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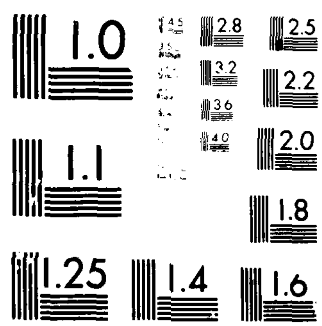

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BASIC CALCULATED RELATIONSHIPS OF ACYCLIC ELECTRICAL MACHINES WITHOUT A FERROMAGNETIC CIRCUIT

by

B. L. Aliyevskiy, A. I. Bertinov, et al.

FOREIGN TECHNOLOGY DIVISION

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А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
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К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere. When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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BASIC CALCULATED RELATIONSHIPS OF ACYCLIC ELECTRICAL MACHINES WITHOUT A FERROMAGNETIC CIRCUIT

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Introduction.

The application of direct-current acyclic machines (UM) which do not contain a ferromagnetic circuit is promising in connection with the use of cryogenic technology and superconductivity for creating a new type of electrical equipment. The design of such UM is complicated by the essentially nonuniform distribution of the magnetic field in their active zone. Estimating methods of calculating the emf of the UM armature are presented in [1, 2]. As an experimental check showed [3], in a number of cases they give satisfactory results. On the basis of these methods and approximate relationships for designing solenoids [4] an attempt was made to graphically assign the range of the most expedient dimensions of excitation coils [5, 6]. Up until now a general method has not been developed for determining the dimension of the UM armature without a ferromagnetic circuit.

Prospects for the use of the examined UM in various branches of technology [7] and the urgency of creating economically advantageous acyclic generators and high-power motors based on the use of superconductors and particularly pure metals for their inductance coils [8, 9] require the development of refined methods for designing these machines. The armature of a UM in the general case may have a collector rod winding with various cooling systems. In the present work we examine the basic problems of the electromagnetic design of a UM without a ferromagnetic circuit with optimization of the geometric parameters of the ring inductance coil performed using numerical methods on a computer.

Determination of the Main Dimensions.

With an assigned rate of rotation, diameter D and the active length l of the UM armature are determined by the electromagnetic load of the machine. Following a known method [1, 10] it is possible to obtain several forms of the basic design equation of UM which are presented below. When designing a UM with a superconducting inductance coil it is convenient to use the maximum value of induction B_m in the magnetic system which must not exceed the critical value B_K. The linear load and current density of the armature are determined by the method of its cooling and the permissible value of the electrical losses of power taking into account the experience of electrical machine building. It was established that independently of the structural diagram (Fig. 1) the diameter D=2R of the armature during the designing of a UM with a single pair of current-collecting circuits may be calculated according to the relationship

$$D=\sqrt[3]{\frac{P_e}{\sigma_n \lambda n}},$$

(1)

where P_e, n are the electromagnetic power and the rate of rotation; λ=l/D is the structural coefficient. The coefficient of use of the active volume

$$\sigma_n=0.5\pi^2(D^*_1+D^*_2)\frac{k_B}{k_{\Phi.\alpha}}B_m A,$$

where $D^*_1=D_1/D=R_1/R$ 1 is the relative diameter of the circuit the current-collecting ring or of the collector nearest to the excitation coil; $D^*_2=D_2/D=R_2/R$ is the relative diameter of the current collector at a distance from the examined coil; $k_{\Phi.\text{я}}$ is the coefficient of the shape of the curve of the current in the rods of the armature; $k_{\text{в}}=B_{cp}/B_{m}\approx 0.1\div 1$ is the coefficient of distribution of induction.

As B_{cp} we take the average value of magnetic induction on surfaces S (Fig. 1), connecting the circuits of the current collectors. The linear load of the armature in the general case

(2)
$$A=\frac{N}{\pi D}\sqrt{\frac{1}{T}\int_{0}^{T} i^{2}(t)\,dt},$$

where i(t) is the current in the rod with a period of change T; N is the number of active rods of the armature winding.

Expression (2) for practical calculations can be conveniently represented in the form

$$A=\frac{k_{\Phi.\text{я}}I_{\text{я}}c_{\text{в}}}{\pi D},$$

where $I_{\text{я}}$ and $c_{\text{в}}$ are the current of the armature circuit and the number of series-connected rods of the armature winding.

For a machine with a continuous rotor $c_{\text{в}}=1$ and $k_{\Phi.\text{я}}=1$. For collector UM with an armature winding of the rod type [9] the values of the coefficient $k_{\Phi.\text{я}}$ may be determined during detailed investigaicn of problems of commutation. In the simplest case, assuming that commutation is rectilinear, the distance between adjacent brushes b_p is equal to the width of the collector plate b_K and on the basis of this, analyzing the form of curve i(t) for the coefficient of brush overlap $\beta_{щ} = 1$ we obtain $k_{ф.п} = 2/\sqrt{3} = 1{,}155$. With values of the coefficient $\beta_{щ}$ greater or less than one the coefficent $k_{ф.п}$ approaches one. In all variations $k_{ф.п}$ increases with $b_p > b_K$, inasmuch as function i(t) has the character of pulses with relatively large pauses. In particular if $\beta_{щ} = 1$, $b_p = 2b_K$, then $k_{ф.п} \approx 1{,}41$.

