

AD-A086 776

NAVAL RESEARCH LAB WASHINGTON DC

F/6 20/5

SLOW WAVE CYCLOTRON INSTABILITY IN DIELECTRIC LOADED WAVEGUIDE --ETC(U)

MAY 80 A K GANGULY, K R CHU

UNCLASSIFIED

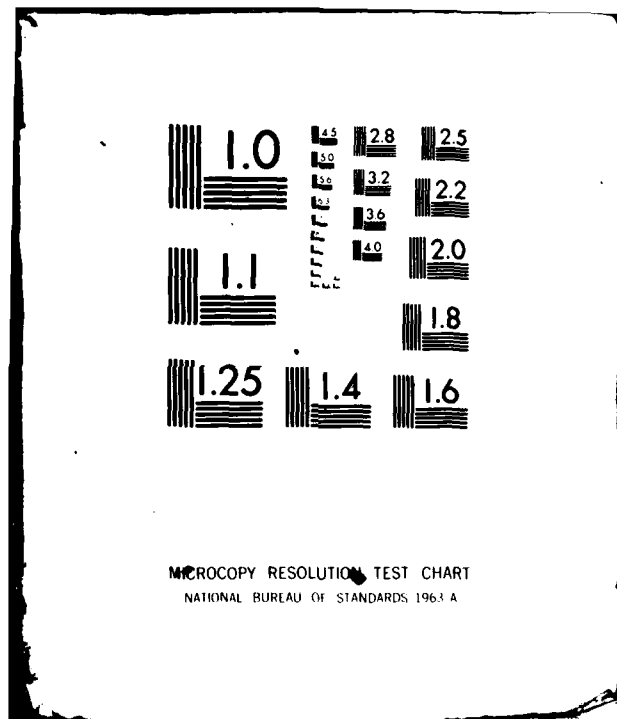
NRL-MR-4215

SBIE-AD-E000 468

NL

1-1  
10  
12 1980-11

END  
DATE  
FILMED  
8 80  
DTIC



ADA086776

14  
UNCLASSIFIED

NRL-MR-4215

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                  | READ INSTRUCTIONS<br>BEFORE COMPLETING FORM   |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------|-----------------------------------------------|
| 1. REPORT NUMBER<br>NRL Memorandum Report 4215                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 2. GOVT ACCESSION NO.<br>AD-A086776                                                              | 3. RECIPIENT'S CATALOG NUMBER<br>(9)          |
| 4. TITLE (and Subtitle)<br>SLOW WAVE CYCLOTRON INSTABILITY IN DIELECTRIC<br>LOADED WAVEGUIDE OF RECTANGULAR<br>CROSS-SECTION.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           | 5. DATE OF REPORT & PERIOD COVERED<br>Final report on one phase of a con-<br>tinuing NRL problem |                                               |
| 7. AUTHOR(s)<br>A. K. Ganguly and K. R. Chu                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 8. CONTRACT OR GRANT NUMBER(s)<br>NRL Problem No. 67-0866-0-0                                    |                                               |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS<br>Naval Research Laboratory<br>Washington, DC 20375                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        | 10. PROGRAM ELEMENT, PROJECT, TASK<br>AREA & WORK UNIT NUMBERS                                   |                                               |
| 11. CONTROLLING OFFICE NAME AND ADDRESS                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 | 12. REPORT DATE<br>18 May 1986                                                                   | 13. NUMBER OF PAGES<br>20                     |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             | 15. SECURITY CLASS. (of this report)<br>UNCLASSIFIED                                             | 15a. DECLASSIFICATION/DOWNGRADING<br>SCHEDULE |
| 16. DISTRIBUTION STATEMENT (of this Report)<br>Approved for public release; distribution unlimited                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                  |                                               |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |                                                                                                  |                                               |
| 18. SUPPLEMENTARY NOTES<br>SBIE                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                  |                                               |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number)<br>Weibel instability<br>Slow wave cyclotron instability<br>Electron beam<br>Electromagnetic waves                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |                                                                                                  |                                               |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number)<br>Slow wave cyclotron instability in dielectric loaded waveguides driven by an anisotropic velocity distribution is considered. The spatial growth rate of the radiation field is calculated by deriving a dispersion relation of the waveguide modes in the presence of a weak electron beam. It is shown that broad band amplification is possible in such structures. Bandwidth in excess of 60% is obtained for a "cold" beam. The bandwidth decreases with increase in the velocity spread of the electrons. At 5% velocity spread, the bandwidth decreases to 12%. The dependence of the gain and bandwidth on other material and electron beam parameters as well as the applied magnetic field is discussed. |                                                                                                  |                                               |

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE  
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

251150

Flu

## CONTENTS

|                                   |    |
|-----------------------------------|----|
| I. INTRODUCTION .....             | 1  |
| II. MODEL AND FORMALISM .....     | 2  |
| III. RESULTS AND DISCUSSION ..... | 8  |
| IV. CONCLUSION .....              | 10 |
| REFERENCES .....                  | 10 |

|                     |                                     |
|---------------------|-------------------------------------|
| Accession For       |                                     |
| NTIS GRA&I          | <input checked="" type="checkbox"/> |
| DDC TAB             | <input type="checkbox"/>            |
| Unannounced         | <input type="checkbox"/>            |
| Justification _____ |                                     |
| By _____            |                                     |
| Distribution/ _____ |                                     |
| Availability Codes  |                                     |
| Dist.               | Avail and/or<br>special             |
| <b>A</b>            |                                     |

S      DTIC      D

ELECTE

JUL 17 1980

D

# **SLOW WAVE CYCLOTRON INSTABILITY IN DIELECTRIC LOADED WAVEGUIDE OF RECTANGULAR CROSS SECTION**

## **I. INTRODUCTION**

The instability of the waveguide modes interacting with a relativistic electron beam guided by an applied uniform magnetic field  $B_0$  may occur due to two different mechanisms, namely, cyclotron maser radiation<sup>1,2</sup> (CMR) and Weibel instability<sup>3</sup>. Both mechanisms depend on an anisotropic velocity distribution of the electrons. The instability in CMR results from the azimuthal bunching of electrons due to the relativistic variation of their masses. Weibel instability, on the other hand, is nonrelativistic in nature and originates from axial bunching of the electrons induced by the Lorentz force  $\vec{v}_\perp \times \vec{B}^{(1)}$  where  $\vec{v}_\perp$  is the velocity of electrons perpendicular to  $B_0$  and  $\vec{B}^{(1)}$  the wave magnetic field. The phase bunching tends to be destroyed if there is any spread in the velocity of the electrons. In a recent study<sup>4</sup>, it was shown that the two mechanisms always combine to offset one another. The azimuthal bunching predominates for waves with phase velocity ( $v_{ph}$ ) greater than the speed of light in vacuum ( $c$ ) while the axial bunching predominates if  $v_{ph} < c$ . Thus cyclotron maser radiation (CMR) is generated in fast wave modes and the Weibel instability in slow wave modes. Weibel instability is capable of producing amplification over a wide bandwidth. In this paper we consider slow wave cyclotron instability in a dielectric loaded waveguide of rectangular cross-section. In this geometry, the instability can be studied in the fundamental mode which couples efficiently with external circuit.

