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ALGORITHMS FOR COMPUTING APPARENT SPEED.(U)

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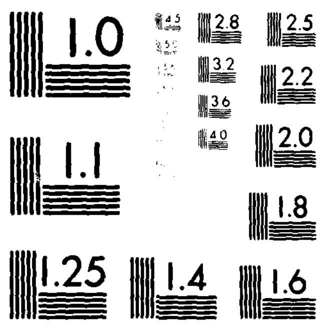


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# FOREIGN TECHNOLOGY DIVISION

AIR FORCE SYSTEMS COMMAND



## ALGORITHMS FOR COMPUTING APPARENT SPEED

by

A. P. Tkachenko



FOREIGN TECHNOLOGY DIVISION

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## U. S. BOARD ON GEOGRAPHIC NAMES TRANSLITERATION SYSTEM

| Block | Italic | Transliteration | Block | Italic | Transliteration |
|---|---|---|---|---|---|
| А а | *А а* | A, a | Р р | *Р р* | R, r |
| Б б | *Б б* | B, b | С с | *С с* | S, s |
| В в | *В в* | V, v | Т т | *Т т* | T, t |
| Г г | *Г г* | G, g | У у | *У у* | U, u |
| Д д | *Д д* | D, d | Ф ф | *Ф ф* | F, f |
| Е е | *Е е* | Ye, ye; E, e* | Х х | *Х х* | Kh, kh |
| Ж ж | *Ж ж* | Zh, zh | Ц ц | *Ц ц* | Ts, ts |
| З з | *З з* | Z, z | Ч ч | *Ч ч* | Ch, ch |
| И и | *И и* | I, i | Ш ш | *Ш ш* | Sh, sh |
| Й й | *Й й* | Y, y | Щ щ | *Щ щ* | Shch, shch |
| К к | *К к* | K, k | Ъ ъ | *Ъ ъ* | " |
| Л л | *Л л* | L, l | Ы ы | *Ы ы* | Y, y |
| М м | *М м* | M, m | Ь ь | *Ь ь* | ' |
| Н н | *Н н* | N, n | Э э | *Э э* | E, e |
| О о | *О о* | O, o | Ю ю | *Ю ю* | Yu, yu |
| П п | *П п* | P, p | Я я | *Я я* | Ya, ya |

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

## RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

| Russian | English | Russian | English | Russian | English |
|---|---|---|---|---|---|
| sin | sin | sh | sinh | arc sh | $\sinh^{-1}$ |
| cos | cos | ch | cosh | arc ch | $\cosh^{-1}$ |
| tg | tan | th | tanh | arc th | $\tanh^{-1}$ |
| ctg | cot | cth | coth | arc cth | $\coth^{-1}$ |
| sec | sec | sch | sech | arc sch | $\text{sech}^{-1}$ |
| cosec | csc | csch | csch | arc csch | $\text{csch}^{-1}$ |

| Russian | English |
|---|---|
| rot | curl |
| lg | log |

# ALGORITHMS FOR COMPUTING APPARENT SPEED

A. P. Tkachenko

In controlling moving objects of several classes, it is expedient to use information on the apparent speed of the object. The apparent speed will be called the vector value, equal to the increase in integral from the vector of the apparent acceleration from the beginning moment in time t=0 to some moment t. This same apparent acceleration is the difference between the acceleration of an object relative to an immobile coordinate system and the acceleration of gravity [1]. (Hereafter, by speed or acceleration of an object we means the speed or acceleration of a certain point of it, such as the center of mass). The speed of an object relative to an immobile coordinate system equals the vector total of the apparent speed of an object, the vector of initial speed of an object at moment t=0 and the integral of acceleration of gravity in the area of origination of the object within t=0 to the current moment in time t.

