

AD-A085 126

NAVAL POSTGRADUATE SCHOOL MONTEREY CA

F/8 17/2

DIFFUSION APPROXIMATIONS FOR THE COOPERATIVE SERVICE OF VOICE A--ETC(U)

FEB 80 J P LEHOCZKY, D P GAVR

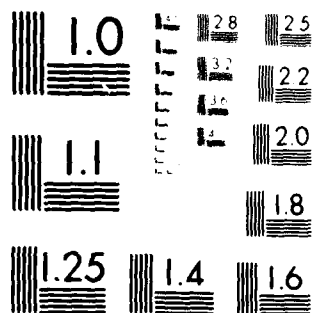
UNCLASSIFIED

NPS55-80-007

NL

1-1
A
A-8500

END
DATE
FILMED
7 80
DTIC



WIDE-AREA RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

LEVEL II

2

NPS55-80-007

NAVAL POSTGRADUATE SCHOOL
Monterey, California

ADA085126



DTIC
ELECTE
JUN 4 1980
S D
A

DIFFUSION APPROXIMATIONS FOR
THE COOPERATIVE SERVICE
OF VOICE AND DATA MESSAGES

by

J. P. Lehoczky

and

D. P. Gaver

February 1980

Approved for Public Release; Distribution Unlimited.

Prepared for:

Naval Postgraduate School
Monterey, California 93940

DOC FILE COPY

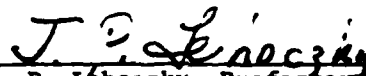
80 5 30 083

NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA

Rear Admiral J. J. Ekelund
Superintendent

J. R. Borsting
Provost

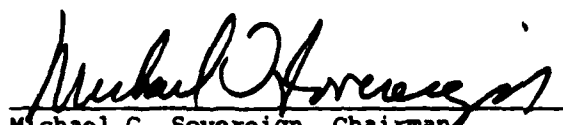
This report was prepared by:


J. P. Lehoczky, Professor
Carnegie-Mellon University


D. P. Gaver, Professor
Department of Operations Research

Reviewed by:

Released by:


Michael G. Sovereign, Chairman
Department of Operations Research


William M. Tolles
Dean of Research

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER NPS55-80-007	2. GOVT ACCESSION NO. AD-A085126	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) Diffusion Approximations for the Cooperative Service of Voice and Data Messages.		5. TYPE OF REPORT & PERIOD COVERED Technical <i>rept.</i>
7. AUTHOR(s) J. P. Lehoczky and D. P. /Gaver		6. PERFORMING ORG. REPORT NUMBER
9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Postgraduate School Monterey, CA 93940		8. CONTRACT OR GRANT NUMBER(s)
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, Ca. 93940		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		12. REPORT DATE Feb 1980 80
		13. NUMBER OF PAGES 20 1242
		15. SECURITY CLASS. (of this report) Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report) Approved for Public Release; Distribution Unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number) Queues Semigroup Theory Communications Probability Modeling Data Transmission Voice Transmission		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) A probability model is presented for a set of communication channels that share the service of data and voice transmissions. A diffusion-theoretic approximation is derived, utilizing new results of Burman (1979). It is shown that the data queue (which is of low priority relative to voice) is approximated by a Wiener process.		

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 68 IS OBSOLETE
S/N 0102-014-6601

Unclassified

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

DIFFUSION APPROXIMATIONS FOR THE COOPERATIVE SERVICE
OF VOICE AND DATA MESSAGES

by

J. P. Lehoczky
Carnegie-Mellon University
Pittsburgh, PA

and

D. P. Gaver
Naval Postgraduate School
Monterey, CA

Accession For	
NTIS Grant	☒
DIC TAB	☐
Unannounced	☐
Justification	
By _____	
Distribution/ _____	
Availability Codes	
Dist	Avail and/or special
A	

INTRODUCTION

In this paper we study the behavior of a queueing system which arises in the study of certain communication networks. Specifically we study a queueing phenomenon which arises with the SENET network, as described by Coviello and Vena (1975) or Barbacci and Oakley (1976). This network allows for both voice and data messages to be transmitted over the same channels by using a special type of integrated circuit and packet-switched multiplexor structure. The two classes of traffic have substantially different performance requirements. Voice messages tend to possess great redundancy, and hence not to be sensitive to channel error rates, while data is very sensitive to channel error, having essentially no redundancy. Voice messages on the other hand have critical timing requirements and cannot be queued, while data is

relatively insensitive to timing and can be queued. Additionally, voice messages tend to be very long relative to data messages which can be broken up into small packets. These special requirements have led to the following queueing network. A node of the network consists of $c + v$ channels or servers. The voice messages are assigned to v channels and do not queue. Thus the voice messages operate as a loss system. Data messages may use c channels exclusively and any unused voice channels; however, voice preempts data using voice channels. Data messages are queued if necessary. Typical performance measures that one may wish to calculate include the loss rate of voice traffic and the mean data queue length.

We make standard probabilistic assumptions. Specifically, we assume voice traffic arrives according to a Poisson(λ) process and each voice message has an independent exponential(μ) service time. Data messages are assumed to have independent exponential(η) service times and arrive according to a Poisson(δ) process. With these assumptions voice is an M/M/v/v loss system, and data is an M/M/S system where $S = c + v - V(t)$ with $V(t)$ = number of voice messages in service. The stochastic process $\{(X(t), V(t)), t \geq 0\}$ is Markov with state space $Z^+ \times \{0, 1, \dots, v\}$ where $X(t)$ = data system size at time t . One can easily write the Kolmogorov forward equations appropriate for this system; however, these equations do not yield a closed form solution. To describe this system

one must either numerically solve the forward equations or introduce approximations.

This system has been studied previously by a number of researchers including Halfin and Segal (1972), Halfin (1972), Fischer and Harris (1976), Bhat and Fischer (1976), Fischer (1977), Chang (1977), and Gaver and Lehoczky (1979a,b). The last two papers introduce a "fluid flow" and a diffusion approximation and derive explicit formulas for data queue behavior. These papers focus on the important case in which $\rho_d = \delta/\eta > c$. In such a situation the data messages must have access to voice channels for the system to be stable. Furthermore, it was assumed that η/μ was large, say 10^4 . Under these circumstances the data flow could be treated deterministically. Suppose we define $\rho_v = \lambda/\mu$ and $q = (\rho_v^v/v!)/\sum_{j=0}^v \rho_v^j/j!$, the Erlang B blocking probability. The total traffic intensity on the $c + v$ channels is given by $\rho_d + \rho_v(1-q)$, or we could define $\rho = (\rho_d + \rho_v(1-q))/(c+v)$. A heavy traffic approximation can be derived for this case $\rho \nearrow 1$. Such an approximation was derived in Gaver and Lehoczky (1979b) assuming η/μ was large; a Wiener process with reflecting boundary was found appropriate. In this paper we derive a heavy traffic approximation for the system without the fluid flow assumption that η/μ is large. The methodology is drawn heavily from the approach of Burman (1979). In this approach one characterizes a Markov process

by its infinitesimal generator. One next suitably normalizes the process so that the generator converges to a limiting infinitesimal generator (in this case to that of a reflected Brownian motion). This convergence allows the conclusion that the finite dimensional distributions of the normalized Markov process converge. The diffusion approximation consists of treating the actual process through its limiting behavior. The details are somewhat complicated by the presence of a boundary.

2.

