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DEVELOPMENT AND APPLICATION
OF A SUBSONIC TRIANGULAR
VORTEX PANEL

THESIS

AFIT/GAE/AE/80J-1 John C. Sparks

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1980

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DEVELOPMENT AND APPLICATION
OF A SUBSONIC TRIANGULAR
VORTEX PANEL

THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology

Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

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Graduate Aerospace - Mechanical Engineering

June 1980

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Preface

I would like to take this opportunity to express my thanks to my advisor, Major Stephen J. Koob of the Department of Aeronautics and Astronautics, Air Force Institute of Technology, for his guidance, support, and criticism during the preparation of this thesis. Thanks are also extended to James R. Snyder of ASD/XRHI for his valuable suggestions and to my wife, Carolyn, for putting up with me during the course of this study.

John C. Sparks

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List of Symbols

Symbol

α	. . . angle of attack
$\vec{\omega}$	. . . vorticity vector
$\delta(x, y), \gamma(x, y), 0$	. . . components of $\vec{\omega}$
A, B, C	. . . coefficients of the assumed "bilinear" form for γ
D, E, F	. . . coefficients of the assumed "bilinear" form for δ
x_1, y_1	. . . panel corner points
δ_1, γ_1	. . . corner vorticity values
x, y, z	. . . chordwise, spanwise, and normal coordinates, respectively
Γ	. . . straight line segment
$\vec{s}$	. . . control point in the plane $z = 0$
u, v, w	. . . perturbation velocities in the respective x, y, and z directions
$\vec{i}, \vec{j}, \vec{k}$	. . . unit vectors in the respective x, y, and z directions
R	. . . region of integration
M	. . . number of chordwise panels or slope (Appendix A only)
N	. . . number of spanwise panels
$\wedge$	. . . leading edge slope (Section III)
$[\theta_j]$	. . . column vector of corner vorticities
$[A_{ij}]$	. . . coefficient matrix premultiplying $[\theta_j]$
M_∞	. . . free stream Mach number
V_∞	. . . free stream velocity

List of Symbols

Symbol

 dc/dx

. . . camber slope

 γ

. . . ratio of specific heats (eqs 3.30 and 3.31 only)

 $f_1(x, y)$

. . . integrands obtained through application of the Biot-Savart Law

 F_1

. . . improper integrals of the form

$$\lim_{L \rightarrow \infty} \int_{y_0}^{y_1} \int_0^L \frac{f_1(x, y) dx dy}{(y - b)/M} \quad i = 1, \dots, 5$$

 S_1

. . . strip function

 I_1

. . . integral over a triangular region

 C_P

. . . pressure coefficient

 C_L

. . . local lift coefficient

 C_M

. . . local pitching moment coefficient

Subscripts and Superscripts

L

. . . leading triangle

T

. . . trailing triangle

X

Abstract

Paneling methods are approximate techniques for solving flow problems over wings and bodies. Vortex panels are used to model flow over wings and other lifting surfaces. The author develops a triangular vortex panel having a vorticity distribution that can vary in magnitude and direction. This panel is used to predict the pressure distribution on a rectangular and a sweptback wing in subsonic flow. Lift distributions obtained compare favorably to Anderson's solution and wind tunnel results except near the wing tip. In this region, the distribution will spike before satisfying the Kutta condition imposed at the tip. Possible remedies for the tip problem are discussed.

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DEVELOPMENT AND APPLICATION
OF A SUBSONIC TRIANGULAR
VORTEX PANEL

I. Introduction

Background

Paneling methods are approximate techniques for solving linearized subsonic and supersonic potential flow problems over wings and bodies. Panels in use today incorporate singularity distributions of the source, vortex, and doublet types. Source panels are often used to model bodies and other non-lifting surfaces and to model thickness of lifting surfaces. Vortex panels are used to model either lifting or non-lifting surfaces. In a typical problem, the airplane is represented by a finite number of panels. Each panel is a singularity distribution of unknown strength that models some part of the aerodynamic surface. These unknown strengths are determined by applying the flow tangency boundary conditions at or near the aerodynamic surfaces. Once the strengths are known, the perturbation velocities can be computed. These are substituted into the Bernoulli equation to obtain the pressure distribution and the corresponding aerodynamic forces and moments.

Problem Statement

In many current paneling methods, the orientation of the vorticity vector is fixed on the panel (Ref 6 and Ref 8). This leads to unacceptable errors in some cases. The purpose of this study is to

derive a subsonic triangular panel having a vorticity distribution that can vary both in magnitude and direction. A panel system is assembled and used to predict the pressure distribution on a planform. Since the computations involved are too lengthy to be performed by hand, a computer code was developed to apply the panels to planar wings.

Approach

The subsonic triangular panel is derived in Section II. The Biot-Savart Law is used to compute the induced velocity at a point due to an assumed bilinear vorticity distribution on the panel. This expression formulates the induced velocity in terms of panel geometry and unknown corner point vorticity.

Section III presents methodology for panel system assembly. Numbering schemes are developed for panels and the unknown corner point vorticities. Planform boundary conditions are applied which reduce the number of unknowns and the remaining unknowns are solved for by formulating a linear system of equations. This system consists of control point equations (one per panel) and a number of edge continuity conditions. The linearized form of the flow tangency boundary condition is then used to effect the solution. Once the corner vorticities are known, the vorticity at any point on the planform can be obtained. Finally, induced velocities and corresponding pressures can be calculated from the known vorticity.

Section IV presents a computer code developed to apply the methodology to planar wings. Brief descriptions of each subroutine

and a detailed input description are provided. Appendix B is a sample output and Appendix C is the program listing.

Section V presents program predictions for a rectangular wing which are compared against Anderson's solution (Ref 1:9-16). Predictions are also presented for a swept untapered wing which are compared against wind tunnel tests (Ref 4:92).

Section VI concludes the report and makes recommendations for the improvement of the aerodynamic model.

II. Panel Derivation

This section presents the development of the subsonic triangular panel. The goal is to derive an expression for the induced velocity at an arbitrarily chosen point in the xy plane due to an assumed bilinear vorticity distribution on the panel.

Geometry

The first step in panel development is the definition of panel geometry. Initially assume the panel is a trapezoid lying in the xy-plane and having two edges parallel to the x-axis. It is then subdivided into two triangles having a common side that joins the upper left hand corner to the lower right hand corner. The panel is oriented so that the root and tip chords lie parallel to the free stream flow direction at $\alpha = 0$. Figure 1 depicts the panel geometry, corner point numbering scheme, and coordinate system.

Singularity Strength Distribution

The general form of the singularity strength distribution on the panel will be

$$\vec{\omega}(x, y) = \delta(x, y)\vec{i} + \gamma(x, y)\vec{j} \quad (2.1)$$

where δ and γ are continuous functions of x and y. For the purpose of this study, δ and γ are assumed to have the "bilinear" forms

$$\delta(x, y) = F + Dx + Ey \quad (2.2)$$

$$\gamma(x, y) = A + Bx + Cy \quad (2.3)$$

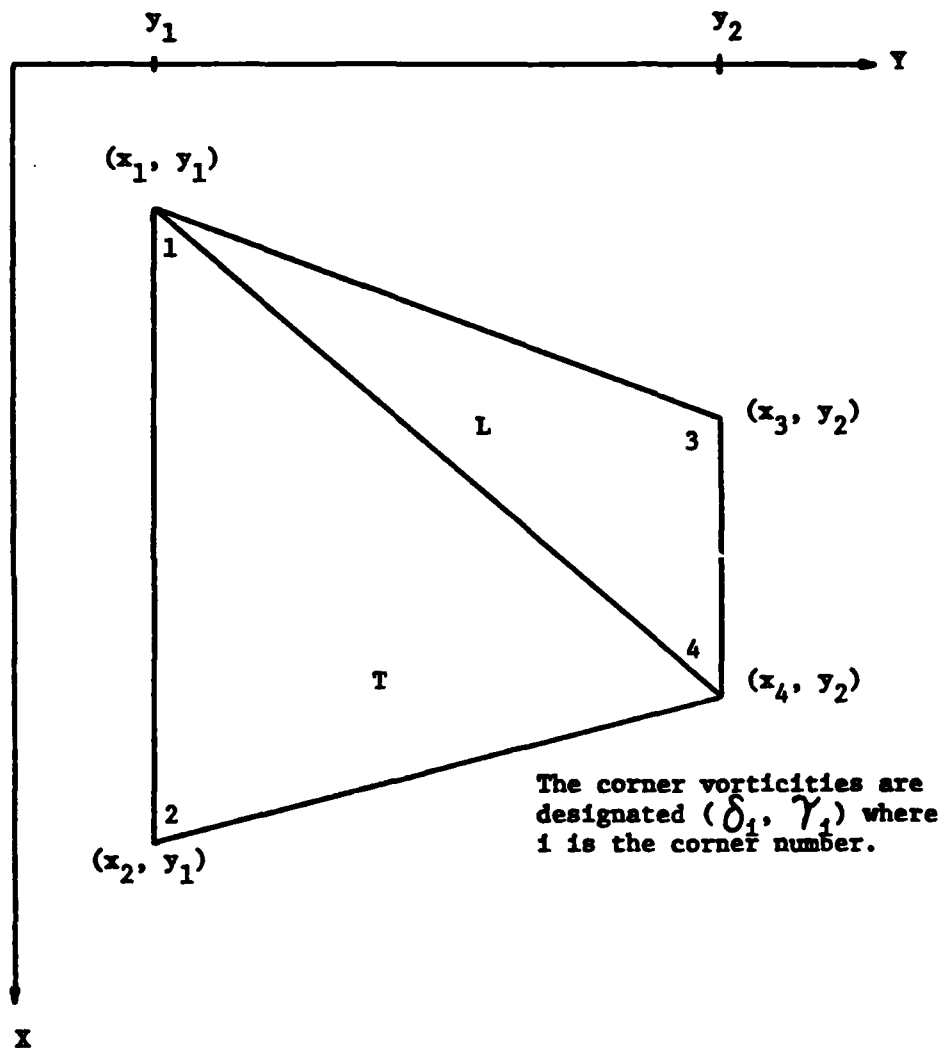


Figure 1. Panel Geometry and Corner Point Numbering Scheme

where the coefficients A, B, C, D, F, D and E are yet to be determined constants. These constants will be expressed as functions of the panel geometry and the unknown singularity strengths at the corner points.

Consequence of the Helmholtz Condition

The vorticity distribution $\bar{\omega}(x, y)$ is required to satisfy the Helmholtz condition that vorticity must be preserved in the fluid. Thus,

$$\nabla(\delta_j + \gamma_j) = 0 \quad (2.4)$$

on the panel. If δ and γ have the forms (2.2) and (2.3), respectively, then

$$\partial(F + Dx + Ey)/\partial x + \partial(A + Bx + Cy)/\partial y = 0 \quad (2.5)$$

which implies

$$D = -C \quad (2.6)$$

such that

$$\delta(x, y) = F - Cx + Ey \quad (2.7)$$

The formulation could be continued in terms of the bilinear coefficients. However, it is conventional to express them in terms of panel corner point vorticities.

Bilinear Coefficients in Terms of Corner Vorticity

In the ensuing discussion, the subscript L refers to the leading triangle and the subscript T refers to the trailing triangle.

The γ component of vorticity (eq (2.3)) is assigned the unknown values γ_1 , γ_3 , γ_4 at the corners (x_1, y_1) , (x_3, y_2) , and (x_4, y_2) of the leading triangle. This leads to the following system of three equations in A_L , B_L and C_L :

$$\gamma_1 = A_L + B_L x_1 + C_L y_1 \quad (2.8)$$

$$\gamma_3 = A_L + B_L x_3 + C_L y_2 \quad (2.9)$$

$$\gamma_4 = A_L + B_L x_4 + C_L y_2 \quad (2.10)$$

The system has the solution:

$$A_L = y_2 \gamma_1 / (y_2 - y_1) + [(y_1 x_4 - x_1 y_2) / (y_2 - y_1)] \gamma_3 / (x_3 - x_4) + [(x_1 y_2 - y_1 x_3) / (y_2 - y_1)] \gamma_4 / (x_3 - x_4) \quad (2.11)$$

$$B_L = (\gamma_3 - \gamma_4) / (x_3 - x_4) \quad (2.12)$$

$$C_L = - \gamma_1 / (y_2 - y_1) + [(x_1 - x_4) / (y_2 - y_1)] \gamma_3 / (x_3 - x_4) + [(x_3 - x_1) / (y_2 - y_1)] \gamma_4 / (x_3 - x_4) \quad (2.13)$$

Since $D_L = -C_L$, only two corner conditions may be used to solve for F_L and E_L in the δ component eq (2.7). Assigning the values δ_1 and δ_3 at the corners (x_1, y_1) and (x_3, y_2) leads to

$$\delta_1 + C_L x_1 = E_L y_1 + F_L \quad (2.14)$$

$$\delta_3 + c_L x_3 = E_L y_2 + F_L \quad (2.15)$$

and

$$E_L = (\delta_1 - \delta_3)/(y_1 - y_2) + [(x_1 - x_3)/(y_1 - y_2)]c_L \quad (2.16)$$

$$F_L = (y_1 \delta_3 - y_2 \delta_1)/(y_1 - y_2) + [(y_1 x_3 - x_1 y_2)/(y_1 - y_2)]c_L \quad (2.17)$$

In a similar way, the trailing triangle coefficient equations are obtained as follows:

$$A_T = y_1 \gamma_4/(y_1 - y_2) + [(y_2 x_1 - x_4 y_1)/(y_1 - y_2)] \gamma_2/(x_2 - x_1) + [(x_4 y_1 - y_2 x_2)] \gamma_1/(x_2 - x_1) \quad (2.18)$$

$$B_T = (\gamma_2 - \gamma_1)/(x_2 - x_1) \quad (2.19)$$

$$C_T = -\gamma_4/(y_1 - y_2) + [(x_4 - x_1)/(y_1 - y_2)] \gamma_2/(x_2 - x_1) + [(x_2 - x_4)/(y_1 - y_2)] \gamma_1/(x_2 - x_1) \quad (2.20)$$

$$E_T = (\delta_2 - \delta_4)/(y_1 - y_2) + [(x_2 - x_4)/(y_1 - y_2)]c_T \quad (2.21)$$

$$F_T = (y_1 \delta_4 - y_2 \delta_2)/(y_1 - y_2) + [(y_1 x_4 - x_2 y_2)/(y_1 - y_2)] C_T \quad (2.22)$$

where E_T and F_T have been expressed in terms of (as functions of) $\vec{\omega}$ components at corners 2 and 4.

Mathematical Continuity

The bilinear vorticity distribution is continuous on each triangular region. In addition, the γ component has been made to be continuous throughout the planform by the representation in terms of corner values. This can be demonstrated as follows. Let Γ be the boundary shared by any two adjacent triangles. Then Γ is a straight line segment and can be described by a linear expression (i.e., y in terms of x or x in terms of y). The γ distribution on each of the adjacent triangles will degenerate to a linear function of a single variable upon substitution of this expression. Both functions assume the same γ values at the endpoints of Γ . Since only two points are needed to determine a straight line or a linear form, we have γ matching identically on Γ .

The δ component has breaks in continuity throughout the planform. This is a consequence of applying the Helmholtz condition (eq (2.4)) which eliminated the constant D and expressing the remaining two unknowns in terms of two δ corner values, out of a possible three. The δ component is continuous on panel leading and trailing edges since it is on these edges that common δ values are assumed at the endpoints. The δ component is discontinuous on panel diagonals.

