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METHODS FOR ASSESSING THE ROBUSTNESS PROPERTIES OF A LINEAR MUL--ETC(U)
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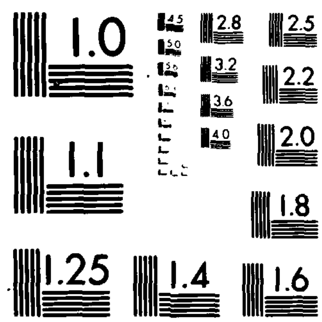

MICROCOPY RESOLUTION TEST CHART

Lecture notes

Methods for assessing the robustness properties of a linear multivariable feedback design

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Abstract

On completing the design of a multivariable feedback system it is always desirable to be able to predict that under certain changes the system will remain stable. In these notes some methods are described for determining the robustness properties of a multivariable feedback system under

(i) simultaneous variation of sensor (or actuator) gains;

(ii) simultaneous nonlinear perturbations of loop gains; and

(iii) plant parameter uncertainty.

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1. Introduction

In the inverse Nyquist array method[1] for multivariable feedback design, the designer,having made the inverse of the open-loop system diagonally dominant,can by examination of the open-loop inverse Nyquist array (with Gershgorin circles) determine bounds on feedback gains for stability.The characteristic locus design method[2], although a procedure which does not require the constraint of diagonal dominance, only gives stability with respect to a single gain common to all the loops. But, since low interaction and accurate tracking are main objectives of this procedure the closed-loop transfer function will almost certainly be diagonally dominant,and in Section 2 it is shown how by looking at appropriate closed-loop Nyquist arrays bounds can be obtained on sensor or actuator changes for stability. This is a technique developed and implemented on the Cambridge multivariable design package by Dr. J.M. Edmunds.[3]

In Section 3 the robustness of a multivariable feedback system is considered under simultaneous nonlinear perturbations of loop gains. A simple interpretation of work by Mees and Rapp[4] provides a bound on the nonlinear perturbations for stability in terms of principal gains[5].

In Section 4 the problem of stability under plant parameter uncertainty is discussed. In particular it is shown how recent developments in the complex variable analysis of multivariable feedback systems[6] can be used to determine stability with respect to any system parameter.

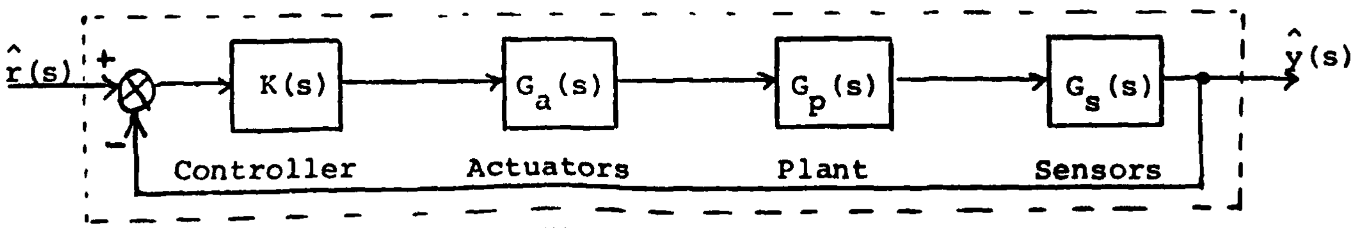
2. Closed-loop Nyquist arrays[3]

The Nyquist array of a transfer function G(s) is formed by drawing the Nyquist diagrams of the individual elements of G(s). If G(s) describes an open-loop system then the array is called the open-loop Nyquist array; a similar diagram for $G(s)^{-1}$ is called the inverse Nyquist array. If G(s) ($G(s)^{-1}$) is diagonally dominant then the diagonal elements of the Nyquist (inverse Nyquist) array, with superposed Gershgorin circles[1], give bounds on the simultaneous variation of loop gains for stability. This is a well known result due to Rosenbrock[1] and is a highly attractive consequence of the inverse Nyquist array design method.

The characteristic locus design method is atechnique which does not require the constraint of diagonal dominance but as such only gives stability with respect to a single gain common to all the loops. However, the main objective of this approach is to obtain a stable closed-loop system with low interaction and accurate tracking, and so the closed-loop transfer function R(s) will almost certainly be diagonally dominant. By forming a Nyquist array for R(s), called the closed-loop Nyquist array, and drawing Gershgorin circles on the diagonal-element diagrams, bounds can be obtained on simultaneous variation of sensor gains for stability. By looking at a similar closed-loop transfer function bounds can be determined for actuator variations. These methods are now described.

2.1 Closed-loop configuration

We will consider the configuration shown in figurel below.



The closed-loop transfer function R(s) is given by

$$R(s)=\left[I_m+G_s(s)G_p(s)G_a(s)K(s)\right]^{-1}G_s(s)G_p(s)G_a(s)K(s)$$

and if K(s) has been designed succesfully then R(s) will be diagonally dominant.

2.2 Stability under simultaneous changes in sensor or controller gains

Consider a situation where feedback is applied around the closed-loop system via a real diagonal operator $F=\mathrm{diag}\{f_i\}$; see figure 2. Then from the diagonal elements of the closed-loop Nyquist array the Gershgorin circles can be used to give bounds on the f_i for stability[1].

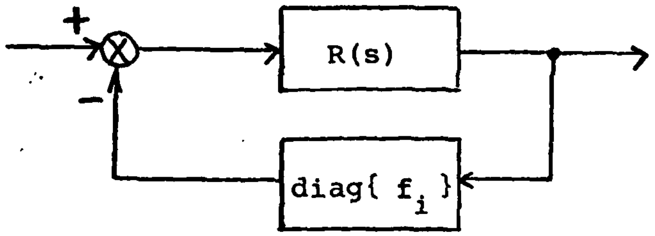


Figure 2

But the configuration of figure 2 is equivalent to figures 3 and 4 :-

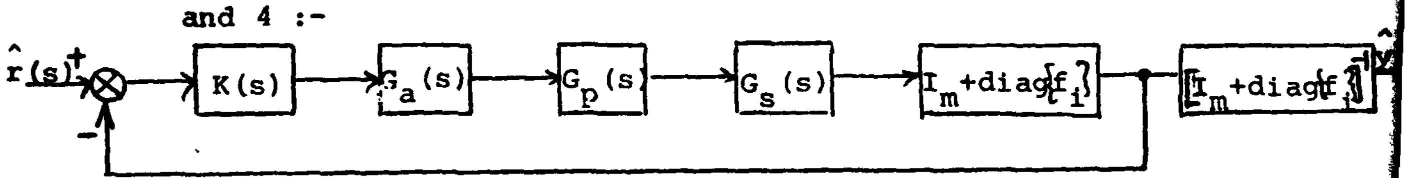


Figure 3

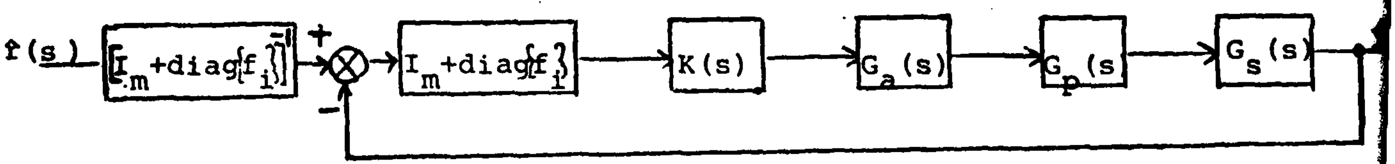


Figure 4

Consequently the bounds on the f_i can be translated into bounds on changes in sensor or controller gains for stability in the original

Remark 1. Note that the diagonal elements of the closed-loop Nyquist array (without Gershgorin circles) give exact information about system stability under changes in a single sensor assuming all other sensors remain unaltered. To see this, suppose that we are interested in varying the jth sensor gain, and let $f_i=0$ for all i except i=j. Then we see from figure 3 that we have a single-input single-output problem described by the j,jth element of R(s). Stability with respect to f_j can therefore be determined exactly using the Nyquist diagram of the j,jth element of R(s) which is the j,jth element of the closed-loop Nyquist array.