Let $\{(X(t), V(t)), t \geq 0\}$ be a bivariate Markov process with state space $S = \mathbb{Z}^+ \times \{0, 1, \dots, v\}$. Here $\{V(t), t \geq 0\}$ is marginally an M/M/v/v loss system with arrival rate λ and service rate μ . Conditional on $V(t)$, $\{X(t), t \geq 0\}$ is an M/M/(c + v - V(t)) queueing system with arrival rate δ and service rate η . We say that the V process subordinates the X process. We let

$$Q = \begin{pmatrix} -\rho_v & \rho_v & & & & & \\ 1 & -(1+\rho_v) & \rho_v & & & & \\ & 2 & -(2+\rho_v) & \rho_v & & & \\ & & \ddots & \ddots & \ddots & \ddots & \\ & & & \ddots & \ddots & \ddots & \rho_v \\ & & & & v-1 & -(v-1+\rho_v) & \rho_v \\ & & & & & v & -v \end{pmatrix}, \quad (2.1)$$

the infinitesimal generator of the V process.

The generator of the (X, V) process is given by

$$Af(x, k) = \begin{cases} Qf(x, k) + \delta(f(x+1, k) - f(x, k)) \\ \quad + \eta(c+v-k)(f(x-1, k) - f(x, k)) \\ \quad \text{if } x \geq c+v-k \\ \\ Qf(x, k) + \delta(f(x+1, k) - f(x, k)) \\ \quad + \eta x(f(x-1, k) - f(x, k)) \\ \quad \text{if } x < c+v-k \end{cases} \quad (2.2)$$

for $f: S \rightarrow R$ continuous where

$$Qf(x,k) = \rho_v f(x,k+1) - (k+\rho_v) f(x,k) + kf(x,k-1) \quad (2.3)$$

$$v \geq k \geq 0$$

and $f(x,-1) = f(x,v+1) = 0$. Clearly $Qf(x) = 0$, that is Q annihilates functions of x alone. We next normalize the (X,V) process by defining $X_n(t) = X(nt)/\sqrt{n}$ and $V_n(t) = V(nt)$. One can calculate the generator of the Markov process $\{(X_n(t), V_n(t)), t \geq 0\}$ having state space $S_n = \{0, 1/\sqrt{n}, 2/\sqrt{n}, \dots\} \times \{0, 1, \dots, v\}$ to be

$$A_n f(x,k) = \begin{cases} nQf(x,k) + \delta_n (f(x + 1/\sqrt{n}, k) - f(x,k)) \\ \quad + \eta_n (f(x - 1/\sqrt{n}, k) - f(x,k)) \\ \quad \text{if } x \geq \frac{c+v-k}{\sqrt{n}} \\ nQf(x,k) + \delta_n (f(x + 1/\sqrt{n}, k) - f(x,k)) \\ \quad + \eta_n \sqrt{nx} (f(x + 1/\sqrt{n}, k) - f(x,k)) \\ \quad \text{if } x = 0, 1/\sqrt{n}, \dots, (c+v-k)/\sqrt{n} . \end{cases} \quad (2.4)$$

We assume $f(x,k)$ has three bounded derivatives in x for each fixed k . With this assumption one can expand terms in (2.4) in a Taylor series and rewrite as

$$A_n f(x, k) = \begin{cases} nQf(x, k) + n^{1/2} f_x(x, k) (\delta - \eta(c+v-k)) \\ \quad + \frac{1}{2} f_{xx}(x, k) (\delta + \eta(c+v-k)) \\ \quad + O(n^{-1/2}), \\ \quad \text{if } x \geq (c+v-k)/\sqrt{n} \\ \\ nQf(x, k) + n^{1/2} f_x(x, k) (\delta - \eta\sqrt{nx}) \\ \quad + \frac{1}{2} f_{xx}(x, k) (\delta + \eta\sqrt{nx}) + O(n^{-1/2}), \\ \quad \text{if } x = 0, 1/\sqrt{n}, \dots, (c+v-k)/\sqrt{n} \end{cases} \quad (2.5)$$

with $f_x(x, k) = \frac{\partial}{\partial x} f(x, k)$ and $f_{xx}(x, k) = \frac{\partial^2}{\partial x^2} f(x, k)$.

We ultimately wish to prove that the finite dimensional distributions of $\{X_n(t), t \geq 0\}$ converge to those of a Wiener process with reflecting barrier at the origin. This can be restated in terms of semi-groups. We let $\{T_t^n, t \geq 0\}$ be the semi-group of operators associated with $\{(X_n(t), V_n(t)), t \geq 0\}$ and $\{T_t^\infty, t \geq 0\}$ be that associated with a Wiener process having reflecting barrier at 0. Let g be a continuous function $g: R' \rightarrow R'$. Knowledge of the semi-group is equivalent to knowledge of the transition functions by taking a sequence of g 's which approximate indicator functions. We wish to prove $|T_t^n g(x, k) - T_t^\infty g(x)| \rightarrow 0$ as $n \rightarrow \infty$ for all (x, k) . Here $T_t^n g(x, k) = E(g(X_t) | X_n(0) = x, V_n(0) = k)$. The presence of the variable k prevents this

from being done directly. The method we use is to construct a convenient sequence of functions $\langle g_n \rangle_{n=1}^{\infty}$ which converge in some sense to g . We write

$$\begin{aligned} \|T_t^n g - T_t^{\infty} g\|_n &\leq \| (T_t^{\infty} g)_n - T_t^{\infty} g \|_n + \|T_t^n g - T_t^n g_n\|_n \\ &\quad + \|T_t^n g_n - (T_t^{\infty} g)_n\|_n \end{aligned} \quad (2.6)$$

where $\| \cdot \|_n$ refers to the sup norm over S_n . Both T_t^n and T_t^{∞} are contraction semigroups.

$\langle (T_t^{\infty} g)_n \rangle_{n=1}^{\infty}$ is the sequence of functions constructed from $T_t^{\infty} g$. Our goal is to show that each of the three terms on the right side of (2.6) converges to 0. The first and second terms can be handled similarly. For any function g , we must guarantee that the constructed $\langle g_n \rangle_{n=1}^{\infty}$ sequence satisfies $\|g_n - g\|_n \rightarrow 0$. It will follow that $\| (T_t^{\infty} g)_n - T_t^{\infty} g \|_n \rightarrow 0$. Moreover, since $\{T_t^n, t \geq 0\}$ is a contraction semi-group $\|T_t^n g - T_t^n g_n\|_n \leq \|g - g_n\|_n$ which also converges to 0. The sequence $\langle g_n \rangle_{n=1}^{\infty}$ will be chosen in such a way that the third term converges to 0.

We focus on a convergence determining class of functions g , those which are bounded and have three bounded derivatives. For such a function $g(x)$ we define

$$g_n(x, k) = g(x) + \frac{1}{\sqrt{n}} u(x, k) + \frac{1}{n} v(x, k) \quad (2.7)$$

where u and v have two bounded derivatives in x for each fixed k . The functions u and v will be determined explicitly later and are chosen to control the third term in (2.6). Clearly when g_n is defined by (2.7), $\|g_n - g\|_n = o(n^{-1/2})$ and therefore converges to 0 as required.