Application of the Biot-Savart Law

Let $\vec{\omega}(\vec{r})$ be a vorticity distribution defined on a finite region R in the xy plane. Let $\vec{s}$ be a fixed point (control point) in the plane. The velocity induced at $\vec{s}$ due to the distribution on R is given by the Biot-Savart Law (Ref 5:526-528):

$$4\pi\vec{v}(\vec{s}) = \iint_R [\vec{\omega}(\vec{r}) \times (\vec{s} - \vec{r})] / |\vec{s} - \vec{r}|^3 dR \quad (2.23)$$

Suppose the control point $\vec{s}$ is located at the origin of the coordinate system. Note that this can be done by performing a simple translation of the plane. Then,

$$\vec{s} = (0, 0, 0) \quad (2.24)$$

$$\vec{s} - \vec{r} = (0, 0, 0) - (x, y, 0) = (-x, -y, 0) \quad (2.25)$$

Assuming the distribution has the form (2.1),

$$\vec{\omega} \times (\vec{s} - \vec{r}) = (x\gamma - y\delta) \vec{k} \quad (2.26)$$

where $\vec{k}$ is the unit vector normal to the xy -plane. Substituting the expressions (2.25) and 2.26) into eq (2.23) yields:

$$4\pi w(0, 0, 0) = \iint_R (x\gamma - y\delta) / (x^2 + y^2)^{3/2} dR \quad (2.27)$$

where w is the normal velocity component induced at the origin.

Substituting the expressions for γ (2.3) and δ (2.7) into eq (2.27) yields:

$$4 \pi w = \iint_R (Ax + Bx^2 + 2Cxy - Ey^2 - Fy) / (x^2 + y^2)^{3/2} dR \quad (2.28)$$

where R is taken as the region defined by a trapezoidal panel.

Let R_L and R_T be the subregions of R which correspond to the leading and trailing triangles. The coefficients A , B , C , F , and E remain constant on each subregion and eq (2.28) is rewritten as:

$$4\pi w = A_L I_1^L + A_T I_1^T + B_L I_2^L + B_T I_2^T + 2C_L I_3^L + 2C_T I_3^T - F_L I_4^L - F_T I_4^T - E_L I_5^L - E_T I_5^T \quad (2.29)$$

where

$$I_3^{L, T} = \iint_{R_{L, T}} (x/(x^2 + y^2)^{3/2}) dR \quad (2.30)$$

$$I_2^{L, T} = \iint_{R_{L, T}} (x^2/(x^2 + y^2)^{3/2}) dR \quad (2.31)$$

$$I_3^{L, T} = \iint_{R_{L, T}} (xy/(x^2 + y^2)^{3/2}) dR \quad (2.32)$$

$$I_4^{L, T} = \iint_{R_{L, T}} (y/(x^2 + y^2)^{3/2}) dR \quad (2.33)$$

$$I_5^{L, T} = \iint_{R_{L, T}} (y^2/(x^2 + y^2)^{3/2}) dR \quad (2.36)$$

Evaluation of these integrals may be found in Appendix A. Substitution of the expressions for A_L through E_T and collecting coefficients of the unknowns ($\delta_1, \delta_2, \delta_3, \delta_4, \gamma_1, \gamma_2, \gamma_3$, and γ_4) yields, after considerable algebraic manipulation,

$$\begin{aligned}
 4 \pi w = & [(y_2 I_4^L - I_3^L)/(y_1 - y_2)] \delta_1 + \\
 & [(y_2 I_4^T - I_5^T)/(y_1 - y_2)] \delta_2 + \\
 & [(I_5^L - y_1 I_4^L)/(y_1 - y_2)] \delta_3 + \\
 & [(I_5^T - y_1 I_4^T)/(y_1 - y_2)] \delta_4 + \\
 & [((x_4 y_1 - y_2 x_2) I_1^T - (x_2 - x_1) y_2 I_1^L - \\
 & (y_1 - y_2) I_2^T + (x_2 - x_1) K_L + \\
 & (x_2 - x_4) K_T)/((x_2 - x_1)(y_1 - y_2))] \gamma_1 + \\
 & [((y_2 x_1 - x_4 y_1) I_1^T + (y_1 - y_2) I_2^T + \\
 & (x_4 - x_1) K_T)/((y_1 - y_2)(x_2 - x_1))] \gamma_2 + \\
 & [((y_1 x_4 - x_1 y_2) I_1^L + (y_2 - y_1) I_2^L + \\
 & (x_1 - x_4) K_L)/((y_2 - y_1)(x_3 - x_4))] \gamma_3 + \\
 & [((x_1 y_2 - y_1 x_3) I_1^L - (x_3 - x_4) y_1 I_1^T - \\
 & (y_2 - y_1) I_2^L + (x_3 - x_1) K_L + (x_3 - x_4) K_T)/
 \end{aligned}$$

$$((y_2 - y_1)(x_3 - x_4)) \gamma_4 \quad (2.35)$$

where

$$\begin{aligned} K_L = & 2I_3^L - [(x_1 - x_3)/(y_1 - y_2)]I_5^L - \\ & [(y_1x_3 - x_1y_2)/(y_1 - y_2)]I_4^L \end{aligned} \quad (2.36)$$

$$\begin{aligned} K_T = & 2I_3^T - [(x_2 - x_4)/(y_1 - y_2)]I_5^T - \\ & [(y_1x_4 - x_2y_2)/(y_1 - y_2)]I_4^T \end{aligned} \quad (2.37)$$

Summary

Expression (2.35) is the normal velocity induced at the origin of the xy plane by a trapezoidal vorticity panel. This velocity is due to a bilinear vorticity distribution which satisfies the Helmholtz condition eq (2.4).

III. Panel Assembly

This section presents the panel assembly procedures needed to model a wing. The goal is to develop the methodology required to predict the pressure distribution and associated forces and moments on a wing.

Panel Numbering

Figure 2 illustrates a paneling arrangement and associated numbering scheme for a 16 panel wing. The panels are numbered consecutively in the chordwise direction starting with the inboard leading edge panels and terminating with the outboard trailing edge panels.

Number of Unknowns, Boundary Conditions and Numbering

Let M be the number of chordwise panels and N be the number of spanwise panels. These are defined using $M + 1$ chordwise cuts and $N + 1$ spanwise cuts. Each intersection determines a panel corner point. Since there are two unknown components at each corner point, the total number of unknowns is given by:

$$2(M + 1)(N + 1) \quad (3.1)$$

Boundary Conditions

Two boundary conditions are imposed on the wing panel system. These reduce the number of unknowns and improve the physical modeling of the flow field.

The z axis is normal to the wing planform.

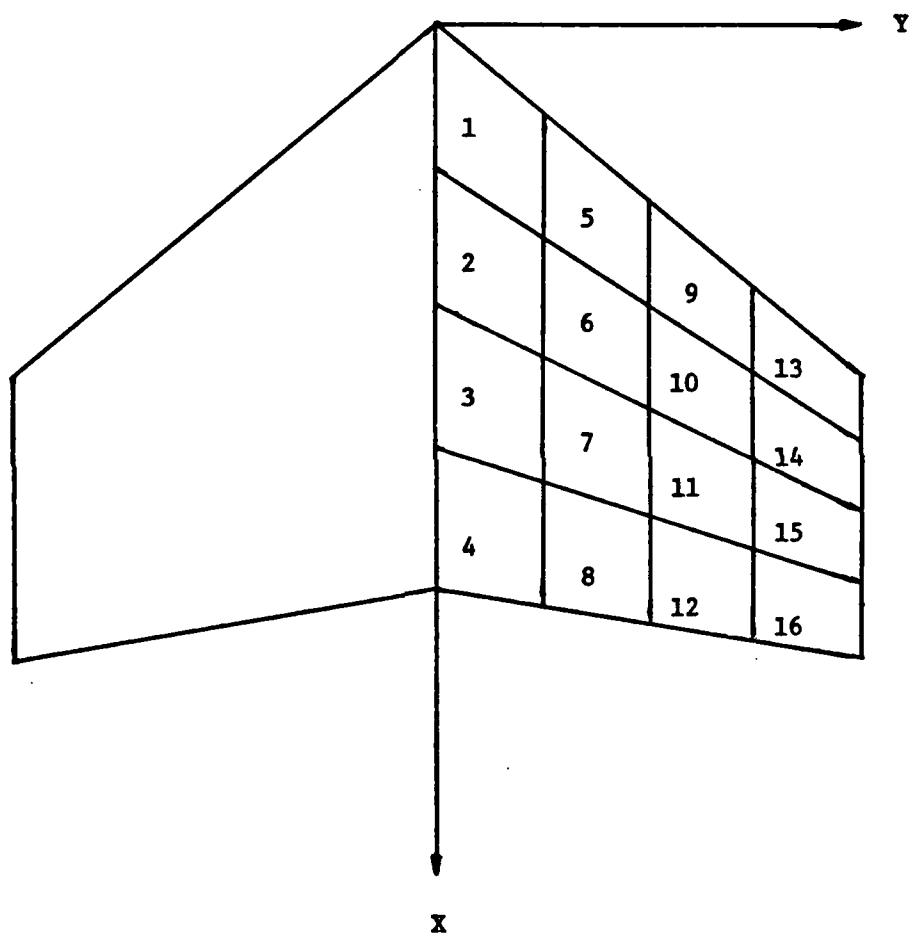


Figure 2. A Paneling Arrangement and Associated Numbering Scheme for a 16 Panel Wing

The Kutta condition (Ref 5:390-399)

$$\gamma(x, y) = 0 \quad (3.2)$$

is imposed at all corner points on the wing trailing and tip edges. Since the γ component of vorticity eq (2.2) is both linear and continuous on these edges, the Kutta condition is satisfied identically. The Kutta condition reduces the total number of unknowns by:

$$M + N + 1 \quad (3.3)$$

(NOTE: The point defining the intersection of the trailing and tip edges is common to both.)

A historically acceptable boundary condition (an outgrowth of Prandtl's lifting-line theory (Ref 5:535-567)) is for the vorticity vector to lie tangent to the wing leading edge. This boundary condition initially orients the vorticity vector so that a positive circulation is produced. The boundary condition is imposed at all leading edge corner points and can be written:

$$\gamma/\delta = \wedge \quad (3.4)$$

or

$$\delta = \gamma/\wedge \quad (3.5)$$

where $\wedge$ is the leading edge slope at the corner point. It reduces the total number of unknowns by:

$$N + 1 \quad (3.6)$$

The unknown corner δ s are denoted by "-" and the γ s by "o."

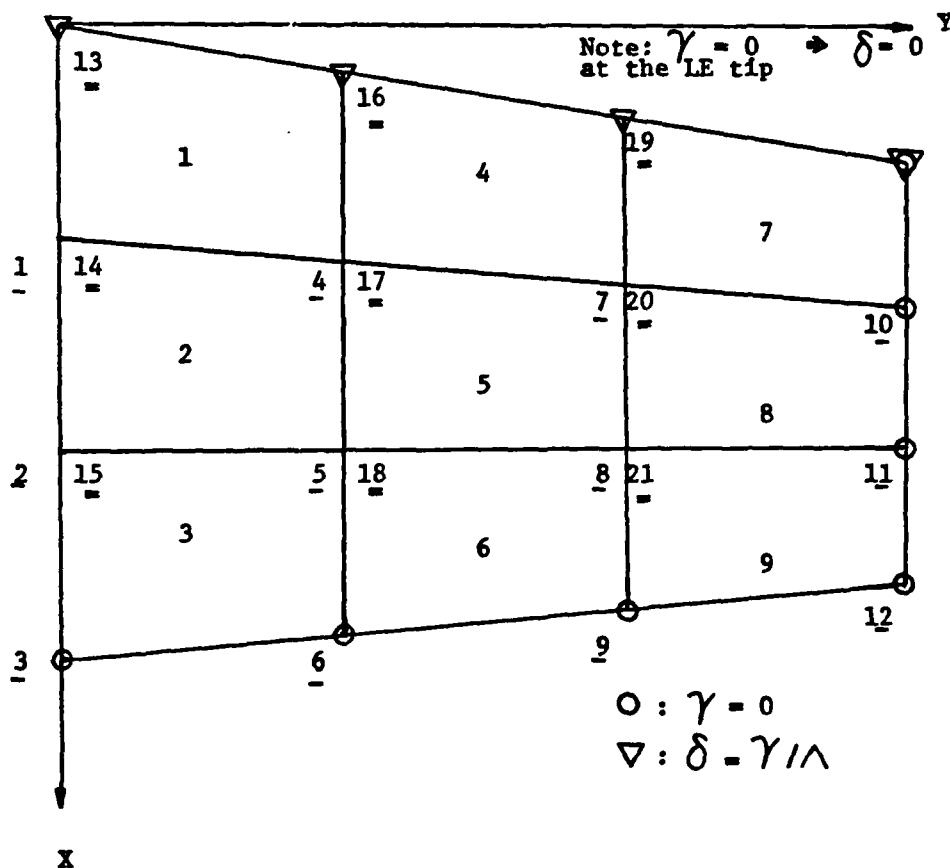


Figure 3. Unknown Numbering Scheme for a 9 Panel Wing

and the total number of unknowns becomes:

$$2MN + M \quad (3.7)$$

after imposing the two boundary condition equations (3.2) and (3.5).

Since the wing is symmetric about the x-axis, it may seem logical to impose a boundary condition at the wing root chord. However, setting

$$\delta = 0 \quad (3.8)$$

at the centerline is redundant for rectangular wings and leads to an ill-conditioned system once planform symmetry is considered.

Unknown Numbering Scheme

Figure 3 illustrates the unknown numbering scheme for a 9 panel wing with applied boundary conditions. The paneling arrangement of Figure 3 is chosen because it represents the smallest number of panels needed to illustrate interior panels and panels having boundary conditions. Let i be the panel number of an interior panel. Then the following numbers (in terms of i , M , and N) are assigned to the eight unknown corner vorticity components:

$$(\delta_1) \quad i - 1 \quad (3.9)$$

$$(\delta_2) \quad i \quad (3.10)$$

$$(\delta_3) \quad i + M - 1 \quad (3.11)$$

$$(\delta_4) \quad i + M \quad (3.12)$$

$$(\gamma_1) \quad MN + M + 1 \quad (3.13)$$

$$(\gamma_2) \quad MN + M + 1 + 1 \quad (3.14)$$

$$(\gamma_3) \quad MN + 2M + 1 \quad (3.15)$$

$$(\gamma_4) \quad MN + 2M + 1 + 1 \quad (3.16)$$

Solving for the Corner Vorticities

The total number of unknowns (after the boundary conditions are applied) is given by eq (3.7) which also specifies the number of conditions needed to solve for the corner vorticities. Two types of conditions will be used; control point conditions and edge or point continuity conditions.

Control Point Equations

Control point equations are obtained using eq (2.35). The velocity component is computed for one point (control point) on each panel comprising the wing.

Let (x_1, y_1) be the control point on panel 1. To obtain the contribution to w_1 due to panel j , express the coordinates of panel j in a coordinate system with (x_1, y_1) at the origin. This is done by performing a translation in the $z = 0$ plane:

$$x' = x - x_1 \quad (3.17)$$

$$y' = y - y_1$$

Equation (2.35) is then applied with appropriate boundary conditions.

Planform symmetry is included by reflecting either the panel or control point about the x-axis and applying eq (2.35). Reflecting the control point is less complicated from a programming viewpoint.

The above process is repeated for each panel on the wing. After all the contributions to w_1 have been calculated, it can be written as:

$$w_1 = \sum_{j=1}^{2MN+M} A_{1j} \theta_j \quad (3.18)$$

where θ_j is the column vector of unknown corner vorticities and the coefficients A_{1j} are functions of panel geometry. The θ_j are numbered using the system given by eqs (3.9) through (3.16). One control point equation (3.18) is obtained for each panel on the wing and together they comprise MN conditions.

Edge Continuity Conditions

The δ component of vorticity eq (2.7) is discontinuous across the panel diagonal (See discussion in Section II.). This can be partially remedied by specifying a point continuity condition at the panel lower right hand corner. This condition is:

$$\delta_L(x_4, y_2) = \delta_4 \quad (3.19)$$

which becomes

$$\begin{aligned} & \delta_4 - \delta_3 + [(x_3 - x_4)/(y_2 - y_1)] \gamma_1 - \\ & [(x_1 - x_4)/(y_2 - y_1)] \gamma_3 - [(x_3 - x_1)/ \\ & (y_2 - y_1)] \gamma_4 = 0 \end{aligned} \quad (3.20)$$

after substituting of (x_4, y_2) and the expressions for A_L (2.11), B_L (2.12), and C_L (2.13) into eq (2.7). One condition eq (3.20) is formulated for each panel on the wing for a total of MN conditions. Note that δ is still not continuous across the panel diagonal due to the remaining discontinuity at the upper left hand corner. Also, δ is not continuous across panel side edges.

