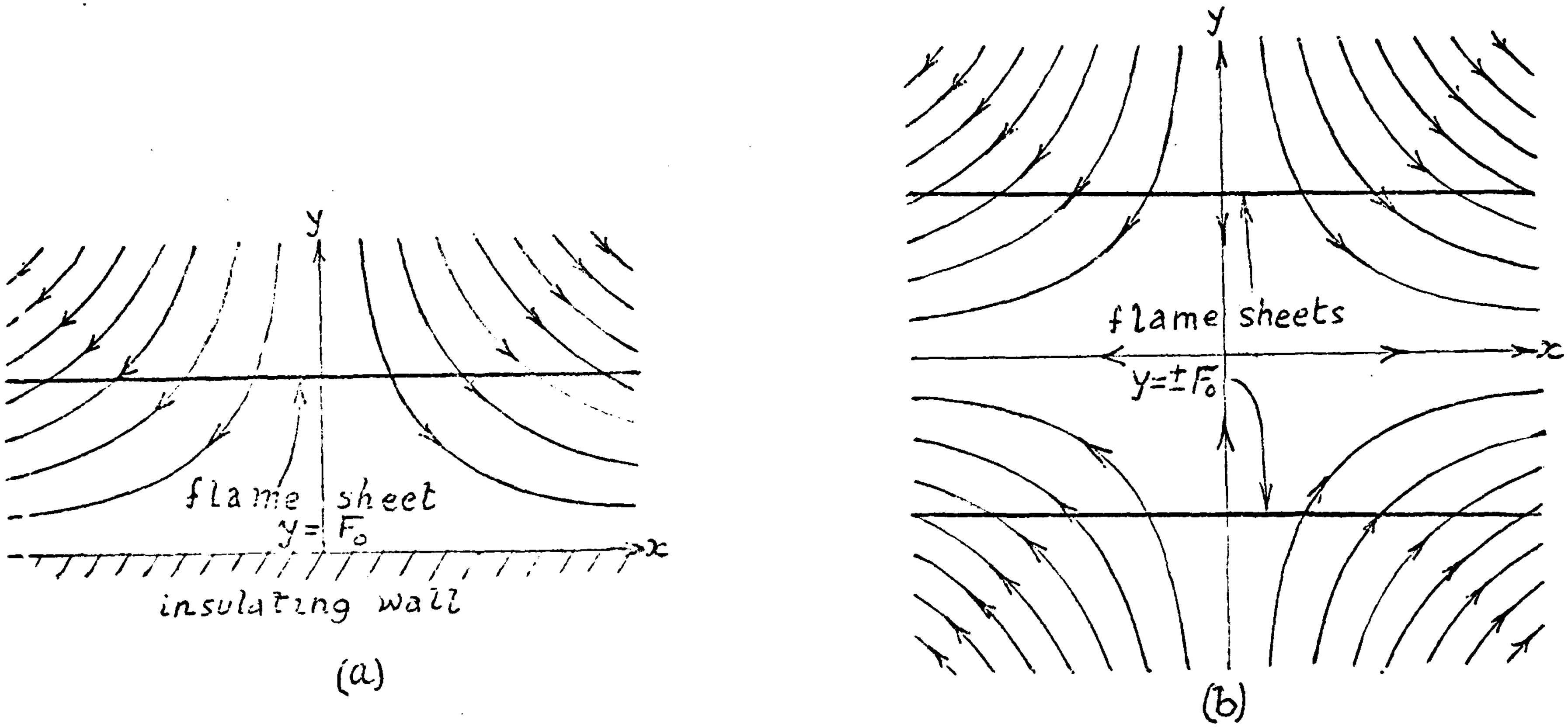

Fig. 4 Notation for flame in stagnation-point flow.
(a) Insulating wall. (b) Equal opposing jets.

