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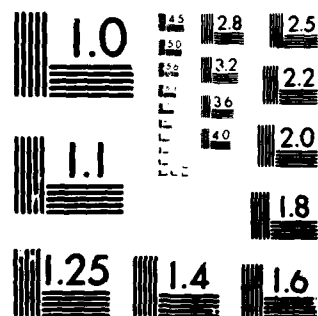
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DYNAMIC CRACK PROPAGATION IN
PRECRACKED CYLINDRICAL VESSELS
SUBJECTED TO SHOCK LOADING

by

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the crack. Crack growth initiation and propagation through the wall is then calculated. It is found that, once initiated, crack propagation will continue until the crack penetrates the wall. Crack arrest within the wall does not appear to be possible under the conditions considered in this paper.

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ABSTRACT

Previous work has shown that a speed-independent dynamic fracture toughness property can be used in an elastodynamic analysis to describe crack initiation and unstable propagation under impact loading. In this paper, a further step is taken by extending the analysis from simple laboratory test specimens to treat more realistic crack-structure geometries. A circular cylinder with an initial part-through wall crack subjected to an impulsive loading on its inner surface is considered. The crack is in a radial-axial plane and has its length in the axial direction long enough that a state of plane strain exists at the center of the crack. Crack growth initiation and propagation through the wall is then calculated. It is found that, once initiated, crack propagation will continue until the crack penetrates the wall. Crack arrest within the wall does not appear to be possible under the conditions considered in this paper.

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NOMENCLATURE

a	crack length, mm
c_R	Rayleigh wave speed, m/sec
c_0	bar wave speed, m/sec
E	elastic modulus, Pa
G	dynamic strain energy release rate, J/m^2
h	cylinder wall thickness, mm
K_I	dynamic stress intensity factor, $MPa\sqrt{m}$
K_{Ia}	static crack arrest toughness, $MPa\sqrt{m}^{1/2}$
K_{ID}	dynamic fracture toughness, $MPa\sqrt{m}^{1/2}$
K_{Id}	dynamic initiation toughness, $MPa\sqrt{m}^{1/2}$
K_{Im}	minimum value of K_{ID} , $MPa\sqrt{m}^{1/2}$
R	cylinder mean radius, mm
T	period of oscillation, μ sec
t	time, μ sec
U	total energy per unit length, J/mm
V	crack speed, m/sec

INTRODUCTION

Fracture mechanics offers significant opportunities for developing failure-safe structures. By relating the fracture-critical flaw sizes at various locations in the structure to the expected applied stresses, more accurate specification of both material toughness requirements and of non-destructive inspection limits will be possible. This must lead towards an effective use of materials without compromising the integrity of the structure. But, first, an extension of the relatively simple fracture mechanics techniques that are now available must be made to treat the materials, geometries, and loading conditions arising in real engineering application.

The specific objective of this work is the development of fracture mechanics analysis procedures for crack growth initiation, unstable propagation and arrest in ship structures subjected to blast loading. Because there are several different aspects of this problem that are beyond current analysis capabilities, a multi-step approach is being followed. A first step was to assess the effect of impact loading in a combined experimental and mathematical analysis program using simple laboratory test specimens. The work reported in this paper extends this work to more complex structures. The results are expected to permit the effects of structural geometry on dynamic crack propagation to be assessed and, in addition, will assist in the design of a critical experimental test of the validity of dynamic fracture mechanics predictions under shock loading conditions.

PRELIMINARY DISCUSSION

Dynamic fracture mechanics encompasses all problems involving crack growth initiation and subsequent unstable propagation up to and including crack arrest. The methodology was developed to treat problems where, for an acceptable solution, inertia forces must be included in the equations of motion of the cracked body. At present, dynamic fracture mechanics treatments are limited to problems where the basic

assumptions of linear elastic fracture mechanics (LEFM) are valid. The essential assumption in this approach is that the plastic deformation attending the propagating crack tip is small enough to be "dominated" by the elastic deformation in the field surrounding the crack tip. In these conditions, the plastic energy absorbed in the fracture process can be taken as characteristic of the material with the body being treated as completely elastic. Problems of crack growth initiation and subsequent rapid unstable crack propagation can then be solved by using elastodynamically computed stress intensity factors coupled with experimentally determined dynamic fracture toughness values.

