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EVALUATIONS OF CERTAIN TRANSMISSION LINE FORCING FUNCTIONS

Sidney Frankel & Associates
Menlo Park, CA 94025

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June 1979

Final Report

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Air Force Systems Command
Kirtland Air Force Base, NM 87117

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This technical report has been reviewed and is approved for publication.

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This study was undertaken to evaluate the relative importance of certain newly-discovered transmission-line forcing functions, the effect of which was expected to depend on the differences in line currents induced by incident electromagnetic waves. Straightforward calculation of these currents and their effects on typical lines showed that the apparent existence of these terms is due to an unconventional resolution of the currents into independent modes. Alternatively, it was found that the usual distributed series-voltage and shunt-current induced sources are generally necessary and sufficient to characterize

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external excitation of a line along its length. It was concluded that when the transmission-line component of the line currents is defined in the conventional manner, no new forcing terms are apparent.

CONTENTS

SUMMARY

This study was undertaken to evaluate in certain simple cases, newly discovered source terms for the differential equations of multiconductor transmission lines. Quantitative measures for these terms were sought for lines of two- and three-conductor coplanar round wires of arbitrary diameters, and of arbitrary spacings initially assumed large compared to wire diameters.

Detailed analysis for the special cases cited yielded null values for the new source terms. Qualitative arguments were used to demonstrate that these terms are absent in general.

The reason for postulating the existence of these terms is a set of definitions for "antenna current," "transmission-line current," and "termination current" in which the second of these terms differs from the corresponding term as used in conventional transmission line theory.

On the other hand, a definition which specifies antenna-mode current as that component of conductor current which cannot flow through the line terminations is in accord with the conventional formulation of series-voltage and shunt-current sources.

In the case of two-conductor lines of arbitrary cross-section excited by a plane wave at arbitrary incidence, either method gives the correct result for the line terminal current.

It was concluded that the conventional formulation is adequate for predicting line behavior when appropriate definitions of transmission-line current and antenna-mode current are used.

SECTION I

INTRODUCTION, HISTORY

For purposes of this report the history of the study of electromagnetic fields coupling to transmission lines and cables begins with the papers by Schelkunoff (ref. 1) and Schelkunoff and Odarenko (ref. 2) in 1934 and 1937 respectively. These workers were concerned with coupling into the interior of a coaxial cable with solid conducting sheath. Their analyses correctly employed a single coupling force in the form of a distributed voltage induced along the interior of the cable shield. The shielding effectiveness of the outer conductor could be expressed in terms of a single parameter.

Perhaps because of the influence of these papers, workers dealing with braided-shield cables and with open-wire lines continued to assume that a single coupling parameter sufficed to characterize the response to external fields (refs. 3-6).

However, within the last 10 years, several workers came to realize that characterization of external coupling by a single parameter is incomplete except for the solid-shield cable (refs. 7-12). It was shown that, for braided-shield cables with associated openings in the conducting sheath, and for open-wire lines, impressed transverse electric fields could induce a distribution of shunt-current sources along the line.

In addition to these excitation sources, Lee (refs. 11, 12) in 1972 suggested that an additional pair of source terms should generally be added to those described above. The origin of these terms may be exemplified as follows:

Consider Figure 1, which shows an electromagnetic wave ($\bar{E}^i$, $\bar{H}^i$, $\bar{k}$) incident on an infinitely long, two-conductor line consisting of two cylinders of radii a_1, a_2, respectively, and with spacing, D, between conductor axes.

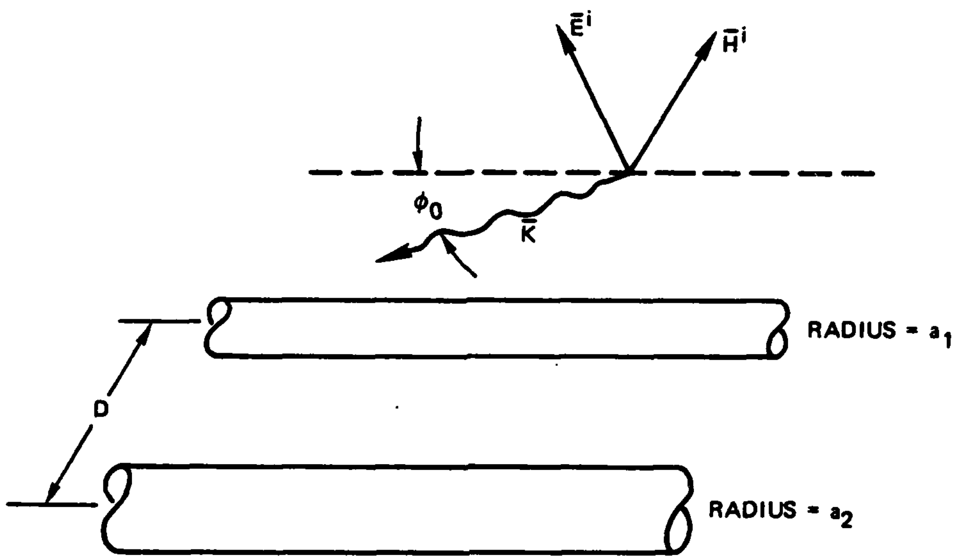

Figure 1. Two-Conductor Transmission Line in Incident Field

The plane of incidence containing $\bar{E}^i$ and $\bar{k}$ is normal to the plane containing the conductor axes. The propagation vector, $\bar{k}$, makes an angle, θ_0, with that plane. The component of $\bar{E}^i$ parallel to the conductors, namely, $E^i \sin \theta_0$, induces currents in the conductors. If the conductor radii are equal, the induced currents are also equal, whence it is clear that the resulting total magnetic flux linking the conductors is zero, and no series distributed electromotive force results. On the other hand, if $a_1 \neq a_2$, then the induced currents are unequal, and the net magnetic flux may be different from zero. Furthermore, with $\theta_0 \neq \pi/2$, the propagation constant of the applied intensity along the line is $\bar{k} \cos \theta_0$; since the induced currents have the same propagation constant, unequal charges are also induced on the conductors, and these may result in a transverse electric field representing a distributed current source.

