

AD-A085 096

CENTER FOR NAVAL ANALYSES ALEXANDRIA VA NAVAL WARFARE--ETC F/G 16/2
A TARGETING PROBLEM EXACT VERSUS EXPECTED-VALUE APPROACHES.(U)
APR 80 M MIZRAHI
CNA-PP-278

UNCLASSIFIED

NL

1 of 1
ADA
CP 108



END
DATE
FILMED
17-80
DTIC

52
ADA 085096

9 PROFESSIONAL PAPER 11 Apr 1986

2

LEVEL II

versus

6 A TARGETING PROBLEM
EXACT ~~vs~~ EXPECTED-VALUE
APPROACHES.

10 Maurice Mizrahi

14 CNA-PP-278

12 31

DTIC
ECTE
JUN 3 1980
D

C

This document has been approved
for public release and sale; its
distribution is unlimited.



CENTER FOR NAVAL ANALYSES

2000 North Beauregard Street, Alexandria, Virginia 22311

252950 *Dec*

80 6 2 107

DOC FILE COPY

PROFESSIONAL PAPER 278 / April 1980

2

A TARGETING PROBLEM EXACT vs. EXPECTED-VALUE APPROACHES

Maurice Mizrahi

DTIC
ELECT
S
C



Naval Warfare Analysis Group

CENTER FOR NAVAL ANALYSES

2000 North Beauregard Street, Alexandria, Virginia 22311

This document has been approved
for public release and sale; its
distribution is unlimited.

ABSTRACT

✓ This paper solves a targeting problem exactly and compares the exact solution with several reasonable expected-value approaches in order to gauge their accuracy. The scenario consists of air-to-surface missiles fired simultaneously at a ship formation defended with point defenses. The problem is to find the probability that a given ship will survive the raid. Each ship is present with a given probability and possesses a targeting "weight" determining the likelihood that it will be selected for targeting among the other ships present. The method can be extended to further layers of defense under certain conditions. Each solution is presented in 2 forms: a finite sum and a definite integral, each being easier to handle than the other under certain conditions. Examples are given. One important conclusion is the spectacular breakdown of reasonable expected-value approaches.

Accession For	
NTIS GRA&I	☒
DDC TAB	☐
Unannounced	☐
Justification	<i>for</i>
<i>its on file</i>	
By _____	
Distribution/ _____	
Availability Codes	
Dist	Avail and/or special
A	

A TARGETING PROBLEM: EXACT VS. EXPECTED-VALUE APPROACHES

INTRODUCTION

The purpose of this paper is twofold: first, to solve a targeting problem exactly, and second, to compare the exact solution with several possible expected-value approximations in order to gauge their accuracy.

The targeting problem consists of a wave of bomber aircraft simultaneously firing missiles at a ship formation. Before the first bomber raid, all the ships are present, but after the first raid a ship is present only with a certain probability -- depending on whether it was sunk or not. Strictly speaking, these probabilities are not independent (a missile can hit only 1 ship: when it does, it is no longer available to hit another ship). However, if they are assumed independent, multiple raids can be analyzed sequentially with the method presented here. This method exactly calculates the probability that a given ship is present after a raid from the probabilities of ship presence before the raid and other relevant parameters (number of missiles launched, ship targeting weights, and ship vulnerability data).

The bombers typically face many layers of defenses, but the model described here considers only those missiles facing the last layer, namely the ships' point defenses, if any. Nevertheless, the method can be extended to further layers of ship defense -- such as SAM barriers or outer air battle of fighters vs. bombers.

Six expected-value approaches are presented as possible alternatives to the "exact" method. Their obvious advantage is that they are easy to use and require far less computer time and space. Two numerical examples help determine which, if any, of the 6 approximations are acceptable and under what conditions they are acceptable.

THE PROBLEM

The object is:

- (1) To calculate the probability that a given ship in a formation is targeted by a given number of incoming antiship missiles (ASMs), and
- (2) To calculate the probability that a given ship will survive the raid.

(We shall see that it is not always necessary to calculate (1) in order to obtain (2).) The inputs are:

n : maximum number of ships in formation

a_i : probability that i th ship is present in formation
(the a_i s are assumed independent)

w_i : weight of i th ship (a positive integer)

M : total number of ASMs launched (the missiles are considered independent)

$L_i(m)$: lethality function, i.e., probability that the i th ship is sunk if hit by m missiles (or, if m missiles attempt to penetrate its point defenses).

The constant weight assigned to each ship determines the probability that the ship will be targeted. For example, the probability that the 5th ship is targeted given that the 2nd, 5th, 7th, and 10th ships alone are present is

$$w_5 / (w_2 + w_5 + w_7 + w_{10}) .$$

THE SOLUTION

All solutions will be expressed both as sums and integrals. In general, the larger the number of ships n the longer it takes to calculate the sum (2^{n-1} terms) and the larger the number of missiles M the longer it takes to calculate the integral (up to M terms in the integrand).

Targeting Probabilities

$P_i(m|M) \equiv$ probability that i th ship is targeted by exactly m missiles given that M missiles are launched and given that ship i is present

$$= \sum_{k=1}^n \{j_2, \dots, j_k\} = \{1, \dots, n; \text{except } i\} a_{j_2} a_{j_3} \dots a_{j_k} \cdot (1-a_{j_{k+1}}) \dots (1-a_{j_n}) \binom{M}{m} p_i^m (1-p_i)^{M-m} \quad (1)$$

where

$$p_i \equiv \frac{w_i}{w_i + w_{j_2} + \dots + w_{j_k}} \quad (2)$$

This sum can be expressed as the following definite integral for

$m \neq 0$:

$$P_i(m|M) = w_i \binom{M}{m} \int_0^1 f_i(x) dx \quad (3)$$

where

$$f_i(x) \equiv g_i(x) x^{w_i-1} \prod_{j=1; j \neq i}^n (a_j x^{w_j} + 1 - a_j) \quad (4)$$

$$g_i(x) \equiv \sum_{\ell=0}^{M-m} \binom{M-m}{\ell} (-1)^\ell \frac{(-w_i \text{Log} x)^{\ell+m-1}}{(\ell+m-1)!} \quad (5)$$

For $m=0$,

$$P_i(0|M) = 1 + w_i \int_0^1 dx h_i(x) x^{w_i-1} \prod_{j=1; j \neq i}^n (a_j x^{w_j} + 1 - a_j) \quad (6)$$

where

$$h_i(x) \equiv \sum_{\ell=1}^M \binom{M}{\ell} (-1)^\ell (-w_i \text{Log} x)^{\ell-1} / (\ell-1)! \quad (6a)$$

It is observed that the distribution is unimodal (rises, peaks, then falls), except for the fact that it may rise abruptly at the endpoint $m=M$. This corresponds to the configuration where ship i is the only one present, in which case it catches all the missiles.

Survival Probabilities

The probability that ship i will be present after the raid is:

$$a'_i = a_i [1 - P_{Ki}(M)] , \quad (7)$$

where

$P_{Ki}(M) \equiv$ probability that ship i is sunk given that it is present and that M missiles are launched on the formation

$$= \sum_{m=0}^M L_i(m) P_i(m|M) \quad (8)$$

Note that, strictly speaking, this scheme cannot be iterated for several missile raids because the a'_i are not independent (as the a_i were assumed to be)¹.

Specific forms for the lethality function $L_i(m)$ often enable one to perform the above sum directly, without having to specifically calculate the targeting distribution $P_i(m|M)$. Consider the following form:

$$L_i(m) = 1 - (1-d_i)^m + c_i m(1-b_i)^m, \quad (9)$$

where d_i , c_i , and b_i are free parameters determined by curve-fitting the lethality data (frequently, a simple power law, i.e., $c_i=0$, is sufficient).

