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Analysis of Conventional and Reflective Butler Matrices with Imperfect Components

J. P. SHELTON AND J. K. HSIAO

*Target Characteristics Branch
Radar Division*

March 18, 1980

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ANALYSIS OF CONVENTIONAL AND REFLECTIVE BUTLER MATRICES WITH IMPERFECT COMPONENTS

INTRODUCTION

A Butler matrix that forms a cluster of beams evenly distributed in the $\sin-\theta$ space is not usually symmetric with respect to a plane midway between the input and output ports. However, by properly adjusting the phase shifts and interconnections one may modify a conventional Butler matrix to be symmetric. Such a matrix may also be folded on itself on the line of symmetry, so that the input and output ports are identical. Such a network not only reduces the number of components required; it also becomes a reflection-type system in which the feed positions are in the plane of the aperture. The synthesis of this network was described previously [1,2]. In this report, we analyze the performance of both conventional and reflective Butler matrices. In particular, we investigate the effect of reflected waves on the beam-forming performance. In a conventional Butler matrix, since the input and output ports are separate, the reflected waves emerging from the input ports have no effect on the beam-forming performance. Multiply reflected waves may emerge from output ports; however, their amplitudes are generally small, and their effects are relatively insignificant. In a reflective Butler matrix, the reflected waves accumulate at the input/output ports; hence, the aperture distribution at the antenna array is significantly modified, and this may degrade the beam-forming performance. These effects are investigated, and computer simulated results are presented together with a listing of the computer program.

SCATTERING MATRIX OF A 3-dB HYBRID COUPLER

The basic building block of a Butler matrix is a 3-dB hybrid coupler. For the ideal hybrid coupler, energy fed into any one of the input ports will be split into two equal components, one with a phase shift of 90° relative to the other. However, practical hybrid couplers will in general exhibit amplitude and phase errors in their transfer coefficients. These amplitude and phase errors will affect the transfer coefficients of both reflective and conventional Butler matrices in the same way. That is, the errors in the overall network input/output transfer coefficients will be the same for both conventional and reflective networks. Practical hybrid couplers will also have nonzero reflection and transfer coefficients to the isolated port. For the conventional network, to a first order, the error components due to these effects will appear at the network inputs. For the reflective network, with its inputs and outputs sharing a single set of ports, all error components affect the input/output transfer coefficients.

Thus, the two types of hybrid coupler errors are forward and reverse. Our investigation will be concentrated on the reverse-error components, and we shall assume that there

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is no amplitude or phase error in the forward-transfer coefficients of the 3-dB coupler. The following analysis is based on the assumption that, when an incident wave of unit amplitude is applied to one of the input ports, two waves of amplitude α will emerge from the two output ports, one with a 90° phase shift and the other with no phase shift. Similarly, waves of amplitude β will be reflected to the two input ports. As shown in Fig. 1(a), when an incident wave of unit amplitude is applied at port 12, reflected waves of $-\beta$ and $-j\beta$ appear at ports 11 and 12 respectively and waves of $-j\alpha$ and α appear at ports 21 and 22. For conservation of energy, one has

$$2\alpha^2 + 2\beta^2 = 1. \quad (1)$$

The isolation factor is defined as the power ratio of the reflected wave to the incident wave. In this case, the isolation is

$$I = \beta^2. \quad (2)$$

Accordingly, in terms of the isolation factor,

$$\alpha = \sqrt{0.5 - I}. \quad (3)$$

If the parameters in Fig. 1(b) are used, the reflected waves are related to the incident waves by the matrix equation

$$\begin{bmatrix} b_{11} \\ b_{12} \\ b_{21} \\ b_{22} \end{bmatrix} = \begin{bmatrix} -j\beta & -\beta & \alpha & -j\alpha \\ -\beta & -j\beta & -j\alpha & \alpha \\ \alpha & -j\alpha & -j\beta & \beta \\ -j\alpha & \alpha & -\beta & -j\beta \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ a_{21} \\ a_{22} \end{bmatrix}, \quad (4)$$

where a_{11} , a_{12} , a_{21} , and a_{22} are incident waves and b_{11} , b_{12} , b_{21} , and b_{22} are scattered waves at ports 11, 12, 21, and 22 respectively.

Let

$$\begin{aligned} \mathbf{b}_1 &= \begin{bmatrix} b_{11} \\ b_{12} \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} b_{21} \\ b_{22} \end{bmatrix}, \\ \mathbf{a}_1 &= \begin{bmatrix} a_{11} \\ a_{12} \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} a_{21} \\ a_{22} \end{bmatrix}, \end{aligned} \quad (5a)$$

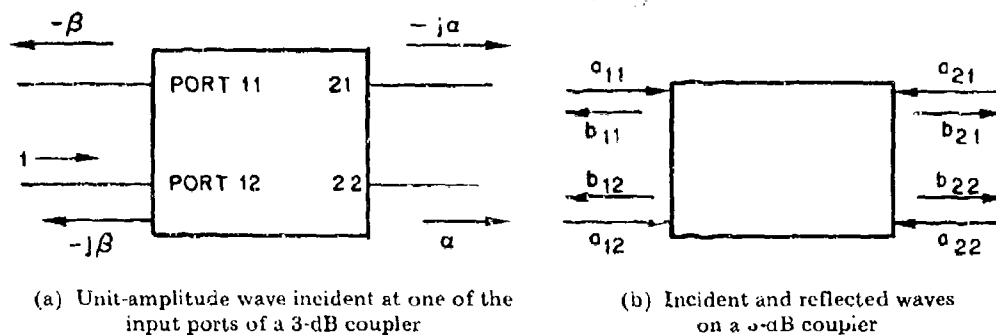


Fig. 1 — Transfer and reflection in four-port networks

and

$$S_{11} = S_{22} = \begin{bmatrix} -j\beta & -\beta \\ -\beta & -j\beta \end{bmatrix}, \quad (5b)$$

$$S_{12} = S_{21} = \begin{bmatrix} \alpha & -j\alpha \\ -j\alpha & \alpha \end{bmatrix}.$$

Matrix Eq. (4) can now be simplified to the form

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}. \quad (6)$$

SCATTERING AND TRANSFER MATRICES OF A BUTLER NETWORK

A Butler network can be represented by a block diagram as shown in Fig. 2.* Blocks in regions 1 and 3 represent the 3-dB couplers described in the previous section, and a phase-shift transfer network is located in region 2. A number of similar networks are

*For the remainder of this report, a network will be considered a physical entity and a matrix a mathematical entity.

connected in cascade to form a complete conventional Butler network. The scattering matrix for regions 1 and 3 is

$$\begin{bmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{1n} \\ b_{21} \\ b_{22} \\ \vdots \\ b_{2n} \end{bmatrix} = \begin{bmatrix} -j\beta & -\beta & 0 & 0 & \dots & \alpha & -j\alpha & 0 & 0 & \dots & 0 \\ -\beta & -j\beta & 0 & 0 & \dots & -j\alpha & \alpha & 0 & 0 & \dots & 0 \\ 0 & 0 & -j\beta & -\beta & 0 & 0 & \dots & \alpha & -j\alpha & 0 & \dots \\ 0 & 0 & -\beta & -j\beta & 0 & 0 & \dots & -j\alpha & \alpha & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \alpha & -j\alpha & 0 & 0 & \dots & -j\beta & -\beta & 0 & 0 & \dots & 0 \\ -j\alpha & \alpha & 0 & 0 & \dots & -\beta & -j\beta & 0 & 0 & \dots & 0 \\ 0 & 0 & \alpha & -j\alpha & 0 & 0 & \dots & j\beta & -\beta & \dots & 0 \\ 0 & 0 & -j\alpha & \alpha & \dots & \dots & -\beta & -j\beta & \dots & 0 & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1n} \\ a_{21} \\ a_{22} \\ \vdots \\ a_{2n} \end{bmatrix} \quad (7)$$

Define

$$b_1 = \begin{bmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{1n} \end{bmatrix}, \quad b_2 = \begin{bmatrix} b_{21} \\ b_{22} \\ \vdots \\ b_{2n} \end{bmatrix}, \quad (8a)$$

$$a_1 = \begin{bmatrix} a_{11} \\ a_{12} \\ \vdots \\ a_{1n} \end{bmatrix}, \quad a_2 = \begin{bmatrix} a_{21} \\ a_{22} \\ \vdots \\ a_{2n} \end{bmatrix}, \quad (8b)$$

$$S_{11} = S_{22} = \begin{bmatrix} -j\beta & -\beta & 0 & 0 & \dots & \dots \\ -\beta & -j\beta & 0 & 0 & \dots & \dots \\ 0 & 0 & -j\beta & -\beta & 0 & 0 \\ 0 & 0 & -\beta & -j\beta & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & -j\beta & -\beta \\ 0 & 0 & \dots & \dots & -\beta & -j\beta \end{bmatrix}, \quad (8c)$$

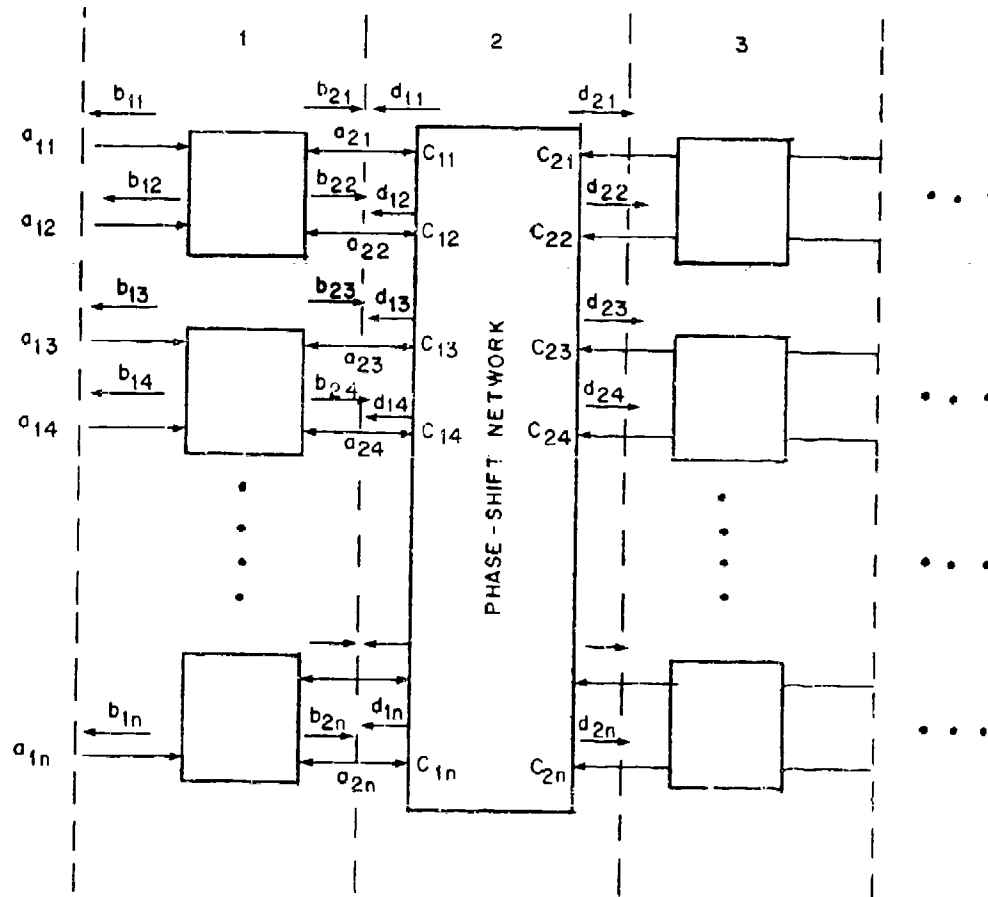


Fig. 2 — Block diagram of a Butler network

and

$$S_{12} = S_{21} = \begin{bmatrix} \alpha & -j\alpha & 0 & 0 & \dots & \dots \\ -j\alpha & \alpha & 0 & 0 & \dots & \dots \\ 0 & 0 & \alpha & -j\alpha & 0 & 0 \\ 0 & 0 & -j\alpha & \alpha & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \dots & \alpha & -j\alpha \\ 0 & 0 & \dots & \dots & -j\alpha & \alpha \end{bmatrix} \quad (8d)$$

Equation (7) can now be simplified to

$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \quad (9)$$

The scattering matrix in region 2, which is a phase-shift and transfer network, can be represented as

$$\begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \quad (10)$$

where d_1 , d_2 , c_1 , and c_2 are vectors such that

$$d_1 = \begin{bmatrix} d_{11} \\ d_{12} \\ \vdots \\ d_{1n} \end{bmatrix}, \quad d_2 = \begin{bmatrix} d_{21} \\ d_{22} \\ \vdots \\ d_{2n} \end{bmatrix} \quad (11a)$$

$$c_1 = \begin{bmatrix} c_{11} \\ c_{12} \\ \vdots \\ c_{1n} \end{bmatrix}, \quad c_2 = \begin{bmatrix} c_{21} \\ c_{22} \\ \vdots \\ c_{2n} \end{bmatrix} \quad (11b)$$

Matrices R_{11} and R_{22} are zero, and matrices R_{12} and R_{21} have identical elements. These matrices describe the phase shifts and interconnections from one row of couplers to the

next. Their elements depend on the configuration of the Butler network. As an example, the R matrix of the 4-port Butler network shown in Fig. 3 is

$$R_{12} = R_{21} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & e^{-j\frac{\pi}{4}} & 0 \\ 0 & e^{-j\frac{\pi}{4}} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (12)$$

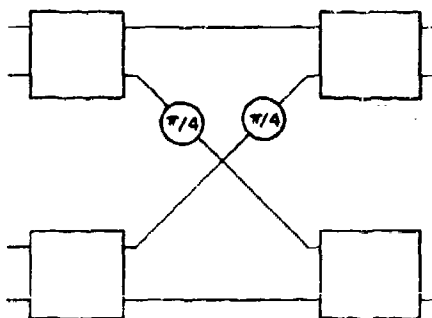


Fig. 3 — Four-port Butler network

Since we are interested in the overall scattering matrix of this network, we must first convert the scattering matrix in each region to a transfer matrix, which in turn can be multiplied to form the overall transfer matrix of the whole network. A transfer matrix can be represented as

$$\begin{bmatrix} b_2 \\ a_2 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}, \quad (13)$$

where a_1 and b_1 are the incident and reflected waves at the left hand ports and a_2 and b_2 are similar waves at the right hand ports.