Calculations show that in a cylindrical UM (Fig. 1a) it is possible to assume that $\lambda = 1.25 - 2.0$ with $D^*_2 \approx 1$; in a disk machine (Fig. 1b) with $D^*_2 = 0.2 - 0.3$ we have $\lambda = (D^*_1 - D^*_2)/2\cos\beta \approx 0{,}35 - 0{,}45$. A further increase of λ is not expedient especially for machines with relatively small dimensions of the inductance coil since in addition to an insignificant increase of the emf there is a significant rise in electrical losses and in the weight of the armature.

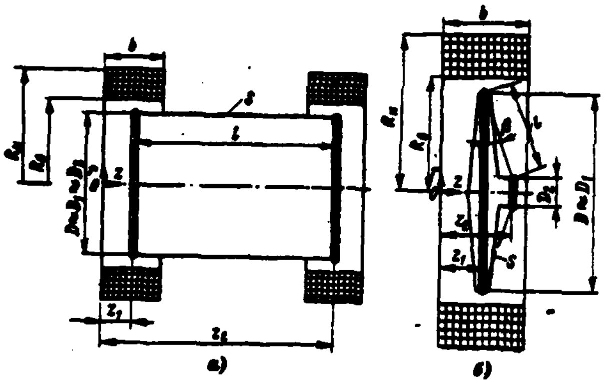

Fig. 1. Design diagrams of a cylindrical (a) and a disk (b) acyclic machine.

For an acyclic motor it is sometimes convenient to write relationship (1) through the electromagnetic moment

(3) $$D=\sqrt[3]{\frac{M_{э}}{s_{м}\lambda}};\ s_{м}=\frac{c_{м}}{2\pi}.$$

In constructions with a solid rotor as an index of the current load it is also possible to use the current density $j_{я}$ in the armature. The equations (1) and (3) assume the form

(4) $$D=\sqrt[4]{\frac{P_*}{\sigma'_{\scriptscriptstyle \text{и}}\lambda n}};$$

(5) $$D=\sqrt[4]{\frac{M_*}{\varepsilon'_{\scriptscriptstyle \text{и}}\lambda}};\ \varepsilon'_{\scriptscriptstyle \text{и}}=\frac{\sigma'_{\scriptscriptstyle \text{и}}}{2\pi}.$$

The coefficient characterizing the use of the active volume of a cylindrical UM

$$\sigma'_{\scriptscriptstyle \text{и}}=0{,}25\,\pi^2 D^*{}_1 k_B B_m j_{\scriptscriptstyle \text{н}},$$

where $j_n=4I_n/\pi D^2$ is the calculated density of current in the cylinder [1].

For a disk machine

$$\sigma'_{\scriptscriptstyle \text{и}}=0{,}5\,\pi^2(D^*{}_1+D^*{}_2)\,b^*{}_D k_B B_m j_{\scriptscriptstyle \text{н}},$$

where $j_n=I_n/\pi D b_D$, $b^*{}_D=b_D/D$ are the current density and the relative width of the disk on the periphery.

In those cases when the rate of rotation n is not a preconditioned value, relationships (1) and (4) may be converted introducing the greatest permissible linear velocity in the contact v_K:

(6) $$D=\sqrt{\frac{P_e}{s_v\lambda}};\ s_v=\frac{s_{н}v_{н}}{\pi D^2_1 c_{н}};$$

(7) $$D=\sqrt[3]{\frac{P_e}{s'_v\lambda}};\ s'_v=\frac{s'_{н}v_{н}}{\pi D^2_1 c_{н}},$$

where $c_{к}=D_{к}/D_1$ is the ccefficient taking into account the difference in the diameter of the real contact surface from that of the equivalent circuit of the current collector

For excitation wirdings manufactured frca especially pure metals (aluminum brand A999 or beryllium) with low-temperature cooling (with hydrogen, neon or nitrcgen) relaticnships (1) and (4) are expediently expressed in a different form. Using the concept of average current density $j_{ш}=F_в/Q$ ($F_в$, Q — the total magnetizing force of all coils and the total area of the transverse section) in the inductance coil consisting of $m_в$ coils we have

(8) $$D=\sqrt[4]{\frac{P_e}{\varkappa n}};\ \varkappa=\frac{\pi\mu_0\dot{E}_0}{8k_{ф.з}}\,j_{ш}A.$$

With a solid rotor for cylindrical UM

(9) $$D=\sqrt[5]{\frac{P_e}{\varkappa' n}},\ \varkappa'=\frac{\pi\mu_0\dot{E}_0}{32}\,j_{ш}l_a.$$

For disk machines

$$D=\sqrt[5]{\frac{P_a}{\varkappa' n}},\quad \varkappa'=\frac{\pi \mu_0 \overset{*}{E}_0}{8} b^*_D j_{s0} j_s. \tag{10}$$

Here $\mu_0=4\pi \cdot 10^{-7}$ H/m; $\overset{*}{E}_0 \approx 0.1-3$ is the dimensionless coefficient (relative emf) taking into account the geometry of the UM and the current distribution in the inductance coil which is functionally connected with k_B by the relationship

$$\overset{*}{E}_0=2\pi k_B \cdot B^*_m (D^*_1+D^*_2)\lambda.$$

The coefficient $B^*_m=B_m/\mu_0 j_{s0} R$ (relative value of greatest induction) with $m_B=1$ may be determined from graphs [4, p. 309]; with $m_B>1$ according to [4] an approximate determination of B_m is possible. A refined value B_m is obtained during calculation of the magnetic field of the inductance coil.