Then

$$P_{Ki}(M) = \sum_{k=1}^n \sum_{\{j_2, \dots, j_k\} = \{1, \dots, n; \text{except } i\}} a_{j_2} \dots a_{j_k} (1-a_{j_{k+1}}) \dots (1-a_{j_n}) \left[1 - (1-d_i p_i)^M + M c_i p_i (1-b_i) (1-b_i p_i)^{M-1} \right] \quad (10)$$

with p_i given in equation (2). The integral form is

$$P_{Ki}(M) = w_i \int_0^1 dx u_i(x) x^{w_i-1} \prod_{j=1, \neq i}^n (a_j x^{w_j} + 1 - a_j) \quad (11)$$

¹To see this, consider a formation of 2 ships with equal weights and $a_i=(1,1)$ targeted by 1 missile with lethality 1 (nuclear missile). Then $a'_i=(.5,.5)$. If the a'_i were independent, the probability that both ships would be sunk $[(1-.5)^2 = .25]$ would be nonzero, clearly an impossibility since the missile can only hit one ship. The results shown here are strictly correct only if the initial probabilities of presence (a_i) are truly independent. This means that the a_i s cannot themselves be the result of a missile raid. They could be, for example, the result of independent attacks on each ship prior to the bombing raid.

where

$$u_i(x) \equiv \sum_{\ell=0}^{M-1} (M-\ell) \binom{M}{\ell} (-1)^\ell \frac{(-w_i \text{Log} x)^\ell}{\ell!} \left[\frac{d_i^{\ell+1}}{\ell+1} + c_i (1-b_i) b_i^\ell \right]. \quad (12)$$

For $c_i=0$, $u(x)$ simplifies to:

$$u_i(x) = -d_i \sum_{\ell=1}^M \binom{M}{\ell} (-1)^\ell \frac{(-w_i d_i \text{Log} x)^{\ell-1}}{(\ell-1)!}. \quad (13)$$

If the lethality data are too irregular to be fitted by (9), other formulas, with more free parameters, can be constructed which might enable one to carry out the sum in (8) exactly. These lethality formulas must satisfy $0 \leq L_i(m) < 1$, $L_i(0) = 0$, and $L_i(\infty) = 1$. An example is

$$L_i(m) = 1 - (1-d_i)^m + P_1(m)(1-b_i)^m + P_2(m)(1-b'_i)^m + \dots,$$

where the $P_j(m)$ are polynomials in m with no constant term whose coefficients are free parameters depending on i .

Even when specific forms for the lethality function $L_i(m)$ allow us to bypass the calculation of the targeting distribution $P_i(m|M)$, some useful statistical information can still be obtained. For example:

- $\bar{m}_i \equiv$ expected number of missiles targeting ship i given that it is present

$$= MP_i(1|1) . \quad (14)$$

Calculation of $P_i(1|1) \equiv \bar{p}_i$ is particularly simple (reference 1).

- The expected number of missile hits on ship i can also be calculated without knowing $P_i(m|M)$. If there are no point defenses, it is simply $\bar{m}_i$. If there are point defenses (which would be included in the lethality functions), it is obtained by matching the probability of sink P_{K1} with the curve giving probability of sink versus number of hits received.

PROOFS

Proof of Targeting Distribution Formulas

To prove the sum result in equation (1), consider a subset of k ships containing the i th ship. The ships in the subset are labelled $i, j_2, \dots, j_k$. The probability that this subset is realized is $a_i a_{j_2} \dots a_{j_k} (1-a_{j_{k+1}}) \dots (1-a_{j_n})$, and the probability that a given missile will target the i th ship from this subset is $p_i = w_i / (w_i + w_{j_2} + \dots + w_{j_k})$. Since the missiles are independent, the distribution for m missiles impacting is binomial with probability p_i . One must then sum over all possible ways of forming a subset of k ships containing the i th ship, and sum also over all possible sizes of the subset (k varying from 1 to n). Each of the 2^{n-1} possible configurations of ships -- from the i th ship alone to all ships present -- will then have been included.

Let us now turn to the integral representation (3). The case for 1 missile ($m=M=1$) was treated in reference 1, and the procedure is generalized here. We can establish the following lemma.

Lemma

The following sum:

$$\sum_{k=1}^n \sum_{\{j_2, \dots, j_k\} = \{1, \dots, n; \text{except } i\}} a_{j_2} \dots a_{j_k} (1-a_{j_{k+1}}) \dots (1-a_{j_n}) \psi(p_i) \quad (15)$$

where ψ is an arbitrary function and $p_i \equiv w_i / (w_i + w_{j_2} + \dots + w_{j_k})$, is equal to the following integral:

$$w_i \int_0^1 dx F_i(x) x^{w_i-1} \prod_{j=1; j \neq i}^n (a_j x^{w_j+1-a_j}), \quad (16)$$

where $F_i(x)$ is the inverse Laplace transform of $\psi(1/s)$ evaluated at $t = -w_i \log x$, when such a transform exists.¹ [The Laplace transform of $q(t)$ is $\int_0^\infty e^{-st} q(t) dt$].

¹ $F_i(x)$ can be a distribution, rather than a function defined at every point. All that is required is that the integral in (16) be defined. See below for an example where F is the distribution called the delta "function".

Proof of Lemma. Both the integral and the sum have 2^{n-1} terms.

A typical term in the integrand of (16) is

$$w_1 a_{j_2} \dots a_{j_k} (1-a_{j_{k+1}}) \dots (1-a_{j_n}) x^{w_1 + w_{j_2} + \dots + w_{j_k} - 1} F_1(x) .$$

To match this with the typical term in the sum (15), we must have

$$w_1 \int_0^1 x^{w_1 + w_{j_2} + \dots + w_{j_k} - 1} F_1(x) dx = \psi \left(\frac{w_1}{w_1 + w_{j_2} + \dots + w_{j_k}} \right) . \quad (17)$$

If we make the change of variable $x^{w_1} = y$, then define $s \equiv 1 + (w_{j_2} + \dots + w_{j_k})/w_1$ and $\phi_1(y) = F_1(y^{1/w_1})$, the problem reduces to finding a function ϕ_1 independent of s such that

$$\int_0^1 y^{s-1} \phi_1(y) dy = \psi\left(\frac{1}{s}\right) .$$

With the change of variable $y=e^{-t}$, this becomes:

$$\int_0^\infty e^{-st} \phi_1(e^{-t}) dt = \psi\left(\frac{1}{s}\right) .$$

The left-hand side is the definition of the Laplace transform of $\phi_1(e^{-t})$. Substituting back $t = -\log y$ and $y = x^{w_1}$ gives the result stated in the lemma.

In this case, we have

$$\psi(p_1) = p_1^m (1-p_1)^{M-m} .$$

so

$$\psi\left(\frac{1}{s}\right) = \frac{(s-1)^{M-m}}{s^M} = \sum_{\ell=0}^{M-m} \binom{M-m}{\ell} (-1)^\ell / s^{m+\ell} \quad (18)$$

Taking the inverse Laplace transform of the right-hand side gives

$$\phi_1(e^{-t}) = \sum_{\ell=0}^{M-m} \binom{M-m}{\ell} (-1)^\ell t^{\ell+m-1} / (\ell+m-1)! \quad (19)$$

for $m \neq 0$. Substituting back $t = -\log y$ and $y = x^{w_1}$ gives the $g_1(x)$ in equation (5). When $m=0$, the $\ell=0$ term in (19) is undefined.

Equation (18) shows that this term is the inverse Laplace transform of 1, which is the delta function¹ $\delta(t)$. Thus,

$$\phi_1(e^{-t}) = \delta(t) + \sum_{\ell=1}^M \binom{M}{\ell} (-1)^\ell t^{\ell-1} / (\ell-1)! \quad (20)$$

for $m=0$. Substituting back to y , then x , and putting the resulting $g_1(x)$ in (4) yields the expression (6) for $P_1(0|M)$. A change of variable $-\log x = u$ and the fact that $\int_a^b \delta(x)f(x)dx = f(0)$ if $0 \in [a,b]$ must be used.

Note that g_1 is essentially a Laguerre polynomial. Indeed, the Laplace transform of $t^{M-1} (M-m)! L_{M-m}^{m-1}(t) / (M-1)!$ is $(s-1)^{M-m} / s^M$, which is the right-hand side of (18) (reference 2, p. 191). Thus, for $m \neq 0$,

$$g_1(x) = (-w_1 \log x)^{M-1} (M-m)! L_{M-m}^{m-1}(-w_1 \log x) / (M-1)! \quad (21)$$

¹The step function $\theta(t-k)$, which is 1 for $t > k$ and 0 otherwise, has the Laplace transform e^{-ks}/s . Its derivative with respect to k , $-\delta(t-k)$, has the Laplace transform $-e^{-ks}$. As $k \rightarrow 0^+$, $\delta(t)$ has the Laplace transform 1.

where L is the Laguerre polynomial, $L_n^a(x) \equiv \sum_{m=0}^n \binom{n+a}{n-m} (-x)^m / m!$

Proof of Survival Probabilities Formula

For a lethality function $L_i(m)$ given by (9), one can readily establish that $P_{Ki}(M)$ is given by (10) by using the following results:

$$\sum_{m=0}^M \binom{M}{m} p_i^m (1-p_i)^{M-m} = 1 \quad (22)$$

$$\sum_{m=0}^M \binom{M}{m} \alpha^m p_i^m (1-p_i)^{M-m} = (\alpha p_i + 1 - p_i)^M \quad (23)$$

$$\sum_{m=0}^M \binom{M}{m} m \alpha^m p_i^m (1-p_i)^{M-m} = M \alpha p_i (\alpha p_i + 1 - p_i)^{M-1} \quad (24)$$

Equations (22) and (23) are simple binomial expansions of $(a+b)^n$.