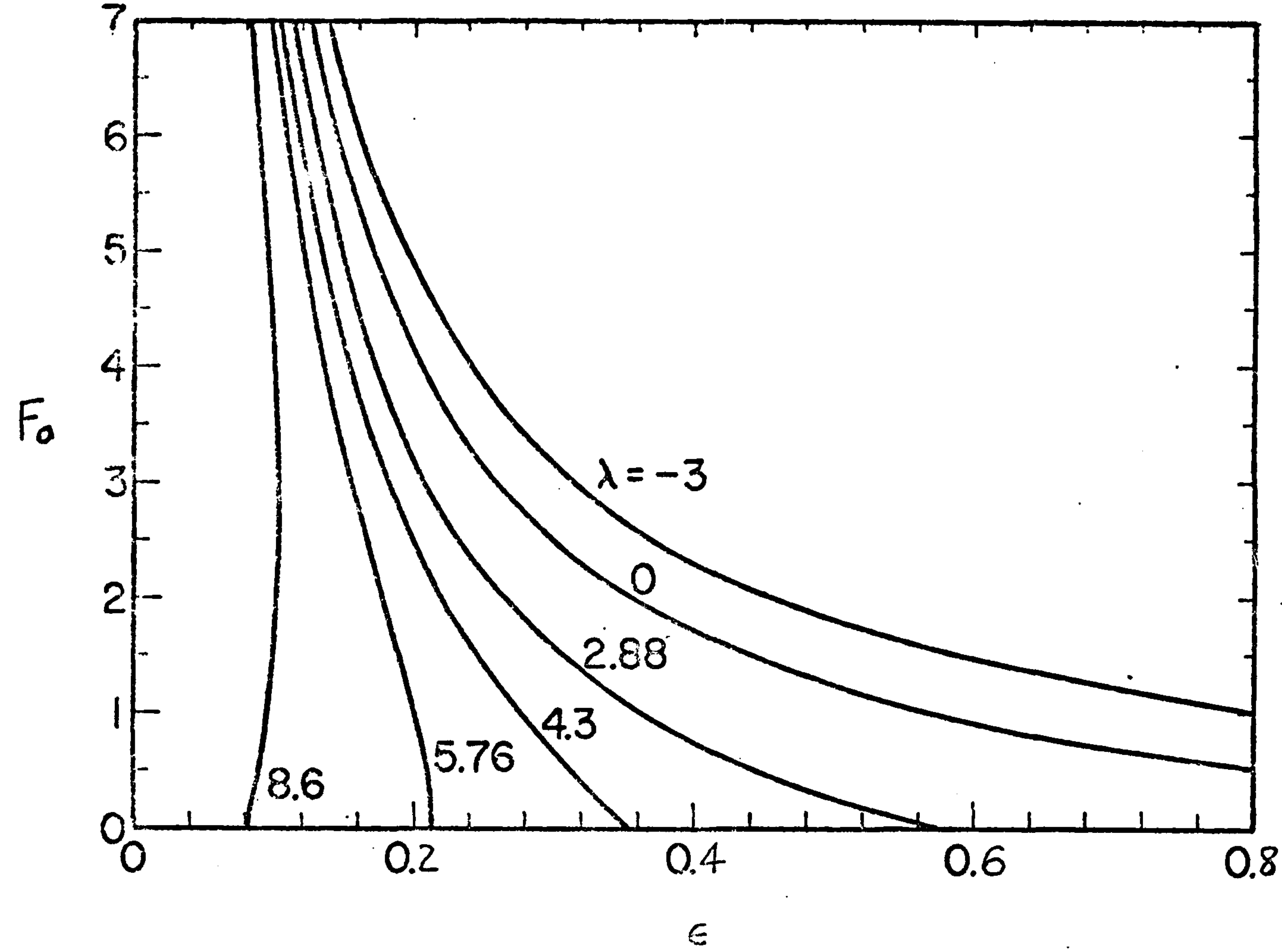

Fig. 5 Variation of stand-off distance F_o with straining rate ϵ. Drawn for $T_1 = 0.2$, $Y_1 = 1$.

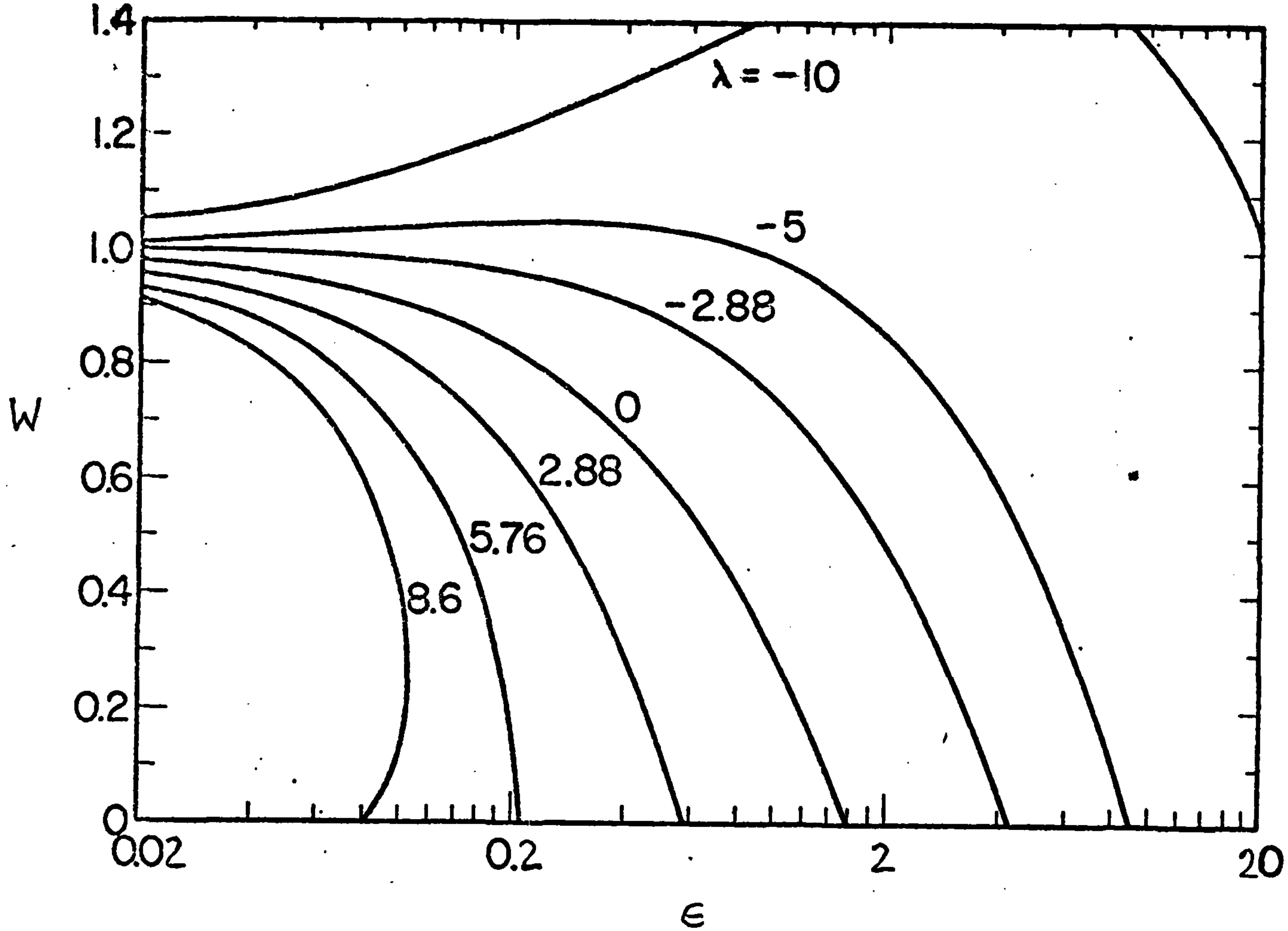

Fig. 6 Variation of flame speed W with straining rate ϵ. Drawn for $T_1 = 0.2$, $Y_1 = 1$.