The most expedient values of coefficients $\sigma_m, \sigma'_m, \varkappa, \varkappa'$ for various types of machines and various types of applications of materials may be refined in proportion to the accumulation of experience in designing and manufacturing UM without a ferromagnetic circuit. The value of the diameter obtained from one of the relationships (19 and (3)-(10) may be used directly in designing taking into account the corresponding limitations with respect to thermal conditions, mechanical strength, etc. and also be considered as the first approximation during subsequent overall optimization of the machine according to the selected criteria. The active length, i.e., the distance between two current collectors of opposite polarity, for all forms of the basic design equation (1) is $l=\lambda D$.

Determination of the Optimum Dimensions of the Inductance Coil.

As a criterion of optimization we select the volume of the excitation coil V since the basic characteristics of the inductance coil - the weight of the winding G and the electrical losses in it (for nonsuperconducting GM) - significantly depend on V. Dependence $G_a(V)$ with a constant charge coefficient of the winding is practically linear. Electrical losses in inductance coils made of especially pure metals for one and the same values of the internal radius of the coil and of the average current density in it with an increase of V, increase faster than the linear function of volume. This is explained by an increase in the intensity of the magnetic field and consequently by a decrease of the average electrical conductivity of the winding material.

Arguments of optimization are the external radius R_M and width b of the axisymmetric inductance coil with a rectangular transverse section. The internal radius of the excitation coil R_B is assigned by the equality $R_B=R^*_B R$ (R^*_B is the structural coefficient). The mutual arrangement of the examined coil and the armature is characterized by parameter z_1 (Fig. 1) and is determined from the condition of the maximum emf. Use of the principle of superposition makes it possible to limit the analysis to the examination of an elementary UM containing one coil and a pair of current-collecting rings. In such a UM emf is assigned by the relationship

(11) $$E=c_B n(\Phi_1-\Phi_2),$$

where Φ_1, Φ_2 are magnetic fluxes permeating the vicinity of the current collectors.

Let us determine the flux through the circuit (r, z) in a linear medium (with $\mu=\mu_0$) from the annular inductance coil with an azimuthal current density j_B using the vector potential of the magnetic field of excitation,

(12) $$\Phi=\oint_L A_\varphi dL=2\pi r A_\varphi.$$

where A_φ is the azimuthal component of the vector potential.

Using the integral representation of the vector potential [1] expression (12) may be written in the form:

(13) $$\Phi=\mu_0\int\limits_{R_1}^{R_2}\int\limits_0^b\int\limits_0^\pi \frac{I_0(\xi,\ \zeta)\,r\xi\cos\varphi}{\rho(\xi,\ \zeta)}\,d\varphi d\zeta d\xi,$$

whereby the distance from the point of integration (ξ,ζ) to the point of observation $(r,\ z)$ $\rho=[r^2+\xi^2-2r\xi\cos\varphi+(z-\zeta)^2]^{0.5}$.

Testing (11) for the extremum by differentiating it with respect to z_1 taking (12) into account leads to the equality

(14) $$R_1B_{r1}=R_2B_{r2}.$$

In the limit with $R_2=0$ or $z_2\to\infty$, i.e., when the radial component of induction from the examined coil B_{r2} in point $(R_2,\ z_2)$ is equal to zero, equation (14) is satisfied with $z_1=b/2$, since in this plane $B_{r1}(R_1,\ z_1)=0$. For satisfaction of equation (14) with $R_2\neq0$ and with finite values z_2 it is necessary that $z_1>b/2$. However, calculations show that for real dimensions of UM this increase of z_1 is insignificant and does not lead to a significant increase of the emf. Therefore in the majority of cases it is recommended that the basic coil be arranged so that its center is in the plane of the circuit of the current collector closest to it. Subsequently such an arrangement is assumed to be preassigned.