One can apply the generator A_n to g_n to derive

$$A_n g_n(x, k) = \begin{cases} nQg(x) + n^{1/2} [Qu(x, k) + g'(x) (\delta - \eta(c+v-k))] \\ + [Qv(x, k) + u_x(x, k) (\delta - \eta(c+v-k)) \\ + \frac{1}{2} g''(x) (\delta - \eta(c+v-k))] + o(n^{-1/2}), \\ \text{if } x \geq (c+v-k)/\sqrt{n} \\ \\ nQg(x) + n^{1/2} [Qu(x, k) + g'(x) (\delta - \eta n^{1/2} x)] \\ + [Qv(x, k) + u_x(x, k) (\delta - \eta n^{1/2} x) + \frac{1}{2} g''(x) (\delta + \eta n^{1/2} x)] \\ + o(n^{-1/2}) \\ \text{if } x = 0, 1/\sqrt{n}, \dots, (c+v-k)/\sqrt{n} \end{cases}$$

where $u_x(x, k) = \frac{\partial}{\partial x} u(x, k)$. Recall that Q annihilates functions of x alone, thus $nQg(x) \equiv 0$. We want to have $A_n g_n(x, k)$ converge to a finite limit and to have that limit be independent of k . For this to occur, the $n^{1/2}$ term must be controlled and the functions u and v must be chosen in such a way as to eliminate the variable k .

The $n^{1/2}$ coefficient in (2.8) can be rewritten by adding and subtracting

$$\sum_{k=0}^v \pi_k g'(x) (\delta - \eta(c+v-k)) = -\eta(c+v)(1-\rho) g'(x) .$$

We next pick $u(x,k)$ to be a solution of

$$\begin{aligned} \Omega u(x,k) &= -(g'(x) \eta(\rho_d - (c+v-k)) + g'(x) \eta(c+v)(1-\rho)) \\ &= -g'(x) \eta(k - \rho_v(1-q)) \end{aligned} \quad (2.9)$$

When $u(x,k)$ is any solution of (2.9), the coefficient of the $n^{1/2}$ term in (2.8) becomes

$$\begin{aligned} &-g'(x) \eta(c+v)(1-\rho) && \text{if } x \geq \frac{c+v-k}{\sqrt{n}} \\ &g'(x) \eta((c+v)\rho - n^{1/2}x - k) && \text{if } 0 \leq x < \frac{c+v-k}{\sqrt{n}} \end{aligned}$$

Equation (2.9) can be solved explicitly. Define $a_k = -g'(x) \eta(k - \rho_v(1-q))/\mu$, so that (2.9) can be written as

$$\begin{aligned} -\rho_v(u(x,k) - u(x,k-1)) - (k-1)(u(x,k-1) - u(x,k-2)) &= a_{k-1}, \quad k = 1, \dots, v \\ -v(u(x,v) - u(x,v-1)) &= a_v \end{aligned} \quad (2.10)$$

Equation (2.10) has a solution since $\sum_{k=0}^v \pi_k a_k = 0$, where $\langle \pi_k \rangle_{k=0}^v$ is the stationary distribution associated with Q , or $\pi_k = (\rho_v^k/k!)/(\sum_{i=0}^v \rho_v^i/i!)$. The solution is given by

$$u(x,k) - u(x,k-1) = \frac{\sum_{i=0}^{k-1} \pi_i a_i}{\rho_v \pi_{k-1}} = \frac{-g'(x)\eta T_{k-1}}{\mu \rho_v \pi_{k-1}}$$

where

$$T_k = \sum_{i=0}^k \pi_i (1 - \rho_v(1-q)) \quad \text{and} \quad T_v = 0.$$

Clearly

$$u(x,k) = u(x,0) - \frac{g'(x)\eta}{\mu \rho_v} \sum_{i=1}^k \frac{T_{i-1}}{\pi_{i-1}}, \quad 1 \leq k \leq v \quad (2.11)$$

where $u(x,0)$ is arbitrary. We let $u(x,0) = \frac{1}{2} g'(x)$ so

$$u(x,k) = g'(x) \left(\frac{1}{2} - \frac{\eta}{\mu \rho_v} \sum_{i=1}^k \frac{T_{i-1}}{\pi_{i-1}} \right), \quad 0 \leq k \leq v \quad (2.12)$$

For the choice of u specified by (2.12) we next wish to insure that the limiting generator is independent of the variable k . The function v is chosen to eliminate the dependence on k . The $O(1)$ term of (2.8) is given, for $x \geq (c+v-k)/\sqrt{n}$, by

$$\begin{aligned}
Qv(x,k) + g''(x) & \left[\frac{1}{2} - \frac{\eta}{\mu\rho_v} \sum_{i=1}^k \frac{T_{i-1}}{\pi_{i-1}} (\delta - \eta(c+v-k)) \right] \\
& + \frac{1}{2} g''(x) (\delta + \eta(c+v-k)) \\
& = Qv(x,k) + H(x,k).
\end{aligned}$$

Let $\bar{H}(x) = \sum_{k=0}^v \pi_k H(x,k)$ and consider $Qv(x,k) + (H(x,k) - \bar{H}(x)) + \bar{H}(x)$. We now let $v(x,k)$ be any solution of

$$Qv(x,k) = -(H(x,k) - \bar{H}(x)). \quad (2.13)$$

Equation (2.13) has a one-parameter family of solutions, since $\sum_{k=0}^v \pi_k (H(x,k) - \bar{H}(x)) = 0$. When $v(x,k)$ is chosen to be any solution of (2.13), the $O(1)$ term of (2.3), for $x \geq (c+v-k)/\sqrt{n}$, will become $\bar{H}(x)$ and will therefore be independent of k . It remains to calculate $\bar{H}(x)$.

$$\begin{aligned}
\bar{H}(x) &= g''(x) \left[\sum_{k=0}^v \pi_k \left\{ \left(\frac{1}{2} - \frac{\eta}{\mu\rho_v} \sum_{i=1}^k \frac{T_{i-1}}{\pi_{i-1}} \right) (\delta - \eta(c+v-k)) \right. \right. \\
&\quad \left. \left. + \frac{1}{2} (\delta + \eta(c+v-k)) \right\} \right] \\
&= g''(x) \left[\delta - \frac{\eta}{\mu\rho_v} \sum_{k=0}^v \pi_k (\delta - \eta(c+v-k)) \sum_{i=1}^k \frac{T_{i-1}}{\pi_{i-1}} \right] \quad (2.14) \\
&= g''(x) \left[\delta - \frac{\eta^2}{\mu\rho_v} \sum_{k=0}^v \pi_k (k - \rho_v(1-q)) \sum_{i=1}^k \frac{T_{i-1}}{\pi_{i-1}} \right. \\
&\quad \left. + \frac{\eta(c+v)(1-\rho)}{\mu\rho_v} \sum_{k=0}^v \pi_k \sum_{i=1}^k \frac{T_{i-1}}{\pi_{i-1}} \right].
\end{aligned}$$

The second term can be rewritten by interchanging the order of summation. The third term is $O(1-\rho)$. We find

$$\begin{aligned}\bar{H}(x) &= g''(x) \left[\delta - \frac{n^2}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i}{\pi_i} \sum_{k=i+1}^v \pi_k (k - \rho_v (1-q)) + O(1-\rho) \right] \\ &= g''(x) \left[\delta - \frac{n^2}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i}{\pi_i} (T_v - T_i) + O(1-\rho) \right]\end{aligned}$$

with $T_v = 0$ or

$$\bar{H}(x) = g''(x) \eta \left[\rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} + O(1-\rho) \right]. \quad (2.15)$$

For the functions u and v specified by (2.12) and (2.13), equation (2.8) can be rewritten as

$$A_n g_n(x, k) = \begin{cases} -n^{1/2} (1-\rho) (\sigma+v) \eta g'(x) \\ \quad + \eta \left[\rho_d + \frac{1}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} + O(1-\rho) \right] g''(x) + O(n^{-1/2}) \\ \quad \text{for } x \geq (c+v-k)/\sqrt{n} \\ \\ n^{1/2} \eta [(c+v)\rho - n^{1/2}x - k] g'(x) \\ \quad + \eta g''(x) \left[\rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} + O(1-\rho) \right. \\ \quad \quad \left. - (c+v-k-n^{1/2}x) \frac{n}{\mu \rho_v} \sum_{i=0}^k \frac{T_i}{\pi_i} \right] + O(n^{-1/2}) \\ \quad \text{for } x \leq (c+v-k)/\sqrt{n} \end{cases} \quad (2.16)$$

We now introduce the "heavy traffic approximation."