Shortly after initiating the study, it became evident that these new forcing functions (i.e., those due to unequal-sized conductors) are nonexistent, except possibly for higher-order effects. The remainder of this report details the technical analysis leading to this conclusion.

SECTION II

TECHNICAL DISCUSSION

Two different methods are used to find currents, or current ratios, on the various conductors; from these the effective magnetic flux linking any conductor with a reference conductor may be determined. Thence, the distributed series voltage can be calculated. Charges on the conductors are then readily determined from the equation of continuity

$$dI/dz + j\omega Q = 0 \tag{1}$$

Given the charges, the electric flux between conductors is determined, and, therefore, the distributed shunt-current sources.

The first method is based on the well-known solution for the scattering of an electromagnetic field with electric intensity parallel to the axis of an infinitely long perfectly conducting cylinder (refs. 13, 14). For two or more parallel infinite cylinders, with radii small compared to their spacing, and with maximum spacing much less than a wavelength of the incident radiation, the basic solution is modified by assuming the incident field on any one conductor to be the sum of the primary incident field plus the secondary fields scattered by all the other conductors.

The second method is based on material in a book by Uchida (ref. 15, Chapter 7). It uses electrostatic ideas justified by the small cross section of the lines under study, and by the transverse magnetic nature of propagation along the line, which permits formulation in terms of a transverse electrostatic potential.

The Line Equations

For a multiconductor line consisting of n conductors plus reference conductor (n-line), the differential equations may be written (ref. 16, Sec. 4.5)

$$\left.\begin{aligned} d\underline{V}/dz + j\omega\underline{L}\,\underline{I} &= \underline{E}^e(z) \\ d\underline{I}/dz + j\omega\underline{C}\,\underline{V} &= \underline{H}^e(z) \end{aligned}\right\} \qquad (2)$$

where $\underline{V}$, $\underline{I}$ are the line voltage- and current vectors, respectively:

$$\left.\begin{aligned} \underline{V} &= (V_i) \\ \underline{I} &= (I_i) \end{aligned}\right\} i = 1, \cdots, n$$

$\underline{L}$ and $\underline{C}$ are the line inductance and capacitance matrices

$$\left.\begin{aligned} \underline{L} &= (L_{ij}) \\ \underline{C} &= (C_{ij}) \end{aligned}\right\} i, j = 1, \cdots, n$$

$\underline{E}^e(z)$ and $\underline{H}^e(z)$ are the series distributed voltage and shunt distributed current sources resulting from excitation by external fields along the line. ω is the angular frequency of the impressed signal and z represents distance in the direction of the line axis.

$\underline{E}^e$ and $\underline{H}^e$ may be quite general. It is sufficient that they be integrable in the Riemannian sense. Impulse functions may be included. In fact, normal line-end excitation may be represented by an impulse function at one end of the line, thus

$$\underline{E}^{e}(z) = \underline{V}_{g}\delta(z) \tag{3}$$

where

$$\left.\begin{aligned} &\delta(z) = 0, \ z \neq 0 \\ &\delta(0) = \infty \\ &\int_{-\infty}^{\infty} \delta(z)dz = 1 \end{aligned}\right\} \tag{4}$$

Without inclusion of antenna-mode components, $\underline{E}^{e}(z)$ depends only on the transverse magnetic intensity of the exciting field, while $\underline{H}^{e}(z)$ depends on the transverse electric intensity, thus (ref. 16, Sec. 4.5)

$$\left.\begin{aligned} \underline{E}^{e}(z) &= j\omega\underline{L}^{e}H_{t}^{e} \\ \underline{H}^{e}(z) &= j\omega\underline{C}^{e}E_{t}^{e} \end{aligned}\right. \tag{5}$$

where H_{t}^{e}, E_{t}^{e} are the transverse components of excitation magnetic and electric intensities, respectively, and $\underline{L}^{e}$, $\underline{C}^{e}$ are proportionality vectors which depend on the line transverse configuration, and which may be evaluated by electrostatic techniques (refs. 11; 16, Chapter 10).

In the sequel, Equation 2, with forcing functions represented by Equation 5, will be termed the "standard" -- or "conventional" -- formulation.

