

AD-A086 659

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F/G 20/4

ANALYSIS OF NUMERICAL METHODS FOR THE EQUATIONS OF FLUID DYNAMICS--ETC(11)

JUN 80 C A HALL, T A PORSCHE

F49620-79-C-0100

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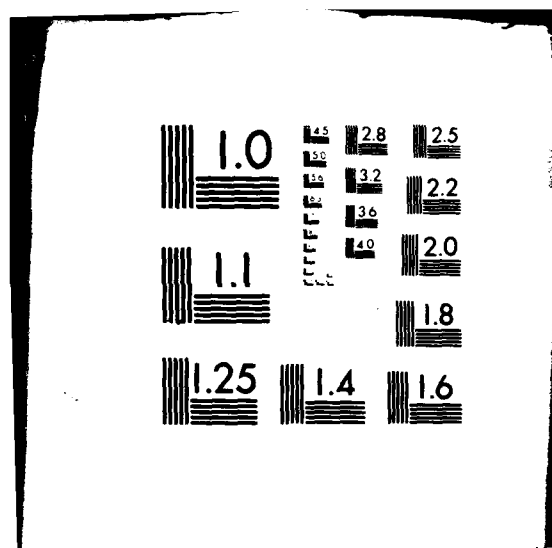
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Final Report

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AFOSR Contract No. F49620-79-C-0100

"Analysis of Numerical Methods for the Equations of Fluid Dynamics"

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June, 1980

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During the last twenty-five years, the published literature on computational fluid dynamics has grown enormously. However, there are few papers dealing with the analysis of computational methods for time dependent multidimensional flow. There also seems to be a polarization of investigators into two groups: advocates of finite difference methods and advocates of finite element methods.

It is our contention that an "ideal" computational model for fluid flow problems may in fact draw on the strengths of both schools of thought. For example, we have recently implemented a frontal solution technique, originally devised for finite element analyses, in a finite difference model of two-phase fluid flow.

Major Accomplishments:

A. Flexible Stencil for Navier Stokes

In 1977, W. Frey developed for potential flow a 9-point finite difference stencil which permits the points of the stencil to lie along

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curved lines, thus promoting straightforward modeling of curved boundaries. This construction involves a standard finite element parametric mapping. We have extended the notion of a flexible stencil (including more global transformations) to the more general, nonlinear Navier Stokes problem. We have derived in detail a consistent finite difference stencil for a linearized discretization of this problem which will accommodate curved boundaries. This work will be included in a Ph.D. thesis of John Ellison (expected December 1980).

B. Stability and Convergence Analysis

In 1966, Krzhivitski and Ladyzhenskaya developed a finite difference discretization of the Navier-Stokes problem for which they prove unconditional stability in the discrete L_2 -norm. Their proof hinges on a "key inequality", the proof of which uses distinctive properties of their discretization. Their scheme also generates approximations which converge to a weak solution of the continuous problem as the discretization parameters tend to zero.

However, unless the region is a union of squares their treatment of boundary conditions utilizes a translation of boundary velocities to the nearest interior mesh point. Although this rather naive procedure is adequate for a convergence analysis, in actual computation it may introduce a major contribution to the error. An obvious way to avoid this objection is to map the flow region onto a square, in the spirit of the 1974 works of Thompson, Thames and Martin or Hall and Gordon.

As a step towards analyzing the flexible stencil and/or the use of curvilinear co-ordinatizations, we first considered the extension of the Krzhivitski-Ladyzhenskaya convergence analysis to a differential system of the form that would arise in such transformed problems. This required the derivation of a new class of finite difference approximations.

To date, we have been able to establish the analogue of the Krzhivitski-Ladyzhenskaya "key inequality" and hence the unconditional stability of the class of finite difference schemes generated. It is anticipated that the work of Krzhivitski-Ladyzhenskaya and Temam can be applied to prove weak convergence of our schemes also. This work will constitute a portion of a Ph.D. thesis of John Ellison (expected December 1980).

C. Fractional Step Methods

The fractional step methods as presented by Chorin in the late 1960's have been re-formulated in terms of Padé approximations to the solution of a matrix-initial-boundary value problem with algebraic constraints. As such, it becomes apparent that these schemes inherently involve a factorization of

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a matrix exponential, which is exact only for a very restrictive class of problems. The error introduced for more general problems does vanish as the discretization parameters tend to zero. An assessment of its magnitude for computationally practical grids is still under investigation.