In order for the generator to converge to a limiting generator we must have $1-\rho = O(n^{-1/2})$. Specifically, we assume $\rho = \rho_n = 1 - (\theta/\sqrt{n})$ for some $\theta \geq 0$. In this case, $n^{1/2}(1-\rho) = \theta$, and (2.16) becomes

$$A_n g_n(x, k) = \begin{cases} -\theta n(c+v) g'(x) + n \left[\rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right] g''(x) + O(n^{-1/2}) \\ \text{for } x \geq (c+v-k)/\sqrt{n} \\ \\ n[(c+v)\rho - n^{1/2}x - k]n^{1/2}g'(x) \\ + ng''(x) \left[\left\{ \rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right\} \right. \\ \left. - (c+v-k - n^{1/2}x) \frac{n}{\mu \rho_v} \sum_{i=0}^k \frac{T_i}{\pi_i} \right] \\ \text{for } x \leq (c+v-k)/\sqrt{n} \end{cases} \quad (2.17)$$

We now define a limiting generator A_∞ with domain consisting of all functions g having three bounded derivatives and $g'(0) = 0$. Let

$$A_\infty g(x) = -\theta n(c+v) g'(x) + n \left[\rho_d + \frac{n}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right] g''(x), \quad x > 0 \quad (2.18)$$

A_∞ is the generator of a Markov process which corresponds to a Wiener process with drift $-\theta\eta(c+v)$, scale

$$2\eta \left[\rho_d + \frac{\eta}{\mu\rho_v} \sum_{i=0}^{v-1} \frac{T_v^2}{\pi_i} \right],$$

and a reflecting barrier at 0. The $O(n^{-1/2})$ terms involve the first three derivatives of g which are bounded. It is clear from a direct comparison of (2.16) and (2.18) that $|A_n g_n(x,k) - A_\infty g| \rightarrow 0$ as $n \rightarrow \infty$ for all $x > 0$ and k arbitrary. In addition, $g'(0) = 0$ is necessary for the generator to converge at $x = 0$. Unfortunately even assuming $g'(0) = 0$,

$$|A_n g(0,k) - A_\infty g(0)| \rightarrow (c+v-k) \frac{\eta^2}{\mu\rho_v} \sum_{i=0}^k \frac{T_i^2}{\pi_i} g''(0) \text{ as } n \rightarrow \infty$$

rather than to 0. One needs a special argument to handle this lack of convergence at the boundary.

We set out to prove the third term in (2.6) converges to 0. Standard semi-group results (see Burman, 1979, p. 33) give

$$(T_t^\infty g)_n - T_t^n g_n = \int_0^t T_{t-s}^n ((A_\infty w)_n - A_n w_n) ds \quad (2.19)$$

where $w = w(t,x) = T_t^\infty g(x)$. Recall that $w_n = w + (1/\sqrt{n})u + (1/n)v$ with u and v defined by (2.12) and (2.13) with g replaced by w . It follows that

$$\|T_t^n g_n - (T_t^n g)_n\|_n$$

$$= \left\| \int_0^t T_{t-s}^n ((A_\infty w)_n - A_\infty w + A_\infty w - A_n w_n) ds \right\|_n$$

$$\leq \int_0^t \|T_{t-s}^n ((A_\infty w)_n - A_\infty w)\|_n ds + \left\| \int_0^t T_{t-s}^n (A_\infty w - A_n w_n) ds \right\|_n$$

$$\leq \int_0^t \|(A_\infty w)_n - A_\infty w\|_n ds + \left\| \int_0^t T_{t-s}^n (A_\infty w - A_n w_n) ds \right\|_n.$$

The first term is clearly $O(n^{-1/2})$. It remains to show that the second is $O(n^{-1/2})$ as well. We have shown $|A_\infty w - A_n w_n| = O(n^{-1/2})$ except at the boundary where it is $O(1)$. We split the integral into two parts, for one of which the process is away from the boundary, and for the other, near the boundary. The integral away from the boundary has an integrand which is $O(n^{-1/2})$. The integral near the boundary is also $O(n^{-1/2})$ since under a heavy traffic assumption the process is rarely near the boundary. The details are merely summarized here; they are based on the ideas of Burman (1979).

Let I_{0n} be the indicator function of

$$\left[0, \frac{c+v-k}{\sqrt{n}}\right)$$

and I_{1n} be the indicator of

$$\left[\frac{c+v-k}{\sqrt{n}}, \infty \right) .$$

We have

$$\begin{aligned} & \left\| \int_0^t T_{t-s}^n (A_\infty w - A_n w_n) ds \right\|_n \\ & \leq \left\| \int_0^t T_{t-s}^n (A_\infty w - A_n w_n) I_{1n} ds \right\|_n + \left\| \int_0^t T_{t-s}^n (A_\infty w - A_n w_n) I_{0n} ds \right\|_n \\ & \leq \left\| \int_0^t (A_\infty w - A_n w_n) I_{1n} ds \right\|_n + \|A_\infty w - A_n w_n\|_n \left\| \int_0^t T_{t-s}^n I_{0n} ds \right\|_n . \end{aligned}$$

The first term is $O(n^{-1/2})$, since $|A_\infty w - A_n w_n| = O(n^{-1/2})$ off the boundary. The factor $\|A_\infty w - A_n w_n\| = O(1)$, thus it remains to show that

$$\left\| \int_0^t T_{t-s}^n I_{0n} ds \right\|_n = O(n^{-1/2}) .$$

This gives the total time in $[0, t]$ spent near the boundary.

We bound

$$\left\| \int_0^t T_{t-s}^n I_{0n} ds \right\|_n$$

by first introducing a function $h(x)$ not in the domain of A_∞ . We let $h(x)$ have bounded support, be infinitely differentiable and be given by $h(x) = x$ for x near 0. One can construct $h_n(x)$ using (2.7) and apply A_n to h_n to find

$$A_n h_n = \begin{cases} O(1) & \text{if } x \geq \frac{c+v-k}{\sqrt{n}} \\ n^{1/2} ((c+v)\rho - n^{1/2}x - k) + O(1) & \text{if } x < \frac{c+v-k}{\sqrt{n}} \end{cases} \quad (2.20)$$

One has

$$\begin{aligned} T_t^n h_n - h_n &= \int_0^t T_S^n A_n h_n dS \\ &= \int_0^t T_S^n A_n h_n I_{1n} dS + \int_0^t T_S^n A_n h_n I_{0n} dS. \end{aligned}$$

It follows that

$$\begin{aligned} \left\| \int_0^t T_S^n A_n h_n I_{0n} dS \right\|_n &\leq \|T_t^n h_n - h_n\|_n + \left\| \int_0^t T_S^n A_n h_n I_{1n} dS \right\|_n \\ &\leq 2\|h_n\|_n + O(1). \end{aligned}$$

We have shown $\left\| \int_0^t T_S^n A_n h_n I_{0n} dA \right\|_n$ to be bounded in n . An application of (2.2) shows

$$\left\| \int_0^t T_S^n A_n h_n I_{0n} dA \right\|_n = n^{1/2} ((c+v)\rho - n^{1/2}x - k + O(1)) \left\| \int_0^t T_S^n I_{0n} dS \right\|_n$$

is bounded in n . It follows that $\left\| \int_0^t T_S^n I_{0n} dS \right\|_n = O(n^{-1/2})$.

