ADA084636

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER 14371.9-M	2. GOVT ACCESSION NO. ARO AD-A084636	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Finite Element Analysis of Transonic Flows		5. TYPE OF REPORT & PERIOD COVERED Final Report. 1 Nov 76 — 31 Oct 79
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) George J. Fix		8. CONTRACT OR GRANT NUMBER(s) DAAG29-77-G-0026
9. PERFORMING ORGANIZATION NAME AND ADDRESS Carnegie-Mellon University Pittsburgh, Pennsylvania 15213		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U. S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE May 80
		13. NUMBER OF PAGES 6
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

A

18. SUPPLEMENTARY NOTES

The view, opinions, and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

algorithms
finite element method
transonic flow

incompressible flow
boundary layers

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The work reported here centered around the development of finite element algorithms for the following class of problems:

1. Transonic flow problems governed by the potential equations.
2. Moving boundary problems
3. Incompressible flow problems governed by the Navier-Stokes equations.

DD FORM JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DTIC FILE COPY

Final Report To

U.S. Army Research Office

on

Contract DAAG 29 77 G 0026

George J. Fix
Department of Mathematics
Carnegie-Mellon University

The work supported by this contract centered around the development of finite element algorithms for the following class of problems:

1. Transonic flow problems governed by the potential equations

2. Moving boundary problems

3. Incompressible flow problems governed by the Navier-Stokes equations.

Work on the first problem

The first major result appeared in reference [1] in the list of papers published during the contract. This paper contains a new variational formulation of transonic flow problems as well as calculations from sample flow problems. These numerical examples indicated that the method yielded second order accuracy in the velocity field if linear elements were used. Further refinement of the algorithm was introduced in reference [4] where a reformulation of the basic variational principle was developed using least square ideas. Sample numerical experiments indicated that it retained the optimal accuracy yet required the inversion of a far simplier matrix in order to compute the fluid velocity field. Subsequent work involved mathematical analysis of these methods in order to determine when optimal accuracy can be expected and to find other strong and weak points of the methods. This work was reported in references [4] - [8], [12] - [14]. It was found

that if a "dash pot" treatment of shocks was used, then second
order accuracy could be achieved in general. Moreover, the
algebraic system could be solved with roughly the same amount
of arithmetic operations as the first order finite difference
schemes in current use. Thus from the point of view of numerical
analysis, these methods introduce an order of magnitute increase
in efficiency over existing methods.

The major drawback of the methods is that they are extremely
hard to program. This may be a prohibitive limitation to small
research groups with limited computational equipment and limited
experience with finite element methods. However, larger groups
with experience with finite element methods should be able to
capture the order of magnitude increase in efficiency provided by
these new methods.

Work on the third problem

During the analysis of the new methods introduced for transonic
flow problemsit became clear that these results applied also to the
standard and widely used finite element approximations to other
flow problems. The most interesting result in this regard concerns
the Navier-Stokes equations. Our analysis indicated that if one
used arbitrary finite element spaces in the approximation, then
instabilities could arise. Only select spaces will work, and
conditions are given with which the suitability of a particular
space can be checked. This in part explains the wide divergence
of opinion concerning the suitability of finite elements for flow

problems. The fact that a given finite element would work with a particular finite element space, and not with a closely related one was not anticipated from the use of the method for problems in solid mechanics.

Work on the second problem

This was work done in the last part of the contract and was reported in references [6] and [17] with the latter being the more important paper. The object was to construct numerical algorithms capable of dealing with two phase problems where effects of supercooling (or superheating) were present. The enthalpy method widely used for Stefan problems is not directly applicable here since the enthalpy in effect becomes multivalued in regions of supercooling. To avoid these problems an alternate phase field model is introduced where the phase is an independent variable satisfying a nonlinear parabolic partial differential equation. Pilot numerical experiments indicated that the method had some promise, yet several computational problems need to be resolved, the most important of which is the fact that the equation for the phase is much stiffer than the diffusion equation for the temperature field. Thus different grids with varying length scales are required for the temperatures and phase. Intuition has suggested some reasonable choices, however a thorough analysis is required before a truly practical algorithm will result.

List of Publications During Contract

* 1. (with M. Gurtin), "On patched variational methods", Numer. Math., 28(1977), 259-271.

* 2. (with B. Neta), "Finite element approximation of a nonlinear diffusion problem", Comp. and Math. with Appls. 3(1977), 287-298.

* 3. (with M. Gunzburger), "Downstream Boundary Conditions for viscous flow problems", Comp. and Math. with Appls. 3(1977), 53-63.

* 4. (with M. Gunzburger), "On least squares approximation to indefinite problems of the mixed types", Int'l. Jour. for Numer. Mtd. in Eng., 12(1978), 453-470.

  5. (with S. Marin), "Variational Methods for underwater acoustic problems", J. of Comp. Phy., 28(1978), 1-18.

* 6. "Numerical Methods for Alloy Solidification Problems", Moving Boundary Problems, Ward, ed., Academic Press, 1978.

* 7. "Variational Methods for Elliptic Boundary Value Problems", A.M.S. Short Course on Numerical Analysis, Oliger, ed., A.M.S. publications, 1978.

* 8. (with M. Gunzburger and R. A. Nicolaides), "Least squares finite element methods", NASA-ICASE Report 77-18, revised version published in Comp. and Math. with Appls., 5 (1979). 87-98.

* 9. (with M. Gunzburger and R. A. Nicolaides), "On mixed finite element methods", NASA-ICASE Reports 77-17 and 78-7; revised version submitted to Numer. Math.

  10. (with M. Gunzburger), "On numerical methods for acoustic problems", NASA-ICASE Report 78-15, revised version accepted for publication in Math. and Computers with Applications, 1979.

* 11. (with C. Coffman), "Constructive Approaches to Mathematical Models", Proceedings of a symposium in honor of R. J. Duffin, Academic Press, 1979.

* 12. (with M. Gunzburger and R. A. Nicolaides), "Theory and applications of mixed finite element methods", to appear in reference [33], 1979.

(continued)

*Papers supported by A.R.O. Contract.

* 13. (with M. Gunzburger and R. A. Nicolaides), "On mixed
    finite element methods for a class of nonlinear boundary
    value problems", to appear T.I.C.O.M. Conference on .
    Nonlinear Mechanics, T. Oden (ed.), 1979.

* 14. "On the structure of errors in mixed finite element
    methods", submitted to R.A.I R.O., 1979.

* 15. (with H. E. Donley), "On Finite Element Approximations
    for a Class of Ill-Posed Problems", accepted for publication,
    SIAM Jour. Numerical Analysis.

* 16. "Finite Element Approximations to Flow Problems," to
    appear, Third International Conference on Finite Elements
    in Water Resources, May 1980.

* 17. "Phase Field Methods for Free Boundary Value Problems,"
    to appear, Second International Symposium on Innovative
    Numerical Analysis in Applied Engineering Science,
    University of Virginia Press.

18. (with R. A. Nicolaides), "An Analysis of Mixed Finite
    Element Approximations for Periodic Acoustic Wave
    Propagation," to appear, SIAM Jour. Numerical Analysis.

*Papers supported by A.R.O. Contract.

Personnel Supported

August 1978

Professor George J. Fix   35%

James Epperson, (Graduate Student)   100%

Degree Awarded

James Epperson, Ph.D., August 1980