To date, we have developed an algebraic proof of convergence for the type of fractional step methods (assuming periodic boundary conditions) studied by Chorin as well as those using a MAC placement of variables and Dirichlet boundary conditions.

D. Network Theory and Implicit Differencing

The implicit finite difference method can with proper identification be regarded as systems defining flows on an associated network. This observation has lead to the dual variable method which, through the introduction of a different set of network variables, significantly reduces the size of the original finite difference system. A computer code DUVAL, (not developed with AFOSR funding) which solves the two-dimensional, homogeneous, two-phase, transient, fluid flow problem has demonstrated the computational savings and robustness of the dual variable method.

Currently, a computer program DUMAS is being developed (partially with AFOSR funding) which combines the computational efficiencies of the dual variable method with new continuation strategies of W. C. Rheinboldt. A nonlinear, implicit, steady state system of finite difference equations is reduced in size by a factor of three via the introduction of the dual variables. A sequence of problems is then solved in which the continuation parameter arises in the equation of state advancing from an incompressible fluid to a thermally expandable fluid.

E. Analysis of Difference Approximations of the Stationary Navier-Stokes Equations.

In her book on viscous incompressible flow, Ladyzhenskaya demonstrates the existence of a generalized solution of the stationary Navier-Stokes equations with homogeneous boundary conditions. For sufficiently large viscosity she also proves that this solution is unique.

We have shown that exactly analogous results hold for a certain consistent system of nonlinear finite difference equations. In the case of inhomogeneous boundary conditions an existence theorem holds for the infinite dimensional problem. We believe that a finite dimensional analog of this result is also true. This and related theoretical questions are investigated in the Ph.D. theses of A. Cha (expected December (1980).

Publications/Theses Supported by F49620-79-C-0100:

1. C. A. Hall and T. A. Porsching, "Padé Approximation, Fractional Step Methods and Navier-Stokes Discretizations", ICMA-80-13 Technical Report, January, 1980. Invited paper for G. D. Birkhoff Special Issue of SIAM J. Num. Anal.
2. R. Amit, C. A. Hall and T. A. Porsching, "An Application of Network Theory to the Solution of Implicit Navier-Stokes Difference Equations", ICMA-79-09 Technical Report, November, 1979. Submitted to J. Comp. Phys.
3. J. Ellison, Ph.D. Thesis, University of Pittsburgh (expected completion December 1980).
4. A. Cha, Ph.D. Thesis, University of Pittsburgh (expected completion December 1980).

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SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER (1) AFOSR-TR-80-0543	2. GOVT ACCESSION NO. AD-A086659	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) (6) ANALYSIS OF NUMERICAL METHODS FOR THE EQUATIONS OF FLUID DYNAMICS,	5. TYPE OF REPORT & PERIOD COVERED (9) Final rept.	
7. AUTHOR(s) (10) C. A./Hall T. A./Porsching	6. PERFORMING ORG. REPORT NUMBER	
	8. CONTRACT OR GRANT NUMBER(s) (15) F49620-79-C-0100-4	
9. PERFORMING ORGANIZATION NAME AND ADDRESS University of Pittsburgh Department of Mathematics and Statistics Pittsburgh, PA 15260	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A3	
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, Washington, DC 20332	12. REPORT DATE June, 1980	
	13. NUMBER OF PAGES Four	
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)	15. SECURITY CLASS. (of this report) UNCLASSIFIED	
15a. DECLASSIFICATION DOWNGRADING SCHEDULE		
16. DISTRIBUTION STATEMENT (of this Report) Approved for public; distribution unlimited.		
17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) (11) Jun 80 L		
18. SUPPLEMENTARY NOTES		
19. KEY WORDS (Continue on reverse side if necessary and identify by block number)		
20. ABSTRACT (Continue on reverse side if necessary and identify by block number) During the last twenty-five years, the published literature on computational fluid dynamics has grown enormously. However, there are few papers dealing with the analysis of computational methods for time dependent multidimensional flow. There also seems to be a polarization of investigators into two groups; advocates of finite difference methods and advocates of finite element methods. It is our contention that an "ideal" computational model for fluid flow problems may in fact draw on the strengths of both schools of thought. For example, we have recently implemented a frontal solution technique, originally devised		

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