Remark 2. Note that the Gershgorin circles define bands within which the characteristic gain loci of R(s) must lie.

2.3 Stability under changes in actuator or plant gains

For the closed-loop system shown in figure 1 consider a new input and output between the actuators and plant as shown below in figure 5.

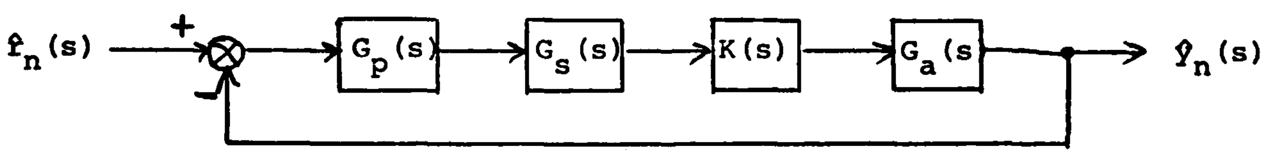


Figure 5

The closed-loop transfer function relating $\hat{r}_n(s)$ and $\hat{y}_n(s)$ is given by

$$R_n(s)=\left[I_m+ G_a(s)K(s)G_s(s)G_p(s)\right]^{-1}G_a(s)K(s)G_s(s)G_p(s)$$

and following exactly the same procedure with $R_n(s)$, as with R(s) in the previous section, bounds can be obtained on changes in actuator or controller gains for stability. These bounds will in general be more conservative than those obtained for sensor variations since there is no explicit objective in the design procedure which aims to make $R_n(s)$ diagonally dominant.

2.4 Example —— PW F100 Jet Engine[7]

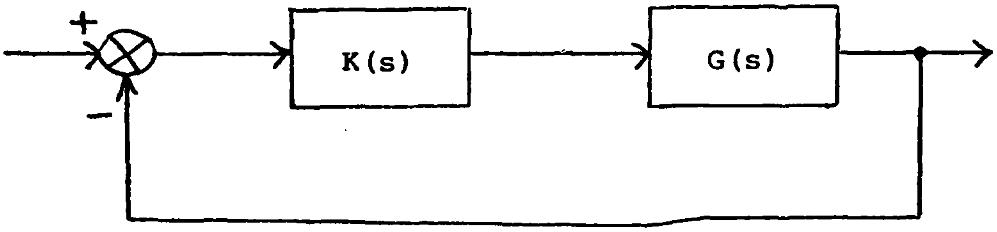


Figure 6

K(s) —— controller obtained via characteristic locus design metho

G(s) —— actuator, plant and sensor dynamics combined

The characteristic gain loci and step responses are shown in figures 7-11, and the open-loop Nyquist array is shown in figure 12 from which it can be seen that the open loop system is not diagonally dominant. The closed-loop Nyquist array is shown in figure 13. If the three sensors have their gains multiplied by k_1, k_2, and k_3 respectively then the Gershgorin bands indicate that the closed-loop system will be stable if

$$k_1 < -0.48 \quad \text{or} \quad 0 < k_1 ,$$
$$k_2 < -0.43 \quad \text{or} \quad 0 < k_2 ,$$
$$\text{and} \qquad 0.33 < k_3 < 11.$$

If $k_3 < -1.2$ then instability will definitely occur.