In Section II we derive the modified dispersion relation of the dielectric-loaded waveguide modes in the presence of a weak electron beam. The response of the electrons is determined by solving linearized Vlasov Equation. Effect of a spread in  $V_1$  and  $V_z$  (components of electron velocity perpendicular and parallel to the applied magnetic field  $B_0$ ) is also considered. We assume that the spread in  $V_1$  and  $V_z$  arise from a spread in the angle at which the electrons enter the magnetic field  $B_0$  but all the electrons have the same energy. Results are discussed in Sec. III.

## II. MODEL AND FORMALISM

Figure 1 shows an end-view of the configuration under consideration. An annular beam of electrons uniformly distributed over a cylinder of radius  $r_0$  is guided by a uniform magnetic field  $B_0$  along helical trajectories inside a waveguide partially filled with a dielectric. The origin of the coordinate system is at the center of the guide. Wave propagation takes place along  $z$ -axis. The field  $B_0$  is parallel to the  $z$ -axis. Region I ( $-d < x < d$ ) is vacuum and region II ( $-L_1 < x < -d, d < x < L_1$ ) dielectric. The linear response of the electrons is obtained under the following simplifying assumption: (i) the beam is sufficiently tenuous so that its self fields may be neglected, (ii) the rf fields are first order with respect to  $B_0$  and the perturbed electron distribution function  $f^{(1)}(\vec{r}, \vec{p}, t)$  caused by these fields is of first order with respect to the equilibrium distribution function  $f_0(\vec{r}, \vec{p})$ , and (iii) the distribution function and the rf fields are independent of  $y$ -coordinate.

The modes of the dielectric loaded waveguide cannot be classified as TE or TM with respect to the propagation direction. However, they may be classified as TEx or TMx with respect to the  $x$ -axis<sup>5</sup>. These modes are also classified as longitudinal section electric (LSE) and longitudinal section magnetic (LSM) modes, respectively. There is no TMx mode which is independent of the  $y$ -coordinate. The modes may be divided into odd and even symmetries

with respect to the reflection at  $yz$ -plane. It will show that the even and odd modes couple, respectively, to the odd and even order cyclotron modes. Since our primary concern is coupling to the fundamental cyclotron mode, we give detailed calculation for the waveguide modes of even symmetry and mention the results in the case of modes of odd symmetry. For  $L_2 < 2L_1$ , the fundamental waveguide mode belongs to even symmetry. In accordance with assumption (iii) we restrict our attention to  $TE_{x,0n}$  modes. These modes with even symmetry are given by<sup>6</sup>

$$B_x^{(1)} = B_{x0}^{(1)} e^{i(k_z z - \omega t)} \begin{cases} \sin l_{II} (L_1 - |x|) & d < |x| < L_1 \\ \frac{l_{II} \cos l_{II} t}{l_{In} \sin l_{In} t} \cos l_{In} x & -0 < |x| < d \end{cases} \quad (1)$$

$$B_z^{(1)} = \frac{l_{II}}{k_z} B_{x0}^{(1)} e^{i(k_z z - \omega t)} \begin{cases} \text{sign}(x) \cos l_{II} (L_1 - |x|) & d < |x| < L_1 \\ \frac{\cos l_{II} t}{\sin l_{In} d} \sin l_{In} x & -0 < |x| < d \end{cases} \quad (2)$$

$$E_y^{(1)} = \frac{\omega B_{x0}^{(1)} e^{i(k_z z - \omega t)}}{k_z} \begin{cases} \sin l_{II} (L_1 - |x|) & d < |x| < L_1 \\ \frac{l_{II} \cos l_{II} t}{l_{In} \sin l_{In} d} \cos l_{In} x & -0 < |x| < d \end{cases} \quad (3)$$

where

$$l_{II}^2 = \mu_0 \epsilon_0 \omega^2 - k_z^2 \quad (4)$$

$$l_{In}^2 = \mu \epsilon \omega^2 - k_z^2 \quad (5)$$

and the unperturbed dispersion relation is

$$l_{In} \tan l_{In} d = l_{II} \cot l_{II} t. \quad (6)$$

$\mu$  and  $\epsilon$  are respectively the permeability and dielectric constant of medium II. The corresponding vacuum values are  $\mu_0$  and  $\epsilon_0$ . Slow waves exist for  $\mu_0 \epsilon_0 \omega < k_z < \mu \epsilon \omega$ . In this case  $l_{In}$  is imaginary. We will assume that  $\mu = \mu_0$ . A typical dispersion curve and field distribution for the fundamental mode are shown in Fig. 2a and 2b, respectively. A bend in the dispersion curve occurs at  $k_z = k_c$  such that  $\omega = ck_c$ . From Eq. (4) and (6), for the fundamental mode

$$k_c = \pi/2t(\epsilon/\epsilon_0 - 1)^{1/2}. \quad (7)$$

For  $k_z < k_e$  we have  $\omega > ck$  (fast wave) and for  $k_z > k_e$ ,  $\omega < ck$  (slow wave). For fast wave, the field is maximum at the center of the wave guide while for slow waves the maximum occurs in the dielectric.

In the presence of a weak electron beam, the results given in Eqs. (1)-(7) will be slightly modified. Maxwell's equations yield for  $B_x$

$$\left[ \frac{\partial^2}{\partial x^2} + \epsilon(x) \frac{\omega^2}{c^2} - k_z^2 \right] B_{x,0n}^{(1)} = i\mu k_z j_y^{(1)} \quad (8)$$

where  $j_y^{(1)}$  is the perturbed current density and

$$\begin{aligned} \epsilon(x) &= \epsilon_0, \quad 0 < |x| < d \\ &= \epsilon, \quad d < |x| < L_1. \end{aligned} \quad (9)$$

$B_{x,0n}^{(1)}$  may be expanded in terms of the unperturbed functions  ${}_0B_{x,0m}^{(1)}$ . Multiplying Eq. (8) by  ${}_0B_{x,0m}^{(1)}$ , integrating over  $x$  and  $y$  and neglecting the off-diagonal terms on the left hand side, we obtain

$$\left[ \frac{\omega^2}{c^2} - l_n^2 - k_z^2 \right] B_{x0}^{(1)} = \frac{i\mu k_z}{L_1 L_2 \Theta_n} \cdot \frac{l_n \sin l_n d}{l_{nn} \cos l_{nn} t} \int_0^{L_2} \int_{-d}^d j_y^{(1)} \cos l_n x dx dy, \quad (10)$$

where the dimensionless quantity  $\Theta_n$  is

$$\Theta_n = \frac{d}{d+t} \left( 1 + \frac{\sin 2l_n d}{2l_n d} \right) + \frac{\epsilon}{\epsilon_0} \frac{t}{d+t} \cdot \frac{l_n^2 \sin^2 l_n d}{l_{nn}^2 \cos^2 l_{nn} t} \left( 1 - \frac{\sin 2l_{nn} t}{2l_{nn} t} \right). \quad (11)$$