The edge continuity conditions and control point equations total 2MN conditions. M additional conditions can be obtained by specifying an edge continuity condition for δ at the upper left hand corner of each panel along the centerline. This condition is:

$$\delta_T(x_1, y_1) = \delta_1 \quad (3.21)$$

which becomes

$$\begin{aligned} \delta_2 - \delta_1 - [(x_2 - x_1)/(y_1 - y_2)] \gamma_4 + \\ [(x_4 - x_1)/(y_1 - y_2)] \gamma_2 + [(x_2 - x_4)/ \\ (y_1 - y_2)] \gamma_1 = 0 \end{aligned} \quad (3.22)$$

after substitution of (x_1, y_1) and the expressions for A_T (2.18), B_T (2.19), and C_T (2.20) into eq (2.7). This choice is based on trial and error, the additional δ continuity on the centerline having the effect of minimizing vorticity oscillations.

Compressibility

Compressibility effects are accounted for by using the Prandtl-Glauert transformation (Ref 2:124-127):

$$\bar{x} = x / \sqrt{1 - M^2} \quad (3.23)$$

The transformation is applied to all x coordinates which are used in either the control point equations or edge continuity conditions.

Matrix Formulation

The control point equations and edge continuity conditions are $2MN + M$ equations in the unknowns, θ_j . This system has the matrix formulation:

$$\begin{matrix} \begin{bmatrix} A_{1j} \end{bmatrix} \cdot \begin{bmatrix} \theta_1 \\ \theta_{2MN+M} \end{bmatrix} \\ (2MN + M) \times (2MN + M) \end{matrix} = \begin{bmatrix} w_1 \\ w_{MN} \\ \circ \end{bmatrix} \quad (3.24)$$

The first MN rows of A_{1j} are the coefficients for the control point equations and are all nonzero. The last $MN + M$ rows of A_{1j} correspond to the homogeneous edge continuity conditions and have no more than five nonzero entries per row.

Solution

The linearized form of the flow tangency boundary condition is (Ref 5:495):

$$w/V_\infty = dc/dx - \alpha \quad (3.25)$$

where α is the wing angle of attack, dc/dx is the local camber slope and V_∞ is the free stream velocity. This expression is substituted for each w_1 in eq 3.24 where dc_1/dx is the panel slope at control point 1. In matrix notation,

$$[A_{1j}] \cdot [\theta_j] = \begin{Bmatrix} V_\infty (dc_1/dx - \alpha) \\ 0 \end{Bmatrix} \quad (3.26)$$

or

$$\theta_j/V_\infty = [A_{1j}]^{-1} \cdot \begin{Bmatrix} dc_1/dx - \alpha \\ 0 \end{Bmatrix} \quad (3.27)$$

Forces and Moments

Once the θ_j/V_∞ are obtained for a given α and camber slope distribution, eqs (2.2) and (2.7) can be used to calculate the vorticity strength at any point on the planform. The surface perturbation velocities in terms of local vorticity are (Ref 5:508):

$$u/V_\infty = \pm \gamma/2 \quad (3.28)$$

$$v/V_\infty = \mp \delta/2 \quad (3.29)$$

The upper sign corresponds to the upper wing surface and visa-versa.

Pressure coefficients are obtained from the perturbation velocities by either using the exact isentropic expression (Ref 3:167):

$$C_p = 2[(1 + (\gamma - 1)M_\infty^2/2(1 - ((V_\infty + u)^2 + v^2 + w^2)/V_\infty^2)) \gamma / (\gamma - 1) - 1] / (\gamma M_\infty^2) \quad (3.30)$$

or the second order approximation (Ref 3:167):

$$C_p = -[2u/V_\infty + (1 - M_\infty^2)u^2/V_\infty^2 + (v^2 + w^2)/V_\infty^2] \quad (3.31)$$

which is adequate for two-dimensional and planar flows. These coefficients are integrated along chord lines to obtain local lift and moment

coefficients. The appropriate expressions are

$$C_L = (1/c) \int_{X_{LE}}^{X_{TE}} (C_{P_1} - C_{P_u}) dx \quad (3.32)$$

$$C_M = (1/c^2) \int_{X_{LE}}^{X_{TE}} (C_{P_1} - C_{P_u}) x dx \quad (3.33)$$

where the subscripts l and u refer to the lower and upper wing surfaces.

This concludes the theoretical section of this report. The next step is the implementation of this panel to predict airloads on wings. This is done by the use of a computer code "WING" which is presented in Section IV.

IV. Computer Code

General Description

A FORTRAN code "WING" has been developed to analyze planform flow using the methodology discussed in Sections II and III. WING is a pilot code and should not be treated as a fully checked-out production program until the problems in the wing tip region are resolved (Section V). Many of the programming techniques used in WING have been previously developed by the author and can be found in Reference 8. Care has been taken to insure correspondence between FORTRAN variable names and the symbol usage in Sections II and III. The listing (Appendix C) contains comment cards that describe program function and logic in detail. Below is a brief description of each of the sub-routines in WING.

WING (Main)

WING is the executive control routine. All geometric data is read by WING. WING initializes panel parameters and calls subroutines MESH, AERO, INVRT, and LOADS in that order.

MESH

MESH generates the x and y coordinates for the panel corner points and control points. The mesh is generated from the information given on the first four data cards. MESH is a FORTRAN version of the mesh generator discussed in Reference 8.

AERO

AERO formulates the control point equations and edge continuity conditions. Subroutine INT is called by AERO.

INT

INT evaluates the panel integrals using equations (A.31) and (A.32). Subroutine STRIP is called by INT.

STRIP

STRIP evaluates the five STRIP function equations (A.24) through (A.28) given two points in the plane.

INVRT

INVRT inverts the coefficient array for the system of equations formulated by AERO. The inversion is performed using Gaussian elimination. INVRT is essentially the same inversion subroutine found in the FASTLODS lifting surface program (Ref 6:73-133).

LOADS

LOADS calculates the planform pressure distributions and aerodynamic coefficients for the loading cases specified by cards 6 and 7. After the last loading case is examined, LOADS will terminate program execution.

Input Description

This section provides a card by card description of input data

along with some helpful "dos and don'ts" of program operation. All input is unformatted and should be separated by commas. Integer data cannot have a decimal point. WING is a nondimensional code which will accept data in any consistent system of units. Presently, WING can analyze planforms having sixty panels or less. This can be increased by changing the dimensions of arrays X, Y, XC, YC, E, A, SG, CBR, and SUM in common blocks A and C. The reader is referred to Appendix B which contains a sample problem.

Card 1 (Span Data)

Variables (In Order)

Description

SSPN

The length of the wing semi-span.

NS

The number of stations needed to define the spanwise panel boundaries. The wing root chord is station 1 and the last station is the wing tip. NS is an integer and must be less than or equal to 12.

S(1), ..., S(NS)

Span stations as a fraction of the semi-span. 0. and 1. will always be the first and last entries. Entries must be in ascending order left to right.

Card 2 (Chord Data)

NC

The number of stations needed to define the chordwise panel boundaries. The leading edge is station 1 and the last station is the trailing edge. NC is an integer and must be less than or equal to 10.

C(1), ..., C(NC)

Chord stations as a fraction of the wing local chord. 0. and 1. will always be the first and last entries. Entries must be in ascending order left to right.

Card 3 (Break Point Data)

Break points are the x coordinates of leading and trailing edge for those chords that define a sweep change. The wing centerline defines the positive x-axis with origin at the leading edge.

NB

The number of break point sets needed to outline the planform geometry. Two sets will be needed to define a four-point wing. NB is an integer and must be less than or equal to 10.

NL(1), B(1, 1), B(1, 2), ...,
NL(NB), B(NB, 1), B(NB, 2)

NL is the number (Not the value!) of the span station where the sweep change occurs. NL is an integer. B(I, 1) and B(I, 2) are the leading and trailing edge x coordinates of the chord line at span station NL(I). Break point sets are entered from inboard to outboard. The first set (NL = 1) will always be the x coordinates of the root chord leading and trailing edge. The last set (NL = NS) will always be the x coordinates of the tip chord leading and trailing edge.

Card 4 (Mach and Control Point Data)

CY, CX

Control point location in terms of local panel span and chord. CY is the fraction of the panel span and CX is the fraction of the panel chord. A recommended control point choice is CY = .15 and CX = .75 which is based on extensive program testing.

MACH

Mach number (not an integer). WING accepts subsonic Mach numbers only.

Card 5 (Number of Loading Cases)

NA

The number of loading cases to be examined - i.e., changes in angle of attack or camber slope distribution. NA is an integer that has no upper bound.

Card 6 (Load Cases)

NA cards are required

ALPHA

The angle of attack in degrees. Plus is nose up.

NCCG

The camber change parameter (integer). Enter 0 to read a new camber slope distribution or 1 to retain the previous distribution. WING initially sets the camber slope array equal to 0.

NPF

The pressure option parameter (integer). Enter 0 to use the exact isentropic expression eq (3.30) or enter 1 to use the 2nd order approximation eq (3.31).

CY, CX

Panel location (fraction of panel span, fraction of panel chord) where the pressures are to be calculated. Pressures may be computed at points other than the control points.

Card 7 (Camber Slope Distribution)

This card is used only for a NCCG value of 0.

CBR(1) ..., CBR(NP)

The value of the local panel slope in degrees for each panel on the plan-form. NP is the total number of panels. Entries must be made in the order corresponding to the panel numbering scheme (Figure 2). The sign rule for camber follows the standard convention.

V. Results

Program WING was exercised for a variety of four point wings having various aspect ratios, taper ratios, and sweep angles. This section presents results for two of these wings.

Two general observations are made first. One, control point location is the major factor controlling bounded numerical oscillations of the vorticity vector as it is in many current paneling routines (ex. Refs 6 and 8). Oscillations are very common if the control point is located anywhere on the leading triangle. Fewer oscillations occur if the control point is located on the trailing triangle with $.1 \leq CY \leq .5$ and $.4 \leq CX \leq .9$. Secondly, the program shows the best results when uniform spanwise paneling is used. Non-uniform spanwise paneling tends to cause oscillations in the vorticity vector. However, non-uniform chordwise paneling seems to have little effect on solution stability. The best total C_L match (with other known solutions) occurs at approximately $CY = .15$ and $CX = .75$.

Rectangular Wing

The first case examined is a rectangular wing; $AR = 8$, $\alpha = 5^\circ$ and $M_\infty = .1$. The wing is modeled using 12 uniformly spaced span stations and 6 non-uniformly spaced chord stations (0., .1, .3, .6, .8, and 1.) which define 55 panels. Figure 4 shows the C_L distribution predicted by WING and Anderson (Ref 1:9-16). The lift coefficient experiences a spiking phenomena near the wing tip. This phenomena always happens in

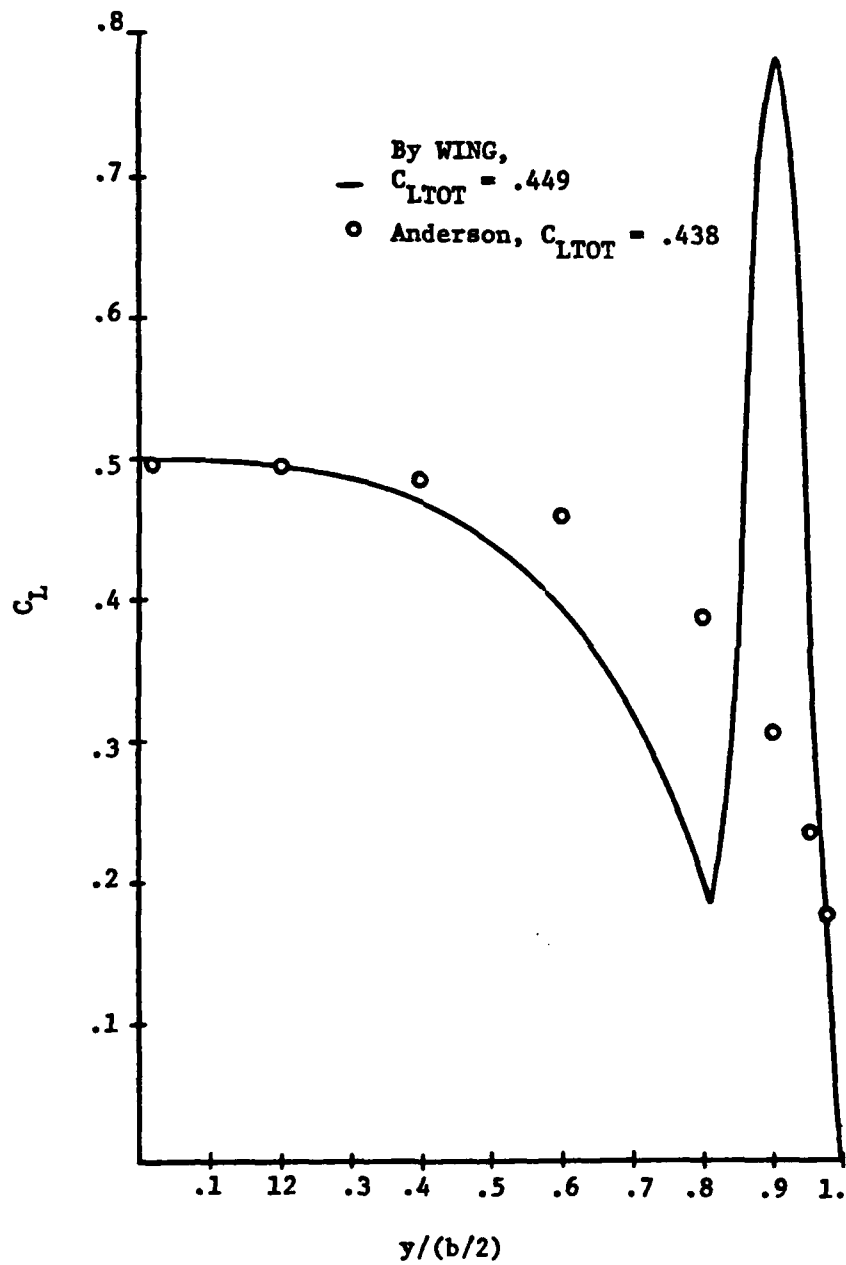


Figure 4. C_L Versus Span Station for a Rectangular Wing; $AR = 8$, $\alpha = 5^\circ$

the region defined by the last three outboard span stations. Also, the C_L distribution falls off faster than it should before reaching the region where spiking occurs. There is very little difference between total lift coefficients indicating the areas under the curves are approximately equal. This shows the solution is possibly trying to compensate for the spike by underpredicting lift in the inboard region.

Figure 5 shows the spanwise distribution of center of pressure. The X_{cp} shifts aft in the region of spiking before traveling forward. The ΔC_p versus chord station curves of Figure 6 exhibit expected behavior for the inboard stations ($0 \leq Y/S \leq .585$). At $Y/S = .9$, the curve has "fattened up" considerably which drives the X_{cp} backwards and creates the C_L spike. The curve is subsiding again at $Y/S = .95$ since the Kutta (no net load) boundary condition is imposed at the tip.

Figure 7 is the γ strength at the root chord. The calculated solution compares favorably with the exact 2-D flat plate solution (Ref 5:515):

$$\gamma(x) = 2\alpha [(c - x)/(cx - x^2)] \quad (5.1)$$

where c is the chord length and α is measured in radius.

