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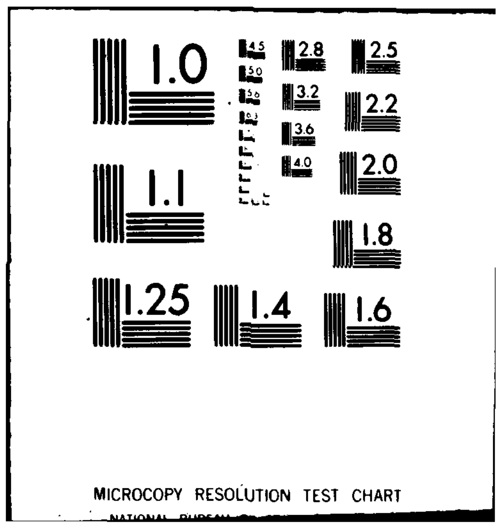
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OPTIMALITY CRITERION TECHNIQUES APPLIED TO STRUCTURES COMPOSED OF DIFFERENT ELEMENT TYPES

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The optimality criterion design method, developed by Khan, Willmert and Thornton which exploits the concept of one most critical constraint, has been extended in this paper to handle more complex structures, specifically ones composed of several different types of elements. The elements considered were the truss and two different plane stress elements: a constant strain triangular element and a rectangular symmetric shear panel. The optimality criterion method treats multiple load conditions and stress and displacement

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OPTIMALITY CRITERION TECHNIQUES APPLIED TO STRUCTURES COMPOSED OF DIFFERENT ELEMENT TYPES

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Abstract

The optimality criterion design method, developed by Khan, Willmert and Thornton which exploits the concept of one most critical constraint, has been extended in this paper to handle more complex structures, specifically ones composed of several different types of elements. The elements considered were the truss, and two different plane stress elements: a constant strain triangular element and a rectangular symmetric shear panel. The optimality criterion method treats multiple load conditions and stress and displacement constraints. A wing-box structure is considered as an example to demonstrate the efficiency and simplicity of the method.

I. Introduction

The application of optimality criterion techniques to structural design has been of great interest to many researchers during the last few years.[1-8] (See Venkayya[9] for an excellent review article in this area.) Khan, Willmert, and Thornton have developed several very efficient techniques based on optimality conditions for the design of both two and three dimensional trusses, frames, and even high speed mechanical mechanisms.[10-14] Numerous constraints have been considered including limits on stress and displacement (and combinations of these), natural frequencies, and upper and lower bounds on the design variables. Recently problems involving more than one design variable per member have also been considered.[15] The techniques, based on the assumption that only one constraint is most critical at any stage of the process, have proven to be very efficient in terms of computational time and core storage requirements. It has been shown through numerous examples that the optimal designs obtained by these new methods were nearly identical (sometimes better) to those obtained by previous methods.

In all of the problems considered thus far, however, the structures were composed of a single element type, that is, either truss or frame elements, or in the case of mechanical mechanisms, a special vibrational element. The purpose of the research presented here was to develop an optimality criterion technique, based on the efficient method of Khan, Willmert, and Thornton, capable of designing structures composed of several different element types.

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II. Theory

The type of structures considered in this work are ones composed of truss elements and two different plane stress elements, particularly a constant strain triangular element and a symmetric rectangular shear panel. The cross sectional areas A_i of the truss elements, shown in Figure 1, and the thicknesses T_j of the plane stress elements, shown in Figures 2 and 3, were used as the design variables. The objective was to minimize the total weight of the structure:

$$W = \rho\left[\sum_{i=1}^{N} A_i L_i + \sum_{j=1}^{J} T_j S_j\right] \qquad (1)$$

where L_i and S_j are the length of the truss element i and surface area of the plane stress element j respectively. N and J are the total number of truss and plane stress elements respectively.