This finally concludes the argument which shows

$$\|T_t^n g_n - (T_t^\infty g)_n\|_n = O(n^{-1/2}), \text{ hence by (2.6) } \|T_t^n g - T_t^\infty g\|_n = O(n^{-1/2}).$$

We have thus shown that the finite-dimensional distributions of the $(X_n(t), V_n(t))$ process converge to those of a Wiener process with drift $-\theta\eta(c+v)$ scale

$$\eta \left(\rho_d + \frac{\eta}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right),$$

and reflection at 0. The diffusion approximation treats $X_n(t)$ as though it were such a Wiener process. For instance, the limiting Wiener process has a stationary exponential distribution with parameter

$$\frac{\theta(c+v)}{\rho_d + \frac{\eta}{\mu \rho_v} \sum_{i=0}^{v-1} (T_i^2/\pi_i)}.$$

This is a distribution for $X(nt)/\sqrt{n}$ and suggests $X(t)$ will have a steady state distribution given approximately by an exponential with parameter

$$(c+v)(1-\rho) \left(\rho_d + \frac{\eta}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right).$$

The steady state mean data queue length would then be

$$E(X(t)) = \frac{\rho_d + \frac{\eta}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i}}{(c + v)(1-\rho)} . \quad (2.21)$$

It is interesting to consider the special case $c = 0$, $v = 1$ where the two types of traffic use the same channel. Under heavy traffic $\rho = \rho_d + \rho_v / (1 + \rho_v)$, so $\rho_d \approx (1 + \rho_v)^{-1}$. The mean data queue length derived from the diffusion approximation (2.21) will be

$$\left(\rho_d + \frac{\eta}{\mu} \frac{\rho_v}{(1 + \rho_v)^3} \right) / (1 - \rho) \approx \frac{\rho_d}{1 - \rho} \left(1 + \frac{\eta}{\mu} \frac{\rho_v}{(1 + \rho_v)^2} \right) .$$

The latter is the exact expression derived by Fisher (1978) for this case. The expression (2.21) represents a generalization of the results of Gaver and Lehoczky (1979b). In this paper, a diffusion approximation is given based on the fluid flow assumption for the data. For this case the result is the same except that the scale is given by

$$\frac{\eta^2}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i}$$

rather than

$$\eta \left(\rho_d + \frac{\eta}{\mu \rho_v} \sum_{i=0}^{v-1} \frac{T_i^2}{\pi_i} \right) .$$

The results derived in this paper therefore definitely generalize

the results of Gaver and Lehoczky (1979b) since the variability in the data queue is now included. When η/μ is large, the second term dominates, and the fluid flow approximation is satisfactory.

The Wiener process approximation for the $X(t)$ process provides a method for studying the dynamics of that process. For instance, suppose the data queue were at level x at time t where x is large. One might wish to study the time that elapses until the queue becomes empty. This is essentially the duration of the busy period under heavy traffic and corresponds to a first-passage time for a Wiener process. Let us denote it by T_x . Straightforward martingale arguments provide for its transform

$$E(e^{-sT_x}) = \exp \left[\left(\frac{x}{\sigma} \right) - \left(\frac{m}{\sigma} \right) - \sqrt{\left(\frac{m}{\sigma} \right)^2 + 2s} \right] \quad (2.22)$$

where

$$m = \theta(c+v)\eta \approx n^{1/2}(1-\rho)(c+v)\eta$$

$$\frac{\sigma^2}{2} = \eta \left(\rho_d + \frac{\eta}{\mu\rho_v} \sum_{i=0}^{n-1} \frac{T_i^2}{\pi_i} \right).$$

It is also easy to find the mean first-passage time

$$E(T_x) = x/m \quad (2.23)$$

One might also be interested in the area beneath the sample path until emptiness occurs, since this area represents the total time waited by all data customers involved in the busy period. If A_x represents this area, simple backward equation arguments give

$$E(A_x) = \frac{x^2}{2m} + \frac{\sigma^2}{2m^2} x \quad (2.24)$$

where m and σ^2 are given in (2.22).

Acknowledgment. This research was supported in part by a contract from the Office of Naval Research.

BIBLIOGRAPHY

Barbacci, M. R. and Oakley, J. D. (1976). "The integration of Circuit and Packet Switching Networks Toward a SENET Implementation," 15th NBS-ACM Annual Technique Symposium.

Bhat, U. N. and Fischer, M. J. (1976). "Multichannel Queueing Systems with Heterogeneous Classes of Arrivals," Naval Research Logistics Quarter 23

Burman, David Y. (1979). "An Analytic Approach to Diffusion Approximations in Queueing," Ph.D. Dissertation, New York University, Courant Institute of Mathematics.

Chang, Lih-Hsing (1977). "Analysis of Integrated Voice and Data Communication Network," Ph.D. Dissertation, Department of Electrical Engineering, Carnegie-Mellon University, November.

Coviello, G. and Vena, P. A. (1975). "Integration of Circuit/ Packet Switching in a SENET (Slotted Envelop NETWORK) Concept," National Telecommunications Conference, New Orleans, December, pp. 42-12 to 42-17.

Fischer, M. J. (1977a). "A Queueing Analysis of an Integrated Telecommunications System with Priorities," INFOR 15

Fischer, M. J. (1977b). "Performance of Data Traffic in an Integrated Circuit- and Packet-Switched Multiplex Structure, DCA Technical Report.

Fischer, M. J. and Harris, T. C. (1976). "A Model for Evaluating the Performance of an Integrated Circuit- and Packet-Switched Multiplex Structure," IEEE Trans. on Comm., Com-24, February.

Gaver, D. P. and Lehoczky, J. P. (1979a). "Channels that cooperatively service a data stream and voice messages," Technical Report, Naval Postgraduate School, Department of Operations Research.

Halfin, S. (1972). "Steady-state Distribution for the Buffer Content of an M/G/1 Queue with Varying Service Rate," SIAM J. Appl. Math., 356-363.

Halfin, S. and Segal, M. (1972). "A Priority Queueing Model for a Mixture of Two Types of Customers," SIAM J. Appl. Math., 369-379.

Lehoczky, J. P. and Gaver, D. P. (1979b). "Channels that Cooperatively Service a Data Stream and Voice Messages, II: Diffusion Approximations," Technical Report, Naval Postgraduate School, Department of Operations Research.