Fig. 7. Variations of flame speed W and stretch q_0' when $\epsilon = 0.35$ and q_0 is
(a) $\epsilon \sin X$,

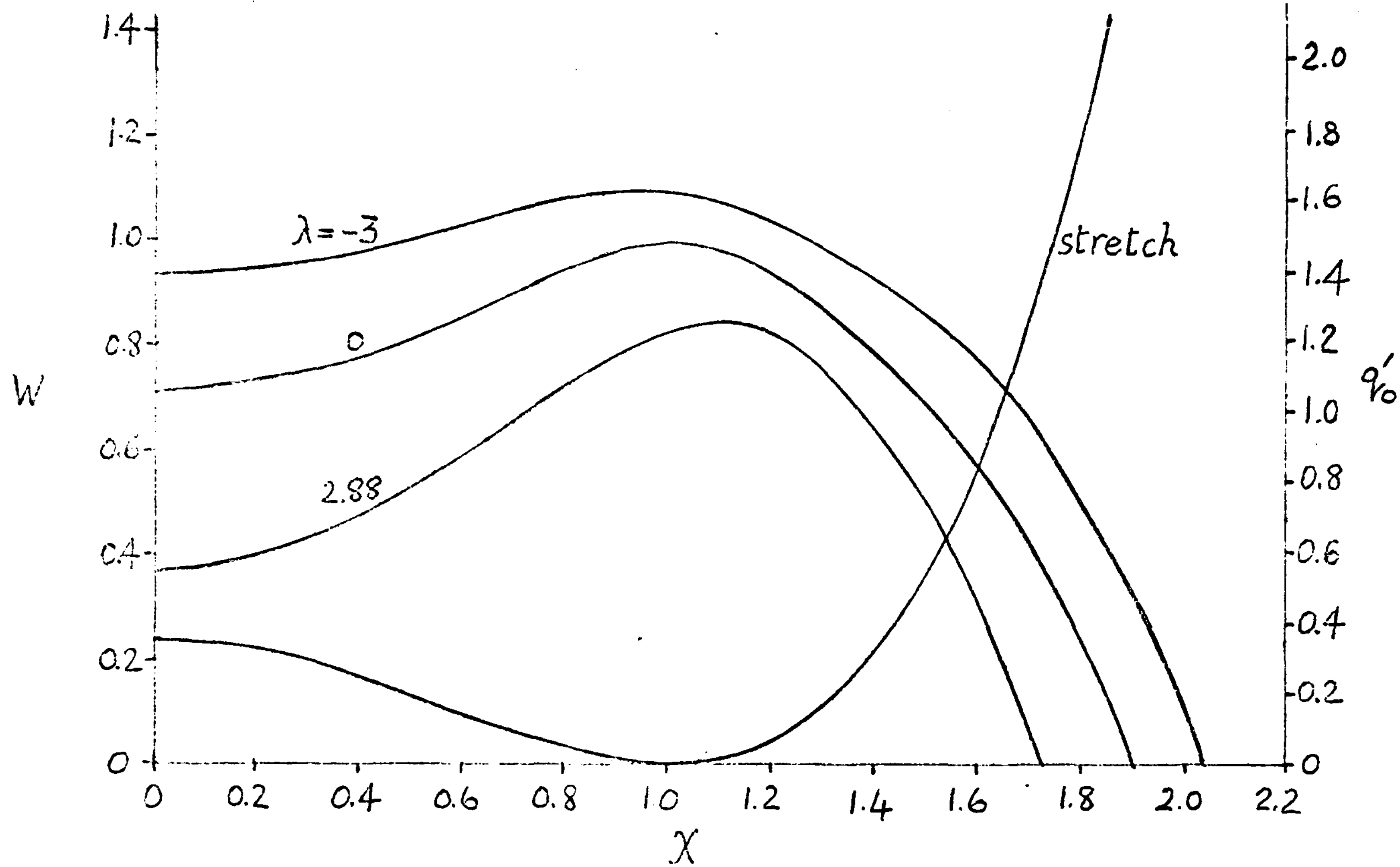

Fig. 7(b) $\epsilon(X - 2X^3/3 + X^5/5)$.