Figures 8 and 9 show γ and δ strength distributions at selected span stations. The δ distribution grows in magnitude relative to γ as we approach either the tip or the trailing edge. This allows the vorticity vector to turn and satisfy the Kutta condition as shown in Figure 10. The δ component is 0 at the leading edge which

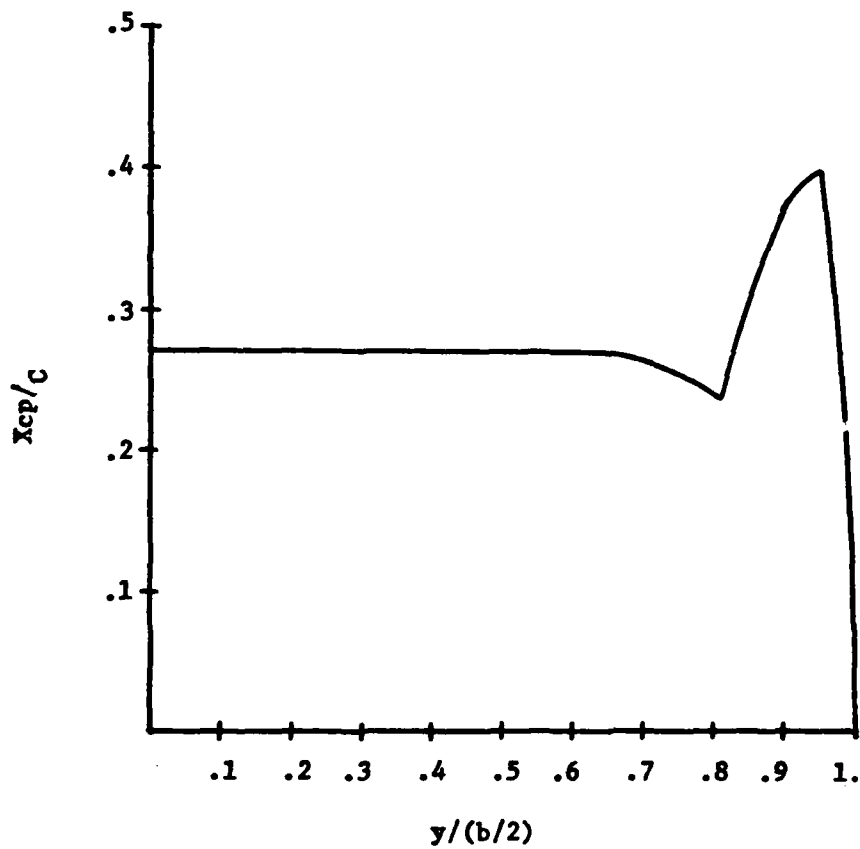


Figure 5. X_{cp}/c Versus Span Station for a Rectangular Wing; $AR = 8$, $\alpha = 5^\circ$

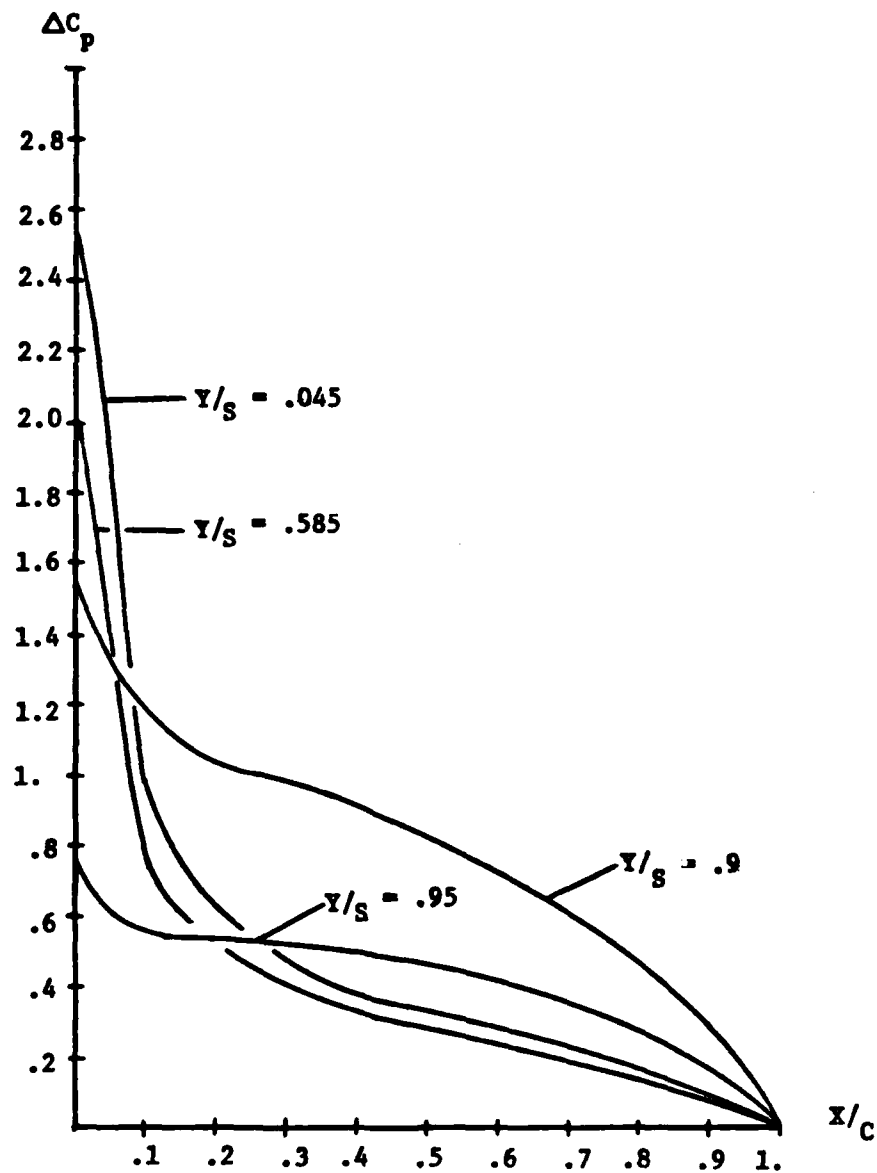


Figure 6. ΔC_p Versus X/c at Selected Span Stations -
Rectangular Wing, $AR = 8$, $\alpha = 5^\circ$

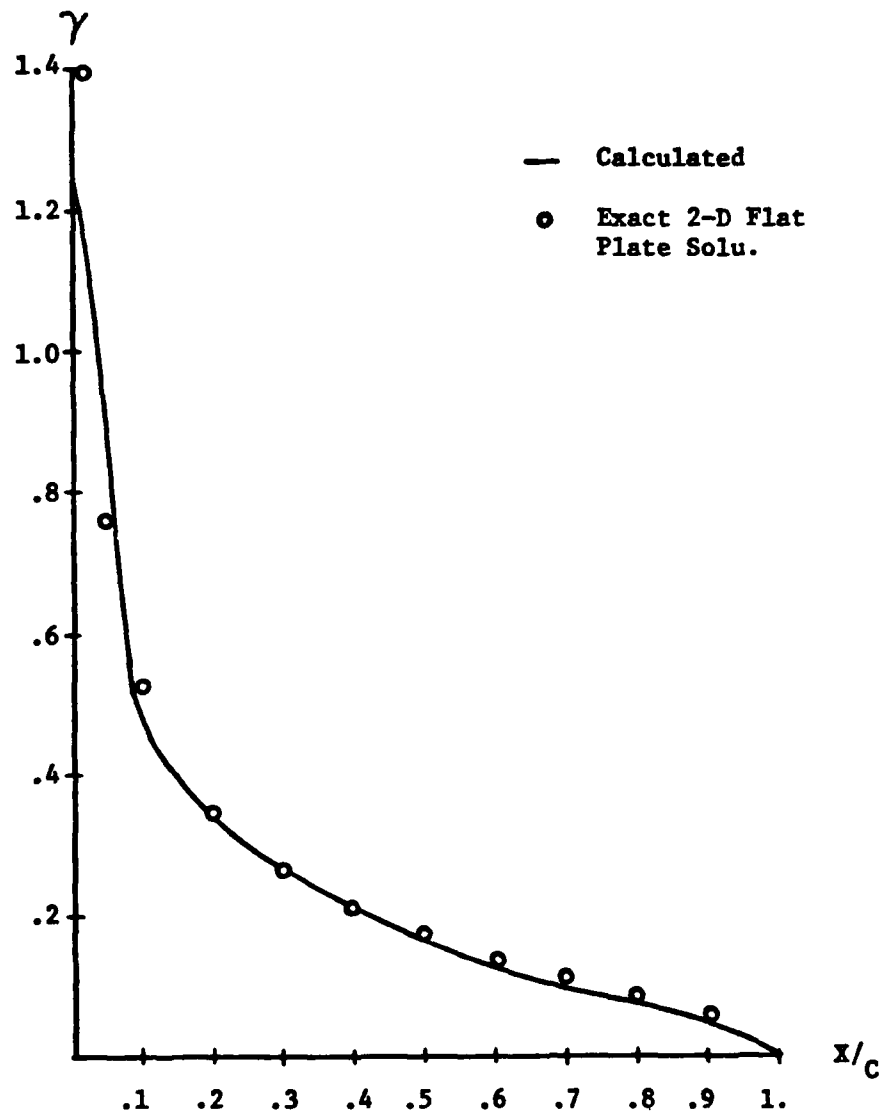


Figure 7. γ Strength Distribution at the Root Chord of a Rectangular Wing; $AR = 8$, $\alpha = 5^\circ$

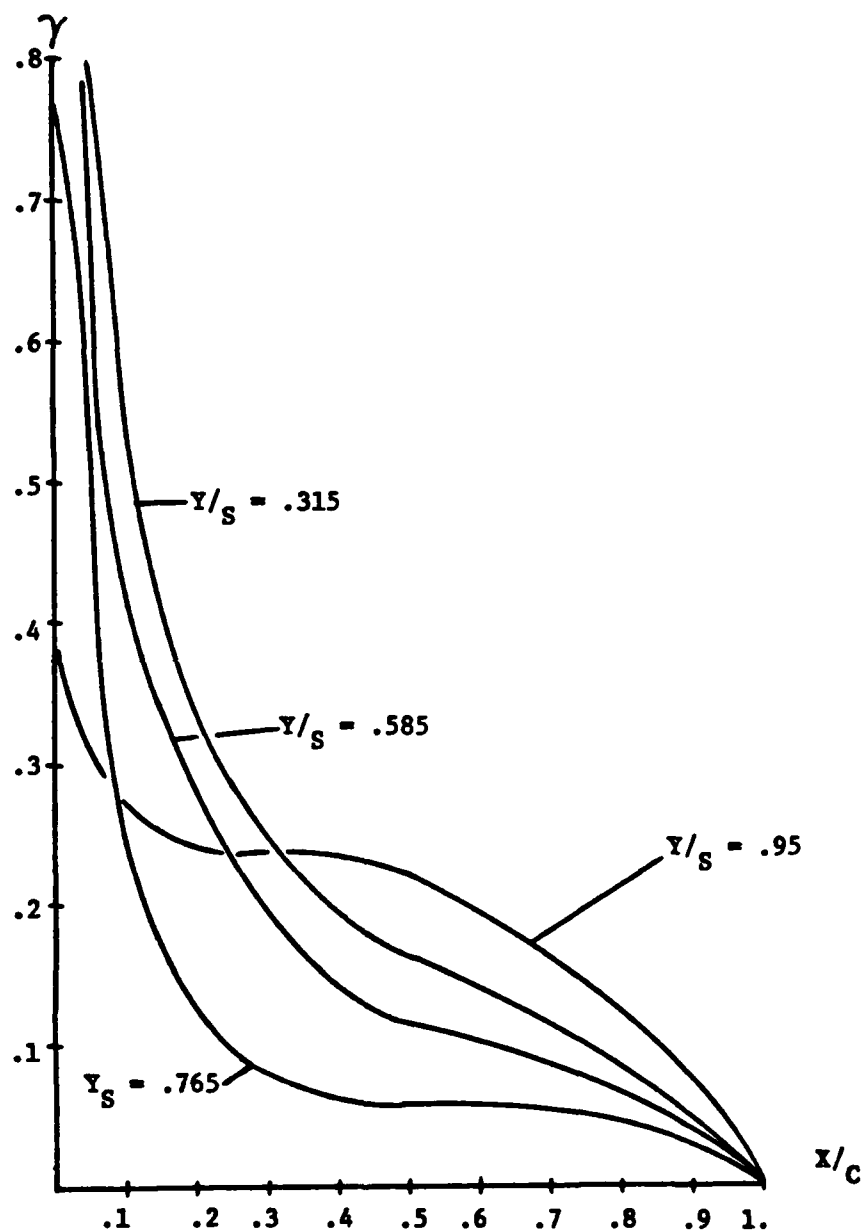


Figure 8. γ Strength Distribution at Selected Span Stations - Rectangular Wing; AR = 8, $\alpha = 5^\circ$

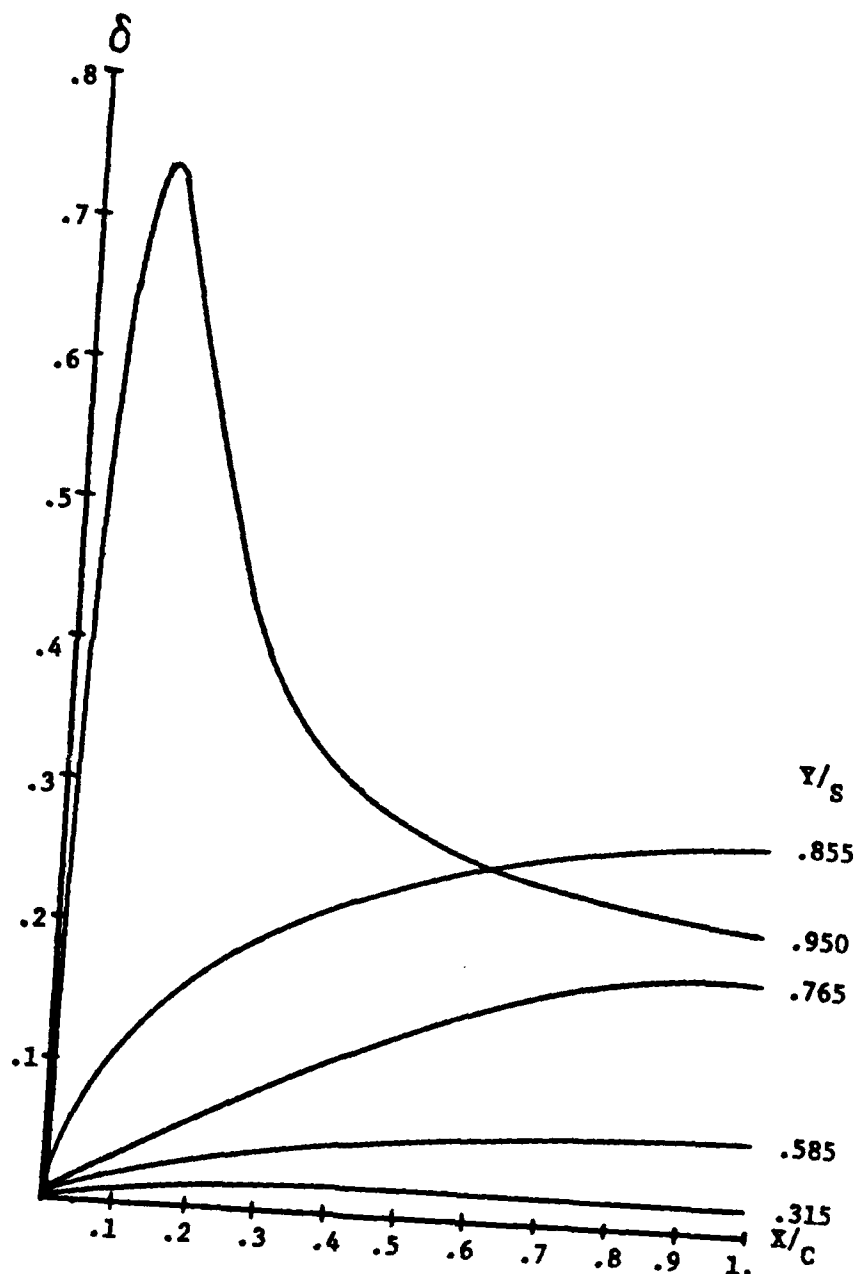


Figure 9. δ Strength Distribution at Selected Span Stations - Rectangular Wing; $AR = 8$, $\alpha = 50$