It can be shown that a matrix T is related to an S matrix by the following relations [3,4]:

$$T_{11} = S_{21} - S_{22} S_{12}^{-1} S_{11} \quad (14a)$$

$$T_{12} = S_{22} S_{12}^{-1} \quad (14b)$$

$$T_{21} = -S_{12}^{-1} S_{11}, \quad (14c)$$

and

$$T_{22} = S_{12}^{-1}. \quad (14d)$$

The overall transfer matrix is

$$T = \prod_{i=1}^k T_i \quad (15)$$

where $T_1, T_2, \dots, T_k$ are transfer matrices in regions 1, 2, ..., k .

The overall transfer matrix can be converted to a scattering matrix by the relations

$$S_{11} = -T_{22}^{-1} T_{21}, \quad (16a)$$

$$S_{12} = T_{22}^{-1}, \quad (16b)$$

$$S_{21} = T_{11} - T_{12} T_{22}^{-1} T_{21}, \quad (16c)$$

and

$$S_{22} = T_{12} T_{22}^{-1}. \quad (16d)$$

Since $S_{12} = S_{21}$, one may use the simpler relation of Eq. (16b) instead of Eq. (16c).

Elements of matrix S_{21} (or S_{12}) represent the transmitted waves at the output ports when a unit incident wave is applied at any one of the input ports. Therefore, matrix S_{21} is the transfer function of a conventional Butler network. Elements of matrix S_{11} (or S_{22}) represent the reflected waves at the input ports when a unit incident wave is applied at any one of the input ports. In a reflective Butler network both the reflected waves and transmitted waves emerge from the same set of ports. Therefore, the scattering matrix of such a network is the sum of matrices S_{12} and S_{11} , or

$$S = S_{11} + S_{12}. \quad (17)$$

In deriving this relation, we have made the assumption that the symmetry plane of a reflective Butler network exhibits an open-circuit unity reflection coefficient.

PATTERNS OF AN ARRAY FED BY A BUTLER NETWORK

Figure 4 shows a schematic diagram of a reflective Butler network, which has half the components of a conventional Butler network. There are n ports, since ports a_{11} ,

$a_{12}, \dots, a_{1n}$ are identical with ports $a_{21}, a_{22}, \dots, a_{2n}$. Using previously developed notation and setting $[b_2] = [a_2] = 0$, this can be represented as

$$[b_1] = [S_{11} + S_{12}][a_1]. \quad (18)$$

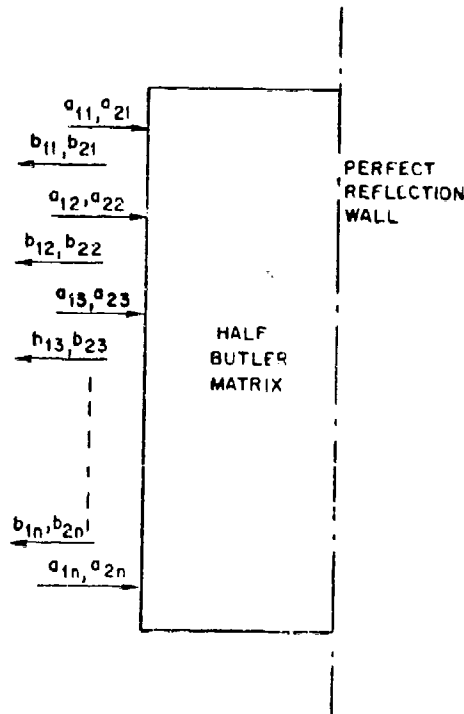


Fig. 4 — Reflective Butler network

The vector input of $[a_1]$ can be represented, for the case of an incident plane wave received by a linear array, by

$$a_{1k} = A_k \exp [j(k-1)u] \quad (19)$$

where $u = 2\pi d \sin \theta / \lambda$,

with λ = wavelength,
 θ = angle of incidence from the normal to the array, and
 d = element spacing.

In the subsequent discussion, we shall assume that the array has a uniform illumination function, that is, that $A_k = 1$. The scattering matrix $[S_{11}] + [S_{12}]$ is computed as a function of isolation factor I . Radiation patterns of the network-fed array are represented by two types of plot. One shows the main beams formed by several ports of the reflective Butler network, and the other shows the complete array pattern of one port of the network, in the range $0 \leq u \leq 180^\circ$.

Figure 5 shows the array patterns of an eight-port reflective Butler network. Figure 5a shows four of the main beams for variation of the isolation factor of the 3-dB hybrid from 10 dB to 40 dB. Figure 5b shows the array pattern when the main beam is at $u = 22.5^\circ$ for the same range of isolation factor. Figure 6 shows the corresponding patterns for a 16-port reflective network. From these figures, it can be seen that the null filling level is roughly equal to the isolation factor of the 3-dB couplers. That is, for the case of 10-dB isolation, the pattern is filled to a level of about 10 dB below its peak; and for the case of 40 dB isolation, the pattern is filled to a level of about 40 dB below its peak.

Tables 1 and 2 show computed results for eight-port and 16-port reflective Butler networks, respectively. The isolation factors in dB are listed in the first column. The transmitted power is the percentage of incident power, averaged over all inputs and outputs, that would emerge from the outputs for the conventional Butler-network configuration. The remaining power emerges from the input ports. It is seen that the transmitted power decreases as the isolation decreases and as the number of rows of couplers in the network increases. For the reflective-network configuration, the input and output ports are combined, and the components emerging from these ports are also combined. The RMS amplitude and phase errors describe the effects of these spurious components on the combined outputs and are defined by

$$\Delta b = \left[\frac{\sum_{k=1}^N \sum_{\ell=1}^N \left(|s_{k\ell}| - \bar{s} \right)^2}{N^2} \right]^{1/2}$$

and

$$\Delta \phi = \left[\frac{\sum_{k=1}^N \sum_{\ell=1}^N \left(\phi_{k\ell} - \phi'_{k\ell} \right)^2}{N^2} \right]^{1/2},$$

where Δb and $\Delta \phi$ are the RMS amplitude and phase errors, respectively, $s_{k\ell}$ is an element of the scattering matrix S ,

$$\bar{s} = \frac{\sum_{k=1}^N \sum_{\ell=1}^N |s_{k\ell}|}{N^2},$$

$\phi_{k\ell}$ is the phase of $s_{k\ell}$, and $\phi'_{k\ell}$ is the phase of $s_{k\ell}$ for the ideal network with no errors. The error components increase with the number of rows of couplers and with decreasing isolation.

A computer program for carrying out these calculations is listed in the appendix. In addition to providing for imperfect reverse parameters of the hybrid couplers, the program provides for imperfect forward parameters and for errors in the interconnecting transmission lines.

CONCLUSIONS

An exact analysis procedure has been developed that is applicable to both conventional and reflective Butler networks with imperfect components. The analytical procedure has been programmed for computation of results for conventional and reflective Butler networks of arbitrary size. Results are presented for eight-port and 16-port reflective networks using hybrid couplers with varying degrees of isolation. The results are given in the form of radiation-pattern factors that would be obtained from a linear antenna array fed by the network and also in terms of the RMS phase and amplitude errors of the network transfer coefficients.

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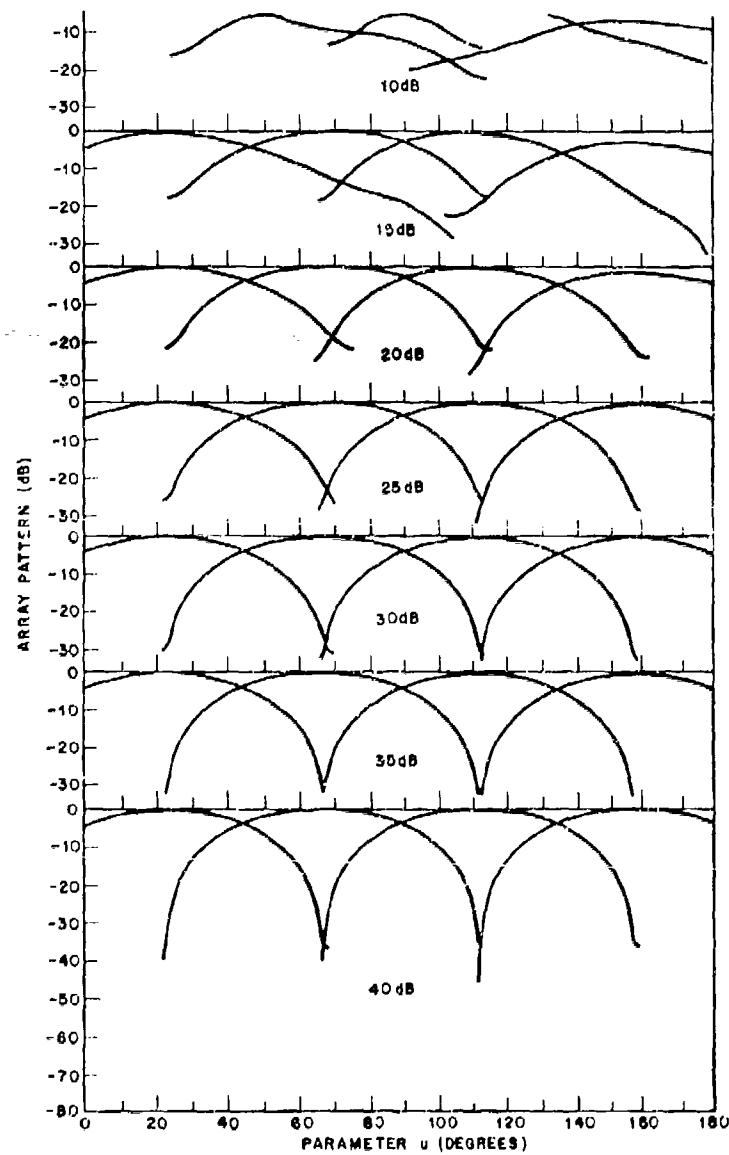


Fig. 5a — Main-beam pattern of an eight-port reflective Butler network; isolation factor varies from 10 dB to 40 dB