The stress intensity factor K_I enters in the computed elastodynamic stress field in the immediate vicinity of the crack tip. It can depend on time t , the crack tip speed, V , the crack length, the external geometry of the cracked body, material constants, and on the applied loads. The conditions governing crack motion in a body can be expressed in terms of $K_I(t, V)$ and experimentally determined critical values that are taken to be properties of the material. Thus, for a propagating crack

$$K_I(t, V) = K_{ID}(V), \quad (1)$$

where K_{ID} is known as the dynamic fracture toughness. Ordinarily, K_{ID} values will be greater than K_{IC} , although it is possible that K_{ID} , the minimum value of K_{ID} , can be less than K_{IC} .

An equality is sometimes used for crack arrest. This is expressed in terms of K_I and an "arrest toughness" parameter, K_{IA} . However, while the concept can be useful as an approximation, crack arrest is more rigorously defined as occurring only when Equation (1) cannot be satisfied. That is, the crack will arrest at a time t when $K_I < K_{IA}$ for all $t > t$. Thus, crack arrest is properly viewed as the termination of a general dynamic crack propagation process, not as a unique event governed solely by material properties. This is the way in which the methodology will be applied here.

Because of the equivalence that exists between them, applications of LEFM can be made either in terms of the stress intensity factor or the energy release rate parameter G . That is, for plane strain conditions

$$G = \frac{1-\nu^2}{E} A(V) K_I^2, \quad (2)$$

where $A(V)$ is a universal geometry-independent function that is unity at zero crack speed, and increases monotonically to become unbounded as $V \rightarrow c_R$, where c_R is the speed of Rayleigh waves in the material. For most practical situations, $A(V)$ can be taken as equal to one.

In addition to the work reported by Kanninen, et al (1), other applications which illustrate the necessity of a dynamic approach have been given by Hahn, et al (2,3). Specifically, a series of thermal shock experiments performed on axially-cracked thick-walled cylinders at the Oak Ridge National Laboratory (4) were analyzed. It was found that the predictions were in quite good agreement with the measured distances of crack propagation prior to arrest. These experiments produced only short crack jump lengths (e.g., penetration to 14% of the wall thickness) and for these a quasi-static approach would also be acceptable. However, for conditions which would produce much larger penetrations, a significant difference was found. Specifically, for a postulated reduced frac-

ture toughness in which a quasi-static calculation would predict crack arrest at a penetration equal to 71% of the wall thickness, the dynamic fracture mechanics calculation of Reference 3 revealed that the crack would completely penetrate the wall.

The growth of a part-through wall crack is generally a three-dimensional problem. An effective simplifying approach is to assume a plane strain idealization; e.g., considering an initial part-through wall flaw to have a much greater length in the direction parallel to the surface of the wall than in the thickness direction. This approach was successfully taken in the finite-difference computation for a run-arrest event in a thermally shocked pressure vessel with an internal part-through wall crack reported in Reference (3). This method has been extended here to take account of rapidly applied loading for application to structures under impact loading.

The following analyses have two primary purposes. The work reported in Reference (1) has demonstrated that the elastodynamic crack propagation and crack arrest methodology can be applied to impact-loaded dynamic tear test specimens. It is therefore desirable to study rapid fracture and crack arrest in an impulsively loaded component that is more realistic than a simple laboratory specimen. Second, because there is a lack of information on stress intensity factors for impulsively loaded structures, these analyses will provide useful quantitative predictions of the magnitude of the impulsive loading required to initiate crack propagation in a precracked or flawed cylindrical vessel. It is expected that this information can be used to design a critical test of the methodology.

ANALYSIS AND RESULTS

The Computational Model

The objectives of immediate interest are to determine the magnitude of the impulsive loading that will initiate propagation of a part-through crack in the wall of a cylindrical vessel and to determine whether or not this crack will arrest before it penetrates the wall. The particular loading envisioned here is an intense pressure spike. The duration of the spike will be much smaller than the fundamental period of oscillation of the cylinder to constitute an impulsive or a shock loading.

The cylindrical vessel to be addressed in the following is depicted in Figure 1. It consists of a

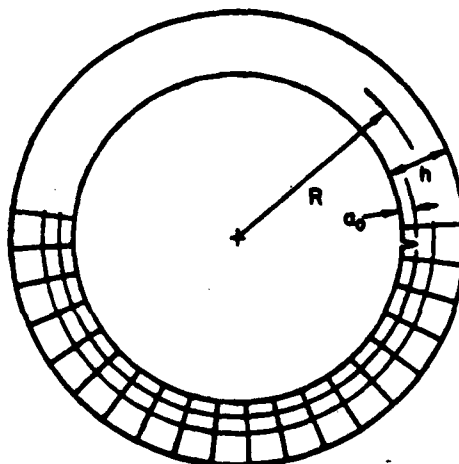


Figure 1. Cracked Cylinder with Finite Difference Grid

very long circular cylinder of mean radius R and wall thickness h . A part-through wall crack of depth a_0 is assumed to exist at the inside surface of the cylinder. The length of the crack, like the cylinder, is taken to be long enough for plane strain conditions to be valid. This type of crack clearly poses a more severe condition than one with a finite axial length. An internal flaw is considered.