For a two-conductor line (n = 1) excited by an incident wave as in Figure 1, Lee formulates the line equations somewhat differently, as follows: If the currents induced on the line conductors by the incident wave are I_1, I_2, respectively, then

$$\left.\begin{aligned} dV/dz + j\omega L I_d &= -j\omega(\bar{h} \times \bar{e}_z) \cdot \bar{B}^{inc} - j\omega L\gamma I_c \\ dI_d/dz + j\omega CV &= -j\omega C\bar{h} \cdot \bar{E}^{inc} + j\omega\gamma Q_c \end{aligned}\right\} \qquad (6)$$

where $\bar{B}^{inc}$ and $\bar{E}^{inc}$ are the incident magnetic flux and electric field intensities, respectively, and

$$\begin{aligned} I_d &= \tfrac{1}{2}(I_2 - I_1) \\ I_c &= \tfrac{1}{2}(I_2 + I_1) \end{aligned} \qquad (7)$$

and where $\bar{e}_z$ is a unit vector in the z-direction and $\bar{h}$ is the vector distance between charge centroids of the two conductors. By Equation 1

$$j\omega Q_c = -dI_c/dz$$

Using this in Equation 6 and rearranging,

$$\left.\begin{aligned} dV/dz + j\omega L(I_d + \gamma I_c) &= -j\omega(\bar{h} \times \bar{e}_z) \cdot \bar{B}^{inc} \\ d/dz(I_d + \gamma I_c) + j\omega CV &= -j\omega C\bar{h} \cdot \bar{E}^{inc} \end{aligned}\right\} \qquad (8)$$

Thus, Lee's equations are the same as conventional Equation 2 for the special case of an incident plane wave on a two-conductor line, provided that the quantity $(I_d + \gamma I_c)$ can be identified as the whole transmission-line component of current in the conventional formulation.

Equation 7 implies

$$I_1 = I_c - I_d$$
$$I_2 = I_c + I_d$$

The two sets of definitions may now be compared quantitatively. The following table summarizes the comparison.

Term	Conventional	Lee
Antenna-mode current	By definition, that component of current that cannot flow through the terminations.	By definition, the antenna-mode current on each conductor is the mean of the total current on the two conductors. By Equation 8, a fraction, γI_c, of this current flows through the terminations.
Transmission-line current	The total current flowing through the terminations; on each conductor, the total current minus the antenna component.	By definition, one-half the difference between the total currents on the two conductors.
Terminal current	Equal to the transmission line current.	Equal to the transmission line current plus the fraction, γI_c, of the antenna current.

The task of this study is to determine whether additional forcing functions, due to differences in currents and charges induced on multiconductor lines by incident fields, are required.

Configurations for Analysis

In general, interest focuses on uniform multiconductor lossless n-lines of conductors with arbitrary cross sections, excited by a plane wave at arbitrary incidence. Initially, however, in order to isolate the particular effects of interest, it is desirable to concentrate on a restricted class of configurations and modes of excitation. To begin with, it is assumed that all conductors are infinitely long, of circular cross section small compared to distances among their axes, with the whole configuration much less than a wavelength of excitation in its maximum cross section. Next, in order to eliminate the coupling effects represented by Equation 5, it is necessary and sufficient that the conductor axes lie in one plane, and that the plane containing the wave normal and the E-vector be normal to the axial plane of the conductors, and parallel to the conductors themselves.

Infinitely Long Circular Cylinder

As mentioned at the beginning of Section II, one method of analysis is based on the scattering of an incident field by an infinitely long circular cylinder. The solution to the problem of excitation by a wave with electric intensity parallel to the cylinder axis is well-documented (refs. 13, 14). Consult Figure 2.

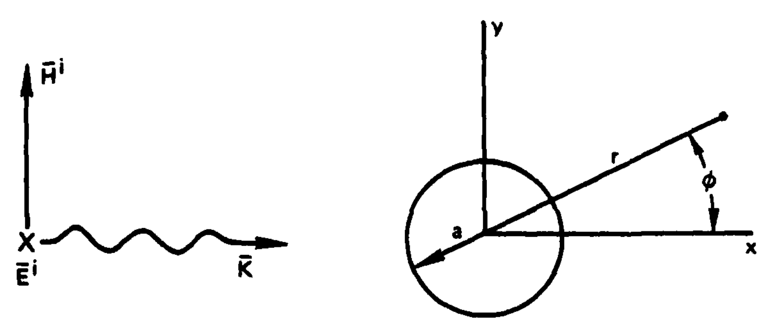

Figure 2. Infinite Circular Cylinder in Broadside Incident Field

The incident field may be written

$$E^i = E_0 \sum_{n=-\infty}^{\infty} j^{-n} J_n(kr)e^{jn\phi} \tag{9}$$

and the scattered field

$$E^s = -E_0 \sum_{n=-\infty}^{\infty} j^{-n} \frac{J_n(ka)}{H_n^{(2)}(ka)} H_n^{(2)}(kr) \cdot e^{jn\phi} \tag{10}$$

where

$J_n(x)$ = Bessel function of the first kind, n<u>th</u> order

$H_n^{(2)}(x)$ = Hänkel function of the second kind, n<u>th</u> order

$k = \omega(\mu\varepsilon)^{\frac{1}{2}}$

$j = (-1)^{\frac{1}{2}}$

For $|x| << 1$, using

$J_0(x) \rightarrow 1$

$J_n(x) \rightarrow (\frac{1}{2}kr)^n/n!$, $n \geqq 1$

$H_0^{(2)}(x) \rightarrow 1 - j \frac{2}{\pi} \ln \frac{\gamma x}{2}$

$H_n^{(2)}(x) \rightarrow j(n-1)!(2/x)^n/\pi$, $n \geqq 1$

where $\gamma = 1.781$, reduces Equations 9 and 10 for $|kr| << 1$, to

$$\begin{aligned} E^i &= E_0\{1 + 2 \sum_{n=1}^{\infty} (-1)^n \frac{j^n}{n!} (\tfrac{1}{2}kr)^n \cos n\phi\} \\ &\simeq E_0(1 - jkr\cos\phi) = E_0(1 - jkx) \end{aligned} \tag{11}$$

and

$$E^s \approx -E_0 \left\{ \frac{1 - j\,\frac{2}{\pi}\ln\frac{\gamma k r}{2}}{1 - j\,\frac{2}{\pi}\ln\frac{\gamma k a}{2}} - jka\cos\phi \right\} \tag{12}$$

