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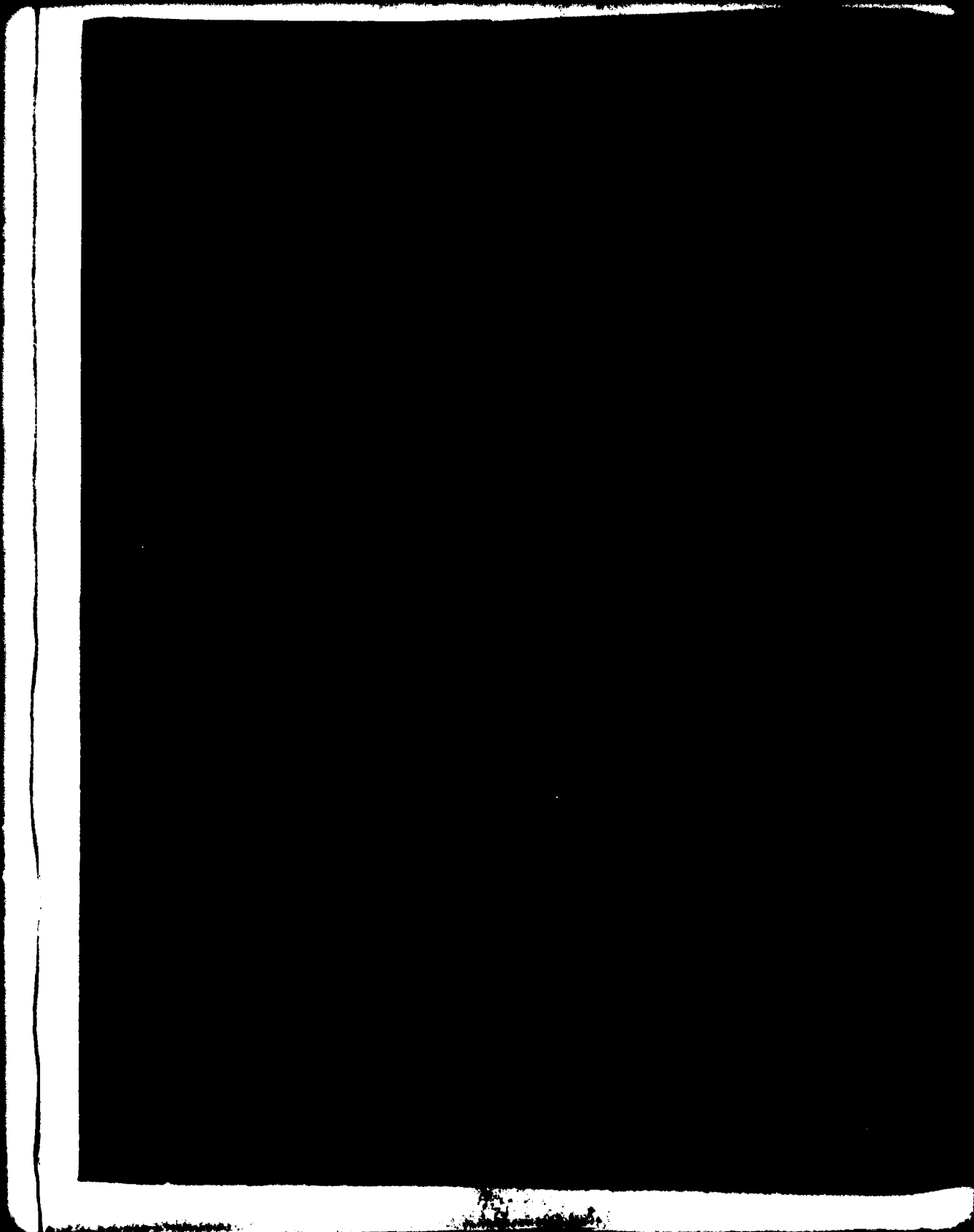
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

SHORT SPACE IMPLEMENTATION OF WIENER FILTERING FOR IMAGE RESTORATION

J. S. LIM

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Abstract

In this report, short space implementation of image restoration systems such as Wiener filtering to avoid the image non-stationarity problem is discussed. It is demonstrated by way of examples that short space implementation leads to a significant performance improvement in reducing wide-band random noise relative to the traditional approach in which the entire image is processed by a linear space invariant filter.