Arguments of optimization of R_H and b are determined as a result of minimization of the criterial function of volume $V(R_n, R_s, b)$ while observing the condition E=const. Taking (11) and (13) into account we write the corresponding equations in dimensionless form:

$$\mathring{V}=\pi(\mathring{R}_{\text{н}}^2-\mathring{R}_{\text{в}}^2)b^*; \qquad (15)$$

$$\mathring{E}=\int_{R^*_{\text{в}}}^{R^*_{\text{н}}}\int_0^{b^*}\int_0^{\pi}\mathring{j}_{\text{в}}(\mathring{\xi},\mathring{\zeta})\mathring{\xi}\left[\frac{R^*_1}{\rho^*_1(\xi^*,\zeta^*)}-\right.$$

$$\left.-\frac{R^*_2}{\rho^*_2(\xi^*\zeta^*)}\right]\cos\varphi d\varphi d\mathring{\zeta}\, d\mathring{\xi}=\text{const}. \qquad (16)$$

The obtained notation makes it possible, with satisfaction of the similarity criteria,

$$R^*_{\text{н}}=R_{\text{н}}/R;\ R^*_{\text{в}}=R_{\text{в}}/R;\ b^*=b/R;\ R^*_1=R_1/R;$$
$$R^*_2=R_2/R;\ \mathring{z}_1=z_1/R;\ \mathring{z}_2=z_2/R;\ \mathring{j}_{\text{в}}=j_{\text{в}}/j_{\text{вх}}\sim\text{idem}$$

to extend the results of calculation to a number of analogs

$$\overset{*}{V}=V/R^{*}\sim \text{idem}; \overset{*}{E}=E/\mu_0 j_{sz} n c_g R^{*}\sim \text{idem}.$$

Here j_g is the current density in a random point of the excitation coil; j_{sz} is the current density in a certain characteristic point of the examined coil, for example $\xi = R_s$, $\zeta = b/2$. Let us note that dimensionless emf E^*_0 from m_g excitation coils is determined by the relationship

(17) $$\overset{*}{E}_0=\sum_{k=1}^{m_g}\overset{*}{E}_k\frac{j_{gk}}{j_{g0}}.$$

The minimum of function (15) with condition (16) is found by the method of indeterminant Lagrange multiplers. As a result of conversions we obtain a system into which, along with (16), enters the connecting equation

(18) $$\overset{*}{j}=\overset{*}{E}\Big|_{\overset{*}{\xi}=R^*_s}-\frac{R^*_s}{R^*_s+R^*_s}\left(\overset{*}{E}\Big|_{\overset{*}{\zeta}=b^*}+\overset{*}{E}\Big|_{\overset{*}{\zeta}=0}=0\right).$$

Expression (18) is written for the case when with possible variations of the excitation coil geometry its median plane remains immobile relative to the armature. It may be shown that if for structural considerations it is necessary to fix the distance between one of the ends of the coil ($\overset{*}{\zeta}=0$ or $\overset{*}{\zeta}=b^*$) and the armature then application of Lagrange's method gives relationships identical to (18) but in this case

$$\mathring{E}|_{\mathring{\zeta}=0}=0$$

or

$$\mathring{E}|_{\mathring{\zeta}=b^*}=0$$

respectively and the multiplier in front of the bracket is doubled.

In expanded form (18) is

$$J^*=\int_0^{b^*}\int_0^{\pi}\mathring{j}_{\text{в}}\,(R^*_{\text{н}},\,\mathring{\zeta})\left(\frac{R^*_1}{\rho^*_1}-\frac{R^*_2}{\rho^*_2}\right)_{\mathring{\xi}=R^*_{\text{н}}}\cos\varphi d\varphi d\mathring{\zeta}-$$

(19)
$$-\frac{b^*}{R_{\text{н}}^2-R_{\text{в}}^2}\int_{R^*_{\text{в}}}^{R^*_{\text{н}}}\int_0^{\pi}\mathring{\xi}\left[j^*_{\text{в}}\,(\mathring{\xi},\,b^*)\left(\frac{R^*_1}{\rho^*_1}-\frac{R^*_2}{\rho^*_2}\right)_{\mathring{\zeta}=b^*}+\right.$$

$$\left.+j^*_{\text{в}}(\mathring{\xi},\,0)\left(\frac{R^*_1}{\rho^*_1}-\frac{R^*_2}{\rho^*_2}\right)_{\mathring{\zeta}=0}\right]\cos\varphi d\varphi d\mathring{\xi}=0.$$

Joint solution of equations (16) and (19) makes it possible to determine the optimum dimensions of the inductance coil with respect to a given emf. For obtaining calculation expressions we integrate (16) and (19) with respect to coordinate ϕ:

$$\mathring{E}=\int_{R^*_{\text{в}}}^{R^*_{\text{н}}}\int_0^{b^*}j^*_{\text{в}}(\mathring{\xi},\,\mathring{\zeta})\sqrt{\xi^*}\,[f\,(k_1)\sqrt{R^*_1}-$$

(20)
$$-f\,(k_2)\sqrt{R^*_2}]\,d\mathring{\zeta}d\,\mathring{\xi}=\text{const};$$

$$J^*=\int_0^{b^*}j^*_{\text{в}}(R^*_{\text{н}},\,\mathring{\zeta})\,[f\,(k_{1R})\sqrt{R^*_1}-f\,(k_{2R})\sqrt{R^*_2}]\,d\mathring{\zeta}-$$

$$-\frac{b^{\circ}\sqrt{R^{\circ}_{\text{н}}}}{\mathring{R}_{\text{н}}^{2}-\mathring{R}_{\text{в}}^{2}}\int\limits_{R^{\circ}_{\text{в}}}^{R^{\circ}_{\text{н}}}\sqrt{\mathring{\xi}}\,\{j^{\circ}_{\text{в}}(\mathring{\xi},\ b^{\circ})\,[f(k_{1b})\sqrt{R^{\circ}_{1}}-$$