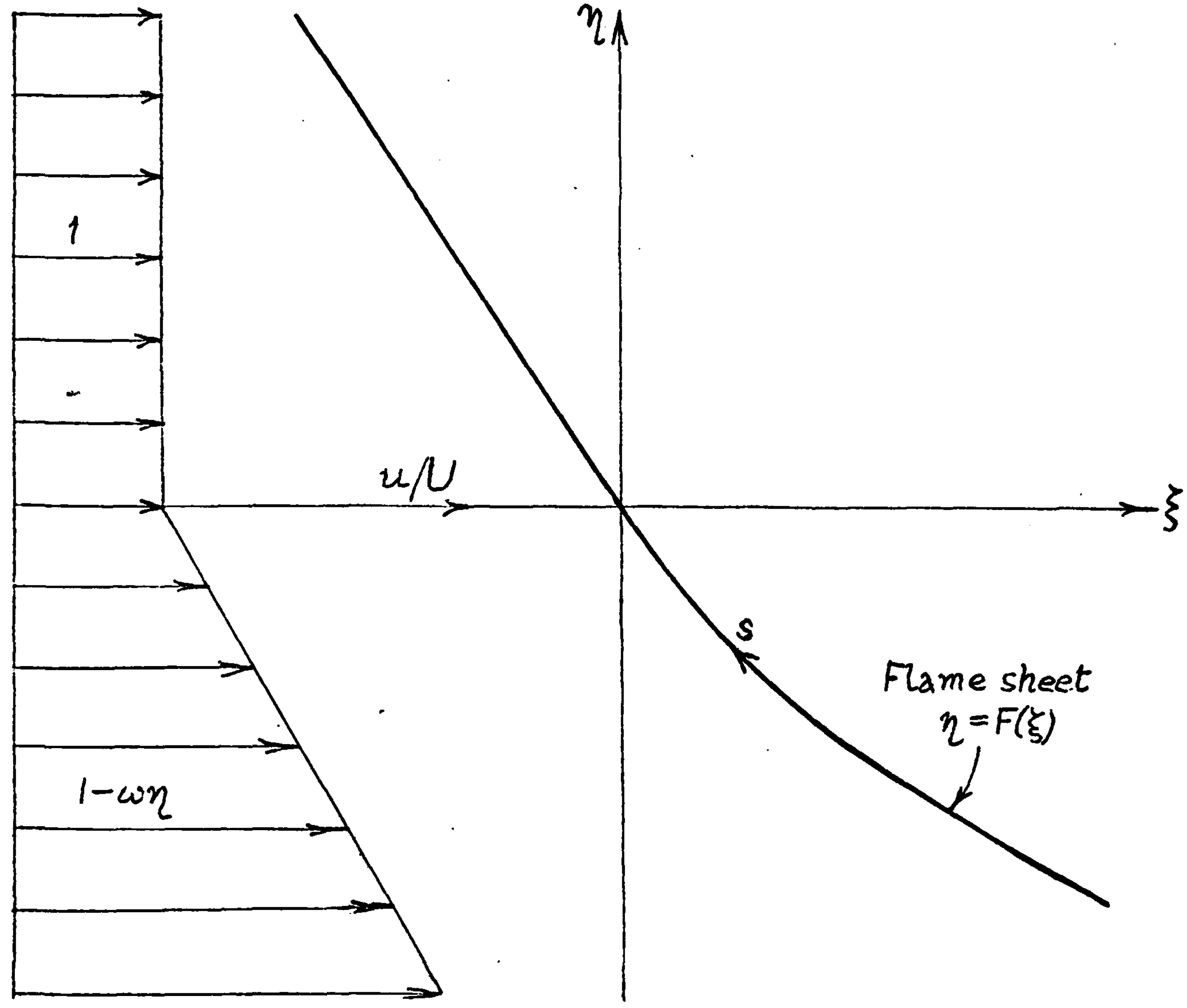

Fig. 8 Notation for slowly varying simple shear.