Let us connect the right orthogonal triangle xyz with the object. Its orientation relative to some nonrotating trihedron $\xi\eta\xi$ will be characterized by a matrix of directrices of cosines L:

$$L = \begin{bmatrix} l_{x\xi} & l_{x\eta} & l_{x\zeta} \\ l_{y\xi} & l_{y\eta} & l_{y\zeta} \\ l_{z\xi} & l_{z\eta} & l_{z\zeta} \end{bmatrix}. \qquad (1)$$

This matrix satisfies the differential equation

$$\dot{L} = -\Omega L, \qquad (2)$$

where

$$\Omega = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix} \quad (3)$$

($\omega_x, \omega_y, \omega_z$ projections of angular speed of an object on axis x,y,z).

Let us indicate through $W = (W_\xi, W_\eta, W_\zeta)$ the vector of apparent speed of an object assigned in the coordinate system $\xi,\eta,\zeta$, and through $w=(w_x,w_y,w_z)$ - the same vector assigned in the coordinate system xyz:

$$W = L^T w \quad (4)$$

(index T indicates transposition).

In the center of mass of the object, let us establish three integrators of linear accelerations, the sensitivity axes of which are parallel to axes x,y,z. Signs $b_x$, $b_y$, $b_z$ of these integrators are the essence of increase of integrals from projections of the apparent acceleration of an object on axis x, y, z for some period of discretion (step) h:

$$b_x = \int_{t_k}^{t_k+h} (l_{x\xi}\dot{W}_\xi + l_{x\eta}\dot{W}_\eta + l_{x\zeta}\dot{W}_\zeta)\, dt = \int_{t_k}^{t_k+h} (\dot{w}_x + \omega_y w_z - \omega_z w_y)\, dt \quad (xyz). \quad (5)$$

For shortening the written form, we use here a symbol of cyclical transposition of indexes. Introducing the symbolic vector $b=(b_x,b_y,b_z)$, we present the equations in (5) in a matrix form:

$$b = \int_{t_k}^{t_k+h} L\dot{W}\, dt = \int_{t_k}^{t_k+h} (\dot{w} + \Omega w)\, dt. \quad (6)$$

Let us also propose that with the aid of the three integrators of angular speed, the sensitivity axes of which are parallel to axes x, y, z, we measure the increase $\theta_x$, $\theta_y$, $\theta_z$ of integrals from values $\omega_x$, $\omega_y$, $\omega_z$ for the same period of discretion:

$$\theta_x = \int_{t_k}^{t_k+h} \omega_x dt \quad (xyz). \quad (7)$$

Below, we examine algorithms for computing the apparent speed of an object by processing signals $b_x$, $b_y$, $b_z$, $\theta_x$, $\theta_y$, $\theta_z$ with the aid of a digital computer.

Let us first examine the problem of finding values $w_x$, $w_y$, $w_z$. Let us introduce matrix $\Theta$ with the aid of equation

$$\Theta = \int_{t_k}^{t_k+h} \Omega dt = \begin{bmatrix} 0 & -\theta_z & \theta_y \\ \theta_z & 0 & -\theta_x \\ -\theta_y & \theta_x & 0 \end{bmatrix}. \quad (8)$$

Let us propose that matrix Ω can be represented by the Taylor series in the area of the beginning of the step $t=t_k$:

$$\Omega = \Omega_k + \dot{\Omega}_k (t - t_k) + \frac{1}{2}\ddot{\Omega}_k (t - t_k)^2 + \cdots \quad (9)$$

$$(t_k < t < t_k + h),$$

then

$$\Theta = h\Omega_k + \frac{1}{2} h^2\dot{\Omega}_k + \frac{1}{6} h^3\ddot{\Omega}_k + \cdots \quad (10)$$

Let us write vector w in the form of the Taylor series in the area of moment in time $t=t_k$:

$$w = w_k + \dot{w}_k (t - t_k) - \frac{1}{2} \ddot{w}_k (t - t_k)^2 + \cdots \quad (11)$$

$$(t_k < t < t_k + h).$$

Using equations (9), (10) and (11), we transform expression (6) to the form

$$b = \Delta w + \Theta\left(w_k + \frac{1}{2}\Delta w\right) + \frac{1}{12} h^3 (\dot{\Omega}_k \dot{w}_k - \Omega_k w_k) + 0(h^4). \quad (12)$$