(21)

$$-f(k_{2b})\sqrt{R^{\circ}_{2}}]+j^{\circ}_{\text{в}}(\mathring{\xi},\ 0)\,[f(k_{10})\sqrt{R^{\circ}_{1}}-$$

$$-f(k_{20})\sqrt{R^{\circ}_{2}}]\}\,d\mathring{\xi}=0.$$

Here function

$$f(k)=\frac{2}{k}[K(k)-E(k)]-kK(k),$$

where K(k), E(k) are the complete elliptical integrals respectively of the first and second kinds with the square of the modulus

$$k^{2}=4\mathring{r}\,\mathring{\xi}/[(\mathring{r}+\mathring{\xi})^{2}+(\mathring{z}-\mathring{\zeta})^{2}],$$

moreover

$$k_{1}=k\Big|_{\substack{\mathring{r}=R^{\circ}_{1}\\ \mathring{z}=\mathring{z}_{1}}},\quad k_{2}=k\Big|_{\substack{\mathring{r}=R^{\circ}_{2}\\ \mathring{z}=\mathring{z}_{2}}};$$

$$k_{iR}=k_{i}\Big|_{\mathring{\xi}=R^{\circ}_{\text{н}}},\quad k_{ib}=k_{i}\Big|_{\mathring{\zeta}=b^{\circ}},$$

$$k_{i0}=k_{i}\Big|_{\mathring{\zeta}=0},\quad i=1;\ 2.$$

With a constant current density $j_{\text{в}}$=const equations (16) and (19) may be represented in a simpler form. As a result of double integration using conversions [11] we have:

(22)
$$\dot{E}=\int\limits_0^{\dot{\pi}}(\mathrm{e}_2-\mathrm{e}_1)\cos\varphi d\varphi=\mathrm{const};$$

(23)
$$J^*=\int\limits_0^{\dot{\pi}}(c_2-c_1)\cos\varphi d\varphi=0,$$

where

$$\mathrm{e}_i=\dot{r}\left[\frac{\dot{z}-\dot{\zeta}}{2}\dot{\rho}+\frac{1}{2}(\dot{\xi}^2-\dot{r}^2\cos 2\varphi)\ln(\dot{z}-\dot{\zeta}+\dot{\rho})+\right.$$
$$+(\dot{z}-\dot{\zeta})\dot{r}(\cos\varphi)\times\ln(\dot{\xi}-\dot{r}\cos\varphi+\dot{\rho})+$$
$$\left.+\dot{r}^2(\sin 2\varphi)\,\mathrm{arctg}\,\frac{\dot{\xi}-\dot{r}\cos\varphi+\dot{z}-\dot{\zeta}+\dot{\rho}}{\dot{r}\sin\varphi}\right]_{\dot{\zeta}=0}^{b^*}\Bigg|_{\dot{\xi}=R^*_{\text{в}}}^{R^*_{\text{н}}};$$

$$c_i=\dot{r}\ln[\dot{z}-\dot{\zeta}+\dot{\rho}(R^*_{\text{н}},\dot{\zeta})]_{\dot{\zeta}=0}^{b^*}+\frac{b^*\dot{r}}{R^{*2}_{\text{н}}-R^{*2}_{\text{в}}}\{\dot{\rho}(\dot{\xi},b^*)+$$
$$+\dot{\rho}(\dot{\xi},0)+\dot{r}(\cos\varphi)\times\ln[\dot{\xi}-\dot{r}\cos\varphi+\dot{\rho}(\dot{\xi},b^*)]+$$
$$+\dot{r}(\cos\varphi)\ln[\dot{\xi}-\dot{r}\cos\varphi+\dot{\rho}(\dot{\xi},0)]\}_{\dot{\xi}=R^*_{\text{в}}}^{R^*_{\text{н}}};$$
$$\mathrm{e}_1=\mathrm{e}_i\Big|_{\substack{\dot{r}=R^*_1\\ \dot{z}=\dot{z}_1}},\quad \mathrm{e}_2=\mathrm{e}_i\Big|_{\substack{\dot{r}=R^*_2\\ \dot{z}=\dot{z}_2}},\quad c_1=c_i\Big|_{\substack{\dot{r}=R^*_1\\ \dot{z}=\dot{z}_1}},$$
$$c_2=c_i\Big|_{\substack{\dot{r}=R^*_2\\ \dot{z}=\dot{z}_2}}.$$

Other forms of notation corresponding to unidimensional distributions of current density $j^*_2=j^*_2(\dot{\xi})$, $j^*_2=j^*_2(\dot{\zeta})$, are given in Appendix 1. Let us note that the solution of the inverse problem (i.e., obtaining the greatest emf in the case of a given volume of the excitation coil) during differentiation of $\overset{*}{E}$ with respect to R^* or b^* and $\overset{*}{V}$=const leads to the same calculation relationships (18), (19), etc. Although all of the dependences given above are directly valid for UM with a single pair of current collector circuits, they may, however, be extended to more general cases (see Appendix 2).