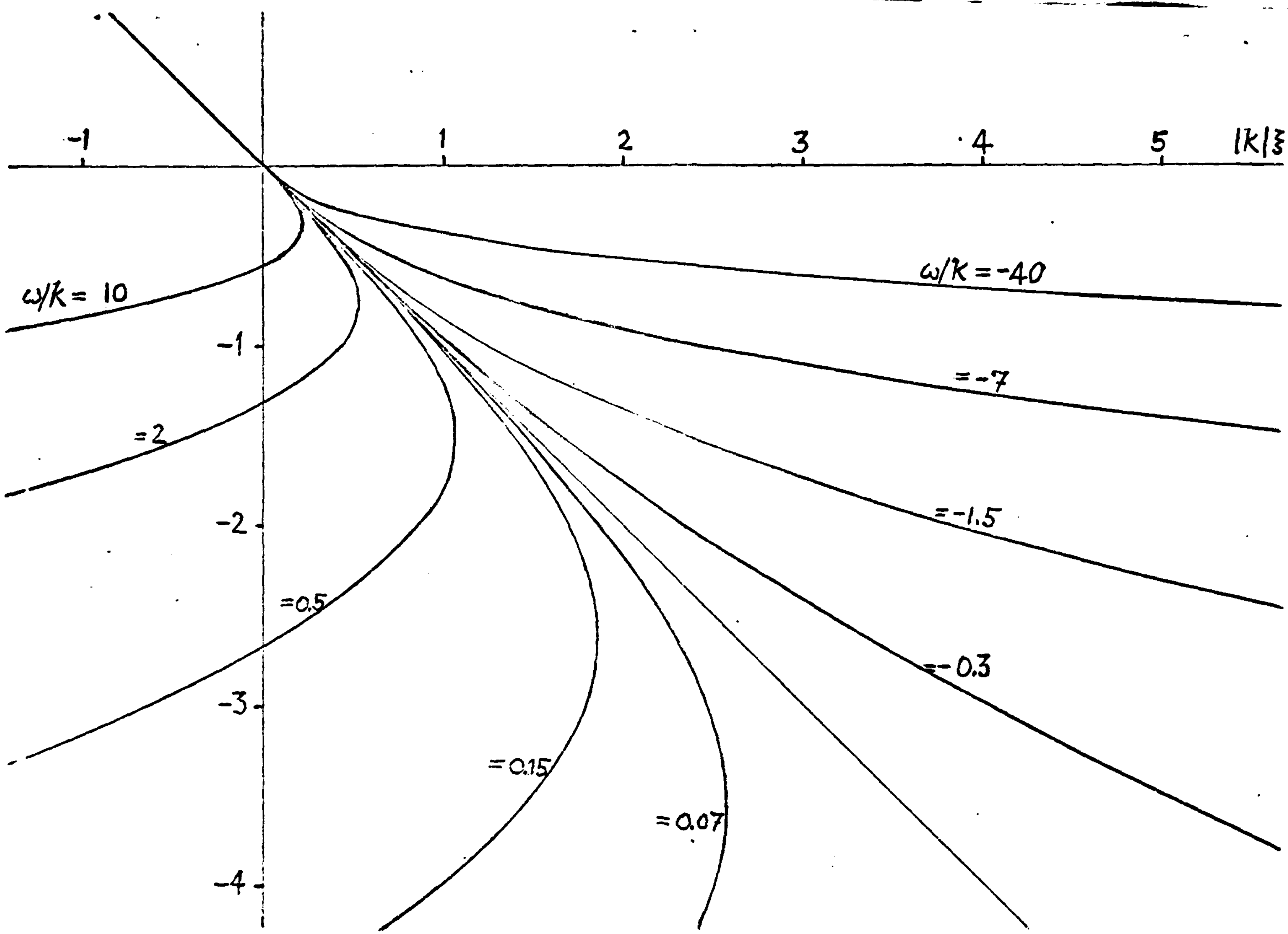

Fig. 9 Effect of slowly varying shear on a plane flame. Drawn for $\rho_1 U = \sqrt{2}$.