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I.   INTRODUCTION

There exist a number of practical situations in which the restoration
of a degraded image is important and consequently the problem of image
restoration has received considerable attention in the literature (1,2).
In many cases such as Wiener filtering (3), power spectrum filtering (4),
etc., the restoration techniques are based on the assumption that an image
can be modelled by a stationary random field, and restoration is achieved
by filtering the degraded image with a linear space invariant restoration
filter.

For a typical image, each part of an image generally differs suffi-
ciently from other parts so that the stationarity assumption over the entire
image is not generally valid. One approach to reduce the effect of the
non-stationarity problem is to implement a restoration filter on a short-
space basis in which an image is divided into many subimages and each sub-
image is restored separately and then combined. The notion to implement an
image restoration system on a short-space basis to reduce the image non-
stationarity problem has been considered in the literature (5,6,7). For
example, Hunt and Trussel (5) segmented the image by overlapping two
dimensional (2-D) rectangular windows for an MAP image restoration system.
Lim (6) segmented the image by overlapping 2-D separable triangular windows
for a spectral subtraction image restoration system. Even though the
importance of short-space processing has been observed in the context of
specific recent image restoration techniques, it has not been demonstrated
in the more traditional restoration techniques such as Wiener filtering.
In this note, we demonstrate by way of examples that short-space imple-
mentation can noticeably enhance the performance of well known image
restoration techniques such as Wiener filtering.

In section II, we discuss the model of image degradation considered
in this note and the Wiener filtering technique for image restoration. In
section III, we discuss short-space implementation of Wiener filtering. In

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section IV, we illustrate and discuss various examples which demonstrate the importance of short-space processing for image restoration by Wiener filtering.

II.   IMAGE DEGRADATION MODEL AND WIENER FILTERING

The degradation that will be considered in this note is additive random noise. Specifically, a degraded image $y(n_1,n_2)$ is represented by

$$y(n_1,n_2) = f(n_1,n_2) + d(n_1,n_2) \qquad (1)$$

where $f(n_1,n_2)$ is a noise-free image and $d(n_1,n_2)$ is additive random noise uncorrelated with $f(n_1,n_2)$. The restoration problem is to restore $f(n_1,n_2)$ from the degraded image $y(n_1,n_2)$.

If $f(n_1,n_2)$ and $d(n_1,n_2)$ are assumed to be samples of stationary random fields uncorrelated with each other, then the optimum linear filter which minimizes the mean square error between $f(n_1,n_2)$ and the processed image is the non-causal Wiener filter whose frequency response $H(\omega_1,\omega_2)$ is given by

$$H(\omega_1,\omega_2) = \frac{P_f(\omega_1,\omega_2)}{P_f(\omega_1,\omega_2) + P_d(\omega_1,\omega_2)} \qquad (2)$$

In equation (2), $P_f(\omega_1,\omega_2)$ and $P_d(\omega_1,\omega_2)$ represent the power spectrum of the image and degrading random noise respectively.

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Image restoration based on equation (2) requires a priori knowledge of $P_f(\omega_1,\omega_2)$ and $P_d(\omega_1,\omega_2)$. In some applications (8,9), $P_d(\omega_1,\omega_2)$ is known exactly and in many others (2) the estimation of $P_d(\omega_1,\omega_2)$ is relatively simple. In this note, we assume that $P_d(\omega_1,\omega_2)$ is known a priori. To estimate $P_f(\omega_1,\omega_2)$, a common procedure used is to average the spectral density over many different images or prototype images. Alternatively, $P_f(\omega_1,\omega_2)$ could be estimated by

$$\hat{P}_f(\omega_1,\omega_2) = S[k \cdot |Y(\omega_1,\omega_2)|^2 - P_d(\omega_1,\omega_2)] \tag{3}$$

where $Y(\omega_1,\omega_2)$ is the discrete space Fourier transform[1] of $y(n_1,n_2)$, "k" is a normalization constant between power and energy spectrum and "S" is some form of smoothing operation. Subtraction of $P_d(\omega_1,\omega_2)$ reduces the bias due to the noise power and the smoothing operation reduces (10) the variance of the spectral estimate. Estimating $P_f(\omega_1,\omega_2)$ in a manner similar to equation (3) has the advantage that the estimated $P_f(\omega_1,\omega_2)$ is derived from some aspect of the signal $f(n_1,n_2)$ and consequently is data-adaptive.

Once $H(\omega_1,\omega_2)$ is determined, $f(n_1,n_2)$ is typically estimated by[2]

$$\hat{f}(n_1,n_2) = F^{-1}[Y(\omega_1,\omega_2) \cdot H(\omega_1,\omega_2)] \tag{4}$$

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1. Definitions for various terms such as discrete space Fourier transform, power spectrum, and energy spectrum, and determination of the normalization constant "k" can be found in (6,10).