In writing the limits of the  $x$  - integration on the right hand side of Eq. (10), we use the relation that  $j_y^{(1)} = 0$  for  $|x| > d$ . The perturbed current  $j_y^{(1)}$  is given by

$$\begin{aligned} j_y^{(1)} &= -e \int v_y f^{(1)}(\vec{r}, \vec{p}, t) d\vec{p} \\ &= -e \int_0^\infty \int_{-\infty}^{+\infty} dp_1 dp_2 \frac{p_1^2}{m} \int_0^\phi \sin \phi f^{(1)}(\vec{r}, \vec{p}, t) dp. \end{aligned} \quad (12)$$

The perturbed distribution function  $f^{(1)}(\vec{r}, \vec{p}, t)$  is obtained from linearized Vlasor equation by integrating along the unperturbed orbits. The result is standard<sup>7</sup> and can be written as

$$f^{(1)}(\vec{r}, \vec{p}, t) = e \int_{-\infty}^t dt' \left[ \vec{E}^{(1)}(\vec{r}', t') + \frac{\vec{r} \times \vec{B}^{(1)}(\vec{r}', t')}{m} \right] \cdot \nabla_{\vec{r}} f_0(\vec{r}', \vec{p}'). \quad (13)$$

where the  $t'$ -integration is along the unperturbed orbits. The primed quantities  $\bar{r}'$  and  $\bar{p}'$  are treated as functions of  $t'$  while  $\bar{r}$  and  $\bar{p}$  are, respectively, the values of  $\bar{r}'$  and  $\bar{p}'$  at  $t' = t$ . In Eq. (13) we need to use only the lowest order fields given by Eqs. (1)-(3). We now assume a specific form for the equilibrium distribution function  $f_0(\bar{r}, \bar{p})$  constructed from the constants of motion,  $p_1, p_2, \bar{x} = x + \frac{p_1}{m_0 \Omega_0} \sin \phi$  and  $\bar{y} = y + \frac{p_2}{m_0 \Omega_0} \cos \phi$  ( $\bar{x}$  and  $\bar{y}$  locate the guiding center of an electron and  $\Omega_0 = eB/m_0$ ):

$$f_0 = \frac{N}{2\pi r_0} \cdot \frac{\{\theta(r_0 - \bar{x}) - \theta(-r_0 - \bar{x})\} \{\theta(r_0 - \bar{y}) - \theta(-r_0 - \bar{y})\}}{\sqrt{r_0^2 - \bar{x}^2}} g_0(p_1, p_2), \quad (14)$$

where  $\int g_0(p_1, p_2) d\bar{p} = 1$ . This represents an annular beam of electrons uniformly distributed on a cylinder of radius  $r_0$ .  $N$  is the number of electrons per unit length of the waveguide. Since we consider waveguide modes which are independent of  $y$ , there is no loss in generalization in assuming that  $\bar{y}$  is distributed uniformly in the range  $-r_0$  to  $r_0$ .

The modified dispersion relation for the waveguide in the presence of the electron beam may now be obtained from Eqs. (10)-(14). The methods for evaluating the integrals in these equations are standard<sup>7</sup> and the result may be written in the form

$$\begin{aligned} \left( \frac{\omega^2}{c^2} - l_n^2 - k_z^2 \right) &= \frac{4\pi^2 \nu [1 + J_0(2l_n r_0)]}{2L_1 L_2 \Theta_n} \\ &\times \sum_{s=-\infty}^{+\infty} \{1 - (-1)^s\} \int dp_2 dp_1 \frac{p_1}{\gamma} g_0(p_1, p_2) \\ &\times \left[ \frac{(\omega - k_z v_z) \left( \frac{s^2}{x} - x \right) \frac{d}{dx} J_s^2(x)}{(\omega - s \Omega_0 / \gamma - k_z v_z)} - \frac{(\omega^2 - k_z^2 c^2) (v_z/c)^2 [J'_s(x)]^2}{\left( \omega - \frac{s \Omega_0}{\gamma} - k_z v_z \right)^2} \right] \end{aligned} \quad (15)$$

where

$$\gamma = (1 + p_1^2/m_0^2 c^2 + p_2^2/m_0^2 c^2)^{1/2}$$

$$\nu = Ne^2/4\pi \epsilon_0 m_0 c^2,$$

$$\Omega_0 = eB_0/m_0,$$

$$x = l_n p_1/m_0 \Omega_0,$$

and  $J_s(x)$  the Bessel function of order  $s$ .

We now specialized to a particular distribution in velocity space given by

$$g_0(p_1, p_z) = \frac{K}{\pi} e^{-(p_z - p_{z0})^2 / 2(\Delta p_z)^2} \theta(p_0 - p_z) \delta(p_0^2 - p_1^2 - p_z^2), \quad (17)$$

where  $p_0^2 = p_{10}^2 + p_{z0}^2$  and the normalizing constant  $K$  is such that  $\int g_0 d\vec{p} = 1$ . Thus

$$K^{-1} = \int_0^{p_0} e^{-(p_z - p_{z0})^2 / 2(\Delta p_z)^2} dp_z. \quad (18)$$

Equation (17) represents a monoenergetic beam with a spread in the angle at which the electrons enter the field  $B_0$ . From Eqs. (15) and (17) we then obtain

$$\left[ \frac{\omega^2}{c^2} - l_n^2 - k_z^2 \right] = \frac{4\pi K}{L_1 L_2 \gamma_0 \Gamma_n} \int_0^{p_0} dp_z e^{-(p_z - p_{z0})^2 / 2(\Delta p_z)^2} \sum_{s=-\infty}^{+\infty} \frac{[1 - (-1)^s]}{2} \times \left[ \frac{(\omega - k_z v_z) \left( \frac{s^2}{x} - x \right) \frac{d}{dx} J_s^2(x)}{\left[ \omega - s \frac{\Omega_0}{\gamma_0} - k_z v_z \right]} - \frac{(\omega^2 - k_z^2 c^2) \left( \frac{v_0^2 - v_z^2}{c^2} \right) [J_s'(x)]^2}{\left[ \omega - s \frac{\Omega_0}{\gamma_0} - k_z v_z \right]^2} \right], \quad (19)$$

where

$$\Gamma_n = \frac{2\Theta_n}{\pi [1 + J_0(2l_n r_0)]}, \quad x = l_n \sqrt{p_0^2 - p_z^2} / m_0 \Omega_0. \quad (20)$$

For waveguide modes of odd symmetry an expression similar to Eq. (19) is obtained except that the factor  $[1 - (-1)^s]$  becomes  $[1 + (-1)^s]$  and in the expression for  $\Theta_n$  in Eq. (11)  $\sin 2l_n d$  changes to  $-\sin 2l_n d$  and  $\sin^2 l_n d$  to  $\cos^2 l_n d$ . Thus even symmetry modes couples with cyclotron modes having odd values of  $s$  and vice versa. The unperturbed dispersion relation in Eq. (6) is replaced by  $l_n \cos l_n d = -l_{nn} \cos l_{nn} d$  for odd symmetry modes.