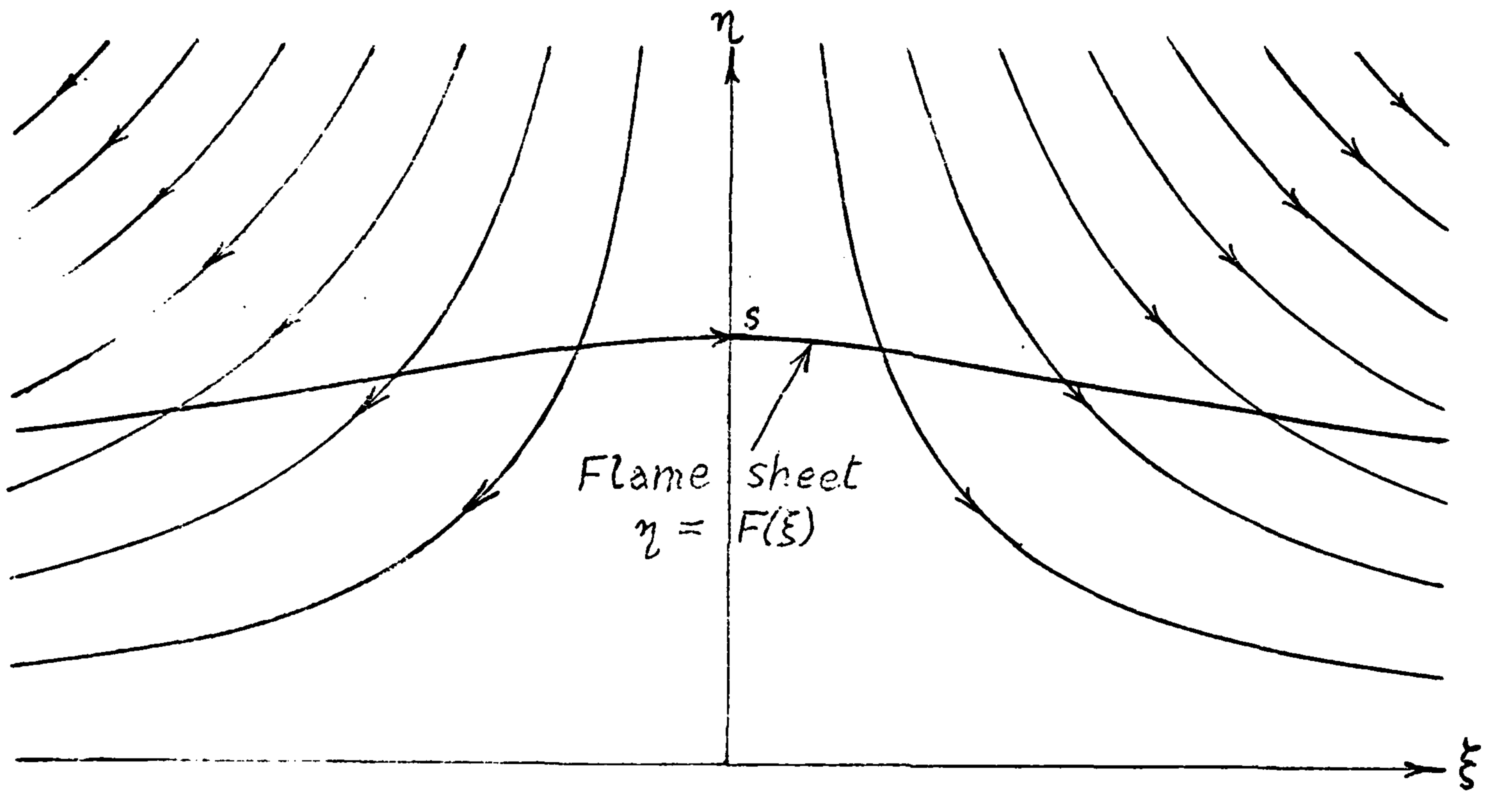

Fig. 10 Notation for slowly varying simple strain

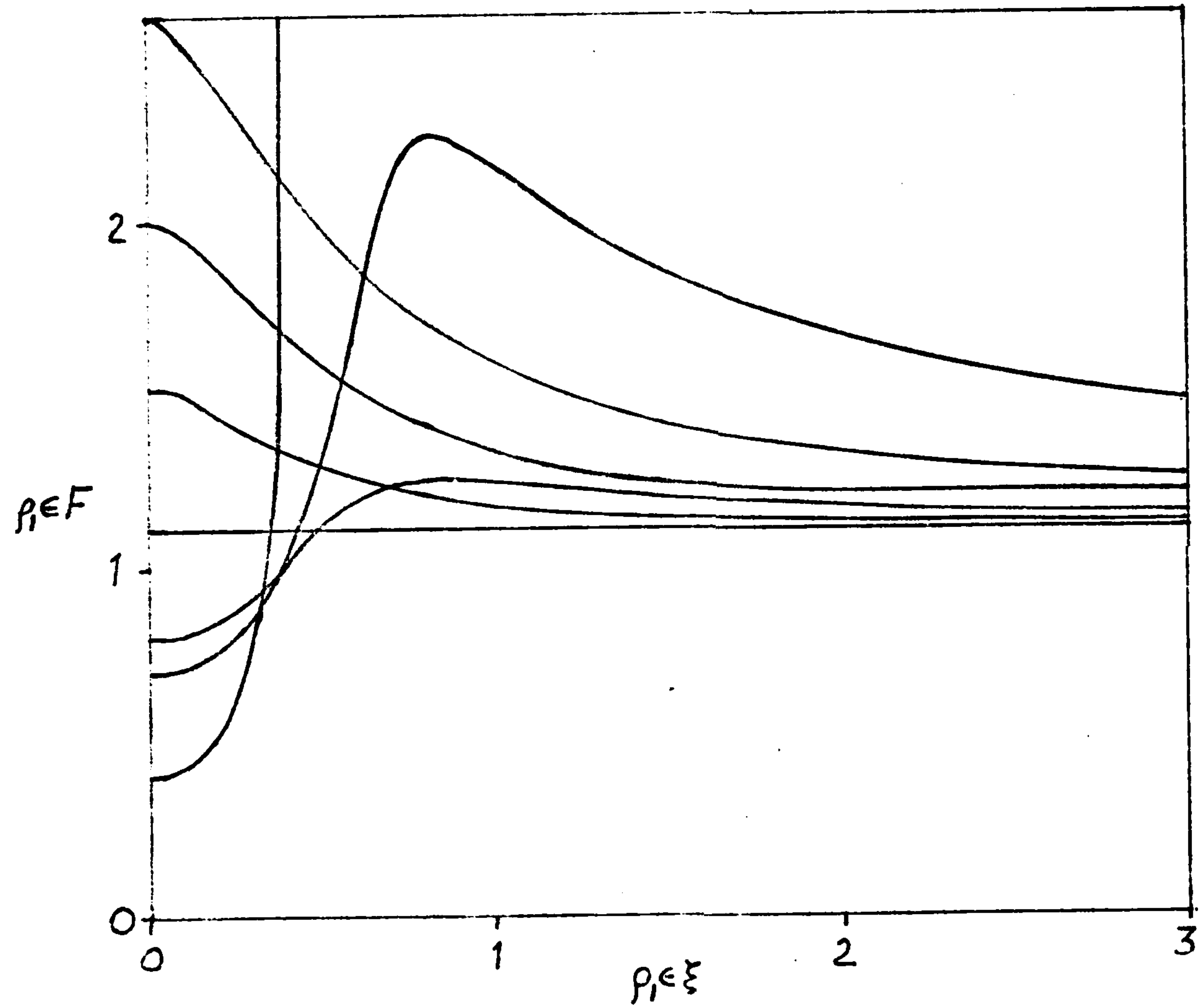

Fig. 11 Flames at a stagnation point. Drawn for $k/\rho_1^2\epsilon^2 =$

(a) -8.