CHARACTERISTIC GAIN LOCI

F100 Engine

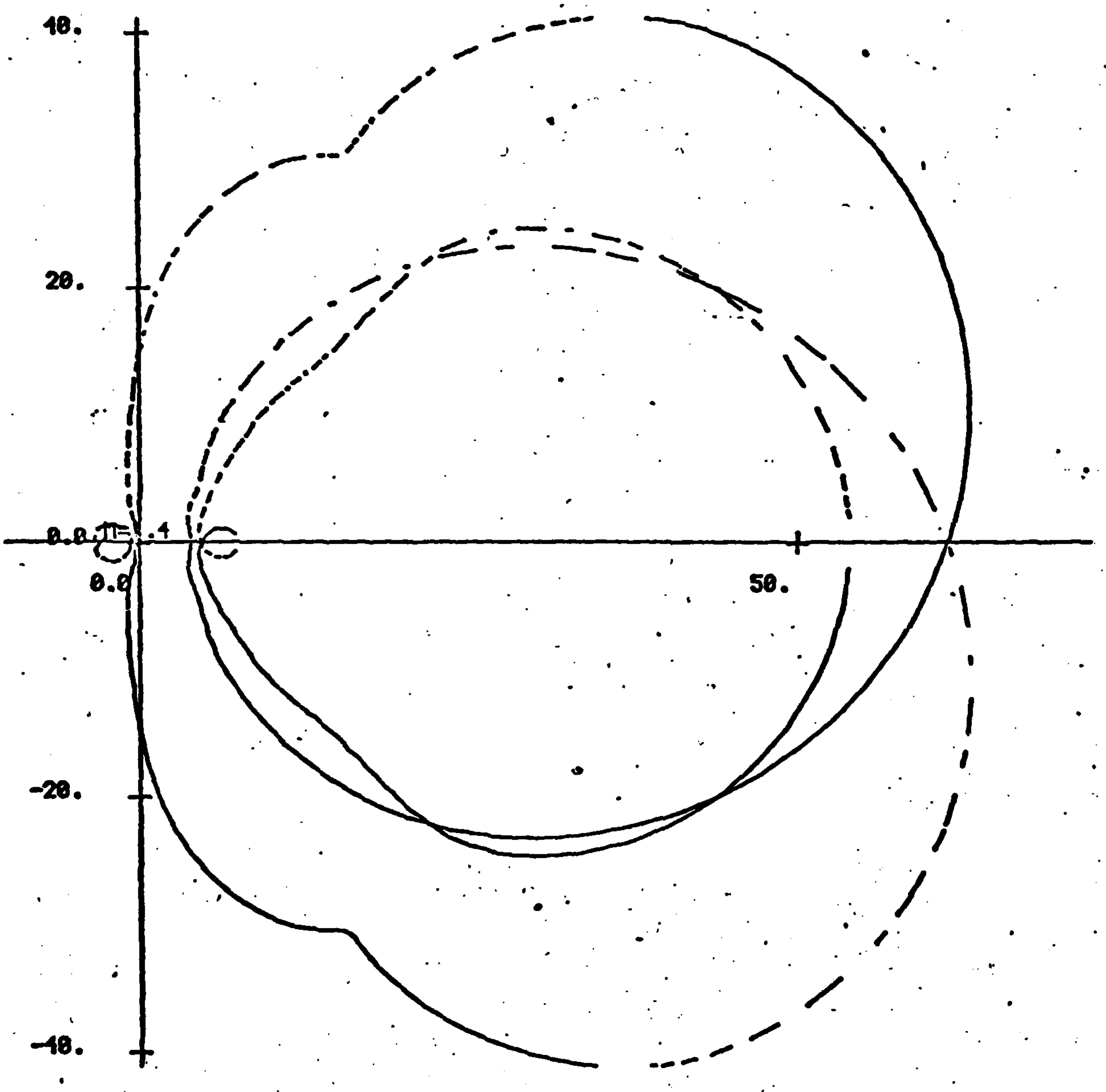


Figure 7

CHARACTERISTIC GAIN LOCI — about the origin

F100 Engine

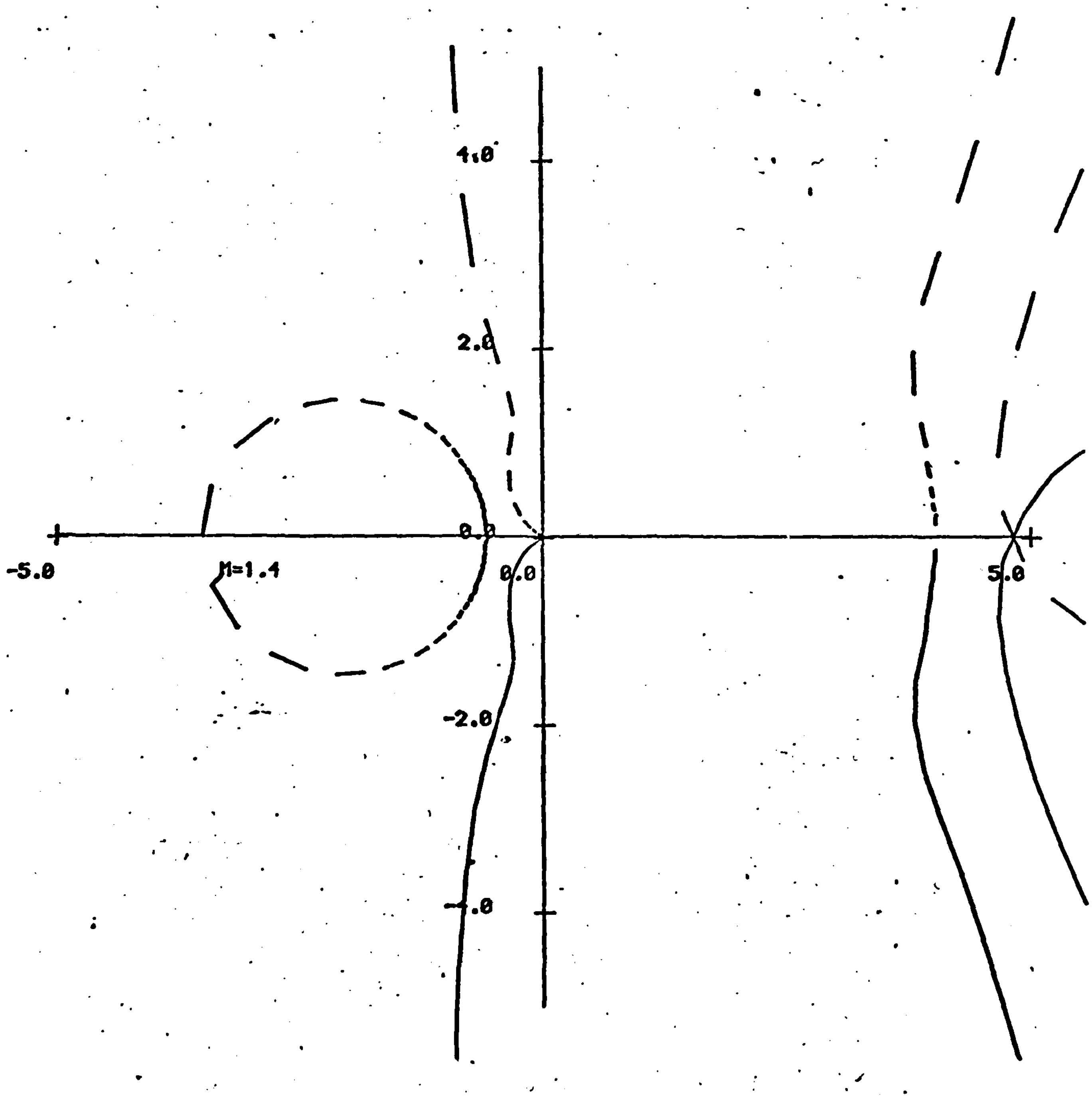


Figure 8

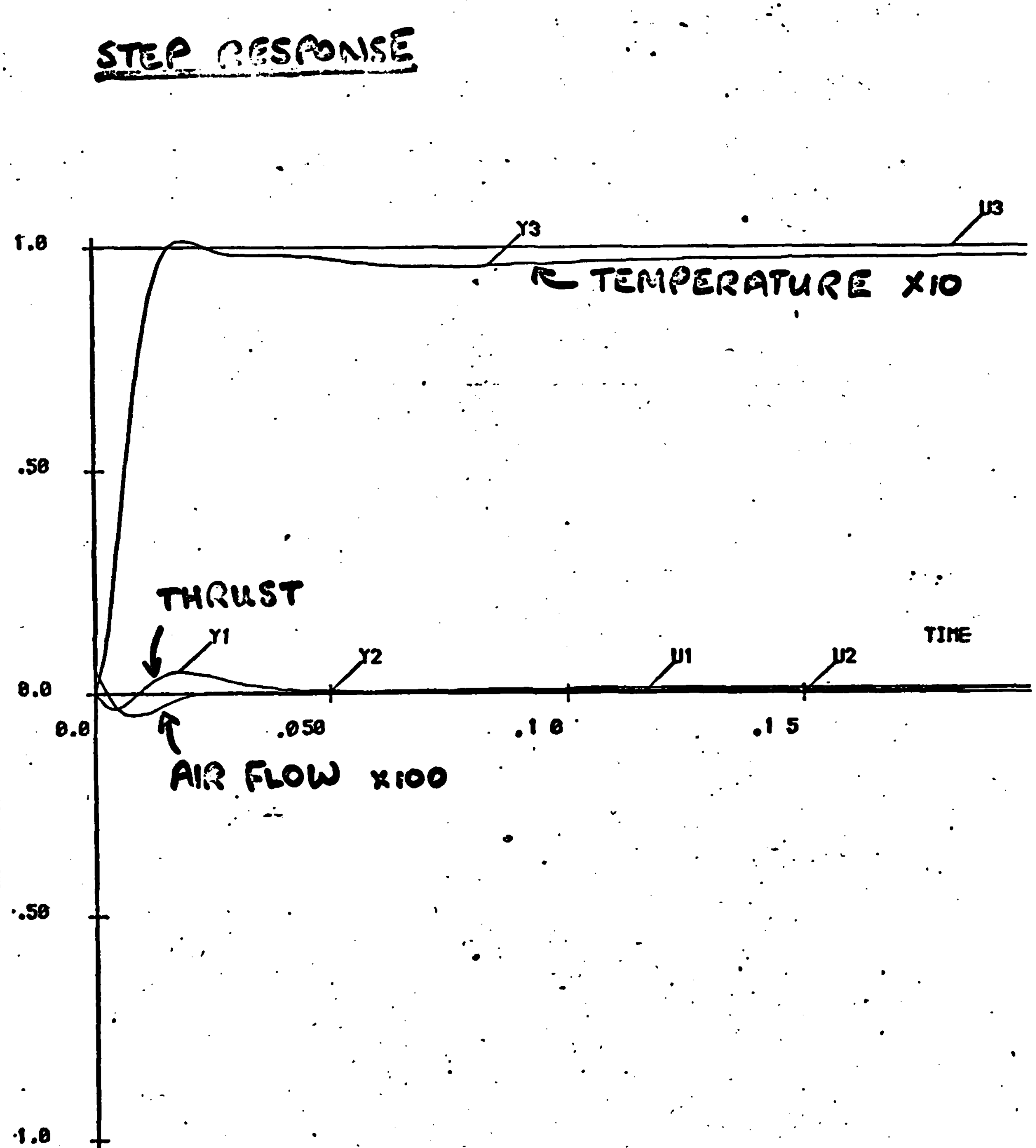

STEP RESPONSE
1.0
.50
0.0
-.50
-1.0
0.0
.050
.1 0
.1 5
TIME
Y3
U3
TEMPERATURE X10
THRUST
Y1
Y2
U1
U2
AIR FLOW X100

Figure 9

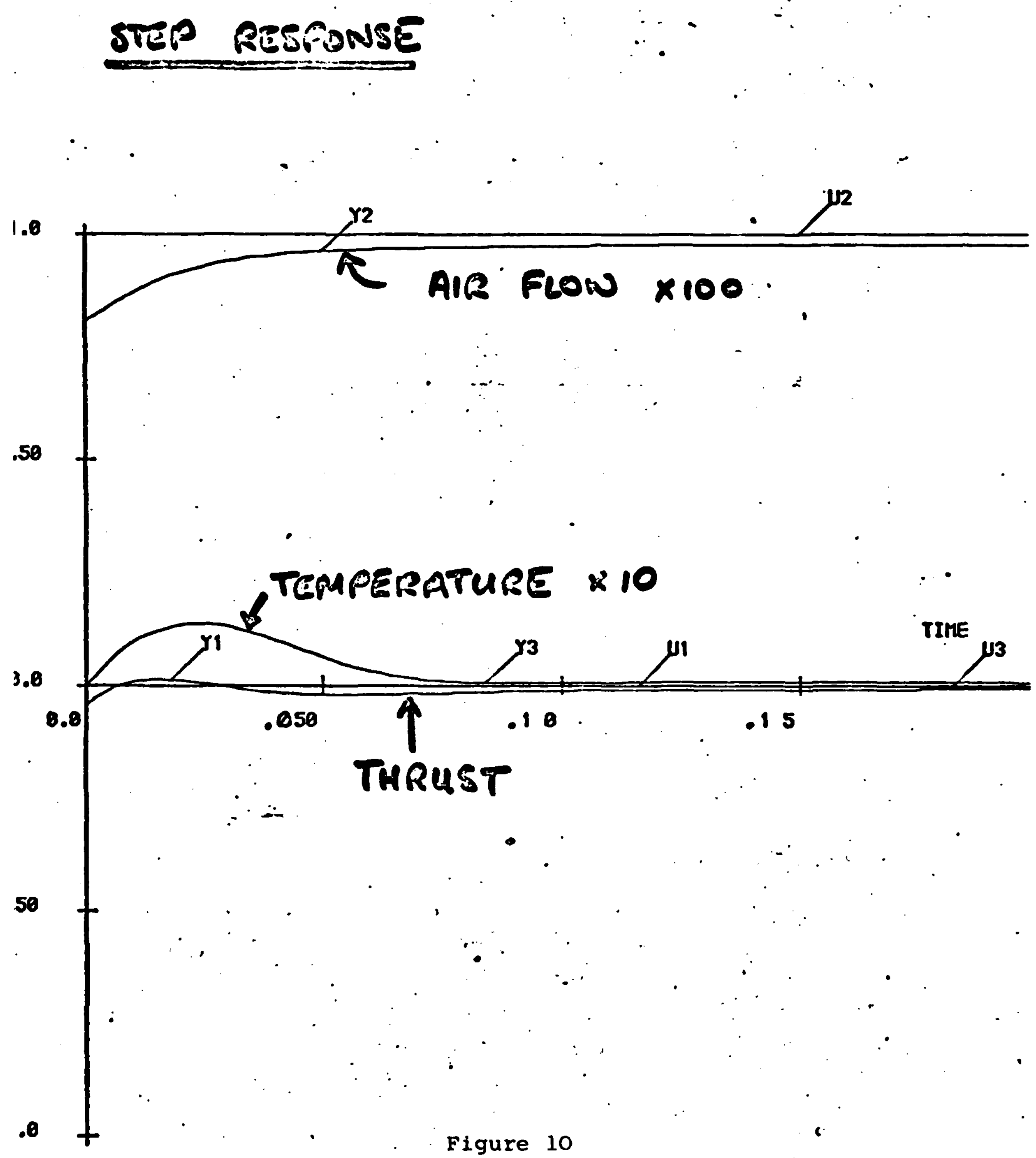

STEP RESPONSE
Y2
U2
AIR FLOW X100
TEMPERATURE x 10