Here Δw - increase in vector w at step

$$\Delta w = h\dot{w}_k + \frac{1}{2} h^2 \ddot{w}_k + \cdots \quad (13)$$

Equation (12) has a matrix form of a system of three linear algebraic equations relative to values $\Delta w_x$, $\Delta w_y$, $\Delta w_z$. The solution to this system of equations is also singular. Considering step h of the value of the first order of smallness, let us reject (in the recalled solution) members up to the third order of smallness inclusively:

$$\Delta w = b + \left(-\Theta + \frac{1}{2}\Theta^2\right)\left(w_k + \frac{1}{2} b\right) - \frac{1}{4}\Theta^3 w_k - \frac{1}{12} h^3 (\dot{\Omega}_k \dot{w}_k - \Omega_k \dot{w}_k). \quad (14)$$

Dropping into the expression $w_{k+1}=w_k+\Delta w$ values of the higher second order of smallness, we acquire an algorithm of the second order for determining vector w:

$$w_{k+1} = w_k + b + \left(-\Theta + \frac{1}{2}\Theta^2\right)\left(w_k + \frac{1}{2} b\right). \quad (15)$$

In the scalar form

$$w_{x,k+1} = \frac{1}{2} b_x + \left[1 - \frac{1}{2}(\theta_y^2 + \theta_z^2)\right]\left(w_{xk} + \frac{1}{2} b_x\right) + \left(\theta_z + \frac{1}{2}\theta_x\theta_y\right) \times$$

$$\times \left(w_{yk} + \frac{1}{2} b_y\right) - \left(\theta_y - \frac{1}{2}\theta_x\theta_z\right)\left(w_{zk} + \frac{1}{2} b_z\right) \quad (xyz).$$

The rejected members of the third order of smallness compose the error of algorithm (15) at step

$$\delta_k w = \frac{1}{12} h^3 (3\Omega_k^3 w_k + \dot{\Omega}_k \dot{w}_k - \Omega_k \ddot{w}_k). \tag{17}$$

The algorithm of the first order of computation of the apparent speed can be acquired by rejecting the members of the second order which enter into algorithm (16)

$$w_{x,k+1} = w_{xk} + b_x + \theta_z \left(w_{yk} + \frac{1}{2} b_y\right) - \theta_y \left(w_{zk} + \frac{1}{2} b_z\right) \quad (xyz). \tag{18}$$

The error of this algorithm on the step is characterized by a matrix expression

$$\delta_k w = -\frac{1}{2} h^2 \Omega_k^2 w_k. \tag{19}$$

Let us examine algorithms of computation of values $W_\xi, W_\eta, W_\zeta$ . For finding vector W we can use relationship (4) after some number of steps of computations according to formula (16) or (18). Here, we must, with the proper accuracy, compute matrix L, for example by means of integration of equation (2). Another method of finding W consists of using relationship

$$W_{n+1} = W_n + L_{n+1}^T \Delta w_n. \tag{20}$$

Here $W_n$, $W_{n+1}$ - values of W at moments in time $t_n$ and $t_{n+1}$; $L_{n+1}$ - value of matrix L at the moment in time $t_{n+1}$; $\Delta w_n$ - increase in vector w in the length of time ($t_n$, $t_{n+1}$). If this length equals in duration several steps h (on the order of 5-10), then for finding increase in $\Delta w_n$ we can use algorithm (18) with step h with the following conditions at moment in time $t_n$: $w_{xn}=w_{yn}=w_{zn}=0$. After accomplishing transformation according to formula (20), we again propose that w=0 and proceed with computations according to formulas (18) with step h [2]. The error of computing w at the step with the same calculation scheme, as we can see from formulas (17) and (19) is estimated approximately by the expression

$$\delta_k w = \frac{1}{12} h^3 \left(\dot{\Omega}_k \dot{w}_k - \Omega_k \ddot{w}_k\right). \tag{21}$$