Numerical Processing and Experimetal Check.

Solution of the system of integral relationships represented in the form of (20) and (21) or (22) and (23) relative to known b^*, R^*_H may be obtained with gradient methods using computers [12], for example, by the method of the fastest start. However, for plotting design curves it is more convenient to solve the system relative to the parameters $\overset{*}{E}, b^*$ or $\overset{*}{E}, R^*_H$. In this case equations (21) and (23) are solved by the method of chords or the method of half division and value $\overset{*}{E}$ is determined directly as a result of numerical integration of relationships (20) and (22). The simplicity of the algorithm and the possibility of using a preliminarily written program for finding the actual roots of a transcendental equation determined the selection of such a method for realization on computers. Solution of the system (22) and (23) was carried out on the "Minsk-22" computer relative to the unknowns b^*, $\overset{*}{E}$. The error of numerical integration in this case was $\delta_H \leqslant 0.1\%$, and the assigned error of solution of equation (23) $\delta_y = b^*_k - b_{k-1} \leqslant 0.05$. Analogous calculations according to formulas (20) and (21) were performed on the "Nairi-2" computer.

Numerical integration in the process of calculations was performed using the Gaussian method. The calculations showed that for achieving the same accuracy as in the preceding case it is sufficient, in the case of single integration, to use the quadratic formula with 6-8 points. As a result of these calculations graphs were plotted (Fig. 2 and 3) which make it possible to determine the optimum width b* and height $h^* = R^*_{н} - R^*_{в}$ of a section of the inductance coil with respect to a given $\overset{*}{E}$.

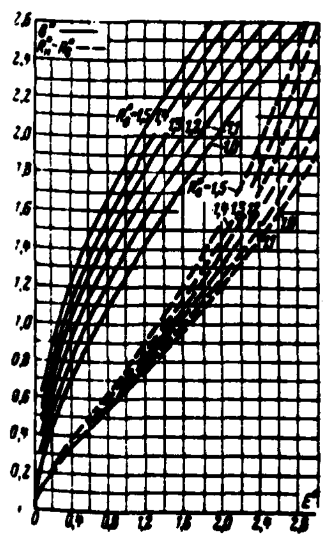

Fig. 2. Optimum dimensions of the inductance coil of a cylindrical UM ($\lambda = 1.5$).

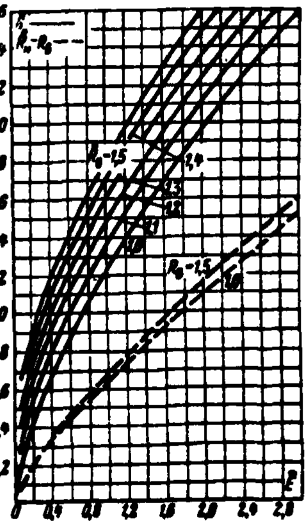

Fig. 3. Optimum dimensions of the inductance coil of a disk UM ($\overset{*}{z}_2-\overset{*}{z}_1=0.2$; $R^*_2=0.2$).

For checking the presented method we calculated and measured the magnetic flux of models of a number of experimental inductance coils. Flux was determined using a webermeter type F-18. In this case it was taken into account that

$$\overset{*}{E}=\frac{E}{\mu_0 I_{a0} n c_E R^2}=\frac{\Phi_1}{\mu_0 I_{a0} R^2}=\Phi^*_1. \quad (24)$$

The results of comparison of experimental data with calculation according to formulas (20) and (22) with $R^*_2=0$ showed their sufficiently good correspondene (see table, where $v=(\Phi^*_{\delta 0}-\Phi^*_{\delta p})/\Phi^*_{\delta 0}$.

$R^*_{в}$	$R^*_{н}$	b^*	z^*_1	$E^*=\Phi_{\delta}$		v, %
				1) Расчет	2) Опыт	
1,11	1,69	1,38	0,69	0,923	0,928	0,54
1,25	1,9	1,55	0,775	1,009	1,015	0,6
1,11	1,38	3,36	1,68	0,692	0,693	0,14
1,25	1,55	3,78	1,89	0,774	0,776	0,26
1,11	1,655	1,48	0,74	0,919	0,927	0,87
1,25	1,863	1,66	0,83	1,007	1,036	2,9
1,11	1,723	1,29	0,645	0,925	0,904	—2,8
1,25	1,94	1,45	0,726	1,009	0,990	—1,89

Table. KEY: 1. calculation. 2. experiment.

Using experimentally checked calculation algorithms curves were plotted (Fig. 4) which show the character of change $\overset{*}{E}$ depending on the geometry of the coil in the case of preservation of the constancy of the relative volume $\overset{*}{V}$. As is evident from fig. 2-4, values b* and $h^*=R^*_{н}-R^*_{в}$, which provide the extremum $\overset{*}{E}$, in all cases coincide with their values obtained from solution of the system (20)-(23). The use of the method developed above is illustrated by the the example of a check calculation of a UE (Appendix 2).