The length  $L_1$  can be scaled out of the dispersion relation in Eq. (19) by introducing the following normalization scheme:

- (i) length normalized to  $L_1$  (e.g.  $\bar{r}_0 = r_0/L_1$ )
- (ii) frequency normalized to  $c/L_1$  (e.g.  $\bar{\omega} = \omega L_1/c$ )
- (iii) momentum normalized to  $m_0 c$  (e.g.  $\bar{p}_1 = p_1/m_0 c$ ).

Other quantities such as  $k_z$ ,  $l_{||n}$  and  $\bar{v}$  are to be normalized consistently with the proceeding procedures (e.g.,  $\bar{k}_z = k_z L_1$ ,  $\bar{l}_{||n} = l_{||n} L_1$ ,  $\bar{v}_z = v_z/c$ ,  $\bar{v}_1 = v_1/c$ ). In terms of these dimensionless variables, the dispersion relation (Eq. (19)) may be written as

$$\begin{aligned}
 & (\bar{\omega}^2 - \bar{l}_{||n}^2 - \bar{k}_z^2) \\
 & - \frac{4\nu K}{\bar{L}_2 \gamma_0 \Gamma_n} \sum_{s=-\infty}^{+\infty} \left\{ \frac{1 - (-1)^s}{2} \right\} \int_0^{\bar{p}_0} d\bar{p}_z e^{-(\bar{p}_z - \bar{p}_{z0})^2 / 2(\Delta \bar{p}_z)^2} \\
 & \times \left[ \frac{(\bar{\omega} - \bar{k}_z \bar{v}_z) \left( \frac{s^2}{x} - x \right) \frac{d}{dx} J_s^2(x)}{\left( \bar{\omega} - s \frac{\bar{\Omega}_0}{\gamma_0} - \bar{k}_z \bar{v}_z \right)} - \frac{(\bar{\omega}^2 - \bar{k}_z^2) (\bar{v}_0^2 - \bar{v}_z^2) \{J_s'(x)\}^2}{\left( \bar{\omega} - s \frac{\bar{\Omega}_0}{\gamma_0} - \bar{k}_z \bar{v}_z \right)^2} \right]
 \end{aligned} \quad (21)$$

where  $x = \bar{l}_{||n} \sqrt{\bar{p}_0^2 - \bar{p}_z^2} / \bar{\Omega}_0$ . For slow wave  $\bar{l}_{||n}$  (and  $x$ ) is imaginary. In this case  $J_s(x) = J_s(i|x|) = iI_s(|x|)$  where  $I_s$  is the modified Bessel function. Also  $\sin \bar{l}_{||n} \bar{d}$  in the expression for  $\Theta_n$  changes to  $i \sinh |\bar{l}_{||n}| \bar{d}$ .

For a "cold" beam (i.e.,  $\Delta \bar{p}_z = 0$ ), the dispersion relation (Eq. (21)) becomes

$$\begin{aligned}
 & (\bar{\omega}^2 - \bar{l}_{||n}^2 - \bar{k}_z^2) = \frac{4\nu}{\bar{L}_2 \gamma_0 \Gamma_n} \sum_{s=-\infty}^{+\infty} \frac{1 - (-1)^s}{2} \cdot \left[ \frac{(\bar{\omega} - \bar{k}_z \bar{v}_{z0}) \left( \frac{s^2}{x_0} - x_0 \right) \frac{d}{dx_0} J_s^2(x_0)}{\bar{\omega} - s \frac{\bar{\Omega}_0}{\gamma_0} - \bar{k}_z \bar{v}_{z0}} \right. \\
 & \left. - \frac{(\bar{\omega}^2 - \bar{k}_z^2) \bar{v}_{10}^2 \{J_s'(x_0)\}^2}{\left( \bar{\omega} - s \frac{\bar{\Omega}_0}{\gamma_0} - \bar{k}_z \bar{v}_{z0} \right)^2} \right]
 \end{aligned} \quad (22)$$

where  $x_0 = \bar{l}_{||n} \bar{p}_{10} / \bar{\Omega}_0$ .

For slow waves  $\omega < \bar{k}_z$  and the second term inside the square bracket in Eq. (22) is positive. Instability can occur if the first term is negative i.e.  $\bar{\omega} \leq s \frac{\bar{\Omega}_0}{\gamma_0} + \bar{k}_z \bar{v}_z$  since  $\bar{v}_z < \bar{\omega}/\bar{k}_z$ . For cyclotron maser radiation, however,  $\bar{\omega} > \bar{k}_z$  and the second term drives the instability and the condition  $\bar{\omega} \geq s \frac{\bar{\Omega}_0}{\gamma_0} + \bar{k}_z \bar{v}_z$  is required for azimuthal bunching.

### III. RESULTS AND DISCUSSION

The dispersion relation Eq. (21) is to be analyzed numerically. For a travelling wave amplifier it is convenient to treat  $\bar{\omega}$  as real and  $\bar{k}_z$  complex. The spatial growth of the rf field is then given by the gain coefficient ( $g$ ):

$$g(\text{in dB/cm}) = -8.69 \text{Im}(\bar{k}_z)/L_1 \quad (23)$$

where  $L_1$  is in cm. Gain occurs if  $\text{Im} \bar{k}_z < 0$ .

In Equation (21) there are ten free parameters  $\nu$ ,  $\bar{L}_2$ ,  $\bar{d}$ ,  $\bar{r}_0$ ,  $\epsilon/\epsilon_0$ ,  $\gamma_0$ ,  $v_{10}/v_{z0}$ ,  $\bar{\Omega}_0$ ,  $\Delta p_z/p_{z0}$ ,  $n$  and  $s$ . We choose the lowest modes with  $n = 1$  and  $s = 1$ . It is found that the bandwidth of the amplifier increases with decrease in  $\bar{d}$  and increase in  $\epsilon_r = \epsilon/\epsilon_0$ . If  $\bar{d}$  is made too small, however, it is difficult to accommodate the electron orbits in region I of Fig. 1. For efficient interaction between the electrons and the rf field,  $v_z$  should be nearly equal to  $v_g$ , the group velocity of the waves.  $v_g$  decreases with increase in  $\epsilon$ . Too low a value of  $v_z$  (and  $v_1$  is not desirable. For numerical calculation we choose

$$\bar{d} = .6, \epsilon_r = 20, \bar{r}_0 = .1 \text{ and } \bar{L}_2 = 2\bar{d}.$$

Figure 3 shows a plot of  $\text{Im} \bar{k}_z$  as a function of  $\bar{\omega}$  for different values of  $\Delta p_z/p_{z0}$  at  $\gamma_0 = 1.0617$  ( $v_{z0} = .2$ ,  $v_{10}/v_{z0} = 1.35$ ) and  $\bar{\Omega}_0/\gamma_0 = .72$ . The bandwidth and gain decrease as the velocity spread increases. Each value of  $\Delta p_z/p_{z0}$ , the magnetic field (i.e.,  $\bar{\Omega}_0$ ) has to be optimized for maximum bandwidth. In Fig. 4 we show  $\text{Im}(\bar{k}_z)$  as a function of  $\bar{\omega}$  for different values of