Both stress and displacement limitations were imposed on the design. The stress limitations on the truss elements were:

$$|\sigma_{ik}| \leq \bar{\sigma}_i \qquad \begin{matrix} i = 1,\ldots,N \\ \text{for all } k \end{matrix} \qquad (2)$$

where $\bar{\sigma}_i$ is the allowable stress for element i, and σ_{ik} is the actual stress in element i under load condition k. For the plane stress elements the Von Mises equivalent stress was used:

$$\sigma_{ejk} = (\sigma_{xjk}^2 + \sigma_{yjk}^2 - \sigma_{xjk}\sigma_{yjk} + 3\tau_{xyjk})^{1/2} \leq \bar{\sigma}_j \qquad (3)$$

$$j = 1,\ldots,J \quad \text{for all } k$$

where σ_{ejk} is the equivalent stress for element j under load consition k. The nodal displacements u_{mk} of the structure were limited by:

$$|u_{mk}| \leq \bar{u}_m \qquad \begin{matrix} m = 1,\ldots,M \\ \text{for all } k \end{matrix} \qquad (4)$$

where M is the total number of displacement constraints. Lower bounds were also placed on the areas and thicknesses.

The constant strain triangular element used is shown in Figure 2. It is assumed that this element has constant strain in the field given by the equations:

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} = \frac{1}{bh} \begin{bmatrix} -(b-s) & 0 & -s & 0 & b & 0 \\ 0 & -h & 0 & h & 0 & 0 \\ -h & -(b-s) & h & -s & 0 & b \end{bmatrix} \begin{bmatrix} U_P \\ V_P \\ U_Q \\ V_Q \\ U_R \\ V_R \end{bmatrix} \qquad (5)$$

where U_P, V_P, U_Q, ..., V_R are the displacements in the local $\tilde{X}$ and $\tilde{Y}$ directions of the nodes P, Q, and R, and b, h, and s are the dimensions of the element as shown in Figure 2. The other plane stress element used in this research was the symmetric rectangular shear panel element shown in Figure 3. This element is assumed to be symmetric with respect to the $\tilde{X}$ axis, with a stress distribution of the form:

$$\sigma_x = D_1\tilde{Y} + D_2$$

$$\sigma_y = 0 \qquad (6)$$

$$\tau_{xy} = D_3$$

where D_1, D_2, and D_3 are constants. It is also assumed that the displacement in the $\tilde{X}$ direction on this $\tilde{X}$ axis is zero, i.e.

$$u(\tilde{X},0) = 0 \qquad (7)$$

Using the above assumptions, the element stiffness matrices can be developed for all three elements; from which the displacements and stresses can be determined which are required for the constraints in the optimization.