The loading is a uniform internal impulsive pressure. The reason for selecting an internal pressure over an external pressure is also one of convenience. The internal loading immediately causes the crack to open. By contrast, if an external impulse were applied, the cylinder would have to experience half an oscillation before any crack opening would occur. Hence, the internal loading requires less computer time. Also, the danger of buckling the cylinder is reduced.

This analysis is restricted to conditions where the time required for a longitudinal wave to propagate through the wall is small compared to the fundamental period of oscillation of the cylinder. Hence, it applies when the thickness of the cylinder is small compared to its radius. However, even though the cylinder is a thin shell, it does not follow that classical shell theory is applicable to the present problem.

The net effect of an impulsive loading on a thin cylinder is to impart a nearly uniform radial velocity v_0 to its walls. (This condition is frequently assumed when analyzing impulsive loading of thin shells.) Under such a loading, the cylinder will respond by oscillating in essentially the breathing mode with superimposed low frequency flexural modes. Had an initial velocity imparted only to the inside surface been considered, then higher frequency modes would have been excited. Computations for the latter case showed that the period of these higher modes is small compared to the fracture event. While they can be included, they do little more than cloud the understanding of the fracture phenomenon. Furthermore, these high frequency modes would be the first to be damped out and, hence, can also be neglected on this basis.

The Solution Procedure

In common with the work presented in Reference (1), the approach is within the confines of LEFM with inertia effects included. The finite difference method is used to integrate the equations of motion expressed in terms of the nodal displacements¹. The finite difference grid is depicted in Figure 1. It can be seen that, because of the symmetry with respect to the diametrical plane containing the crack, only half the cylinder need be analyzed. The faces of the crack may open up but they are prevented from penetrating each other. The stress intensity factor is determined from (2) using the energy release rate calculated from the displacements at the nodes in the near vicinity of the crack tip.

At time $t = 0$, each node is given only a radial initial velocity v_0 . The impulse per unit of surface area is

$$I = \rho h v_0 \quad (3)$$

where ρ is the density of the material. The equations

¹ A detailed description of the method appears in Reference (2) with an abbreviated account given in (3).

of motion are solved with stepwise increasing time with the fracture criterion, Equation (1), being tested in each time step. When the fracture criterion is satisfied, the crack is permitted to advance one half of the radial nodal spacing. For the computations described herein, the number of nodes through the thickness was kept constant at thirteen. The number of nodes in the circumferential direction was selected to keep the aspect ratio (the ratio of the radial grid dimension to the circumferential grid dimension) of the grid at approximately 0.04. Favorable results have been obtained previously for this aspect ratio.

Computational Results for a Stationary Crack

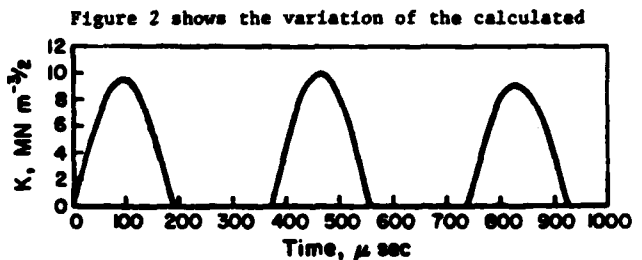


Figure 2. Stress Intensity Factor Versus Time For A Stationary Crack

stress intensity factor with time for an impulsively loaded steel cylinder for a stationary crack. These results are for $R = 305$ -mm, $h = 26$ -mm, $a_0 = 6$ -mm, and $v_0 = 1$ m/s. Note that the intervals where $K = 0$ correspond to the closing of the crack. Of most importance is the maximum stress intensity factor ($K_{max} = 9.5 \text{ MNm}^{-3/2}$ in this instance) determined in this way. This value is required in order to determine the minimum load necessary to initiate crack propagation.

The period T of oscillation in the breathing mode of an unflawed cylinder is

$$T = \frac{2\pi R(1 - v_0^2)^{1/2}}{C_0} \quad (4)$$

where $C_0 = \sqrt{E/\rho}$ is the bar wave speed which is 5000 m/s for steel. For $v_0 = 0.3$, the period for $R = 305$ -mm is 365 μ s. This is very nearly equal to the period of the stress intensity factor in Figure 2. It is also apparent from this figure that, while the amplitude varies slightly, it can be taken as essentially constant. Therefore, subsequent analyses of the stationary crack need only consider the first half period of oscillation to determine reasonable values of K_{max} .