The total field is

$$E^t = E^i + E^s$$

$$\approx E_0 \left\{ \frac{j\,\frac{2}{\pi}\ln\frac{r}{a}}{1 - j\,\frac{2}{\pi}\ln\frac{\gamma k a}{2}} - jkr[1 - (a/r)]\cos\phi \right\} \tag{13}$$

Two Infinite Circular Cylinders of Unequal Radii

Consult Figure 3.

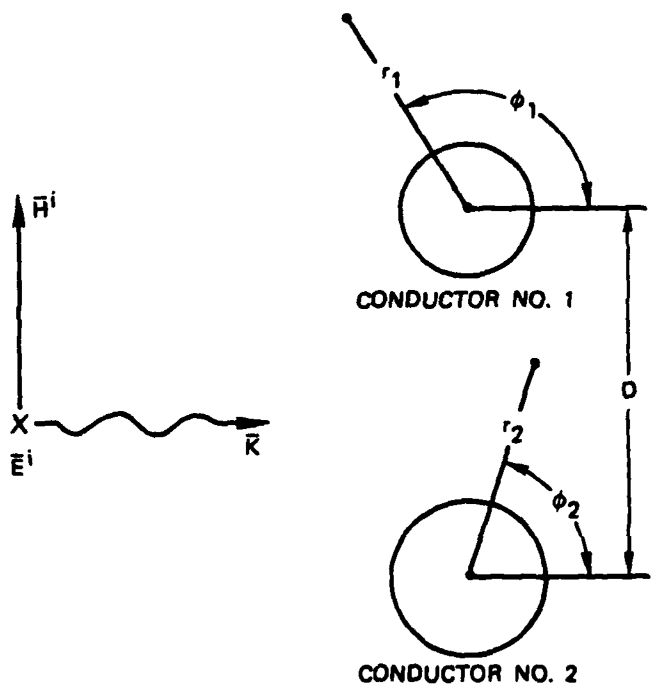

Figure 3. Two Circular Cylinders
in Broadside Incident Field

Write, for the field scattered from conductor No. 1,

$$E_1^s = E_1^s(r_1, \phi_1)$$

and for that scattered from No. 2,

$$E_2^s = E_2^s(r_2, \phi_2)$$

Let E_m^i = total field incident on conductor No. m, m = 1,2. Therefore,

$$\left.\begin{aligned} E_1^i &= E^i + E_2^s(D, \tfrac{1}{2}\pi) = E_0 - F_2 E_2^i \\ E_2^i &= E^i + E_1^s(D, -\tfrac{1}{2}\pi) = E_0 - F_1 E_1^i \end{aligned}\right\} \qquad (14)$$

where, from Equation 12,

$$\left.\begin{aligned} F_1 &= \frac{1 - j\,\frac{2}{\pi}\ln\frac{\gamma k D}{2}}{1 - j\,\frac{2}{\pi}\ln\frac{\gamma k a_1}{2}} \\ F_2 &= \frac{1 - j\,\frac{2}{\pi}\ln\frac{\gamma k D}{2}}{1 - j\,\frac{2}{\pi}\ln\frac{\gamma k a_2}{2}} \end{aligned}\right\} \qquad (15)$$

Solving Equation (14) simultaneously yields

$$\left.\begin{aligned} E_1^i &= (1 - F_2)E_0/K \\ E_2^i &= (1 - F_1)E_0/K \\ K &= 1 - F_1 F_2 \end{aligned}\right\} \qquad (16)$$

and

$$1 - F_m = j\,\frac{2}{\pi}\,\frac{\ln(D/a_m)}{1 - j\,\frac{2}{\pi}\,\ln(\tfrac{1}{2}\gamma k a_m)}\ , \quad m = 1,2 \tag{17}$$

With the knowledge of the effective values of $E_m^i (m = 1,2)$ given by Equation 16, the conductor currents may be calculated.

Conductor Currents

The surface current density on a conductor of radius = a is (ref. 13)

$$J_z = H_\phi \Big|_{r=a} = \frac{1}{j\omega\mu} \cdot \frac{\partial E_t}{\partial r}\Big|_{r=a}$$

From Equation 13,

$$\frac{\partial E_t}{\partial r}\Big|_{r=a} = E_0 \left\{ \frac{j\,\frac{2}{\pi a}}{1 - j\,\frac{2}{\pi}\,\ln\frac{\gamma k a}{2}} + jk\cos\phi \right\}$$

so that

$$J_z = \frac{E_0}{\omega\mu} \left\{ \frac{\frac{2}{\pi a}}{1 - j\,\frac{2}{\pi}\,\ln\frac{\gamma k a}{2}} + k\cos\phi \right\}$$

and the total current is

$$I = \int_0^{2\pi} J_z a\,d\phi = \frac{E_0}{\omega\mu} \left\{ \frac{4}{1 - j\,\frac{2}{\pi}\,\ln(\tfrac{1}{2}\gamma k a)} \right\}$$

Thus, for the two-conductor line,

$$I_m = \frac{4E_m^i}{\omega\mu} \frac{1}{1 - j\,\frac{2}{\pi}\,\ln(\frac{1}{2}\gamma k a_m)}\ , \quad m = 1,2 \tag{18}$$

with $E_m^i (m = 1,2)$ given by Equation 16.