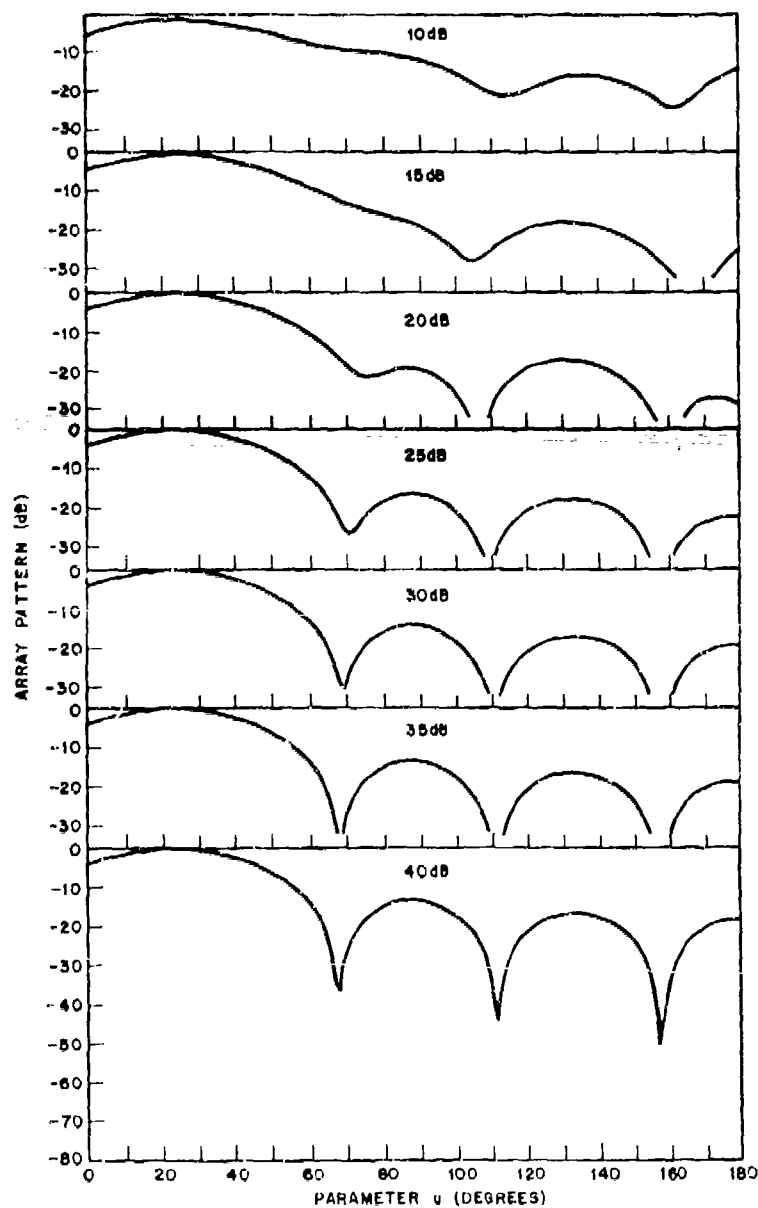


Fig. 5b — Array pattern of an eight-port reflective Butler network; isolation factor varies from 10 dB to 40 dB; main beam at $u = 22.50^\circ$

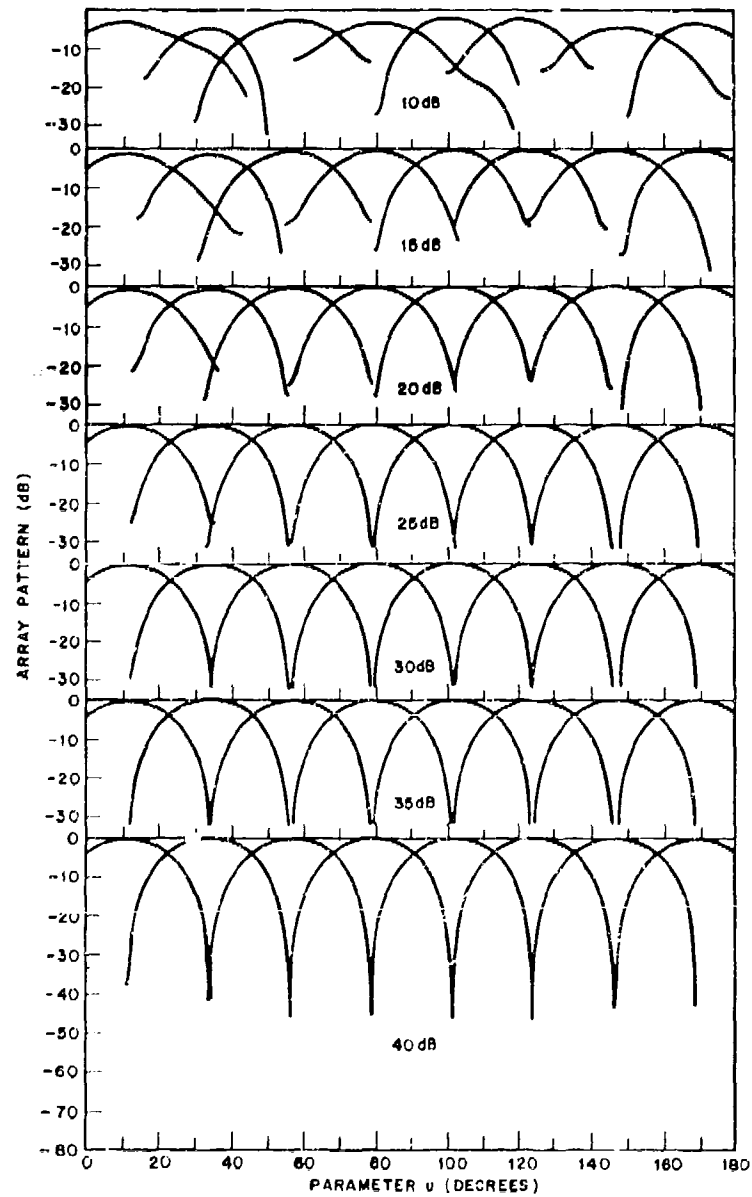


Fig. 6a — Main beam pattern of a 16-port reflective Butler network;
isolation factor varies from 10 dB to 40 dB

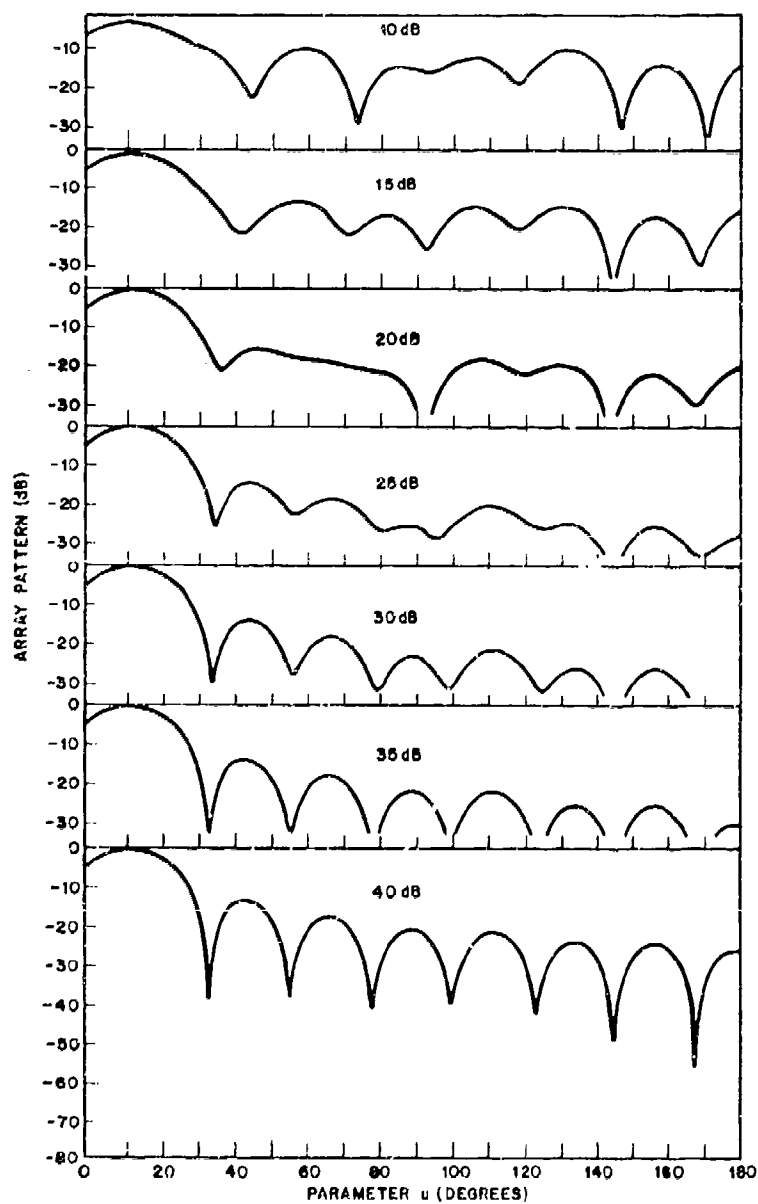


Fig. 6b -- Array pattern of a 16-port reflective Butler network; isolation factor varies from 10 to 40 dB; main beam at $u = 11.25^\circ$

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Table 1 — Computed Statistical Parameters for
Eight-Port Reflective Network

Isolation (dB)	Transmitted Power (percent of incident)	RMS Amplitude Error (percent)	RMS Phase Error (degrees)
10	54.85	30.25	38.69
15	80.76	19.42	23.53
20	93.20	12.30	12.97
25	97.77	7.30	7.16
30	99.29	4.21	3.99
35	99.77	2.39	2.24
40	99.93	1.35	1.25

Table 2 — Computed Statistical Parameters for
16-Port Reflective Network

Isolation (dB)	Transmitted Power (percent of incident)	RMS Amplitude Error (percent)	RMS Phase Error (degrees)
10	44.92	47.29	50.58
15	76.30	31.96	30.95
20	91.49	20.41	12.66
25	97.19	11.67	6.91
30	99.10	6.61	3.84
35	99.71	3.73	2.16
40	99.91	2.10	1.22

Appendix

COMPUTER PROGRAM FOR ANALYSIS

This computer program computes the coupling coefficients from the input ports to the output ports and the power transmitted and reflected; it also plots the array radiation pattern if it is desired. The type of Butler matrix analyzed by this program can be either a conventional or a reflective type as described in this report. For this program three input data cards are required. The first data card enters the following fixed-point (I5 format) data:

NPT — Number of ports of the Butler matrix to be computed.

NROW — Number of rows of this network.

KLL — Absolute value of KLL represents the beam index whose pattern is to be plotted. If KLL = 0, there is no plot. If KLL is less than 0, the program plots the array pattern and also plots all main beams formed by the Butler matrix network.

LPRINT — Printout control. If LPRINT = 0, the program prints all detailed output at each computation step.

The second data card, which is also in a fixed-point I5 format, specifies the number of ports in each basic coupling network in each row. This implies that identical coupling networks are used in each row. However, coupling networks of different ports may be used in different rows.

The third input data card, which has a F10.6 floating-point format, specifies the coupling coefficients of the 3-dB coupler used as the basic building block of the Butler matrix network. These coefficients are read in the sequence A1, B1, C1, D1. These numbers are related to the coupling coefficient of the 3-dB coupler by the relations (see Fig. 1a)

$$\begin{aligned}\beta_1 &= 10^{-(0.05 \times A1)}, \\ \beta_2 &= 10^{-(0.05 \times B1)}, \\ \alpha_1 &= 10^{-(0.05 \times C1)},\end{aligned}$$

and

$$\alpha_2 = 10^{-(0.05 \times D1)}.$$

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0001      PROGRAM RFBMTX
C          THIS PROGRAM FIRST FIGURES OUT BUTLER MATRIX CONNECTION AND PHASE
C          ANGLE , COMPUTES THE TRANSFER FUNCTION AND THEN PLOT THE PATTERN
C          MATRIX LIMIT TO THE SIZE OF 64
C          COMPILED ON JULY 13,1976 BY J. K. HSIAO
C          REVISED ON AUGUST 18,1976 BY J. K. HSIAO
C          ABSOLUTE VALUE OF KLL REPRESENTS THE BEAM INDEX WHOSE PATTERN IS
C          TO BE PLOTTED
C          KLL=0 NO PLOT
C          KLL GRATER THAN 0 PLOT PATTERN ONLY
C          KLL LESS THAN 0 PLOT BOTH PATTERN AND MAIN BEAMS
C          LLL=1, FULL MATRIX, LLL=0 REFLECTIVE MATRIX
C          LPRINT =0, PRINT ALL DETAILED OUTPUTS
C          IF LPRINT NOT EQUAL 0 NO MATRIX MULTIPLICATION RESULT IS PRINTED
C          IF LPRINT LT 0 PRINT ONLY THE TRANSFER FUNCTION
0002      COMMON/C$1/PLTAY(500)
0003      COMMON/C$4/A1,A2,B1,B2
0004      DIMENSION NBP(16),NBK(16)
0005      DIMENSION MC(8,64),PHA(8,64)
0006      DIMENSION S11(32,32),S12(32,32),S21(32,32),S22(32,32)
0007      COMPLEX S11,S12,S21,S22
0008      CALL PLOTS(PLTAY,500,0.)
0009      NMAX=32
0010      KC=0
0011      1  READ 100,NTP,NROW,KLL,LPRINT,LLL
0012          IF(NTP.EQ.0)GO TO 2
0013      3  READ 100,(NBP(I),I=1,NROW)
0014      100  FORMAT(16I5)
0015          READ 101, A1,A2,B1,B2
0016      101  FORMAT(8F10.6)
0017          IF(KC.GT.0)CALL ORIGIN(14.,0.)
0018          KC=KC+1
0019          NRI=NROW+1
0020          CALL NTWK(NTP,NRI,NBP,NBK,MC,PHA)
0021          IF(LLL.GT.0)GO TO 4
0022          CALL HLFMTX(NTP,NRI,NBP,NBK,MC,PHA)
0023      4  CALL TRFMTX(NMAX,NTP,NRI,NBP,NBK,MC,PHA,S11,S12,S21,S22)
0024          LL=0
0025          CALL PRTOU(NTP,S21,S11,LL,NMAX,LTFP,LPRINT)
0026          LTFP=1
0027          IF(KLL.EQ.0)GO TO 1
0028          NPAV=1
0029          CALL PATERN (NTP,S21,S11,KLL,NPAV,NMAX)
0030          GO TO 1
0031      2  CALL ENDPLOT
0032      END