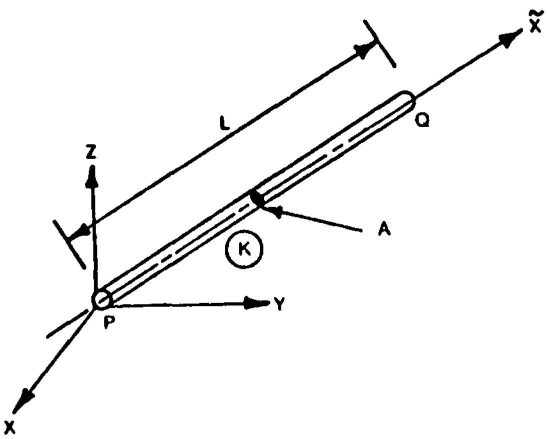

Figure 1: Truss Element With Uniform Cross-Sectional Area

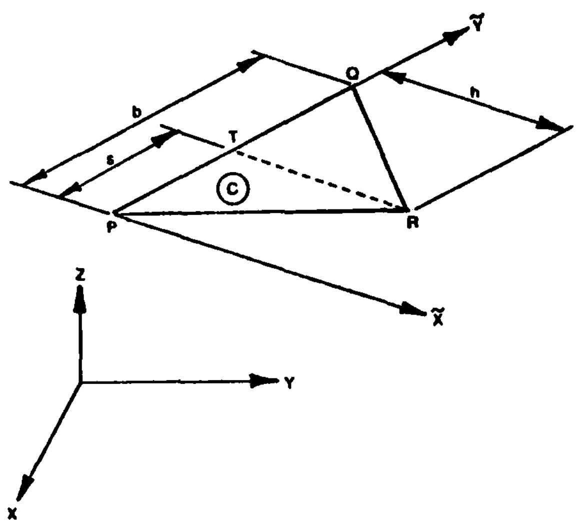

Figure 2: Constant Strain Triangular Element

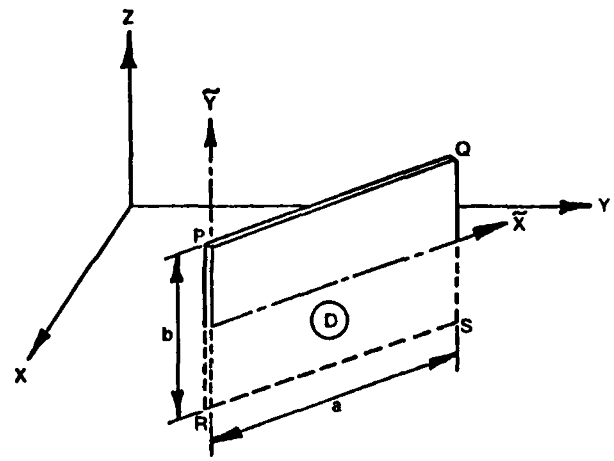

Figure 3: Symmetric Shear Panel Element

III. Design Algorithm

Khan, Willmert and Thornton[10] derived recursive relations for the design of structures composed of truss or frame elements based on the concept of one most critical constraint at any iteration. From these they developed an algorithm for the design of trusses or frames under multiple stress and displacement limitations. In their work, the cross-sectional areas were the design variables. In the research presented here, the areas of the truss elements and thicknesses of the plane stress elements were used. Thus recursion relations for the thicknesses were derived in this research. These were combined with those developed by Khan, Willmert and Thornton to obtain the following design algorithm.

1. Choose any design, specifically values for all the design variables namely areas A_i, i=1, 2,...,N and thicknesses T_j, j=1,2,...,J, such that all the areas are the same and similarly for the thicknesses. Choose a value of the relaxation parameter (η). (Say 0.05 to 0.15).

2. Analyze the design for each load condition.

3. Check displacements in each load condition at those nodes where displacement limitations are imposed and determine the node and direction for which the calculated displacement most closely approaches (or exceeds) the allowable displacement. This is the most critical displacement (u_{pq}).

4. Knowing the magnitude of the most critical displacement (u_{pq}) from Step 3 and the value of the allowable displacement ($\bar{u}_p$), scale the chosen design for all the design variables so that the most critical displacement becomes active. All other displacement constraints will then be inactive. Let the scaled design be denoted by A_i' and T_j' where

$$A_i' = \frac{|u_{pq}|}{\bar{u}_p} A_i \qquad i = 1,2,\ldots,N \tag{8}$$

and

$$T_j' = \frac{|u_{pq}|}{\bar{u}_p} T_j \qquad j = 1,2,\ldots,J \tag{9}$$

If the structure was analyzed with the scaled design then the displacement vectors calculated at Step 2 would have been:

$$\vec{u}_k' = \frac{\bar{u}_p}{|u_{pq}|} \vec{u}_k \qquad \text{for all } k \tag{10}$$

and stiffness matrix from the scaled design would have been:

$$[\underline{K}'] = \frac{|u_{pq}|}{\bar{u}_p} [\underline{K}] \tag{11}$$

5. From the scaled displacements $\vec{u}_k'$ and design variables A_i' and T_j', compute the maximum stress $\max_k |\sigma_{ik}|$ in each truss member i, and $\max_k \sigma_{ejk}$ for each plane stress element j. Also determine the stress response ratio (stress obtained at Step 2 divided by the limiting stress) for each member and let the most critical response ratio be obtained for the nth member. This is denoted by R_n.

$$\text{If } R_n > 1 \text{ compute } V_1 = R_n \left[\sum_{i=1}^{N} A_i' L_i + \sum_{j=1}^{J} T_j' S_j \right]$$

$$\text{If } R_n \le 1 \text{ compute } V_1 = \left[\sum_{i=1}^{N} A_i' L_i + \sum_{j=1}^{J} T_j' S_j \right] \tag{12}$$

6. Using the scaled design, apply a unit load only at the node and in the direction of the active displacement constraint. Let the set of resulting nodal displacements be denoted by $\hat{u}_i'$. Note that this is the only unit load that must be applied, and that the structural stiffness matrix inverted at Step 2 is used here as scaled in Step 4 to compute $\hat{u}_i'$.