Net Magnetic Flux Between Conductors

Consult Figure 4. The magnetic intensity is everywhere normal to the y-axis at x = 0. We have

$$H_x \Big|_{x=0} = (1/2\pi)[I_2/(\tfrac{1}{2}D+y) - I_1/(\tfrac{1}{2}D-y)]$$

and the total flux per meter of line between conductors is

$$\phi_m = \mu \int_{-\frac{D}{2}+a_2}^{\frac{D}{2}-a_1} H_x dy$$

$$\simeq (\mu/2\pi)[I_2 \ln(D/a_2) - I_1 \ln(d/a_1)] \tag{19}$$

for a_1, a_2 << D.

Then, from Equation 18,

$$\phi_m = (2/\pi\omega) \left\{ \frac{\ln(D/a_2)}{1 - j\,\frac{2}{\pi}\,\ln(\frac{1}{2}\gamma k a_2)} E_2^i - \frac{\ln(D/a_1)}{1 - j\,\frac{2}{\pi}\,\ln(\frac{1}{2}\gamma k a_1)} E_1^i \right\}$$

whence, upon substitution for E_1^i, E_2^i from Equations 16 and 17, one obtains

$$\phi_m = 0 \tag{20}$$

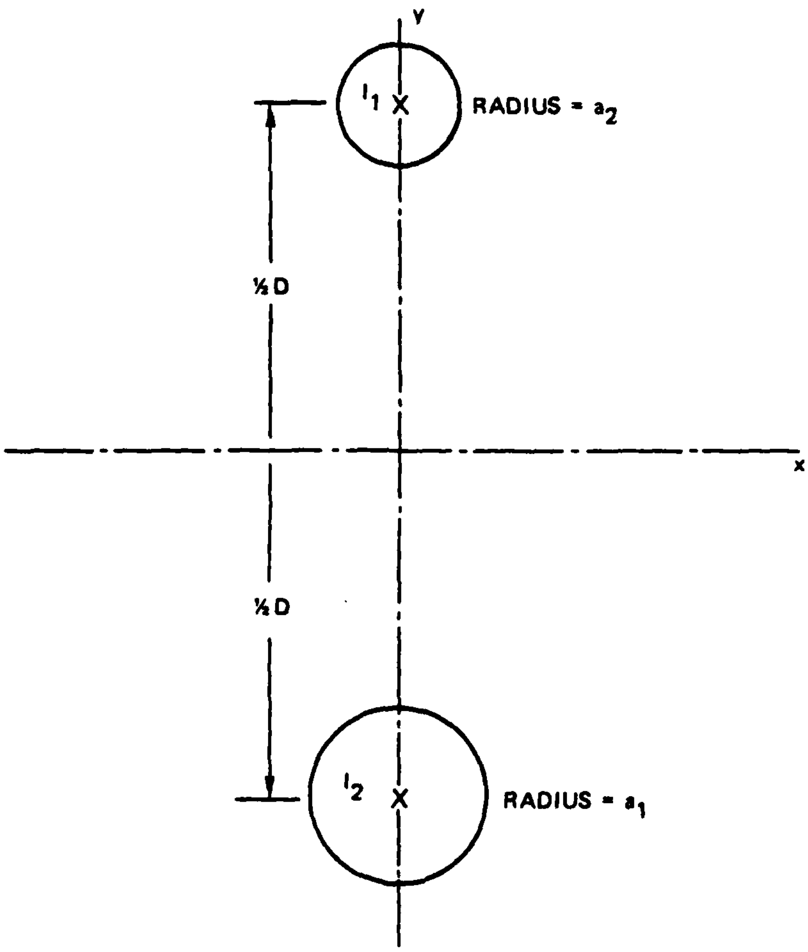

Figure 4. Two Current-Carrying Cylinders

The conclusion is that no transverse magnetic flux is added by the antenna-current component, and, therefore, no net series voltage.

At this point it is of interest to note the relative values of I_1 and I_2. From Equations 16, 17, and 18,

$$I_2/I_1 = \ln(D/a_1)/\ln(D/a_2) \qquad (21)$$

$$= Q_2/Q_1 \qquad (22)$$

by Equation 1.

To calculate the shunt-current source distribution, first find the potential difference due to Q_1, Q_2. By standard electrostatics,

$$\left.\begin{aligned} V_1 &= p_{11}Q_1 + p_{12}Q_2 \\ V_2 &= p_{12}Q_1 + p_{22}Q_2 \end{aligned}\right\} \qquad (23)$$

where

$$-2\pi\varepsilon\ p_{mm} = \ln a_m,\ m = 1,2$$

$$-2\pi\varepsilon\ p_{12} = \ln D$$

Then

$$V_2 - V_1 = (2\pi\varepsilon)^{-1}[Q_2 \ln(D/a_2) - Q_1 \ln(D/a_1)]$$

$$= 0$$

by Equation 22.

Thus, no shunt-current source is added by the antenna-mode charges.

We conclude that, for the two-conductor infinite line, no added forcing terms are required in the line differential equations by virtue of the presence of antenna-mode currents and charges.