INITIAL DISTRIBUTION LIST

	Number of Copies
Defense Technical Information Center Cameron Station Alexandria, VA 22314	2
Library Code Code 0142 Naval Postgraduate School Monterey, CA 93940	2
Library Code 55 Naval Postgraduate School Monterey, Ca. 93940	1
Dean of Research Code 012A Naval Postgraduate School Monterey, Ca. 93940	1
Attn: A. Andrus, Code 55	1
D. Gaver, Code 55	25
D. Barr, Code 55	1
P. A. Jacobs, Code 55	1
P. A. W. Lewis, Code 55	1
P. Milch, Code 55	1
R. Richards, Code 55	1
M. G. Sovereign, Code 55	1
R. J. Stampfel, Code 55	1
R. R. Read, Code 55	1
J. Wozencraft, Code 74	1
Mr. Peter Badgley ONR Headquarters, Code 102B 800 N. Quincy Street Arlington, VA 22217	1
Dr. James S. Bailey, Director Geography Programs, Department of the Navy ONR Arlington, VA 93940	1
Prof. J. Lehoczky Dept. of Statistics Carnegie Mellon University Pittsburgh, PA. 15213	10

DISTRIBUTION LIST

	No. of Copies
STATISTICS AND PROBABILITY PROGRAM OFFICE OF NAVAL RESEARCH CODE 426 ARLINGTON VA	1 22217
OFFICE OF NAVAL RESEARCH NEW YORK AREA OFFICE 715 BROADWAY - 5TH FLOOR ATTN: DR. ROGER GRAFTON NEW YORK, NY	1 10003
DIRECTOR OFFICE OF NAVAL RESEARCH BRANCH OFF 536 SOUTH CLARK STREET ATTN: DEPUTY AND CHIEF SCIENTIST CHICAGO, IL	1 60605
LITERARY NAVAL OCEAN SYSTEMS CENTER SAN DIEGO CA	1 92152
NAVY LIBRARY NATIONAL SPACE TECHNOLOGY LAB ATTN: NAVY LIBRARIAN BAY ST. LOUIS MS	1 39522
NAVAL ELECTRONIC SYSTEMS COMMAND NAVELEX 320 NATIONAL CENTER NO. 1 ARLINGTON VA	.1 20360
DIRECTOR NAVAL RESEARCH LABORATORY ATTN: LIBRARY (JNRL) CODE 2026 WASHINGTON, D.C.	1 20375
TECHNICAL INFORMATION DIVISION NAVAL RESEARCH LABORATORY WASHINGTON, D. C.	1 20375

DISTRIBUTION LIST

No. of Copies

PROF. C. R. BAKER
DEPARTMENT OF STATISTICS
UNIVERSITY OF NORTH CAROLINA
CHAPEL HILL
NORTH CAROLINA
27514

1

PROF. R. E. DECHOFER
DEPARTMENT OF OPERATIONS RESEARCH
CORNELL UNIVERSITY
ITHACA
NEW YORK 14850

1

PROF. A. J. BERSHAC
SCHOOL OF ENGINEERING
UNIVERSITY OF CALIFORNIA
IRVINE
CALIFORNIA
92664

1

P. J. BICKEL
DEPARTMENT OF STATISTICS
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA

1

54720

PROF. F. W. BLOCK
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF PITTSBURGH
PITTSBURGH
PA

1

15260

PROF. JOSEPH BLUM
DEPT. OF MATHEMATICS, STATISTICS
AND COMPUTER SCIENCE
THE AMERICAN UNIVERSITY
WASHINGTON
DC

1

20016

PROF. R. A. BRADLEY
DEPARTMENT OF STATISTICS
FLORIDA STATE UNIVERSITY

1

TALLAHASSEE, FLORIDA 32306

PROF. R. E. BARLOW
OPERATIONS RESEARCH CENTER
COLLEGE OF ENGINEERING
UNIVERSITY OF CALIFORNIA
BERKLEY
CALIFORNIA 94720

1

MR. C. N. EBBETT
NAVAL COASTAL SYSTEMS LABORATORY
CODE P761
PANAMA CITY,
FLORIDA
32401

1

26

THIS PAGE IS NOT NECESSARILY PRACTICABLE
FROM THE POINT OF VIEW OF THE USER

DISTRIBUTION LIST

No. of Copies

PROF. L. N. PHAT
COMPUTER SCIENCE / OPERATIONS
RESEARCH CENTER
SOUTHERN METHODIST UNIVERSITY
DALLAS
TEXAS 75275

1

PROF. W. R. ELTSCHKE
DEPT. OF QUANTITATIVE
BUSINESS ANALYSIS
UNIVERSITY OF SOUTHERN CALIFORNIA
LOS ANGELES, CALIFORNIA
90007

1

DR. DERRILL J. BERDELON
NAVAL UNDERWATER SYSTEMS CENTER
CODE 21
NEWPORT
RI

02840

1

J. E. ROYER JR
DEPT. OF STATISTICS
SOUTHERN METHODIST UNIVERSITY
DALLAS
TX

75275

1

DR. J. CHANDRA
U. S. ARMY RESEARCH
P. O. BOX 12211
RESEARCH TRIANGLE PARK
NORTH CAROLINA
27706

1

PROF. P. CHERNOFF
DEPT. OF MATHEMATICS
MASS INSTITUTE OF TECHNOLOGY
CAMBRIDGE,
MASSACHUSETTS 02139

1

PROF. C. GERMAN
DEPARTMENT OF CIVIL ENGINEERING
AND ENGINEERING MECHANICS
COLUMBIA UNIVERSITY
NEW YORK
NEW YORK

10027

1

PROF. R. L. DISNEY
VIRGINIA POLYTECHNIC INSTITUTE
AND STATE UNIVERSITY
DEPT. OF INDUSTRIAL ENGINEERING
AND OPERATIONS RESEARCH
BLACKSBURG, VA

24061

1

DISTRIBUTION LIST

MR. GENE F. GLEISSNER
APPLIED MATHEMATICS LABORATORY
DAVID TAYLOR NAVAL SHIP RESEARCH
AND DEVELOPMENT CENTER
BETHESDA
MD

20084

1

PROF. S. S. GLPTA
DEPARTMENT OF STATISTICS
PURDUE UNIVERSITY
LAFAYETTE
INDIANA 47907

1

PROF. C. L. HANSEN
DEPT OF MATH. SCIENCES
STATE UNIVERSITY OF NEW YORK,
BINGHAMTON
BINGHAMTON
NY

13901

1

Prof. M. J. Hinich
Dept. of Economics
Virginia Polytechnic Institute
and State University
Blacksburg, VA 24061

1

Dr. D. Depriest,
ONR, Code 102B
800 N. Quincy Street
Arlington, VA 22217

1

Prof. G. E. Whitehouse
Dept. of Industrial Engineering
Lehigh University
Bethlehem, PA 18015

1

Prof. M. Zia-Hassan
Dept. of Ind. & Sys. Eng.
Illinois Institute of Technology
Chicago, IL 60616

1

Prof. S. Zacks
Statistics Dept.
Virginia Polytechnic Inst.
Blacksburg, VA 24061

1

Head, Math. Sci Section
National Science Foundation
1800 G Street, N.W.
Washington, D.C. 20550