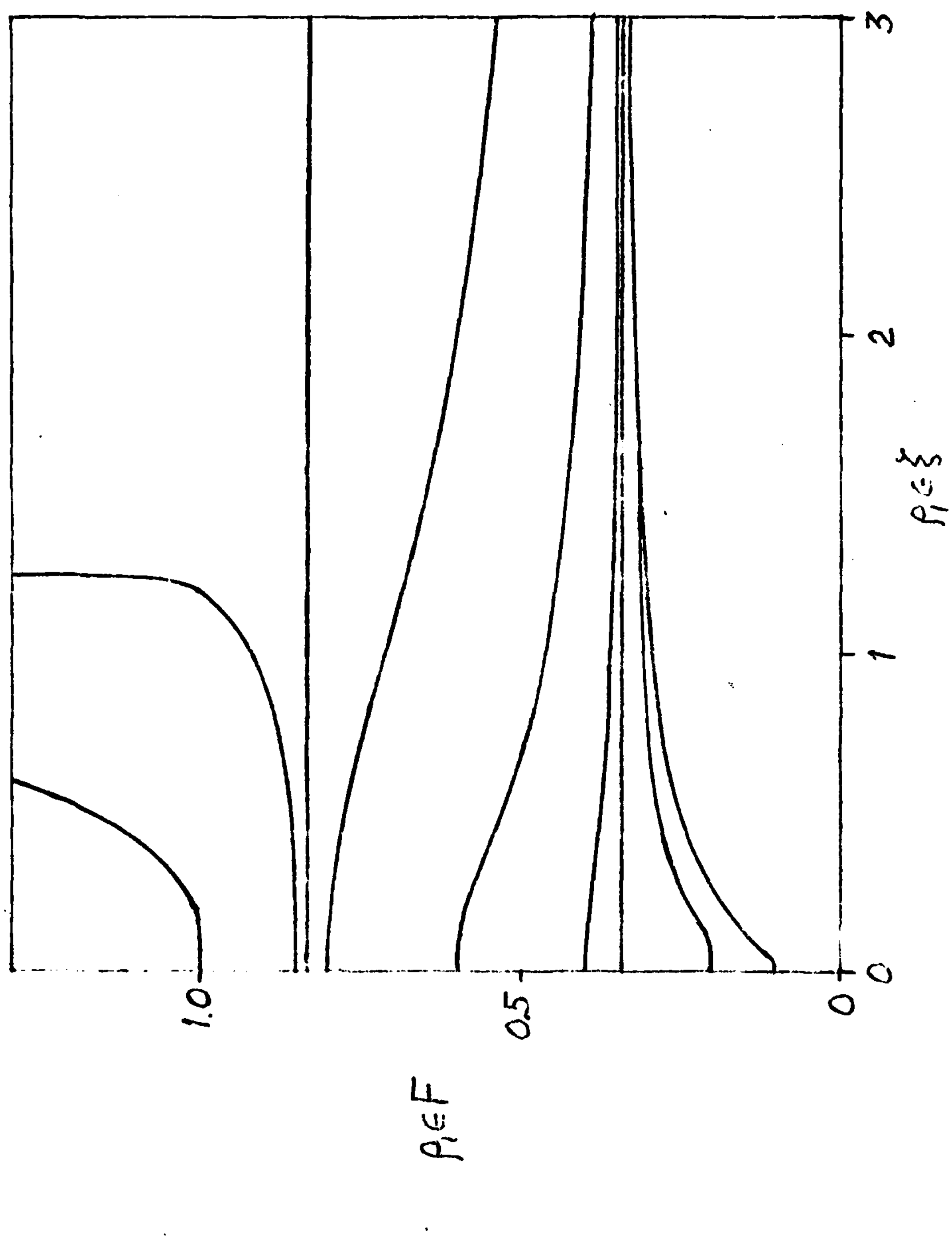

Fig 11 (b) 8.