$\Delta \bar{p}_z / \bar{p}_{z0}$  with the corresponding  $\bar{\Omega}_0$  for maximum bandwidth. The values of  $\Delta \bar{p}_z / \bar{p}_{z0}$ ,  $\bar{\Omega}_0$  and the bandwidth  $\Delta \bar{\omega} / \bar{\omega}$  are given in Table I. Fig. 5 shows a plot of  $\Delta \bar{\omega} / \bar{\omega}$  vs  $\Delta \bar{p}_z / \bar{p}_{z0}$ . The bandwidth decreases rapidly from 64% at  $\Delta \bar{p}_z / \bar{p}_{z0} = 0$  to 12% at  $\Delta \bar{p}_z / \bar{p}_{z0} = .05$ . The calculated bandwidths correspond to a change in gain coefficient given by the relation  $[g_{\max} - g(\bar{\omega})] / g_{\max} = \frac{\log_e 2}{2 \log_e 10}$ . In Figs. 6-8, we plot  $\text{Im } \bar{k}_z$  as a function of  $\bar{\omega}$  to show the changes in gain and bandwidth with variation in the parameters  $\bar{\Omega}_0$ ,  $\bar{v}_{z0}$  and the ratio  $\alpha = \bar{v}_{10} / \bar{v}_{z0}$ , respectively. It is seen that the gain and the bandwidth are sensitive functions of  $\bar{\Omega}_0$  and  $\bar{v}_{z0}$  and increases slowly with increase in  $\alpha$ . The ratio  $\alpha$ , however, cannot be made too large since for a given  $v_{z0}$  and  $\Omega_0$ , the Larmor radius  $r_L$  increases with  $\alpha$ .

Table I

| $\Delta \bar{p}_z / \bar{p}_{z0}$ | $\bar{\Omega}_0 / \gamma_0$ | $\Delta \bar{\omega} / \bar{\omega}$ | $\bar{r}_L$ |
|-----------------------------------|-----------------------------|--------------------------------------|-------------|
| 0                                 | .680                        | .64                                  | .3970       |
| .001                              | .705                        | .45                                  | .3830       |
| .01                               | .720                        | .28                                  | .3750       |
| .02                               | .728                        | .205                                 | .3711       |
| .03                               | .734                        | .165                                 | .3678       |
| .04                               | .737                        | .140                                 | .3664       |
| .05                               | .739                        | .125                                 | .3654       |

$$\begin{aligned} \bar{L}_2/2 = \bar{d} &= .6, & \epsilon_r &= 20, & \bar{r}_0 &= .1, \\ \bar{v}_{z0} &= 2, & \bar{v}_{10}/\bar{v}_{z0} &= 1.35, & \nu &= .002, \\ \gamma_0 &= 1.062 \end{aligned}$$

To convert normalized quantities  $\bar{\Omega}_0$  and  $\bar{\omega}$  to design parameters, the following relations should be used. For a fixed frequency  $f$  (in Hz)

$$L_1 = \frac{\bar{\omega} c}{2\pi f} \text{ cm},$$

$$H_0 = 1.707 \frac{\bar{\Omega}_0}{L_1} \text{ kOe},$$

while for fixed magnetic field  $H_0$  (in kOe)

$$L_1 = 1.707 \times \bar{\Omega}_0 / H_0 \text{ cm}$$

$$f = \bar{\omega} c / 2\pi L_1 \text{ Hz.}$$

We give below the design parameters of a slow wave amplifier for operation at frequency  $f = 24$  GHz. For  $\nu = .002$ ,  $\bar{v}_{z0} = .2$  and  $\bar{v}_{L0}/v_{z0} = 1.35$ , the parameters are: beam energy of  $V_b$  (kV) = 31.52, beam current  $I_b$  (amp) = 3.41,  $2L_1 = .426$  cm,  $2d = L_2 = .256$  cm,  $r_0 = .0213$  cm,  $H_0 = 6.245$  kOe,  $\epsilon/\epsilon_0 = 20$ . In this case  $\Delta\bar{\omega}/\bar{\omega} = .165$  and  $g_{\max} = 1.346$  dB/cm when  $\Delta\bar{p}_z/\bar{p}_{z0} = .03$ .

#### IV. CONCLUSION

We have shown that slow wave cyclotron instability may be used to obtain broadband amplification in the millimeter range. The bandwidth deteriorates with increase in the velocity spread of the electrons. The velocity spread should not exceed 2% to obtain a bandwidth of 30% or more. Our results are based on a linear theory. A nonlinear analysis is necessary to calculate the efficiency and the saturated output power level of the system.

#### REFERENCES

1. V.A. Flyagin, A.V. Yaponov, M.I. Petelin and V.K. Yulpaten, IEEE Trans. Microwave Theo. Tech. **25**, 514 (1977) and references therein
2. J.L. Hirschfield and V.L. Granatstein, IEEE Trans. Microwave Theo. Tech. **25**, 522 (1977) and references therein
3. E.S. Weibel, Phys. Rev. Letters, **2**, 83 (1959)
4. K.R. Chu and J.L. Hirschfield, NRL Memorandum Report 3607 (Sept. (1977)), Naval Research Laboratory, Washington, D.C.

NRL MEMORANDUM REPORT 4215

5. R.F. Harrington, "Time Harmonic Electromagnetic Fields" (McGrawHill, New York, 1961) p. 158
6. A.M. Woodward, Wireless Engineer, 24, 192, (1947)
7. D.E. Baldwin, I.B. Bernstein and M.P.H. Weenik, "Kinetic Theory of Plasma Waves in a Magnetic Field" in Advances in Physics (Interscience, New York 1969), Vol. 3, p. 1

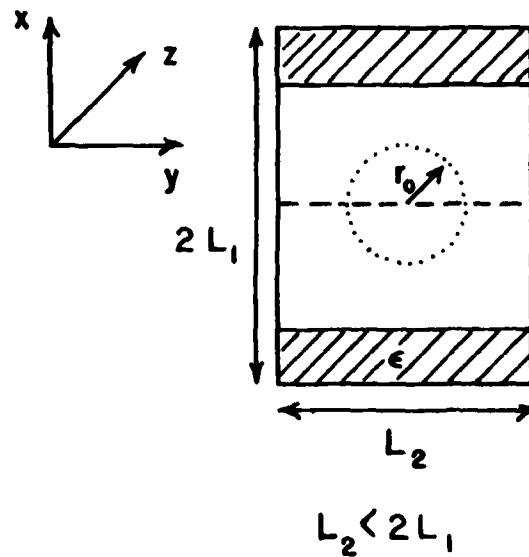


Fig. 1 — End view of the dielectric loaded guide. Guiding centers of the electrons are uniformly distributed on the circle of radius  $r_0$ .