2. Since equation (4) is implemented using the Discrete Fourier Transform (DFT) and Inverse Discrete Fourier Transform (IDFT), some aliasing may occur at the image boundaries. Such aliasing can be reduced by increasing the size of the DFT and IDFT. The smoothing operation in equation (3) also reduces the aliasing problem, since from equation (2) a smoother spectral estimate of $P_f(\omega_1,\omega_2)$ generally corresponds to a shorter unit sample response of the resulting filter.

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where $\hat{f}(n_1,n_2)$ represents an estimate of $f(n_1,n_2)$ and F^{-1} represents the inverse discrete space Fourier transform operation. The implementation of non-causal Wiener filtering by equation (4) corresponds to processing the entire image with a linear space invariant filter. As will be discussed in section IV, the performance of this implementation is poor in reducing additive random noise partly due to the image non-stationarity problem.

III. SHORT SPACE IMPLEMENTATION

To reduce the effect of the image non-stationarity problem, a reasonable approach is to implement Wiener filtering on a short space basis in which the degraded image is divided into many subimages and each subimage is restored separately and then combined. One such approach is to first apply a short space window function $w_{ij}(n_1,n_2)$ to $y(n_1,n_2)$ in equation (1) so that

$$y(n_1,n_2) \cdot w_{ij}(n_1,n_2) = f(n_1,n_2) \cdot w_{ij}(n_1,n_2) \tag{5}$$
$$+ d(n_1,n_2) \cdot w_{ij}(n_1,n_2)$$

Denoting $y(n_1,n_2) \cdot w_{ij}(n_1,n_2)$ by $y_{ij}(n_1,n_2)$ and using similar notation for the remaining two terms, equation (5) can be written as

$$y_{ij}(n_1,n_2) = f_{ij}(n_1,n_2) + d_{ij}(n_1,n_2) \tag{6}$$

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Equation (6) is identical in form to equation (1). By substituting the roles of $f(n_1,n_2)$, $d(n_1,n_2)$ and $y(n_1,n_2)$ in equation (1) for $f_{ij}(n_1,n_2)$, $d_{ij}(n_1,n_2)$ and $y_{ij}(n_1,n_2)$ in equation (5), the discussions in section II can be applied to estimate $f_{ij}(n_1,n_2)$ from $y_{ij}(n_1,n_2)$. Since the essence of short space processing is data-adaptive filtering, the estimation of $P_f(\omega_1,\omega_2)$ from $y_{ij}(n_1,n_2)$ in constructing $H(\omega_1,\omega_2)$ should be performed in a manner similar to equation (3). Once each subimage $f_{ij}(n_1,n_2)$ is estimated, the entire image can be constructed by

$$\hat{f}(n_1,n_2) = \sum_i \sum_j \hat{f}_{ij}(n_1,n_2) \tag{7}$$

To successfully implement Wiener filtering on a short space basis, the window function $w_{ij}(n_1,n_2)$ must be carefully chosen. For example, to reconstruct an image from its subimages by equation (7), $w_{ij}(n_1,n_2)$ has to satisfy the following equation;

$$\sum_i \sum_j w_{ij}(n_1,n_2) = 1 \quad \text{for all } n_1, n_2 \text{ of interest.} \tag{8}$$

In addition, $w_{ij}(n_1,n_2)$ is desired to be a smooth function to avoid some possible discontinuities or degradations that may appear at the subimage boundaries in the processed image. Two window functions which have the above properties are 2-D separable triangular or Hamming window overlapped with its neighboring window by half the window duration in each dimension (6).

In summary, in estimating $f(n_1,n_2)$ from $y(n_1,n_2)$ by short space implementation of Wiener filtering, we first divide $y(n_1,n_2)$ into subimages and the non-causal Wiener filter is applied separately to each subimage. The resulting subimages are then combined to form an estimate of $f(n_1,n_2)$. In the next section, we illustrate by way of examples the performance improvement that can be achieved by short space implementation.

IV.   EXAMPLES AND DISCUSSIONS

In this section, we illustrate and discuss various examples which demonstrate the importance of short space processing for image restoration by Wiener filtering. In Figure 1 are shown two noise-free images of 256 x 256 pixels with each pixel represented by 8 bits. In Figure 2 are shown two degraded images at S/N ratio of 10 dB that were generated by adding zero mean white Gaussian noise to the images of Figure 1. The Gaussian noise was digitally generated and S/N ratio is defined by

$$\text{S/N ratio} = 10. \log \frac{\text{Variance of } f(n_1,n_2)}{\text{Variance of } d(n_1,n_2)} \qquad (9)$$

In Figure 3 are shown typical images obtained by filtering the degraded images of Figure 2 with a linear space invariant non-causal Wiener filter. In Figure 4 are shown the results of short space implementation of a non-causal Wiener filter.