Equation (24) can be obtained from (23) by differentiating with respect to α then multiplying by α .

To find the $u_i(x)$ in the integral form, we proceed as before. With reference to the lemma established earlier, we have:

$$\psi(p_i) = 1 - (1-d_i p_i)^M + M c_i p_i (1-b_i) (1-b_i p_i)^{M-1} \quad (25)$$

so

$$\begin{aligned} \psi\left(\frac{1}{s}\right) &= 1 - \left(\frac{s-d_i}{s}\right)^M + M c_i (1-b_i) \frac{(s-b_i)^{M-1}}{s^M} \\ &= -\sum_{\ell=1}^M \binom{M}{\ell} \frac{(-d_i)^\ell}{s^\ell} + M c_i (1-b_i) \sum_{\ell=0}^{M-1} \binom{M-1}{\ell} \frac{(-b_i)^\ell}{s^{\ell+1}} \end{aligned} \quad (26)$$

Taking inverse Laplace transforms as before gives $\phi_i(e^{-t})$, and hence $u_i(x)$. A change of index in the first sum then puts it in the form shown in (12).

Formula (14) is proved by observing that

$$\sum_{m=0}^M m \binom{M}{m} p_i^m (1-p_i)^{M-m} = M p_i \quad (\text{i.e., (24) with } \alpha=1).$$

PROPERTIES OF THE INTEGRAL REPRESENTATIONS

The integrand $f_i(x)$ in (3) has the following properties:

- (1) $f_i(0) = 0$ if $w_i > 1$ because of the presence of x^{w_i-1} and the fact that for any positive integer N , $[(\log x)^N x]$ tends to 0 when x tends to 0. If $w_i = 1$, then $f_i(0)$ can be infinite. Since this should be avoided in numerical integration, it is preferable to take all weights w_i larger than 1, which does not change the end result.

$$(2) f_i(1) = \begin{cases} 0 & \text{if } m > 1 \\ 1 & \text{if } m = 1 \end{cases}$$

- (3) The polynomial in x entering the expression of $f_i(x)$ has positive coefficients, and hence no roots for x in $[0,1]$. The function $g_i(x)$, which is a polynomial in the positive quantity $y = -\log x$ whose coefficients have alternating signs, has $M-m$ positive roots y_r (reference 2, p. 204). Therefore, $f_i(x)$ has $M-m$ roots $(x_r = e^{-y_r})$ between 0 and 1, excluding 0 and 1. Thus, the graph of $f_i(x)$ is oscillatory with $(M-m)/2$ cycles. The graphs of $h_i(x)$ in (6) and $u_i(x)$ in (12) are also oscillatory.

(4) The case $M=m=1$ is particularly simple to integrate numerically and was treated in reference 1. $f(x)$ is a polynomial with positive coefficients, and hence its graph is concave upwards. The maximum error using the trapezoidal rule or better was given as $[2f(0) + f(1) - f(1-1/N)]/2N$, where N is the number of (equal) intervals used. A refinement¹, due to David A. Perin, gives a maximum error of only

$$E = [f(0) + f(1) - f(1-1/4N) - f(1/4N)]/2N. \quad (27)$$

Verifications

Any program which calculates $P_i(m|M)$ should include the following simple checks:

$$\bullet \sum_{m=0}^M P_i(m|M) = 1 \quad (28)$$

$$\bullet \sum_{m=0}^M m P_i(m|M) = M P_i(1|1) \quad (29)$$

The quantity $P_i(1|1) \equiv \bar{p}_i$ must be calculated separately as described earlier.

¹ Instead of extending the chord from x_i to x_{i+1} to obtain a lower bound on the next interval, as was done in reference 1, consider the tangent to the curve at each interval point x_i (except the endpoints 0 and 1), and extend it until it crosses the mid-interval lines on either side [$y = (x_i + x_{i-1})/2$ and $y = (x_i + x_{i+1})/2$]. The area of the resulting right trapezoid is simply $f(x_i)/N$. This gives a lower bound for all but 2 half-intervals, one at each endpoint. Drawing the tangent at the mid-point of each half-interval [at $x_1/4$ and $(3 + x_{N-1})/4$] gives the total lower bound:

$$\int_0^1 f(x) dx > \frac{1}{N} [f(\frac{1}{N}) + \dots + f(1-\frac{1}{N})] + \frac{1}{2N} [f(\frac{1}{4N}) + f(1-\frac{1}{4N})].$$

EXPECTED-VALUE APPROACHES

Expected-value constructions of the targeting distribution

$P_i(m|M)$ consist in calculating an average probability $\bar{p}_i$ that a single missile will target ship i given that ship i is present and then constructing a binomial distribution based on this average probability. Thus

$$P_i(m|M) = \binom{M}{m} \bar{p}_i^m (1-\bar{p}_i)^{M-m} \quad (30)$$

Some possibilities for $\bar{p}_i$ are:

Expected-Value Approach No. 1: (Semi-Exact)

$$\begin{aligned} \bar{p}_i &= P_i(1|1) = \sum_{k=1}^n \{j_2, \dots, j_k\} \sum_{\{1, \dots, n; \text{except } i\}} a_{j_2} \dots a_{j_k} \\ &\quad \cdot (1-a_{j_{k+1}}) \dots (1-a_{j_n}) \frac{w_1}{w_1 + w_{j_2} + \dots + w_{j_k}} \\ &= w_1 \int_0^1 x^{w_1-1} \prod_{j=1, \neq i}^n (a_j x^{w_j+1-a_j}) dx \quad (31) \end{aligned}$$

Expected-Value Approach No. 2:

$$\bar{p}_i = \frac{a_i w_i}{\sum_{j=1}^n a_j w_j} \quad (32)$$

Expected-Value Approach No. 3:

$$\bar{p}_1 = \frac{w_1}{\sum_{j=1}^n a_j w_j} [1 - \prod_{s=1}^n (1 - a_s)] \quad (33)$$

Expected-Value Approach No. 4:

$$\bar{p}_1 = \frac{w_1}{w_1 + \sum_{j=1, \neq i}^n a_j w_j} \quad (34)$$

Expected-Value Approach No. 5:

$$\bar{p}_1 = \frac{k w_1}{w_1 + \sum_{j=1, \neq i}^n a_j w_j} ; \quad k \equiv \frac{1 - \prod_{s=1}^n (1 - a_s)}{\sum_{i=1}^n \left(\frac{a_i w_i}{w_i + \sum_{j=1, \neq i}^n a_j w_j} \right)} \quad (35)$$

Expected-Value Approach No. 6:

$$\bar{p}_1 = \frac{w_1}{\sum_{j=1}^n w_j} \quad (\text{all } a_j = 1) \quad (36)$$

No. 1 is referred to as "semi-exact" because the $\bar{p}_i$ it uses is exact. No. 2 takes the ratio of the expected weight of ship i to the total expected weight of the formation. No. 3 takes the full weight of the i th ship (since $\bar{p}_i$ assumes ship i is present) divided by the expected weight, and normalizes the result, so that the probability of targeting something, namely $\sum_{i=1}^n a_i \bar{p}_i$, is equal to the probability that something is present, namely $1 - \prod_s (1 - a_s)$. Its drawback is that $\bar{p}_i$ can be larger than 1 (if the a_j s are small enough). No. 4 argues that since the i th ship is assumed present in $\bar{p}_i$, it should contribute its full weight when the expected weight of the formation is computed in the denominator. No. 5 normalizes No. 4. No. 6 assumes that all ships are present for purposes of calculating the conditional probability, with a view of correcting this assumption when all $P_i(m|M)$ are multiplied by a_i at the end to yield the unconditional probability. All approaches, exact or expected-value, must reduce to No. 6 when all a_j s are indeed 1. No. 6 gauges the sensitivity of the resulting probability of targeting ship i given that it is present to whether or not ship i is, in fact, present.