TIME
Y1
Y3
U1
U3
THRUST
1.0
.50
0.0
.50
.0
0.0
.050
.1 0
.1 5

Figure 10

STEP RESPONSE

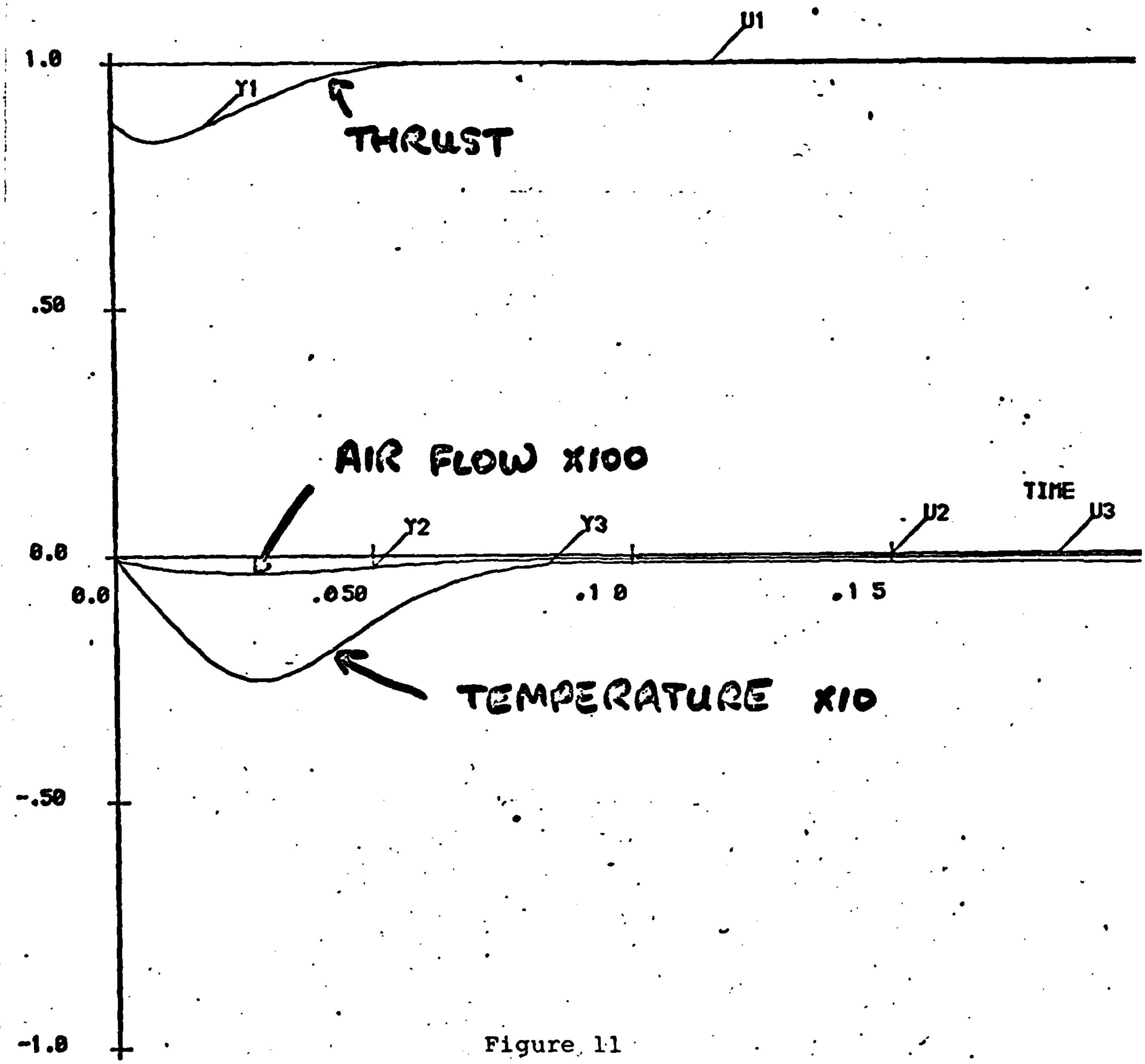


Figure 11

<u>Open-loop Nyquist array</u>

<u>F100 Engine</u>

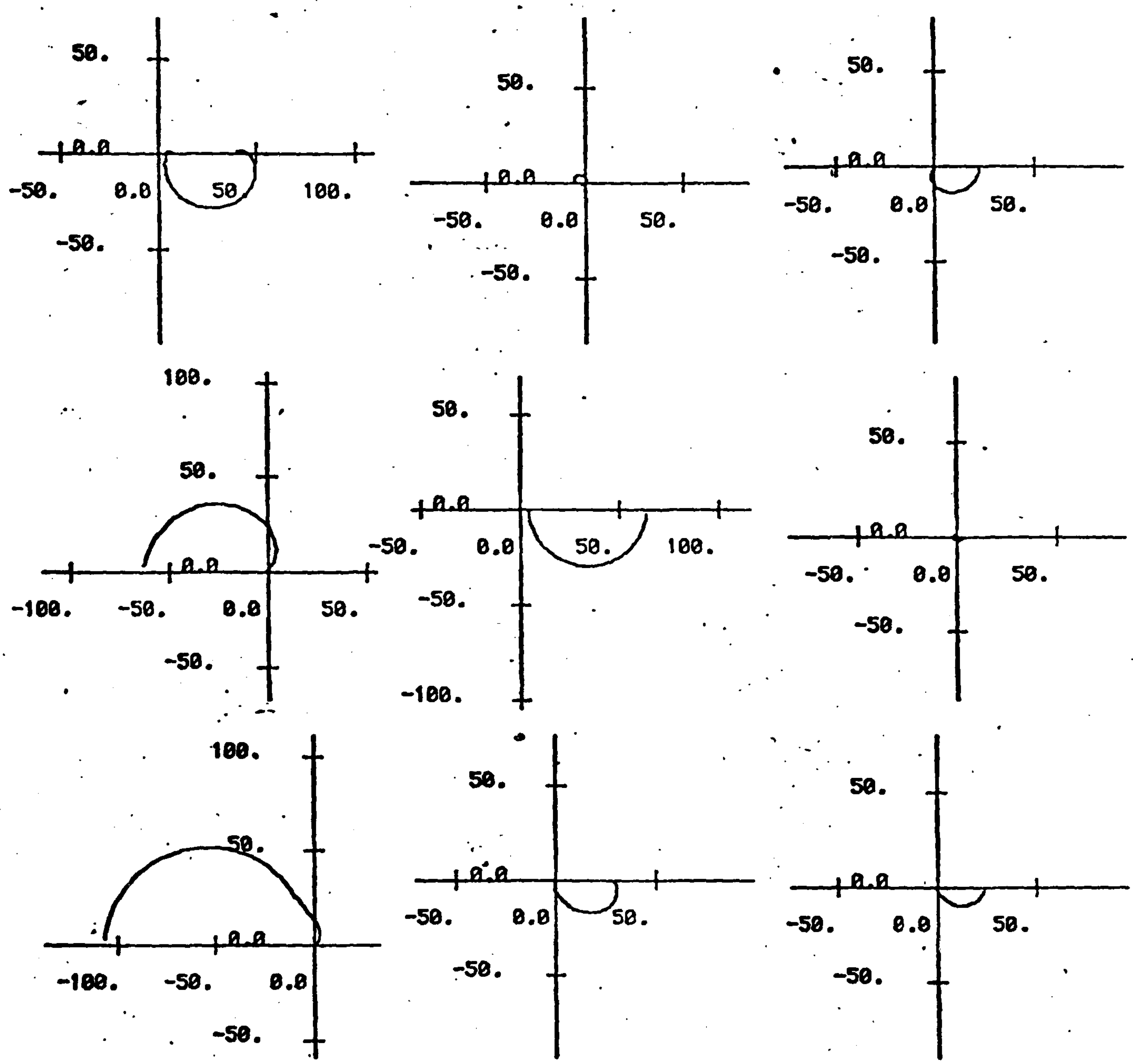


Figure 12

CLOSED-LOOP NYQUIST ARRAY (SENSOR GAINS)

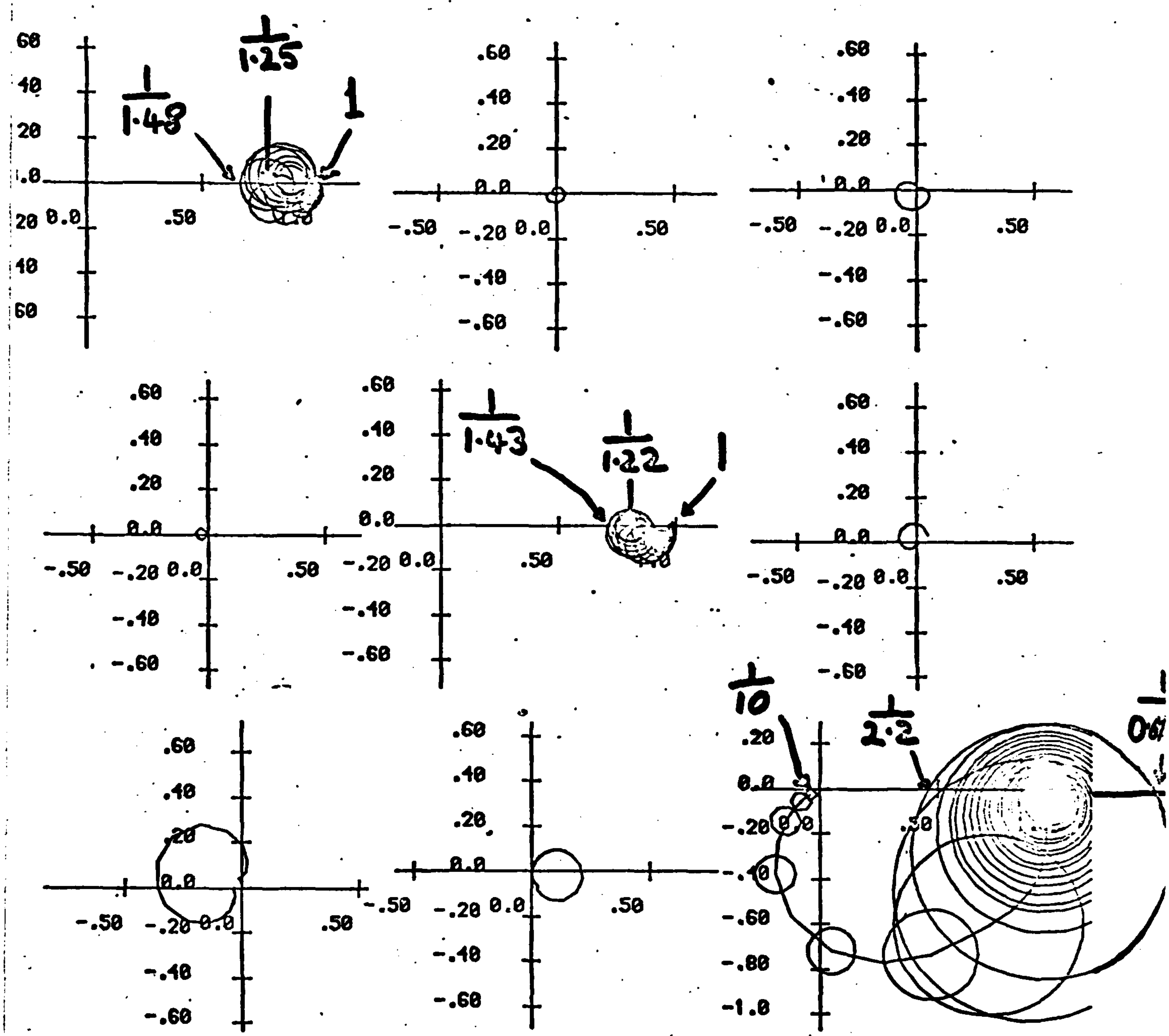


Figure 13

The closed-loop Nyquist array for actuator variations is shown in figure 14. Because the actuator dynamics is included in $G(s)$ the closed-loop transfer function is ~~from~~ for an input and output between actuator and controller and not between plant and actuator as described in the theory. An extra scaling pre-compensator (1,0.001,0.01) and an extra scaling post-compensator (1,1000,100) have been added because of the different orders of magnitude of input signals. This does not affect the stability since in the closed loop these operations effectively cancel each other out.