In estimating the image power spectrum $P_f(\omega_1,\omega_2)$ to generate the images in Figure 3, a variety of different ways including those discussed in section I have been considered. In all cases, we have observed that the background noise can be noticeably reduced only at the expense of noticeable blurring of the resulting images. The images shown in Figure 3 are typical
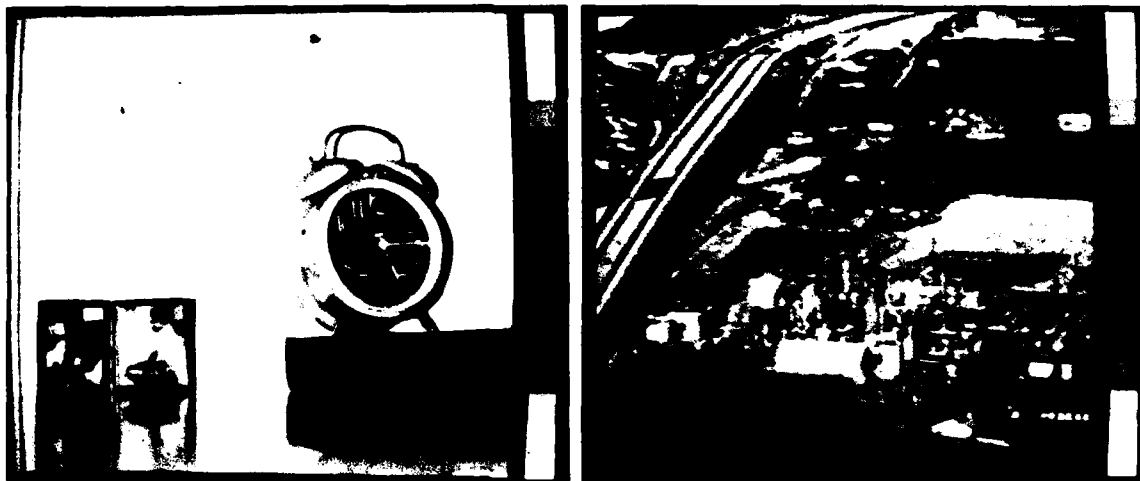
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Fig. 1. Original images.

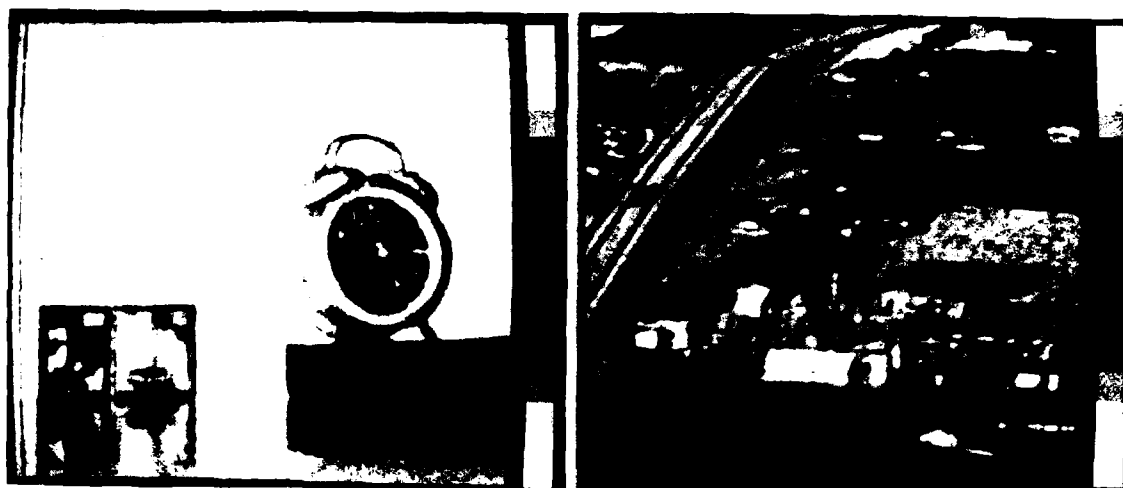Fig. 2. Images in Fig. 1 degraded by additive Gaussian noise at S/N ratio of 10 dB.