Critique of the Expected-Value Approaches: Examples

We will compare the various approaches in 2 examples. Both include $n=11$ ships. In the first the formation is attacked with $M=12$ missiles. In the second the weights are slightly changed and $M=40$ missiles attack. The data are:

Example 1:

$a_i = .379, .249, .605, .553, .458, .695, .258, .695, .437, .460, .553$

$w_i = 2, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1$

$M = 12$

Example 2:

$a_i = .379, .249, .605, .553, .458, .695, .258, .695, .437, .460, .553$

$w_i = 4, 4, 1, 1, 1, 1, 1, 1, 1, 1, 1$

$M = 40$

The exact and the 6 expected-value targeting distributions for ship 1, $P_1(m|M)$, are shown in figures 1 and 2¹. We can make the following observations:

- When only 12 missiles are launched, many expected-value approaches come reasonably close to the correct result. When as many as 40 missiles are launched, all expected-value approaches break down. They all lead to narrow, sharply-peaked distributions whereas the correct distribution is noticeably spread out.

¹The $\bar{p}_1$ on which the binomial distributions are based are:

Expected-value method number →	1	2	3	4	5	6
$\bar{p}_1$ (example 1)	.295	.127	.335	.277	.306	.154
$\bar{p}_1$ (example 2)	.435	.210	.553	.412	.474	.235