7. Compute the displacement derivatives for area design variables as follows:

$$\frac{\partial u_{pq}}{\partial A_i'} = -\frac{\vec{u}_{iq}'^{\,T} [\underline{K}_i'] \hat{u}_i'}{A_i'} \qquad i = 1,2,\ldots,N \tag{13}$$

where $\vec{u}_{iq}'$ is a vector of the actual displacements for member i, and $[\underline{K}_i']$ is the scaled element stiffness matrix for element i. Compute displacement derivatives for thickness design variables using:

$$\frac{\partial u_{pq}}{\partial T_j'} = -\frac{\vec{u}_{jq}'^{\,T} [\underline{K}_j'] \hat{u}_j'}{T_j'} \qquad j = 1,2,\ldots,J \tag{14}$$

Also the Lagrange multiplier associated with the critical displacement for area and thickness design variables is calculated as follows:

$$\lambda_{pq} = \frac{1}{\bar{u}_p} \left[\sum_{i=1}^{N} A_i' L_i + \sum_{j=1}^{J} T_j' S_j \right] \tag{15}$$

8. Group the memebers as follows:

i. If $\partial u_{pq}/\partial A_i' \ge 0$ or $\sigma_i \ge \bar{\sigma}_i$, then truss member i belongs to group G_1. If $\partial u_{pq}/\partial T_j' \ge 0$ or $\sigma_{ej} \ge \bar{\sigma}_j$ then plane stress member j belongs to G_1.

ii. Otherwise, member i or j belongs to group G_2. Note that either group could be empty and a particular member would belong to only one group at a time.

9. Use the stress ratio formula to resize the elements of group G_1. Resizing of the area design variables is accomplished as follows:

$$[A_i]_{\nu+1} = \left[\frac{\max_k |\sigma_{ik}|}{\bar{\sigma}_i} A_i' \right]_\nu \qquad i = 1,2,\ldots,N \tag{16}$$

and resizing of the thickness design variables:

$$[T_j]_{\nu+1} = \left[\frac{\max_k \sigma_{ejk}}{\bar{\sigma}_j} T_j' \right]_\nu \qquad j = 1,2,\ldots,J \tag{17}$$

10. Resize the elements of group G_2 for the area design variables as follows:

$$(A_i)_{\nu+1} = \left[\lambda_{pq} \frac{(-\partial u_{pq}/\partial A_i')}{L_i} \right]_\nu^{\eta} (A_i')_\nu \qquad i = 1,2,\ldots,N \tag{18}$$

and resize the elements of group G_2 for thickness design variables as follows:

$$(T_j)_{\nu+1} = \left[\lambda_{pq} \frac{(-\partial u_{pq}/\partial T_j')}{S_j} \right]_\nu^{\eta} (T_j')_\nu \qquad j = 1,2,\ldots,J \tag{19}$$

11. Compute the volume V_1' from areas and thicknesses obtained in Step 9 & 10. If the quantity $|(V_1 - V_1')/V_1'|$ is less than ϵ (a small number ranging between .001 to .050) then stop, this design is the optimum design; otherwise go to Step 2 and repeat the process.

IV. Examples

To show the effectiveness and efficiency of the design algorithm, a wing box structure as shown in Figure 4 was considered. This structure was assumed to be symmetric with respect to the middle surface. As a result only the upper half of the structure was considered in optimization. In this half there were 5 truss elements, 5 constant strain triangular elements and 8 rectangular shear panel elements. The stress limit used for all elements was 10,)00 psi, lower bounds on the truss areas was 0.1 in^2 and on the plane stress element thicknesses was 0.0? in. Two different load conditions were considered. These were loads of 5000 lbs. at node 7 (as shown in Figure 4), and 10,000 lbs. at node 5, both in the Z direction. The problem was solved three different times with three different displacement limits. These were 2.0, 1.0 and 4.0 inches at all nodes in all directions.