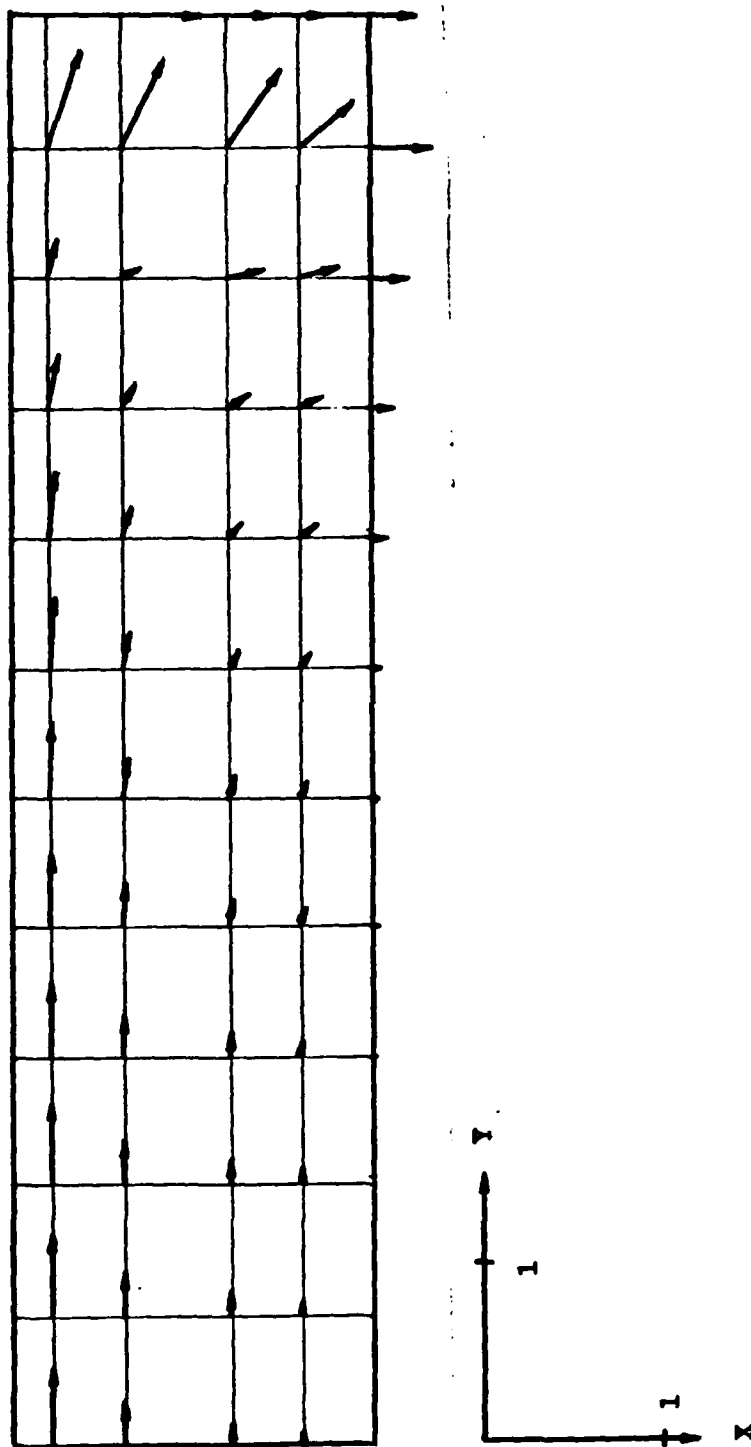


Figure 10. Relative Magnitude and Direction of the Vorticity Vector at Selected Corner Points - Rectangular Wing; $AR = 8$, $\alpha = 5^\circ$

is the condition of infinite slope (eq (3.5)). Also, the δ component is small near the wing root chord which illustrates the effect of plan-form symmetry. Both the δ and γ components exhibit unusual behavior in the tip region. Possible remedies for this problem are discussed in Section VI.

Swept Wing

The second case examined is a swept untapered wing; $AR = 4.5$, $\Lambda = 40^\circ$, $\alpha = 5^\circ$, and $M_\infty = .1$. The wing modeling is the same as the rectangular wing. Figure 11 shows the C_L distribution predicted by WING and by wind tunnel tests (Ref 4:92). Again, a spiking phenomena occurs. The predicted C_L curve is showing the proper curvature in the region inboard of the spike. Also, the total lift coefficients again show close agreement. Figure 12 shows the x center of pressure versus span station which again shifts aft in the region of spiking.

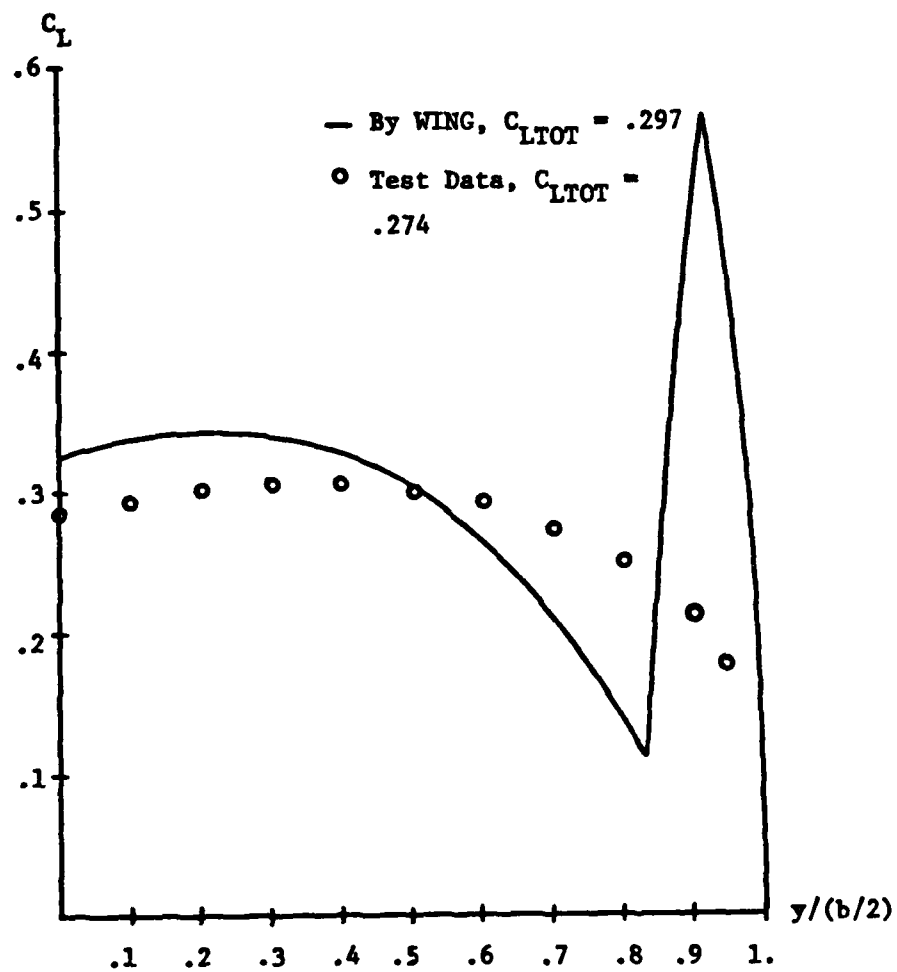


Figure 11. C_L Versus Span Station for a Sweptback
 Untapered Wing; $AR = 4.5$, $\Lambda = 40^\circ$, $\alpha = 5^\circ$

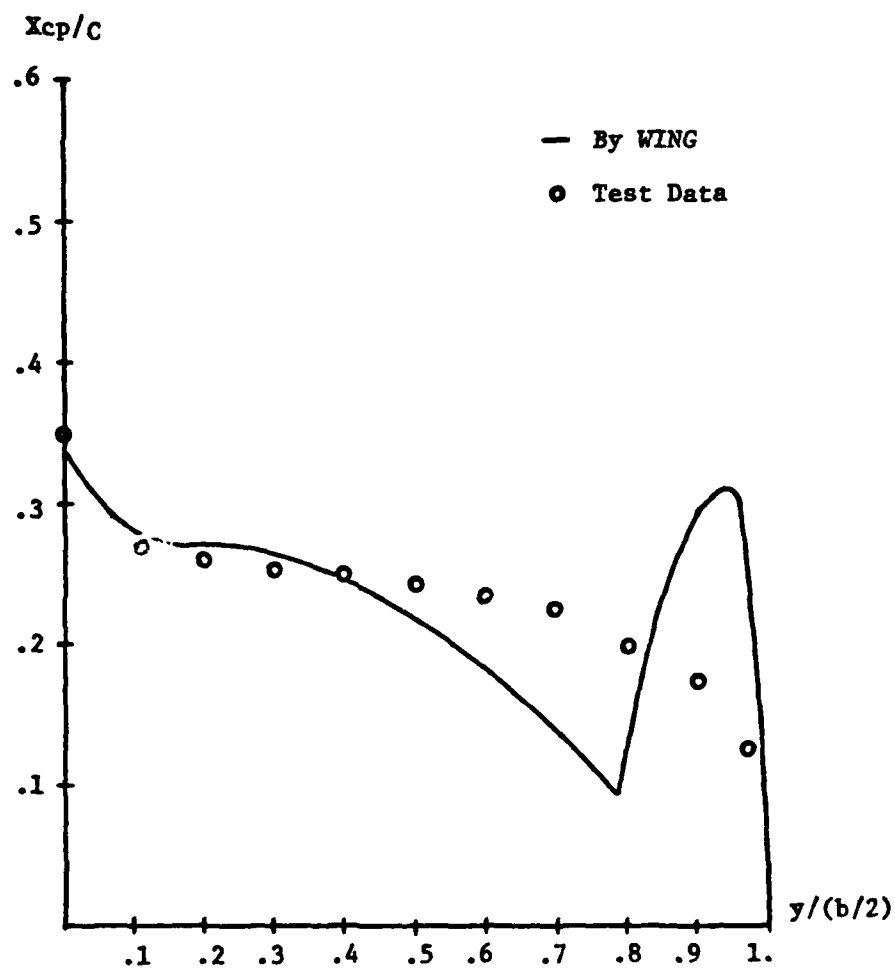


Figure 12. X_{cp}/c Versus Span Station for a Sweptback
Untapered Wing; $AR = 4.5$, $\Lambda = 40^\circ$, $\alpha = 5^\circ$

VI. Conclusions and Recommendations

Conclusions

A paneling technique which allows the vorticity vector to change direction has been demonstrated. The proper choice of control point (see Section V) will guarantee a solution free of numerical oscillations in the vorticity vector. This method has been implemented on a computer and introduces no new complexities to an experienced programmer. Existing mesh generators and other aerodynamic modules were incorporated into this technique (Ref 8). Computer run times are of the same order as programs incorporating "fixed direction vorticity" panels and no problems involving extensive "run times" were encountered.

The method gives good aerodynamic results near the centerline of the wing. However, the method will underpredict lift as we move outboard. A gross overprediction of lift occurs in the region defined by the three most outboard span stations. Total lift coefficients as predicted by this technique agree well with existing solutions.

Recommendations

The following ideas are suggested for the improvement of the method and hopefully will lead to the elimination or minimization of the "spiking" problem.

- a. A wake model should be incorporated into the program.
- b. The flat plate panels should have a provision for leading edge suction.

c. Higher-order panels might be needed at the leading edge and tip. This would turn the vorticity vector faster and minimize the spiking. An elliptic vorticity distribution is suggested since many classic lift distributions are elliptic near the tip.

d. Change or modify the leading edge boundary condition. The "classical" tangency condition may be inappropriate for this kind of panel. Reference 9 suggests the Kutta condition be applied at the leading edge.

e. Interchange the role of γ and δ with respect to panel boundary continuity conditions. Perhaps the γ distribution enjoys too much continuity on the planform. This could be creating problems by forcing the γ distribution to undergo unusual "warping" in order to satisfy the planform boundary conditions.

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APPENDIX A

Evaluation of the Panel Integrals

The methodology for evaluating the integrals I_1^L, T of section 2 is presented here.

Consider the semi-infinite region shown in Figure 13. This strip is bound by the lines $y = y_0$, $y = y_1$, and the line segment connecting (x_0, y_0) to (x_1, y_1) . The equation for the line segment is:

$$y = Mx + b \quad (A.1)$$

where

$$M = ((y_1 - y_0)/(x_1 - x_0)) \quad (A.2)$$

and

$$b = y_0 - x_0 M \quad (A.3)$$

Define the following improper integrals on the semi-infinite strip

$$F_i = \lim_{L \rightarrow \infty} \int_{y_0}^{y_1} \int_{(y-b)/M}^L f_i(x, y) dx dy \quad (i = 1, \dots, 5) \quad (A.4)$$

where

$$f_1(x, y) = x/(x^2 + y^2)^{3/2} \quad (A.5)$$

$$f_2(x, y) = x^2/(x^2 + y^2)^{3/2} \quad (A.6)$$

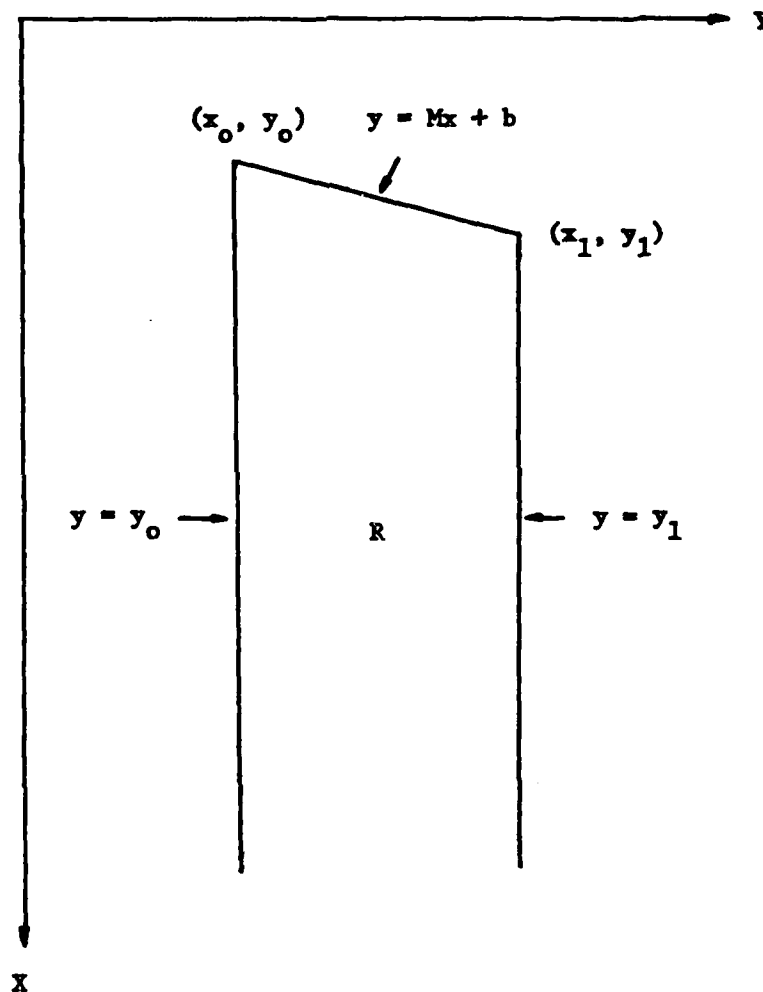


Figure 13. Semi-Infinite Strip R Used In Integral Evaluation

$$f_3(x, y) = xy/(x^2 + y^2)^{3/2} \quad (A.7)$$

$$f_4(x, y) = y/(x^2 + y^2)^{3/2} \quad (A.8)$$

and

$$f_5(x, y) = y^2/(x^2 + y^2)^{3/2} \quad (A.9)$$

To illustrate the methodology used in evaluating the integrals (A.4), consider

$$F_5 = \lim_{L \rightarrow \infty} \int_{y_0}^{y_1} \int_{(y-b)/M}^L [y^2/(x^2 + y^2)^{3/2}] dx dy \quad (A.10)$$