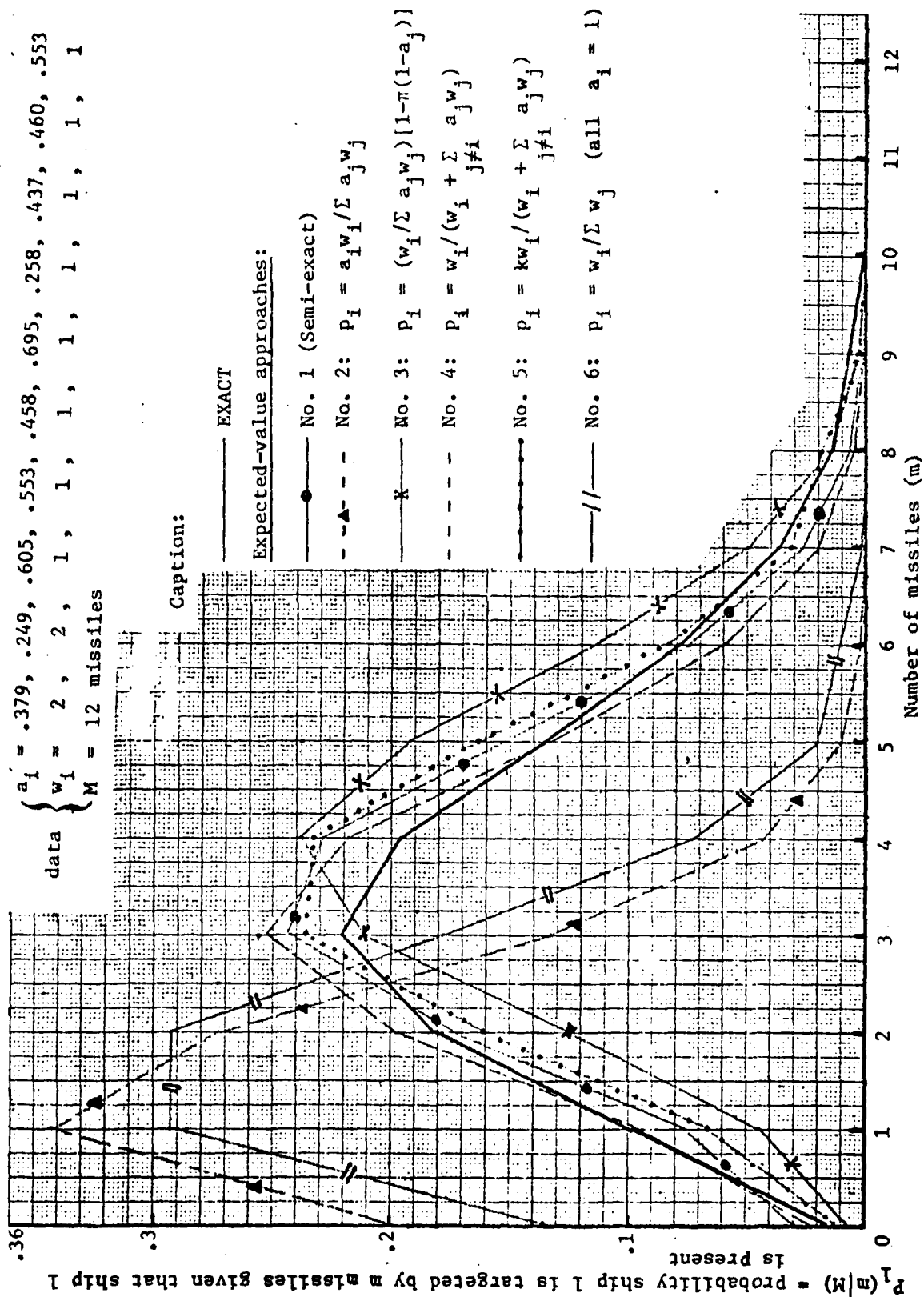


FIG. 1: Targeting Distribution in Example 1

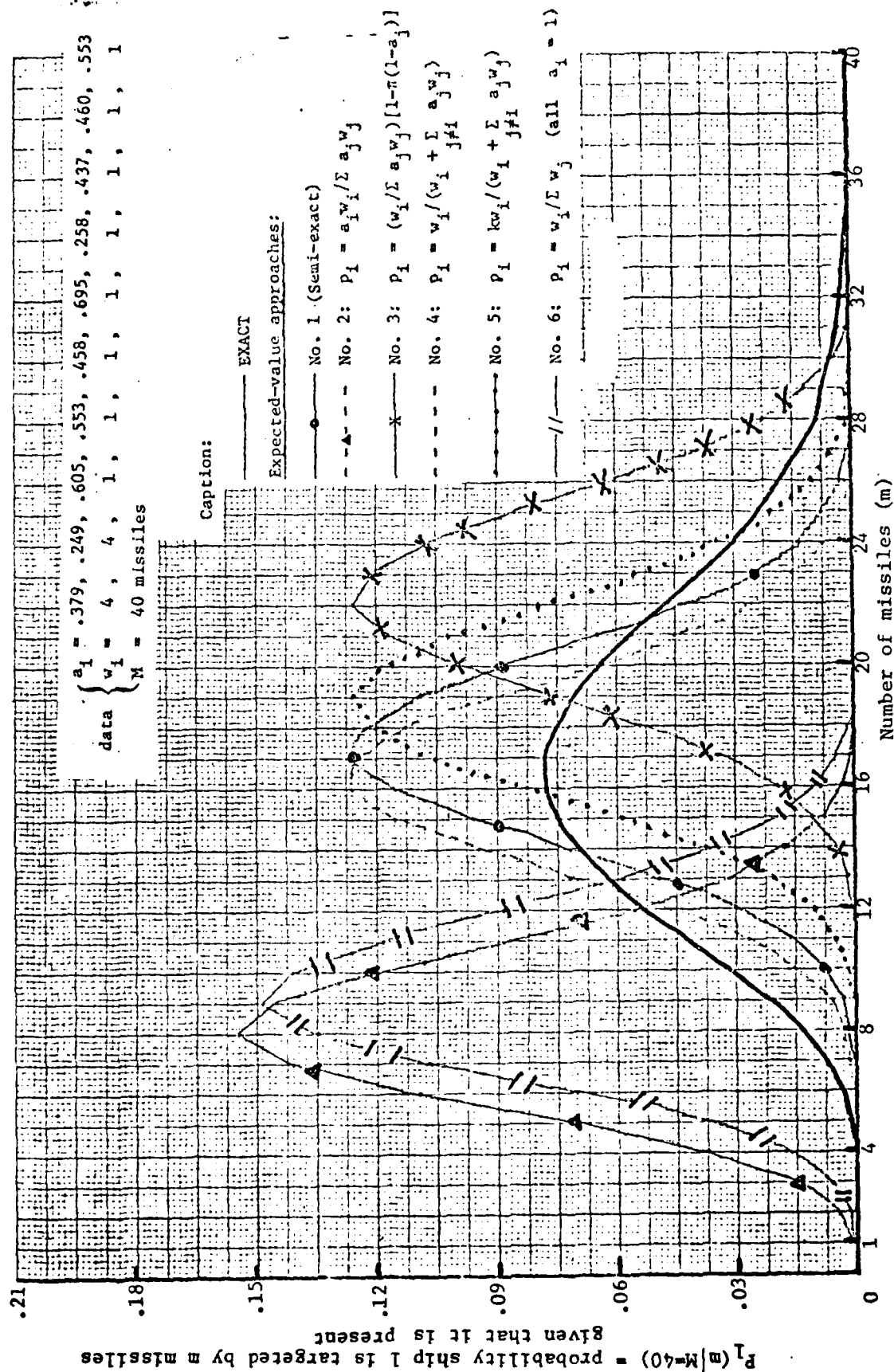


FIG. 2: Targeting Distribution in Example 2

- Approaches Nos. 2 and 6 lead very far from the correct result in both cases.
- The semi-exact result (No. 1) is closest to the exact result in both cases.
- No. 3 peaks later than all the others.
- Nos. 1, 4, and 5 are close to one another in both cases.

The wider spread of the exact distribution can be predicted by calculating variances. For example, the variance of the exact distribution (calculated by James K. Tyson) is:

$$\sigma_m^2 = M \bar{p}_i (1 - \bar{p}_i) + M(M-1) \sigma_p^2 ,$$

where $\sigma_p^2 \equiv p_i^2 - \bar{p}_i^2$ is independent of M, but the variance of the semi-exact distribution, which is a single binomial, is

$$\sigma_m^2 = M \bar{p}_i (1 - \bar{p}_i) .$$

Thus, the exact distribution is always more spread out than the semi-exact expected-value distribution, and the difference between the 2 standard deviations grows as M, the total number of missiles. For a very large number of missiles, one would witness a spectacular breakdown of all the expected-value approaches considered here.

Let us now introduce the lethality (Equation (8)) to find the probability P_{K1} that ship 1 is sunk given it is present. Two lethality functions are considered: $L_1(m) = 1 - (1 - .01)^m$ and $L_1(m) = 1 - (1 - .03)^m$. The results for example 2 are given in the table below.

Lethality function $L_1(m)$	P_{K1}						
	No. 1 (semi-exact)		No. 2	No. 3	No. 4	No. 5	No. 6
	Exact						
$1 - (1 - .01)^m$	.159	.160	.081	.199	.152	.173	.090
$1 - (1 - .03)^m$	.404	.408	.223	.488	.392	.436	.247

As can be seen, the semi-exact result is extremely close to the exact result, and Nos. 4 and 5 are not far behind. The other expected-value methods break down. It appears therefore, that, in this particular case, expected-value distributions having the same mean as the exact one yield results close to the exact one for power-law lethality functions. However, this is not true for other realistic lethality functions. For example, consider the case of the cookie-cutter lethality function,

$L_i(m) = 0$ for $m \leq m_0$ and 1 for $m > m_0$, corresponding to a saturation effect with nuclear missiles (the point defenses destroy any number of missiles smaller than m_0 , but if more than m_0 engage, the excess will hit the ship and sink it with certainty). The sink probability is then simply $P_{Ki}(M) = \sum_{m=m_0}^M P_i(m|M)$, which can vary widely depending on m_0 and the approach used, as figures 1 and 2 testify.

The reason why the sink probabilities given by some expected-value results are close the exact sink probability in this case is that in the interval where most of the distribution is concentrated (m between 12 to 24) the lethality function does not vary dramatically, so that the shape of the distribution is not a dominant consideration.

Conclusion on Expected-Value Approaches

The conclusions to be drawn from the above are:

- When the object is to calculate the targeting distribution, all expected-value approaches fail significantly for large numbers of missiles (~40), but some are successful for small numbers of missiles (~12).

- When the object is to calculate the probability of sinking a ship, all expected-value approaches fail significantly for some lethality functions (such as a cookie-cutter), but some are successful for other lethality functions (such as power laws).
- The semi-exact expected-value approach (No. 1) yields results closest to the exact approach. Approaches Nos. 4 and 5 are not far behind. The other approaches fail significantly in all cases.

As can be seen, results yielded by reasonable expected-value approaches can show large errors when compared to results yielded by the corresponding exact approach. This is all the more sobering as in many cases the problem is so complex (e.g., a full campaign) that an exact approach is almost impossible to formulate, let alone to use for calculations.

More details and computer programs can be found in reference 3.

References

1. Center for Naval Analyses, Memorandum for the Record (CNA)79-0680, "Calculation of ASM Targeting Probabilities for the Sea War 85 Study," by M. Mizrahi and D. Perin, Unclassified, 30 Apr 1979
2. Harry Bateman, et al., "Higher Transcendental Functions, Vol. II," New York: McGraw Hill, 1953
3. Center for Naval Analyses, Professional Paper 255, "A Targeting Problem: Exact vs. Expected-Value Approaches," by M. Mizrahi, Unclassified, Aug 1979