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0001      SUBROUTINE PRIOUT(NTP,TRFF,TRFB,LL,NMX,LTFP,LPRINT)
C          LLGT.0 FOR BLOCK, AND LLIS THE BLOCK NUMBER
C          LL=0 FOR OVERAL TRANSFER FUNCTION
0002      DIMENSION TRFF(NMX,NMX),TRFB(NMX,NMX)
0003      COMMON/CS4/A1,A2,B1,B2
0004      COMMON/CS6/AMPT(32,32),ANGL(32,32),ANGT(32),TRFF2(32,32), TR(32,32
C),AMPAY(32),ANGAY(32),AMX(32),ANX(32),AMPRMS(32),ANGRMS(32),
C      SUMR(1824)
0005      COMPLEX TRFF,TRFB,TRFF2,TR,SR
0006      KC=0
0007      PI=3.1415926536
0008      RAC=180./PI
0009      K6=6
0010      LLL=0
0011      IF(A1.LE.0..OR.LL.GT.0)LLL=1
0012      IF(LL.LE.0)GO TO 1
0013      PRINT 101,LL
0014      101  FORMAT(/,20X,"THIS IS THE TRANSFER FUNCTION OF BLOCK",I5)
0015      GO TO 4
0016      1  PRINT 111
0017      PRINT 106
0018      106  FORMAT(/,20X,"OVERAL TRANSFER FUNCTION")
0019      2  IF(A1.GT.0.)GO TO 3
0020      PRINT 119
0021      119  FORMAT(/,10X,"ZERO REFLECTION")
C          GENERATE TRANSFER FUNCTION FOR AN IDEAL BUTLER MATRIX
0022      3  PRINT 124,NTP,A1
0023      PRINT 117
0024      124  FORMAT(/,20X,"NUMBER OF PORTS",I5,5X,"ISOLATION(DB)",F10.4,/)
0025      CALL TRFIDL(NTP)
0026      IF(LTFP.GT.0)GO TO 4
0027      IF(LPRINT .GT.0)GO TO 4
0028      PRINT 107,((AMPT(I,J),J=1,NTP),I=1,NTP)
0029      PRINT 117
0030      PRINT 107,((ANGL(I,J),J=1,NTP),I=1,NTP)
0031      PRINT 117
0032      4  IF(A1.LE.0.)K6=2
0033      DO 60 K=1,K6
0034      SUM=0.
0035      DO 15 I=1,NTP
0036      15  SUMR(I)=0.
0037      IF(LPRINT .GT.0)GO TO 75
0038      GO TO (71,72,73,74,76,77)K
0039      71  PRINT 102
0040      102  FORMAT(/,20X,"AMPLITUDE OF FORWARD TRANSFER FUNCTION")
0041      PRINT 117
0042      117  FORMAT(/)
0043      GO TO 75
0044      72  PRINT 103
0045      103  FORMAT(/,20X,"PHASE ANGLE OF FORWARD TRANSFER FUNCTION")

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0046      PRINT 117
0047      GO TO 75
0048      73 PRINT 104
0049      104 FORMAT(/,20X,"AMPLITUDE OF REFLECTIVE TRANSFER FUNCTION")
0050      PRINT 117
0051      GO TO 75
0052      74 PRINT 105
0053      105 FORMAT(/,20X,"PHASE ANGLE OF REFLECTIVE TRANSFER FUNCTION")
0054      PRINT 117
0055      GO TO 75
0056      76 PRINT 109
0057      109 FORMAT(/,20X,"AMPLITUDE OF THE RESULTANT TRANSFER FUNCTION")
0058      PRINT 117
0059      GO TO 75
0060      77 PRINT 110
0061      110 FORMAT(/,20X,"PHASE ANGLE OF THE RESULTANT TRANSFER FUNCTION")
0062      PRINT 117
0063      75 DO 67 I=1,NTP
0064          DO 70 J=1,NTP
0065              GO TO(61,62,63,64,65,66)K
0066      61  ANG1(J)=CABS(TRFF(I,J))
0067          ANG2=ANG1(J)**2
0068          SUM=SUM+ANG2
0069          SUMR(J)=SUMR(J)+ANG2
0070          GO TO 70
0071      62  IF(LPRINT .GT.0)GO TO 70
0072          ANG1(J)=CANG(TRFF(I,J))*RAC
0073          GO TO 70
0074      63  ANG1(J)=CABS(TRFB(I,J))
0075          ANG2=ANG1(J)**2
0076          SUM=SUM+ANG2
0077          SUMR(J)=SUMR(J)+ANG2
0078          GO TO 70
0079      64  IF(LPRINT .GT.0)GO TO 70
0080          ANG1(J)=CANG(TRFB(I,J))*RAC
0081          GO TO 70
0082      65  TR(I,J) =TRFF(I,J)+TRFB(I,J)
0083          ANG1(J)=CABS(TR(I,J))
0084          ANG2=ANG1(J)**2
0085          SUMR(J)=SUMR(J)+ANG2
0086          SUM=SUM+ANG2
0087          IF(LLL.GT.0)GO TO 70
0088          AMPT(I,J)=(ANG1(J)-AMPT(I,J))/AMPT(1,J)
0089          GO TO 70
0090      66  ANG1(J)=CANG(TR(I,J))*RAC
0091          IF(LLL.GT.0)GO TO 70
0092          AG =ANG1(J)-ANGL(I,J)
0093          ANGL(J,1)=AG
0094          IF(ABS(AG).LE.180.)GO TO 70
0095          NSIGN=1

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0096      IF(AG.GT.0.)NSIGN=-1
0097      ANGL(I,J)=NSIGN*(360.-ABS(AG))
0098  70      CONTINUE
0099      IF(LPRINT .GT.0)GO TO 67
0100      PRINT 107,(ANGT(J),J=1,NTP)
0101  107     FORMAT(10X, 8F10.4)
0102  67      CONTINUE
0103      KMOD=MOD(K,2)
0104      IF(KMOD.LE.0)GO TO 60
0105      PRINT 122,SUM
0106  122     FORMAT(/,10X,"TOTAL POWER OUTPUT",F10.4)
0107      PRINT 123,(SUMR(I),I=1,NTP)
0108  123     FORMAT(/,10X,"POWER FROM EACH PORT",/,(10X,10F10.4))
0109  60      CONTINUE
0110      IF(LL.LT.0)GO TO 7
0111      IF(LPRINT .GT.0)GO TO 8
0112      DO 50 L=1,2
0113      GO TO (51,52)L
0114  51      PRINT 120
0115  120     FORMAT(/,20X,"ERROR FUNCTION",/,20X,"AMPLITUDE",/)
0116      GO TO 53
0117  52      PRINT 121
0118  121     FORMAT(/,20X,"PHASE ANGLE",/)
0119  53      DO 50 I=1,NTP
0120      GO TO (54,55)L
0121  54      PRINT 107, (AMPT(J,I),J=1,NTP)
0122      GO TO 50
0123  55      PRINT 107, (ANGL(J,I),J=1,NTP)
0124  50      CONTINUE
0125      IF(LPRINT .LT.0)RETURN
0126      PRINT 111
0127  8       DO 54 L=1,2
0128      DO 57 I=1,NTP
0129      IF(L.GT.1.AND.I.GT.1)GO TO 58
0130      ANG5=0.
0131      AMP5=0.
0132      ANGX=0.
0133      AMPX=0.
0134  58      DO 56 J=1,NTP
0135      AMPS=AMPS+AMPT(J,I)
0136      ANG5=ANG5+ANGL(J,I)
0137      IF(AMPT(J,I).GT.AMPX)AMPX=AMPT(J,I)
0138  56      IF(ABS(ANGL(J,I)).GT.ABS(ANGX))ANGX=ANGL(J,I)
0139      IF(L.GT.1)GO TO 57
0140      AMPAV(I)=AMPS/NTP
0141      ANGAV(I)=ANG5/NTP
0142      AMX(I)=AMPX
0143      ANX(I)=ANGX
0144  57      CONTINUE
0145      IF(L.GT.1)GO TO 59

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0146      PRINT 117
0147      PRINT 107, (AMPV(K), K=1, NTP)
0148      PRINT 117
0149      PRINT 107, (ANGV(K), K=1, NTP)
0150      PRINT 117
0151      PRINT 107, (AMX(K), K=1, NTP)
0152      PRINT 117
0153      PRINT 107, (ANX(K), K=1, NTP)
0154      PRINT 117
0155      59 CONTINUE
0156      AMPS=AMPS/NTP**2
0157      ANGSS=ANGS/NTP**2
0158      ANGSSST=0.
0159      AMPSST=0.
0160      DO 80 I=1, NTP
0161      ANGSS=0.
0162      AMPSS=0.
0163      DO 81 J=1, NTP
0164      AMPSS=AMPSS+(AMPT(J,I)-AMPV(I))**2
0165      ANGSS=ANGSS+(ANGL(J,I)-ANGV(I))**2
0166      AMPSST=AMPSST+(AMPT(J,I)-AMPS)**2
0167      81 ANGSSST=ANGSSST+(ANGL(J,I)-ANGS)**2
0168      AMPRMS(I)=SQRT(AMPSS /NTP)
0169      ANGRMS(I)=SQRT(ANGSS /NTP)
0170      80 CONTINUE
0171      PRINT 107, (AMPRMS(K), K=1, NTP)
0172      PRINT 117
0173      PRINT 107, (ANGRMS(K), K=1, NTP)
0174      AMPSS=SQRT(AMPSST/NTP**2)
0175      ANGSS=SQRT(ANGSSST/NTP**2)
0176      PRINT 117
0177      PRINT 107, AMPS, ANGSS, AMPX, ANGX, AMPSS, ANGSS
0178      7 IF(LLL.GT.0)RETURN
0179      IF(LPRINT .NE.0)RETURN
0180      PRINT 111
0181      111 FORMAT(1H1)
0182      L3=K6/2
0183      6 DO 10 L=1, L3
0184      GO TO (11, 12, 13)L
0185      11 PRINT 112
0186      112 FORMAT(/, 20X, "IDEAL CASE", /)
0187      GO TO 14
0188      12 PRINT 113
0189      113 FORMAT(/, 20X, "ACTUAL CASE", /)
0190      GO TO 14
0191      13 PRINT 114
0192      114 FORMAT(/, 20X, "DIFFERENCE", /)
0193      14 DO 30 I=1, NTP
0194      DO 30 J=1, NTP
0195      SR=CMPLX(0., 0.)