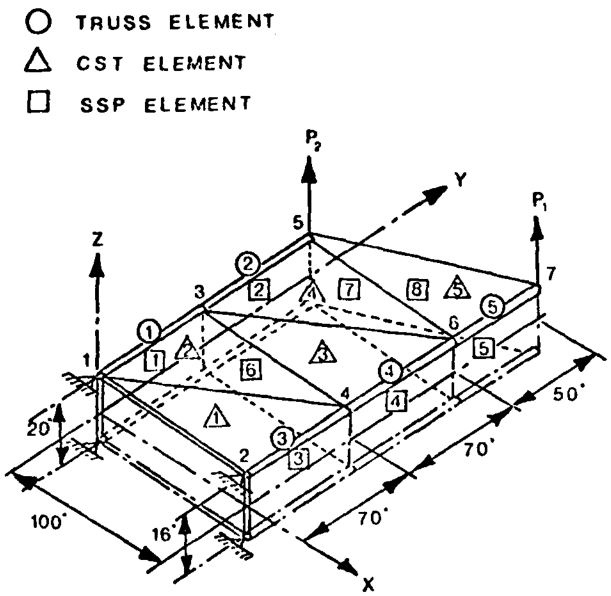

Figure 4: Finite Element Model For Eighteen Element Wing Box Structure

The 2.0 inch displacement limit case has been solved by other researchers including Gallatly and Berke[16], Gallatly[17], and Schmit and Miura[18]. The final designs obtained are shown in Table 1. It is difficult to compare these results directly since different finite element models of the structure were used in each case, and thus some variation in optimal designs are observed. Although Schmit and Miura's model I was identical to that used in the research presented here. The optimal structures are similar however. The greatest variation among the designs shown appears in the area of truss element 1. The optimal design obtained by the technique of this research, although slightly higher in weight, is similar to the other designs.

In order to provide a means of directly comparing results, this example problem was also solved using two other methods in addition to the technique presented in section III of this paper. These were the SUMT technique using Powell's method[19], and by the stress ratio method with displacement scaling. In the latter technique, the stress ratios alone, equations (16) and (17), were used to change the design variables from one iteration to the next, but after each iteration the design was scaled so that the displacement constraints were all satisfied. Table 2 presents the resulting optimal designs for an allowable displacement of 2.0 inches obtained by the three methods of this research. Two different values of the relaxation parameter η were used with the algorithm of section III as shown in this table.

The optimal design obtained b_ Powell's method was almost identical to that reported by Schmit and Miura using their Model 1; however, the technique required 4605 analyses to obtain this design. The stress ratio method with displacement scaling also gave a similar design with only slightly higher weight. It is noted that the final design obtained by this method was not fully stressed, because of the displacement scaling present. The optimal designs generated by the technique of section III of this paper for two different relaxation parameters had a slightly higher weight yet. This seems to indicate that the stress ratio method with displacement scaling is perhaps the best method to use in this case, since it is very simple and is able to locate very nearly the optimal design in a small number of iterations.

The wing box problem was also solved under a reduced allowable displacement of 1.0 inch using Powell's method, stress ratio method with displacement scaling and the design algorithm of section III. The final designs using the three techniques are presented in Table 3. Since there are no previously published results available for comparison, the results of the latter two methods are compared with those obtained by Powell's method. The final design obtained by Powell's method is believed to be close to the optimum design, since the method compared favorably with the one by Schmit and Miura for the 2.0 inch case. The design obtained by the algorithm of section III was very close to the Powell's method design. The final design obtained by stress ratio method with displacement scaling is considerably different with a much higher weight; in fact it has almost twice the minimum weight obtained for an allowable displacement of 2.0 inches using the same technique. It has been observed in this wing box problem that, if a displacement limit of a little over 2.0 inches was imposed, then the optimal design would be one in which no displacement constraint was active. Thus imposing a displacement limit of 2.0 inches or less would result in an active displacement constraint at the optimal design, but with a limit of over 2.0 inches no displacement constraint would be active. Since the stress ratio method only considers the displacement limit in a crude way, by simple scaling, then it might be expected to have difficulty with a problem where the displacement constraint was critical at the optimal design, which is the case for this example with a displacement limit of 1.0 inch. On the other hand, with a displacement limit of 2.0 inches, even though it is active at the optimal design, it is not a critical constraint. Thus the stress ratio method should work well.