Note that for the particular case of broadside incidence discussed, the line charges are zero, since

$$dI/dz = 0$$

However, if the E-vector is at an angle to the line axis (with the plane of incidence still normal to the plane of the conductor axes), then the charges are different from zero and Equation 22 holds.

Three Circular Cylinders

An attempt was made to investigate the three-coplanar-conductor case with arbitrary radii and spacings, but the algebra proved excessively burdensome. An alternate approach was located in Uchida's Chapter 7 (ref. 15), which contains a good deal of the information sought. In particular, Equation 21 was confirmed for the two-conductor case. To illustrate the method using the two-conductor case one starts with Equation 23, but with Q_m replaced by I_m, and p_{mn} replaced by

$$z_{mn} = p_{mn}(\mu\varepsilon)^{-\frac{1}{2}}$$

(ref. 16, Sec. 2.2.2).

Thus

$$\left.\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{12}I_1 + z_{22}I_2 \end{aligned}\right\} \quad (24)$$

where

$$\left.\begin{aligned} -2\pi(\varepsilon/\mu)^{\frac{1}{2}} z_{mm} &= \ln a_m, \ m = 1,2 \\ -2\pi(\varepsilon/\mu)^{\frac{1}{2}} z_{12} &= \ln D \end{aligned}\right\} \quad (25)$$

For axial incident electric intensity in a plane normal to the conductors, there is no incident transverse electric intensity, and no induced transverse electric intensity due to magnetic linkage. Consequently, there is no potential difference between the conductors, so that $V_1 = V_2$. Thus,

$$z_{11}I_1 + z_{12}I_2 = z_{12}I_1 + z_{22}I_2$$

or

$$I_2/I_1 = (z_{11} - z_{12})/(z_{22} - z_{12}) = \ln(D/a_1)/\ln(D/a_2)$$

as in Equation 21.

Turning now to the three-conductor bisymmetric line of Figure 5, where the conductors are equally spaced at distance D; and where the two outside conductors have equal radii, a_1,

$$I_2/I_1 = I_2/I_3 = \ln(D/2a_1)/\ln(D/a_2) \quad (26)$$

(ref. 15, Chapter 7).

Refer to Figure 6 to compute the magnetic flux linking conductors 2 and 3. By symmetry, $I_3 = I_1$. On the x-axis, in

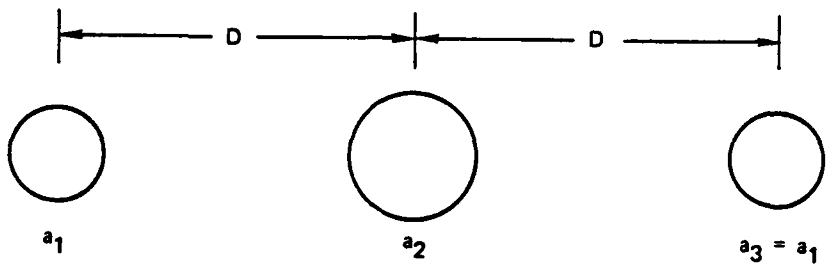

Figure 5. Three Circular Cylinders

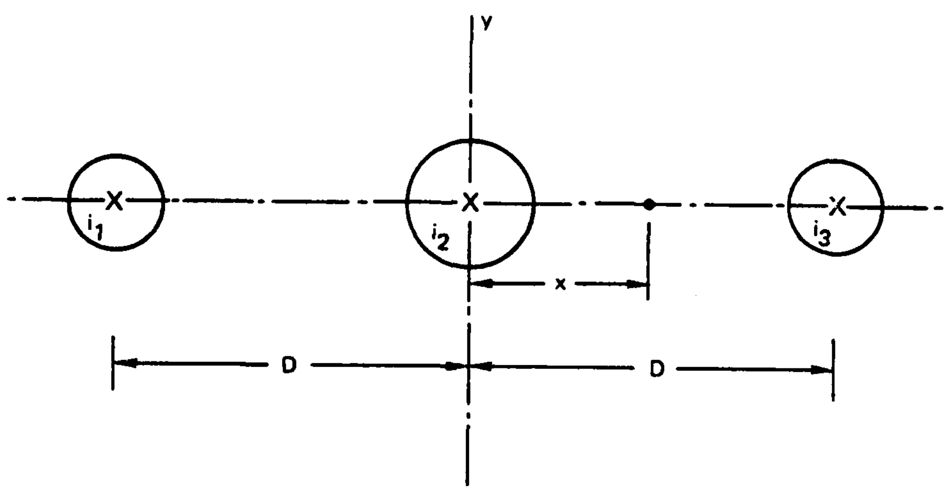

Figure 6. Three Current-Carrying Circular Cylinders

the interval $a_2 \leqq x \leqq a_3$, the magnetic intensity is in the y-direction and given by

$$H_y = (2\pi)^{-1}[I_1/(x + D) + I_2/x - I_3/(D - x)]$$

and the magnetic flux linkage by

$$\begin{aligned}\phi_m &= \mu \int_{a_2}^{D-a_3} H_y dx \\ &\doteq (\mu/2\pi)[I_2 \ln(D/a_2) - I_1 \ln(D/2a_1)] \\ &= 0\end{aligned}$$

with the help of Equation 26.

A somewhat laborious calculation shows the same result for three coplanar circular cylinders of arbitrary (small) radii and arbitrary spacings (Appendix A).