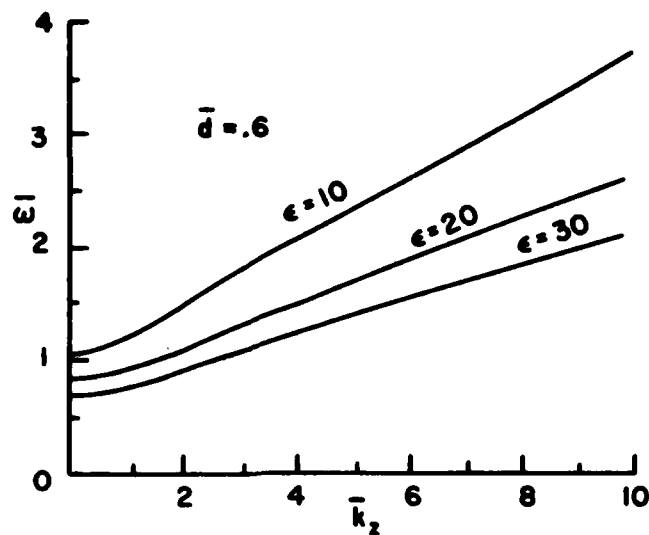


Fig. 2a - Dispersion relation for dielectric loaded waveguide of rectangular cross-section.

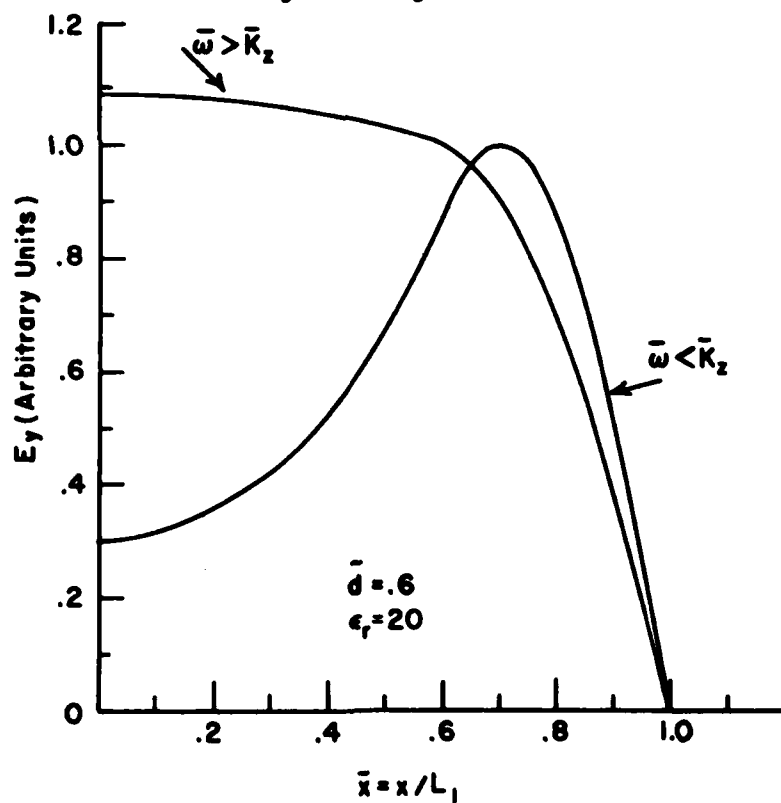


Fig. 2b -  $E_y$  as a function of  $x$ .

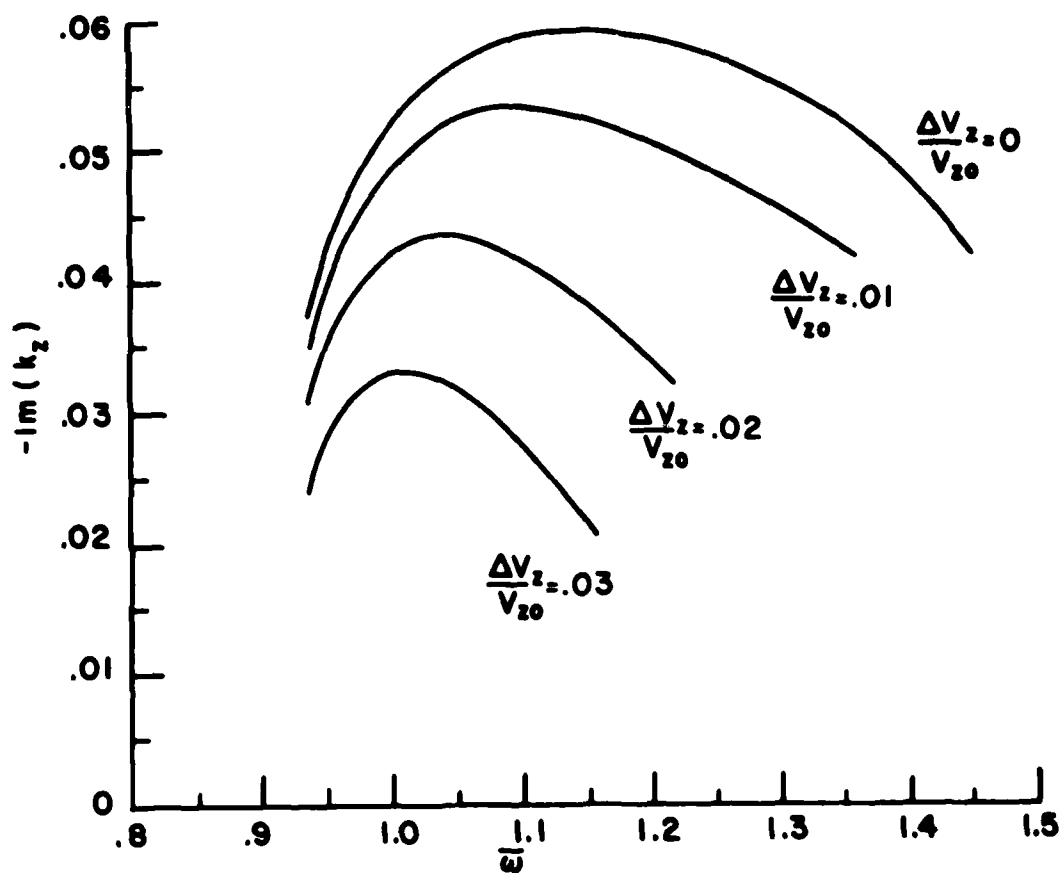


Fig. 3 —  $\text{Im}(\bar{k}_z)$  vs  $\bar{\omega}$  for different values of velocity spread at a fixed magnetic field ( $\Omega_0/\gamma_0 = .72$ ).  
Parameters used:  $n = s = 1$ ,  $\gamma_0 = 1.062$ ,  $\bar{v}_{10}/\bar{v}_{z0} = 1.35$ ,  $\bar{v}_{z0} = .2$ ,  $\nu = .002$ ,  $\bar{d} = .6$ ,  $\epsilon_r = 20$ ,  $\bar{L}_z = 2\bar{d}$ .