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0196      DO 40 K=1,NTP
0197      GO TO (41,42,43)L
0198  41    SR=SR+TRFF(I,K)*CONJG(TRFF2(J,K))
0199      GO TO 40
0200  42    SR=SR+TR(I,K)*CONJG(TRFF2(J,K))
0201      GO TO 40
0202  43    SR=SR+TRFB(I,K)*CONJG(TRFF2(J,K))
0203  40    CONTINUE
0204      ANGL(I,J)=CANG(SR)*RAC
0205  30    AMPT(I,J)=CABS(SR)
0206      DO 20 K=1,2
0207      GO TO (21,22)K
0208  21    PRINT 115
0209  115   FORMAT(20X,"AMPLITUDE",/)
0210      GO TO 23
0211  22    PRINT 116
0212  116   FORMAT(//,20X,"PHASE ANGLE",/)
0213  23    DO 20 I=1,NTP
0214      GO TO (24,25)K
0215  24    PRINT 107, (AMPT(I,J),J=1,NTP)
0216      GO TO 20
0217  25    PRINT 107, (ANGL(I,J),J=1,NTP)
0218  20    CONTINUE
0219  10    CONTINUE
0220      IF( KC.GT.0)RETURN
0221      IF(A1.LE.0.)RETURN
0222      PRINT 118
0223  118   FORMAT(1H1,10X,"REFLECTION MATRIX IS USED",/)
0224      DO 5 I=1,NTP
0225      DO 5 J=1,NTP
0226  5     TRFF2(I,J)=TRFF(I,J)
0227      KC=KC+1
0228      GO TO 6
0229      END

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0001      SUBROUTINE TRFMIX(NM,NN,NR1,NBP,NBK,MC,PHA,S11,S12,S21,S22)
0002      DIMENSION NBP(16),NBK(16)
0003      DIMENSION MC(NR1,NN),PHA(NR1,NN)
0004      DIMENSION S11(NM,NN),S12(NM,NN),S21(NM,NN),S22(NM,NN)
0005      COMMON/C45/T11(32,32),T12(32,32),T21(32,32),T22(32,32)
0006      COMMON/C46/R11(32,32),R12(32,32),R21(32,32),R22(32,32)
0007      DIMENSION SS11(8,8),SS12(8,8),SS21(8,8),SS22(8,8)
0008      DIMENSION MCT(32)
0009      COMPLEX S11,S12,S21,S22,T11,T12,T21,T22,R11,R12,R21,R22,AR,SS11,
C      CSS12,SS21,SS22
C      FIRST INDEX ROW
C      SECOND INDEX COLUMN
0010      DO 10 I=1,NR1
C      TRANSFER MATRIX IN CONNECTION REGION
0011      DO 11 L=1,NN
0012      LL=MC(I,L)
0013      11 MCT(LL)=L
0014      PRINT 102,(MCT(I,L),L=1,NN)
0015      PRINT 102,(MCT(L),L=1,NN)
0016      PRINT 101,(PHA(I,L),L=1,NN)
0017      102 FORMAT(/,(10X,8Y5))
0018      101 FORMAT(/,(10X,8F10.4))
0019      DO 20 J=1,NN
0020      DO 20 K=1,NN
0021      T11(J,K)=CMPLX(0.,0.)
0022      T12(J,K)=CMPLX(0.,0.)
0023      T21(J,K)=CMPLX(0.,0.)
0024      T22(J,K)=CMPLX(0.,0.)
0025      IF(MCT(J).NE.K)GO TO 20
0026      T11(J,K)=AR(PHA(I,J))
0027      T22(J,K)=CONJG(T11(J,K))
0028      20 CONTINUE
0029      PRINT 100,((T11(M,N),N=1,NN),M=1,NN)
0030      PRINT 100,((T22(M,N),N=1,NN),M=1,NN)
0031      IF(I.GT.1)GO TO 21
0032      DO 22 J=1,NN
0033      DO 22 K=1,NN
0034      R11(J,K)=T11(J,K)
0035      R12(J,K)=CMPLX(0.,0.)
0036      R21(J,K)=CMPLX(0.,0.)
0037      22 R22(J,K)=T22(J,K)
0038      GO TO 23
0039      21 CALL MTXMLT(NM,NN,T11,T12,T21,T22,R11,R12,R21,R22)
0040      23 PRINT 100,((R11(M,N),N=1,NN),M=1,NN)
0041      PRINT 100,((R12(M,N),N=1,NN),M=1,NN)
0042      PRINT 100,((R21(M,N),N=1,NN),M=1,NN)
0043      PRINT 100,((R22(M,N),N=1,NN),M=1,NN)
0044      100 FORMAT(/,(10X,8F10.4))
0045      IF(I.EQ.NR1)GO TO 10
C      TRANSFER MATRIX IN BLOCK REGION

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0046      NP=NBPC(I)
0047      IF(I.LE.1)GO TO 26
0048      IF(NP.EQ.NBPC(I-1))GO TO 27
0049      26 CALL BLK(8,NP,SS11,SS12,SS21,SS22)
C      RESET S MATRIX
0050      DO 24 J=1,NN
0051      DO 24 K=1,NN
0052      24 S11(J,K)=CMPLX(0.,0.)
0053      S12(J,K)=CMPLX(0.,0.)
0054      S21(J,K)=CMPLX(0.,0.)
0055      S22(J,K)=CMPLX(0.,0.)
0056      DO 25 J=1,NN,NP
0057      DO 25 JJ=1,NP
0058      J1=JJ-1
0059      DO 25 KK=1,NP
0060      K1=KK-1
0061      S11(J+J1,J+K1)=SS11(JJ,KK)
0062      S12(J+J1,J+K1)=SS12(JJ,KK)
0063      S21(J+J1,J+K1)=SS21(JJ,KK)
0064      S22(J+J1,J+K1)=SS22(JJ,KK)
0065      25 CONTINUE
0066      PRINT 100,((S11(M,N),N=1,NN),M=1,NN)
0067      PRINT 100,((S12(M,N),N=1,NN),M=1,NN)
0068      PRINT 100,((S21(M,N),N=1,NN),M=1,NN)
0069      PRINT 100,((S22(M,N),N=1,NN),M=1,NN)
C      INVERSE S-MATRIX
0070      CALL INVS1(NM,NN,S12)
0071      CALL STYRF(NM,NN,S11,S12,S21,S22,T11,T12,T21,T22)
0072      PRINT 100,((S11(M,N),N=1,NN),M=1,NN)
0073      PRINT 100,((S12(M,N),N=1,NN),M=1,NN)
0074      PRINT 100,((S21(M,N),N=1,NN),M=1,NN)
0075      PRINT 100,((S22(M,N),N=1,NN),M=1,NN)
0076      27 DO 50 J=1,NN
0077      DO 50 K=1,NN
0078      T11(J,K)=S11(J,K)
0079      T12(J,K)=S12(J,K)
0080      T21(J,K)=S21(J,K)
0081      50 T22(J,K)=S22(J,K)
0082      CALL MTXMLT(NM,NN,T11,T12,T21,T22,R11,R12,R21,R22)
0083      PRINT 100,((R11(M,N),N=1,NN),M=1,NN)
0084      PRINT 100,((R12(M,N),N=1,NN),M=1,NN)
0085      PRINT 100,((R21(M,N),N=1,NN),M=1,NN)
0086      PRINT 100,((R22(M,N),N=1,NN),M=1,NN)
0087      10 CONTINUE
0088      CALL INVS1(NM,NN,R22)
0089      DO 40 J=1,NN
0090      DO 40 K=1,NN
0091      S12(J,K)=R22(J,K)
0092      S21(J,K)=R22(J,K)
0093      S11(J,K)=CMPLX(0.,0.)
0094      S22(J,K)=CMPLX(0.,0.)
0095      DO 40 L=1,NN
0096      S11(J,K)=S11(J,K)-R22(J,L)*R21(L,K)
0097      S22(J,K)=S22(J,K)+R12(J,L)*R22(L,K)
0098      40 CONTINUE
0099      PRINT 100,((S11(M,N),N=1,NN),M=1,NN)
0100      PRINT 100,((S12(M,N),N=1,NN),M=1,NN)
0101      PRINT 100,((S21(M,N),N=1,NN),M=1,NN)
0102      PRINT 100,((S22(M,N),N=1,NN),M=1,NN)
0103      RETURN
0104      END

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0001  SUBROUTINE STRF(NM,NN,NR1,NBP,NBK,MC,PHA,S11,S12,S21,S22)
0002      DIMENSION NBP(16),NBK(16)
0003      DIMENSION MC(NR1,NN),PHA(NR1,NN)
0004      DIMENSION S11(NM,NN),S12(NM,NN),S21(NM,NN),S22(NM,NN)
0005      COMMON/CL5/T11(8,8),T12(8,8),T21(8,8),T22(8,8),R11(8,8),R12(8,8),
C      R21(8,8),R22(8,8),SPACE(7168)
0006      DIMENSION MCT(32)
0007      DIMENSION SS11(2,2),SS12(2,2),SS21(2,2),SS22(2,2)
0008      COMPLEX S11,S12,S21,S22,T11,T12,T21,T22,R11,R12,R21,R22,AR,SS11,
C      SS12,SS21,SS22
0009      CALL TWOPT(SS11,SS12,SS21,SS22,2)
C      1ST INDEX,COLUMN
C      2ND INDEX,ROW
0010      DO 10 I=1,NR1
C      TRANSFER MATRIX IN CONNECTION REGION
0011      DO 11 L=1,NN
0012      LL=MC(I,L)
0013      11  MCT(LL)=L
0014      DO 20 J=1,NN
0015      DO 20 K=1,NN
0016      T11(J,K)=CMPLX(0.,0.)
0017      T12(J,K)=CMPLX(0.,0.)
0018      T21(J,K)=CMPLX(0.,0.)
0019      T22(J,K)=CMPLX(0.,0.)
0020      IF(MCT(J).NE.K)GO TO 20
0021      T11(J,K)=AR(PHA(I,J))
0022      T22(J,K)=CONJG(T11(J,K))
0023      20  CONTINUE
0024      IF(I.GT.1)GO TO 21
0025      DO 22 J=1,NN
0026      DO 22 K=1,NN
0027      R11(J,K)=T11(J,K)
0028      R12(J,K)=T12(J,K)
0029      R21(J,K)=T21(J,K)
0030      R22(J,K)=T22(J,K)
0031      GO TO 23
0032      21  CALL MTXHLT(NM,NN, T11,T12,T21,T22,R11,R12,R21,R22)
0033      IF(I.EQ.NR1)GO TO 10
C      TRANSFER MATRIX IN BLOCK REGION
C      RESET S MATRIX
0034      23  NP=NBP(I)
0035      IF(I.LE.1)GO TO 26
0036      IF(NP.EQ.NBP(I-1))GO TO 27
0037      26  DO 24 J=1,NN
0038      DO 24 K=1,NN
0039      24  S11(J,K)=CMPLX(0.,0.)
0040      S12(J,K)=CMPLX(0.,0.)
0041      S21(J,K)=CMPLX(0.,0.)
0042      S22(J,K)=CMPLX(0.,0.)
0043      DO 25 J=1,NN,NP

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0044      00 25 JJ=1,NP
0045      J1=JJ-1
0046      00 25 KK=1,NP
0047      K1=KK-1
0048      S11(J+J1,J+K1)=SS11(JJ,KK)
0049      S12(J+J1,J+K1)=SS12(JJ,KK)
0050      S21(J+J1,J+K1)=SS21(JJ,KK)
0051      S22(J+J1,J+K1)=SS22(JJ,KK)
0052      25 CONTINUE
C
0053      INVERSE S-MATRIX
0054      CALL INVS2(NM,NN,S12)
0055      27 CALL STTRF(NM,NN,S11,S12,S21,S22,T11,T12,T21,T22)
0056      00 50 J=1,NN
0057      00 50 L=1,NN
0058      T11(J,K)=S11(J,K)
0059      T12(J,K)=S12(J,K)
0060      T21(J,K)=S21(J,K)
0061      50 T22(J,K)=S22(J,K)
0062      CALL MTXMLT(NM,NN,T11,T12,T21,T22,R11,R12,R21,R22)
0063      10 CONTINUE
0064      CALL INVS2(NM,NN,R22)
0065      00 40 J=1,NN
0066      00 40 K=1,NN
0067      S12(J,K)=R22(J,K)
0068      S21(J,K)=R22(J,K)
0069      S11(J,K)=CMPLX(0.,0.)
0070      S22(J,K)=CMPLX(0.,0.)
0071      00 40 L=1,NN
0072      S11(J,K)=S11(J,K)-R22(J,L)*R21(L,K)
0073      S22(J,K)=S22(J,K)+R12(J,L)*R22(L,K)
0074      40 CONTINUE
0075      RETURN
0076      END

0001      SUBROUTINE STTRF(NM,NN,S11,S12,S21,S22,T11,T12,T21,T22)
C
C      THIS SUBROUTINE INVERSES S MATRIX AND STORES IN T
0002      DIMENSION S11(NM,NN),S12(NM,NN),S21(NM,NN),S22(NM,NN)
0003      DIMENSION T11(NM,NN),T12(NM,NN),T21(NM,NN),T22(NM,NN)
0004      COMPLEX S11,S12,S21,S22,T11,T12,T21,T22
0005      00 30 J=1,NN
0006      00 30 K=1,NN
0007      T22(J,K)=S12(J,K)
0008      T21(J,K)=CMPLX(0.,0.)
0009      00 30 L=1,NN
0010      30 T21(J,K)=T21(J,K)-S12(J,L)*S11(L,K)
0011      00 31 J=1,NN
0012      00 31 K=1,NN
0013      T12(J,K)=CMPLX(0.,0.)
0014      T11(J,K)=S21(J,K)
0015      00 31 L=1,NN
0016      T12(J,K)=T12(J,K)+S22(J,L)*S12(L,K)
0017      31 T11(J,K)=T11(J,K)+S22(J,L)*T21(L,K)
0018      00 20 K=1,NN
0019      00 20 J=1,NN
0020      S11(J,K)=T11(J,K)
0021      S12(J,K)=T12(J,K)
0022      S21(J,K)=T21(J,K)
0023      20 S22(J,K)=T22(J,K)
0024      RETURN
0025      END