CNA Professional Papers — 1976 to Present*

- PP 141
Mizrahi, Maurice M., "Generalized Hermite Polynomials," 5 pp., Feb 1976 (Reprinted from the Journal of Computational and Applied Mathematics, Vol. 1, No. 4 (1975), 273-277).
*Research supported by the National Science Foundation
- PP 143
Horowitz, Stanley and Sherman, Allan (LCdr., USN), "Maintenance Personnel Effectiveness in the Navy," 33 pp., Jan 1976 (Presented at the RAND Conference on Defense Manpower, Feb 1976) AD A021 581
- PP 144
Dureh, William J., "The Navy of the Republic of China — History, Problems, and Prospects," 66 pp., Aug 1976 (Published in "A Guide to Asiatic Fleets," ed. by Barry M. Blechman and Robert Berman, Naval Institute Press) AD A030 460
- PP 145
Kelly, Anne M., "Port Visits and the "Internationalist Mission" of the Soviet Navy," 36 pp., Apr 1976, AD A023 436
- PP 147
Kessler, J. Christian, "Legal Issues in Protecting Offshore Structures," 33 pp., Jun 1976 (Prepared under task order N00014-68-A-0091-0023 for ONR) AD A028 389
- PP 149
Squires, Michael L., "Counterforce Effectiveness: A Comparison of the Tsipis "K" Measure and a Computer Simulation," 24 pp., Mar 1976 (Presented at the International Study Association Meetings, 27 Feb 1976) AD A022 591
- PP 150
Kelly, Anne M. and Petersen, Charles, "Recent Changes in Soviet Naval Policy: Prospects for Arms Limitations in the Mediterranean and Indian Ocean," 28 pp., Apr 1976, AD A 023 723
- PP 151
Horowitz, Stanley A., "The Economic Consequences of Political Philosophy," 8 pp., Apr 1976 (Reprinted from Economic Inquiry, Vol. XIV, No. 1, Mar 1976)
- PP 152
Mizrahi, Maurice M., "On Path Integral Solutions of the Schrodinger Equation, Without Limiting Procedure," 10 pp., Apr 1976 (Reprinted from Journal of Mathematical Physics, Vol. 17, No. 4 (Apr 1976), 566-576).
*Research supported by the National Science Foundation
- PP 153
Mizrahi, Maurice M., "WKB Expansions by Path Integrals, With Applications to the Anharmonic Oscillator," 137 pp., May 1976, AD A025 440
*Research supported by the National Science Foundation
- PP 154
Mizrahi, Maurice M., "On the Semi-Classical Expansion in Quantum Mechanics for Arbitrary Hamiltonians," 19 pp., May 1976 (Published in Journal of Mathematical Physics, Vol. 18, No. 4, pp. 789-790, Apr 1977), AD A025 441
- PP 155
Squires, Michael L., "Soviet Foreign Policy and Third World Nations," 26 pp., Jun 1976 (Prepared for presentation at the Midwest Political Science Association meetings, Apr 30, 1976) AD A028 388
- PP 156
Stallings, William, "Approaches to Chinese Character Recognition," 12 pp., Jun 1976 (Reprinted from Pattern Recognition (Pergamon Press), Vol. 8, pp. 87-98, 1976) AD A028 692
- PP 157
Morgan, William F., "Unemployment and the Pentagon Budget: Is There Anything in the Empty Pork Barrel?" 20 pp., Aug 1976 AD A030 455
- PP 158
Haskell, LCdr. Richard D. (USN), "Experimental Validation of Probability Predictions," 25 pp., Aug 1976 (Presented at the Military Operations Research Society Meeting, Fall 1976) AD A030 458
- PP 159
McConnell, James M., "The Gorshkov Articles, The New Gorshkov Book and Their Relation to Policy," 93 pp., Jul 1976 (Published in Soviet Naval Influence: Domestic and Foreign Dimensions, ed. by M. McGwire and J. McDonnell: New York: Praeger, 1977) AD A029 227
- PP 160
Wilson, Desmond P., Jr., "The U.S. Sixth Fleet and the Conventional Defense of Europe," 50 pp., Sep 1976, AD A030 457
- PP 161
Melich, Michael E. and Peet, Vice Adm. Ray (USN, Retired), "Fleet Commanders: Afloat or Ashore?" 9 pp., Aug 1976 (Reprinted from U.S. Naval Institute Proceedings, Jun 1976) AD A030 456
- PP 162
Friedheim, Robert L., "Parliamentary Diplomacy," 106 pp. Sep 1976 AD A033 306
- PP 163
Lockman, Robert F., "A Model for Predicting Recruit Losses," 9 pp., Sep 1976 (Presented at the 84th annual convention of the American Psychological Association, Washington, D.C., 4 Sep 1976) (Published in Defense Manpower Policy (Richard V. L. Conber, ed.), The Rand Corporation, 1979), AD A030 459
- PP 164
Mahoney, Robert B., Jr., "An Assessment of Public and Elite Perceptions in France, The United Kingdom, and the Federal Republic of Germany," 31 pp., Feb 1977 (Presented at Conference "Perception of the U.S. — Soviet Balance and the Political Uses of Military Power" sponsored by Director, Advanced Research Projects Agency, April 1976) AD A036 599
- PP 165
Jondrow, James M., "Effects of Trade Restrictions on Imports of Steel," 67 pp., November 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 166 — Revised
Feldman, Paul, "Why It's Difficult to Change Regulation," Oct 1976, AD A037 682
- PP 167
Kleinman, Samuel, "ROTC Service Commitments: A Comment," 4 pp., Nov 1976, (Published in Public Choice, Vol. XXIV, Fall 1976) AD A033 305
- PP 168
Lockman, Robert F., "Revalidation of CNA Support Personnel Selection Measures," 36 pp., Nov 1976
- PP 169
Jacobson, Louis S., "Earnings Losses of Workers Displaced from Manufacturing Industries," 38 pp., Nov 1976, (Delivered at ILAB Conference in Dec 1976), AD A039 809
- PP 170
Brechtling, Frank P., "A Time Series Analysis of Labor Turnover," Nov 1976, (Delivered at ILAB Conference in Dec 1976)
- PP 171
Jordan, A. S. and Ralston, J. M., "A Diffusion Model for GaP Red LED Degradation," 10 pp., Nov 1976, (Published in Journal of Applied Physics, Vol. 47, pp. 4518-4527, Oct 1976)
*Bell Laboratories
- PP 172
Classen, Kathleen P., "Unemployment Insurance and the Length of Unemployment," Dec 1976, (Presented at the University of Rochester Labor Workshop on 16 Nov 1976)
- PP 173
Kleinman, Samuel D., "A Note on Racial Differences in the Added-Worker/Discouraged-Worker Controversy," 2 pp., Dec 1976, (Published in the American Economist, Vol. XX, No. 1, Spring 1976)
- PP 174
Mahoney, Robert B., Jr., "A Comparison of the Brookings and International Incidents Projects," 12 pp. Feb 1977 AD A037 206
- PP 175
Levine, Daniel; Stoloft, Peter and Spruill, Nancy, "Public Drug Treatment and Addict Crime," June 1976, (Published in Journal of Legal Studies, Vol. 5, No. 2)
- PP 176
Felix, Wendi, "Correlates of Retention and Promotion for USNA Graduates," 38 pp., Mar 1977, AD A039 040
- PP 177
Lockman, Robert F. and Warner, John T., "Predicting Attrition: A Test of Alternative Approaches," 33 pp., Mar 1977, (Presented at the OSD/ONR Conference on Enlisted Attrition, Xerox International Training Center, Leesburg, Virginia, 4-7 April 1977), AD A039 047
- PP 178
Kleinman, Samuel D., "An Evaluation of Navy Unrestricted Line Officer Accession Programs," 23 pp., April 1977, (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977), AD A039 048

*CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia 22151. Other papers are available from the Management Information Office, Center for Naval Analyses, 2000 North Beauregard Street, Alexandria, Virginia 22311. An Index of Selected Publications is also available on request. The index includes a Listing of Professional Papers, with abstracts, issued from 1969 to February 1978.