If the transformation in (20) is done at each step ($t_{n+1}=t_n+h$) and value h is sufficiently small, then in the first approximation $\Delta w_n=b$. Hence, we acquire the algorithm of the first order

$$W_{k+1} = W_k + L_{k+1}^T b. \tag{22}$$

In estimating the error of this algorithm, we can propose that $b \approx w_k h$. The error which corresponds to algorithm (22) of finding the vector w at the step, as we can see from a comparison with formulas in (18) with $w_k=0$, is determined as

$$\delta_k w = \frac{1}{2} h^2 \Omega_k \dot{w}_k. \quad (23)$$

A more accurate expression for the increase in $\Delta w_k$ at the step can be acquired by proposing in formula (14) w=0:

$$\Delta w_k = b - \frac{1}{2}\left(\Theta - \frac{1}{2}\Theta^2\right) b + 0(h^3). \quad (24)$$

Examining the same formulas of numerical integration of equation (2), we can be sure that values $L_k$ and $L_{k+1}$ of matrix L at the beginning and at the end of the step satisfy the relationship

$$L^T_{k+1} = \frac{1}{2}(L^T_{k+1} + L^T_k) + \frac{1}{4}(L^T_{k+1} + L^T_k)\Theta + 0(h^3). \quad (25)$$

Substituting expressions (24) and (25) into formula (20) and dropping the values of the third order of smallness, we acquire the algorithm of the second order of computation of W:

$$W_{k+1} = W_k + \frac{1}{2}(L^T_{k+1} + L^T_k) b. \quad (26)$$

In the scalar form, this algorithm has the form

$$W_{\xi,k+1} = W_{\xi k} + \frac{1}{2}[(l_{x\xi,k+1} + l_{x\xi k}) b_x + (l_{y\xi,k+1} + l_{y\xi k}) b_y + (l_{z\xi,k+1} + l_{z\xi k}) b_z] \quad (\xi\eta\zeta). \quad (27)$$

The error of algorithm (26) at the step in projections to the axes of the trihedron xyz is determined by expression (21).

We can show that the methodological (accumulated) error of $\delta W$ of computing W with sufficient accuracy is approximated by formula

$$\delta W = \frac{1}{h}\int_0^t L^T \delta_t w\, dt, \quad (28)$$

where $\delta_t w$ - expression for the error of computations at the step in which the current values of kinematic parameters are figured. So, with the use of algorithm (18)

$$\delta W = -\frac{1}{2} h \int_0^t L^T \Omega^2 w\, dt. \quad (29)$$

With the use of the same algorithm (22) the methodological error has the form

$$\delta W = \frac{1}{2} h \int_0^t L^T \Omega \dot{w}\, dt. \quad (30)$$

Accomplishing in equation (30) integration by parts and using the equation in (2), we acquire

$$\delta W' = \frac{1}{2} h L^T \Omega w - \frac{1}{2} h \int_0^t L^T \dot{\Omega} w dt + \frac{1}{2} h \int_0^t L^T \Omega^2 w dt. \qquad (31)$$

With considerable angular fluctuations of the object, the last member of formula (31), distinguished only by sign from expression (29), is dominating, since it has the tendency to grow with time. Thus, the accumulated errors of algorithms of the first order (18) and (22) are close in value and opposite in sign.

Algorithms (26) and (18) with recalculation according to the formula in (22) and with a return to null of the initial conditions provide an error

$$\delta W = \frac{1}{12} h^2 \int_0^t L^T (\dot{\Omega} \dot{w} - \Omega \ddot{w})\, dt. \qquad (32)$$

Finally, an estimate of the error of algorithm in (15) has the form

$$\delta W' = \frac{1}{12} h^2 \int_0^t L^T (\dot{\Omega} \dot{w} - \Omega \ddot{w} + 3\Omega^3 w)\, dt. \qquad (33)$$

The selection of one or another of these algorithms is determined by the characteristics of the measured elements and the computer and the requirements for accuracy in determining the apparent speed.

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