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SHELTON AND HSIAO

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0001      SUBROUTINE MTXMLT(NM,NN,R11,R12,R21,R22,T11,T12,T21,T22)
C          THIS SUBROUTINE MULTIPLE SUBMATRICES R*T THEN STORE THE RESULT
C          IN R
C          S=R*T
C          S11=R11*T11+R12*T21
C          S12=R11*T12+R12*T22
C          S21=R21*T11+R22*T21
C          S22=R21*T12+R22*T22
0002      DIMENSION TT1(32,32),TT2(32,32)
0003      DIMENSION T11(NM,NN),T12(NM,NN),T21(NM,NN),T22(NM,NN)
0004      DIMENSION R11(NM,NN),R12(NM,NN),R21(NM,NN),R22(NM,NN)
0005      COMPLEX T11,T12,T21,T22,R11,R12,R21,R22,TT1,TT2
0006      PRINT 101
0007      PRINT 100,((R11(M,N),N=1,NN),M=1,NN)
0008      PRINT 100,((R12(M,N),N=1,NN),M=1,NN)
0009      PRINT 100,((R21(M,N),N=1,NN),M=1,NN)
0010      PRINT 100,((R22(M,N),N=1,NN),M=1,NN)
0011      PRINT 100,((T11(M,N),N=1,NN),M=1,NN)
0012      PRINT 100,((T12(M,N),N=1,NN),M=1,NN)
0013      PRINT 100,((T21(M,N),N=1,NN),M=1,NN)
0014      PRINT 100,((T22(M,N),N=1,NN),M=1,NN)
0015      PRINT 101
0016      100  FORMAT(//,(10X,8F10.4))
0017      101  FORMAT(//,'.....',//)
0018      DO 10 J=1,NN
0019      DO 10 K=1,NN
0020      TT1(J,K)=CMPLX(0.,0.)
0021      TT2(J,K)=CMPLX(0.,0.)
0022      DO 10 L=1,NN
0023      TT1(J,K)=TT1(J,K)+R11(J,L)*T11(L,K)+R12(J,L)*T21(L,K)
0024      TT2(J,K)=TT2(J,K)+R11(J,L)*T12(L,K)+R12(J,L)*T22(L,K)
0025      10  CONTINUE
0026      DO 20 J=1,NN
0027      DO 20 K=1,NN
0028      R11(J,K)=TT1(J,K)
0029      R12(J,K)=TT2(J,K)
0030      20  DO 30 J=1,NN
0031      DO 30 K=1,NN
0032      TT1(J,K)=CMPLX(0.,0.)
0033      TT2(J,K)=CMPLX(0.,0.)
0034      DO 30 L=1,NN
0035      TT1(J,K)=TT1(J,K)+R21(J,L)*T11(L,K)+R22(J,L)*T21(L,K)
0036      TT2(J,K)=TT2(J,K)+R21(J,L)*T12(L,K)+R22(J,L)*T22(L,K)
0037      30  CONTINUE
0038      DO 40 J=1,NN
0039      DO 40 K=1,NN
0040      R21(J,K)=TT1(J,K)
0041      R22(J,K)=TT2(J,K)
0042      40  DO 50 J=1,NN
0043      DO 50 K=1,NN
0044      T11(J,K)=R11(J,K)
0045      T12(J,K)=R12(J,K)
0046      T21(J,K)=R21(J,K)
0047      T22(J,K)=R22(J,K)
0048      50  RETURN
0049      END

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0001      SUBROUTINE PATERN (NTP,TRFF,TRFB,KLL,NPAV,NMX)
          C      ABSOLUTE VALUE OF KLL REPRESENTS THE BEAM INDEX WHOSE PATTERN IS
          C      TO BE PLOTTED
          C      KLL=0 NO PLOT
          C      KLL GRATER THAN 0 PLOT PATTERN ONLY
          C      KLL LESS THAN 0 PLOT BOTH PATTERN AND MAIN BEAMS
0002      COMMON/C$1/PLTAY(500)
0003      DIMENSION TRFF(NMX,NMX),TRFB(NMX,NMX)
0004      COMMON/C$5/PEAK(64,100),PMAX(64),PAV(64),PMAV(64),KIND1(64),
          C      KIND2(64),PEAKDB(100),SPACE(1372)
0005      COMMON/C$6/CONTA(4096),SINTA(4096)
0006      COMPLEX TRFF,TRFB,S
0007      Z(X)=10.*ALOG10(X)
0008      PRINT 104
0009      104  FORMAT(1H1)
0010      PI=3.1415926536
0011      ATR=PI/180.
0012      KPLOT=IABS(KLL)
0013      NTP2=NTP/2
          C      PLOT FRAME
0014      YSL=80.
0015      NY=YSL
0016      XSL=180.
0017      NX=XSL
0018      HN=5.
0019      SY=2.
0020      XM=10.
0021      YN=5.
0022      YS=2.
0023      YSM=YS+YM
0024      NTA=20*NTP
0025      NTA1=NTA+1
0026      TAINC=PI/NTA
0027      PNOR=NTP
0028      CALL PHASAN(TAINC,IKA)
0029      KL=1
0030      IF(KLL.LT.0)KL=2
0031      NTA1N=2*NTA+1
0032      DO 1 I=1,NTP
0033      DO 1 J=1,NTP
0034      1  TRFF(I,J)=TRFF(I,J)+TRFB(I,J)
0035      DO 20 IL=1,KL
0036      IF(IL.LT.2)GO TO 25
0037      CALL PLOT(XM+4.,0.,-3)
0038      NTA1N=NTA1
0039      NSIGN=1
0040      25  CALL FRAME(XM,YM,XSL,YSL,SY,HN,NX,NY)
0041      DO 20 K=1,NTP2
0042      KK=0
0043      KFLAG=0

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0044      KMI=1
0045      KPCONT=1
0046      KMIND=1
0047      KMA=0
0048      LEDGE=0
0049      DO 30 IJ=1,NTA1N
0050      IF(IL.GT.1)GO TO 23
0051      NSIGN=-1
0052      IF(IJ.GE.NTA1)NSIGN=1
0053      I=IJ-NTA1+NSIGN
0054      GO TO 24
0055      23 I=IJ
0056      24 II=IABS(I)
0057      II=II-1
0058      PAR=0.
0059      PAI=0.
0060      IF(IL.LT.2)GO TO 21
0061      IF(I.LT.KIND1(K).OR.I.GT.KIND2(K))GO TO 30
0062      IF(I.EQ.KIND1(K))I=-1
0063      21 DO 40 J=1,NTP
0064      S=TRFF(J,K)
0065      JI=(J-1)*II+1
0066      JMOD=MOD(JI,IKA)
0067      IF(JMOD.EQ.0)JMOD=IKA
0068      PAR=CENTA(JMOD)*REAL(S)-SINTA(JMOD)*AIMAG(S)*NSIGN+PAR
0069      PAI=CENTA(JMOD)*AIMAG(S)+SINTA(JMOD)*REAL(S)*NSIGN+PAI
0070      40 CONTINUE
0071      PAT=PAR**2+PAI**2
0072      PAT=PAT/PNCR
0073      IF(IL.EQ.2)GO TO 22
0074      IF(IJ.LE.1)GO TO 31
0075      IF(PAT-PAT1)32,31,33
0076      C EXAMINE IF A MAXIMUM IS PASSED
0077      32 KMI=1
0078      IF(IJ.EQ.2)LEDGE=1
0079      IF(KMA.LE.0)GO TO 31
0080      IF(PAT1.LE.PEAK(K,KPCONT )) GO TO 34
0081      KIND1(K)=KMIND
0082      KFLAG=1
0083      KPCONT =KK+1
0084      34 KK=KK+1
0085      PEAK(K,KK)=PAT1
0086      KMA=0
0087      GO TO 31
0088      C EXAMINE IF A MINIMUM IS PASSED
0089      33 KMA=1
0090      IF(KMI.LE.0)GO TO 31
0091      35 KMIND=I-1
0092      IF(KFLAG.GT.0)KIND2(K)=I-1
0093      KFLAG=0

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0092      KMI=0
0093      31  PAT1=PAT
           C  PLOT PATTERN FOR A GIVEN BEAM
0094      IF(K.NE.KPLOT)GO TO 30
0095      IF(IJ.LT.NTA1)GO TO 30
0096      22  DB=Z(PAT)
0097      Y=(1.+DB/YSL)*YM+SY
0098      IF(Y.GT.YSM)Y=YSM
0099      IF(Y.LT.SY)Y=SY
0100      P=I-1
0101      X=P*XM/NTA
0102      IF(II.EQ.1)GO TO 3
0103      CALL PLOT(X,Y,2)
0104      GO TO 30
0105      3    CALL PLOT(X,Y,3)
0106      30  CONTINUE
0107      IF(IL.GE.2)GO TO 20
0108      IF(KMA.GT.0)GO TO 42
0109      IF(KPCONT .EQ.KK)KIND2(K)=NTA1
0110      IF(KIND2(K).LE.0)KIND2(K)=NTA1
           C  DELETE THE MAIN LOBE
0111      GO TO 43
0112      42  IF(LEGE.LE.0)GO TO 43
0113      KK=KK+1
0114      PEAK(K,KK)=PAT1
0115      DO 44 I=1,KK
0116      44  PEAKDB(I)=10.*ALOG10(PEAK(K,I))
0117      PRINT 102,K
0118      102  FORMAT(/,20X,"BEAM INDEX",I5)
0119      PRINT 101,(PEAKDB(I),I=1,KK)
0120      KK=KK-1
0121      DO 53 L=1,KK
0122      IF(1.LT.KPCONT )GO TO 53
0123      PEAK(K,L)=PEAK(K,L+1)
0124      53  CONTINUE
0125      PMAX(K)=0
0126      PSUM=0.
0127      PRINT 101,(PEAK(K,I),I=1,KK)
0128      101  FORMAT(/,(10X,10E12.4))
0129      DO 50 L=1,KK
0130      IF(PEAK(K,L).GT.PMAX(K))PMAX(K)=PEAK(K,L)
0131      PSUM=PSUM+PEAK(K,L)
0132      50  CONTINUE
0133      PAV(K)=PSUM/KK
0134      PMAX(K)=Z(PMAX(K))
0135      PAV(K)=Z(PAV(K))
0136      PRINT 103,PMAX(K),PAV(K)
0137      103  FORMAT(/,10X,"PEAK",F10.4,5X,"AVERAGE",F10.4)
0138      20  CONTINUE
0139      RETURN
0140      END

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0001      SUBROUTINE HLFMIX(NTP,NR1,NBP,NBK,MC,PHA)
0002      DIMENSION NBP(16),NBK(16)
0003      DIMENSION MC(NR1,NTP),PHA(NR1,NTP)
0004      COMMON/C$3/MCT(64)
0005      DIMENSION ANG(64),ATEMP(64)
0006      NN=NR1/2
0007      LL=(NR1+1)/2-NN
      C      LL=1 NUMBER OF ROWS IS EVEN
      C      LL=0 NUMBER OF ROWS IS ODD
0008      CALL PHASUM(NR1,NTP,NBP,MC,PHA,ANG)
0009      DO 10 I=1,NTP
0010      II=I
0011      DO 11 J=1,NR1
0012      KK=MC(J,II)
0013      IF(J.EQ.NN)KKP=KK
0014      II=KK
0015      11 CONTINUE
      C      FIND THE JOINT POINT THEN STORE IN MCT ARRAY
0016      DO 12 J=1,NN
0017      KKS=MC(J,KK)
0018      12 KK=KKS
0019      MCT(KKP)=KKS
      C      AVERAGE THE PHASE ANGLES FOR SYMMETRICAL MATRIX
0020      DO 13 J=1,NN
0021      JJ=NR1-J+1
0022      IMC=MC(J,I)
0023      AVG=(PHA(J,IMC)+PHA(JJ,I))/2.
0024      13 PHA(J,IMC)=AVG
0025      PHA(JJ,I)=AVG
0026      10 CONTINUE
      C      CORRECT PHASE ANGLE OF THE MIDDLE ROW WHEN THE NUMBER OF ROWS IS
      C      EVEN
0027      IF(LL.LE.0)GO TO 1
0028      N1=NN+1
0029      DO 20 I=1,NTP
0030      II=MC(N1,I)
0031      IN=MC(N1,MCT(I))
0032      ATEMP(II)=PHA(N1,II)
0033      IF(PHA(N1,IN).GT.ATEMP(II))ATEMP(II)=PHA(N1,IN)
0034      20 CONTINUE
      C      CORRECT THE PHASE ANGLE BY ADDING THE SAME EXTRA PHASE TO EACH
      C      PORT IN A BLOCK
0035      NMP=NBP(N1)
0036      NMB=NBK(N1)
0037      DO 21 I=1,NMB
0038      IMB=(I-1)*NMP
0039      AA=0.
0040      DO 22 J=1,NMP
0041      KK=IMB+J
0042      A=ATEMP(KK)-PHA(N1,KK)
0043      IF(A.GT.AA)AA=A
0044      22 CONTINUE
0045      IF(AA.LE.0.) GO TO 21
0046      DO 23 J=1,NMP
0047      KK=IMB+J
0048      23 PHA(N1,KK)=PHA(N1,KK)+AA
0049      21 CONTINUE
0050      RETURN
      C      CORRECT THE PHASE ANGLES FOR THE CASE WHEN THE NUMBER OF ROWS IS
      C      ODD