1

	No. of Copies
Dr. H. Sittrop Physics Lab., TNO P.O. Box 96964 2509 JG, The Hague The Netherlands	1
DR. R. ELASHOFF BIOMATHEMATICS UNIV. OF CALIF. LOS ANGELES CALIFORNIA	1
90024	
PROF. GEORGE S. FISHMAN UNIV. OF NORTH CAROLINA CUR. IN CR AND SYS. ANALYSIS PHILLIPS ANNEX CHAPEL HILL, NORTH CAROLINA	1
20742	
DR. R. GNANAGESIKAN BELL TELEPHONE LAB MOLDELT, N. J.	1
07733	
DR. A. J. COLEMAN CHIEF, CR DIV. 2CS.CZ, ADMIN. A428 U.S. DEPT. OF COMMERCE WASHINGTON, D.C.	1
20234	
DR. H. FIGGINS 53 BONN 1, POSTFACH 585 NASSESTRASSE 2	1
WEST GERMANY	
DR. P. T. HOLMES DEPT. OF MATH. CLEMSON UNIV. CLEMSON SOUTH CAROLINA	1
29631	
Dr. J. A. Hocke Bell Telephone Labs Whippany, New Jersey	1
07733	
Dr. Robert Hooke Box 1982 Pinehurst, No. Carolina	1
28374	

DR. D. L. IGLEHART
DEPT. OF C.E.
STANFORD UNIV.
STANFORD
CALIFORNIA

1

94305

Dr. D. Trizna, Mail Code 5323
Naval Research Lab
Washington, D.C. 20375

1

Dr. E. J. Wegman,
ONR, Code 436
Arlington, VA 22217

1

DR. H. KOBAYASHI
IBM
VERMONT HEIGHTS
NEW YORK

1

10598

DR. A. LEMOINE
1020 GUINCA ST.
FALO ALTO,
CALIFORNIA

1

94301

DR. J. MACQUEEN
UNIV. OF CALIF.
LOS ANGELES
CALIFORNIA

1

90024

Prof Kneale Marshall
Scientific Advisor to DCNO (MPT)
Code Op-QIT, Room 2705
Arlington Annex
Washington, D.C. 20370

1

DR. M. MAZUMBAR
PATH. DEPT.
ESTINGHOUSE RES. LABS
CHURCHILL BCFE
PITTSBURGH
PENNSYLVANIA

1

15235

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

DISTRIBUTION LIST

No. of Copies

70
 PROF. W. M. FIRSCH
 INSTITUTE OF MATHEMATICAL SCIENCES
 NEW YORK UNIVERSITY
 NEW YORK
 NEW YORK 10453

1

PROF. J. B. KACANE
 DEPARTMENT OF STATISTICS
 CARNEGIE-MELLON
 PITTSBURGH,
 PENNSYLVANIA
 15213

1

DR. RICHARD LAU
 DIRECTOR
 OFFICE OF NAVAL RESEARCH BRANCH OFF
 1030 EAST GREEN STREET
 PASADENA
 CA

1

91101

DR. A. R. LAUFER
 DIRECTOR
 OFFICE OF NAVAL RESEARCH BRANCH OFF
 1030 EAST GREEN STREET
 PASADENA
 CA

1

91101

PROF. M. LEADBETTER
 DEPARTMENT OF STATISTICS
 UNIVERSITY OF NORTH CAROLINA
 CHAPEL HILL
 NORTH CAROLINA 27514

1

DR. J. S. LEE
 J. S. LEE ASSOCIATES, INC.
 2001 JEFFERSON DAVIS HIGHWAY
 SUITE 802
 ARLINGTON
 VA

1

22202

PROF. L. C. LEE
 DEPARTMENT OF STATISTICS
 VIRGINIA POLYTECHNIC INSTITUTE
 AND STATE UNIVERSITY
 BLACKSBURG
 VA

1

24061

PROF. R. S. LEVENWORTH
 DEPT. OF INDUSTRIAL AND SYSTEMS
 ENGINEERING
 UNIVERSITY OF FLORIDA
 GAINESVILLE,
 FLORIDA 32611

1

31

THIS PAGE IS BEST QUALITY PRACTICABLE
 FROM COPY FURNISHED TO EDC

DISTRIBUTION LIST

	No. of copies
FRCE G. LIEPERMAN STANFORD UNIVERSITY DEPARTMENT OF OPERATIONS RESEARCH STANFORD CALIFORNIA 94305	1
DR. JAMES R. MAAR NATIONAL SECURITY AGENCY FORT MEADE, MARYLAND 20755	1
FPCF. R. W. MAESEN DEPARTMENT OF STATISTICS UNIVERSITY OF MISSOURI COLUMBIA MO 65201	1
DR. N. R. MAAN SCIENCE CENTER ROCKWELL INTERNATIONAL CORPORATION P.O. BOX 1085 THOUSAND OAKS, CALIFORNIA 91320	1
DR. W. H. MARLOW PROGRAM IN LOGISTICS THE GEORGE WASHINGTON UNIVERSITY 707 22ND STREET, N. W. WASHINGTON, D. C. 20037	1
PROF. E. MASRY DEPT. APPLIED PHYSICS AND INFORMATION SERVICE UNIVERSITY OF CALIFORNIA LA JOLLA CALIFORNIA 92093	1
DR. BRUCE J. MCCONALD SCIENTIFIC DIRECTOR SCIENTIFIC LIAISON GROUP OFFICE OF NAVAL RESEARCH AMERICAN EMBASSY - TOKYO APO SAN FRANCISCO 96503	1

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

	No. of copies
Dr. Leon F. McGinnis School of Ind. And Sys. Eng. Georgia Inst. of Tech. Atlanta, GA 30332	1
DR. D. R. MCNEIL DEPT. OF STATISTICS PRINCETON UNIV. PRINCETON NEW JERSEY	1
	08540
DR. F. MOSTELLER STAT. DEPT. HARVARD UNIV. CAMBRIDGE MASSACHUSETTS	1
	02139
DR. M. REISER IBM THOMAS J. WATSON RES. CTR. YORKTOWN HEIGHTS NEW YORK	1
	10598
DR. J. RICMAN DEPT. OF MATHEMATICS ROCKEFELLER UNIV. NEW YORK NEW YORK	1
	10021
DR. LINUS SCHRAGE UNIV. OF CHICAGO GRAD. SCHOOL OF BLS. 5836 GREENWICH AVE. CHICAGO, ILLINOIS	1
	60637
Dr. Paul Schweitzer University of Rochester Rochester, N.Y. 14627	1
Dr. V. Srinivasan Graduate School of Business Stanford University Stanford, CA. 94305	1
Dr. Roy Welsch M.I.T. Sloan School Cambridge, MA 02139	1

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

DISTRIBUTION LIST

No. of Copies

DR. JANET M. MYHRE
THE INSTITUTE OF DECISION SCIENCE
FOR BUSINESS AND PUBLIC POLICY
CLAREMONT MEN'S COLLEGE
CLAREMONT
CA 91711

1

MR. F. NISSELSCH
BUREAU OF THE CENSUS
ROOM 2025
FEDERAL BUILDING 3
WASHINGTON
D. C. 20533

1

MISS B. S. CRLEANS
NAVAL SEA SYSTEMS COMMAND
(SEA 03F)
RM 103CD
ARLINGTON VIRGINIA 20360

1

FRCF. C. E OWEN
DEPARTMENT OF STATISTICS
SOUTHERN METHODIST UNIVERSITY
DALLAS
TEXAS
75222

1

Prof. E. Parzen
Statistical Science Division
Texas A & M University
College Station TX 77843