If the three actuators have their gains multiplied simultaneously by k_{a1}, k_{a2}, and k_{a3} respectively then the Gershgorin circles indicate that stability is maintained if

$$2 > k_{a1} > 0.04 \quad ,$$

$$k_{a2} < -1.7 \quad \text{or} \quad 0.12 < k_{a2} \quad ,$$

$$\text{and} \quad k_{a3} < -0.24 \qquad 0.02 < k_{a3} \; .$$

If $-0.24 < k_{a3} < -0.07$ then instability will definitely occur.

GAINS)

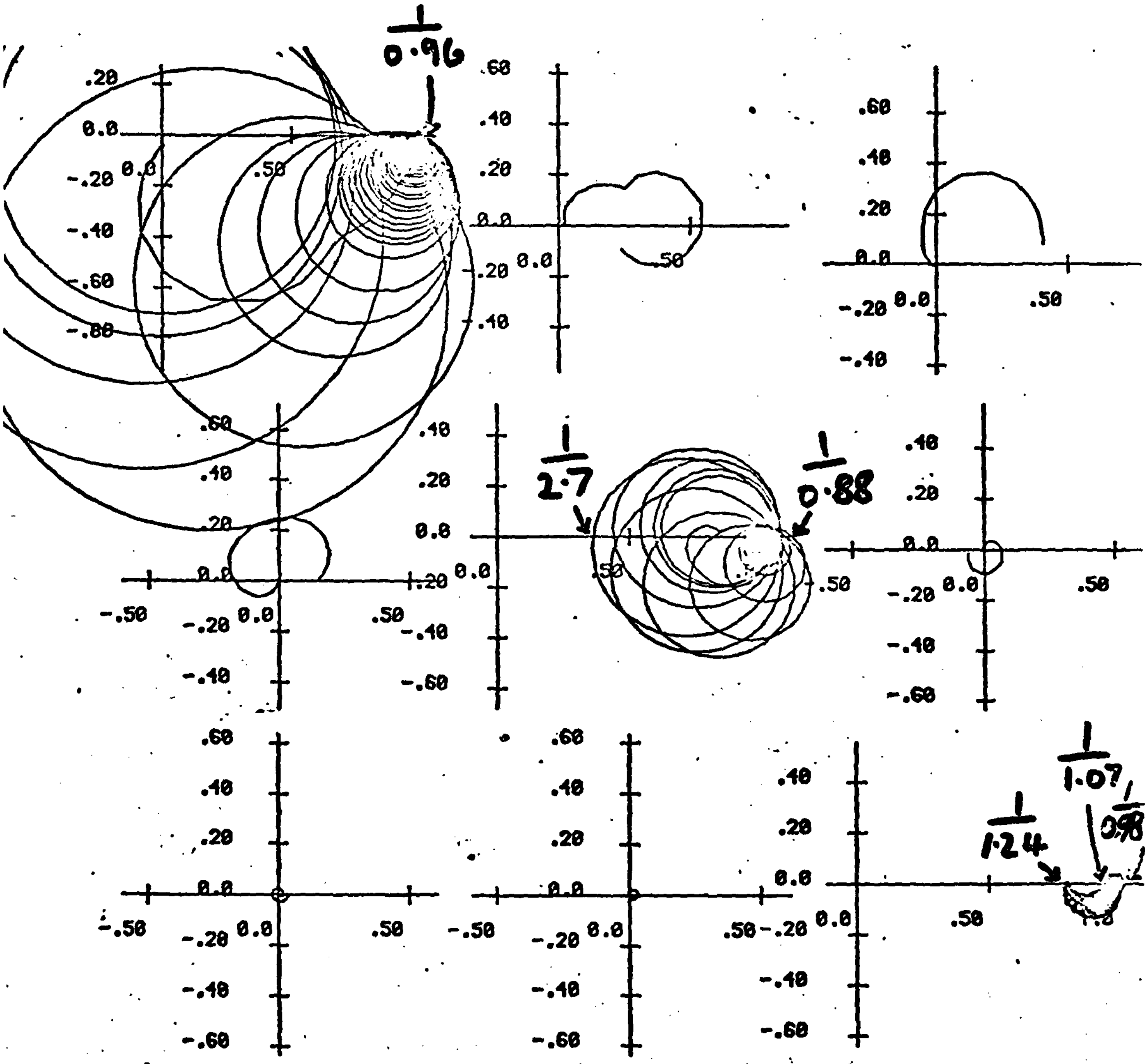


Figure 14

3. Stability with nonlinear perturbations

The results in this section are based on work by Mees and Rapp[4] which in turn is founded on the small-gain theorem and related results[8]. In simple terms the small-gain theorem says that if the open-loop 'gain' is less than unity then the closed-loop system will be stable. No attempt to be rigorous is made in the following.

Consider the closed-loop configuration of figure 1 and let us assume that associated with the sensors there are some nonlinear perturbations as illustrated in figure 15 below.

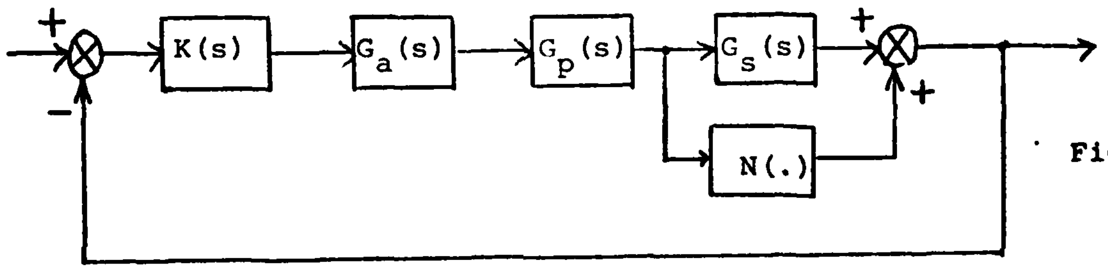


Figure 15

Assume also that the nonlinear perturbation is diagonal i.e. $N(.)=\mathrm{diag}(n_i(.))$, and further that the slope of $n_i(x_i)$ is bounded below by $-\varepsilon_i$ and above by ε_i, as illustrated below.

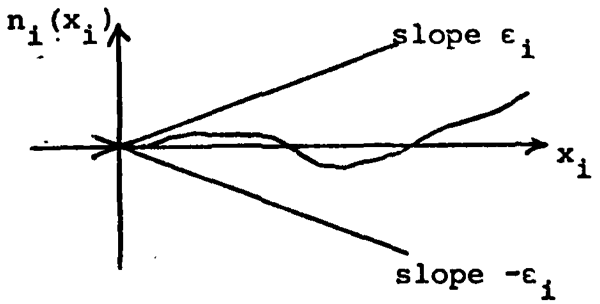


Figure 16

It is also required that $N(0)=0$.

Figure 15 can be redrawn to give the figure below

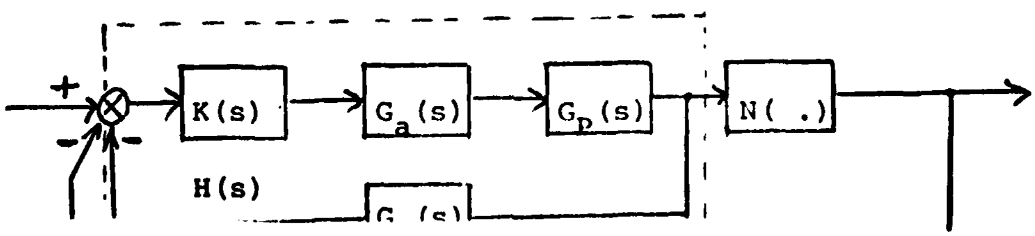


Figure 17

and applying the small-gain theorem to this we have that for stability H(s) must be stable,which it is by design, and the gain round the loop must be less than unity. To check this we must first give suitable definitions for the gains of the linear and nonlinear operators.