The maximum stress in an unflawed cylinder subjected to a uniform impulsive loading is readily shown to be

$$\sigma_0 = \frac{E v_0}{C_0 (1 - v_0^2)^{1/2}} \quad (5)$$

which is independent of R/h . The maximum stress intensity factor can be related to σ_0 by a relation valid in quasi-static conditions. This is

$$K_{max} = \sigma_0 \sqrt{a_0} f(a_0/h, R/h) \quad (6)$$

where f is a function of the geometry of the flawed cylinder. In Table 1 the computed K_{max} values for various geometries are shown. These have also been

Table 1. Computational Results For A Stationary Crack In An Impulsively Loaded Radially Cracked Cylinder

R/h	$K_{\max}$ [MN/m ^{3/2}]	$\sigma_{\max}$ [N/mm ²]	$\frac{K_{\max}}{\sigma_o \sqrt{a_o}}$	$\frac{\sigma_{\max}}{\sigma_o}$
$a_o/h = 0.23$				
11.7	9.44	50.1	1.70	1.24
30.	9.40	49.5	1.69	1.22
60.	9.42	49.4	1.70	1.22
$a_o/h = 0.46$				
11.7	14.0	62.5	1.78	1.54
30.	14.5	62.7	1.85	1.55

normalized with respect to $\sigma_o \sqrt{a_o}$. It appears from these results that the normalized stress intensity factor is virtually independent of R/h for these thin walled cylinders. Hence, for design purposes, it could be considered that

$$K_{\max} = 1.75 \frac{E}{(1-\nu^2)^{1/2}} \frac{v_o}{C_o} (\pi a_o)^{1/2} \quad (7)$$

where the result given in Table 1 has been used.

The maximum stress intensity factor also appears to be nearly independent of a_o/h . However, only relatively small values of a_o/h were used here and it is likely that the edge effect is small in this range. Results for steady state vibrations of center-crack plates and an infinite plane with a periodic system of cracks—see Reference (5)—also indicate that edge effects do not become significant until $a_o/h > 0.6$. For larger values of a_o/h , $K_{\max}/\sigma_o \sqrt{a_o}$ would be expected to be greater than the value found here.

Table 1 also shows the maximum stress attained at the point two nodes (4-mm) ahead of the crack tip. This value is denoted as $\sigma_{\max}$. Here again, there is only a slight dependence on R/h. But, $\sigma_{\max}$ does depend upon a_o/h . By comparing $\sigma_{\max}$ to the yield stress, these results can be used to predict the maximum impulse that would satisfy the conditions of LEFM.

Computational Results for Unstable Crack Propagation

For the same initial loading, the computations for a stationary crack show that $K_{\max}$ increases with increasing crack depth. If crack propagation were to initiate, it would therefore appear that the crack would be propagating in an increasing K field. Under such circumstances the crack would not arrest, but would certainly penetrate the wall. On the other hand, the impulsive loading only imparts a finite amount of energy to the cylinder. And, as the crack propagates, it must consume some of this energy in the fracture process. Hence, there is a question as to whether or not this energy loss is sufficient to reduce the amplitude of the stress intensity factor sufficiently to arrest the crack. This question can be properly addressed only with a dynamic analysis.

A quarter-through-wall crack is frequently taken to be minimum identifiable crack length. If, for

reasons of safety, cracks exceeding half the wall thickness are deemed not permissible, a question of concern is whether or not a quarter-through wall crack which initiates will arrest before it propagates half way through the wall. To study this question, assume that the material has a speed independent toughness $K_{ID} = K_{IC}$ and the amplitude of the final stress intensity factor is K_{ID} ; i.e., the crack just arrests. A balance of energy per unit length requires that

$$U_o - U_f = \frac{K_{ID}^2 (1 - \nu^2)}{E} (a_f - a_o) \quad (8)$$

where U denotes the total energy per unit length and the subscripts o and f are used to denote initial and final quantities. Using (2), the energies can be written as

$$U_o = \pi \rho v_o^2 R h = \frac{(K_o)_{\max}^2 (1 - \nu^2) R h}{3.2 E a_o}$$

$$U_f = \frac{K_{ID}^2 (1 - \nu^2) R h}{3.2 E a_f} ; \frac{a_f}{h} < 1/2 \quad (9)$$