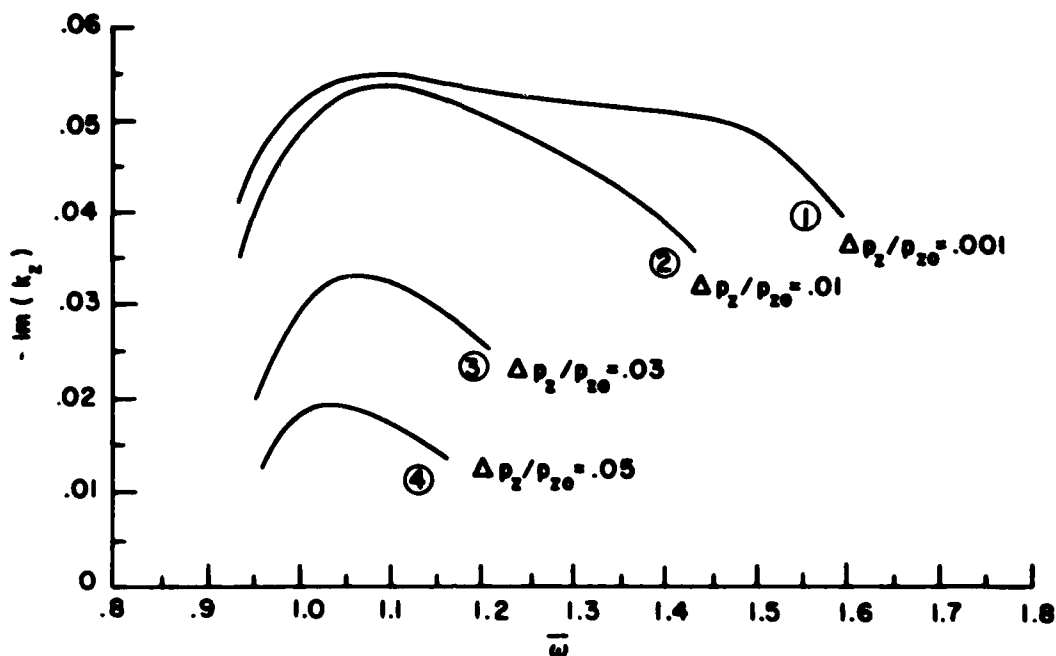


Fig. 4 -  $\text{Im } \bar{k}_z$  as a function of  $\bar{\omega}$  for different  $\frac{\Delta p_z}{p_{z0}}$  with corresponding values of  $\bar{\Omega}_0$  to optimize the bandwidth. Other parameters are same as in Fig. 2. Values of  $\bar{\Omega}_0$  and  $\Delta \bar{\omega} / \bar{\omega}$  are given in Table I.

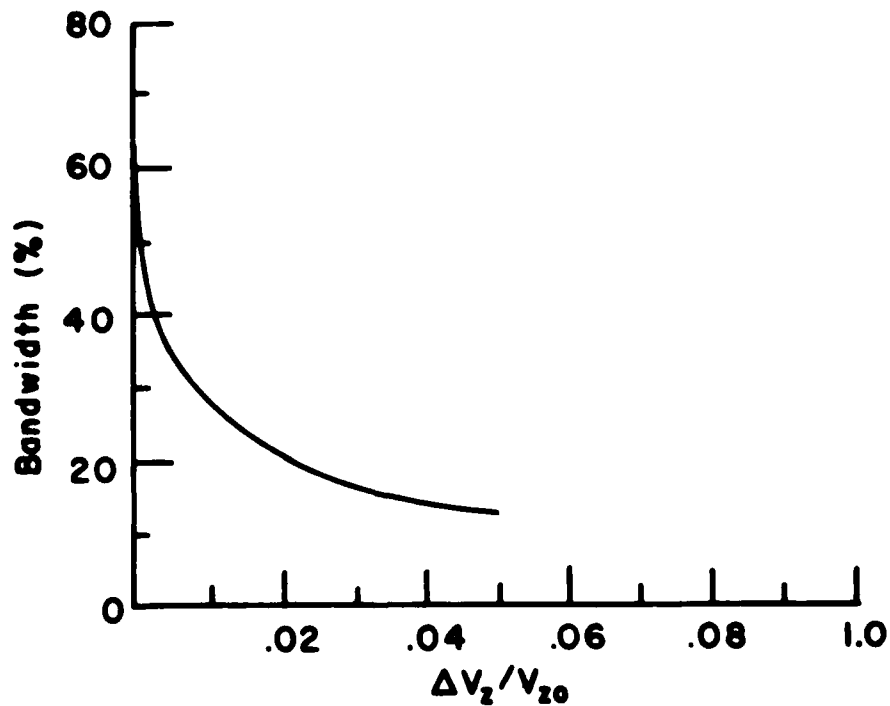


Fig. 5 - Optimized bandwidth as a function of velocity spread. Parameters are same as in Fig. 4.