The gain of the linear part H(s) can be defined as

$$\sup_{\omega \geq 0} \| H(j\omega) \|$$

$$\text{where } \| A \| = \sqrt{(\text{max. eigenvalue of } A^{\dagger}A)}$$

and so is the maximum principal gain[5] of H(s).

The gain of the nonlinear part will be taken as

$$\max_{i} \{\varepsilon_i\}$$

Consequently the inverse of the maximum principalgain of H(s) gives an upper bound to the slopes of the nonlinear perturbations for stability to be maintained.

By taking a slightly different definition for the gain of the linear operator H(s) a graphical test, analagous to the multivariable circle criteria , can be obtained. The resulting bound on the ε_i however is more conservative; this is now explained.

At a particular value of s

$$H = R \Lambda R^{-1}$$

R—matrix of eigenvectors
Λ—diagonal matrix of eigenvalues

therefore

$$\begin{aligned} \| H \| &= \| R \Lambda R^{-1} \| \\ &\leq \| R \| \; \| \Lambda \| \; \| R^{-1} \| \\ &= \gamma \, \rho \end{aligned}$$

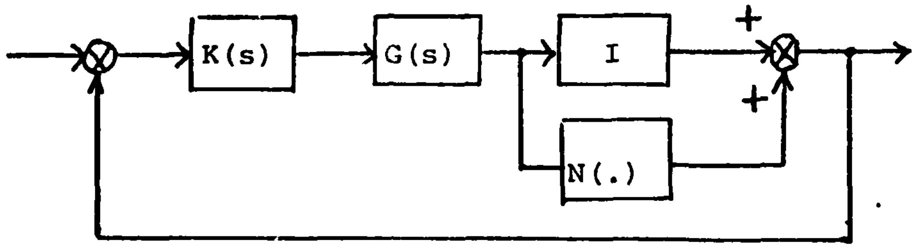
where $\gamma = \| R \| \; \| R^{-1} \|$ is the ~~square root of the~~ ratio of the largest to the smallest principal gain of R, and ρ is the largest eigenvalue of H. Since $\gamma\rho$ is an upper bound on $\| H \|$, thegain of

H(s) can be taken as

$$\sup_{\omega \geq 0} \gamma(j\omega)\ \rho(j\omega)$$

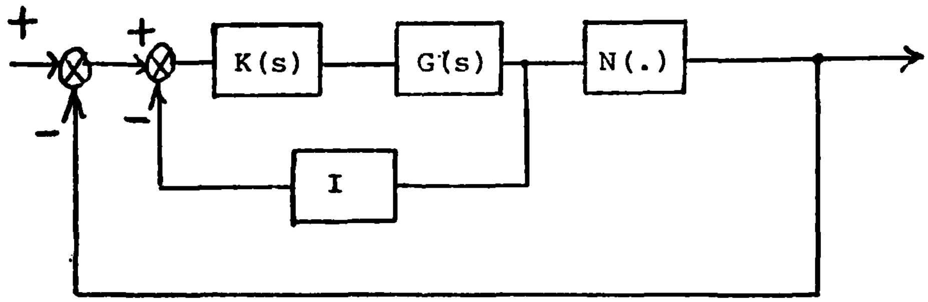
Therefore if the eigenvalue-loci of H(jω) multiplied by γ(jω) all lie within a circle centre the origin radius ε_{max}^{-1} then the system remains stable. The radius of the circle which just touches these loci is therefore a maximum bound on the nonlinear perturbations.

3.1 Example —— PW F100 Engine[7]



(a)

which is equivalent to



Note
$H(s)=[I+G(s)K(s)]^{-1}G(s)K(s)$ (b)

Figure 18

The principal gains of H(s) are shown in figure 19. The maximum principal gain is 1.22 which gives an upper bound on the magnitude of the ε_i of 0.82.

The eigen-loci of H(jω) modified by γ(jω) are shown in figure 20, and γ(jω) is shown separately on figure 21. From figure 20 we see that the loci are completely enclosed by a circle