Substituting Equations (9) into (8) with $a_o/h = 1/4$ and $a_f/h = 1/2$ gives

$$\frac{(K_o)_{\max}}{K_{ID}} = (0.20 \frac{h}{R} + .5)^{1/2} < 1. \quad (10)$$

But, for the crack to initiate, $K_{ID} < (K_o)_{\max}$. However, this is inconsistent with (10) and the hypothesis that the crack arrests. Therefore, this contradiction implies that the crack will not arrest before it propagates half-way through the wall. Because of the impact loading, the crack might initiate with $K_{ID} \leq (K_o)_{\max} < K_{ID}$ and no inconsistency would appear. Also, as noted earlier, viscous damping, which could have a significant effect under certain circumstances, is not included.

In order to determine the character of crack propagation in a cylinder under impulsive or shock loading, computations were performed for a steel cylinder of radius $R = 305$ -mm and thickness $h = 26$ -mm having an initial radial crack of depth $a_o = 6$ -mm ($a_o/h = 0.23$). An initial uniform radial velocity of 1m/s was imparted to the cylinder. These are the same conditions upon which the results of Figure 2 are based. The fracture toughness was taken as a speed independent value equal to 98% of the maximum stress intensity factor experienced by a stationary crack for these same conditions; i.e., $K_{ID} = 9.25$ MN/m^{3/2} per unit of initial velocity.

A plot of computed crack length versus time is shown in Figure 3. It can be seen that approximately 84 μ sec are required for the stress intensity to build up to the critical value and to initiate crack growth. During this interval, the variation of the stress intensity factor with time is depicted in Figure 2. After initiation, the crack propagates at a speed of approximately 300 m/s. As the crack tip propagated further into the wall, its speed increases. This reflects the unstable nature of the crack growth. Some 20 μ sec after initiation the crack tip finally penetrated the exterior surface. The speed of the crack tip during the final stages of the event was approximately 2200 m/s.

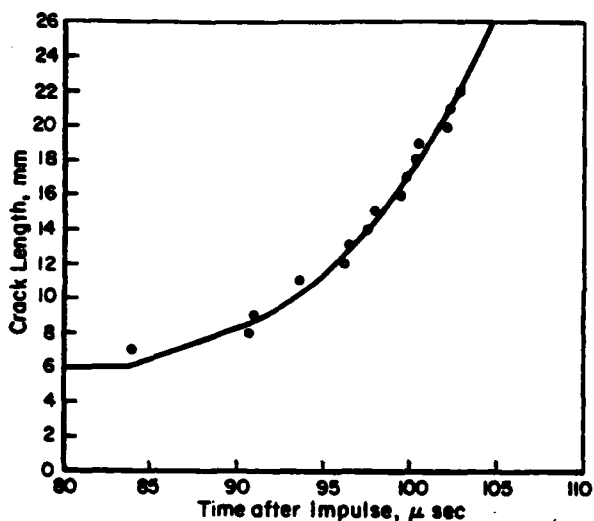


Figure 3. Crack Propagation in Impulsively Loaded Crack Cylinder

For all practical purposes this computation confirms the earlier suspicion that, if growth of an internal crack commences, the crack tip will eventually penetrate the exterior surface. This conclusion is predicated upon the assumption that the material is elastic-brittle and has a crack-speed-independent fracture toughness. In reality, as the remaining ligament becomes sufficiently small, inelastic behavior becomes important. Then, the fracture process is no longer K-dominated as required for LEFM to be applicable. While such inelastic behavior is important and of interest, its consideration is beyond the scope of this paper and is reserved for further research.

CONCLUSIONS

Elastodynamic crack propagation calculations have been performed for a part-through cracked circular cylinder subjected to a uniform impulsive loading. These calculations have demonstrated that, while actual crack-structure geometries are more complicated than are the simple laboratory test specimens previously analyzed, they can be treated effectively with dynamic fracture mechanics. Specifically, for the particular geometry considered in this paper:

1. The minimum impulse required to initiate unstable crack propagation for a given crack depth has been determined in terms of material, mechanical, and fracture properties.
2. Once initiated, crack propagation in the through-wall direction will continue until the crack penetrates the wall--crack arrest within the wall does not appear to be possible.
3. Crack length time predictions can be made for comparison with experimentally measured times of crack growth initiation and of crack penetration.

It is clear that more complicated geometries can be addressed with this analysis procedure (with a corresponding increase in computer costs). However, it

should be recognized that the specific conclusions cited are restricted to LEFM conditions and for a material with a relatively constant dynamic fracture toughness.

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