By Pierce's integral tables (Ref 7),

$$F_5 = \lim_{L \rightarrow \infty} \int_{y_0}^{y_1} [y^2 x / (y^2 \sqrt{x^2 + y^2})] \Big|_{(y-b)/M}^L dy = \quad (A.11)$$

$$\lim_{L \rightarrow \infty} \int_{y_0}^{y_1} L dy / \sqrt{L^2 + y^2} - \int_{y_0}^{y_1} ((y-b)/M) dy / \sqrt{((y-b)/M)^2 + y^2} \quad (A.12)$$

Substituting

$$z = (y - b)/M \quad (A.13)$$

and

$$dy = M dz \quad (A.14)$$

into the rightmost integral of (A.12) yields:

$$F_5 = \lim_{L \rightarrow \infty} \int_{y_0}^{y_1} (Ldy/\sqrt{L^2 + y^2}) - M \int_{x_0}^{x_1} zdz / \sqrt{z^2 + (Mz + b)^2} \quad (A.15)$$

The integrals (A.15) can be evaluated using Ref 7. One obtains, after some algebraic manipulation,

$$\begin{aligned} F_5 = \lim_{L \rightarrow \infty} L \ln[(y_1 + \sqrt{y_1^2 + L^2})/(y_0 + \sqrt{y_0^2 + L^2})] + \\ [bM^2/(1 + M^2)^{3/2}] \ln[(\sqrt{x_1^2 + y_1^2} + ((x_1 + My_1)/\sqrt{1 + M^2})) / (\sqrt{x_0^2 + y_0^2} + ((x_0 + My_0)/\sqrt{1 + M^2}))] + \\ [M/(1 + M^2)] (\sqrt{x_0^2 + y_0^2} - \sqrt{x_1^2 + y_1^2}) \end{aligned} \quad (A.16)$$

recalling

$$y_0 = Mx_0 + b \quad (A.17)$$

and

$$y_1 = Mx_1 + b \quad (A.18)$$

Notice the limit

$$\lim_{L \rightarrow \infty} L \ln[(y_1 + \sqrt{y_1^2 + L^2})/(y_0 + \sqrt{y_0^2 + L^2})] \quad (A.19)$$

is not a function of x_0 or x_1 which leads to the following observation.

If the integral F_5 is evaluated on any other semi-infinite strip bounded by the lines $y = y_0$ and $y = y_1$, the limit (A.19) is invariant. Evaluating the integral F_5 using any other line segment connecting $y = y_0$ to $y = y_1$ and formulating the difference between this result and (A.16) leads to cancellation of the limit (A.19).

The other four integrals, F_1 through F_4 , can be evaluated by a similar procedure using Ref 7. Each integral has a limit term given by:

$$(F_1) \quad \lim_{L \rightarrow \infty} \ln[(y_0 + \sqrt{y_0^2 + L^2}) / (y_1 + \sqrt{y_1^2 + L^2})] \quad (A.20)$$

$$(F_2) \quad \lim_{L \rightarrow \infty} [y_1 \ln(\sqrt{L^2 + y_1^2} + L) - y_0 \ln(\sqrt{L^2 + y_0^2} + L)] \quad (A.21)$$

$$(F_3) \quad \lim_{L \rightarrow \infty} [\sqrt{L^2 + y_0^2} - \sqrt{L^2 + y_1^2}] \quad (A.22)$$

$$(F_4) \quad \lim_{L \rightarrow \infty} \ln[(L + \sqrt{L^2 + y_0^2}) / (L + \sqrt{L^2 + y_1^2})] \quad (A.23)$$

These terms cancel upon the formulation of integral differences.

Retaining the finite terms from each of the integral evaluations, we can define "Strip Functions" S_1 for the functions f_1 and the points (x_0, y_0) and (x_1, y_1) by

$$S_1[(x_0, y_0), (x_1, y_1)] = (M/\sqrt{1 + M^2})G \quad (A.24)$$

$$\begin{aligned}
S_2[(x_0, y_0), (x_1, y_1)] &= y_0 \ln[\sqrt{x_0^2 + y_0^2} + x_0] - \\
& y_1 \ln[\sqrt{x_1^2 + y_1^2} + x_1] + (M/(1 + M^2))H + \\
& [b/(1 + M^2)^{3/2}]G
\end{aligned} \tag{A.25}$$

$$\begin{aligned}
S_3[(x_0, y_0), (x_1, y_1)] &= (M^2/(1 + M^2))H + \\
& [bM/(1 + M^2)^{3/2}]G
\end{aligned} \tag{A.26}$$

$$\begin{aligned}
S_4[(x_0, y_0), (x_1, y_1)] &= \ln[(x_1 + \sqrt{x_1^2 + y_1^2})/ \\
& (x_0 + \sqrt{x_0^2 + y_0^2})] - G/\sqrt{1 + M^2}
\end{aligned} \tag{A.27}$$

$$\begin{aligned}
S_5[(x_0, y_0), (x_1, y_1)] &= [bM^2/(1 + M^2)^{3/2}]G - \\
& (M/(1 + M^2))H
\end{aligned} \tag{A.28}$$

where

$$\begin{aligned}
G &= \ln[(\sqrt{x_1^2 + y_1^2} + ((x_1 + My_1)/\sqrt{1 + M^2}))/ \\
& (\sqrt{x_0^2 + y_0^2} + ((x_0 + My_0)/\sqrt{1 + M^2}))]
\end{aligned} \tag{A.29}$$

and

$$H = \sqrt{x_1^2 + y_1^2} - \sqrt{x_0^2 + y_0^2} \tag{A.30}$$

Examining Figure 1, it is obvious that each of the panel integrals (2.30) through (2.34) can be obtained by formulating the difference of

two corresponding strip functions. This leads to the following fundamental results:

$$I_1^L = S_1[(x_1, y_1), (x_3, y_2)] - S_1[(x_1, y_1), (x_4, y_2)] \quad (A.31)$$

$$I_1^T = S_1[(x_1, y_1), (x_4, y_2)] - S_1[(x_2, y_1), (x_4, y_2)] \quad (A.32)$$

where i ranges from 1 to 5.

APPENDIX B

SAMPLE OUTPUT

SPAN STATIONS

YLOC	Y/S
1	0.000
2	.250
3	.500
4	.750
5	1.000

SEMI-SPAN = 10.00

CHORD STATIONS

XLOC	X/C
1	0.000
2	.250
3	.500
4	.750
5	1.000

BREAK POINTS

LE	TE	YLOC
0.00	2.00	1
0.00	2.00	5

MACH NUMBER = .100

CY = .150
CX = .750

1

PANEL	X1	X2	X3	X4	V1	V2	XC	VC
1	0.00	.50	0.00	.50	0.00	2.50	.38	.38
2	.50	1.00	.50	1.00	0.00	2.50	.68	.38
3	1.00	1.50	1.00	1.50	0.00	2.50	1.38	.38
4	1.50	2.00	1.50	2.00	0.00	2.50	1.88	.38
5	0.00	.50	0.00	.50	2.50	5.00	.38	2.88
6	.50	1.00	.50	1.00	2.50	5.00	.88	2.88
7	1.00	1.50	1.00	1.50	2.50	5.00	1.38	2.88
8	1.50	2.00	1.50	2.00	2.50	5.00	1.88	2.88
9	0.00	.50	0.00	.50	5.00	7.50	.38	5.38
10	.50	1.00	.50	1.00	5.00	7.50	.88	5.38
11	1.00	1.50	1.00	1.50	5.00	7.50	1.38	5.38
12	1.50	2.00	1.50	2.00	5.00	7.50	1.88	5.38
13	0.00	.50	0.00	.50	7.50	10.00	.38	7.88
14	.50	1.00	.50	1.00	7.50	10.00	.88	7.88
15	1.00	1.50	1.00	1.50	7.50	10.00	1.38	7.88
16	1.50	2.00	1.50	2.00	7.50	10.00	1.88	7.88

ALPHA = 5.00

PRESSURE AND SINGULARITY STRENGTH DISTRIBUTION

XP = .750

YP = .150

LINEAR

Y/S = .038						
X/C	CAMBER	DEL	GAM	CPU	CPL	DELCP
.188	0.00	.00215	.41582	-.46623	.36541	.83164
.438	0.00	.00357	.19382	-.21074	.17691	.38765
.688	0.00	.00452	.12030	-.13151	.10910	.24061
.938	0.00	.00484	.02656	-.03436	.01877	.05313

Y/S = .288						
X/C	CAMBER	DEL	GAM	CPU	CPL	DELCP
.188	0.00	.00618	.38896	-.43403	.34389	.77792
.438	0.00	.01164	.17955	-.19518	.16392	.35910
.688	0.00	.01598	.11280	-.12363	.10197	.22560
.938	0.00	.01862	.02556	-.03343	.01770	.05113

Y/S = .538						
X/C	CAMBER	DEL	GAM	CPU	CPL	DELCP
.188	0.00	.05032	.32041	-.35407	.28675	.64082
.438	0.00	.07086	.14394	-.15793	.12994	.28787
.688	0.00	.08090	.09528	-.10678	.08378	.19057
.938	0.00	.08360	.01946	-.02892	.01000	.03892

Y/S = .788						
X/C	CAMBER	DEL	GAM	CPU	CPL	DELCP
.188	0.00	.21639	.33951	-.38736	.29166	.67902
.438	0.00	.17284	.21860	-.24551	.19169	.43720
.688	0.00	.17610	.15057	-.17154	.12959	.30113
.938	0.00	.18774	.03844	-.05524	.02165	.07689

LOADS SUMMARY

YS	X-CP	CHORD	CL	CM
.04	.28	2.00	.48	.134
.29	.28	2.00	.45	.126
.54	.27	2.00	.37	.102
.79	.34	2.00	.45	.152

APPENDIX C

PROGRAM LISTING

```

1      PROGRAM WING(INPUT,CUTPLY,TAPE5=INPUT,TAPE6=OUTPUT)
      C
      C      FFCGRAP WING CALCULATES THE AIRLOADS ON A
      C      WING OF ARBITRARY PLANFORM AND CAMBER IN
      C      SUBSONIC FLOW. A TRIANGULAR PANEL HAVING
      C      A BILINEAR VORTICITY DISTRIBUTION IS USED
      C      TO PREDICT THE PRESSURE FIELD. COMPRESS-
      C      IILITY EFFECTS ARE ACCOUNTED FOR THROUGH
      C      THE PRANDTL-GLAUERT TRANSFORMATION.
10     C
      CCMPCN/PLCCKA/>(60,4),Y(60,2),XC(60),YC(60),E(60,2),CY,CX
      CCMPCN/PLCCKB/NS,SSPA,S(15),NC,C(10),NB,NL(10),B(10,2),PACH
      CCMPCN/PLCCKC/>(120,130),SG(130),CBR(60),SUP(60),ALPHA,N
      CCMPCN/PLCCKD/IL(5),IT(5)
15     CCMPCN/PLCCKE/SPF(5,3)
      REAL PACH
      REAL IL,IT
      C
      C      BEGIN INPUT SEQUENCE FOR GEOMETRIC DATA.
20     C      ALL DATA IS READ USING FREE FORMAT.
      C      READ SPAN DATA.
      READ(5,*) SSPA,NS,(S(I),I=1,NS)
      WRITE(6,100)
100    FORMAT(///29),*SPAN STATIONS*/26X,*YLOC*,11X,*Y/S*)
      DO 10 I=1,NS
25     10 WRITE(6,110) I,S(I)
      110 FORMAT(26X,I3,10X,F6.3)
      WRITE(6,120) SSPA
      120 FORMAT(27X,*SEMI-SPAN =*,F7.2)
30     C      READ CHORD DATA.
      READ(5,*) NC,(C(I),I=1,NC)
      WRITE(6,130)
      130 FORMAT(//28),*CHORD STATIONS*/26X,*YLOC*,11X,*Y/C*)
      DO 20 I=1,NC
35     20 WRITE(6,140) I,C(I)
      140 FORMAT(26X,I3,10X,F6.3)
      C      READ BREAK POINT DATA.
      READ(5,*) NB,(B(1,I),B(1,2),B(1,3)),I=1,NB)
      WRITE(6,150)
40     150 FORMAT(//29),*BREAK POINTS*/22X,*LE*,10X,*TE*,10X,*YLOC*)
      DO 30 I=1,NB
      30 WRITE(6,160) B(1,1),B(1,2),B(1,3)
      160 FORMAT(19X,F7.2,5X,F7.2,7X,13)
      C      READ CONTROL POINT LOCATIONS AND PACH
45     READ(5,*) CY,CX,PACH
      WRITE(6,170) PACH,CY,CX
      170 FORMAT(//27),*PACH NUMBER =*,F6.3//31X,*CY =*,F6.3//31X,
      10CX =*,F6.3)
      C
      C      READ THE NUMBER OF LOADING CASES TO BE
      C      EXAMINED. THE ANGLE OF ATTACK AND CAMBER
      C      DISTRIBUTION FOR EACH CASE WILL BE READ
      C      LATER IN THE PROGRAM. THIS IS DONE TO
      C      PRESERVE STORAGE SPACE.
      READ(5,*) NA
      WRITE(6,*)NA
55     C
      C      CALCULATE P AND N. M IS THE NUMBER OF
      C      CIRCULAR PANELS AND N IS THE NUMBER

```

	C		CF SPANWISE PANELS
		P=NC-1	
6C		N=NS-1	
	C		CALCULATE NUMBER OF PANELS NP.
		NP=P*N	
	C		CALCULATE NUMBER OF UNKNOWN SINGULARITIES.
		NP2=2*NP+P	
65	C		CALL MESH GENERATOR
		CALL MESH(NF)	
		CALL AERO(NP,P,N,NP2)	
		CALL INVRT(NP2)	
		CALL LCACS(NP,P,N,NP2)	
7C		END	

```

1      SUBROUTINE PEST(RP)
      C      SUBROUTINE TO CALCULATE PANEL CORNER PCIN
      C      TS AND CONTRCL PCIKTS.
      C      CCMPPEN/PLCCRA/>(60,4),Y(60,2),XC(60),YC(60),E(60,2),CY,CX
5      CCMPPEN/PLCCRP/NS,SSPN,S(15),NC,C(10),NB,NL(10),B(10,2),PACH
      NC=1
      NE1=NE-1
      NC1=NC-1
      DO 4C K=1,NB1
1C      DO 1C I=1,NC
      E(I,1)=B(K,1)+C(I)*(B(K,2)-B(K,1))
      IC      E(I,2)=B(K+1,1)+C(I)*(B(K+1,2)-B(K+1,1))-E(I,1)
      IC1=NL(K)
      IC2=NL(K+1)
15      IC3=IC2-1
      DO 3C J=IC1,IC3
      D2=S(IC2)-S(IC1)
      D3=S(IC1)
      DO 2C I=1,NC1
2C      X(NC,1)=E(I,1)+E(I,2)*(S(J)-D3)/D2
      X(NC,2)=E(I+1,1)+E(I+1,2)*(S(J)-D3)/D2
      X(NC,3)=E(I,1)+E(I,2)*(S(J+1)-D3)/D2
      X(NC,4)=E(I+1,1)+E(I+1,2)*(S(J+1)-D3)/D2
      Y(NC,1)=S(J)+SSPN
25      Y(NC,2)=S(J+1)+SSPN
      NC=NC+1
      2C CONTINUE
      3C CONTINUE
      4C CONTINUE
3C      C      CALCULATE CONTROL POINTS.
      DO 5C I=1,NB
      YC(I)=(1-CY)*Y(I,1)+CY*Y(I,2)
      5C      XC(I)=CY*X(I,2)+(1-CY)*X(I,1)+CY*(CY*(X(I,4)-X(I,3))+(1-CY)*
      1(X(I,2)-X(I,1)))
35      C      PRINT A TABLE CONSISTING OF THE PANEL NO.
      C      AND THE CORRESPONDING CORNER AND CONTROL
      C      POINTS.
      WRITE(6,100)
10C      FORF(1,777)5X,'PANEL',7X,'X1',9X,'X2',8X,'X3',6X,'X4',9X,'Y1',
4C      19X,'Y2',9X,'XC',8X,'YC'
      DO 2C I=1,NB
      WRITE(6,101) 1,X(I,1),X(I,2),X(I,3),X(I,4),Y(I,1),Y(I,2),XC(I),
      1YC(I)
101      FORF(1,6X,I2,E(14X,F7,2))
45      6C CONTINUE
      RETRN
      END