1

DR. A. PETRASOVITS
ROOM 207B, FCCC AND CRIG BLDG.
TUNNEY'S PASTURE
OTTAWA, ONTARIO K1A-CL2,
CANADA

1

FRCF. S. L. PHOENIX
SIELEY SCHOOL OF MECHANICAL AND
AEROSPACE ENGINEERING
CORNELL UNIVERSITY
ITHACA
NY 14850

1

DR. A. L. POWELL
DIRECTOR
OFFICE OF NAVAL RESEARCH BRANCH OFF
495 SUMMER STREET
BOSTON
MA 02210

1

MR. F. R. FRICFI
CODE 224 OPERATIONAL TEST AND ONRS
EVALUATION FORCE (OPTVFJR)
NORFOLK
VIRGINIA
20360

1

DISTRIBUTION LIST

No. of Copies

PROF. M. L. PURI
DEPT. OF MATHEMATICS
P.O. BOX F
INDIANA UNIVERSITY FOUNDATION
BLOOMINGTON
IN 47401

1

PROF. H. ROBBINS
DEPARTMENT OF MATHEMATICS
COLUMBIA UNIVERSITY
NEW YORK,
NEW YORK 10027

1

PROF. M. ROSENBLATT
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF CALIFORNIA SAN DIEGO
LA JOLLA
CALIFORNIA

1

92093

PROF. S. M. ROSS
COLLEGE OF ENGINEERING
UNIVERSITY OF CALIFORNIA
BERKELEY
CA

1

94723

PROF. I. RUBIN
SCHOOL OF ENGINEERING AND APPLIED
SCIENCE
UNIVERSITY OF CALIFORNIA
LOS ANGELES
CALIFORNIA 90024

1

PROF. I. R. SAVAGE
DEPARTMENT OF STATISTICS
YALE UNIVERSITY
NEW HAVEN,
CONNECTICUT
06520

1

PROF. L. L. SCHARF JR
DEPARTMENT OF ELECTRICAL ENGINEERING
COLORADO STATE UNIVERSITY
FT. COLLINS,
COLORADO
80521

1

PROF. R. SERFLING
DEPARTMENT OF STATISTICS
FLORIDA STATE UNIVERSITY

1

TALLAHASSEE FLORIDA 32306

PROF. W. R. SCHLANY
DEPARTMENT OF STATISTICS
SOUTHERN METHODIST UNIVERSITY
DALLAS,
TEXAS
75222

1

DISTRIBUTION LIST

No. of Copies

PROF. C. C. SIEGMUND
DEPT. OF STATISTICS
STANFORD UNIVERSITY
STANFORD
CA

1

54305

PROF. M. L. SEGOMAN
DEPT. OF ELECTRICAL ENGINEERING
POLYTECHNIC INSTITUTE OF NEW YORK
BRUCKLYN,
NEW YORK
11201

1

DR. A. L. SLAFKOSKY
SCIENTIFIC ADVISOR
COMMANDANT OF THE MARINE CORPS
WASHINGTON,
D. C.
20380

1

DR. C. E. SMITH
DESMATICS INC.
P.O. BOX 618
STATE COLLEGE
PENNSYLVANIA
16801

1

PROF. W. L. SMITH
DEPARTMENT OF STATISTICS
UNIVERSITY OF NORTH CAROLINA
CHAPEL HILL
NORTH CAROLINA 27514

1

Dr. H. J. Solomon
ONR
223/231 Old Marylebone Rd
London NW1 5TH, ENGLAND

1

MR. GLENN F. STAPLY
NATIONAL SECURITY AGENCY
FORT MEADE
MARYLAND 20755

1

Mr. J. Gallagher
Naval Underwater Systems Center
New London, CT

1

Dr. E. C. Monahan
Dept. of Oceanography
University College
Galway, Ireland

1

THIS PAGE IS BEST QUALITY PRACTICABLE
FROM COPY FURNISHED TO DDC

DISTRIBUTION LIST

No. of Copies

DR. R. M. STARK
STATISTICS AND COMPUTER SCI.
UNIV. OF DELAWARE
NEWARK
DELAWARE

1

19711

PROF. JOHN W. TUKEY
FINE HALL
PRINCETON UNIV.
PRINCETON
NEW JERSEY

1

08540

DR. THOMAS C. VARLEY
OFFICE OF NAVAL RESEARCH
CODE 424
ARLINGTON
VA

1

22217

PROF. G. WATSON
FINE HALL
PRINCETON UNIV.
PRINCETON
NEW JERSEY

1

08540

MR. DAVID A. SWICK
ADVANCED PROJECTS GROUP
CODE 8103
NAVAL RESEARCH LAB.
WASHINGTON
DC

1

20375

MR. WENDELL G. SYKES
ARTHUR C. LITTLE, INC.
ACORN PARK
CAMBRIDGE
MA

1

02140

PROF. J. R. THOMPSON
DEPARTMENT OF MATHEMATICAL SCIENCE
RICE UNIVERSITY
HOUSTON,
TEXAS
77001

1

PROF. W. A. THOMPSON
DEPARTMENT OF STATISTICS
UNIVERSITY OF MISSOURI
COLUMBIA,
MISSOURI
65201

1

DISTRIBUTION LIST

	No. of Copies
PROF. F. A. TILLMAN DEPT. OF INDUSTRIAL ENGINEERING KANSAS STATE UNIVERSITY MANHATTAN KS	1
66506	
PROF. A. F. VEINOTT DEPARTMENT OF OPERATIONS RESEARCH STANFORD UNIVERSITY STANFORD CALIFORNIA 94305	1
DANIEL H. WAGNER STATION SQUARE ONE FAIRBANKS, PENNSYLVANIA 15301	1
PROF. GRACE WAMBA DEPT. OF STATISTICS UNIVERSITY OF WISCONSIN MADISON WI	1
53706	
PROF. K. T. HALLENUS DEPARTMENT OF MATHEMATICAL SCIENCES CLEMSON UNIVERSITY CLEMSON SOUTH CAROLINA 29631	1
PROF. BERNARD WIDSCW STANFORD ELECTRONICS LAB STANFORD UNIVERSITY STANFORD CA	1
94305	

THIS PAGE IS BEING REPRODUCED FOR THE
 RECORD OF THE

DISTRIBUTION LIST

No. of Copies

OFFICE OF NAVAL RESEARCH
SAN FRANCISCO AREA OFFICE
760 MARKET STREET
SAN FRANCISCO CALIFORNIA 94102

1

TECHNICAL LIBRARY
NAVAL ORDNANCE STATION
INDIAN HEAD MARYLAND 20640

1

NAVAL SHIP ENGINEERING CENTER
PHILADELPHIA
DIVISION TECHNICAL LIBRARY
PHILADELPHIA PENNSYLVANIA 19112

1

BUREAU OF NAVAL PERSONNEL
DEPARTMENT OF THE NAVY
TECHNICAL LIBRARY
WASHINGTON D. C. 20370

1

PROF. M. AEDEL-FAHEED
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF NORTH CAROLINA
CHARLOTTE
NC

1

28223

PROF. T. W. ANDERSON
DEPARTMENT OF STATISTICS
STANFORD UNIVERSITY
STANFORD, CALIFORNIA 94305

1

PROF. F. J. ANSCOMBE
DEPARTMENT OF STATISTICS
YALE UNIVERSITY
NEW HAVEN
CONNECTICUT 06520

1

PROF. L. A. ARCIA
INSTITUTE OF INDUSTRIAL
ADMINISTRATION
UNION COLLEGE
SCHENECTADY,
NEW YORK 12308

1

FILMED

7-8