Discussion of Preceding Results

Generalization to Arbitrary Coplanar Arrays

In the absence of transverse electric and linking magnetic incident fields, it has been shown that, for two- and three-conductor coplanar arrays of circular cross section conductors, no new magnetic or electric coupling results from the presence of conductors of unequal radii carrying unequal antenna-mode currents. Rather, the inequalities in radii and currents appear exactly to cancel each other.

That this result is generally true for any number of arbitrarily spaced coplanar circular conductors should be evident from a physical fact disclosed in those proofs. The fact is that when the incident electric intensity is parallel to the conductor axes, and no applied magnetic intensity links the conductors, no potential difference exists among them in any transverse plane. Consequently, a load placed across any pair of conductors at any transverse plane will not draw any current.

Generally, when the plane of the wave normal and E-vector is normal to the plane of the conductor axes, $dI_i/dz \neq 0$ ($i = 1, \cdots, n$) by virtue of a propagation component along the line different from zero. Therefore charges, Q_i, now exist on the conductors proportional to I_i (Equation 22). Therefore the resulting transverse electric intensity is proportional to the magnetic intensity linking the conductors by virtue of the currents, I_i. Since the E-vector is normal to the H-vector, the line integral of the electric intensity between any pair of conductors is proportional to the integral of the magnetic intensity linking those conductors. However, the latter has been shown to be zero.

Thus we conclude that, for coplanar arrays, no series- or shunt distributed sources are induced in the line when the incident electric vector and wave normal lie in the plane normal to the plane of the conductors.

Generalization to Coplanar Arrays with Arbitrary Field Incidence

Consider an incident field with E-vector at arbitrary incidence. The E-vector may be resolved into a component, $\bar{E}_t$, in the transverse plane, and a component, $\bar{E}_a$, parallel to the that when the incident electric intensity is parallel to the conductor axes, and no applied magnetic intensity links the conductors, no potential difference exists among them in any transverse plane. Consequently, a load placed across any pair of conductors at any transverse plane will not draw any current.

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Generalization to Coplanar Arrays with Arbitrary Field Incidence

Consider an incident field with E-vector at arbitrary incidence. The E-vector may be resolved into a component, $\bar{E}_t$, in the transverse plane, and a component, $\bar{E}_a$, parallel to the conductor axes. $\bar{E}_a$, itself, may be resolved into a fixed part, $\bar{E}_0$, and a variable part, $\bar{E}_v$, representing phase variation of $\bar{E}_a$ between conductors. Thus, the incident E-vector may be written

$$\bar{E}^i = \bar{E}_0 + \bar{E}_v + \bar{E}_t \tag{27}$$

In Equation 27, $\bar{E}_t$ represents that portion of the incident field contributing to the shunt distributed sources, while $\bar{E}_v$, by Faraday's law, measures the magnetic linkages inducing the series distributed voltage sources (ref. 16, Sec. 4.3.1). The remaining component, $\bar{E}_0$, is just the antenna mode component that induces the antenna-mode currents and charges discussed in earlier paragraphs of this report.

We conclude that the transmission-line differential equations are complete as stated in Equation 2 for coplanar arrays.

Generalization to Arbitrary Arrays of Circular Conductors with Arbitrary Field Incidence

The argument is the same as in the preceding section. The impressed electric intensity is resolved as in Equation 27. The remaining discussion is independent of the coplanarity assumed in that section.

Finally, it is clear that a change to conductors of arbitrary cross section does not change the essential argument.

We conclude that Equation 2 is sufficient for predicting the transmission-line response of any uniform multiconductor line to an arbitrary incident field.

Furthermore, the argument used in the foregoing holds for lines of any length (ref. 15).

Determination of antenna-mode currents, as defined here, is a separate problem. The reader is referred to Uchida (ref. 15) for an adequate discussion of this subject.

The total current on each conductor is, of course, the sum of the transmission-line mode and the antenna-mode currents on that conductor.

Reconciliation with Lee's Formulation

To date, Lee's formulation has been applied only to two-conductor lines. For the special case in which the incident E-vector is in a plane of incidence normal to the axial plane of two round conductors, and parallel to the conductors, the conventional solution yields

$$I = I_d + \gamma I_c = 0 \tag{28}$$

which implies

$$\gamma = -I_d/I_c = (1 - (I_2/I_1))/1 + (I_2/I_1)) \tag{29}$$

With the help of Equation 21 this becomes

$$\gamma = \ln(a_1/a_2)/[2 \ln(D/(a_1 a_2)^{\frac{1}{2}})] \tag{30}$$

In his analysis, Lee (ref. 11) gives, as the exact formula for any conductor radii,

$$\gamma = C(2\pi\varepsilon)^{-1}[\operatorname{csch}^{-1}(2a_2/D) - \operatorname{csch}^{-1}(2a_1/D)] \tag{31}$$

where

$$2\pi\varepsilon/C = \cosh^{-1}[(D^2 - a_1^2 - a_2^2)/(2a_1a_2)] \quad (32)$$

For the small wire approximation,

$$\mathrm{csch}^{-1}(2a_m/D) \approx \ln(D/a_m), \; m = 1,2 \quad (33)$$

and

$$2\pi\varepsilon/C \approx \ln(D/(a_1a_2)^{\frac{1}{2}}) \quad (34)$$

Then substitution of Equations 33 and 34 in Equation 31 yields Equation 30.

Thus both formulations lead to the same result.