The final designs using the three different techniques for an allowable displacement of 4.0 inches are presented in Table 4. The design obtained by the algorithm of section III is close to the optimum design obtained by Powell's method, but is slightly heavier; however, most of the design variables are very similar. The optimum design obtained by the stress ratio method with displacement scaling is nearly the same as Powell's method except the areas of truss elements 3, 4 are slightly higher. It is interesting to note that in this method different starting points made no difference in the number of iterations or minimum weight achieved. It is also observed that the stress ratio method with displacement scaling worked better than the design algorithm of section III as it did in 2.0 inch case. This is due to the fact that the displacement constraint is not active at the optimal design, thus it need not be considered in the optimization, which is the case for the stress ratio method.

In conclusion, the design algorithm presented in section III gave good results for all three allowable displacements used, i.e. 2.0 inch, 4.0 inch, and 1.0 inch. Most of the design variables obtained in this method for all three cases were a good match with those determined by Schmit and Miura and by Powell's method. Also, the constraints that were active were almost always identical to those of Schmit and Miura, and Powell's method. In the design algorithm of section III, a wide range of starting points were used, most of which were far from optimum design, nevertheless the optimum design was obtained in from 11-13 iterations for all cases, while in Schmit and Miura, and Powell's method the initial guess was chosen very close to the optimum design. If starting points had been selected farther from the optimum design it would likely had taken more analyses than indicated in Tables 1-4. In the design algorithm of this paper, a wide range of relaxation parameters η were tried. It was found that η in the range from 0.05 to 0.15 gave the best results for all cases. The stress ratio method worked well and gave designs similar to Schmit and Miura and Powell's method when the displacement constraints at the optimum design were not active (or when just barely active) but it did not work when the displacement constraints were critical. The design algorithm of section III gave good results regardless of whether the displacement constraint was active or not, therefore use of this method is recommended over the stress ratio method with displacement scaling.

Table 1 Final Designs for 18 Element Wing Box, Allowable Displacement 2.0 Inches

Member No.	Schmit and Miura: CST Model 1 SSP	CST Model 2 SSP	CST Model 1 Shear Web	Gallatly & Berke	Gallatly*	This Research
	A_i (in^2)	A_i (in^2)	A_i (in^2)	A_i (in^2)	A_i (in^2)	A_i (in^2)
TRUSS						
1	4.045	3.151	2.229	0.6505	1.0431	0.9735
2	0.1001	0.1000	0.1001	0.1001	0.1036	0.1301
3	0.1001	0.1000	0.1000	0.2366	0.3508	0.2339
4	0.1330	0.2324	0.3202	0.2352	0.3315	0.3652
5	0.1002	0.1000	0.1001	0.1001	0.1035	0.1479
	T_j (in)	T_j (in)	T_j (in)	T_j (in)	T_j (in)	T_j (in)
CST						
1,2	0.08286	0.08641	0.1093	0.1328	**0.1441	0.11131
3,4	0.05363	0.05733	0.05911	0.0702	**0.0599	0.05139
5	0.03786	0.03932	0.04098	0.0449	0.0435	0.03555
	T_j (in)	T_j (in)	T_j (in)	T_j (in)	T_j (in)	T_j (in)
SSP						
1	0.3636	0.3851	0.09345	0.0876	0.0876	0.53075
2	0.2236	0.2152	0.09437	0.0889	0.0895	0.22626
3	0.1310	0.1361	0.07687	0.0808	0.0664	0.11120
4	0.1156	0.1004	0.07293	0.0768	0.0553	0.16274
5	0.09166	0.09113	0.07570	0.0815	0.0537	0.10408
6	0.02000	0.02000	0.02001	0.0200	0.0219	0.02070
7	0.02000	0.02000	0.02001	0.0200	0.0215	0.02070
8	0.03096	0.03090	0.02804	0.0337	0.0256	0.02994
Final Wt. (lbs)	402.97	403.35	357.92	387.67	389.8	429.38
Analyses Needed	9	11	9	4***	193	13

*The original design obtained by Gallatly was scaled up so that the triangular idealization of the cover plates satisfies stress constraints.
**Each portion was modelled by a quadrilateral element in the original work by Gallatly.
***Subsequent iterations gave heavier designs.