- PP 179
Stoloff, Peter H. and Balut, Stephen J., "Vacate: A Model for Personnel Inventory Planning Under Changing Management Policy," 14 pp., April 1977. (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977). AD A039 049
- PP 180
Horowitz, Stanley A. and Sherman, Allan, "The Characteristics of Naval Personnel and Personnel Performance," 16 pp., April 1977. (Presented at the NATO Conference on Manpower Planning and Organization Design, Stresa, Italy, 20 June 1977). AD A039 050
- PP 181
Balut, Stephen J. and Stoloff, Peter, "An Inventory Planning Model for Navy Enlisted Personnel," 35 pp., May 1977 (Prepared for presentation at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Science, 9 May 1977, San Francisco, California). AD A042 221
- PP 192
Murray, Russell, 2nd, "The Quest for the Perfect Study or My First 1138 Days at CNA," 57 pp., April 1977
- PP 183
Kassing, David, "Changes in Soviet Naval Forces," 33 pp., November, 1976. (Published as part of Chapter 3, "General Purpose Forces: Navy and Marine Corps," in Arms, Men, and Military Budgets, Francis P. Hoebler and William Schneider, Jr. (eds.), (Crane, Russak & Company, Inc.: New York), 1977). AD A040 106
- PP 184
Lockman, Robert F., "An Overview of the OSD/ONR Conference on First Term Enlisted Attrition," 22 pp., June 1977. (Presented to the 39th MORS Working Group on Manpower and Personnel Planning, Annapolis, Md., 28-30 Jun 1977). AD A043 618
- PP 185
Kassing, David, "New Technology and Naval Forces in the South Atlantic," 22 pp. (This paper was the basis for a presentation made at the Institute for Foreign Policy Analyses, Cambridge, Mass., 28 April 1977). AD A043 619
- PP 186
Mizrahi, Maurice M., "Phase Space Integrals, Without Limiting Procedure," 31 pp., May 1977. (Invited paper presented at the 1977 NATO Institute on Path Integrals and Their Application to Quantum Statistical, and Solid State Physics, Antwerp, Belgium, July 17-30, 1977) (Published in Journal of Mathematical Physics 19(1) pp 298-307, Jan 1978). AD A040 107
- PP 187
Coile, Russell C., "Nomography for Operations Research," 35 pp., April 1977 (Presented at the Joint National Meeting of the Operations Research Society of America and The Institute for Management Services, San Francisco, California, 9 May 1977). AD A043 620
- PP 188
Durch, William J., "Information Processing and Outcome Forecasting for Multilateral Negotiations: Testing One Approach," 53 pp., May 1977 (Prepared for presentation to the 18th Annual Convention of the International Studies Association, Chase-Park Plaza Hotel, St. Louis, Missouri, March 16-20, 1977). AD A042 222
- PP 189
Coile, Russell C., "Error Detection in Computerized Information Retrieval Data Bases," July, 1977, 13 pp. (Presented at the Sixth Cranfield International Conference on Mechanized Information Storage and Retrieval Systems, Cranfield Institute of Technology, Cranfield, Bedford, England, 26-29 July 1977). AD A043 580
- PP 190
Mahoney, Robert B., Jr., "European Perceptions and East West Competition," 96 pp., July 1977 (Prepared for presentation at the annual meeting of the International Studies Association, St. Louis, Mo. March, 1977). AD A043 661
- PP 191
Sawyer, Ronald, "The Independent Field Assignment: One Man's View," August 1977, 25 pp.
- PP 192
Holen, Arlene, "Effects of Unemployment Insurance Entitlement on Duration and Job Search Outcome," August 1977, 6 pp., (Reprinted from Industrial and Labor Relations Review, Vol., 30, No. 4, Jul 1977)
- PP 193
Horowitz, Stanley A., "A Model of Unemployment Insurance and the Work Test," August 1977, 7 pp. (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)
- PP 194
Classen, Kathleen P., "The Effects of Unemployment Insurance on the Duration of Unemployment and Subsequent Earnings," August 1977, 7 pp. (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)
- PP 195
Brechling, Frank, "Unemployment Insurance Taxes and Labor Turnover: Summary of Theoretical Findings," 12 pp. (Reprinted from Industrial and Labor Relations Review, Vol. 30, No. 40, Jul 1977)
- PP 196
Ralston, J. M. and Lorimer, O. G., "Degradation of Bulk Electroluminescent Efficiency in Zn, O-Doped GaP LED's," July 1977, 3 pp. (Reprinted from IEEE Transactions on Electron Devices, Vol. ED-24, No. 7, July 1977)
- PP 197
Wells, Anthony R., "The Centre for Naval Analyses," 14 pp., Dec 1977, AD A049 107
- PP 198
Classen, Kathleen P., "The Distributional Effects of Unemployment Insurance," 25 pp., Sept. 1977 (Presented at a Hoover Institution Conference on Income Distribution, Oct 7-8, 1977). AD A054 423
- PP 199
Durch, William J., "Revolution From A F.A.R. - The Cuban Armed Forces in Africa and the Middle East," Sep 1977, 16 pp., AD A046 268
- PP 200
Powers, Bruce F., "The United States Navy," 40 pp. Dec 1977 (Published as a chapter in The U.S. War Machine by Salamander Books, England, 1978). AD A049 108
- PP 201
Durch, William J., "The Cuban Military in Africa and The Middle East: From Algeria to Angola," Sep 1977, 67 pp., AD A045 675
- PP 202
Feldman, Paul, "Why Regulation Doesn't Work," (Reprinted from Technological Change and Welfare in the Regulated Industries, Brookings Reprint 219, 1971, and Review of Social Economy, Vol. XXIX, March, 1971, No. 1.) Sep 1977, 8 pp.
- PP 203
Feldman, Paul, "Efficiency, Distribution, and the Role of Government in a Market Economy," (Reprinted from The Journal of Political Economy, Vol. 79, No. 3, May/June 1971.) Sep 1977, 19 pp., AD A045 675
- PP 204
Wells, Anthony R., "The 1967 June War: Soviet Naval Diplomacy and The Sixth Fleet - A Re-appraisal," Oct 1977, 36 pp., AD A047 236
- PP 205
Coile, Russell C., "A Bibliometric Examination of the Square Root Theory of Scientific Publication Productivity," (Presented at the annual meeting of the American Society for Information Science, Chicago, Illinois, 29 September 1977.) Oct 1977, 6 pp., AD A047 237
- PP 206
McConnell, James M., "Strategy and Missions of the Soviet Navy in the Year 2000," 48 pp., Nov 1977 (Presented at a Conference on Problems of Sea Power as we Approach the 21st Century, sponsored by the American Enterprise Institute for Public Policy Research, 6 October 1977, and subsequently published in a collection of papers by the Institute). AD A047 244
- PP 207
Goldberg, Lawrence, "Cost-Effectiveness of Potential Federal Policies Affecting Research & Development Expenditures in the Auto, Steel and Food Industries," 36 pp., Oct 1977, (Presented at Southern Economic Association Meetings beginning 2 November 1977). AD A046 269
- PP 208
Roberts, Stephen S., "The Decline of the Overseas Station Fleets: The United States Asiatic Fleet and the Shanghai Crisis, 1932," 18 pp., Nov 1977 (Reprinted from The American Neptune, Vol. XXXVII, No. 3, July 1977). AD A047 245
- PP 209 - Classified.
- PP 210
Kassing, David, "Protecting The Fleet," 40 pp., Dec 1977 (Prepared for the American Enterprise Institute Conference on Problems of Sea Power as We Approach the 21st Century, October 6-7, 1977). AD A049 109
- PP 211
Mizrahi, Maurice M., "On Approximating the Circular Coverage Function," 14 pp., Feb 1978, AD A054 429
- PP 212
Mangel, Marc, "On Singular Characteristic Initial Value Problems with Unique Solutions," 20 pp., Jun 1978. AD A058 535
- PP 213
Mangel, Marc, "Fluctuations in Systems with Multiple Steady States. Application to Lancaster Equations," 12 pp., Feb 78, (Presented at the First Annual Workshop on the Information Linkage Between Applied Mathematics and Industry, Naval PG School, Feb 23-25, 1978). AD A071 472

