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ANALYTIC REPRESENTATION OF THE JACCHIA 1977 MODEL ATMOSPHERE

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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>Modified Jacchia-Walker temperature profiles are developed to accurately represent the Jacchia 1977 temperature profiles above 125 km. The diffusive equilibrium equations are analytically integrable for these profiles, greatly reducing the need for tabular storage of solutions.<br>409990 Ym |                                     |                                                                                                |  |

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## Analytic Representation of the Jacchia 1977 Model Atmosphere.

### 1. INTRODUCTION

The diffusion equations for the Jacchia 1977 (J77) density model<sup>1</sup> cannot be integrated in closed form. Hence numerical results must be tabulated, requiring extensive storage, if all molecular constituents are tabulated individually. Fortunately this storage requirement can be reduced considerably if one notes that the solutions for constituents in diffusive equilibrium can be expressed as<sup>2</sup>:

$$n_i(h, T_\infty) = n_i(h_0, T_\infty) \left[ \frac{T(h_0, T_\infty)}{T(h, T_\infty)} \right]^{1+\alpha_i} \cdot \exp[-M_i F(h_0, h, T_\infty)]$$

where

$n_i$  = density of  $i^{\text{th}}$  molecular constituent

$h$  = height

$T_\infty$  = exospheric temperature

$T$  = local temperature

$\alpha_i$  =  $i^{\text{th}}$  constituent thermal diffusion coefficient

$M_i$  =  $i^{\text{th}}$  constituent molecular mass

$F = \int_{h_0}^h g(z) / [R^* T(z, T_\infty)] dz$

$g(z)$  = acceleration of gravity at height  $z$ .

$R^* = 8.31432 \times 10^3 \text{ J(Kg-mol)}^{-1}/\text{K}$

In general one then need tabulate only  $F$  as a function of  $h$  and  $T_\infty$  and the  $n_i$  at the boundary  $h_0$ . If  $h_0$  is chosen to be the homopause, 100km, only one  $n_i$  need be stored, from which the others are derived.

In this paper we explore alternative profiles for a large portion of the diffusive region ( $h \geq 125\text{km}$ ) for which  $F$  can be expressed analytically, thus further reducing storage requirements.

## 2. TEMPERATURE PROFILES ( $h \geq 125\text{km}$ )

Since a fully analytic solution for the temperature profiles specified in J77 is not possible it seems worthwhile to explore other forms. In particular the Jacchia-Walker (JW) profile<sup>3</sup>:

$$T_{JW}(h, T_\infty) = T_\infty - (T_\infty - T_0) e^{-\sigma \xi}$$

leads to the solution

$$F(h_0, h, T_\infty) = \frac{g_e R_e^2}{R^* (R_e + h_0)^2 T_\infty} \left\{ \ln \left[ \frac{T(h, T_\infty)}{T_0} \right] + \sigma \xi \right\}$$

where

$$\xi = (h - h_0) (R_e + h_0) / (R_e + h) \text{ (geopotential altitude)}$$

$$R_e = 6356.766\text{km}$$

$$g_e = \text{gravitational acceleration at Earth's surface} \\ = 9.80665 \text{ m/sec}^2$$

$$T_0 = T(h_0, T_\infty)$$

$$\sigma = \text{gradient parameter}$$

For application to J77, it is prudent to choose  $h_0 = 125\text{km}$ , since this is the inflection point, although the various constituent densities at this height cannot be conveniently derived from one of them, as at 100 km. The gradient parameter may be chosen to fit the exact J77 profile in some fashion, such as to match the slope at 125km; i.e.

$$\sigma = G_0 / (T_\infty - T_0) = (dT/dh) / (T_\infty - T_0)$$

Results from this model deviate somewhat from the exact J77 model; In particular, for an exospheric temperature of 900K the local temperature deviation is 20K at 160km and the mass and argon densities differ by 6% and 12% at 225km.

To obtain better fits the form

$$T = [T_{JW} + f]^{-1}$$

has been explored, where the function  $f$  is chosen to retain the closed-form integrability of the diffusion equation. In addition it is desirable that the resulting total temperature fit smoothly the exact J77 profile at  $h = h_0 = 125\text{km}$  and maintain the inflection point character ( $d^2T/dh^2 = 0$ ). It is evidently the lack of this latter characteristic which makes the simple JW form undesirable and possibly accounts for much of the error resulting from its use. These modifications lead to the following boundary conditions at  $\xi = 0$ .

$$f = df/d\xi = 0$$

$$d^2f/d\xi^2 = G_0^2 \zeta / T_0^2$$

where

$$G_0 = [dT/dh]_{h=h_0}$$

$$\zeta = \sigma + 2/(R_e + h_0) = G_0/(T_\infty - T_0) + 2/(R_e + h_0)$$

For convenience the functional form is chosen to have an exponentially decreasing character with parameters hopefully related to  $\sigma$ . One such form is

$$f = - \left[ G_0 \zeta / (2\beta_1^2 T_0^2) \right] f_{\beta_1}(\xi)$$

where

$$f_{\beta}(\xi) = e^{-\beta\xi} (1 - e^{-\beta\xi})^2$$

and  $\beta$  is an adjustable parameter. To provide further flexibility the form

$$f(\xi) = - \left[ G_0 \zeta / (2\beta_1^2 T_0^2) \right] f_{\beta_1}(\xi) + C_2 \xi f_{\beta_2}(\xi)$$

has been chosen. The additional term  $j = C_2 \xi f_{\beta_2}$  satisfies

$$j = j' = j'' = 0$$

at  $\xi = 0$ .