```

1		SUBROUTINE INT(X1,X2,X3,X4,Y1,Y2)
	C	THIS SUBROUTINE EVALUATES THE PANEL INTE-
	C	GRALS. IT ALSO FUNCTIONS AS THE EXECUTIVE
	C	CONTROL ROUTINE FOR SUBROUTINE STRIP.
5		COMMON/PLCCMC/IL(5),IT(5)
		COMMON/PLCCMC/SPF(5,3)
		REAL IL,IT
	C	CALCULATE STRIP FUNCTIONS FOR THE PANEL
	C	LEADING EDGE WITH CORNER POINTS (X1,Y1)
10	C	AND (X3,Y2).
		I=1
		CALL STRIP(X1,Y1,X3,Y2,I)
	C	CALCULATE STRIP FUNCTIONS FOR THE PANEL
	C	MAIN DIAGONAL WITH CORNER POINTS (X1,Y1)
15	C	AND (X4,Y2).
		I=2
		CALL STRIP(X1,Y1,X4,Y2,I)
	C	CALCULATE STRIP FUNCTIONS FOR THE PANEL
	C	TRAILING EDGE WITH CORNER POINTS (X2,Y1)
20	C	AND (X4,Y2).
		I=3
		CALL STRIP(X2,Y1,X4,Y2,I)
	C	EVALUATE PANEL INTEGRALS.
		DO 10 I=1,5
25		IL(I)=SPF(I,1)-SPF(I,2)
		IT(I)=SPF(I,2)-SPF(I,3)
	10	CONTINUE
		RETURN
		END

```

1      SLP=CLTIME STRIP(X0,YC,X1,Y1,1)
      C      SLRRCUTINE TO CALCULATE STRIP FUNCTIONS.
      C      THERE ARE 15 STRIP FUNCTION CALCULATIONS
      C      PER QUADRILATERAL PANEL.
5      CCMPCN/BLOCME/SFF(15,3)
      REAL PC
      C      USE STRIP FUNCTION EQUATIONS FOR AN IN-
      C      FINITE SLOPE IF X1=XC.
10     IF(ABS(X1-XC).LT.1E-8) GO TO 100
      PC=(Y1-Y0)/(X1-XC)
      BC=YC-PC*PC
      CUM1=SCRT(XC*02+Y0*02)
      DLP2=SCRT(X1*02+Y1*02)
      CLP3=SCRT(1+PC*02)
15     CLM4=(CLM1+(XC+PC*YC)/CLP3)/(CUM2+(X1+PC*Y1)/CLM3)
      CLM4=ABS(CUM4)
      CLM4=ALCG(CLP4)
      CLM5=ABS(CUM1+XC)
      CLM5=ALCG(CLP5)
20     CLM6=ABS(CUM2+Y1)
      CLM6=ALCG(CLP6)
      SFF(1,1)=-PC*CLM4/DLP3
      SFF(2,1)=YC*CLP5-Y1*CLP6-PC*(CUM1-CUM2)/DLP3*02-BC*CLP4/DLP3*03
      SFF(3,1)=PC*02*(CLM2-CUM1)/DLP3*02-PC*PC*CLM4/CUM3*03
25     SFF(4,1)=CLP4/CUM3*CLP6-CUM5
      SFF(5,1)=-BC*PC*02*CLP4/CUM3*03+PC*(CUM1-CUM2)/DLP3*02
      REYLNK
100    CONTINUE
      CLP1=SCRT(XC*02+YC*02)
30     CUM2=SCRT(XC*02+Y1*02)
      CLP3=ABS((CLP1+YC)/(CUM2+Y1))
      SFF(1,1)=-ALCG(CUM3)
      CLP2=ABS(CUM1+XC)
      DUM3=ALCG(CLP3)
35     CLP4=ABS(CUM2+XC)
      CLP4=ALCG(CLP4)
      SFF(2,1)=YC*CLP3-Y1*CLP4
      SFF(3,1)=CUM2-CUM1
      SFF(4,1)=CLP4-CLP3
40     SFF(5,1)=-XC*SFF(1,1)
      REYLNK
      END

```


1		SUBROUTINE INVERT(NP2)
	C	SUBROUTINE TO INVERT THE AERODYNAMIC IN-
	C	FLUENCE COEFFICIENT ARRAY USING GAUSSIAN
	C	ELIMINATION.
5		COMMON/PLCCNC/A(130,130),SG(130),CBR(60),SUP(60),ALPHA,NA
		DC 20 I=1,NP2
		PIVCT=A(I,I)
		A(I,I)=1.
10	10	DC 10 L=1,NP2
		A(I,L)=A(I,L)/PIVCT
		DC 20 P=1,NP2
		IF(P.EC.I) CC 10 20
		TT=A(P,I)
		A(P,I)=C.C
15		DC 15 L=1,NP2
	15	A(P,L)=A(P,L)-A(I,L)*TT
	20	CONTINUE
		RETURN
		END

```

1      SUBROUTINE AERFC(NP,P,N,NP2)
      C
      C
      C      THIS SUBROUTINE CALCULATES THE AERODYNA-
5      C      MIC INFLUENCE COEFFICIENTS ASSUMING A
      C      VORTICITY DISTRIBUTION OF  $(-CX+EY+F)I +$ 
      C       $(A+BX+CY)J$ . THERE ARE 2(NP) UNKNOWNNS
      C      AFTER THE BOUNDARY CONDITIONS HAVE BEEN
10     C      APPLIED.
      C
      C      CCPPCN/PLCCKA/X(60,4),Y(60,2),XC(60),YC(60),E(60,2),CY,CX
      C      CCPPCN/PLCCKB/AS,SSPA,S(15),NC,C(10),NB,NL(10),R(10,2),PACH
      C      CCPPCN/PLCCKC/A(130,130),SC(130),CRR(60),SUM(60),ALPHA,NA
15     C      CCPPCN/PLCCKD/IL(5),IT(5)
      C      REAL PACH,IL,IT,KL,KT
      C      INTEGER CI,C2,D3,D4,G1,G2,G3,G4
      C      PI=3.141592
      C      APLF=SCRY(1-FZCF#02)
20     C      PC=C
      C
      C      ZERO OUT THE AERODYNAMIC INFLUENCE
      C      COEFFICIENT AND CONTINUITY CONDITION
      C      ARRAY A(J,I).
25     C
      C      DO 10 I=1,NP2
      C      DO 5 J=1,NP2
      C      A(J,I)=C.
30     C      5 CONTINUE
      C      10 CONTINUE
      C
      C      START GRID CONTROL POINT ECLATION LOOP
      C
      C      DO 400 J=1,NP
35     C      RETRIEVE CONTROL POINTS AT PANEL J
      C      XCJ=XC(J)
      C      YCJ=YC(J)
      C
      C      CALCULATE THE INDUCED VELOCITY AT CONTROL
40     C      POINT J DUE TO PANEL I.
      C
      C      DO 370 I=1,PP
      C
      C      APPLY A LINEAR TRANSFORMATION TO THE CO-
45     C      ORDINATES OF PANEL I WHERE THE NEW ORIGIN
      C      IS (XCJ,YCJ).
      C
      C      X1=X(I,1)-XCJ
      C      X2=X(I,2)-XCJ
      C      X3=X(I,3)-XCJ
      C      X4=X(I,4)-XCJ
50     C
      C      CALCULATE THE 4 SINGULARITY NUMBERS AS A
      C      FUNCTION OF PANEL NUMBER.
55     C
      C      DELTA1
      C      D1=I-1

```

	C		DELTA2
		D2=1	
60	C		DELTA3
		D3=1-1+P	
	C		DELTA4
		D4=1+P	
65	C		CAPPA1
		G1=AP+P+I	
	C		CAPPA2
		G2=AP+P+I+1	
	C		CAPPA3
		G3=AP+2PM+I	
70	C		CAPPA4
		G4=AP+2PM+I+1	
	C		
	C		APPLY THE PRANETL-GLAUERT TRANSFORMATION
	C		
75		X1=X1/APCH	
		X2=X2/APCH	
		X3=X3/APCH	
		X4=X4/APCH	
	C		CALCULATE LEADING EDGE PANEL CHECK PARAP.
80	C		PRACTICE THE USE OF INTEGER ARITHMETIC.
		N8=1-(I/M)*P-1	
	C		CALCULATE TRAILING EDGE PANEL CHECK PARAP.
		N9=1-(I/M)*P	
85	C		
	C		THIS LOOP COMPUTES THE INDUCED VELOCITY
	C		AT CONTROL POINT J DUE TO PANEL I (K=1)
	C		AND THE IMAGE PANEL OF I (K=2).
	C		
		DO 36C K=1,2	
90		CUF=(-1.0)*P	
		V1=V(I,1)+VCJ*CUF	
		V2=V(I,2)+VCJ*CUF	
	C		
	C		EVALUATE THE TEN PANEL INTEGRALS.
95	C		
		CALL INT(X1,X2,X3,X4,V1,V2)	
	C		
		NL=(2*(V1-V2)*IL(3)-(X1-X3)*IL(5)-(V1*X3-X1*V2)*IL(4))/(V1-V2)	
		NT=(2*(V1-V2)*IT(3)-(X2-X4)*IT(5)-(V1*X4-X2*V2)*IT(4))/(V1-V2)	
100	C		CHECK IF PANEL I IS A WING TIP PANEL.
		IF(I.GT.(AP-P)) GO TO 200	
	C		
	C		PANEL EQUATIONS IN THIS SECTION OF THE
	C		CODE ARE FOR RECT CHORD AND INTERIOR PANELS.
105	C		
		IF(AB.AE.PC) GO TO 20	
	C		APPLY FLOW TANGENCY CONDITION AT THE L.E.
		V3=V(I+P,2)+VCJ*CUF	
		X5=(X(I+M,3)-XCJ)/APCH	
110	C		
		A(J,C1)=A(J,C1)+(V2*IL(4)-IL(5))*(X3-X1)/(V1-V2)/(V2-V3)/4/PI	
	C		
		A(J,C2)=A(J,C2)+(IL(5)-V1*IL(4))*(X5-X3)/(V1-V2)/(V2-V3)/4/PI	
		GO TO 30	

119	2C	CONTINUE
	C	$A(J,C1) = A(J,C1) + (Y2 * IL(4) - IL(5)) / (V1 - V2) / 4 / PI$
	C	$A(J,C3) = A(J,C3) + (IL(5) - V1 * IL(4)) / (V1 - V2) / 4 / PI$
120	C	
	3C	CONTINUE
	C	$A(J,C1) = A(J,C1) + ((X4 * V1 - Y2 * X2) * IT(1) - (X2 - X1) * Y2 * IL(1) - (V1 - V2) * IT(2) + (X2 - X1) * K1) * (X2 - X4) * K1 / (X2 - X1) / (V1 - V2) / 4 / PI$
129	C	$A(J,C2) = A(J,C2) + (Y2 * IT(4) - IT(5)) / (V1 - V2) / 4 / PI$
	C	$A(J,C3) = A(J,C3) + ((V1 * X4 - X1 * Y2) * IL(1) + (Y2 - V1) * IL(2) + (X1 - X4) * K1) * (V2 - V1) / (X2 - X4) / 4 / PI$
130	C	$A(J,C4) = A(J,C4) + (IT(5) - V1 * IT(4)) / (V1 - V2) / 4 / PI$
	C	IF (N4.EC.PC) CC TC 36C
139	C	$A(J,C2) = A(J,C2) + ((Y2 * X1 - X4 * V1) * IT(1) + (V1 - V2) * IT(2) + (X4 - X1) * K1) * (V1 - V2) / (X2 - X1) / 4 / PI$
	C	$A(J,C4) = A(J,C4) + ((X1 * Y2 - V1 * X3) * IL(1) - (X3 - X4) * V1 * IT(1) - (V2 - V1) * IL(2) + (X3 - X1) * K1) * (X3 - X4) * K1 / (V2 - V1) / (X3 - X4) / 4 / PI$
140	C	CC TC 36C
	20C	CONTINUE
	C	
149	C	PANEL EQUATIONS IN THIS SECTION OF THE CODE ARE FOR TIP CHORD PANELS
	C	
	C	IF (N4.EC.PC) CC TC 22C
	C	APPLY FLOW TANGENCY CONDITION AT THE L.E.
	C	ACVE - GAMMA3 = 0. APPLIES DELTA3 = 0.
150	C	AT THE L.E.
	C	$A(J,C1) = A(J,C1) + (Y2 * IL(4) - IL(5)) * (X3 - X1) / (V1 - V2) / (Y2 - V1) / 4 / PI$
	C	CC TC 23C
	22C	CONTINUE
159	C	$A(J,C1) = A(J,C1) + (Y2 * IL(4) - IL(5)) / (V1 - V2) / 4 / PI$
	C	$A(J,C3) = A(J,C3) + (IL(5) - V1 * IL(4)) / (V1 - V2) / 4 / PI$
	C	
160	23C	CONTINUE
	C	$A(J,C2) = A(J,C2) + (Y2 * IT(4) - IT(5)) / (V1 - V2) / 4 / PI$
	C	$A(J,C4) = A(J,C4) + (IT(5) - V1 * IT(4)) / (V1 - V2) / 4 / PI$
169	C	$A(J,C1) = A(J,C1) + ((X4 * V1 - Y2 * X2) * IT(1) - (X2 - X1) * Y2 * IL(1) - (V1 - V2) * IT(2) + (X2 - X1) * K1) * (X2 - X4) * K1 / (X2 - X1) / (V1 - V2) / 4 / PI$
	C	
	C	IF (N4.EC.PC) CC TC 36C
170	C	$A(J,C2) = A(J,C2) + ((Y2 * X1 - X4 * V1) * IT(1) + (V1 - V2) * IT(2) + (X4 - X1) * K1) * (V1 - V2) / (X2 - X1) / 4 / PI$

```

170      1/(Y1-Y2)/(X2-X1)/4/PI
      C
      36C CONTINUE
      37C CONTINUE
      40C CONTINUE
      C
      C
      C      THIS SECTION OF SUBROUTINE AERO FORMULATES
      C      THE AP DELTA SPECIAL CONTINUITY CONDITIONS
      C      AT THE LOWER RIGHT HAND PANEL CORNER.
      C
      C
      C      CC ECC I=1,AP
      C      IAP=I+AP
      C
      185      X1=Y(I,1)/APCH
      C      X2=Y(I,2)/APCH
      C      X3=Y(I,3)/APCH
      C      X4=Y(I,4)/APCH
      C      Y1=Y(I,1)
      C      Y2=Y(I,2)
      C
      19C      C3=I-1
      C      C4=I
      C      C1=AP+I
      C      C3=AP+I+M
      C      G4=AP+I+M+1
      C
      195      C
      C      CALCULATE YE AND LE CHECK PARAMETERS.
      C      NE=I-(I/PI)*P-1
      C      NS=I-(I/PI)*P
      C
      20C      C
      C      CHECK IF PANEL I IS A WING TIP PANEL.
      C      IF(I.EY.(IAP-P)) GO TO 500
      C
      C
      C      PANEL CONTINUITY EQUATIONS IN THIS SECTION
      C      OF THE CODE ARE FOR INTERIOR AND ROOT
      C      CHORD PANELS.
      C
      205      C
      C      IF(NS.NE.PC) GO TO 410
      C      Y3=Y(I+P,2)
      C      X5=Y(I+P,3)/APCH
      C      A(IAP,G3)=-(X5-X3)/(Y3-Y2)
      C
      210      GO TO 420
      C
      410      CONTINUE
      C
      A(IAP,G3)=-1.
      C
      420      CONTINUE
      C
      A(IAP,G4)=1.
      C
      A(IAP,G3)=A(IAP,G3)-(X1-X4)/(Y2-Y1)
      C
      A(IAP,G1)=(X3-X4)/(Y2-Y1)
      C
      IF(IAG.EQ.PC) GO TO 590
      C
      A(IAP,G4)=-(X3-X1)/(Y2-Y1)
      C
      GO TO 590
      C
      220      C
      C
      C      PANEL CONTINUITY EQUATIONS IN THIS SECTION
      C      OF THE CODE ARE FOR TIP CHORD PANELS.
      C
      C
      C
      500      CONTINUE
      C
      225      IF(NS.EQ.PC) GO TO 510
      C
      A(IAP,G3)=-1.
      C
      510      CONTINUE
      C
      A(IAP,G4)=1.