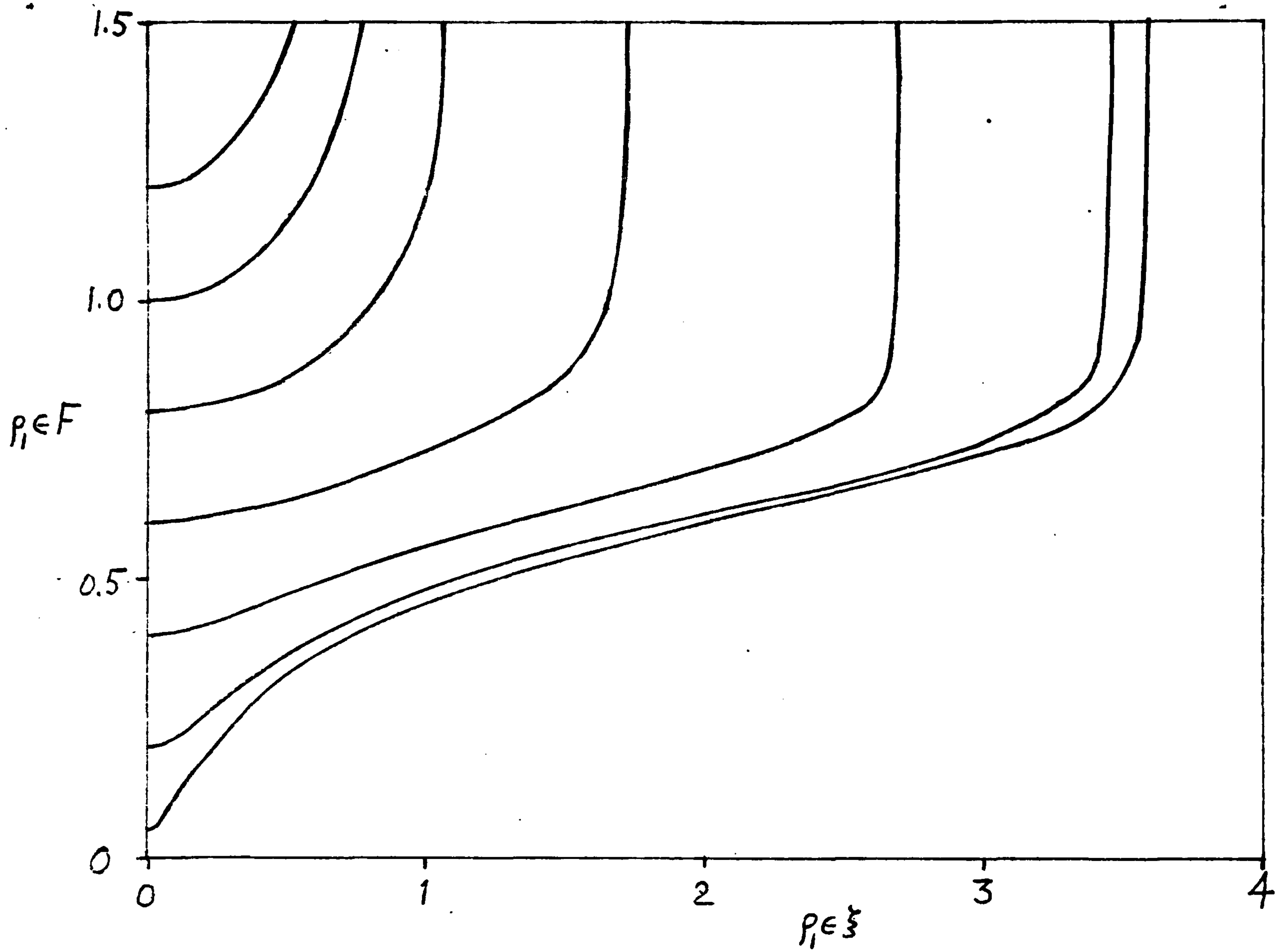

Fig. 11(c) 4.

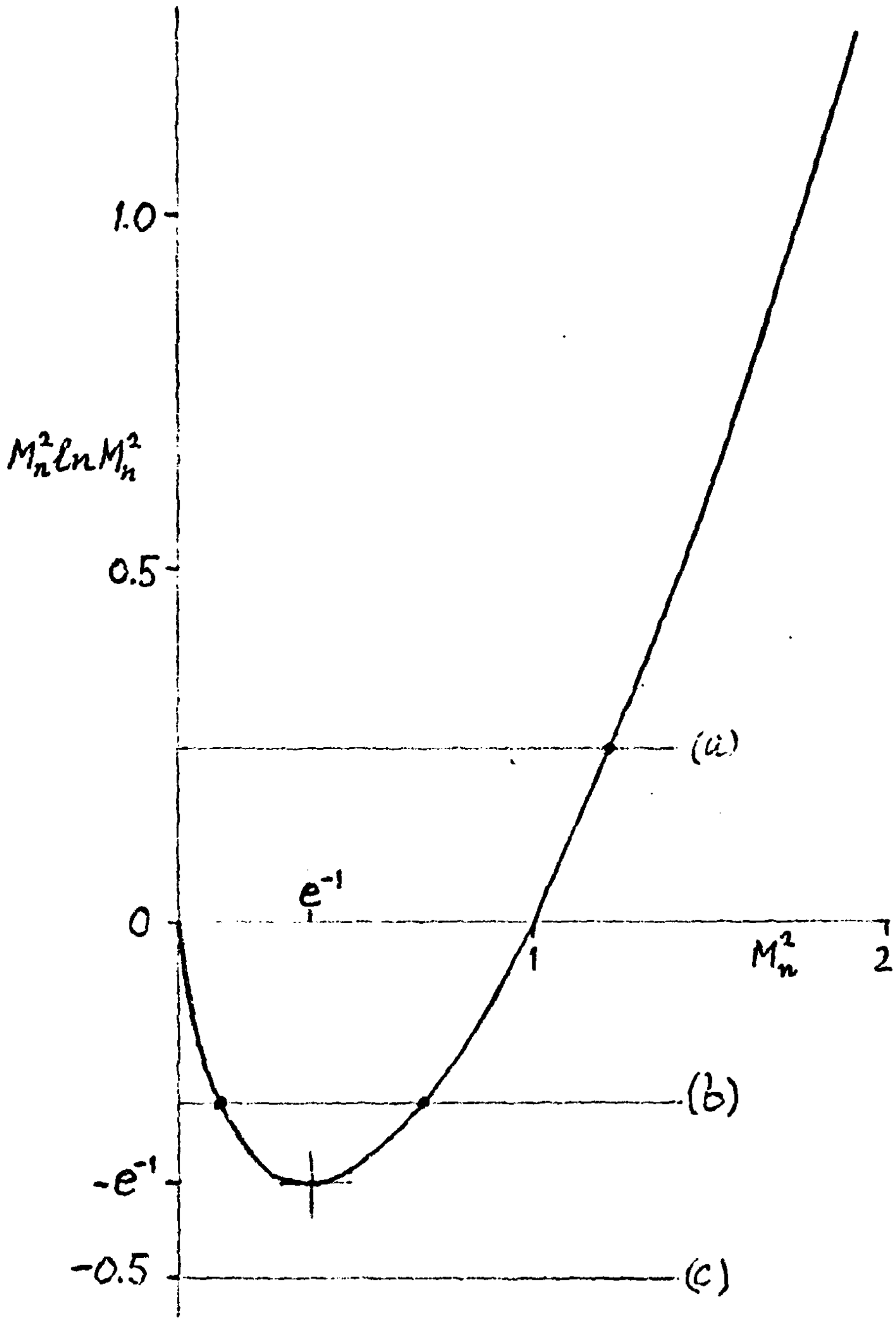

Fig. 12 Plot of $M_n^2 \ln M_n^2$. The levels (a), (b), (c) correspond to Fig. 11.

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18. SUPPLEMENTARY NOTES

The findings in this report are not to be construed as an official Department of the Army position, unless so designated by other authorized documents.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Non-uniformities, simple shear, simple strain, stagnation-point flow, Karlovitz number, stretch, quenching, general strain, slow variations

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report is Chapter X of the twelve in a forthcoming research monograph on the mathematical theory of laminar combustion. The effect of flow non-uniformities on a flame is investigated by first considering simple shear and strain, and then general straining motion. Evidence for the supposed connection between flame stretch and extinction is found to be mixed.