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0051      1  CALL PHASUM(NR1,NTP,NBP,MC,PHA,ATEMP)
0052      DO 30 I=1,NTP
0053      AA=ANG(I)-ATEMP(I)
0054      JJ=MC (1,I)
0055      30  PHA(1,JJ)=AA
0056      PHA(NR1,I)=AA
0057      RETURN
0058      END

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```

0001      SUBROUTINE PHASUM(NR1,NTP,NBP,MC,PHA,AS)
0002      DIMENSION NBP(16)
0003      DIMENSION MC(NR1,NTP),PHA(NR1,NTP),AS(NTP)
0004      DIMENSION LAP(2,64) ,A(64)
0005      C    SET THE PHASE SHIFT OF THE BOTTOM ROW
0006      NROW=NR1-1
0007      NN=NBP(NROW)
0008      DO 1 J=1,NN
0009      1  LAP(2,J)=J
0010      KK=NN
0011      DO 10 I=1,NROW
0012      II=NROW-I
0013      II=II+1
0014      NN=NBP(II)
0015      IF(II.LE.0)NN=1
0016      DO 12 J=1,NTP
0017      12  LAP(1,J)=LAP(2,J)
0018      A(J)=AS(J)
0019      KN=0
0020      DO 20 L=1,KN
0021      LL=LAP(1,L)
0022      DO 21 N=1,NTP
0023      21  IF(MC(II,N).NE.LL)GO TO 21
0024      JJ=N
0025      GO TO 22
0026      21  CONTINUE
0027      22  NMD=MOD(JJ,NN)
0028      IF(NMD.EQ.0)NMD=NN
0029      DO 30 K=1,NN
0030      IND=JJ-NMD+K
0031      IF(NN.EQ.1)IND=JJ
0032      KN=KN+1
0033      LAP(2,KN)=IND
0034      30  AS(IND)=A(LL)+PHA(II,LL)
0035      20  CONTINUE
0036      KK=KN
0037      10  CONTINUE
0038      RETURN
0039      END

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0001      SUBROUTINE NTWK(NTP,NR1,NBP,NBK,MC,PHA)
C*****THIS SUBROUTINE FINDS THE CONNECTION OF A BUTLER MATRIX OR FFT
C      GIVEN THE NUMBER OF ROWS AND THE NUMBER OF PORTS IN EACH BLOCK IN
C      EACH ROW
C*****COMPILED BY J. K. HSIAO
C*****FIRST VERSION IS COMPILED ON MAY 3,1976
C*****NTP, NUMBER OF TOTAL INPUT PORTS OR SAMPLES
C*****NR0W, NUMBER OF ROWS REQUIRED TO PERFORM THE TRANSFORMATION
C*****NBP, AN ARRAY STORES THE NUMBER OF PORTS IN EACH BLOCK AT EACH
C      ROW. EACH BLOCK IN A ROW HAS THE SAME NUMBER OF PORTS
C*****NBK, AN ARRAY STORES THE NUMBER OF BLOCKS IN EACH ROW.
C*****MC, A TWO DIMENSIONAL ARRAY STORES THE CONNCTIONS OF THE NETWORK.
C      FIRST INDEX OF THE ARRAY REPRESENTS THE NUMBER OF CURRENT ROW. THE
C      LOCATION OF THE SECOND INDEX REPRESENTS THE PHYSICAL LOCATION OF
C      THE PREVIOUS ROW WHILE THE CONTENTS OF IT IS THE CONNECTION TO THE
C      CURRENT ROW
0002      DIMENSION MC(NR1,NTP),PHA(NR1,NTP)
0003      DIMENSION NPTS(64),NBK(16),NBP(16)
C      COMPUTES THE NUMBER OF PORTS IN EACH BLOCK
0004      NR0W=NR1-1
0005      PI=3.1415926536
0006      PI2=PI*2.
0007      NBP(NR1)=1
0008      NTP2=NTP/2
0009      DO 10 I=1,NR1
0010      10  NBK(I)=NTP/NBP(I)
C***** NPTS ARRAY STORES THE LOCATION OF THE SAMPLES IN EACH BEAM(CUR
C      FREQUENCY SAMPLE). THE STRUCTURE IS CHARACTERIZED BY TWO NUMBERS,
C      NTS,NUMBER OF TIME SAMPLES(OR INPUT PORTS) AND NFS, NUMBER OF
C      FREQUENCY SAMPLES(OR NUMBER OF BEAMS). FOR EXAMPLE, NPTS((3-1)*
C      NTS+1) IS THE PHYSICAL LOCATION OF THE FIRST TIME SAMPLE IN THE
C      THIRD FREQUENCY GROUP( OR OF THE THIRD BEAM),THIS IS REPRESENTED
C      BY LMC
C
C      SET THE INITIAL NPTS ARRAY
0011      DO 11 I=1,NTP
0012      11  NPTS(I)=I
C***** NTS1 IS THE PREVIOUS VALUES OF THE NUMBER OF TIME SAMPLES(OR INPUT
C      PORTS)
C***** NTS2 IS THE CURRENT VALUE
C***** NFS1 IS THE PREVIOUS VALUE OF THE NUMBER OF FREQUENCY SAMPLES(OR
C      BEAMS)
C*****NFS2 IS THE CURRFNT VALUE
C
C
C      SET THE INIAL VALUES OF NTS AND NFS
0013      NTS1=NTP
0014      NFS1=1
0015      DO 20 I=1,NR1
C      MM THE NUMBER OF BLOCKS OF THE CURRENT ROW

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0016 C NN, THE NUMBER OF PORTS IN EACH BLOCK OF THE CURRENT ROW
0017 MM=N8K(I)
NN=N8P(I)
0018 C SET NTS2 AND NFS2
0019 NTS2=NTS1/NN
NFS2=NTP/NTS2
C*** THE ACTUAL REQUIRED PHASE GRADIENT BETWEEN SUCCESSIVE ELEMENT FOR
C THE FIRST BEAM IS
0020 PAG=PI/NFS2
C*** THAVAILABLE PHASE GRADIENT FOR THE FIRST BEAM IN EACH BLOCK IS
0021 PSG=PI/NN
0022 KK=0
0023 DO 30 J=1,MM
0024 MODJ=MOD(J,NFS1)
0025 IF(MODJ.EQ.0)MODJ=NFS1
0026 JJ=(J-1)/NFS1+1
0027 PAGG=PAG*(MODJ+2-1)
0028 PASGD=PSG-PAGG
0029 DO 30 K=1,NN
0030 K1=K-1
0031 KK=KK+1
0032 LMC=(MODJ-1)*NTS1+(K-1)*NTS2+JJ
0033 MCLOC=NFTS(LMC)
0034 MC(I,MCLOC)=KK
0035 IF(KK.LE.NTP2)GO TO 31
0036 KKI=NTP-KK+1
0037 PHA(I,KK)=PHA(I,KKI)
0038 GO TO 30
0039 31 IF(PASGD.GT.0.)GO TO 32
0040 PHA(I,KK)=ABS(PASGD)*(NN-K)
0041 GO TO 30
0042 32 PHA(I,KK)=PASGD*K1
0043 30 CONTINUE
C RECORDING THE FREQUENCY SAMPLE OR BEAM POSITION INTO NFTS ARRAY
0044 NTS1=NTS2
0045 NFS1=NFS2
0046 KK=0
C MNS IS THE NUMBER OF BLOCKS WITHIN EACH GROUP OF FREQUENCY SAMPLES
0047 MNS=MM/NTS1
0048 DO 40 J=1,NFS1
0049 JM0D=MOD(J,MNS)
0050 IF(JM0D.EQ.0)JM0D=MNS
0051 JJ=(J-1)/MNS+1
0052 DO 40 K=1,NTS1
0053 KK=KK+1
0054 40 NFTS(KK)=(K-1)*NFS1+(JM0D-1)*NN+JJ
0055 20 CONTINUE
0056 RETURN
0057 END

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0001      SUBROUTINE FRAME(XM,YM,XSL,YSL,SY,HN,NX,NY)
0002      COMMON/C81/PLTAY(500)
0003      YMSY=YM+SY
0004      HLAB=HN*.035
0005      HLAS=HLAB+.035
0006      WLAB=4.*HLAB/7.
0007      XSCL=XSL/NX
0008      YSCL=YSL/NY
0009      DY=YM/NY
0010      Y=SY
0011      NNY=NY+1
0012      CALL PLOT(0.,SY,3)
0013      CALL PLOT(XM,SY,2)
0014      CALL PLOT(XM,YMSY,2)
0015      CALL PLOT(0.,YMSY,2)
0016      CALL PLOT(0.,SY,2)
0017      DO 10 I=1,2
0018      Y=SY
0019      IF(I.GT.1)GO TO 12
0020      X1=0.
0021      X2=-.2
0022      X3=-.1
0023      GO TO 13
0024      12  X1=XM
0025          X2=XM+.2
0026          X3=XM+.1
0027      13  DO 10 J=1,NNY
0028          CALL PLOT(X1,Y,3)
0029          MODY=MOD(J-1,10)
0030          IF(MODY.NE.0)GO TO 11
0031          CALL PLOT(X2,Y,2)
0032          IF(I.GT.1)GO TO 10
0033          A=YSCL*(J-1-NY)
0034          CALL NUMBER(-6.5*WLAB,Y-HLAB/2.,HLAB,A,0.,4HF3.0)
0035          GO TO 10
0036      11  CALL PLOT(X3,Y,2)
0037      10  Y=Y+DY
0038          DX=X/NX
0039          NXX=NX+1
0040          DO 20 I=1,2
0041          X=0.
0042          IF(I.GT.1)GO TO 22
0043          Y1=SY
0044          Y2=Y1-.2
0045          Y3=Y1-.1
0046          GO TO 23
0047      22  Y1=YMSY
0048          Y2=Y1+.2
0049          Y3=Y1+.1
0050      23  DO 20 K=1,NXX
0051          KK=K-1
0052          CALL PLOT(X,Y1,3)
0053          MODX=MOD(KK,10)
0054          IF(MODX.NE.0)GO TO 21
0055          CALL PLOT(X,Y2,2)
0056          IF(I.GT.1)GO TO 20
0057          A=KK*XSCL
0058          CALL NUMBER(X-2.5*WLAB,SY-HLAB*3.0,HLAB,A,0.,4HF3.0)
0059          GO TO 20
0060      21  CALL PLOT(X,Y3,2)
0061      20  X=X+DX

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0062      CALL SYMBOL(.5+XM-17.5*WLAB,-5.*HLAB+SY,HLAS,22HPARAMETER U IN DEG
        CREFS,0.,22)
0063      35 CALL SYMBOL(-7.*WLAB,YM/2.+SY-15.*WLAB,HLAS,18HARRAY PATTERN (DB),
        *90.,18)
0064      32 CALL PLOT(0.,0.,3)
0065      END