Which Formulation is Preferable?

It is evident that, by a proper choice of γ, both the conventional and Lee's formulations lead to the same results. This fact has been checked in a number of instances (see the examples in ref. 11). Thus it is reasonable to inquire at this point which formulation is preferable. To answer this question it is desirable to set up a number of objective criteria. The following questions should be examined:

1. Which definition is simpler in concept and implementation?
2. Which definition offers the clearer physical interpretation?
3. Which definition is more useful?

4. Which definition can readily be generalized to multi-conductor lines?

It is likely that addition questions might be posed. Barring that possibility, it is difficult to avoid gravitating toward the conventional formulation. It is certainly simpler in concept since it uses one less pair of forcing terms. It is simpler in implementation since it does not require determination of the special coefficient, γ. It offers a clear and exact picture regarding the origins of the total distributed forcing functions. The definitions of transmission-line current components and antenna-mode current components are physically meaningful.

Additionally, in the conventional definition, transmission-line current and termination current are identical.

A judgment regarding relative usefulness of the two formulations cannot be made at this time.

Finally, while generalization of the standard formulation concepts to multiconductor lines is straightforward, we have been unable to find a corresponding generalization for the Lee formulation.

SECTION III

CONCLUSION

It has been shown that Lee's formulation of the transmission-line differential equations and the conventional formulation give the same results when applied to two-conductor lines. However, since the conventional formulation is simpler to implement, clearer in physical interpretation, and readily generalized to multiconductor lines, we consider it to be the preferable formulation.

SECTION IV

RECOMMENDATIONS

In Section I we stated that the purpose of this study was to obtain preliminary data regarding certain new transmission-line forcing terms. The unexpected result of this study has been the conclusion that if the line current components are defined appropriately no additional forcing terms are required.

APPENDIX A

THREE-CONDUCTOR COPLANAR LINE WITH ARBITRARY (SMALL) RADII AND ARBITRARY SPACINGS

Consult Figure A-1. By previously established principles,

$$\begin{bmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} V \qquad \text{(A-1)}$$

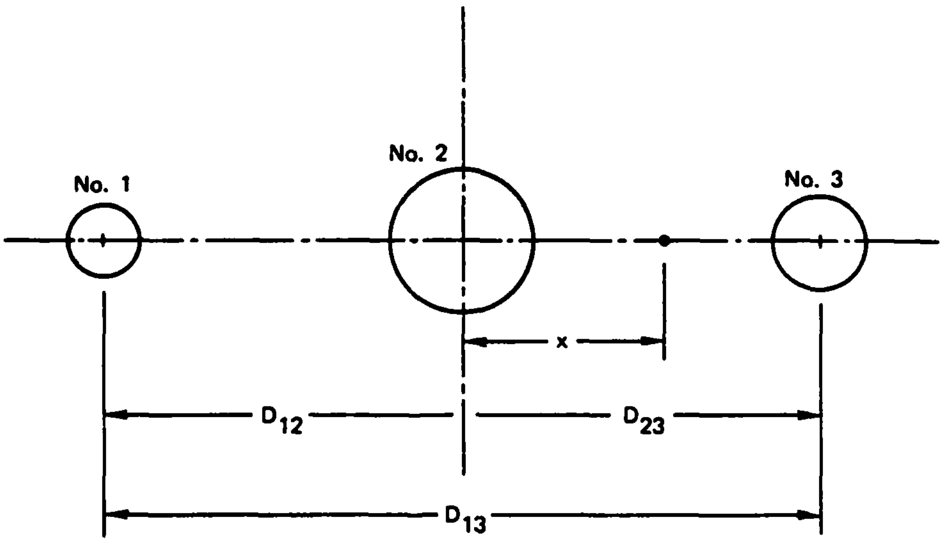

Figure A-1. Three Circular Cylinders of Arbitrary Radii and Spacings where

$$\left.\begin{aligned} z_{mm} &= -K \ln a_m \\ z_{mn} &= -K \ln D_{mn} \\ K &= (2\pi)^{-1}(\mu/\varepsilon)^{1/2} \end{aligned}\right\} m,n = 1,2,3 \tag{A-2}$$

The magnetic flux linking conductors 2 and 3 is

$$\begin{aligned} \phi_m &= (\mu/2\pi)\int_{a_2}^{D-a_3} [I_1/(D_{12}+x) + I_2/x + I_3/(x-D_{23})]dx \\ &\simeq (\mu/2\pi)[I_1 \ln(D_{13}/D_{12}) + I_2 \ln(D_{23}/a_2) + I_3 \ln(D_{23}/a_3)] \end{aligned} \tag{A-3}$$

But (Equation A-2),

$$\left.\begin{aligned} \ln(D_{13}/D_{12}) &= (z_{12} - z_{13})/K \\ \ln(D_{23}/a_2) &= (z_{22} - z_{23})/K \\ \ln(D_{23}/a_3) &= (z_{33} - z_{23})/K \end{aligned}\right\} \tag{A-4}$$

Using Equation A-4 in Equation A-3 yields

$$\phi_m = (\mu/2\pi K)[(z_{12} - z_{13})I_1 + (z_{22} - z_{23})I_2 - (z_{33} - z_{23})I_3] \tag{A-5}$$

The currents I_m $(m = 1,2,3)$ are found by solving all parts of Equation A-1 simultaneously. When these values are substituted for the I_m in Equation A-5, it is found again that $\phi_m = 0$.

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