```

		$A(INP,G1)=(X3-X4)/(Y2-Y1)$
230	590	CONTINUE
	600	CONTINUE
	C	
	C	
	C	THIS SECTION OF SUBROUTINE AERO FORMULATES
	C	M ADDITIONAL DELTA CONTINUITY CONDITIONS
235	C	ON THE ROOT CHORD PANELS.
	C	
		DO 700 I=1,P
		INP=2*NP+I
		J=1
240		X1=X(J,1)/APCH
		X2=X(J,2)/APCH
		X3=X(J,3)/APCH
		X4=X(J,4)/APCH
		Y1=Y(J,1)
245		Y2=Y(J,2)
	C	
		O1=J-1
		O2=J
		G1=NP+M+J
250		G2=NP+M+J+1
		G4=NP+2*M+J+1
	C	CALCULATE TE AND LE CHECK PARAMETERS.
		N8=J-(J/M)*M-1
		N9=J-(J/M)*M
255		IF(N8.NE.MC) GO TO 710
		A(INP,G1)=-X3-X1/(Y2-Y1)
		GO TO 720
	710	CONTINUE
		A(INP,O1)=-1.
260	720	CONTINUE
		A(INP,O2)=1.
		A(INP,G1)=A(INP,G1)+(X2-X4)/(Y1-Y2)
		IF(N9.EQ.MC) GO TO 700
		A(INP,G4)=-X2-Y1/(Y1-Y2)
265		A(INP,G2)=(X4-X1)/(Y1-Y2)
	700	CONTINUE
		RETURN
		END

1	C	SUBROUTINE LOADS(NP,M,N,NP2)
	C	SURROUTINE TO CALCULATE PRESSURE DISTRI-
	C	BUTIONS AND AERODYNAMIC COEFFICIENTS.
5		COMMON/BLOCKA/X(60,4),Y(60,2),XC(60),YC(60),E(60,2),CY,CX
		COMMON/BLOCKB/NS,SSPN,ST15),KC,C(10),N9,NL(10),B(10,2),MACH
		COMMON/BLCKC/A(130,130),SG(130),CBR(60),SUM(60),ALPHA,NA
		DIMENSION DEL(60),CAM(60),CPU(60),CPL(60),CLL(60),CM(60)
		INTEGER D1,D2,D3,D4,G1,G2,G3,G4
10		REAL *ACH
		G=1.4
		AMCH=SQRT(1-MACH**2)
		PC=0
	C	SET CAMBER SLOPES EQUAL TO ZERO.
		DO 10 I=1,NP
15		CRR(I)=0.
	10	CONTINUE
	C	READ NA SETS OF ANGLE OF ATTACK AND
	C	CAMBER SLOPE DISTRIBUTION DATA.
		DO 800 L=1,NA
20		READ(5,*) ALPHA,NCCG,NPF,CY,CX
	C	
	C	NCCG IS THE CAMBER CHANGE PARAMETER.
	C	ENTER 0 TO READ A NEW CAMBER SLOPE DISTRI-
25	C	BUTION OR ENTER 1 TO RETAIN THE PREVIOUS
	C	DISTRIBUTION.
	C	
	C	NPF IS THE PRESSURE OPTION PARAMETER.
	C	ENTER 0 TO USE THE EXACT ISENTROPIC
30	C	EXPRESSION OR ENTER 1 TO USE THE LINEAR-
	C	IZED FORM.
	C	
	C	COMPUTE PRESSURES AT PANEL LOCATION CX
	C	AND CY. NOTE - PRESSURES MAY BE COMPUTED
	C	AT POINTS OTHER THAN THE CONTROL POINTS.
35	C	
	C	CALCULATE PRESSURE EVALUATION POINTS.
		CC 1= 1),PF
		YC(1)=Y-CY*(X(1,1)-X(1,1))
40		XC(1)=X-CX*(X(1,1)-X(1,1))
		IF(XC(1).GT.0) XC(1)=0
		IF(YC(1).GT.0) YC(1)=0
		IF(XC(1).LT.0) XC(1)=0
		IF(YC(1).LT.0) YC(1)=0
		IF(XC(1).GT.0) XC(1)=0
		IF(YC(1).GT.0) YC(1)=0
		IF(XC(1).LT.0) XC(1)=0
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		IF(YC(1).GT.0) YC(1)=0
		IF(XC(1).LT.0) XC(1)=0
		IF(YC(1).LT.0) YC(1)=0
		IF(XC(1).GT.0) XC(1)=0
		IF(YC(1).GT.0) YC(1)=0

```

      50 C(1)=XCP
      55 C(1)=XCP
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      990 C
      995 C

```


115		GAM1=SG(G1) GAM3=SG(G3) IF(N9.FO.MC) GO TO 400 GAM2=SG(G2) GAM4=SG(G4)
120		GO TO 400
	300	CONTINUE
	C	
	C	
	C	SINGULARITIES DETERMINED IN THIS SECTION ARE FOR TIP CHORD PANELS.
125	C	
		IF(NR.NE.MC) GO TO 310 DEL1=(X3-X1)/(Y2-Y1)*SG(G1) GO TO 320
	310	CONTINUE
130		DEL1=SG(D1)
		DEL3=SG(D3)
	320	CONTINUE
		DEL2=SG(D2)
		DEL4=SG(D4)
135		GAM1=SG(G1) IF(N9.FO.MC) GO TO 400 GAM2=SG(G2)
	400	CONTINUE
	C	
140		AL=(Y2*(X3-X4)*GAM1+(Y1*X4-X1*Y2)*GAM3+(X1*Y2-Y1*X3)*GAM4) 1/(Y2-Y1)/(X3-X4)
	C	
		BL=(GAM3-GAM4)/(X3-X4)
145		CL=(-(X3-X4)*GAM1+(X1-X4)*GAM3+(X3-X1)*GAM4)/(Y2-Y1)/(X3-X4)
	C	
		AT=(Y1*(X2-X1)*GAM4+(Y2*X1-X4*Y1)*GAM2+(X4*Y1-Y2*X2)*GAM1) 1/(X2-X1)/(Y1-Y2)
	C	
150		BT=(GAM2-GAM1)/(X2-X1)
	C	
		CT=(-(X2-X1)*GAM4+(X4-X1)*GAM2+(X2-X4)*GAM1)/(Y1-Y2)/(X2-X1)
	C	
	C	CALCULATE LIFT AND MOMENT INCREMENTS
155	C	
		X5=X1+(X3-X1)*(Y1-Y1)/(Y2-Y1) X4=X1+(X4-X1)*(Y1-Y1)/(Y2-Y1) X7=X2+(X4-X2)*(Y1-Y1)/(Y2-Y1)
	C	
160		IF(NR.NE.MC) GO TO 410 XLE=X5
	410	CONTINUE
	C	
		CLL(I)=2*(AL+CL*Y1)*(X6-Y5)+BL*(X6**2-X5**2)
165		CLL(I)=CLL(I)+2*(AT+CT*Y1)*(X7-X6)+BT*(X7**2-X6**2)
	C	
		CM(I)=(AL+CL*Y1)*(X6**2-X5**2)+2*BL*(X6**3-X5**3)/3
		CM(I)=CM(I)+(AT+CT*Y1)*(X7**2-X6**2)+2*BT*(X7**3-X6**3)/3
		I-XLE*CLL(I)
170	C	
	C	

	C	DETERMINE IF UPPER OR LOWER PANEL VORTICITY EQUATIONS ARE TO BE USED
	C	
175	C	CALCULATE THE X CO-ORDINATE OF THE PANEL MAIN DIAGONAL AT YI.
	C	$XD = X1 + (X4 - X1) / (Y2 - Y1) * (YI - Y1)$
	C	CHECK XI TO DETERMINE IF IT LIES ABOVE OR BELOW THE MAIN DIAGONAL. NOTE - IF
180	C	$XI < XD$, THEN IT LIES BELOW THE MAIN DIAGONAL.
	C	IF $(XI > XD)$ GO TO 450
	C	UPPER PANEL EQUATIONS ARE IN THIS SECTION.
185	C	$EL = (DEL1 - DEL3 + (X1 - Y3) * CL) / (Y1 - Y2)$
	C	$FL = (-Y2 * DEL1 + Y1 * DEL3 + (Y1 * X3 - X1 * Y2) * CL) / (Y1 - Y2)$
	C	$GAM(I) = AL * BL * XI + CL * YI$
190	C	$DEL(I) = FL - CL * XI + EL * YI$
	C	GO TO 500
	450	CONTINUE
	C	LOWER PANEL EQUATIONS ARE IN THIS SECTION
195	C	
	C	$ET = (DEL2 - DEL4 + (X2 - X4) * CT) / (Y1 - Y2)$
	C	$FT = (-Y2 * DEL2 + Y1 * DEL4 + (Y1 * X4 - X2 * Y2) * CT) / (Y1 - Y2)$
200	C	$GAM(I) = AT * RT * XI + CT * YI$
	C	$DEL(I) = FT - CT * XI + ET * YI$
	500	CONTINUE
	600	CONTINUE
205	C	CALCULATE UPPER AND LOWER SURFACE CPS.
	C	
	C	IF $(NPF.EQ.1)$ GO TO 604
	C	EXACT ISENTROPIC PRESSURE EXPRESSION.
210	C	$DUM1 = 27 * G * MACH ** 2$
	C	$DUM2 = (G - 1) * MACH ** 2 / 2$
	C	$DUM3 = G / (G - 1)$
	C	DO 601 I=1,NP
	C	$DUM4 = TSUM(I) ** 2 * (GAM(I) / 2) ** 2 + (DEL(I) / 2) ** 2$
215	C	$CPU(I) = DUM1 * ((1 - DUM2 * (GAM(I) * DUM4)) * DUM3 - 1)$
	C	$CPL(I) = DUM1 * ((1 - DUM2 * (-GAM(I) * DUM4)) * DUM3 - 1)$
	601	CONTINUE
	C	GO TO 606
	604	CONTINUE
220	C	LINEARIZED PRESSURE EXPRESSION.
	C	$DUM1 = (1 - MACH ** 2)$
	C	DO 605 I=1,NP
	C	$DUM2 = DUM1 * (GAM(I) / 2) ** 2 + (SUM(I)) ** 2 + (DEL(I) / 2) ** 2$
	C	$CPU(I) = (-GAM(I) * DUM2)$
225	C	$CPL(I) = (-GAM(I) * DUM2)$
	605	CONTINUE
	606	CONTINUE
	C	

	C	START OUTPUT SEQUENCE
230	C	
		WRITE(6,610) ALPHA
	610	FORMAT(////29X,*ALPHA =*,F6.2)
		WRITE(6,620)
	620	FORMAT(//12X,*PRESSURE AND SINGULARITY STRENGTH DISTRIBUTION*)
235		WRITE(6,625) CX,CY
	625	FORMAT(/30X,*XP =*,F6.3/30X,*YP =*,F6.3)
		IF(NPF.EQ.1) GO TO 627
		WRITE(6,626)
	626	FORMAT(/33X,*EXACT*)
240		GO TO 629
	627	CONTINUE
		WRITE(6,628)
	628	FORMAT(/32X,*LINEAR*)
	629	CONTINUE
245		N1=1
		DO 760 J=1,N
		DUM=S(J)+CY*(S(J+1)-S(J))
		WRITE(6,630) DUM
	630	FORMAT(//30X,*Y/S =*,F6.3/ 6X,*X/C*,4X,*CAMBER*,5X,*DEL*,6X,*GAM*,
250		16X,*CPU*,6X,*CPL*,6X,*DELCP*)
		DO 750 I=1,M
		DUM1=C(I)+CX*(C(I+1)-C(I))
		DUM2=CPL(N1)-CPU(N1)
		WRITE(6,640) DUM1,CBR(N1),DEL(N1),GAM(N1),CPU(N1),CPL(N1),DUM2
255	640	FORMAT(13X,F6.3,3X,F6.2,2X,F8.5,3(1X,F8.5),2X,F8.5)
		N1=N1+1
	750	CONTINUE
	760	CONTINUE
260	C	
	C	THIS SECTION OF THE OUTPUT TABULATES THE
	C	LOADS SUMMARY WHICH INCLUDES CL AND CH.
	C	
		WRITE(6,765)
	765	FORMAT(//29X,*LOADS SUMMARY*/17X,*YS*,5X,*X-CP*,4X,*CHORD*,
265		16X,*CL*,6X,*CH*)
		N91=N9-1
		N4=0
		DO 790 K=1,N91
		N1=NL(K)
270		N2=NL(K+1)-1
		N3=NL(K+1)
		DO 790 J=N1,N2
		DUM=S(J)+CY*(S(J+1)-S(J))
		DUM1=(DUM-S(N1))/(S(N3)-S(N1))
275		DUM2=(R(K,2)-R(K,1))*(1-DUM1)+DUM1*(R(K+1,2)-R(K+1,1))
		DUM3=0.
		DUM4=0.
		DO 770 I=1,M
		N4=N4+1
280		DUM3=DUM3+CLL(N4)
		DUM4=DUM4+CM(N4)
	770	CONTINUE
		DUM5=DUM4/DUM3/DUM2
		DUM3=DUM3/DUM2
285		DUM4=DUM4/(DUM2**2)

		WRITE(6,775) DUM,DUM5,DUM2,DUM3,DUM4
	775	FORMAT(14X,F5.2,3X,F5.2,2X,F7.2,2X,F7.2,2X,F6.3)
	780	CONTINUE
	790	CONTINUE
290	800	CONTINUE
		STOP
		END

Vita

John Charles Sparks was born on 24 October 1947 in Elwood, Indiana. He graduated from high school in Xenia, Ohio, in 1965 and attended Wright State University in Dayton, Ohio. He received the degrees of Bachelor of Science (Mathematics) in June of 1969 and Master of Science (Mathematics) in June of 1971. In January of 1974, he accepted employment as a Mathematician for the Air Force Flight Dynamics Laboratory at Wright-Patterson Air Force Base, Ohio. He is currently assigned to the Aeronautical Systems Division (ASD/ENFSL) as an Aerospace Engineer.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Paneling methods are approximate techniques for solving flow problems over wings and bodies. Vortex panels are used to model flow over wings and other lifting surfaces. The author develops a triangular vortex panel having a vorticity distribution that can vary in magnitude and direction. This panel is used to predict the pressure distribution on a rectangular and a sweptback wing in subsonic flow. Lift distributions obtained compare favorably to Anderson's solution and wind tunnel results except near the wing tip. In this region, the		

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distribution will spike before satisfying the Kutta condition imposed at the tip. Possible remedies for the tip problem are discussed.

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