Fig. 3. Images in Fig. 2 processed by a space
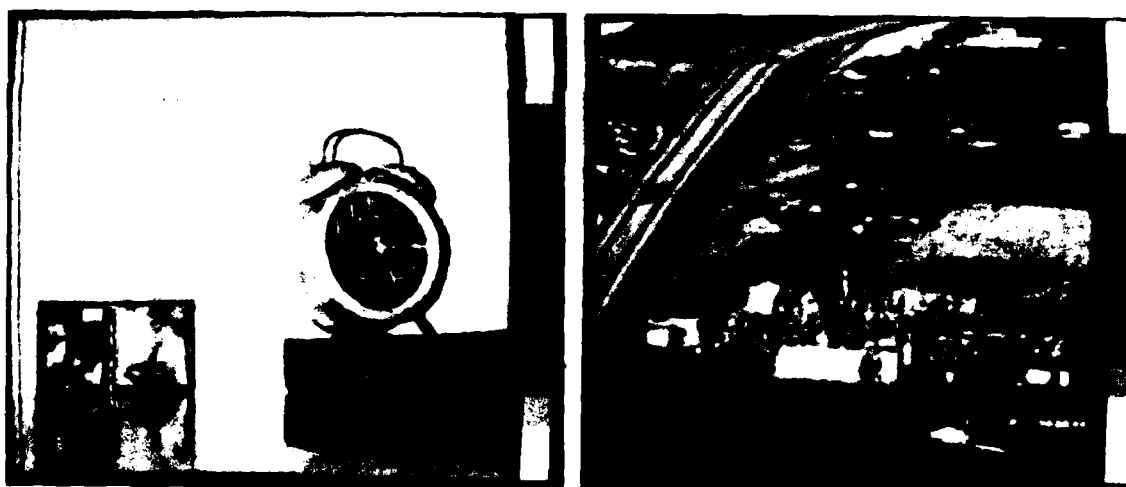invariant Wiener filter.

Fig. 4. Images in Fig. 2 processed by short space
implementation of a Wiener filter.

examples. This seriously diminishes the usefulness of Wiener filtering in reducing background noise in the context of image restoration.

The images in Figure 4 were obtained by estimating the power spectrum of each subimage in a manner similar to equation (3) so that the estimated power spectrum is data adaptive. The subimage size (window size) used in the short space implementation is 8x8 pixels for the "clock" picture and 32x32 pixels for the "aerial view of a village" picture and the window shape used is a 2-D separable triangular window. The subimage size was chosen based on the considerations that a too large subimage size has the image non-stationarity problem and a too small subimage size leads to performance degradation due to the lack of available data to process. For the types of images that we considered such as the two images used in this note, the subimage size in the range of 8x8 pixels to 32x32 pixels produced the best results.

For all the images in Figures 3 and 4, the amount of noise reduction computed corresponds to S/N ratio improvement of approximately 4 or 5 dB in the normalized mean square sense (8). The traditional approach of Wiener filtering achieves this noise reduction at the expense of noticeable blurring of the resulting images. Short space implementation, however, achieves the same noise reduction with significantly less blurring of the resulting images than the traditional approach.

Even though the performance obtained by short space implementation of Wiener filtering is clearly superior to that obtained by the traditional Wiener filtering approach in the context of image restoration, the usefulness of short space Wiener filtering in improving image quality or intelligibility when the degradation is additive noise is difficult to judge from the comparison of Figures 2 and 4. However, in those applications in which S/N ratio improvement in the mean square sense

without significant image blurring is important, short space implementation of Wiener filtering discussed in this note may be useful. Furthermore, there is considerable room for further improvement in the short space implementation technique. For example, in this note we have divided an image into sub-images by imposing arbitrary boundaries. If an image is divided into sub-images at more natural boundaries such as image edges, the system performance may improve. These and other potential improvements are under consideration.

In this note, we have considered short space processing to reduce the image non-stationarity problem in Wiener filtering. In addition to Wiener filtering, we have also considered other image restoration systems such as power spectrum filtering (4), geometrical mean filtering (4), etc. which are based on the assumption that an image can be modelled by a stationary random field. In all these cases, we have also observed that short space implementation similar to that discussed in this note leads to a significant performance improvement over the traditional approach in which the entire image is processed by a linear space invariant filter. From these results, it appears that any image restoration system based on the stationarity assumption of an image will benefit from short space implementation in which each subimage rather than the entire image is assumed to be stationary.

ACKNOWLEDGMENTS

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