- PP 214
Weinland, Robert G., "A Somewhat Different View of The Optimal Naval Posture," 37 pp., Jun 1978 (Presented at the 1976 Convention of the American Political Science Association (APSA/IUS Panel on "Changing Strategic Requirements and Military Posture"), Chicago, Ill., September 2, 1976), AD A056 228
- PP 215
Coile, Russell C., "Comments on: Principles of Information Retrieval by Manfred Kochen," 10 pp., Mar 78, (Published as a Letter to the Editor, Journal of Documentation, Vol. 31, No. 4, pages 298-301, December 1976), AD A054 426
- PP 216
Coile, Russell C., "Lotka's Frequency Distribution of Scientific Productivity," 18 pp., Feb 1978, (Published in the Journal of the American Society for Information Science, Vol. 28, No. 6, pp. 366-370, November 1977), AD A054 425
- PP 217
Coile, Russell C., "Bibliometric Studies of Scientific Productivity," 17 pp., Mar 78, (Presented at the Annual meeting of the American Society for Information Science held in San Francisco, California, October 1976), AD A054 442
- PP 218 - Classified.
- PP 219
Huntzinger, R. LeVar, "Market Analysis with Rational Expectations: Theory and Estimation," 60 pp., Apr 78, AD A054 422
- PP 220
Maurer, Donald E., "Diagonalization by Group Matrices," 26 pp., Apr 78, AD A054 443
- PP 221
Weinland, Robert G., "Superpower Naval Diplomacy in the October 1973 Arab-Israeli War," 76 pp., Jun 1978 (Published in Seapower in the Mediterranean: Political Utility and Military Constraints, The Washington Papers No. 61, Beverly Hills and London: Sage Publications, 1979) AD A055 564
- PP 222
Mizrahi, Maurice M., "Correspondence Rules and Path Integrals," 30 pp., Jun 1978 (Invited paper presented at the CNRS meeting on "Mathematical Problems in Feynman's Path Integrals," Marseille, France, May 22-26, 1978) (Published in Springer Verlag Lecture Notes in Physics, 106, (1979), 234-253) AD A055 536
- PP 223
Mangel, Marc, "Stochastic Mechanics of Molecule-Molecule Reactions," 21 p., Jun 1978, AD A056 227
- PP 224
Mangel, Marc, "Aggregation, Bifurcation, and Extinction in Exploited Animal Populations," 48 pp., Mar 1978, AD A058 536
*Portions of this work were started at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, B.C., Canada
- PP 225
Mangel, Marc, "Oscillations, Fluctuations, and the Hopf Bifurcation," 43 pp., Jun 1978, AD A058 537
*Portions of this work were completed at the Institute of Applied Mathematics and Statistics, University of British Columbia, Vancouver, Canada.
- PP 226
Ralston, J. M. and J. W. Mann*, "Temperature and Current Dependence of Degradation in Red-Emitting GaP LEDs," 34 pp., Jun 1978 (Published in Journal of Applied Physics, 50, 3630, May 1979) AD A058 538
*Bell Telephone Laboratories, Inc.
- PP 227
Mangel, Marc, "Uniform Treatment of Fluctuations at Critical Points," 50 pp., May 1978, AD A058 539
- PP 228
Mangel, Marc, "Relaxation at Critical Points: Deterministic and Stochastic Theory," 54 pp., Jun 1978, AD A058 540
- PP 229
Mangel, Marc, "Diffusion Theory of Reaction Rates. I: Formulation and Einstein-Smoluchowski Approximation," 50 pp., Jan 1978, AD A058 541
- PP 230
Mangel, Marc, "Diffusion Theory of Reaction Rates. II Ornstein-Uhlenbeck Approximation," 34 pp., Feb 1978, AD A058 542
- PP 231
Wilson, Desmond P. Jr., "Naval Projection Forces: The Case for a Responsive MAF," Aug 1978, AD A058 543
- PP 232
Jacobson, Louis, "Can Policy Changes Be Made Acceptable to Labor?" Aug 1978 (Submitted for publication in Industrial and Labor Relations Review), AD A061 528
- PP 233
Jacobson, Louis, "An Alternative Explanation of the Cyclical Pattern of Quites," 23 pp., Sep 1978
- PP 234 - Revised
Jondrow, James and Levy, Robert A., "Does Federal Expenditure Displace State and Local Expenditure: The Case of Construction Grants," 25 pp., Oct 1979
- PP 235
Mizrahi, Maurice M., "The Semiclassical Expansion of the Anharmonic-Oscillator Propagator," 41 pp., Oct 1978 (Published in Journal of Mathematical Physics 20 (1979), pp. 844-855), AD A061 538
- PP 237
Maurer, Donald, "A Matrix Criterion for Normal Integral Bases," 10 pp., Jan 1979 (Published in the Illinois Journal of Mathematics, Vol. 22 (1978), pp. 672-681)
- PP 238
Utgoff, Kathleen Classen, "Unemployment Insurance and The Employment Rate," 20 pp., Oct 1978 (Presented at the Conference on Economic Indicators and Performance: The Current Dilemma Facing Government and Business Leaders, presented by Indiana University Graduate School of Business), AU A061 527
- PP 239
Trost, R. P. and Warner, J. T., "The Effects of Military Occupational Training on Civilian Earnings: An Income Selectivity Approach," 38 pp., Nov 1979, AD A077 831
- PP 240
Powers, Bruce, "Goals of the Center for Naval Analyses," 13 pp., Dec 1978, AD A063 759
- PP 241
Mangel, Marc, "Fluctuations at Chemical Instabilities," 24 pp., Dec 1978 (Published in Journal of Chemical Physics, Vol. 69, No. 8, Oct 15, 1978), AD A063 787
- PP 242
Simpson, William R., "The Analysis of Dynamically Interactive Systems (Air Combat by the Numbers)," 160 pp., Dec 1978, AD A063 760
- PP 243
Simpson, William R., "A Probabilistic Formulation of Murphy Dynamics as Applied to the Analysis of Operational Research Problems," 18 pp., Dec 1978, AD A063 761
- PP 244
Sherman, Allan and Horowitz, Stanley A., "Maintenance Costs of Complex Equipment," 20 pp., Dec 1978 (Published By The American Society of Naval Engineers, Naval Engineers Journal, Vol. 91, No. 6, Dec 1979) AD A071 473
- PP 245
Simpson, William R., "The Accelerometer Methods of Obtaining Aircraft Performance from Flight Test Data (Dynamic Performance Testing)," 403 pp., Jun 1979, AD A075 226
- PP 246
Brechtling, Frank, "Layoffs and Unemployment Insurance," 35 pp., Feb 1979 (Presented at the NBER Conference on "Low Income Labor Markets," Chicago, Jun 1978)
- PP 248
Thomas, James A., Jr., "The Transport Properties of Dilute Gases in Applied Fields," 183 pp., Mar 1979
- PP 249
Glasser, Kenneth S., "A Secretary Problem with a Random Number of Choices," 23 pp., Mar 1979
- PP 250
Mangel, Marc, "Modeling Fluctuations in Macroscopic Systems," 26 pp., Jun 1979
- PP 251
Trost, Robert P., "The Estimation and Interpretation of Several Selectivity Models," 37 pp., Jun 1979, AD A075 941
- PP 252
Nunn, Walter R., "Position Finding with Prior Knowledge of Covariance Parameters," 5 pp., Jun 1979 (Published in IEEE Transactions on Aerospace & Electronic Systems, Vol. AES-15, No. 3, March 1979)
- PP 253
Glasser, Kenneth S., "The d-Choice Secretary Problem," 32 pp., Jun 1979, AD A075 225
- PP 254
Mangel, Marc and Quanbeck, David B., "Integration of a Bivariate Normal Over an Offset Circle," 14 pp., Jun 1979
- PP 255 - Classified
- PP 256
Maurer, Donald E., "Using Personnel Distribution Models," 27 pp., Feb 1980

- PP 257
Thaler, R., "Discounting and Fiscal Constraints: Why Discounting is Always Right," 10 pp., Aug 1979, AD A876 224
- PP 258
Mangel, Marc S. and Thomas, James A., Jr., "Analytical Methods in Search Theory," 86 pp., Nov 1979, AD A077 832
- PP 259
Gless, David V.; Hsu, Ih-Ching; Nunn, Walter R. and Parin, David A., "A Class of Commutative Markov Matrices," 17 pp., Nov 1979, AD A077 833
- PP 260
Mangel, Marc S. and Cope, Davis K., "Detection Rate and Sweep Width in Visual Search," 14 pp., Nov 1979, AD A077 834
- PP 261
Vila, Carlos L.; Zvijac, David J. and Ross, John, "Franck-Condon Theory of Chemical Dynamics. VI. Angular Distributions of Reaction Products," 14 pp., Nov 1979 (Reprinted from Journal Chem. Phys. 70(12), 18 Jun 1979), AD A076 287
- PP 262
Peterson, Charles C., "Third World Military Elites in Soviet Perspective," 50 pp., Nov 1979, AD A077 836
- PP 263
Robinson, Kathy I., "Using Commercial Tankers and Containerships for Navy Underway Replenishment," 25 pp., Nov 1979, AD A077 836
- PP 264
Weinland, Robert G., "The U.S. Navy in the Pacific: Past, Present, and Glimpses of the Future," 31 pp., Nov 1979 (Delivered at the International Symposium on the Sea, sponsored by the International Institute for Strategic Studies, The Brookings Institution and the Yomiuri Shimbun, Tokyo, 16-20 Oct 1978) AD A066 837
- PP 265
Weinland, Robert G., "War and Peace in the North: Some Political Implications of the Changing Military Situation in Northern Europe," 18 pp., Nov 1979 (Prepared for presentation to the "Conference of the Nordic Balance in Perspective: The Changing Military and Political Situation," Center for Strategic and International Studies, Georgetown University, Jun 15-16, 1978) AD A077 838
- PP 266
Utgoff, Kathy Classen, and Brechling, Frank, "Taxes and Inflation," 25 pp., Nov 1979
- PP 267
Trost, Robert P., and Vogel, Robert C., "The Response of State Government Receipts to Economic Fluctuations and the Allocation of Counter-Cyclical Revenue Sharing Grants," 12 pp., Dec 1979 (Reprinted from the Review of Economics and Statistics, Vol. LXI, No. 3, August 1979)
- PP 268
Thomason, James S., "Seaport Dependence and Inter-State Cooperation: The Case of Sub-Saharan Africa," 141 pp., Jan 1980
- PP 269
Weiss, Kenneth G., "The Soviet Involvement in the Ogaden War," 42 pp., Jan 1980 (Presented at the Southern Conference on Slavic Studies in October, 1979)
- PP 270
Remnek, Richard, "Soviet Policy in the Horn of Africa: The Decision to Intervene," 52 pp., Jan 1980 (To be published in "The Soviet Union in the Third World: Success or Failure," ed. by Robert H. Donaldson, Westview Press, Boulder, Co., Summer 1980)
- PP 271
McConnell, James, "Soviet and American Strategic Doctrines: One More Time," 43 pp., Jan 1980
- PP 272
Weiss, Kenneth G., "The Azores in Diplomacy and Strategy, 1940-1945," 46 pp., Mar 1980
- PP 273
Nakade, Michael K., "Labor Supply of Wives with Husbands Employed Either Full Time or Part Time," 39 pp., Mar 1980
- PP 275
Goldberg, Lawrence, "Recruiters Advertising and Navy Enlistments," 34 pp., Mar 1980
- PP 277
Mangel, Marc, "Small Fluctuations in Systems with Multiple Limit Cycles," 19 pp., Mar 1980 (Published in SIAM J. Appl. Math., Vol. 38, No. 1, Feb 1980)
- PP 278
Mizrahi, Maurice, "A Targeting Problem: Exact vs. Expected-Value Approaches," 23 pp., Apr 1980
- PP 280
Goldberg, Lawrence, "Estimation of the Effects of A Ship's Steaming on the Failure Rate of its Equipment: An Application of Econometric Analysis," 25 pp., April 1980