For this profile the solution is given by

$$F(h_0, h, T_\infty) = F_{JW} + K \left\{ C_1 \left( \sum_{i=1}^3 a_i q_1^i + 1/3 \right) / \beta_1 \right. \\ \left. + C_2 \left[ \sum_{i=1}^3 (\beta_2 \xi a_i - b_i) q_2^i + 11/18 \right] / \beta_2^2 \right\}$$

where  $F_{JW}$  = Jacchia-Walker solution

$$C_1 = -G_0 \xi / (2 \beta_1^2 T_0^2)$$

$$a_1 = -a_2 = -1, a_3 = -1/3$$

$$q_i = e^{-\beta_i \xi}$$

$$b_i = a_i / i$$

$$K = g_e R_e^2 / [R^* (R_e + h_0)^2]$$

### 3. RESULTS

The Fletcher-Powell non-linear function minimization method<sup>4,5</sup> has been used to determine the parameters  $B_1$ ,  $B_2$ ,  $C_2$  to minimize the sum of squared residuals

$$Q = \sum (T - T_{J77})^2$$

for  $T_\infty = 500, 700, 900, 1100, 1300, 1500, 1700, 1900$ . By inspection the results are found to be reasonably well represented by:

$$B_1 = 2\sigma \left[ 1 + 10^{-4} (T_\infty - 800) \right] \quad T_\infty \leq 1100K$$

$$B_2 = 0.0215 - 0.005 (T_\infty - 500)/200$$

$$C_2 = 10^{-5} (0.0566 - 0.08x + 0.04x^2); \quad x = (T_\infty - 900)/200$$

$$1100K \leq T_\infty \leq 1500K$$

$$B_1 = 0.0385 - 0.012y + 0.0123y^2; \quad y = (T_\infty - 1100)/400$$

$$B_2 = 0.0065 + 0.0167y$$

$$C_2 = 10^{-5} (0.0166 - 0.4548y^2)$$

$$T_\infty \geq 1500K$$

$$B_1 = 0.0388 - 0.0045z; \quad z = (T_\infty - 1500)/400$$

$$B_2 = 0.0232 - 0.0040z$$

$$C_2 = -10^{-5} (0.4382 + 0.0387z)$$

Mass and constituent densities were computed using the Jacchia-Bass (JB) formulation presented here for the  $T_{\infty}$  region 500K - 1900K at 100K steps and the  $h$  region 130km - 1000km at 10km steps. The results were compared with those from the exact J77 temperature profiles, except that vertical flux is ignored for H. Table I summarizes maximum % deviations for a 6-constituent gas ( $O_2$ , O,  $N_2$ , He, Ar, H) with the total mass density given by

$$\rho = \sum_{i=1}^6 M_i n_i / A; \quad A = \text{Avogadro's Number}$$

It should be noted that a true J77 calculation would call for different exospheric temperatures for the different constituents to model the different diurnal phases; here all the  $n_i$  are computed for the same  $T_{\infty}$ . Argon, with the largest mass, yields the largest disagreement among the constituents. The agreement is quite good, well within typical model-experiment differences such as those published by Forbes, Marcos and Gillette<sup>6</sup>, and Sharp and Brag.<sup>7</sup> It should be noted that the maximum deviations are all within the 1100K - 1500K region, which proved to be the most difficult to fit. Outside this region all JB mass densities agree with J77 within 1% at all heights. Maximum temperature deviations are 13K at  $T_{\infty} = 1300K$ ,  $h = 350km$ . Outside the 1100K - 1500K region the maximum disagreement is 5K.

A FORTRAN deck encompassing the formulation presented here is available on request.

Table I. Maximum Absolute Value of JB-J77 Deviations (%)

|                        | <u>TOTAL DENSITY</u>                 | <u>Ar DENSITY</u>                     |
|------------------------|--------------------------------------|---------------------------------------|
| $h \leq 300\text{Km}$  | 1.1 ( $h = 200, T_{\infty} = 1200$ ) | 1.5 ( $h = 190, T_{\infty} = 1200$ )  |
| $h \leq 500\text{Km}$  | 2.1 ( $h = 500, T_{\infty} = 1300$ ) | 5.6 ( $h = 500, T_{\infty} = 1300$ )  |
| $h \leq 1000\text{Km}$ | 3.0 ( $h = 750, T_{\infty} = 1300$ ) | 8.5 ( $h = 1000, T_{\infty} = 1300$ ) |

#### 4. CONCLUSIONS

It has been shown that analytically solvable temperature profiles can be adjusted to realistically represent temperature vs. height variations above 125km, allowing one to reduce tabular storage requirements and/or avoid the inconvenience of interpolation. It is therefore recommended that similar profiles be used in future atmospheric density modelling. The possibility of developing analytically integrable profiles below 125km will be studied at a later date.

5. REFERENCES

1. Jacchia, L. G., "Thermospheric Temperature, Density, and Composition: New Models", Smithsonian Astrophysical Observatory, Special Report Number 375, 1977.
2. Bass, J. N., "Condensed Storage of Diffusion Equation Solutions for Atmospheric Density Model Computations", AFGL-TR-80-0038, 1978.
3. Walker, J. C. G., "Analytic Representation of Upper Atmospheric Densities Based on Jacchia's Static Diffusion Models" J. Atm. Sci. 22, p. 462, 1965.
4. Fletcher, R., and Powell, M. J. D., "A Rapidly Convergent Descent Method for Minimization", Computer Journal 6, p. 163, 1963.
5. International Business Machines Corp., "System/360 Scientific Subroutine Package", Fifth Edition, p. 221, 1970.
6. Forbes, J. M., Marcos, F. A., and Gillete, D. F., "An Evaluation of Thermospheric Models", AFGL-TR-78-0140, 1979.
7. Sharp, L. R., and Prag, A. B., "The Construction of Thermospheric Density Correction Tables for Use in Satellite Trajectory and Lifetime Predictions", SAMSO-TR-78-87, 1978.