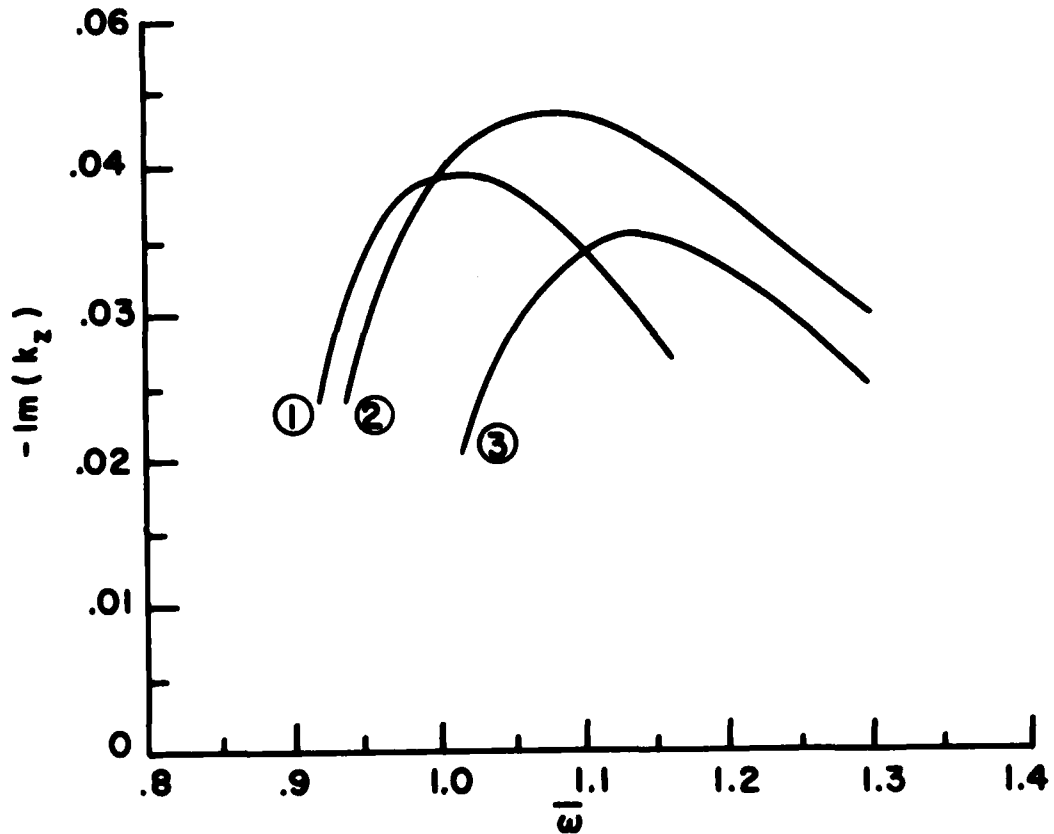


Fig. 6 — Gain vs frequency for three different values of  $\bar{\Omega}_0$  at  $\Delta\bar{p}_z/\bar{p}_{z0} = 0.2$ . Other parameters are same as in Fig. 3. Curve 1:  $\bar{\Omega}_0/\gamma_0 = .71$  and  $\Delta\bar{\omega}/\bar{\omega} = .16$ ; curve 2:  $\bar{\Omega}_0/\gamma_0 = .728$  and  $\Delta\bar{\omega}/\bar{\omega} = .2$ ; and curve 3:  $\bar{\Omega}_0/\gamma_0 = .74$  and  $\Delta\bar{\omega}/\bar{\omega} = .15$ .

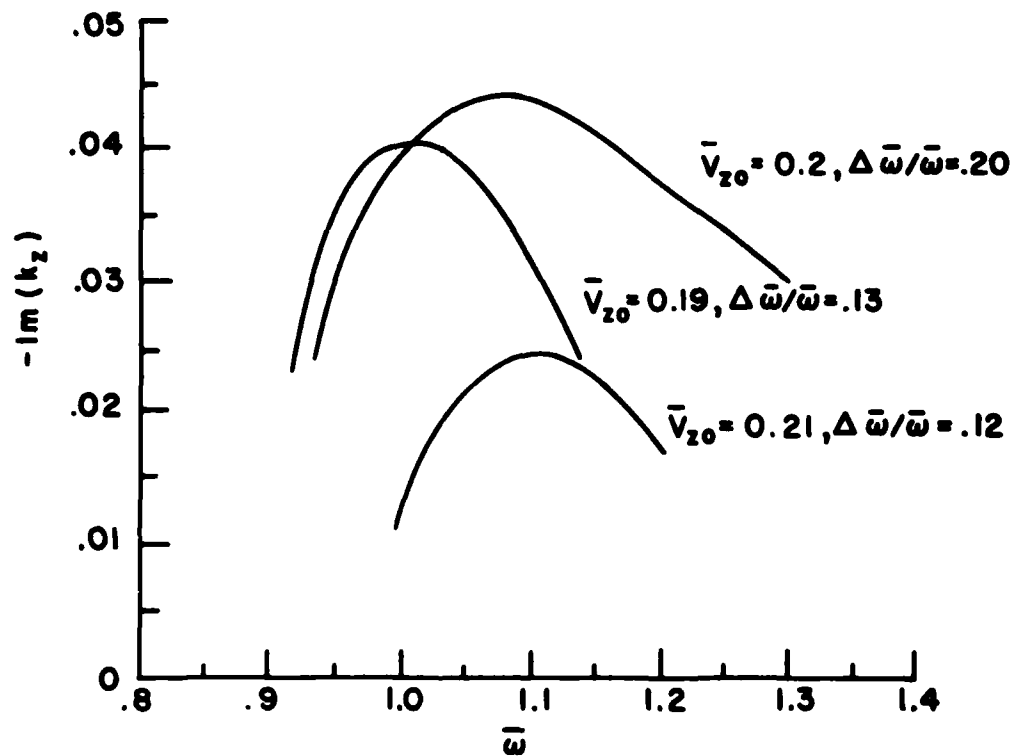


Fig. 7 — Gain vs frequency for different  $\bar{V}_{z0}$  at  $\bar{\Omega}_0/\gamma_0 = .728$  and  $\Delta \bar{p}_z/\bar{p}_{z0} = .02$ .  
Other parameters are same as in Fig. 3.

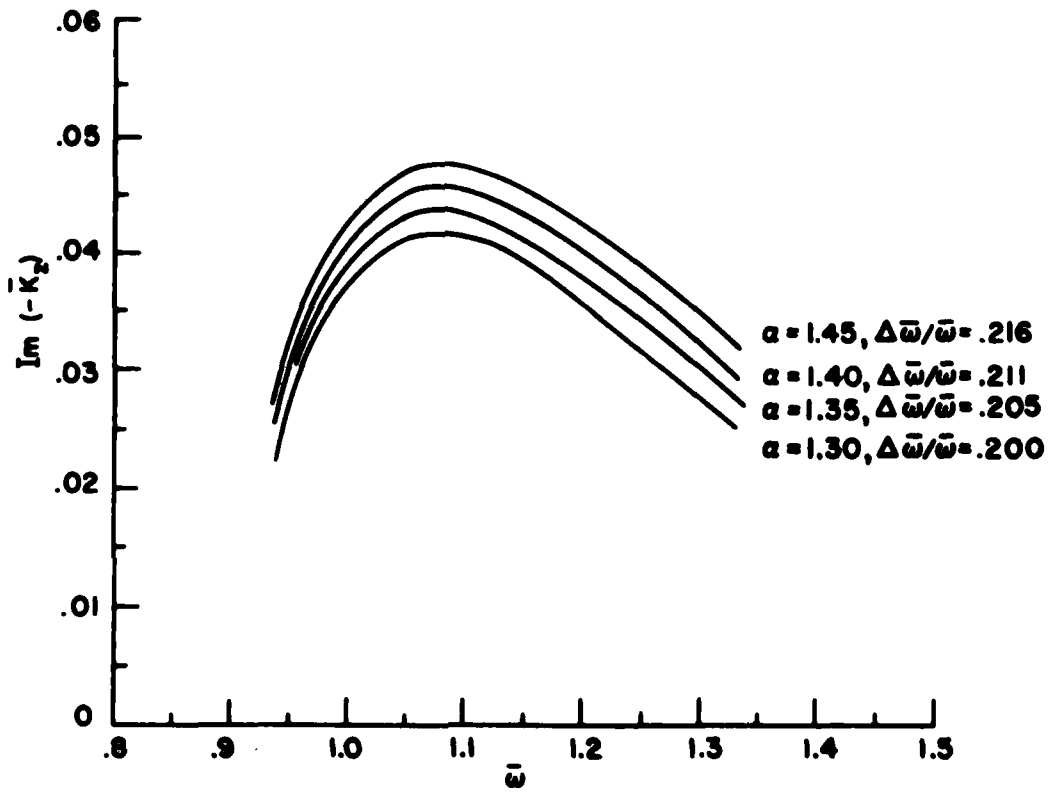


Fig. 8 -  $\text{Im } \bar{k}_z$  vs  $\bar{\omega}$  for different values of  $\alpha$  at  $\bar{\Omega}_p/\gamma_e = .728$ .  
Other parameters are same as in Fig. 3.

EN

DAT  
FILM

8-8

DT