Evaluation of the accuracy of the previously suggested approximate methods of optimization of the inductance coil showed the following. Optimization according to the well-known method of Fabry (with respect to the greatest field on the axis), suggested as applied to UM [3], gives an error with respect to the the sides of the section of the coil for the versions of a disk machine calculated above from 90-15 $^{0}/_{0}$. The error here decreases in proportion to the increase in the dimensions of the inductance coil. For cylindrical machines, especially in the case when using relatively short armatures ($\lambda<1.5$) this method leads to even greater errors. The relationship of sides, determined according to [5] (coinciding in individual cases with the optimum) for various R^*, may give a deviation up to 40-50 $^{0}/_{0}$ (Fig. 5). Both of these methods, however, [5 and 13] give a satisfactory qualitative characteristic of the change of the geometry of the inductance coil with an increase of its volume (especially for disk UM) and in light of the sufficiently plane extremum emf they may be used in estimating calculations.

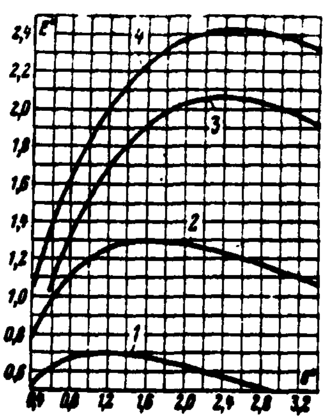

Fig. 4. Results of the calculation of the relative emf $\overset{*}{E}$ for various

dimensions of coils while maintaining constancy of volume V=cont. 1, 3 - cylindrical UM, $\lambda=1.8$; $\overset{\circ}{V}_1=5$; $R^{\circ}_{\text{з}1}=1.1$; $\overset{\circ}{V}_3=40$; $R^{\circ}_{\text{з}3}=1.2$; 2, 4 - disk UM $\overset{\circ}{z}_2-\overset{\circ}{z}_1=0.2$; $\overset{\circ}{V}=12.5$; $R^{\circ}_{\text{з}2}=1.1$; $\overset{\circ}{V}_4=40$; $R^{\circ}_{\text{з}4}=1.2$.

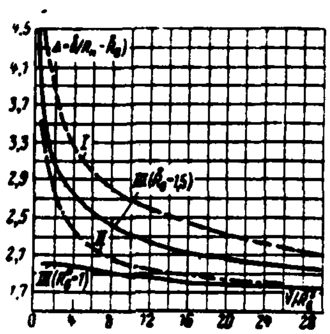

Fig. 5. Dependence of the relationship of sides of an inductance coil of a disk UM on the relative volume of the excitation coil. I - calculated according to [14]; II - calculated according to [5]; III - calculated according to (18).

Conclusions.

Expressions were obtained for determination of the basic dimensions of the armature and inductance coil of UM without a ferromagnetic circuit. Integral relationships assigning the optimum geometry of the inductance coil were numerically processed on a computer. As a result calculation curves were plotted from which the geometric parameters of an annular excitation coil with a given relative emf are determined. The presented method is more universal and accurate than those suggested earlier and corresponds well to experimental results.

Appendix 1.

For $\dot{j}_\text{в}=\dot{j}_\text{в}(\dot{\xi})$

$$\dot{E}=\int\limits_{R^*_\text{в}}^{R^*_\text{н}}\int\limits_0^{\pi}\dot{j}_\text{в}(\dot{\xi})[\dot{R}_2\ln(\dot{z}_2-\dot{\xi}+\rho^*_2)-$$

$$-\dot{R}_1\ln(\dot{z}_1-\dot{\zeta}+\dot{\rho}_1)]_{\dot{\zeta}=0}^{\dot{b}}\cos\varphi d\varphi d\dot{\xi}=\text{const};$$

$$\dot{J}=\int\limits_0^{\pi}\dot{j}_\text{в}(\dot{R}_\text{н})\{\dot{R}_1\ln[\dot{z}_1-\dot{\zeta}+\dot{\rho}_1(R^*_\text{н})]-R^*_2\ln[\dot{z}_2-$$

$$-\dot{\zeta}+\dot{\rho}_2(\dot{R}_\text{н})]\}_{\dot{\zeta}=0}^{\dot{b}}\cos\varphi d\varphi+$$

$$+\frac{\dot{b}}{\dot{R}^2_\text{н}-\dot{R}^2_\text{в}}\int\limits_{R^*_\text{в}}^{R^*_\text{н}}\int\limits_0^{\pi}\dot{j}_\text{в}(\dot{\xi})\dot{\xi}\left[\frac{R^*_1}{\rho^*_1(\xi^*,b^*)}-\frac{R^*_2}{\rho^*_2(\dot{\xi},b^*)}+\right.$$

$$\left.+\frac{R^*_1}{\dot{\rho}_1(\dot{\xi},0)}-\frac{R^*_2}{\rho^*_2(\dot{\xi},0)}\right]\cos\varphi d\varphi d\dot{\xi}=0.$$