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0001      SUBROUTINE SIMCX(IS,ORIG,NN,MAT,MCT,ANS,LK)
        C      IDENT NUMBER - F1002R00
        C      TITLE - COMPLEX MATRIX INVERSION, SOLUTION OF LINEAR EQUATIONS
        C      IDENT NAME - F1-NRL-SIMCX
        C      LANGUAGE - FORTRAN
        C      COMPUTER - CDC-3800
        C      CONTRIBUTOR - JANET P. MASON, CODE 7813, RESEARCH COMPUTATION
        C                      CENTER, HIS DIVISION
        C      ORGANIZATION - NRL - NAVAL RESEARCH LABORATORY - WASHINGTON, D.C.
        C                                  20390
        C      DATE - 16 DECEMBER 1970
        C      PURPOSE - TO SOLVE THE COMPLEX MATRIX EQUATION AX=B WHERE A IS A
        C                      SQUARE COEFFICIENT MATRIX AND B IS A MATRIX OF CONSTANT
        C                      VECTORS. THE DETERMINANT AND INVERSE OF A ARE ALSO
        C                      OBTAINED.
0002      COMPLEX SUM, MAT,ORIG,ANS,B0,B2,B4,B6,B8,B10,B11,B13,B15,CC,CC2,B2
0002      EQU,VALENCE(B2,C),(CC,CX),(CC2,CX2)
0004      DIMENSION MAT(MCT,1),ORIG(NN,1),ANS(MCT),C(2),CX(2),CX2(2)
0005      10 FJRMAT(1X, 2E12.6)
0006      15 FORMAT(25H THIS MATRIX IS SINGULAR/)
0007      18 FORMAT(1H0," VALUE OF DETERMINANT IS ",2E12.6,/)
0008      21 FORMAT(1X,2E12.6,5X,2E12.6)
0009      23 FORMAT(8X,"ORIGINAL CONSTANTS",21X,"DERIVED CONSTANTS"/)
0010      26 FORMAT(1H1,6X,"THE INVERSE (BY COLUMNS)")
0011      27 FORMAT(1H0)
0012      28 FORMAT(1H1,6X,"VALUES OF THE UNKNOWNNS")
0013      35 FORMAT(9X,"IDENTITY MATRIX")
0014      B3=(-1.0,0.0)
0015      B4=(0.0,0.0)
0016      B11=(1.0,0.0)
0017      ICT=MCT
0018      JSING=MCT
0019      MT=MCT+1
0020      MCT=MCT+MCT
        C      PUT ORIGINAL MATRIX INTO MAT
0021      IF(CIS.EQ.0)GO TO 39
0022      ICT=MT+15
0023      MCT=ICT
0024      39 DO 2 J=1,ICT
0025      DO 2 I=1,MCT
0026      MAT(I,J)=ORIG(I,J)
0027      2 CONTINUE
0028      IF(CIS.NE.0)GO TO 30
        C      PUT IDENTITY MATRIX INTO RIGHT HALF OF MAT
0029      31 DO 32 J=MT,MCT
0030      DO 32 I=1,MCT
0031      MAT(I,J)=B4
0032      DO 33 J=1,MCT
0033      33 MAT(J,J+MCT)=B3
        C      FORM TRIANGULARIZED MATRIX

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0034      30 JCT=MCT-1
0035      DO 3 J=1,JCT
0036      KK=J+1
0037      GOTO 25
0038      24 DO 4 K=KK,MCT
0039      BB=MAT(K,J)/MAT(J,J)
0040      DO 5 L=J,MCT
0041      B10=BB*MAT(J,L)
0042      5 MAT(K,L)=MAT(K,L)-B10
0043      4 CONTINUE
C      VALUE OF DETERMINANT
0044      3 B11=B11+MAT(J,J)
0045      B11=B11+MAT(MCT,MCT)
0046      LOW=-MCT
0047      M0=-1
C      TO DO ONE OR MORE BACK SOLUTIONS
0048      DO 6 MINC=MT,MCT
0049      JFIN=MCT
0050      IX=0
C      BACK SOLUTION
0051      DO 6 INN=LOW,M0
0052      M=IABS(INN)
0053      B0=-MAT(M,MINC)
0054      B2=MAT(M,M)
0055      B4=(0.0,0.0)
0056      IF(IX) 7,22,7
0057      22 IX=IX+1
0058      GOTO 8
0059      7 M02=-JFIN
0060      DO 9 INN=LOW,M02
0061      N=IABS(INN)
0062      9 B4=B4+MAT(M,N)*MAT(N,MINC)
0063      B0=B0-B4
0064      JFIN=JFIN-1
0065      8 IF(C(1).EQ.0.0.AND.C(2).EQ.0.0)GO TO 13
0066      29 MAT(M,MINC)=B0/B2
0067      ANS(M)=B0/B2
0068      6 CONTINUE
0069      DO 40 J=MT,MCT
0070      JJ=J-MCT
0071      DO 40 I=1,MCT
0072      40 ORIG(I,JJ)=MAT(I,J)
0073      IF(LK.GT.0)RETURN
0074      IF(IS.EQ.0)GO TO 34
0075      GO TO 41
C      CHECK FOR SINGULARITY AND TO SEE IF FIRST TERM = 0
0076      25 JV=J
0077      CC=MAT(J,J)
0078      IF(CC(1).NE.0.0.OR. CC(2).NE.0.0)GO TO 12
0079      11 IF(JV.NE.JSING)GO TO 14

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```

0080      13 PRINT 15
0081      PRINT 100, J, (MAT(K, J), K=1, MCT)
0082      100 FORMAT(10X, I5, 8F10.4)
0083      PRINT 21, ((MAT(I, J), I=1, MCT), J=1, MCT)
0084      RETURN
0085      14 JV=JV+1
0086      CC2=MAT(JV, J)
0087      IF(CX2(1).EQ.0.0.AND.CX2(2).EQ.0.0)GO TO 11
0088      16 DO 17 JJ=J, MCT
0089      B6=MAT(J, JJ)
0090      MAT(J, JJ)=MAT(JV, JJ)
0091      17 MAT(JV, JJ)=B6
0092      B11=-B11
0093      12 CONTINUE
0094      GOTO 24
C      PRINT SUBSTITUTIONS BACK INTO ORIGINAL MATRIX
0095      45 DO 20 NNV=1, IS
0096      PRINT 27
0097      44 PRINT 23
0098      DO 20 LL=1, MCT
0099      B13=(0.0, 0.0)
0100      DO 19 MM=1, MCT
0101      19 B13=ORIG(LL, MM)*MAT(MM, MCT+NNV)+B13
0102      B15=-ORIG(LL, MCT+NNV)
0103      PRINT 21, B15, B13
0104      20 CONTINUE
0105      RETURN
C      PRINT TITLE - THE INVERSE
0106      34 PRINT 26
0107      GO TO 43
C      PRINT TITLE - VALUES OF UNKNOWNNS
0108      41 PRINT 28
0109      43 DO 38 JJ=MT, MCT
0110      PRINT 27
0111      DO 38 II=1, MCT
0112      38 PRINT 10, MAT(II, JJ)
C      PRINT VALUE OF DETERMINANT
0113      PRINT 10, B11
0114      IF(IS.NE.0)GO TO 45
C      PRINT IDENTITY MATRIX
0115      PRINT 35
0116      DO 36 K=1, MCT
0117      PRINT 27
0118      DO 36 I=1, MCT
0119      SUM=(0.0, 0.0)
0120      DO 37 J=1, MCT
0121      37 SUM=ORIG(K, J)*MAT(J, MCT+I)+SUM
0122      36 PRINT 10, SUM
0123      RETURN
0124      END

```

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```

0001 SUBROUTINE BLK(NM,NN,S11,S12,S21,S22)
0002 DIMENSION NBP(16),NBK(16)
0003 DIMENSION MC(8,16),PHA(8,16)
0004 DIMENSION S11(NM,NM),S12(NM,NM),S21(NM,NM),S22(NM,NM)
0005 COMMON/C$5/T11(8,8),T12(8,8),T21(8,8),T22(8,8),R11(8,8),R12(8,8),
C R21(8,8),R22(8,8),SPACE(7168)
0006 COMPLEX S11,S12,S21,S22
0007 COMPLEX T11,T12,T21,T22,R11,R12,R21,R22
0008 IF(NN.GT.2)GO TO 1
0009 CALL TWOPT(S11,S12,S21,S22,NM)
0010 RETURN
0011 1 II=0
0012 N2=NN
0013 3 N2=N2/2
0014 IF(N2.LE.0)GO TO 2
0015 II=II+1
0016 GO TO 3
0017 2 DO 10 I=1,II
0018 10 NBP(I)=2
0019 II=II+1
0020 CALL NTWK(NM,II,NBP,NBK,MC,PHA)
0021 CALL STRF(NM,NN,II,NBP,NBK,MC,PHA,S11,S12,S21,S22)
0022 RETURN
0023 END

```

```

0001 SUBROUTINE INVS1(NM,NN,S12)
0002 COMMON/C$5/A1(32),T(32,64),SPACE(4032)
0003 DIMENSION S12(NM,NM)
0004 COMPLEX A1,T,S12
0005 CALL SIMCX(0,S12,NM,T,NN,A1,1)
0006 RETURN
0007 END

```

```

0001 SUBROUTINE INVS2(NM,NN,S12)
0002 COMMON/C$5/A1(8),T(8,16),SPACE(7920)
0003 DIMENSION S12(NM,NM)
0004 COMPLEX A1,T,S12
0005 CALL SIMCX(0,S12,NM,T,NN,A1,1)
0006 RETURN
0007 END

```

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```

0001      SUBROUTINE TWOPT (S11,S12,S21,S22,M)
0002      DIMENSION S11(M,M),S12(M,M),S21(M,M),S22(M,M)
0003      COMPLEX S11,S12,S21,S22
0004      COMMON/C$4/A,B,C,D
0005      BCD=B+C+D
0006      IF(BCD.GT.0.)GO TO 1
0007      AR=10.**(-A*.05)
0008      IF(A.LE.0.)AR=0.
0009      A1=SQRT(.5-AR*AR)
0010      A2=SQRT(.5-AR*AR)
0011      B1=AR
0012      B2=AR
0013      GO TO 2
0014      1      B1=10.**(-A*.05)
0015      IF(A.LE.0.)B1=0.
0016      B2=10.**(-B*.05)
0017      IF(B.LE.0.)B2=0.
0018      A1=10.**(-C*.05)
0019      A2=10.**(-D*.05)
0020      2      S11(1,1)=B1*CMPLX(0.,-1.)
0021      S11(2,2)=B1*CMPLX(0.,-1.)
0022      S22(1,1)=B1*CMPLX(0.,-1.)
0023      S22(2,2)=B1*CMPLX(0.,-1.)
0024      S11(1,2)=B2*CMPLX(-1.,0.)
0025      S11(2,1)=B2*CMPLX(-1.,0.)
0026      S22(1,2)=B2*CMPLX(-1.,0.)
0027      S22(2,1)=B2*CMPLX(-1.,0.)
0028      S12(1,1)=A1*CMPLX(1.,0.)
0029      S12(2,2)=A1*CMPLX(1.,0.)
0030      S21(1,1)=A1*CMPLX(1.,0.)
0031      S21(2,2)=A1*CMPLX(1.,0.)
0032      S12(1,2)=A2*CMPLX(0.,-1.)
0033      S12(2,1)=A2*CMPLX(0.,-1.)
0034      S21(1,2)=A2*CMPLX(0.,-1.)
0035      S21(2,1)=A2*CMPLX(0.,-1.)
0036      RETURN
0037      END

```

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```

0001      SUBROUTINE TRT(NTP,TRFF,LL,MXX)
0002      COMMON/C%6/APMT(32,32),ANGL(32,32),ANGT(32),TRID(32,32),SPACE(4064
*)
0003      DIMENSION TRFF(MXX,MXX)
0004      COMPLEX TRFF,TRID
0005      IF(ALL.GT.0)GO TO 1
0006      DO 10 I=1,NTP
0007      DO 10 J=1,NTP
0008      10  TRID(I,J)=TRFF(I,J)
0009      RETURN
0010      1  DO 20 I=1,NTP
0011      DO 20 J=1,NTP
0012      20  TRFF(I,J)=TRID(I,J)
0013      RETURN
0014      END

```

```

0001      SUBROUTINE TRFIOL(NTP)
0002      COMMON/C%6/APMT(32,32),ANGL(32,32),ANGT(32),TRFF2(32,32), TR(32,32
C),SUMR(2016)
0003      COMPLEX TRFF2,TR
0004      PI=3.1415926536
0005      PI2=PI*2.
0006      RTA=199./PI
0007      A=SQRT(1./NTP)
0008      P=-PI/NTP
0009      DO 10 I=1,NTP
0010      PP=(I-1)*P
0011      PR=P*(I-.5)*2.
0012      DO 10 J=1,NTP
0013      PP=AMOD(PP,PI2)
0014      RE=A*COS(PP)
0015      RI=A*SIN(PP)
0016      TRFF2(I,J)=CMPLX(RE,RI)
0017      AMPT(I,J)=A
0018      ANGL(I,J)=PP*RTA
0019      10  PP=PP+PR
0020      RETURN
0021      END

```

```

0001      SUBROUTINE PHASAM (TAINC,I)
0002      COMMON/C%6/CONTAC(4096),SINTA(4096)
0003      PI=3.1415926536
0004      PI2=PI*2.
0005      TA=0.
0006      I=0
0007      1  I=I+1
0008      CONTAC(I)=COS(TA)
0009      SINTA(I)=SIN(TA)
0010      TA=TA+TAINC
0011      IF(TA.GE.PI2)RETURN
0012      GO TO 1
0013      END

```

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```
0001      COMPLEX FUNCTION AR(AUG)
0002      AMP=1.
0003      AG=AUG
0004      RE=AMP*COS(AG)
0005      RI=AMP*SIN(AG)
0006      AR=CMPLX(RE,RI)
0007      RETURN
0008      END
```

```
0001      FUNCTION CANG(SR)
0002      COMPLEX SR
0003      A1=REAL(SR)
0004      A2=AIMAG(SR)
0005      CANG=ATAN2(A2,A1)
0006      RETURN
0007      END
```