V. Conclusions

As a result of solving several example cases it has been shown that the optimality criterion method developed by Khan, Willmert, and Thornton can be extended to structures composed of different element types. Starting from any initial design, the efficient technique is capable of locating a near optimal design in very few analyses, with each iteration of the method requiring little additional calculation beyond that required for the analysis. That is, no extensive derivatives are required, nor the determination of large numbers of Lagrange multipliers.

Acknowledgement

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Table 2 Final Designs for 18 Element Wing Box, Allowable Displacement 2.0 Inches

Member No.	Powell's Method	Stress Ratio Method With Displacement Scaling	Design Algorithm of Section III	
			η=.15	η=.10
	(in^2)	(in^2)	(in^2)	(in^2)
TRUSS				
1	3.1768	3.8082	0.8629	0.9735
2	0.1017	0.1211	0.1037	0.1301
3	0.1016	0.8209	0.1785	0.2339
4	0.1016	0.4745	0.4291	0.3652
5	0.1021	0.1056	0.1277	0.1479
	(in)	(in)	(in)	(in)
CST				
1,2	0.08575	0.07120	0.11434	0.11131
3,4	0.05556	0.05003	0.05108	0.05139
5	0.03724	0.04000	0.03493	0.03555
SSP				
1	0.44214	0.46342	0.51675	0.53075
2	0.21703	0.24850	0.22939	0.22626
3	0.12819	0.16012	0.11050	0.11120
4	0.11548	0.13341	0.16876	0.16274
5	0.09427	0.09816	0.10848	0.10408
6	0.02015	0.02112	0.02075	0.02070
7	0.02018	0.02112	0.02075	0.02070
8	0.02036	0.03254	0.02741	0.02994
Final Weight (lbs)	406.72	418.50	429.80	429.38
Analyses Needed	4605	9	13	13

Table 3 Final Designs Using Different Techniques For Allowable Displacement 1.0 Inch

Member No.	Powell's Method	Design Algorithm of Section III	Stress Ratio Method With Displacement Scaling
	(in^2)	(in^2)	(in^2)
TRUSS			
1	2.2897	1.0110	7.5117
2	0.1100	0.2993	0.2107
3	1.1754	1.5429	1.4704
4	1.1921	0.8385	0.7881
5	0.1102	0.2402	0.2107
	(in)	(in)	(in)
CST			
1,2	0.17404	0.18478	0.14092
3,4	0.11533	0.13036	0.10193
5	0.09889	0.08873	0.08051
SSP			
1	0.20066	0.20034	0.97677
2	0.16501	0.15930	0.49612
3	0.25517	0.21330	0.33075
4	0.18624	0.18628	0.26438
5	0.22221	0.17286	0.19369
6	0.02054	0.02000	0.04215
7	0.02029	0.02000	0.04215
8	0.07849	0.06690	0.065673
Final Weight (lbs)	663.96	669.26	838.84
Analyses Needed	4083	11	10

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Table 4 Final Designs Using Different Techniques For Allowable Displacement 4.0 Inches

Member No.	Powell's Method	Design Algorithm of Section III	Stress Ratio Method With Displacement Scaling
	(in^2)	(in^2)	(in^2)
TRUSS			
1	3.5682	1.0626	3.6869
2	0.1052	0.1041	0.1045
3	0,1067	0.1041	0.6594
4	0.1163	0.1638	0.3241
5	0.1049	0.1041	0.1045
	(in)	(in)	(in)
CST			
1,2	0.08377	0.11289	0.06935
3,4	0.05234	0.05423	0.05143
5	0.03816	0.03601	0.04024
SSP			
1	0.41270	0.49950	0.50901
2	0.22714	0.22511	0.24523
3	0.13464	0.11124	0.16935
4	0.13326	0.16008	0.12981
5	0.09455	0.10697	0.09508
6	0.02047	0.02082	0.02091
7	0.02044	0.02082	0.02091
8	0.03119	0.02944	0.03293
Final Weight (lbs)	406.04	426.72	417.98
Analyses Needed	4074	11	11