For $\dot{j}_\text{в}=\dot{j}_\text{в}(\dot{\zeta})$

$$\dot{E}=\int\limits_0^{\dot{b}}\int\limits_0^{\pi}\dot{j}_\text{в}(\dot{\zeta})[I_1-I_2]_{\dot{\xi}=R^*_\text{в}}^{\dot{R}_\text{н}}\cos\varphi d\varphi d\dot{\zeta}=\text{const};$$

$$\mathring{I} = \int_0^{b^*}\int_0^{\pi} \mathring{i}_n(\mathring{\zeta}) \left[\frac{R^*_1}{\rho^*_1(R^*_{\text{я}}, \mathring{\zeta})} - \frac{R^*_2}{\rho^*_2(R^*_{\text{я}}, \mathring{\zeta})}\right] \cos\varphi \, d\varphi \, d\mathring{\zeta} -$$

$$- \frac{b^*}{\mathring{R}^2_{\text{я}} - \mathring{R}^2_{\text{в}}} \int_0^{\pi} \mathring{i}_n(\mathring{b}) \left[l_1(\mathring{\xi}, \mathring{b}) - l_2(\mathring{\xi}, \mathring{b})\right] +$$

$$+ \mathring{i}_n(0)\left[l_1(\xi, 0) - l_2(\mathring{\xi}, 0)\right]_{\mathring{\xi}=R^*_{\text{в}}}^{\mathring{R}_{\text{я}}} \cos\varphi \, d\varphi = 0,$$

where $l_t = \mathring{r}\,\mathring{\rho} + \mathring{r}^2 \cos\varphi \ln(\mathring{\rho} + \mathring{\xi} - \mathring{r}\cos\varphi)$,

$$l_1 = l_t \Big|_{\substack{\mathring{r} = R^*_1 \\ \mathring{z} = \mathring{z}_1}}, \quad l_2 = l_t \Big|_{\substack{\mathring{r} = R^*_2 \\ \mathring{z} = \mathring{z}_2}}.$$

Appendix 2. Checking calculaticn of the unipolar motor of the firm IRD [9].

1. Diameter and active length of the armature. The examined motor has two pairs of cellectors ($m_{\text{я}}=2$) connected in series and symmetrically arranged relative tc the excitaticn ccil. Therefore the electromagnetic moment due to one active section comprises half of the mcment of the motor: $M_э = M_{эд}/m_{\text{я}} = 0{,}596 \cdot 10^6$ Nm. Acccrding to (3)

$$D = \sqrt[3]{\frac{M_э}{\lambda e_{\text{я}}}} = \sqrt[3]{\frac{0{,}596 \cdot 10^6}{0{,}375 \cdot 15\,750}} = 2{,}16 \text{ м},$$

$$l = \lambda D = 0{,}375 \cdot 2{,}16 = 0{,}81 \text{ м},$$

where $e_{\text{я}} = e_{\text{я}}/2\pi = 15\,750$ J/m³,

$$e_{\text{в}} = \frac{0{,}5\pi^2 (D^*_1 + D^*_2)\, k_B B_m A}{k_{\Phi.\text{в}}} =$$
$$= \frac{0{,}5\pi^2 (1 + 0{,}25)\, 0{,}553 \cdot 3{,}4 \cdot 9\,850}{1{,}155} = 9{,}9 \cdot 10^4 \quad J/m^3$$

A=9850 A/m (winding of the armature with direct water cooling).

2. The internal radius of the excitation coil

$$R_{\text{в}} = R^*_{\text{в}} R = 1{,}11 \cdot 1{,}08 = 1{,}2 \ \text{м},$$

3. The relative emf. Considering that $m_{\text{в}} \neq 1$, we have:

$$\overset{*}{E} = \frac{E}{\mu_0 J_{\text{в0}} n c_E m_{\text{в}} m_{\text{в}} R^3} =$$
$$= \frac{430}{4\pi \cdot 10^{-7} \cdot 24{,}4 \cdot 10^6 \cdot 3{,}33 \cdot 10 \cdot 1 \cdot 2 \cdot 1{,}08^3} = 0{,}167.$$

4. The external radius and the width of the excitation coil. Disregarding the insignificant deviations of parameters $\overset{*}{z}_1, \overset{*}{z}_2, R^*_1$ from those calculated according to the curves of Fig. 3 we find $R^*_{\text{н}} = 1{,}3$, $b^* = 0{,}51$; then $R_{\text{н}} = R^*_{\text{н}} R = 1{,}405$ m, $b = b^* R = 0{,}55$ m. The coefficient $k_{\text{в}}$, determined according to the found dimensions using [4] coincides with the value accepted during the check calculation $k_{\text{в}} = 0{,}553$.

Parameters of the real motor [9]: D=2.16 m; l=0.81 m; $R_{\text{в}}$ =1.2 m; $R_{\text{н}}$ =1.42 m; b=0.535 m. The values calculated above correspond well to this data.

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