PRINICIPAL GAINS

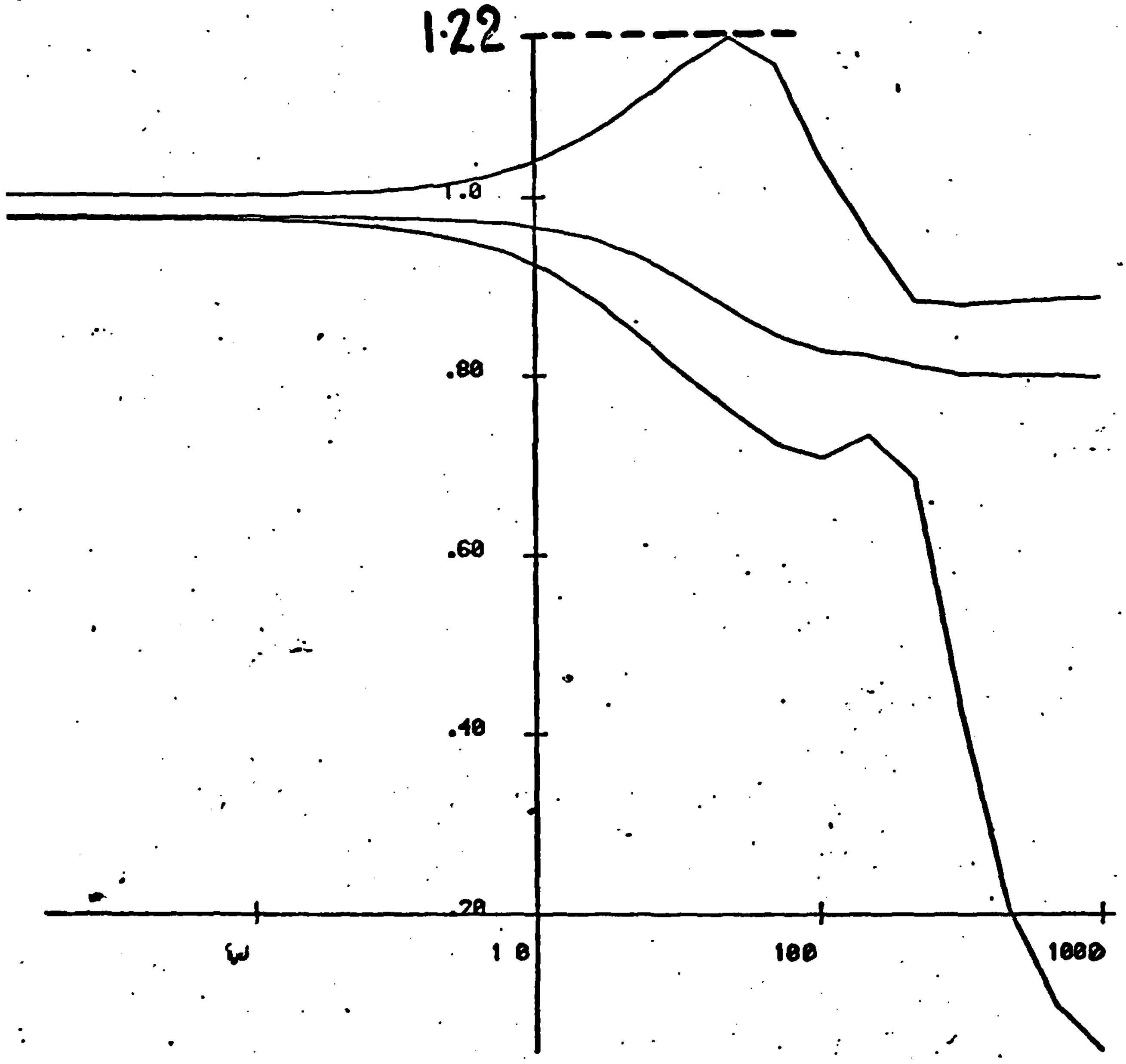


Figure 19

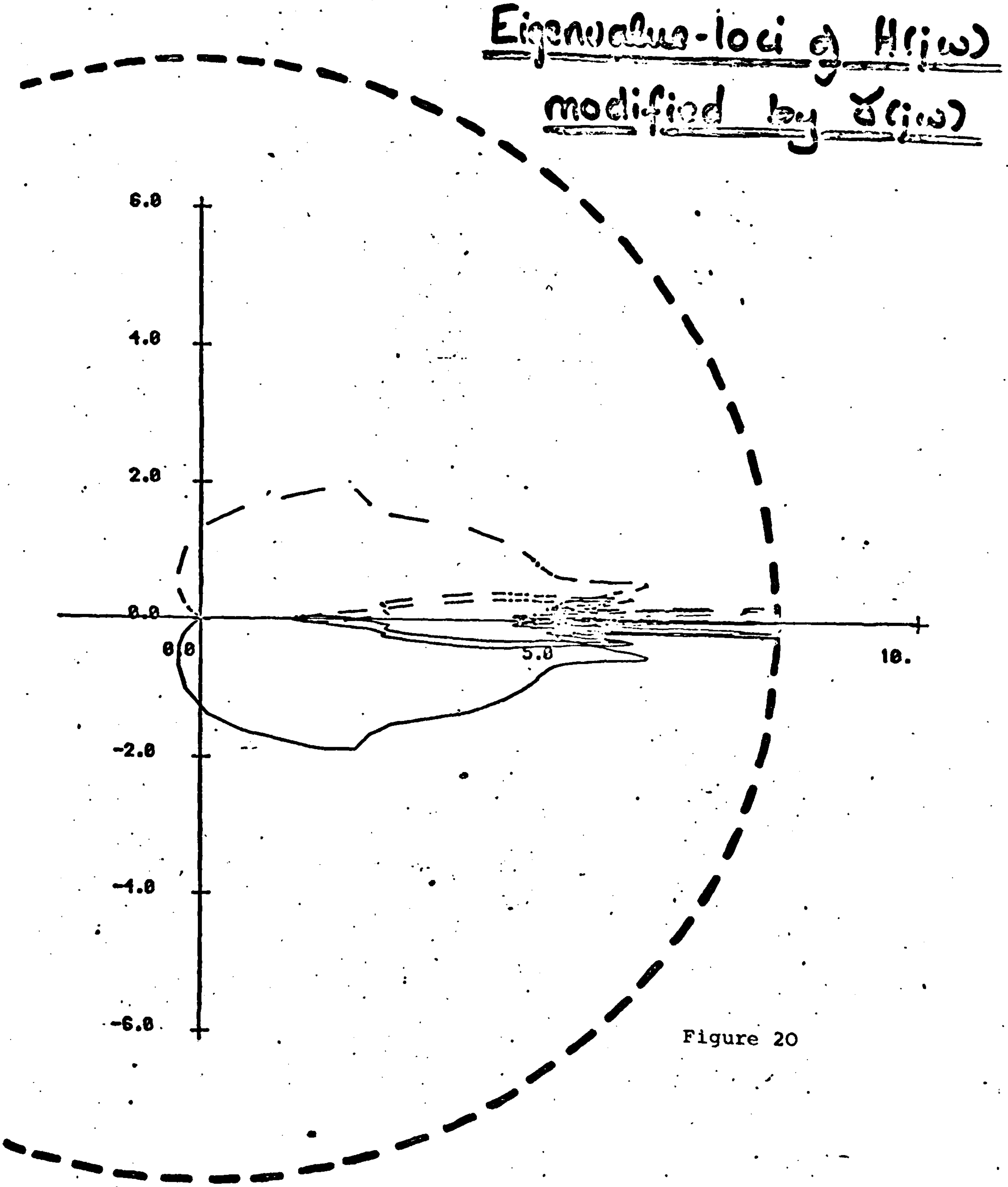

Eigenvalue-loci of H(jω)
modified by γ(jω)
6.0
4.0
2.0
0.0
-2.0
-4.0
-6.0
0.0
5.0
10.

Figure 20

Plot of $\gamma(j\omega)$

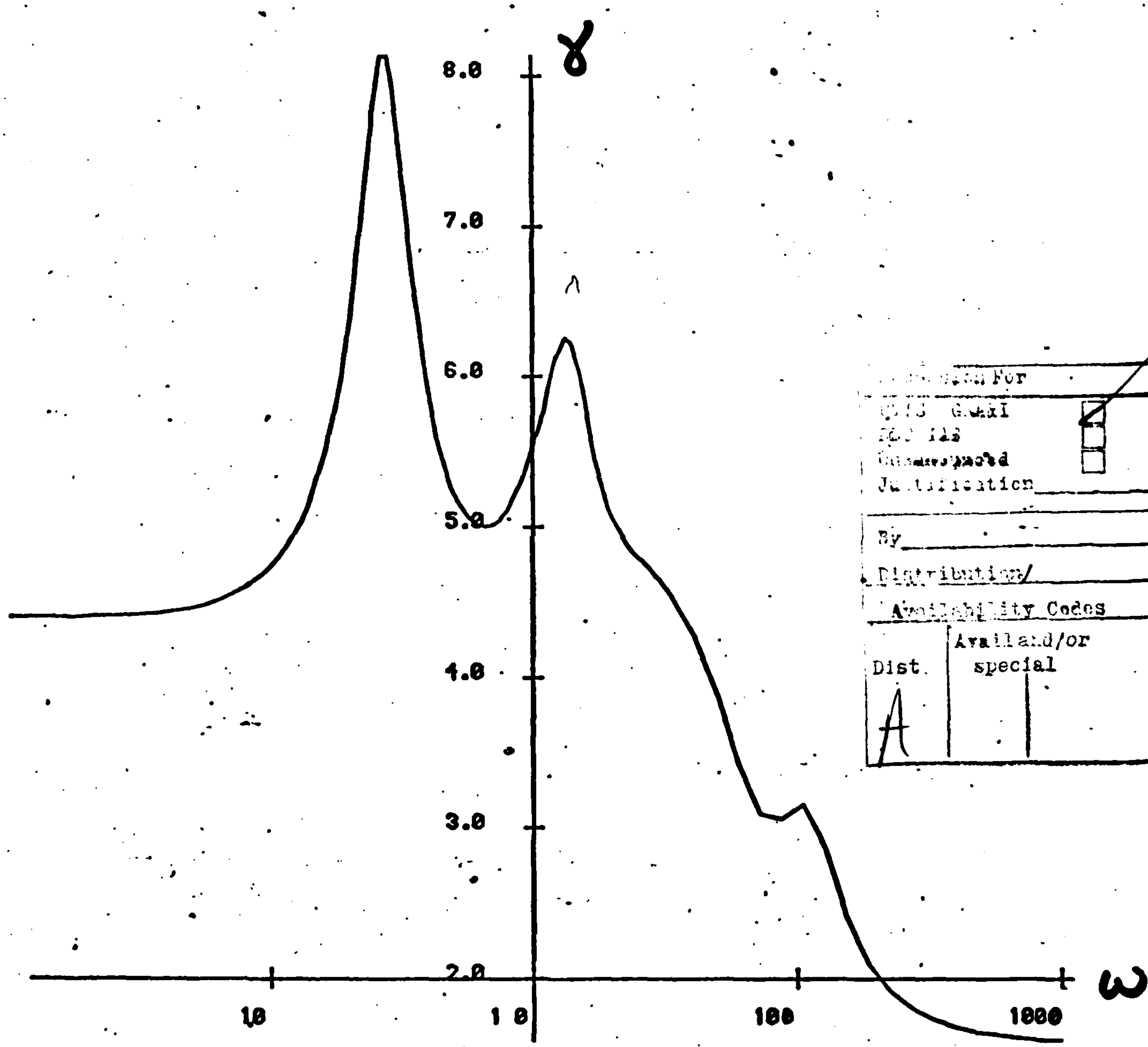


Figure 21

of radius 8 which gives an upper bound of 0.125 on the magnitude of the ε_i.

4. Stability under plant parameter uncertainty

See enclosed paper entitled "A note on parametric stability".

5. References

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